Manipulation of market equilibrium via endowments

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Abstract

In this paper we show that in an exchange economy with quasi-linear preferences it is possible to manipulate market equilibrium by destroying and withholding ones initial endowments.

1. Introduction: The most well known solution prescribed by modern economic theory for decentralized allocation of resources is Walrasian equilibrium. Agents are initially endowed with non-negative quantities of resources and are assumed to engage in mutually compatible exchanges, after maximizing utility or preferences subject to a budget constraint. The budget constraint that an agent faces in such problems imposes a liquidity constraint on their transactions. The Pareto Optimality of such solutions make them very appealing both as prescriptive and descriptive models of market behavior. Campbell (1987) contains a contemporary discussion of issues related to the Walrasian mechanism.

The Walrasian mechanism of resource allocation suffers from severe incentive problems. The possibility of an agent being able to improve by misrepresenting its preferences has often been cited as the main drawback of the Walrasian solution. However, the issue of misrepresenting individual preferences, rests on (i) the realization of an economic environment allowing such a misrepresentation; (ii) the agent who stands to benefit from the deviation being aware of such a possibility as also of the entire profile of preferences and initial endowments. This latter information being prohibitive and dispersed among the agents, adds to the computational complexity of preference misrepresentation. Thus, while the problem of preference misrepresentation is surely a theoretical possibility, the sheer computational complexity of the entire exercise makes the threat posed by it somewhat unlikely.

A more likely possibility is that an agent may tamper around with its available endowments, in order to secure an improvement in welfare for itself. Non-disclosure or false disclosure of taxable income is an extremely common phenomenon that governments often need to deal with. While money income is not the same as a bundle of goods initially endowed with an agent, they are related concepts. In any event, misrepresentation of ones physical assets for accounting or tax purposes is not uncommon either. The question that naturally arises is the following: Is it possible to manipulate the Walrasian mechanism by either destroying or withholding a portion of ones initial endowment? The answer to this question is in the affirmative, as shown in Postlewaite (1979). In fact Postlewaite (1979) contains the more general result that any social choice mechanism in the pure exchange setting, which satisfies Pareto Optimality and individual rationality is manipulable via endowments. Atlamaz and Klaus (2007), consider exchange markets with heterogeneous indivisible goods and investigate the possibility of exchange rules that are efficient and immune to manipulation via endowments. They consider manipulability either with respect to withholding or destroying part of the initial endowment or transferring part of the endowment to another trader. They show that in general no exchange rule exists which is both efficient and immune to manipulation via endowments.

What concerns us here is the ability of an agent to influence prices by taking into consideration the effects of its decision on demand or supply. The basic purpose of this paper is to show that vulnerability of the competitive mechanism to destruction or withholding of endowments prevails even if we restrict our attention to agents who have quasi-linear preferences. Each agent is now equipped with a concave valuation function which gives the maximum amount that the agent is willing to pay for each bundle of goods. The market clearing equilibrium price quantity configuration that now emerges is referred to here as market equilibrium. Market equilibrium obtains when the total quantity of each good demanded is equal to its total availability within the economy.

The possibility of gaining by misrepresenting or manipulating ones initial endowments is not very uncommon in the real world. For manufacturing firms, manipulation via endowments is especially problematic as firms can keep inputs idle or destroy them or not sell them on the market. We will show that a single agent can do this in a way that influences prices to its advantage.

The difference between our analysis and that of Postlewaite (1979) is the following: We provide an example of an economy with quasi-linear preferences that is manipulable via destroying as well as withholding initial endowments, whereas the earlier analysis provided an example of an economy with non-quasi-linear preferences such that every Pareto Optimal and individually rational allocation therein was manipulable via destroying as well as withholding initial endowments. The quasi-linearity assumption in our example prevents us from obtaining our result as a consequence of the apparently more general results established by Postlewaite (1979). *On the contrary (as we show in the last section of this paper) once we restrict ourselves to quasi-linear preferences there are allocations (called dictatorial allocations) that are both individually rational and Pareto optimal and yet are not manipulable via destruction of endowments.*

While the model that we consider here is static, it has some connection with the problem with hoarding. Although hoarding is primarily a dynamic problem, manipulation via endowment is an approximation of such a phenomenon in the static context.

