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Abstract

The difficulties of the classical and Marxian labour theory of value are overcome when labour is measured in terms of marginal labour value. Marginal labour value is the inverse of the marginal productivity of labour. Relative prices are equal to the ratio of marginal labour values. This article presents the marginal approach to the labour theory of value.

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1. Introduction

It is a widespread belief that the labour theory of value has to be abandoned due to numerous inconsistencies. Those economists who still adhere to it are typically regarded as 'heretics'. In the history of economic thought the abandonment has been interpreted as having been executed by the marginalist (counter) revolutionaries. Marginal analysis is presented as incompatible with the labour theory of value. Closer reading of Jevons (1871, p. 167 ff.) shows that this interpretation of the Marginal Revolution is wrong. Also Schumpeter who declared the labour theory of value to be dead and buried in a footnote points out that it would be easy to resurrect it (Schumpeter, 1943, p.24).

Of course the abandonment of the labour theory of value is wanted to avoid the terrible accusation of the exploitation of the labourers. But could orthodox economic theory really be so powerful in guiding the organisation of society without having a labour theoretical foundation? The answer is clearly: no. So let's ignore the apologetic lip service of orthodox economists and instead let's investigate what they do!

2. Labour Values are Marginal Labour Values

A basic proposition concerning the optimal allocation of resources is that the value of the marginal product of labour must be equal in all its employments and under perfect competition this is equal to the uniform wage rate.

Assume a socialist economy with a planning authority. The planners, in order to maximize the value of production have to allocate a marginal increment of a factor of production to that employment which yields the highest value of incremental output. By this the value of the marginal product becomes equal in all employments.

In a market economy (capitalist or socialist) the allocation of the factors of production is performed by the individual firms. One result of microeconomic analysis is that a profit maximizing or cost minimizing firm under conditions of perfect competition equalizes the value of the marginal product of an input to the price (more precise the rental) of the input (Appendix I). The argument implies production functions exhibiting positive but decreasing marginal productivities in all sectors. We apply this to labour which is assumed to be homogeneous.

We have an economy divided in sectors. Each sector produces one type of commodity as output. The wage rate is equal to the value of the marginal product of labour.

$$w = p_i \, \partial x_i / \partial L, \; for \; i = 1, 2, \ldots, n-1 \qquad (1)$$

where $\boldsymbol{w}$ – uniform wage rate, $\boldsymbol{p_i}$ – price of commodity $\boldsymbol{i}$, $\boldsymbol{x_i}$ – quantity of commodity $\boldsymbol{i}$, $\boldsymbol{L}$ – labour

In equilibrium, under conditions of perfect competition, the value of some quantity $\boldsymbol{x}$ of commodity $\boldsymbol{i}$ in terms of labour value is obtained by dividing the money value by the wage rate w, that is the value of the marginal product of labour:

$$\frac{p_i x_i}{p_i \, \partial x_i / \partial L} = \frac{x_i}{\partial x_i / \partial L} = \partial L / \partial x_i \; x_i \qquad (2)$$

The result yields marginal labour units which stand in contrast to the classical concept of average labour units

$$L / x_i \; x_i \qquad (3)$$

One should regard marginal labour values as socially necessary labour and average labour value as the value of the labour force.

Both concepts coincide if there are no capitalistic means of production as in the case of simple commodity production and/or the rate of profit is zero, that is there is no surplus labour.

The precise relationship between the classical concept of average labour values and marginal labour values is as follows:

$$\frac{\delta L}{\delta x_i} = \frac{L / x_i}{a_i} \qquad , where \quad a_i = \frac{\delta x_i}{\delta L} \frac{L}{x_i} \qquad (3a)$$

a_i is the output elasticity of labour. (In the case of the Cobb-Douglas production function it is a constant.)

One might observe that we have divided the money value of some quantity of a commodity by the wage rate ($w = pi\ \delta x_i/\delta L$). The result is commonly known as 'labour commanded'. Here it should be clear that marginal analysis overcomes the discrepancy between 'labour commanded' and 'labour embodied' as the result $\delta L/\delta x_i$ is not only 'labour commanded' but is a pure technical term of production representing marginal labour values which is embodied labour.

Relative prices can be obtained by equating 2 values of the marginal product. Taking 2 commodities it is:

$$w = p_1 \partial x_1 / \partial L = p_2 \partial x_2 / \partial L \qquad (4)$$

This can also be written in terms of marginal labour values as

$$w = \frac{p_1}{\partial L / x_1} = \frac{p_2}{\partial L / \partial x_2} \qquad (5)$$

Prices are proportional to labour values. From this follows that

$$\frac{p_1}{p_2} = \frac{\partial L / \partial x_1}{\partial L / \partial x_2} \qquad (6)$$

Relative prices are equal to the ratio of marginal labour values.

