# Modeling the evolution of age-dependent Gini coefficient for personal incomes in the U.S. between 1967 and 2005

Kitov, Ivan

Institute for the Geospheres' Dynamics

20 August 2008

**In memory of Professor V.N.Rodionov**

# Modeling the evolution of age-dependent Gini coefficient for personal incomes in the U.S. between 1967 and 2005

Ivan O. Kitov


## Abstract

This study validates the microeconomic model defining the evolution of personal incomes in the U.S. Because of a large portion of population not reporting any income, any comprehensive modeling of the overall personal income distribution (PID) is complicated. Age-dependent PIDs allow overcoming this shortcoming since the portion of population without income is very low (<4 %) for ages over 45 years. It is demonstrated that the evolution of Gini coefficient, for the years with a good PID resolution, can be accurately (<0.005) predicted.

As the overall PIDs, the empirical age-dependent (density) PIDs collapse to practically one curve when normalized to cumulative growth in personal income and total population in given age groups for the period between 1967 and 2005. This allows exact prediction of Gini coefficient and other measures of inequality, which are defined by PID. Therefore, these measures of income inequality are only of secondary importance

In all age groups, the model predicts slightly decreasing Gini coefficients between 1977 and 2005. The overall $G$ is approximately constant, however. The Pareto law index, $k$, undergoes significant changes over age: increases from the youngest age to approximately 67 years of age, and then drops. This index defines the roll-off at the highest incomes.


Key words: personal income distribution, age, mean income, microeconomic modelling, USA, real GDP, macroeconomics

JEL Classification: D01, D31, E17, J1, O12

## Introduction

Economic inequality is an apparently inevitable and multi-dimensional phenomenon in any social system. Due to practical and emotional importance for everyone, inequality attracts high attention of economists and politicians. The former ones are aimed at revealing potential deterministic or statistical links between economic inequality and numerous micro- and macroeconomic variables. There is no clear understanding, however, is economic inequality a positive or negative factor for such fundamental economic parameters as real economic growth, inflation, and unemployment.

Income inequality is one of quantitative measures of economic inequality. There are many theories of inequality arising from income distribution. Neal and Rosen (2000) presented a comprehensive overview of state-of-art in this field. In spite of the efforts associated with the development of a consistent model of income distribution there are some problems yet to resolve. Moreover, modern economic theories do not meet some fundamental requirements applied to scientific theories - a concise description of accurately measured variables and prediction of their evolution beyond the period of currently available measurements.

Understanding and modeling of age-dependent personal income distribution (PID) deserves special attention. Dramatic changes in the shape of PID are observed with age. Kitov (2005c) successfully modeled the age-dependent PIDs in the U.S. for the period between 1994 and 2002. This quantitative analysis and prediction was based on a microeconomic model (Kitov, 2005a) and results of the overall PID modeling (Kitov, 2005b). For a given economy, this microeconomic model quantitatively describes the evolution (with age and over time) of each and every personal income as a function of individual capacity to earn money and real economic growth. The sum of all personal incomes predicted by the model (with the simplest assumption about the distribution of the capacity to earn money) builds a macroeconomic model. This modeling of age-dependent PID (Kitov, 2006) was not accompanied by any explicit prediction of the level of income inequality.

The most popular aggregate measure of income inequality is the Gini coefficient, $G$. This coefficient is characterized by a number of advantages such as relative simplicity, anonymity, scale independence, and population independence. On the other hand, the Gini coefficient belongs to the group of operational measures: its evolution in time is not theoretically linked to macroeconomic variables and the differences in Gini coefficient observed between various countries are not well explained. These caveats make the Gini coefficient more useful in political and social applications but not in economics as a potentially quantitative (hard) science.

As a rule, the Gini coefficient is estimated from household surveys and inequality as reported at family and household levels. Such an aggregation is affected by social and demographic processes, which may bias true economic mechanisms driving income inequality. Theoretically, the indivisible level for the study of income inequality is personal income. In our framework, personal incomes are presumed to be sensitive only to macroeconomic variables.

In 2006, the U.S. Census Bureau (CB, 2006) opened an Internet access to the results of PID measurements since 1947. Kitov (2007a) analyzed and modeled the behavior of Gini coefficient for working age population (over 16 years of age) during the period between 1967 and 2005. There were several important findings related to the estimation of empirical Gini coefficients associated with the U.S. income distribution:

1. These estimates of Gini coefficient critically depend on definition of income.
2. The Gini coefficient associated with whole population of 16 years of age and over and that associated only with people with (reported) income *converge* with time as the portion of people without income decreases.
3. Resolution of the measured PIDs (i.e. a proportional coverage of population with income bins) and interpolation of the PIDs inside these bins influences the estimates of Gini coefficient.
4. The empirical PIDs practically collapse to one curve when normalized to the cumulative growth in nominal Gross Personal Income (GPI) for the studied period between 1967 and 2005.

Slight changes observed in these PIDs and the evolution of the *G*-values are related to economic growth and changes in the age structure. Kitov (2007a) also demonstrated that the empirical estimates of Gini coefficient converge to theoretical ones when all individuals in working age population have income. Such convergence might be clearly observed in age-dependent PIDs, since the portion of population without income decreases with age.

The age-dependent PID in the youngest group is characterized by large differences from the overall PIDs. Obviously, all individuals start with zero income and the initial part of income trajectory in time, as personal income observations show, is close to an exponential growth. In the mid-age groups, PIDs are similar to the overall PID. In the oldest age group, PID is also different and is closer to that in the youngest group. Accordingly, Gini coefficient undergoes a substantial evolution from the youngest to the oldest age groups.

The purpose of this paper is to present accurate estimates of the Gini coefficients associated with the PIDs provided by the U.S. Census Bureau. We also model the evolution of *G* in various age groups between 1967 and 2005, i.e. during the period where the estimates of total income in each of these age groups are available.

The paper is organized as follows. Section 1 introduces the model for the evolution of individual incomes in the U.S. Section 2 describes the data on personal income distribution in various age groups, presents estimates of actual Gini coefficients and elaborates on empirical PIDs. Section 3 compares the evolution of the observed Gini coefficients with that predicted by our model.

## 1. The model

This Section repeats practically one-to-one Section 1 in Kitov (2007a). Historically, our model is based on a geo-mechanical model of solid with non-elastic inhomogeneities, introduced by V.N. Rodionov with co-authors (1982).

The principal assumption of the microeconomic model is that every person above fifteen years of age has a capability to work or earn money using some means, which can be a job, bank interest, stocks, interfamily transfers, etc. An almost complete list of the means is available in the U.S. Census Bureau technical documentation (2002) as the sources of income are included in the survey list. Some principal sources of income are not included, however, what results in the observed discrepancy between aggregate (gross) personal income, GPI, and GDI.

Here we introduce the model described by Kitov (2005a). The rate of income, i.e. the overall income a person earns per unit time, is proportional to her/his capability to earn money, $\sigma$. (An equivalent term for earning money is "work", because work is the only source of any goods and services denominated in monetary units.) The person is not isolated from the surrounding world and the work (money) s/he produces dissipates (conventional economic term for the process would be depreciation, but physical terms are more appropriate in this case) through interaction with the outside world, decreasing the final income rate. The counteraction of external agents, which might be people or any other externalities, determines the price of the goods and services a person creates. The price depends not on some absolute measure of quality of the goods but on the aggregate opinion of the surrounding people on relative merits (expressed in monetary units) of the producers not goods. For example, the magic of famous brands provides a significant increase in incomes for their owners without proportional superiority in quality because people appreciate the creators not goods. As a whole, an equilibrium system of prices arises from the aggregate opinions on relative merits of each and every person not from the physical quantities and qualities of goods and services. The personal incomes are ranked in some fixed hierarchy and, when expressed in monetary units, the hierarchy is transformed in the dynamic system of prices. Since the hierarchy of incomes is fixed, the amounts and qualities of goods can only reorder individuals not change the final aggregate price of everything produced – GDP.

Analogously to many cases observed in natural sciences, the rate of dissipation is proportional to the attained income (per unit time) level and inversely proportional to the size of the means used to earn the money, $\Lambda$. Bulk heating of a body accompanied by cooling through its surface is the case. For a uniform distribution of heating sources, the energy released in the body is proportional to its volume or cube of characteristic linear size and the energy lost through its surface is proportional to the square of the linear size. In relative terms, the energy balance or the ratio of cooling and heating is inversely proportional to the linear size. As a result, a larger body undergoes a faster heating because loses relatively less energy and also reaches a higher equilibrium temperature. Therefore one can write an ordinary differential equation for the changing rate of income earned by a person in the following form:

$$dM(t)/dt= \sigma(t)-\alpha M(t)/\Lambda(t) \tag{1}$$

where $M(t)$ is the rate of money income denominated in dollars per year [\$/y], $t$ is the work experience expressed in years [y], $\sigma(t)$ is the capability to earn money [\$/y$^2$]; and $\alpha$ is the dissipation coefficient expressed in units [\$/(y$^2$)]. The size of the earning means, $\Lambda$**,** is also expressed in [\$/y]. The general solution of equation (1), if $\sigma(t)$ and $\Lambda(t)$ are considered to be constant (because these two variables evolve very slowly with time), is as follows:

$$M(t)=(\sigma/\alpha)\Lambda(1-exp(-\alpha t/\Lambda)) \tag{2}$$

In the modeling, we integrate (1) numerically in order to include the effects of the changing $\sigma(t)$ and $\Lambda(t)$. Equations (2) through (4) are derived and discussed in detail below to demonstrate some principal features of the model. These equations represent the solutions of (1) in the case where the observed change in $\sigma(t)$ and $\Lambda(t)$ in all the terms is neglected.

One can introduce the concept of a modified capability to earn money as a dimensionless variable $\Sigma(t)=\sigma(t)/\alpha$. The absolute value of the modified capability, $\Sigma(t)$**,** and the size of earning means evolves with time as the square root of real GDP per capita:

$$\Sigma(t)= \Sigma(t_0)sqrt(GDP(t)/GDP(t_0))$$

and

$$\Lambda(t)= \Lambda(t_0)sqrt(GDP(t)/GDP(t_0)),$$

where $GDP(t_0)$ and $GDP(t)$ are the per capita values at the start point of the modeling, $t_0$, and at time $t$, respectively. Then the capacity of a "theoretical" person to earn money, defined as $\Sigma(t)\Lambda(t)$, evolves with time as real GDP per capita. Effectively, equation (2) states that the evolution in time of a personal income rate depends only on the personal capability to earn money, the size of the means used to earn money, and the economic growth.