The manipulability results that we obtain here are not meant to be surprising at all. In fact the analysis reported here reflects a common threat to the smooth delivery of the market mechanism, as we have already indicated above. In a finite economy everyone has a little bit of monopoly power, which is often exercised by manipulating one’s endowments. This gets translated in an enhancement of the scarcity value of the commodity in question, leading to a price rise. The interesting point that the existing literature highlights is that (and without speculating on what would be the consequences in a continuum artifact) in a finite competitive economy this effect is very small.

A significant byproduct of this analysis would be the ability to convey a considerable amount of economic theory concerned with the strategic behavior of individual agents using simple demand and supply curves, instead of an Edgeworth box diagram.

2. The Model: Consider an economy with H > 0 agents and L + 1 > 1 goods. The first L commodities are non-monetary consumption goods and the L+1th commodity is money. Each agent i = 1,…,H is equipped with a valuation function $f^i: \Re^L_+ \rightarrow \Re_+$ which is concave, non-decreasing and continuous. Further, $f^i(0) = 0$ for all i = 1,…,H. Each agent i is initially endowed with a vector $w^i \in \Re^L_+$ of the non-monetary consumption goods While this need not be the case for a more general statement of the model, ***<u>for the purposes of this paper we assume that</u>*** $w = \sum_{i=1}^{H} w^i \in \Re^L_{++}$.

A (non-monetary) consumption bundle of agent i is denoted by an L-vector $X^i \geq 0$.
A price vector p is an element of $\Re^L_+ \backslash \{0\}$, where for j = 1,…,L, p_j denotes the price of commodity j. Clearly a price vector ***<u>does not allow</u>*** all goods to be available for free.
At a price vector p, the objective of agent i is to maximize surplus:
Maximize $[f^i(X^i) - p^T X^i]$

Note: If for some $i \in \{1,\ldots,H\}$, f^i is homogeneous of degree one, then a maximum surplus if it exists for i, must be zero.

An (non-monetary) allocation is an array $X = \langle X^i / i = 1,\ldots,H \rangle$ such that $X^i \in \Re^L_+$ for all i = 1,…,H.

An allocation $X = \langle X^i / i = 1,\ldots,H \rangle$ is said to be feasible if $\sum_{i=1}^{H} X^i = w$.

A market equilibrium is a pair <p*,X*> where p* is a price vector, X* a feasible allocation and for all i = 1,…,H, X^{*i} maximizes surplus for agent i.

Given a price-allocation pair <p*,X*>: [for all i = 1,…,H, X^{*i} maximizes surplus for agent i] if and only if [X* maximizes aggregate surplus at p* i.e. $\sum_{i=1}^{H} f^i(X^{*i}) - p^* \sum_{i=1}^{H} X^{*i} \geq \sum_{i=1}^{H} f^i(X^i) - p^* \sum_{i=1}^{H} X^i$, whenever $\langle X^i / i = 1,\ldots,H \rangle$ is any allocation].

Hence, a market equilibrium can alternatively be defined as a price-allocation pair <p*,X*> such that X* is a feasible allocation and it maximizes aggregate surplus at p*.
A feasible allocation $X^* = \langle X^{*i} / i = 1,\ldots,H \rangle$ is said to be efficient if $\sum_{i=1}^{H} f^i(X^{*i}) \geq \sum_{i=1}^{H} f^i(X^i)$, whenever $X = \langle X^i / i = 1,\ldots,H \rangle$ is any feasible allocation.

Observation 1: Let <p*,X*> be a market equilibrium. Then X* is an efficient allocation.

