Tariff and Equilibrium Indeterminacy–(II)

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Abstract

We establish conditions under which indeterminacy can occur in a small open economy oil-in the production RBC model with lump sum tariff revenue transfers. The indeterminacy would require that the steady state tariff rates be in an open interval. This means that as long as the government revenues are exogenous, our indeterminacy result will be robust to the usage of the government revenue.

Key Words: Indeterminacy, Endogenous Tariff Rate, Small Open Economy, Lump Sum Transfers

JEL Classification Number: Q43, F41

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1. Introduction

It is well understood by now that under some conditions open economy RBC models can be subject to indeterminacy, in the sense that there exist a continuum of equilibrium trajectories converging to a steady state. The literature on indeterminacy in open economy emphasizes different channels of generating indeterminacy. Weder (2001), Meng and Velasco (2003, 2004) prove that indeterminacy is easier to obtain for a small open economy due to perfect or nearly perfect world capital markets that keep interest rate more or less constant. Wen and Aguiar-Conraria (2005, 2006 henceforth WAC) supply another way of generating indeterminacy through importing oil as a third production factor, in which it is easier for them to have indeterminacy.

Those early models relied on increasing returns or external effects to generate indeterminacy. Benhabib and Farmer (1999) provide five sources of indeterminacy in closed and open economies.[1] Tariff as a kind of transaction costs in international trade belongs to the second category which they mentioned. Schmitt-Grohe and Uribe (1997, in short SGU) prove that within a standard neoclassical growth model (under closed economy), a balanced budget rule can make expectations of higher tax rates self fulfilling if the fiscal authority relies on changes in labor income taxes to eliminate the short run fiscal imbalances. In Zhang (2008a), we prove that in the open economy, tariff and factor income taxes share similar channel of generating indeterminacy in the form of

[1] See Benhabib and Farmer (1999) page 390.

endogenizing rates and making the government revenue exogenous. The intuition for endogenous factor income taxes and tariff to generate indeterminacy is that both of them are countercyclical with respect to the output.

One remaining issue in our work is that although we show that factor income taxes and tariff are channel equivalent to generate indeterminacy, we didn't check if our result is robust to the usage of the government revenue. SGU (1997) in their paper mentioned (pp 985):

> " On the other hand, the assumption that all government expenditures consist of purchases of goods is not important for our indeterminacy result. It can be shown that if all taxes revenues were returned to the public in the form of lum-sum transfers, indeterminacy would still occur for steady-state tax rates greater than s_k and ... Laffer curve."

In this paper we extend our research on indeterminacy to a small open economy RBC model, in the way of relaxing the assumption that all tariff revenues are consumed by the government. Ask the similar question as SGU and bring back this feature into the picture. We let all of the revenues returned to the agent in the form of lump-sum transfers and validate that the indeterminacy result is robust to this extension as long as the government revenue is exogenous.

SGU (1997) explicitly solve the upper bound of the indeterminate region for the steady state labor income tax rate, which is 0.5, if they assume that the government transfers the income tax revenue to the agent (see page 985). In our model, we can't do that since relaxation of the assumption that the government consumes the revenue will make the determinant of the Jacobian matrix in my former model become more complicated.

This paper is also a realistic extension of SGU and related work in the literature, in that we incorporate the energy taxes or tariff on the imported production factor to an otherwise standard Ramsey model of a small open economy. SGU modify the Benhabib and Farmer (1994) structure by replacing the production externality with labor income taxes, we modify WAC model by replacing the production externality with tariffs. Remember that in our model tariff is imposed on the energy income ($o_t p^o$), for example, we can imagine that it is a special kind of factor income taxes in open economies.

2. The One-Sector Open Economy With Lump-Sum Transfers

Consider a modified small open economy version of Benhabib and Farmer (1994) competitive model without production externality. A representative agent maximizes the intertemporal utility function

$$\int_0^\infty e^{-\rho t}(\log c_t - bn_t)dt \tag{1}$$

where c_t is consumption of the single goods which is the numeraire and tradeable, n_t labor supply and $\rho \in (0,1)$ is the subjective discount rate in the continuous time model. Assume that the economy is open to importing oil so that the agent can use the tradeable goods to buy oil. The oil price is assumed to be exogenous as many authors do, for instance, Rotemberg and Woodford (1996), Wen and Aguiar-Conraria (2005, 2006). The oil supply from the rest of the world is assumed to be perfectly elastic.