The modified capability to earn money, $\Sigma(t)$, and the size of earning means, $\Lambda(t)$, obviously have positive minimum values among all the persons, $\Sigma_{min}(t)$ and $\Lambda_{min}(t)$**,** respectively. One can now introduce relative and dimensionless values of the defining variables in the following way: $S(t)=\Sigma(t)/\Sigma_{min}(t)$ and $L(t)=\Lambda(t)/\Lambda_{min}(t)$**.**

A fundamental assumption is made that the possible relative values of $S(t_0)$ and $L(t_0)$ can be represented as a sequence of integer numbers from 2 to 30, i.e. only 29 different integer values of the relative $S(t_0)$ and $L(t_0)$ are available: $S_1$=2,…, $S_{29}$=30; $L_2$=2,…, $L_{29}$=30. This discrete range results from the calibration process described by Kitov (2005a). The largest possible relative value of $S_{max}=S_{29}=30=L_{max}=L_{29}$ is only 15 (=30/2) times larger than the smallest possible $S=S_1$ and $L=L_1$ (in the model, the minimum values $\Lambda_{min}$ and $\Sigma_{min}$ are chosen to be two times smaller than the smallest observed values of $\Lambda_1$ and $\Sigma_1$). Because the absolute values of variables $\Lambda_i$, $\Sigma_i$, $\Lambda_{min}$, and $\Sigma_{min}$ evolve with time according to the same law, the relative and dimensionless variables $L_i(t)$ and $S_i(t)$, $i$=1,…,29, do not change with time retaining the discrete distribution of relative values. This means that the distribution of the relative capability to earn money and the size of earning means is fixed as a whole over calendar years and also over ages. This assumption on the rigid character of the hierarchy of incomes is supported by observations, as presented by Kitov (2005a, 2005b) for the period between 1994 and 2002. This study extends the set of observations to the period between 1947 and 2005.

In equation (2), one can substitute the product of the relative values $S$ and $L$ and the time dependent minimum values $\Lambda_{min}$ and $\Sigma_{min}$ for $\Sigma(t)$ and $\Lambda(t)$**.** We also normalize the equation to the maximum values $S_{max}$ and $L_{max}$**.** The normalized equation for the rate of income, $M_{ij}(t)$**,** for a person with the capability, $S_i$ and the size of earning means, $L_j$ is as follows**:**

$$M_{ij}(t)/(S_{max}L_{max}) = (\Sigma_{min}\,\Lambda_{min})(S_i/S_{max})(L_j/L_{max})(1 - exp(-(\alpha/\Lambda_{min}\,L_{max})t/(L_j/L_{max}))) \quad (3)$$

or

$$M'_{ij}(t) = \Sigma_{min}(t)\Lambda_{min}(t)S'_iL'_j\{1 - exp[-(1/\Lambda_{min})(\alpha' t/L'_j)]\} \quad \textbf{(3′)}$$

where $M'_{ij}(t)=M_{ij}(t)/(S_{max}L_{max})$; $S'_i=(S_i/S_{max})$; $L'_j=(L_j/L_{max})$; $\alpha'=\alpha/L_{max}$, $S_{max}$=30, and $L_{max}$=30. Below we omit the prime indices. The term $\Sigma_{min}(t)\Lambda_{min}(t)$ corresponds to the total (cumulative) growth of real GDP per capita from the start point of a personal work experience, $t$ $(t_0=0)$, and is different for different years of birth. This term might be considered as a coefficient defined for every single year of work experience because this is a predefined external variable. Thus, one can always measure the personal income in units $\Sigma_{min}(t_0)\Lambda_{min}(t_0)$**.** Then equation (3') becomes a dimensionless one and the coefficient changes from 1.0 as the real GDP per capita evolves relative to the start year.

Equation (3′) represents the rate of income for a person with the defining parameters $S_i$ and $L_j$ at time $t$ relative to the maximum possible personal income rate. The maximum possible income rate is obtained by a person with $S_{29}$=30/30=1 and $L_{29}$=30/30=1 at the same time $t$. The term $1/\Lambda_{min}$ in the exponential term evolves inversely proportional to the square root of real GDP per capita. This is the defining term of the personal income evolution, which accounts for the differences between the start years of work experience. The numerical value of the ratio $\alpha/\Lambda_{min}$ is obtained by calibration for the start year of the modeling. This calibration assumes that $\Lambda_{min}(t_0)$=1 (and $\Sigma_{min}(t_0)$=1 as well) at the start point of the modeling and only the dimensionless factor $\alpha$ has to be empirically determined. In this case, absolute value of $\alpha$ depends on start year.

As numerous observations show, the money earning capacity, $S_iL_j$, drops to zero at some critical time, $T_{cr}$, in a personal history (Kitov, 2005a), the solution of (1) is:

$$M_{ij}(t) =$$

$$M_{ij}(T_{cr})exp(-\alpha(t-T_{cr})/ \Lambda_{min} L_j) =$$

$$= \{\Sigma_{min}(t)\Lambda_{min}(t)S_iL_j(1-exp(-\alpha T_{cr}/\Lambda_{min} L_j))\} exp(-\alpha_1(t-T_{cr})/ \Lambda_{min} L_j) \quad (4)$$

The first term is equal to the level of income rate attained by the person at time $T_{cr}$, and the second term represents an exponential decay of the income rate for work experience above $T_{cr}$. The exponent index $\alpha_1$ is different from $\alpha$ and varies with time. It was found that the exponential decrease of income rate above $T_{cr}$ results in the same relative (as reduced to the maximum income for this calendar year) income rate level at the same age. It means that the index can be obtained according to the following relationship:

$$\alpha_1 = -ln(C)/(A - T_{cr})$$

where $C$ is the constant relative level of income rate at age $A$. Thus, when current age reaches $A$ the maximum possible income rate $M_{ij}$ (for $i$=29 and $j$=29) drops to $C$. Income rates for other values of $i$ and $j$ are defined by (4). For the period between 1994 and 2002, empirical estimates are as follows: $C$=0.72 and $A$=64 years. The observed exponential roll-off for individual and the mean income beyond $T_{cr}$ corresponds to a zero-value work applied to earn money in the model. People do not exercise any effort to produce income starting from some predefined (but growing) point in time, $T_{cr}$, and enjoy exponential decay of their incomes. A physical analog of such decay is cooling of a body, for example, the Earth. When all sources of internal heating (gravitational, rotational, and radioactive decay) disappear, the Earth only will be loosing the internal heat through the surface before reaching an equilibrium temperature with the outer space. This process of cooling is also described by an exponential decay because the heat flux from the Earth is proportional to the difference of the temperatures between the Earth's surface and the outer space.

The probability for a person to get an earning means of relative size $L_j$ is constant over all 29 discrete values of the size. The same is valid for $S_i$, i.e. all people of 15 years of age and above are distributed evenly among the 29 groups for the capability to earn money. Thus, the relative capacity for a person to earn money is distributed over the working age population as the product of independently distributed $S_i$ and $L_j$ - $S_iL_j$ = {2×2/900, 2×3/900, …, 2×30/900, 3×2/900, …, 3×30/900, …, 30×30/900}. There are only 841 (=29x29) values of the normalized capacity available between 4/900 and 900/900. Some of these cases seem to be degenerate (for example, 2x30=3x20=4x15= …= 30x2), but actually all of them define different time histories according to (3'), where $L_j$ is also present in the exponential term. In the model, no individual (in sense of real people) future income trajectory is predefined, but it can only be chosen from the set of the 841 predefined individual futures for each single year of birth.

It is not possible to quantitatively estimate the value of the dissipation factor, $\alpha$, using some independent measurements. Instead, a standard calibration procedure is applied. By definition, the maximum relative value of $L_j$ $(L_{29})$ is equal to 1.0 at the start point of the studied period, $t_0$. The value of $\Lambda_{min}(t_0)$ is also assumed to be 1.0. Thus, one can vary $\alpha$ in order to match predicted and observed PIDs, and the best-fit value of $\alpha$ is used for further predictions. The range of $\alpha/\Lambda_{min}$ from 0.09 to 0.04 approximately corresponds to that obtained in the modeling of the U.S. PIDs during the period between 1960 and 2002 (Kitov, 2005a). Actual initial value of $\alpha$ is found to be 0.086 for $t_0$=1960. The value of $\Lambda_{min}$ changes during this period from 1.0 to 1.49 according to the square root of the real GDP per capita growth. The cumulative growth of the real GDP per capita from 1960 to 2002 is 2.22 times.

Because the exponential term in (2) includes the size of earning means growing as the root square of the real GDP per capita, longer and longer time is necessary for a person with the maximum relative values $S_{29}$ and $L_{29}$ to reach the maximum income rate. There is a critical level of income rate, however, which separates two income zones with different properties. This level is called the Pareto threshold of income. Below this threshold, in sub-critical income zone, PID is accurately predicted by the model for the evolution of individual income. One can crudely approximate the PID by an exponent with a small negative index, as shown later on in the paper. Above the Pareto threshold, in supercritical income zone, PID is governed by a power (equivalent to the Pareto) law. The presence of a high-income zone with the Pareto distribution allows any person reaching the threshold to obtain any income in the distribution, with rapidly decreasing probability, however.

The mechanisms driving the power law distribution and defining the threshold are not well understood not only in economics but also in physics as well for similar transitions.

The absence of the explicit description of the driving mechanisms does not prohibit using well established empirical properties of the Pareto distribution in the U.S. – constancy of the exponential index through time and the evolution of the threshold in sync with the cumulative value of the real GDP per capita (Kitov, 2005a, 2005c). Therefore we include the Pareto distribution with empirically determined parameters in our model for the description of the PID above the Pareto threshold. The power law distribution of incomes implies that we do not need to follow each and every individual income as we did in the sub-critical income zone. All we need to know the number of people in the Pareto zone, i.e. the number of people with incomes above the Pareto threshold, as defined by relationships (3) and (4).

The initial dimensionless Pareto threshold is found to be $M_P(t_0)$=0.43 (Kitov, 2005a) and it evolves in time as per capita real GDP:

$$M_P(t)=M_P(t_0)(GDP(t)/GDP(t_0)).$$

When a personal income reaches the Pareto threshold, it undergoes a transformation and obtains a new quality to reach any income with a probability described by the power law distribution. This approach is similar to that applied in the modern natural sciences involving self-organized criticality. Due to the exponential (with a small negative index) character of the growth of income rate the number of people with incomes distributed according to the Pareto law is very sensitive to the threshold value. However, people with high enough $S_i$ and $L_j$ can eventually reach the threshold and obtain an opportunity to get rich, i.e. to occupy a position at the high-income end of the Pareto distribution.

There is a principal feature of the real PID, which is not described by the model so far, but has an inherent relation to the studied problem. The real income distribution spans the range from \$0 to several hundred million dollars, and the theoretical distribution extends only from \$0 to about \$100,000, i.e. the income interval used in (Kitov, 2005a) to match the observed and predicted distributions. The power law distribution starting from the Pareto threshold income (from \$40,000 to \$60,000 during last fifteen years) describes incomes of about ten per cent of the population. The theoretical threshold of 0.43 was introduced above, partly, in order to match this relative number of people distributed by the Pareto law. The model provides an excellent agreement between the real and theoretical distributions below the Pareto threshold. Above the threshold, the theoretical and real distributions diverge.

Above the Pareto threshold, the model distribution drops with an increasing rate to zero at about \$100,000. This limit corresponds to the absence of the theoretical capacity to earn money, $S_iL_j$, above 1.0. The dimensionless units can be converted into actual 2000 dollars by multiplying factor of \$120,000, i.e. one dimensionless unit costs \$120,000. The observed distribution decays above the Pareto threshold inversely proportional to income in the power of ~3.5. Hence, actual and theoretical absolute income intervals are different above the Pareto threshold and retain the same portion of the total population (~10%). Thus, the total amount of money earned by people in the Pareto distribution income zone, i.e. the sum of all personal incomes, differs in the real and theoretical cases.