Observation 2: Let <p*,X*> be a market equilibrium and let X be an efficient allocation. Then, <p*,X> is also a market equilibrium.

An extended allocation is a pair (X,Y), where X is an (non-monetary) allocation and Y is an H-vector of real numbers. The i^{th} component of Y denoted Y^i is the amount of money allocated to agent i.
An extended allocation (X,Y) is said to be feasible if X is a feasible allocation and
$\sum_{i=1}^{H} Y^i = 0$.
A feasible extended allocation (X,Y) is said to be efficient if X is an efficient allocation.

A feasible extended allocation (X*,Y*) is said to be competitive if there exists a price vector p* such that <p*,X*> is a market equilibrium and for all i = 1,…,H: $Y^{*i} = p^{*T}w^i - p^{*T}X^{*i}$.
The pair <p*, (X*,Y*)> is then called a competitive equilibrium.

Observation 3: If <p*, X*> is a market equilibrium and $Y^{*i} = p^{*T}(w^i - X^{*i})$ for all i = 1,…,H, then $f^i(X^{*i}) + Y^{*i}$ is the value that accrues to agent i, at <p*, X*>. Let S be a non-empty subset of {1,…,H} and for $i \in S$, let $X^i \in \Re_+^L$ be such that $\sum_{i\in S} X^i = \sum_{i\in S} w^i$. Since <p*,(X*,Y*)> is a competitive equilibrium, $f^i(X^i) - p^{*T}X^i \le f^i(X^{*i}) - p^{*T}X^{*i}$ for all $i \in S$.
Thus, $\sum_{i\in S}[f^i(X^i) - p^* w^i] = \sum_{i\in S}[f^i(X^i) - p^* X^i] \le \sum_{i\in S}[f^i(X^{*i}) - p^* X^{*i}]$.
Hence $\sum_{i\in S} f^i(X^i) \le \sum_{i\in S}[f^i(X^{*i}) + Y^{*i}] = \sum_{i\in S} f^i(X^{*i})$.
This shows that the total value accruing to agents in S at a market equilibrium is at least at much as they could generate for themselves by simply using their own resources. Thus, the pay-offs to the agents at a market equilibrium belong to the transferable utility core of the underlying market game as in Shapley and Shubik (1969, 1976), thereby establishing that the set of core allocations are non-empty.

3. Manipulation via endowments: In this section we provide two examples in order to establish the main result of this paper.
Example 1: Let f: $\Re_+ \to \Re$ be a function defined as follows for positive real numbers a, b and positive integer K where a > (K+1) b and $K \ge 2$:
For all x = [0,K-1] let f(x) = ax; for $x \ge K - 1$, let f(x) = (K-1)a + b(x-K + 1).
Clearly, f is concave.
Consider an economy with L = 1 and H = 2. Let $f^i = f$ for i = 1,2, $w^1 = 2K$ and $w^2 = 0$.
Thus, w = 2K.
Let p* = b.
Let $X^* = \langle X^{*i} / i = 1,2\rangle$ where $K+1 \ge X^{*i} \ge K-1$ for i = 1,2 and $X^{*1} + X^{*2} = 2K$.
Clearly <p*,X*> is a market equilibrium and <p*,X*> is a market equilibrium if and only if it satisfies the above properties.
The value accruing to agent 1 in money units at <p*,X*> is $f^1(X^{*1}) - bX^{*1} + 2Kb = (K-1)a + b(X^{*1} - K+1) - X^{*1}b + 2Kb = (a + b)K - (a - b)$.