The difficulty of determining the amount of labour embodied in a quantity of some commodity under capitalistic production conditions comes about as many commodities enter into the production that are themselves the product of labour and in addition the productivity of direct labour is increased by this indirect labour. Marginal analysis resolves precisely this problem. One measures the marginal productivity of labour as the marginal increase of output as a consequence of a marginal increase of labour. The inverse of this marginal productivity that is the marginal labour value identifies how much labour is embodied in the increment of output. The multiplication of this amount of labour with the total quantity of output yields the total amount of embodied labour. This amount of labour is clearly greater than that part of the directly applied labour force that is paid labour, or in Marxian terminology variable capital. It contains also the indirect labour of the other inputs which is transferred to the product as well as surplus labour as the result of increased productivity. The effect of the use of capital goods in production is to increase the productivity of the labour force employed and by that it reduces the socially necessary labour, marginal labour value. But the use of capital goods is costly. More capital goods are used as long as the cost of using it is smaller or equal to the savings made by reducing the labour force required to produce a given output. As long as capitalism is progressive there will be no unemployment and the redundant labour force will be employed elsewhere or in the same industry increasing output and reducing the price of the product. If capitalism becomes regressive unemployment comes about through monopolistic practices and prices are kept high.

3. Exploitation

We can distinguish the 3 forms of value a commodity consists of: paid labour ***L***, surplus labour ***S*** and indirect, stored-up labour ***K***. This corresponds to the Marxian concepts of variable capital, surplus labour and constant capital. But we should realize at this point that our definition of labour is not some average labour hour but is more general as it also takes into account the capital intensity of labour. Labour hours are weighted according to their efficiency. The weight is the inverse of the output elasticity of labour.

Total value ***V*** of a commodity ***i*** is

$$V = \partial L / \partial x_i \; x_i \tag{7}$$

The reader should check that the dimension of **V** is labour hours.

The value of constant capital in terms of labour value is

$$K = \sum \partial L / \partial x_j \, x_j \tag{8}$$

where $\boldsymbol{x_j}$ is the quantity of the means of production **j**

Surplus labour is the difference between total value and the sum of paid labour and the value of the means of production.

$$S = V - (L + K) = \partial L / \partial x_i \, x_i - \left(L + \sum \partial L / \partial x_j \, x_j\right) \tag{9}$$

This surplus value represents labour value the labourer has performed by his work but he does not receive it as wages. The capitalist owner of the means of production is able to appropriate this surplus as it comes about through the application of his means of production in the working progress. We see here that this exploitation can be overcome if the labourers own the means of production.

Capitalist competition assures that the rate of profit $\boldsymbol{r}$ is the same in all industries. We assume that wages are paid at the end of the production period. Then $\boldsymbol{r}$ is defined as

$$r = S/K \tag{10}$$

We should observe here that there exists a technical relationship between the rate of profit and the marginal productivity of labour which depends only on the capital labour ratio **k**. When the rate of profit changes, so does the marginal productivity of labour and its inverse, marginal labour values and therefore the price changes too.

Here we have the labour theory of value reformulated along marginalist lines and shown how exploitation can be explained in this framework. The results are obtained from orthodox economics using standard assumptions.

4. Marginal labour values are proportional to prices of production

This result of marginal analysis as outlined above should also come about if one analyses the interdependency of the system of production in an equilibrium situation. This type of analysis has been carried out by Piero Sraffa and the Neo-Ricardians. In the Appendix II is shown that the Sraffian vector of prices of production under conditions of optimally allocated resources is equal to the vector of marginal labour values.

$$v = \left[\delta L/\delta x_1, \ldots, \delta L/\delta x_{n-1}\right] = a_n\left[I - (1+\pi)A\right]^{-1} \tag{11}$$

Prices of production are defined by Pasinetti. “Production prices are physical quantities of labour, weighted with the compounded rate of profit appropriate to their conceptual dates of application.” (Pasinetti, 1977, p. 189).

5. Conclusion

It is surely a sobering insight that there is a labour theoretical foundation of orthodox economics. Modern economics has overcome various difficulties of classical economics. This is not to say that there are no difficulties any more, as for example the capital controversy has shown. But most important is to acknowledge that there is some considerable continuity in the development of economic theory. In the opinion of the great majority of modern economist modern economics has abandoned the labour theory of value. These economists should have blushing faces when they open Adam Smith’s ‘Wealth of Nations’. We have shown that modern economics is build upon the labour theory of value and in fact has made it more consistent. It is high time to turn the theory in this sense against the apologetics of capitalism.

Appendix I

The Profit Maximizing Producer

A profit maximizing producer is endowed with some quantities of factor inputs $\boldsymbol{x_1, \ldots, x_{n-1}}$.

Also given to him are the prices $\boldsymbol{p}$ of output $\boldsymbol{q}$, the prices of the inputs $p_1,\ldots p_{n-1}$ as well as the rental $\boldsymbol{r}$ of the capitalistic inputs and $\boldsymbol{w}$, the wage rate (perfect competition).