Here one can introduce a concept distinguishing below-threshold (sub-critical) and above-threshold (supercritical) behavior of the income earners. Using analogs from statistical physical, Yakovenko (2003) associates the sub-critical interval for personal incomes with the Boltzmann-Gibbs law and the extra income in the Pareto zone with the Bose condensate. In the framework of geomechanics as adapted to the modeling of personal income distribution (Kitov, 2005a), one can distinguish between two regimes of tectonic energy release (Rodionov *et al*., 1982) – slow sub-critical dissipation on inhomogeneites of various sizes and fast energy release in earthquakes. The latter process is more efficient in terms of tectonic energy dissipation and the frequency distribution of earthquake sizes also obeys the Pareto power law.

Therefore for personal incomes in the sub-critical zone, the income earned by a person is proportional to her/his efforts or capacity $S_iL_j$. In the super-critical zone, a person can earn any amount of money between the Pareto threshold and the highest possible income. A probability to get a given income drops with income according to the Pareto law. Total amount of money earned in the supercritical zone (or extra income is of 1.33 times larger than the amount that would be earned if incomes were distributed according to the theoretical curve, in which every income is proportional to the capacity. This multiplication factor is very sensitive to the definition of the Pareto threshold. In order to match the theoretical and observed total amount of the money earned in the supercritical zone one has to multiply every theoretical personal income in the zone by a factor of 1.33. This is the last step in equalizing the theoretical and the observed number of people and incomes in both zones: sub- and supercritical. It seems also reasonable to assume that the observed difference in distributions in the zones is reflected by some basic difference in the capability to earn money.

So, the model is finalized. An individual income grows in time according to relationship (3') until some critical age $T_{cr}(t)$. Above $T_{cr}$, an exponential decrease according to (4) is observed. When the income is above the Pareto threshold it gains 33% of its theoretical value (Kitov, 2005b) in order to fit the overall income above the Pareto threshold. Above the Pareto threshold, incomes are distributed according to a power law with an index to be determined empirically. It is obvious that if a personal income has not reached the Pareto threshold before $T_{cr}$, it never reaches the threshold because it starts to exponentially decay. A personal income above the Pareto threshold at critical work experience $T_{cr}$ starts to decrease and can reach the Pareto threshold at some point. Then it loses its extra 33% value.

All people above 15 years of age are divided into 841 groups according to their capacity to earn money. Any new generation has the same distribution of $L_j$ and $S_i$ as the previous one, but different start values of $\Lambda_{min}$ and $\Sigma_{min}$ which evolve with the real GDP per capita. Thus, actual PID depends on the single year of age population distribution. The population age structure is an external parameter evolving according to its own rules. The critical work experience, $T_{cr}(t)$ also grows proportionally to the square root of per capita real GDP. Based on independent measurements of population age distribution and GDP one can model the evolution of the PID below and above the Pareto threshold.

Since the model defines the evolution of all individual incomes in the U.S. economy one can use it for calculation of the Gini coefficient for personal incomes for any given age. At the same time, comparison of predicted and measured Gini coefficients obtained for the PIDs is of importance for the model calibration. For example, the Gini coefficient depends on the Pareto law index, $k$, which is also a key parameter of our model. As shown in Section 2, index $k$ varies with age.

## 2. **Estimates of age-dependent Gini coefficient**

The U.S. Census Bureau published age-dependent PIDs since 1947 (U.S. CB, 2006). Despite high data quality, there are some important caveats in these distributions. The methodology of income measurements and sample size has been varying over time (U.S. CB, 2002). Therefore, one has to bear in mind potential incompatibility of CPS results obtained in different years. Changes in income definitions, sample coverage and routine processing also influence the estimation of various derivatives from PIDs, for example, measures of inequality. Moreover, such changes in procedures and definitions are likely accompanied by some real changes in true PIDs - the latter changes are hardly distinguished from the former ones.

The portion of population with income varies over age and time. These variations affect the estimation of Gini coefficient associated with personal income distribution (Kitov, 2007a). When some new income sources are introduced, more people with lower incomes are usually included in CPS statistics. Therefore, one could expect a slight decline in relevant Gini coefficient with the introduction of new income sources.

In the youngest age group between 16 (15 before 1987) and 24 years of age, only 65% to 80% reported some income, as presented in Figure 1. Corresponding curve has a peak in 1979. Since then, the portion of people with income in this age group has been decreasing. This effect, obviously, needs a thorough consideration and might be induced by the appearance of some new actual sources of income not included in contemporary CPS questionnaires. An increasing level of intra-family income redistribution is a potential mechanism to consider.

Figure 1 demonstrates that the portion of people with income increases with age before reaching its peak and then falls again. In 2005, the largest potion was measured in the group between 55 and 64 years of age and reached 98%. This observation is consistent with the fact that critical work experience, $T_{cr}$, in our model was moved in this age group (see Section 1 for details). Between 1967 and 1977, the curves for all age groups except the youngest one were converging, and after 1979 the scatter of the curves is almost constant. An important observation is the step in all distributions (except the youngest one) between 1977 and 1979. According to the U.S. Census Bureau (2005), this step is related to the introduction of new income definitions and significant changes to the CPS methodology. In average, this step is around 10%. For example, the portion of population with income in the working age population as a whole jumped from 83% in 1977 to 92% in 1979. One can expect that further elaboration of income definition will result in 100% participation in income distribution: there should be no persons without income.

For people of 44 years of age and above, the portion with income is more than 95%. Therefore, in corresponding age groups, the difference between Gini coefficient associated with people having income and that associated with the working age population as a whole has to be the smallest among all age groups. These age groups provide the best opportunity to test our model because almost everybody has some reported income.

Information on each and every individual income is not available. In this situation, Gini coefficient can be estimated approximately. For example, if $(X_i,Y_i)$ are the values obtained from the CPS, with the $X_i$ indexed in increasing order ($X_{i-1} < X_i$ ), where $X_i$ is the cumulated proportion of the population variable, and $Y_i$ is the cumulated proportion of the income variable, then the Lorenz curve can be approximated on each interval as a straight line between consecutive points and

$$G = 1- \Sigma (X_i-X_{i-1})(Y_{i-1} + Y_i),\ i=1,\ldots,n \qquad (5)$$

is the resulting approximation for *G*.

In addition, one can approximate the Lorenz curve between consecutive points $(X_i,Y_i)$ using exponential function or power law (where it is appropriate) for the interpolation of underlying PIDs, as proposed by Dragulesku and Yakovenko (2001). The choice of appropriate function for the PID interpolation reveals an important problem of the CPS - the usage of the same income bins during relatively long periods of time. The growth rate of nominal GDP in the U.S. has been high - more people obtained larger incomes above the predefined upper income limit in CPS questionnaire and found themselves in the group " $MAX and over". So, the coverage of population below and above the Pareto threshold, which has been also proportionally growing, has been changing significantly. This variation in the coverage might potentially result in an increasing or decreasing overall resolution and corresponding bias in the estimations of Gini coefficient. Also, under-coverage of the highest incomes is a potential source of on-going discussion about increasing income inequality in the U.S.. The absence of good quantitative estimates results in a wrong interpretation of income inequality (Kitov, 2007b).

In the income range below the Pareto threshold, one can use quasi-exponential distribution and estimate mean income in relevant bins. In the high-income zone, a power low approximation is a natural choice for the PIDs. Theoretically, the cumulative distribution function, *CDF*, for the Pareto distribution is defined by the following relationship:

$$CDF(x) = 1 - (x_m / x)^k$$

for all $x>x_m$, where $k$ is the Pareto index. Then, the probability density function, *pdf*, is defined as

$$pdf(x) = kx_m^k / x^{k+1} \tag{6}$$

Functional dependence of the probability density function on income allows an exact calculation of total population in any income bin, total and average income in this bin, and the input of the bin to corresponding Gini coefficient because the pdf defines the Lorenz curve. Thus, if populations are counted in some predefined income bins, then relevant Lorenz curve can be constructed for a given value of the Pareto index, $k$. We use (6) in the following calculations of empirical Gini coefficients in the Pareto zone in all age groups. By definition, the Pareto threshold evolves proportionally to nominal GDP per capita, and does not depend on age.

Index $k$, however, depends on age, as Figure 2 demonstrates. The evolution of the Pareto law index (slope) with age is as follows: $k$=-1.91 for the age group between 25 and 34 years; $k$=-1.48 between 35 and 44; $k$=-1.38 between 45 and 54; $k$=-1.14 in the age group between 55 and 64. It is clear that index $k$ declines with age. Obviously, smaller index $k$ corresponds to elevated PID density at higher incomes and larger Gini coefficient. The decrease in $k$ deserves a special study because it should be inherently linked to some age-dependent dynamic processes above the Pareto threshold.

Also, the declining $k$ is a specific feature of the age-dependent PIDs, which should be incorporated in our model. Kitov (2007a) found that $k$=-1.35 for the population of 15 years of age and over, i.e. within the range of its change with age. One can expect, however, that the age-dependent and overall $k$ might also undergo some changes over time. The latter index may vary just because of changing age pyramid, i.e. changing input of various ages to the net $k$. For the empirical estimates of Gini coefficients carried out below, the observed variation in this index plays insignificant role because we use actual income distributions. For the theoretical estimates in Section 3, Gini coefficient might be overestimated for the youngest age group and underestimated in the oldest age group when one uses $k$=-1.35.

The U.S. Census Bureau (2006) presents two versions of PID – for total working age population and for that with some reported income. We have calculated empirical $G$-values in several (fixed) age groups between 1967 and 2005. Figure 3 displays the evolution of Gini coefficient in all groups except in the youngest one. The latter group is characterized by severe variations in methodology and definition of income. This makes it impossible to distinguish actual and artificial features in the evolution of $G$. The curves associated with all people aged in given ranges are marked “all”, and those including only people with reported incomes – “w/income”. The major revision to income definition between 1977 and 1979, which dramatically increased the portion of people with income, induced sharp decrease in the curves named “all”, and opposite changes in the curves “w/income”. For obvious reasons, the Gini coefficients for people with income are systematically lower than those for the entire population. The curves in Figure 3 have to be predicted by our model.

One important feature of the empirical Gini curves was also mentioned in (Kitov, 2007a). Before 1977, the portion of population without income was big enough to introduce a significant bias in the estimates of Gini coefficient. It was overestimated for the entire population and was underestimated for the population with income. Same effect is observed for the age-dependent Gini. Before 1977, one can observe large changes over time. After 1977, all curves are approximately horizontal, with only a slight decline. Hence, one can expect large deviation between these empirical curves and theoretical ones before 1977.

The accuracy of theoretical estimates of Gini coefficient is related to the quality of PIDs' prediction. Figure 3 indicates that the Gini coefficients for the age groups over 34 years vary in a narrow range. This observation presumes that underlying PIDs are very similar.

Kitov (2007a) demonstrated that the PIDs for the entire working age population (with income) for the years between 1967 and 2005 collapse practically to one curve when normalized to populations and nominal GDI (instead of GDP). Real GDP drives two key parameters in our model: critical work experience, $T_{cr}$, and the size of earning tools, $\Lambda(t)$. However, when GDI is not equal GDP (as assumed in the model) one should use the former variable for the normalization of the PIDs. GPI/GDP ratio has been varying through time since the start of the CPS. Introduction of new sources of income in the CPS questionnaires results in some increase in the GPI in addition to true changes in true GPI. The portion of GPI (BEA, 2008) in the U.S. GDP increased from 0.76 in 1951 to 0.86 in 2001. A drop in the portion was observed between 2001 and 2005 – from 0.86 to 0.82.