The value accruing to agent 2 in money units at $<p^*, X^*>$ is $f^2(X^{*2}) - bX^{*2} = (K-1)a + b(X^{*2} - K+1) - X^{*2}b = (a - b)(K-1)$.
Now suppose agent 1 destroys half its endowments so that its new initial endowment is $\overline{w}^1 = K$. Let agent 2's initial endowment remain unchanged.
Let $\overline{p} = a$.
Let $\overline{X} = <\overline{X}^i / i = 1,2>$ where $K - 1 \geq \overline{X}^i \geq 1$ for i = 1,2 and $\overline{X}^1 + \overline{X}^2 = K$.
$<\overline{p}, \overline{X}>$ is a market equilibrium for the economy with aggregate endowments being K.
In fact if K > 2, then $<\overline{p}, \overline{X}>$ is a market equilibrium for the economy with aggregate endowments being K, if and only if it satisfies these properties.
The value accruing to agent 1 in units of the numeraire consumption good at $<\overline{p}, \overline{X}>$ is $a\overline{X}^1 - a\overline{X}^1 + Ka = Ka > (a + b)K - (a - b)$, since $a > b(K + 1)$.
The value accruing to agent 2 in units of the numeraire consumption good at $<\overline{p}, \overline{X}>$ is $a\overline{X}^2 - a\overline{X}^2 = 0$.
Thus, agent 1 is better off after having destroyed half its endowment.
Further, if agent 1 had withheld half its endowment instead of destroying it, then the value that accrues to agent 1 in units of the numeraire consumption good at $<\overline{p}, \overline{X}>$ is $a(K-1) + (K + \overline{X}^1 - K + 1)b - a\overline{X}^1 + aK = 2aK - (a - b)(\overline{X}^1 + 1) > 2aK > (a + b)K - (a - b)$, since in this situation agent 1 gets to use 3 units of the input for production.
This example clearly reveals that agent 1 benefits by destroying half its initial endowments, and considerably more by withholding it.

Discussion of Example 1: (a) In Example 1, the maximum (in money units) that agent 1 can get corresponding to the original initial endowment is 2a(K-1) + 2b obtained by consuming K units of the good and by extracting from agent 2 the latter's marginal valuation for K units of the good. After destroying half its initial endowments, the maximum that agent 1 can get is worth Ka units of money. This it can obtain by consuming K-1 units of the good and extracting from agent 2 the latter's valuation of 1 unit of the good. This maximum is achieved by agent 1 simply by withholding half its initial endowment.
(b) In Example 1, agent 1's valuation of its initial endowment is (K-1)a + (K+1)b, whereas its valuation of the non-monetary consumption bundle that it obtains at any market equilibrium (prior to manipulation) is less than or equal to (K-1)a + 2b. The latter is in turn less than (K-1)a + (K+1)b since K is assumed to be at least 2.
(c) In Example 1, if agent 1 instead of revealing the valuation function f, had revealed g: $\Re_+ \rightarrow \Re$ where $g(x) = ax$ for $x \geq 0$, then there exists a market equilibrium where the price of the good is 'a' and agent 1 gets to consume K+1 units of the good, the remaining K – 1 units of the good being consumed by agent 2. At such an equilibrium the surplus of agent 1 is a(K-1) + 2b + 2aK – a(K+1) = 2a(K-1) + 2b units of money. This amount exceeds the surplus agent 1 earns by correctly revealing its preferences.
Thus, Example 1 is an instance of a very simple economy, where every market equilibrium is vulnerable to manipulation via both misrepresentation of preferences as well as destruction or withholding of endowments.

We now provide another example of manipulation via endowments using a two good model with valuation functions is homogeneous of degree one. This function is available in Campbell (1987).

Example 2: As in Campbell (1987), let f: $\Re_+^2 \rightarrow \Re$ be a function defined as follows:

For all x = $(x_1,x_2) \in \Re_+^2$: (a) $x_2 \leq \frac{2}{3}x_1$ implies $f(x) = x_1 + 8x_2$; (b) $x_2 \geq \frac{2}{3}x_1$ implies f(x) = $\frac{19}{5}(x_1 + x_2)$.

Clearly, f(0) = 0.

Let $x = (x_1,x_2)$, $y = (y_1,y_2) \in \Re_+^2$ and $\alpha \in [0,1]$. Let $z = (z_1,z_2) = \alpha y + (1-\alpha)x$.