On the production side, there is a single good produced with a Cobb-Douglas production technology with three inputs–capital (k_t), labor (n_t) and non-reproducible natural resources (o_t) :

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0} \tag{2}$$

where the third factor in the production, non-reproducible natural resources, say oil (o_t), is imported, and the technology displays the constant returns to scale ($a_k + a_n + a_0 = 1$). Assuming the firms are price takers in the factor markets, the profits of the firms are given by

$$\pi = y - (r + \delta)k - wn - p^o(1 + \tau)o \tag{3}$$

where $(r+\delta)$ denotes the user cost of renting capital[2], w denotes the real wage, and p^o denotes the real price of oil (the imported goods). τ is the tariff rate imposed on the imported oil, which is uniform to all firms.[3] Perfect competition in factor and product markets implies that factor demands are given by:

$$w_t = a_n \frac{y_t}{n_t}$$

$$r_t + \delta = a_k \frac{y_t}{k_t}$$

and

$$p^o(1+\tau_t) = a_0 \frac{y_t}{o_t}$$

Since we assume that the foreign input is perfectly elastically supplied, the factor price, p_o, is independent of the factor demand for o, we can substitute out o in the production function using

[2] $\delta \in (0,1)$ denotes the depreciation rate of capital, r_t is the rental rate of capital.

[3] Here the tariff rate can be endogeneous. We can also see the endogenous tariff rate in Loewy (2004) and Mourmouras (1991) in a two-country open economy endogenous growth model and a small open economy OLG model respectively. This approach originates from Ramsey (1927).

$$o_t = a_0 \frac{y_t}{p^o(1+\tau_t)}$$

to obtain the following reduced-form production function:

$$y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{4}$$

where $A = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$.

The agent budget constraint is

$$\dot{k}_t = r_t k_t + w_t n_t - c_t + G$$

here $G = p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ is the exogenous revenue collected by the government through imposing tariffs on the oil.[4] We assume that the government transfers the revenue to the agent **in the form of lump-sum**. The first order conditions become

$$\frac{1}{c_t} = \Lambda_t$$

[4] As we see in Zhang (2008a), the exogenous government revenue will require the endogenous tariff rate to be contercyclical with respect to the output since $p^o \tau_t o_t = G = \frac{\tau_t \lambda a_0 y_t}{(1+\tau_t)}$ implies $\frac{\partial \tau}{\partial y} < 0$.

$$b = \Lambda_t w$$

$$\dot{\Lambda_t} = (\rho - r_t)\Lambda_t$$

where Λ_t denotes the marginal utility of income.

Market clearing requires that aggregate demand equal aggregate supply, that is,

$$c_t + \dot{k_t} + \delta k_t + o_t p^o = y_t \tag{4'}$$

Note that the international trade balance is always zero. Foreigners are paid in goods. This is clear in equation (4'), according to which domestic production is divided between consumption, investment and imports ($c_t + i_t + p_t^o o_t = y_t$, $i_t = k_{t+1} - (1-\delta)k_t$). So part of what is produced domestically is used to pay for the imports.

When we replace the consumption with $\frac{1}{\Lambda_t}$, transform wage rate and rental rate into functions of capital and labor, the equilibrium conditions can be reduced to four equations:

$$b = \Lambda_t a_n A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} \tag{5}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - a_k A k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} \tag{6}$$