Figure 4 presents the evolution of various measures of mean income (i.e. GPI per capita) using: GDP; GPI reported by the BEA; and GPI reported by the Census Bureau, as estimated in annual CPS. Two population estimates are used for calculations of these mean values – total working age population (all) and people reporting income (with income). According to current income definitions, the GPI reported by the BEA is larger than that estimated by the CB because the former includes additional sources of income. By mistake, the BEA's GPI was used in the modeling of the overall PID (Kitov, 2007a).

In the case of age-dependent PIDs, one should separately estimate total personal income in each age group. Similar to Figure 4, in Figure 5 we present the evolution of mean personal income in various age groups. There are two cases shown: for all people of given age (including those with no income) and for people who reported income. (By definition, mean personal income is the ratio of total personal income and relevant population.) One can observe dramatic differences between the youngest people and those in the groups with the largest mean income. The curves obtained from the estimates provided by the Census Bureau are used to normalize the age dependent PIDs.

Before normalizing the age-dependent PIDs to total income and population one needs to reduce them to the same units. Originally, the PIDs are obtained in income bins of different width. For example, Figure 6a displays the PIDs for the age group between 35 and 44 years in 1967 and 2005. Income bins are not uniform in 1967 creating local troughs and peaks. In 2005, income bins are uniform between \$0 and \$100,000. Obviously, the number of people in a given bin depends on its width and position in the distribution. A reasonable way to reduce these inhomogeneous distributions to the same units is to divide the number of people in a given bin by its width. This mathematical operation defines population density, i.e. the number of people per \$1 at given income level. For the sake of simplicity, we assign the obtained readings of density to the centers of relevant bins. Figure 6b depicts the PIDs (shown in Figure 6a) normalized to the width of relevant income bins. The troughs and peaks are essentially smoothed in the density curves. It is likely that true population density distribution can be represented by an exponent undergoing a smooth transformation into a power law function near the Pareto threshold (Yakovenko, 2003).

Finally, we have population density curves, which are defined in the same units. To reduce them to one scale we normalize them to total population in relevant age intervals and to the increase in total income over years, as presented in Figure 5. We expect that the normalized curves should collapse to one within the bounds of uncertainty related to measurement errors. Figure 7 shows the normalized PIDs in various age groups for people for years 1967, 1993, and 2005. There is no significant difference between the curves except in the age group between 16 and 24 years of age. We exclude the latter age group from the modeling due to very high uncertainty in income measurements. The overall PIDs are also shown as a corrigendum to those in (Kitov, 2007a).

This is the similarity between the normalized PIDs that results in practically constant Gini coefficients in given age groups between 1967 and 2005. On the other hand, this similarity supports our basic assumption that relative distribution of personal income has not been changing over time not only overall, as shown by Kitov (2007a), but also for any given age. One can conclude that there exist internal (economic, social, etc.) forces, which return personal income distribution to its fixed shape. In other words, PID is an invariant in the

U.S. economy. This is an observation, not an assumption. To obtain theoretical estimates of age-deponent Gini coefficients we start with the modeling of corresponding PIDs.

## 3. Comparison of observed and predicted Gini coefficients

Following our analysis of the observed age-dependent PIDs in Section 2 we have predicted PIDs in the same age groups. The model is characterized by a resolution of 1 year of age and 1 year in calendar time. Therefore, we aggregated all personal incomes in the age groups predefined by the CPS. The start year of the model is 1967 with the following defining parameters: $\alpha_0$=0.071; $T_{cr}(1967)$=32.0 years; $M_P(1967)$=0.43. Index *k* is taken for given age group from the empirical estimates in Section 2. Other parameters are the same as in (Kitov, 2007a) and described in Section 2. The age distribution of population was obtained from the Census Bureau (2007). This distribution is prone to revisions, however. For selected age groups, such revisions may reach several percentage points. This might result in slight deviations in the predicted Gini coefficients.

Figure 8 displays predicted and observed PIDs for the age groups between 24 and 35 years of age, between 45 and 54 years of age, and for the whole population over 15 years of age. For the narrow age groups, the PIDs measured in 1993 were chosen, and for the whole working age population the year of 2005 was modeled. Corresponding indices are those estimated empirically and are as follows: *k*=-1.91; *k*=-1.38, and *k*=-1.35. These values fit the roll-off in relevant PIDs' above the Pareto threshold. In the low-income zone, the best fit is observed for the whole population. This is likely the result of better resolution in the entire population curve al lower incomes in 2005. In 1993, the resolution at low incomes was poor. This was one of the reasons for new questionnaire and methodology introduced in 1994. The number of income bins underwent a dramatic increase from 23 (including the open-end one for the highest incomes) to 42. All in all, the observed and modeled PIDs demonstrate very good similarity, which should be reflected in Gini coefficients.

The model predicts the evolution of each and every personal income. Therefore, it allows the prediction of an exact Gini coefficient for a given set of defining parameters because the construction of the Lorenz curve is possible. The empirical Gini coefficients were obtained separately for the whole working age population and for people with income.

These empirical coefficients provide only some estimates of the range, within which true age-dependent Gini coefficients are likely to reside.

Figure 9 presents the evolution of the observed and predicted Gini coefficient in four age groups and for the whole population over 16 years of age. For the sake of simplicity, we predicted Gini using the same index $k$=-1.35. In the age group between 25 and 34 years of age, the predicted curve is close to that obtained for the entire population in this group. Because actual index is $k$=-1.91, there is a slight overestimation of the predicted coefficient, but it still resides between the empirical curves. Since 1994, the predicted curve has been deviating from the curve for the whole population and approaching that for population with income. This might be an effect of a higher resolution related to the introduction of new income bins.

In the age group between 35 and 44 years of age, the empirical curves are closer to each other. The predicted curve stays between them, but much close to the curve for population with income. In the age group between 45 and 54, where theoretical index $k$ is close to actual one, the predicted curve reproduces the decline in both empirical curves observed after 1983 and lays much close to the empirical curve for population with income. This is likely that the true Gini coefficient in this age group is consistent with the predicted one. In the age group between 55 and 64, the predicted curve is also close to that for the working age population, but still between the empirical curves. As expected, the level of income inequality in this age group is larger than in any other age group.

It is worth noting that the gap between the empirical Gini curves is between 0.02 and 0.03. The gap between the predicted and empirical curves is usually less than 0.01. This is less than the uncertainty of the estimation of Gini coefficient as related to the discrete representation of the observed PIDs.

In all age groups, the level of personal income inequality, as expressed by Gini coefficient, has been decreasing (with small local peaks) since 1967. This empirical and theoretical observation is especially important for the age groups above 45 years, where the portion of population with income is close to 100%.

The predicted and measured curves demonstrate that the true Gini coefficient is age dependent. Figure 10 displays two empirical curves of Gini dependence on age obtained in this study for 1967 and 2005, two theoretical curves predicted by our model, and a curve reported by the Census Bureau. Due low resolution large measurement errors in the youngest and oldest age groups we limit our study to the age between 25 and 65. In this range, all curves are very close.  However, the CB's curve goes beyond the limits and demonstrates that there is a turning point at the age between 65 and 70. Our model supports this observation and the average income for people above the critical age, $T_{cr}$, rolls-off exponentially with age. This fast decay is also reflected in a severe drop in the number of people with income above the Pareto threshold and corresponding decrease in Gini coefficient.

## Conclusion

This study was primarily carried out for validation of the microeconomic model defining the evolution of personal incomes in the U.S. Our previous paper (Kitov, 2007a) revealed some problems with income definition, which did not allow a comprehensive description of the overall PID. The most important problem was that a large portion of population did not report any income. Another problem is a poor resolution of PIDs before 1977.

In the model, everyone is assigned a non-zero income. This discrepancy results in a significant deviation between observed and predicted Gini coefficients. The age-dependent PIDs allow overcoming this discrepancy because the portion of population without income is very low (~2%) for ages over 45 years. Therefore, one could assume a more precise prediction of Gini coefficient in these age groups. This paper confirms this assumption: the evolution of Gini coefficient for the years with a good PID resolution was accurately (<0.005) predicted.

I would not like to repeat here all conclusions from (Kitov, 2007a). They are all right and validated in this paper. In order to unbiased estimates of (also age-dependent) Gini coefficient, one needs precise definition of personal income, i.e. the definition under which GPI equals GDP, higher and uniform resolution, and accurate measurements.

As expected, the gap between the Gini coefficient associated with the entire working population in a given age interval and that associated with people reporting income converge with the decreasing portion of people without income. The true Gini coefficient had to be somewhere between these two estimates. In the group between 45 and 54 years of age, this portion is approximately 3% and the gap is less than 0.02.

As the overall PIDs, the empirical age-dependent (density) PIDs collapse to practically one curve when normalized to cumulative growth in personal income and total population for the period between 1967 and 2005.

In all age groups, the model predicts slightly decreasing Gini coefficients between 1967 and 2005. The overall $G$ is approximately constant, however. Minor changes in the overall $G$ are related to economic growth and changes in the age structure of American population.

The Pareto law index, $k$, undergoes significant changes over age: increases from the youngest age to approximately 67 years of age, and then drops. Such an evolution could be expected but its actual behavior deserves a deeper study.

The age-dependent PIDs demonstrate a fixed hierarchy during a very long period between 1967 and 2005. It is unlikely that this hierarchy will be destroyed in near future. The shape and the evolution of the measured PIDs are well predicted for the whole period. This allows exact prediction of Gini coefficient and other measures of inequality, which are defined by personal income distribution. Therefore, these measures of income inequality are only of secondary importance.

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**Figures**

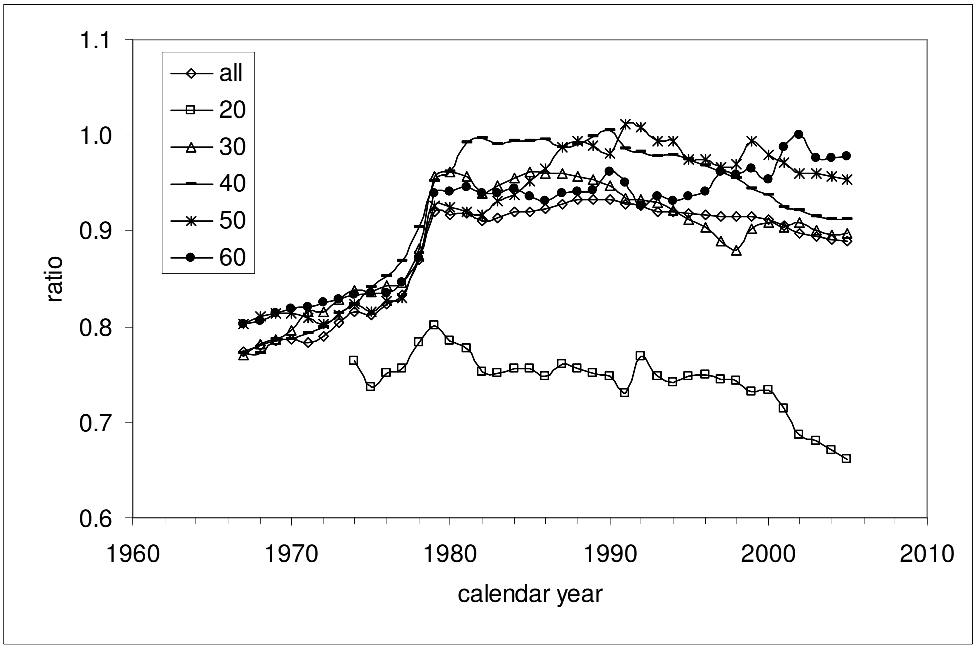



Figure 1. Evolution of the portion of population with income in various age groups: all – above 15 years of age, 20 – from 16 to 24 years of age, 30 – from 25 to 34 years, 40 – from 35 to 44 years, 50 – from 45 to 54 years, 60 – from 55 to 64 years. In the group between 16 and 24 years of age, the portion has been falling since 1979. Notice the break in the distributions between 1977 and 1979 induced by large revisions implemented in 1980 – "Questionnaire expanded to show 27 possible values from 51 possible sources of income."