If $x_2 \leq \frac{2}{3}x_1$ and $y_2 \leq \frac{2}{3}y_1$ then $z_2 \leq \frac{2}{3}z_1$.

Thus, $f(z) = z_1 + 8z_2 = (\alpha y_1 + (1-\alpha)x_1) + 8(\alpha y_2 + (1-\alpha)x_2) = \alpha[y_1 + 8y_2] + (1-\alpha)[x_1 + 8x_2]$ $= \alpha f(y) + (1-\alpha) f(x)$.

If $x_2 \geq \frac{2}{3}x_1$ and $y_2 \geq \frac{2}{3}y_1$ then $z_2 \geq \frac{2}{3}z_1$.

Thus, $f(z) = \frac{19}{5}(z_1 + z_2) = \frac{19}{5}(\alpha y_1 + (1-\alpha)x_1) + \frac{19}{5}(\alpha y_2 + (1-\alpha)x_2) = \alpha[\frac{19}{5}(y_1 + y_2)] +$ $(1-\alpha)[\frac{19}{5}(x_1 + x_2)] = \alpha f(y) + (1-\alpha) f(x)$.

Hence suppose $x_2 \geq \frac{2}{3}x_1$ and $y_2 \leq \frac{2}{3}y_1$. Thus, $f(x) = \frac{19}{5}(x_1 + x_2)$ and $f(y) = (y_1 + 8y_2)$.

Case 1: $z_2 \leq \frac{2}{3}z_1$.

Thus, $\alpha y_2 + (1-\alpha)x_2 \leq \frac{2}{3}[\alpha y_1 + (1-\alpha)x_1]$ and $f(z) = z_1 + 8z_2 = [\alpha y_1 + (1-\alpha)x_1] + 8[\alpha y_2 + (1-\alpha)x_2]$.

Further, $f(z) - (1-\alpha)f(x) - \alpha f(y) = [\alpha y_1 + (1-\alpha)x_1] + 8[\alpha y_2 + (1-\alpha)x_2] - (1-\alpha)\frac{19}{5}(x_1 + x_2)$ $- \alpha(y_1 + 8y_2) = (1-\alpha)[(x_1+8x_2) - \frac{19}{5}(x_1 + x_2)] = (1-\alpha)[\frac{21}{5}x_2 - \frac{14}{5}x_1] = \frac{7(1-\alpha)}{5}[3x_2 - 2x_1]$ ≥ 0, since $x_2 \geq \frac{2}{3}x_1$.

Case 2: $z_2 \geq \frac{2}{3}z_1$.

Thus, $\alpha y_2 + (1-\alpha)x_2 \geq \frac{2}{3}[\alpha y_1 + (1-\alpha)x_1]$ and $f(z) = \frac{19}{5}(z_1 + z_2) = \frac{19}{5}([\alpha y_1 + (1-\alpha)x_1] + [\alpha y_2 + (1-\alpha)x_2])$.

Further, $f(z) - (1-\alpha)f(x) - \alpha f(y) = \frac{19}{5}([\alpha y_1 + (1-\alpha)x_1] + [\alpha y_2 + (1-\alpha)x_2]) - (1-\alpha)\frac{19}{5}(x_1 + x_2) - \alpha(y_1 + 8y_2) = \alpha[\frac{19}{5}(y_1 + y_2) - (y_1+8y_2)] = \alpha[\frac{14}{5}y_1 - \frac{21}{5}y_2] = \frac{7\alpha}{5}[2x_1 - 3x_2] \geq 0$, since $x_2 \leq \frac{2}{3}x_1$.

Thus, f is concave. Also observe that f is homogeneous of degree one.

Consider an economy with $L = 2$ and $H = 2$. Let $f^i = f$ for $i = 1,2$, $w^1 = (2,0)^T$ and $w^2 = (0,1)^T$. Thus, $w = (2,1)^T$.