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau_t}) y_t - \delta k_t - \frac{1}{\Lambda_t} \tag{7}$$

and

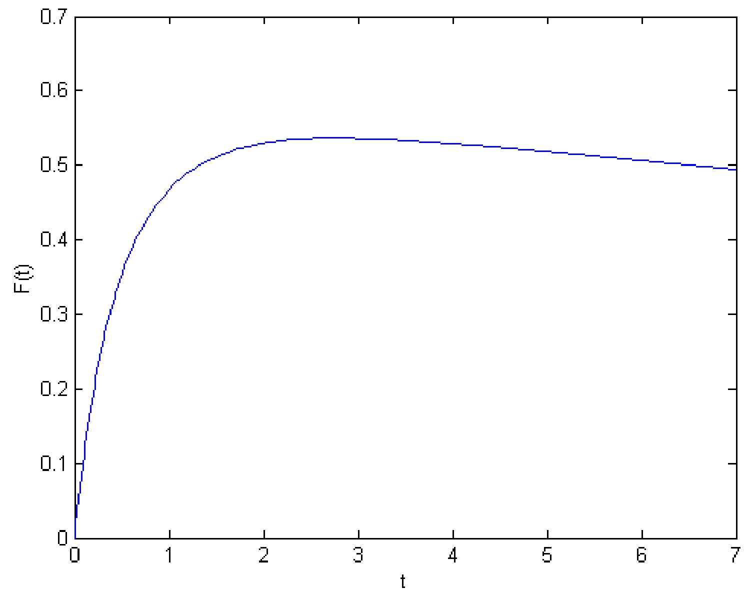
$$G = \frac{\tau_t a_0 y_t}{(1+\tau_t)},\ y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{8}$$

We claim that the number of the steady state tariff rate that generates enough revenue to finance a given level of government revenue can be 0, 1 or 2.[5]

Claim 1. *The steady state in the continuous-time dynamic system (5)-(8) exists, given the proper level of government expenditure.*

We can derive steady state $\frac{k}{n} = (\frac{\rho+\delta}{a_k A})^{\frac{1-a_0}{-a_n}}$, $\Lambda = \frac{b}{a_n A}(\frac{\rho+\delta}{a_k A})^{\frac{a_k}{a_n}}$, $k = \frac{\frac{a_n A}{b}(\frac{\rho+\delta}{a_k A})^{-\frac{a_k}{a_n}}}{[\frac{1-\frac{a_0}{1+\tau}}{a_k}(\rho+\delta)-\delta]}$, $G = \frac{\tau}{[\frac{1-\frac{a_0}{1+\tau}}{a_k}(\rho+\delta)-\delta](1+\tau)^{\frac{a_n+a_0}{a_n}}} cons = F(\tau)$, $constant = (\frac{a_0}{p^o})^{\frac{a_0}{a_n}} \frac{a_0(\rho+\delta)a_n(\frac{\rho+\delta}{a_k})^{-\frac{a_k}{a_n}}}{a_k b}$. We can see $F(\tau)$ is non-monotone and the number of the steady state tariff rate that gen-

[5] SGU (1997) show that the revenue maximizing tax rate is the least upper bound of the set of taxes rate for which the rational expectations equilibrium is indeterminate. But in our endogenous tariff rate case, this property doesn't hold.



erates enough revenue to finance a given level of government purchases can be 0, 1 or 2.

Example 2. *We give an example for* $a_0 = 0.21$, $a_n = 0.7$, $a_k = 0.09$, $\delta = 0.025$, $\rho = 0.065$. ρ *is taken as Benhabib and Farmer (1994). Other parameters are taken from WAC (2005). We can see that given the proper level of the government revenue, the number of the steady state tariff rate usually is 0 or 2.*

Consider the log linear approximation of the equilibrium conditions (5)–(8) around the steady state. Let $\hat{k}_t$, $\hat{n}_t$, $\hat{\tau}$, λ_t denote the log deviations of k_t, n_t and τ, Λ_t from their respective steady states.[6] The log linearized equilibrium conditions then are

$$0 = \lambda_t - \frac{\tau_{ss}\hat{\tau}}{\frac{1-a_0}{a_0}(1+\tau_{ss})} + \frac{a_k}{1-a_0}(\hat{k}_t - \hat{n}_t) \tag{9}$$

$$\dot{\lambda}_t = (\rho+\delta)[\frac{a_n}{1-a_0}(\hat{k}_t - \hat{n}_t) + \frac{\tau_{ss}\hat{\tau}}{\frac{1-a_0}{a_0}(1+\tau_{ss})}] \tag{10}$$

$$\dot{\hat{k}}_t = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta]\hat{k}_t + \frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}\hat{n}_t + \{-\delta + \frac{[1-\frac{a_0}{(1+\tau_{ss})}]}{a_k}(\rho+\delta)\}\lambda_t \tag{11}$$

$$\hat{y}_t = -\frac{1}{1+\tau_{ss}}\hat{\tau} = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t{}^{7} \tag{12}$$