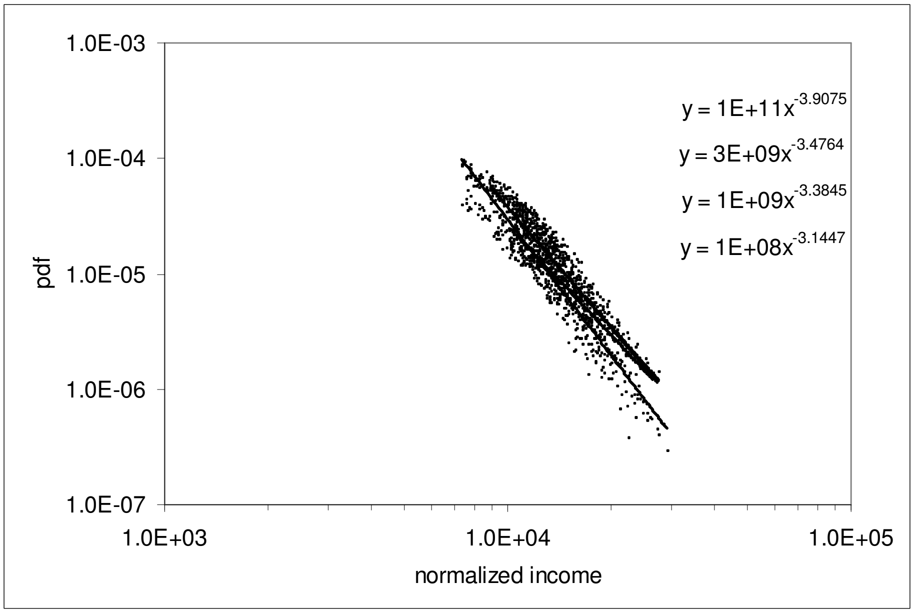



Figure 2. Evolution of the Pareto law index (slope) with age: *k*=-1.91 for the age group between 25 and 34 years, *k*=-1.48 between 35 and 44, *k*=-1.38 between 45 and 54, and *k*=-1.14 in the age group between 55 and 64.

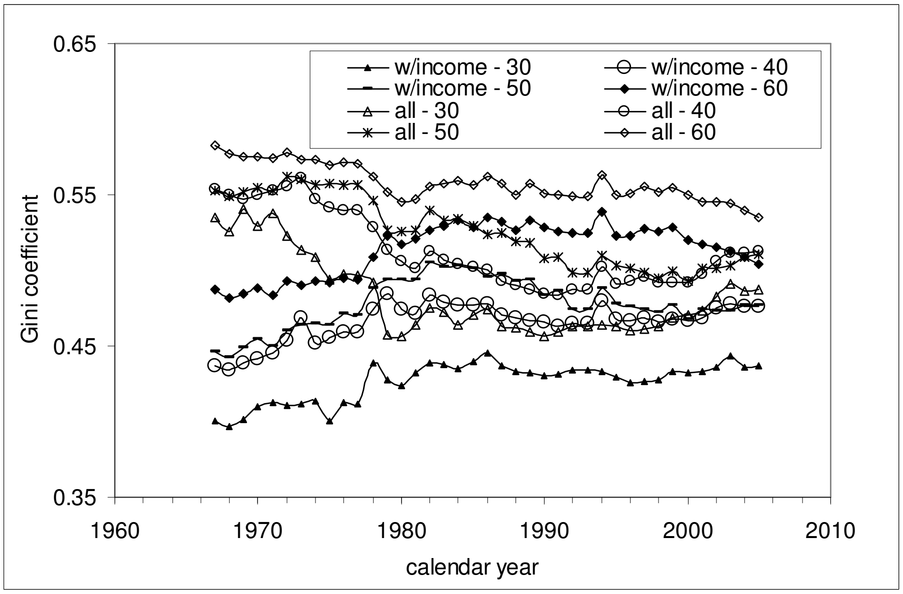



Figure 3. Evolution of estimated Gini coefficient for personal incomes in various age groups between 1967 and 2005. There are two versions in each age group - first includes all people aged in given range (all), and second includes only those with nonzero income (w/income). There was a large revision of income definition between 1977 and 1979, which dramatically changed the portion of people with income and induced sharp decrease in the curves named "all". Obviously, the Gini coefficients for people with income are systematically lower than those including all population with given ages.

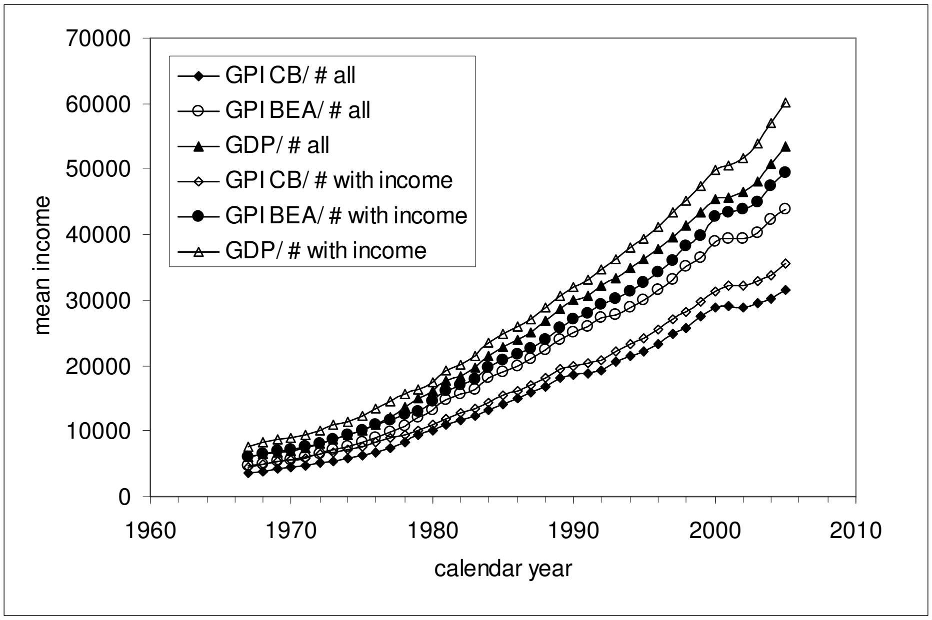
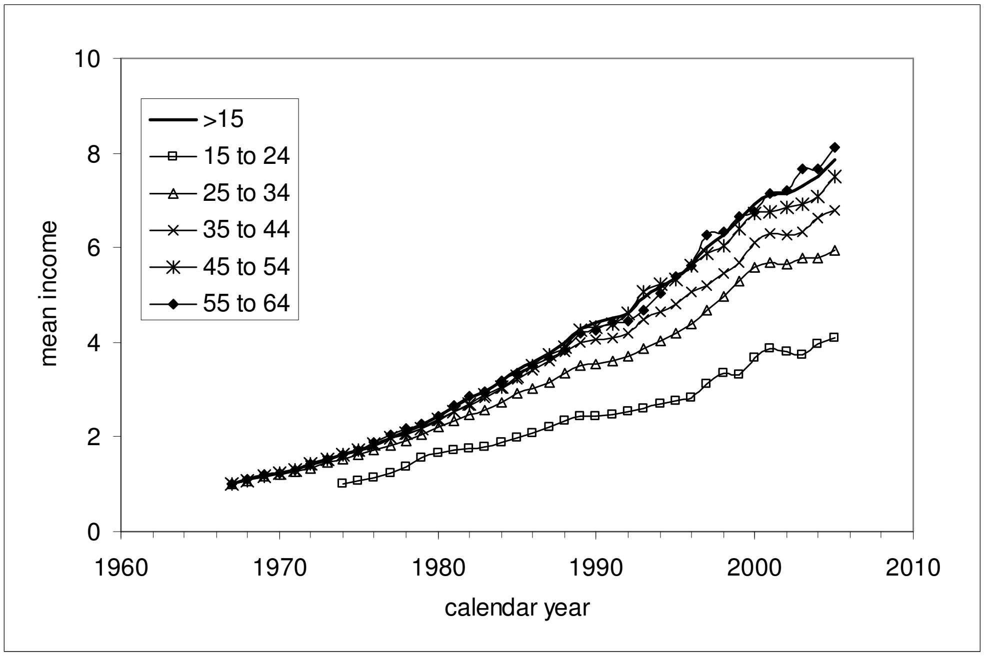



Figure 4. Evolution of various measures of the overall mean income: using GDP; GPI reported by the BEA; and GPI reported by the Census Bureau as estimated in annual CPS. Two population estimates are used for calculations of the mean values – total working age population (all) and people reporting income (with income). According to current income definitions the GPI-BEA is larger than the GPI-CB because the former includes additional sources of income.

a)




b)