Let $p^* = (1,8)^T$. If $X^i \in \Re^2_+$ is such that $X_2^i > \frac{2}{3}X_1^i$, then $f^i(X^i) - p^{*T}X^i = \frac{19}{5}(X_1^i + X_2^i) - (X_1^i + 8X_2^i) = (\frac{14}{5}X_1^i - \frac{21}{5}X_2^i) = \frac{7}{5}(2X_1^i - 3X_2^i) < 0$.

However, if $X_2^i \leq \frac{2}{3}X_1^i$, then $f^i(X^i) - p^{*T}X^i = (X_1^i + 8X_2^i) - (X_1^i + 8X_2^i) = 0$.

Let $X^* = \langle X^{*i} / i = 1,2\rangle$ where $X^{*1} = (\frac{2}{5}, \frac{1}{5})^T$ and $X^{*2} = (\frac{8}{5}, \frac{4}{5})^T$.

$\langle p^*, X^*\rangle$ is a market equilibrium.

The value accruing to agent 1 in money units at $\langle p^*, X^*\rangle$ is $f^1(X^{*1}) - p^{*T}X^{*1} + p^{*T}w^1 = 2$.

The value accruing to agent 2 in money units is $f^2(X^{*2}) - p^{*T}X^{*2} + p^{*T}w^2 = 8$.

Now suppose agent 1 destroys half its endowments so that its new initial endowment is $\overline{w}^1 = (1,0)^T$. Let agent 2's initial endowment remain unchanged.

Let $\overline{p} = (\frac{19}{5}, \frac{19}{5})^T$. If $X^i \in \Re^2_+$ is such that $X_2^i < \frac{2}{3}X_1^i$, then $f^i(X^i) - p^{*T}X^i = (X_1^i + 8X_2^i) - \frac{19}{5}(X_1^i + X_2^i) = (\frac{21}{5}X_2^i - \frac{14}{5}X_1^i) = \frac{7}{5}(3X_2^i - 2X_1^i) < 0$.

However, if $X_2^i \geq \frac{2}{3}X_1^i$, then $f^i(X^i) - p^{*T}X^i = \frac{19}{5}(X_1^i + X_2^i) - \frac{19}{5}(X_1^i + X_2^i) = 0$.

Let $\overline{X} = \langle \overline{X}^i / i = 1,2\rangle$ where $\overline{X}^i = (\frac{1}{2}, \frac{1}{2})^T$ for $i = 1,2$.

$\langle \overline{p}, \overline{X}\rangle$ is a market equilibrium for the economy with aggregate endowments being $(1,1)^T$.

The value accruing to agent 1 in money units at $\langle \overline{p}, \overline{X}\rangle$ is $f^1(\overline{X}^1) - \overline{p}^T\overline{X}^1 + \overline{p}^T\overline{w}^1 = \frac{19}{5} > 2$.

The value accruing to agent 2 in money units at $\langle \overline{p}, \overline{X}\rangle$ is $f^2(\overline{X}^2) - \overline{p}^T\overline{X}^2 + \overline{p}^T w^2 = \frac{19}{5} < 8$.

Thus, agent 1 is better off after having destroyed half its endowment.

Further, if agent 1 had withheld half its endowment instead of destroying it, then the value that accrues to agent 1 in units of the numeraire consumption good at $<\bar{p}, \bar{X}>$ is $\frac{11}{2} - \bar{p}^{T}\bar{X}^{1} + \bar{p}^{T}\bar{w}^{1} = \frac{11}{2} - \frac{19}{5} + \frac{19}{5} = \frac{11}{2} > \frac{19}{5} > 2$, since in this situation agent 1 gets to consume $\frac{3}{2}$ units of the first and $\frac{1}{2}$ units of the second commodity.

This example again reveals that agent 1 benefits by destroying half its initial endowments, and considerably more by withholding it.