Combining the (9) and (12), we can imply

$$\hat{n}_t = \frac{\lambda_t}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}} + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\hat{k}_t$$

[6] τ_{ss} is the steady state tariff rate.

[7] Note that $\frac{a_k+a_n}{1-a_0(1+\tau_{ss})} > 1$, the increasing returns to scale comes from the endogenous tariff rate.

Using this expression to eliminate the $\hat{n}_t$in the (10) and (11) results in the following system:

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{\hat{k}}_t \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \begin{bmatrix} \lambda_t \\ \hat{k}_t \end{bmatrix}, J = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}$$

where

$$\mathbf{J}_{11} = -(\rho+\delta)\frac{\frac{a_n}{1-a_0} + \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}$$

$$J_{12} = (\rho+\delta)\{[\frac{a_n}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_k}{1-a_0(1+\tau_{ss})}] - \frac{[\frac{a_n}{1-a_0} + \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}]\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\}$$

$$J_{21} = \{-\delta + \frac{[1-\frac{a_0}{(1+\tau_{ss})}]}{a_k}(\rho+\delta)\} + \frac{\frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}$$

$$J_{22} = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta] + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}} \frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}$$

After some tedious algebra, we can have, $J_{11} = -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}}$, $J_{22} = (\rho+\delta)\frac{1-a_0}{a_k-a_0\tau_{ss}} - \delta$, $J_{12} = (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k-a_0\tau_{ss}}$, $J_{21} = \frac{(\rho+\delta)}{a_k}\frac{(1-a_0)^2+\tau_{ss}[(1-a_0)^2-a_na_0]-\tau_{ss}^2a_0}{(a_k-a_0\tau_{ss})(\tau_{ss}+1)} - \delta$.

Proposition 3. *The equilibrium is indeterminate iff* $trace(J) = J_{11} + J_{22} < 0 < J_{22}J_{11} - J_{12}J_{21} = \det(J)$, *or,* $\tau_2 < \tau_{ss} < \tau_3$, *where* $\frac{a_k}{a_0} < \tau_2 < \tau_3$, τ_2, τ_3 *are determined by the system parameters.*

The indeterminacy requires that $trace(J) = \frac{a_k}{a_k-a_0\tau_{ss}}(\rho+\delta) - \delta < 0$ if and only if $\tau_{ss} > \frac{a_k}{a_0}$

After some manipulations, the determinant of the Jacobian can be written as

$$\det(J) = \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}\Delta$$

where

$$\Delta = -a_n(\rho+\delta)\frac{1-a_0}{a_k - a_0\tau_{ss}} + \delta a_n + a_0\tau_{ss}\{\frac{(\rho+\delta)}{a_k}\frac{(1-a_0)^2 + \tau_{ss}[(1-a_0)^2 - a_n a_0] - \tau_{ss}^2 a_0}{(a_k - a_0\tau_{ss})(\tau_{ss}+1)} - \delta\}$$

The positive $det(J)$ requires that (conditional on $a_k - a_0\tau_{ss} < 0$) $\Delta < 0$. We define