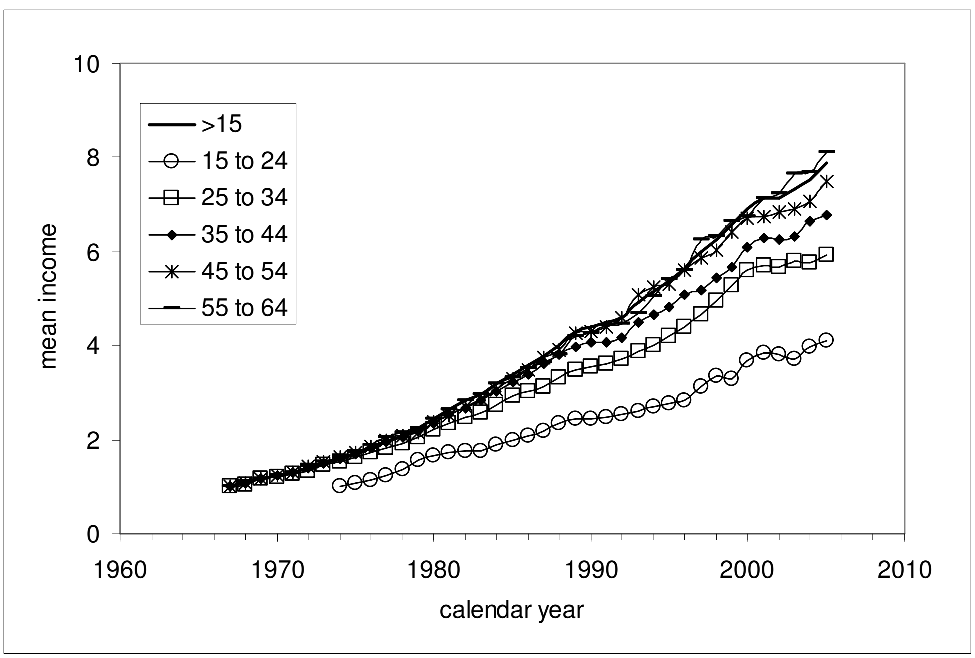
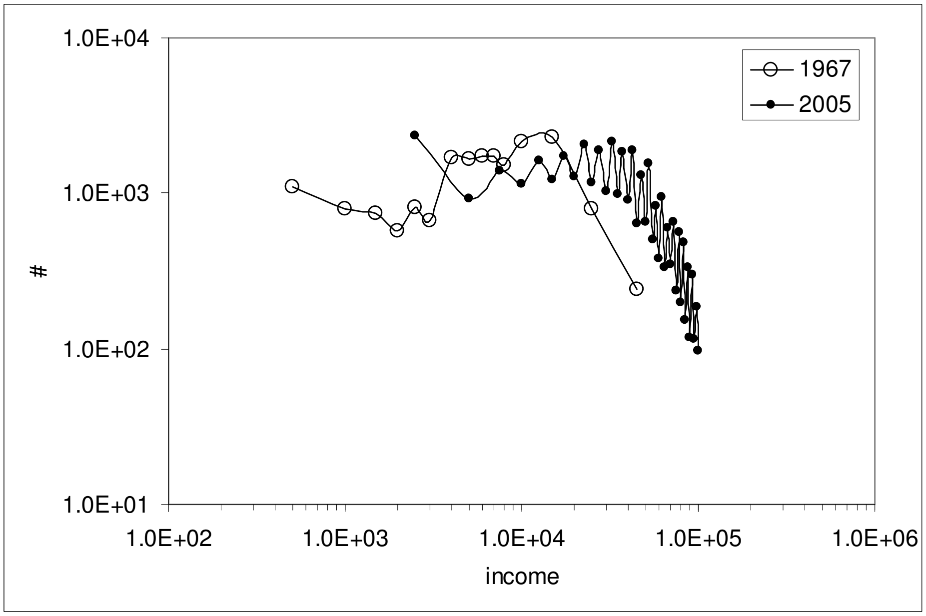



Figure 5. Evolution of mean income (normalized to that in 1967) in various age groups as estimated using: a) total working age population; b) only people with income. The curves are used to normalize corresponding PIDs.

a)




b)

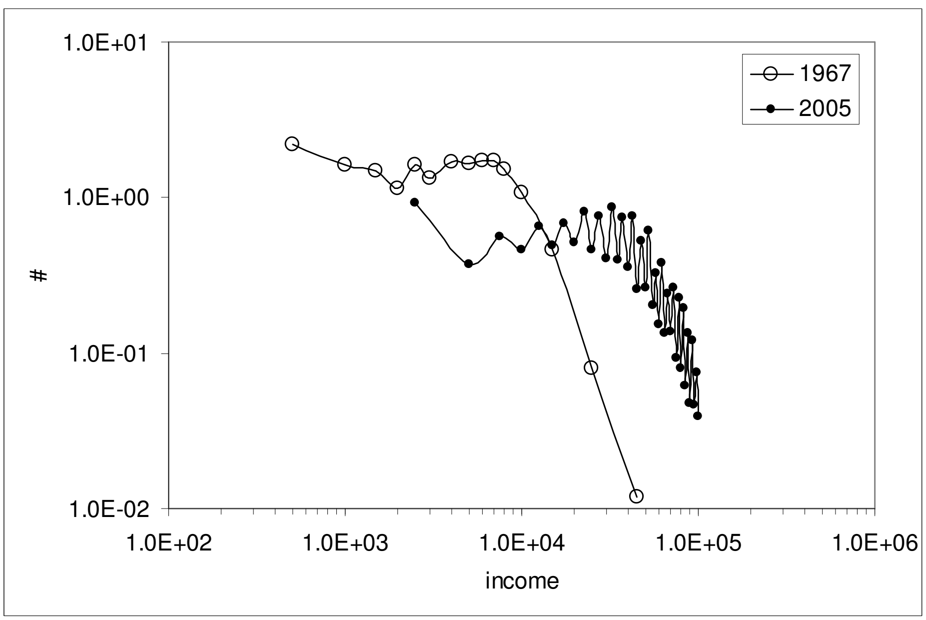
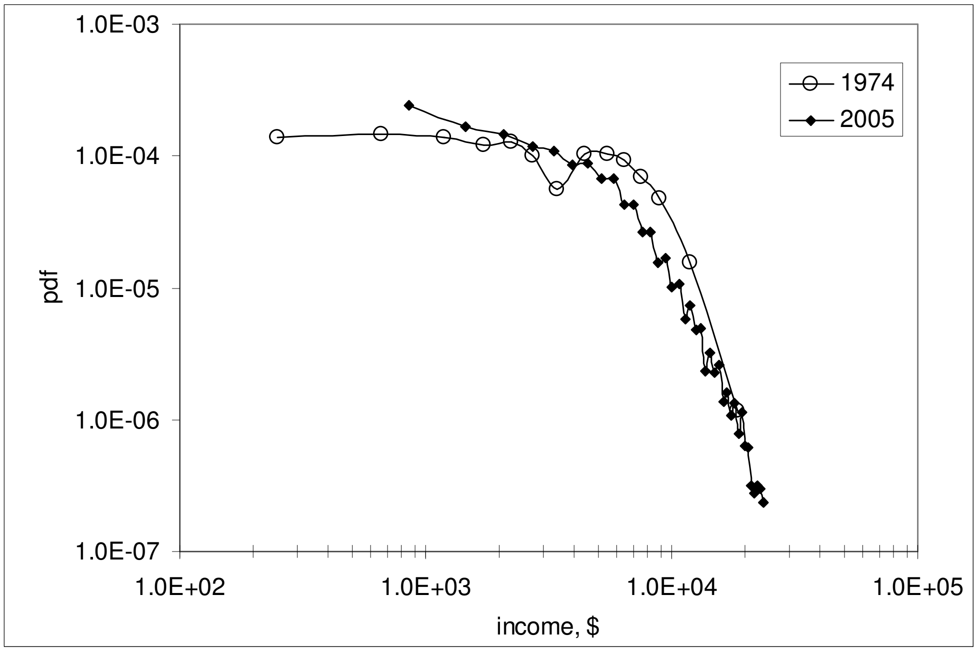
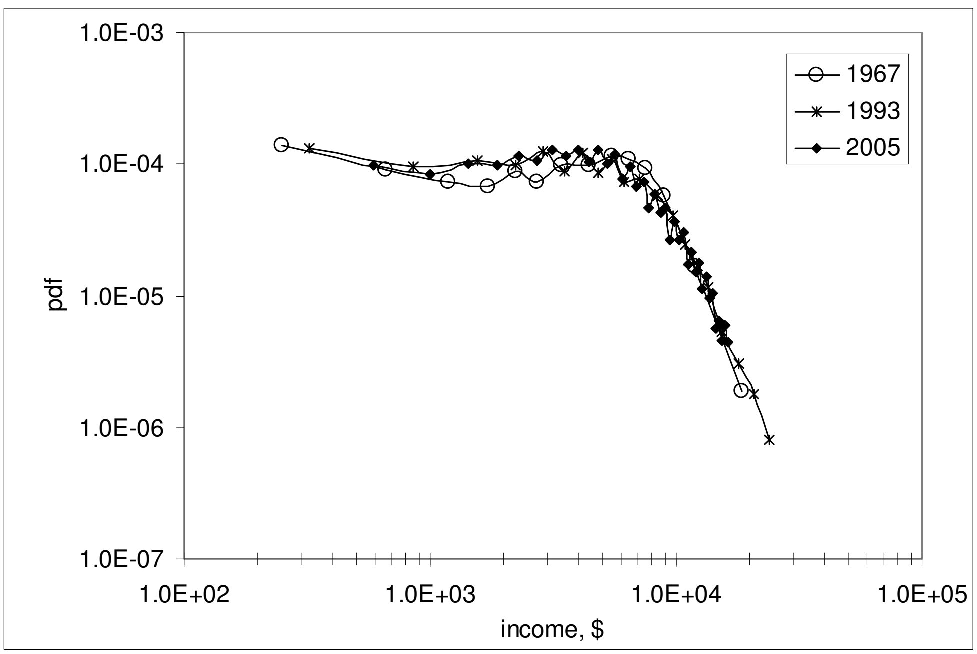



Figure 6. Comparison of a) original; and b) normalized personal income distribution (in current dollars) in the age group between 35 and 44 years in 1967 and 2005. Original distributions published by the U.S. Census Bureau are normalized to the width of relevant income bins in order to obtain population density distribution. Income bins are not uniform in 1967 creating local troughs and peaks. In 2005, income bins are uniform between $0 and $100,000. Three $50,000-wide bins above $100,000 are not shown. More people and larger GPI in 2005 are observed.