4. Manipulation via Endowments of the Dictatorial Allocation: A feasible extended allocation (X,Y) may be said to be ***individually rational*** if for all i = 1,…,H: $f^i(X^i) + Y^i \geq f^i(w^i)$.

Let (X*,Y*) be a efficient extended allocation such that for some $i \in \{1,\ldots,H\}$: $Y^{*i} = \sum_{k \neq i}[f^k(X^{*k}) - f^k(w^k)]$ whilst $Y^{*k} = f^k(w^k) - f^k(X^{*k})$ for $k \neq i$.

In this extended allocation, agent i, receives the aggregate valuation/willingness-to-pay for X* and pays each of the other agents the latter's valuation of its initial endowment.

The final pay-off to agent i is $\sum_{i=1}^{H} f^i(X^{*i}) - \sum_{k \neq i} f^k(w^k)$ and the final pay-off to an agent k $\neq$ i, is $f^k(w^k)$.

Since $\sum_{i=1}^{H} f^i(X^{*i}) \geq f^i(w^i) + \sum_{k \neq i} f^k(w^k)$, we get $\sum_{i=1}^{H} f^i(X^{*i}) - \sum_{k \neq i} f^k(w^k) \geq f^i(w^i)$.

Thus (X*,Y*) is individually rational. Further, agent i, can do no better than at (X*,Y*), and given the weak monotonicity assumption of the production functions, at (X*,Y*) agent i does not stand to gain by destroying any of its initial endowment either. Further, any agent k($\neq$i) receives the output that it can produce by using its own initial endowments. Hence given the weak monotonicity of the valuation functions, no one can benefit by destroying its own initial endowment.

Thus this extended allocation (which we may refer to as the dictatorial extended allocation with agent i as the dictator) is not manipulable via destruction of initial endowment.

If agent i withheld a portion $\alpha \in \Re^{L}_{+}$, $\alpha \leq w^i$ of its initial endowment, then at an efficient allocation $X = <X^i / i = 1,\ldots,H>$ that may arise due to concealment it would receive $\sum_{i=1}^{H} f^i(X^i) - \sum_{k \neq i} f^k(w^k) + f^i(X^i + \alpha) - f^i(X^i) \leq \sum_{i=1}^{H} f^i(X^{*i}) - \sum_{k \neq i} f^k(w^k)$ since X* is an efficient allocation, whilst the allocation where agent i is allocated $X^i + \alpha$ and agent any k ($\neq$ i), is allocated X^k is feasible for the economy prior to concealment. Thus i does not benefit from withholding any of its initial endowment.

On the other hand if an agent k ($\neq$ i), withheld a fraction α of its initial endowment, then at an efficient allocation $X = <X^i / i = 1,\ldots,H>$ that may arise due to concealment it would receive $f^k((1-\alpha)w^k) + f^k(X^k + \alpha w^k) - f^k(X^k)$.

We will now show by means of an example that it may be possible for an agent k ($\neq$ i) to benefit by withholding a portion of its initial endowment.

Let L = 1, $f^1(x) = 2x$ for all $x \in \Re_+$, $f^2(x) = x$ for all $x \in [0,1]$, $f^2(x) = \frac{1}{2}x$ for all $x \geq 1$. Let $w^1 = 0$, $w^2 = 2$.

Let (X*,Y*) be any efficient extended allocation where $Y^{*2} = f^2(w^2) - f^2(X^{*2})$. Since X* is efficient, $X^{*1} = 2$ and $X^{*2} = 0$. The final pay-off to agent 2 is $\frac{3}{2}$.

If agent 2 withholds one unit of the good, at any efficient extended allocation (X,Y) with respect to revealed endowments, where $Y^2 = f^2(\frac{1}{2}w^2) - f^2(X^2)$ its final pay-off is $f^2(1) + f^2(1) = 2 > 1$. Thus agent 2 is better off by withholding half its initial endowment.

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