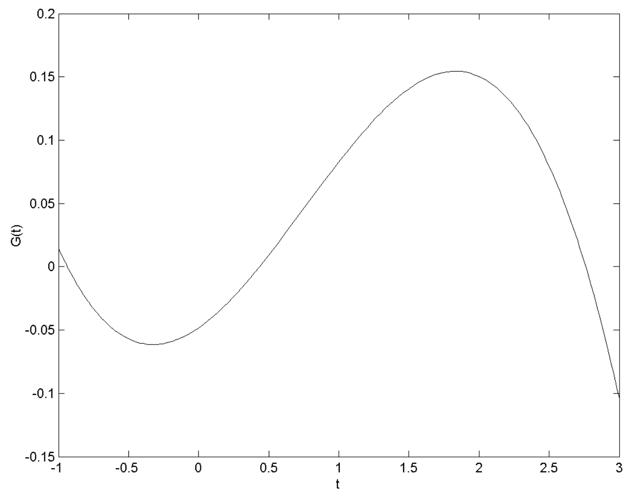
$$G(\tau_{ss}) = \Delta_1\tau_{ss}^3 + \Delta_2\tau_{ss}^2 + \Delta_3\tau_{ss} + \Delta_4$$

where $\Delta_1 = a_0^2[\delta - \frac{(\rho+\delta)}{a_k}] < 0$, $\Delta_2 = -\delta a_0 a_n + a_0\{\frac{(\rho+\delta)[(1-a_0)^2 - a_n a_0]}{a_k} + \delta a_0 - \delta a_k\}$, $\Delta_3 = -\delta a_0 a_n + [-(\rho+\delta)a_n(1-a_0) + \delta a_n a_k]$ $+ a_0\frac{(\rho+\delta)}{a_k}(1-a_0)^2 - \delta a_k$, $\Delta_4 = -(\rho+\delta)a_n(1-a_0) + \delta a_n a_k$. $\Delta < 0$ is equivalent to $G(\tau_{ss}) > 0$.

We can easily find that $G(0) < 0$, as $\delta = 0$, $G(\frac{a_k}{a_0}) = 0$. As $\delta > 0$ but close to zero, $G(\frac{a_k}{a_0}) < 0$. There are three roots for $G(\tau_{ss}) = 0$. Let us order them $\tau_1 < 0 < \tau_2 < \tau_3$. We can see that $\frac{a_k}{a_0} < \tau_2$, as $\tau_2 < \tau_{ss} < \tau_3$, $G(\tau_{ss}) > 0$, indeterminacy arises in the tariff model.

2.1. *Calibrated example*

The purpose of this section is to illustrate the main result of the proposition–that indeterminacy in fact occurs with the empirical tariff rate-by one numerical experiment.



We adopt the following "standard" values in RBC models: $a_0 = 0.21$, $a_n = 0.7$, $a_k = 0.09$, $\delta = 0.025$, $\rho = 0.065$[8].

Case 1: $\tau_{ss} = \frac{\text{import tariff}}{\text{import price}} = \frac{15.6\$/bbl}{26\$/bbl} = 0.6$ which is the optimal tariff rate of oil from David Newbery (2005), consistent with the one in EU (2002).

We draw $G(t)$ graph for the numerical experiment and see that $\tau_1 = -0.9353$, $\frac{a_k}{a_0} = 0.4286 < \tau_2 = 0.4341$, $\tau_3 = 2.7605$. As $\tau_2 < \tau_{ss} = 0.6 < \tau_3$, $G(\tau_{ss}) > 0$.

Different from SGU (1997), we cannot explicitly get the indeterminate region because we suppose that the government transfers the revenue to the agent in a lump-sum way. Both of the two bounds for the indeterminate region change since relaxation of the

[8] The factor weights are taken from WAC (2005). They are of the country, Netherlands, based on input-output tables from OECD (1995) reports. $\rho = 0.065$, see Benhabib and Farmer (1994).

assumption that the government consumes the revenue will make the determinant of the Jacobian matrix become more complicated, up to a third order polynomial. But the indeterminacy result generated by the endogenous tariff rate is still robust to the usage of the government revenue.

3. Discussion and extensions

It has been shown that an otherwise standard one-sector oil -in the production real business cycle model may exhibit indeterminacy and sunspots under a balanced-budget rule that consists of fixed and "wasteful" government spending (or lump-sum transfers) and endogenous tariff rate. However, the economy always displays saddle-path stability and equilibrium uniqueness if the government finances endogenous public expenditures with a constant tariff rate. We may extend this paper by allowing for productive or utility-generating government purchases in either of these specifications. It may turn out that the earlier determinacy results are overturned when public expenditures generate sufficiently strong production or consumption externalities.

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