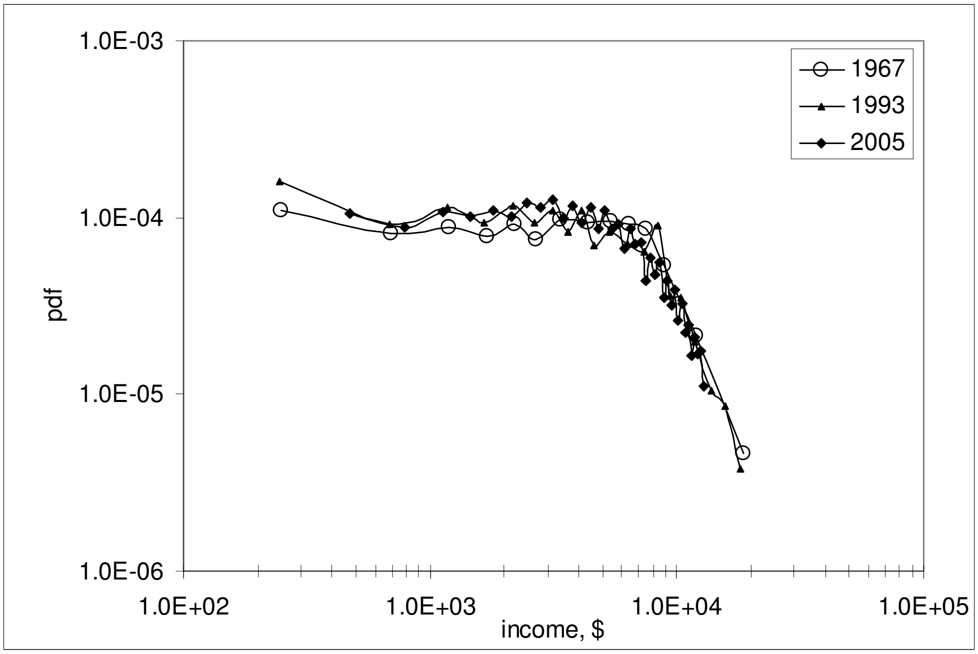
a) 16 to 24



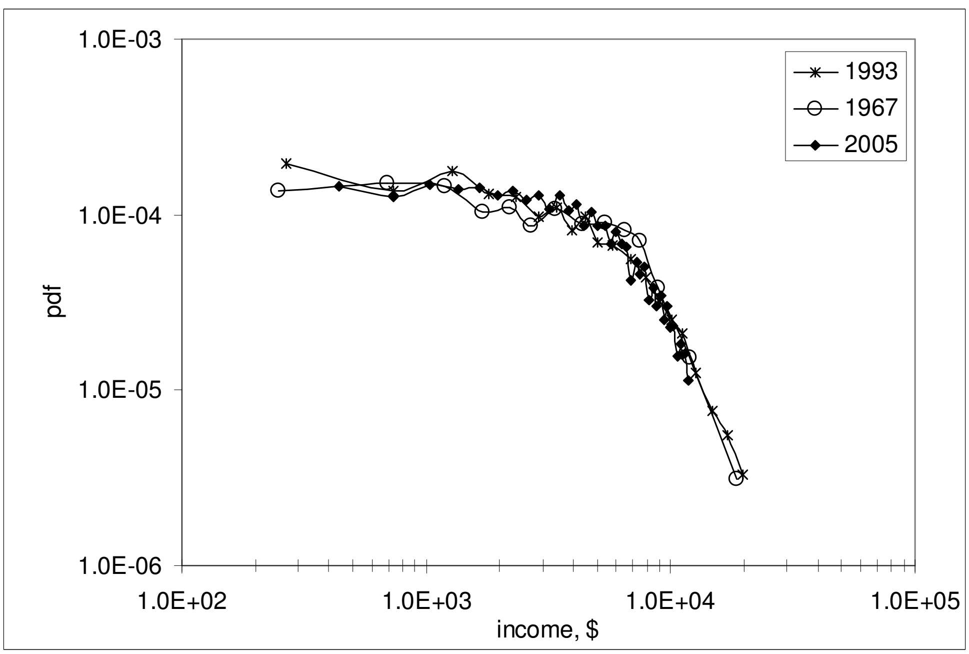

b) 25 to 34



c) 45 to 54




d) 55 to 64



e) 16 years of age and over

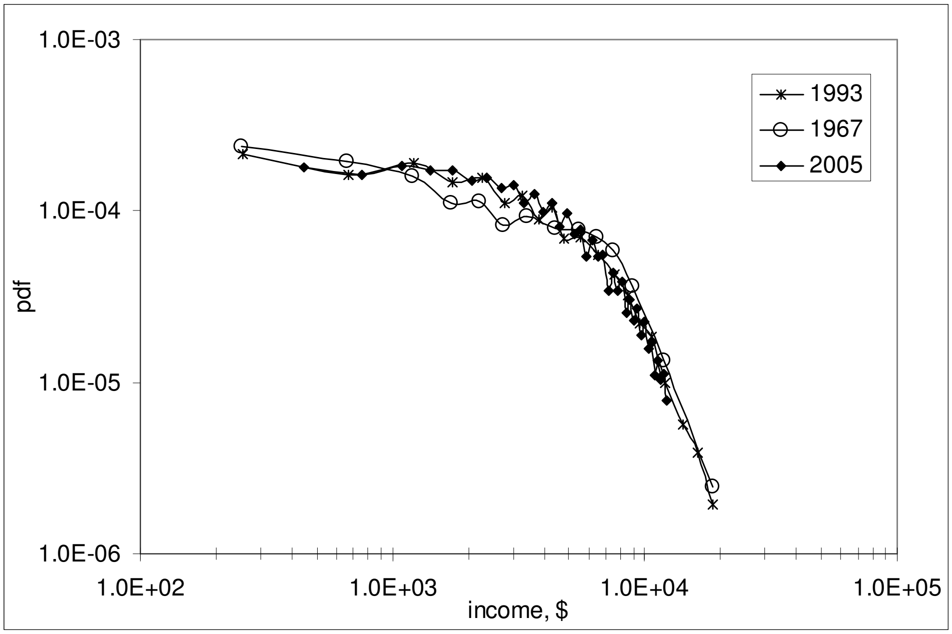
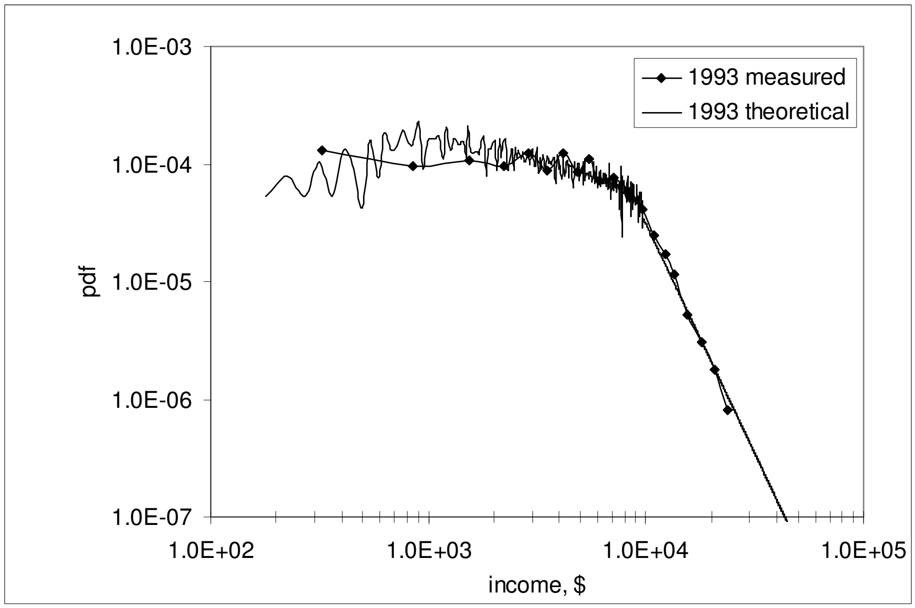
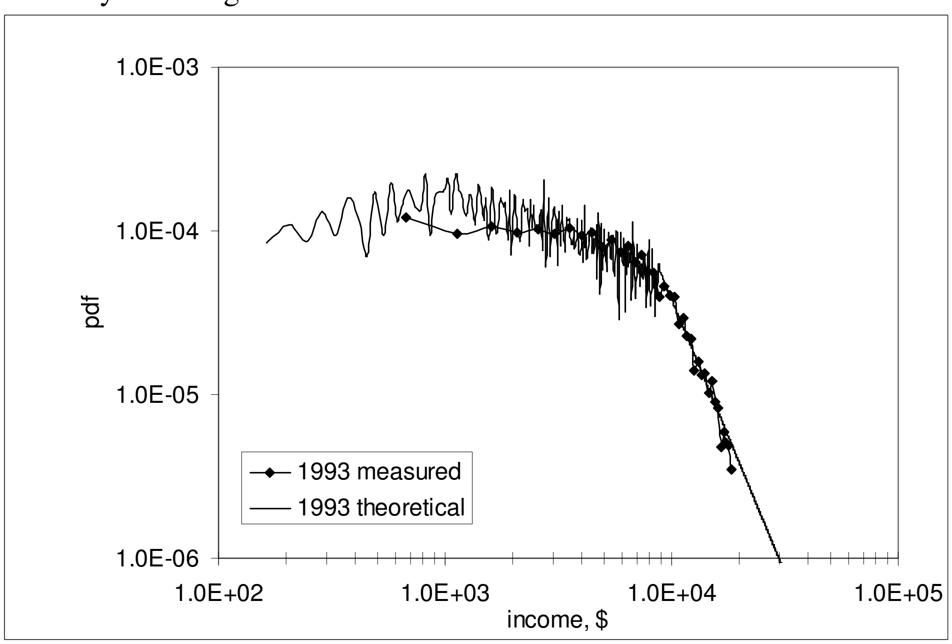



Figure 7. PIDs in various age groups for people with income normalized to the increase in total income and total population in given group. Years 1967, 1993, and 2005 are presented. There is no significant difference between the curves except in the age group between (15) 16 and 24 years of age. We exclude the latter age group from the modeling due to very high uncertainty in income measurements.

a) 25 to 34 years of age




b) from 45 to 54 years of age



c) over 16 years of age

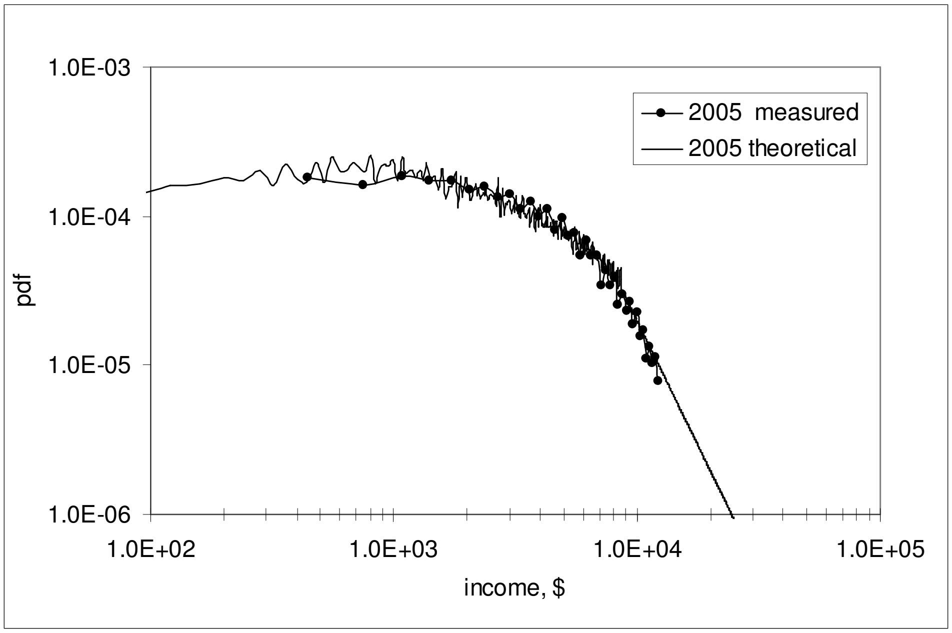
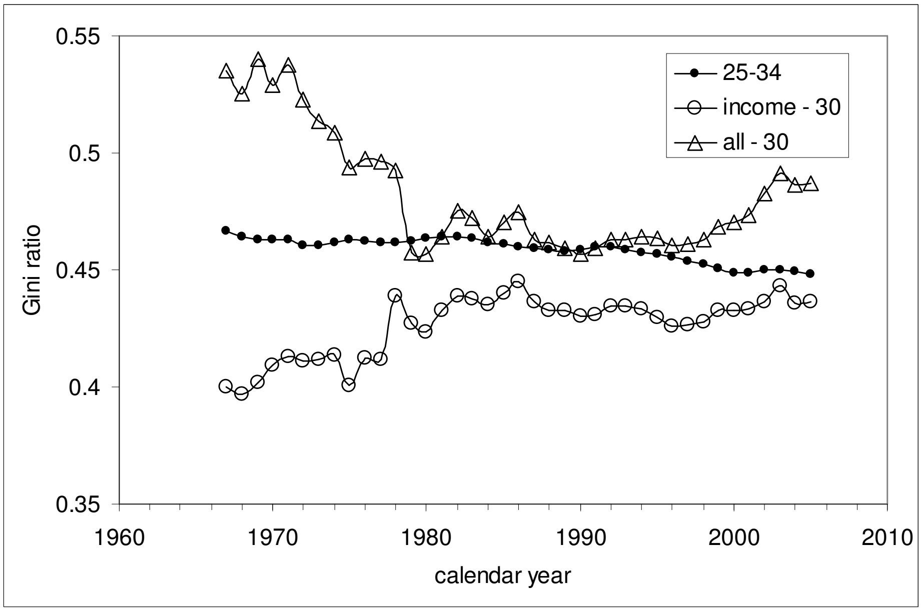
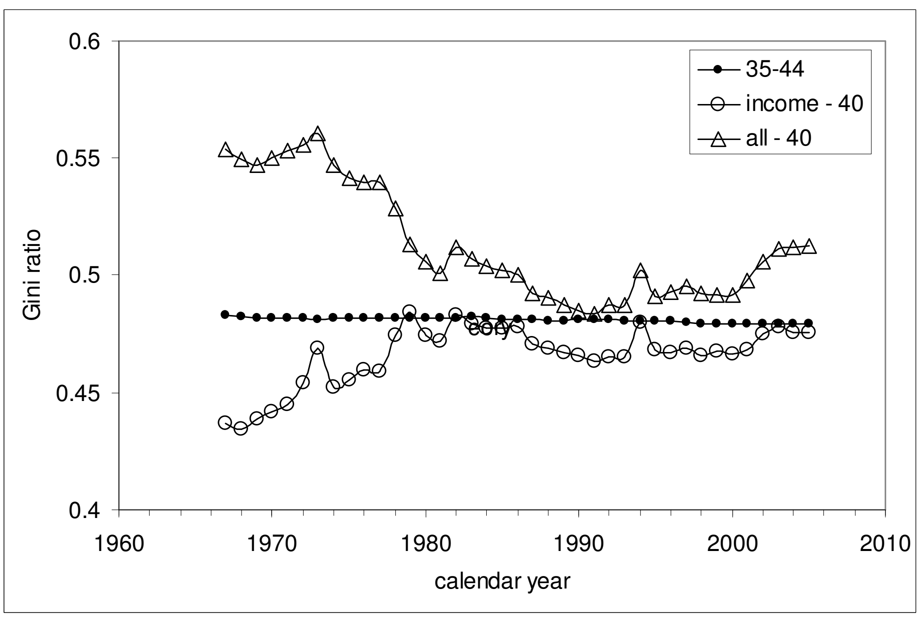



Figure 8. Comparison of measured and predicted PIDs in some age groups. High incomes are describes by a power (Pareto) law with index *k*=-1.91; *k*=-1.38; and *k*=-1.35, respectively.

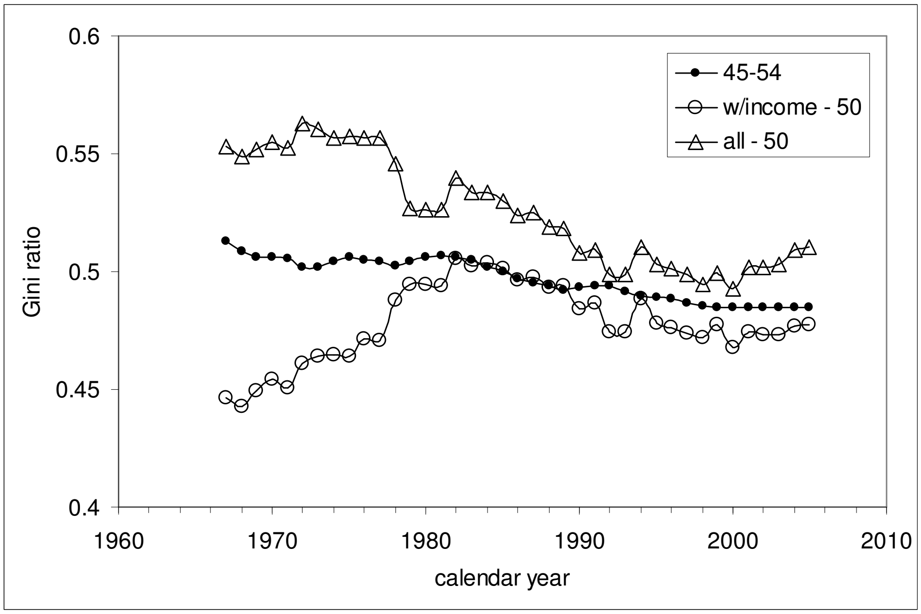
a)


25-34
income - 30
all - 30
Gini ratio
0.55
0.5
0.45
0.4
0.35
1960
1970
1980
1990
2000
2010
calendar year

b)

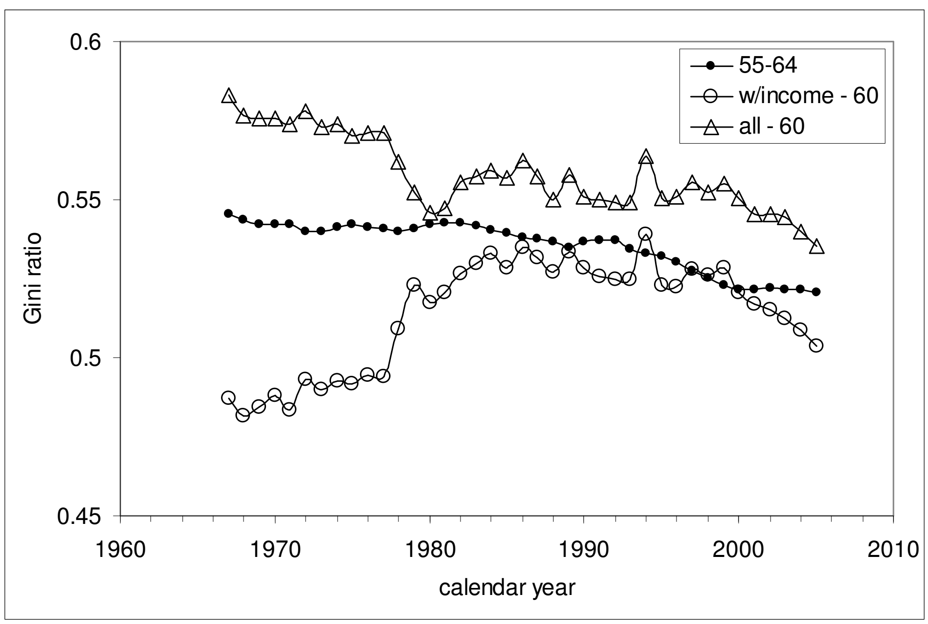

35-44
income - 40
all - 40
Gini ratio
0.6
0.55
0.5
0.45
0.4
1960
1970
1980
1990
2000
2010
calendar year

c)



0.6
0.55
0.5
0.45
0.4
Gini ratio
1960
1970
1980
1990
2000
2010
calendar year
45-54
w/income - 50
all - 50


d)



0.6
0.55
0.5
0.45
Gini ratio
1960
1970
1980
1990
2000
2010
calendar year
55-64
w/income - 60
all - 60

e)

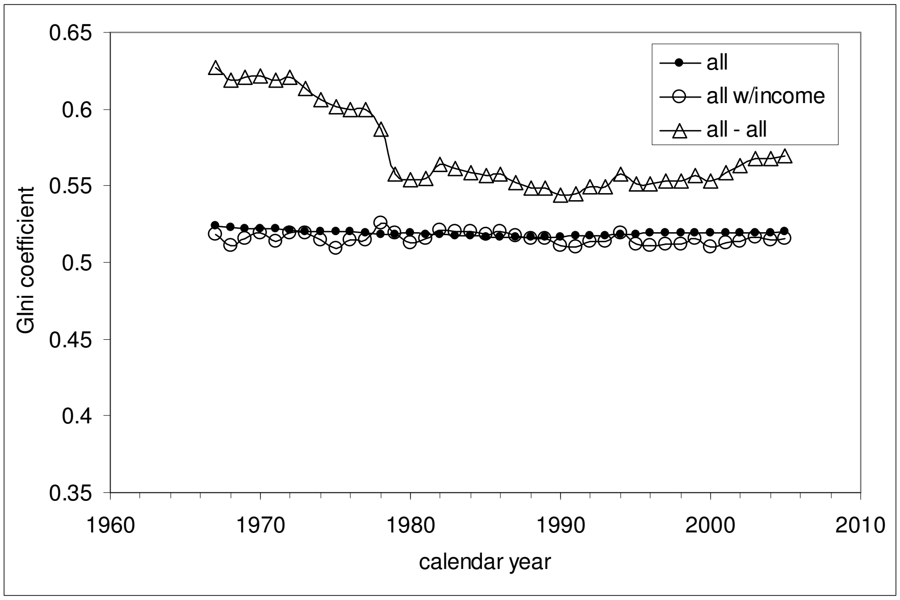



Figure 9. Comparison of predicted and empirical Gini coefficient in various age groups for the period between 1967 and 2005. In all cases *k*=-1.35.

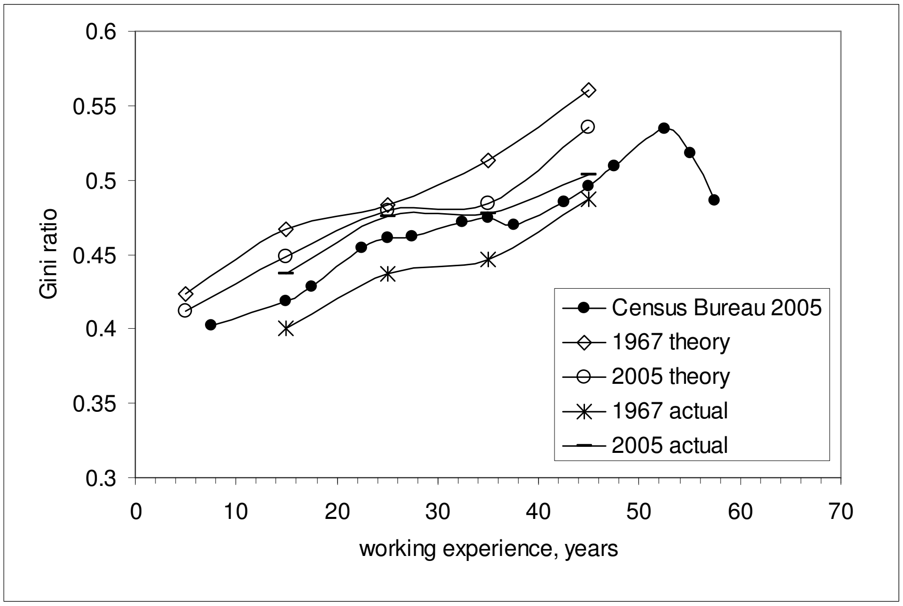



Figure 10. Comparison of Gini coefficient dependence on age, as estimated by the U.S. Census Bureau and in this study from personal income distributions in 1967 and 2005 (curves marked – actual). The Gini coefficients predicted by our model for 1967 and 2005 are also shown.