The producers' profits $\boldsymbol{\pi}$ equals the difference between total revenue $\boldsymbol{pq}$ (turn over) and costs $\boldsymbol{C}$. Profits are defined here in a very restrictive sense as 'entrepreneurial profits' or what Marx calls extra profits. Profits in the sense of surplus value would include the returns to the value of the means of production too.

The profit function is:

$$\pi = pq - C \qquad (\text{AI.1})$$

The production function is

$$q = f(x_1, \ldots, x_{n-1}, L) \qquad (\text{AI.2})$$

and the cost function is

$$C = \sum (1 + \mathrm{r})\, p_i x_i + wL \qquad (\text{AI.3})$$

Inserted into the profit function leads to

$$\pi = pf(x_1, \ldots, x_{n-1}, L) - \left[\sum (1 + r)\, p_i x_i + wL\right] \qquad (\text{AI.4})$$

This function is to be maximized. The first order condition for its maximization is that the partial derivatives are equal to zero.

$$\partial\pi/\partial x_1 = pf'_{x_1} - (1+r)\,p_1 = 0 \qquad \text{(AI.5)}$$

$$\vdots$$

$$\partial\pi/\partial x_{n-1} = pf'_{x_{n-1}} - (1+r)\,p_{n-1} = 0$$

$$\partial\pi/\partial L = pf'_L - w = 0$$

The producer adjusts his factor endowments via the market according to his budget constraint. The second order condition for **π** to be a maximum are

$$f''_{x_1} < 0, \ldots, f''_{x_{n-1}} < 0, and\, f''_L < 0 \qquad \text{(AI.6)}$$

This is assumed to be the case. This condition is essentially what Marx has called 'the tendency of the rate of profit to fall'. It means that an increase of the capital inputs relative to labour inputs causes the rate of profit to decline.

From the first order conditions follow

$$(1+r) = p/p_1 f'_{x_1} \qquad \text{(AI.7)}$$

$$\vdots$$

$$(1+r) = p/p_{n-1} f'_{x_{n-1}}$$

and in particular

$$w = pf'_L \qquad \text{(AI.8)}$$

Equation **(AI.8)** applied to all sectors of the economy leads to equation **(1).**

Regarding the value of maximum profits it is interesting to observe the following:

If the production function of the producer is homogeneous of degree **1** that is if there are constant returns to scale which is a necessary condition for perfect competition we have

$$f(x_1,\ldots,x_{n-1},L) = \sum f'_{x_i}\, x_i + f'_L\, L \tag{AI.9}$$

and

$$pf(x_1,\ldots,x_{n-1},L) = \sum pf'_{x_i}\, x_i + pf'_L\, L \tag{AI.10}$$

(AI.7) and **(AI.8)** substituted into this equation yields

$$pf(x_1,\ldots,x_{n-1},L) = \sum (\gamma + r) p_i x_i + wL \tag{AI.11}$$

But this is the equality of the value of total output to total costs. In equilibrium the (entrepreneurial) profits are zero!

Appendix II

The Vector of Labour Values

In his book "Lectures on the Theory of Production" Luigi Pasinetti introduces the concept of *the vector of vertically integrated labour coefficients*

$$v = a_n[I - A]^{-1}$$

The meaning of this concept is limited to the case where the rate of profit is zero. But it should be appropriate to interpret the vector

$$v = a_n[I - (1 + r)A]^{-1}$$

as *the vector of labour values* also in the general case of a positive rate of profit **r.**

To put the point more clearly let's suppose that there is an economy with continuous production functions in all of its ***n-1*** sectors. In equilibrium the allocation of its resources is optimal. Assuming constant returns to scale one can still analyse this economy in terms of linear algebra and describe it with the technology matrix ***A*** and the vector of labour inputs $\boldsymbol{a_n}$ as $\boldsymbol{[A, a_n]'}$ as long as there are constant returns to scale. We then arrive at equation **(V.3.1a)** p. 73 in the "Lectures",

$$pA(1 + r) + a_n w = p \qquad \text{(AII.1)}$$

and this can be written as

$$p = a_n[I - (1 + r)A]^{-1} w \qquad \text{(AII.2)}$$

This corresponds to equation **(V.5.18)** p. 80 in the 'Lectures'.

It is important to realize that under the assumptions above the row vector

$$v = a_n[I - (1+r)A]^{-1}$$

is equal to the vector of marginal labour values

$$v = a_n[I - (1+r)A]^{-1} = [\partial L/\partial x_1, \ldots, \partial L/\partial x_{n-1}] \qquad \text{(AII.3)}$$

where $\partial L/\partial x_i$ are marginal labour values of sector **i**.

That this must be so can easily be shown. If labour is optimally allocated the uniform wage rate is equal to the value of the marginal product of each sector.

$$w = p_i \, \partial x_i/\partial L, \; for \; i = 1,2,\ldots,n-1 \qquad \text{(AII.4)}$$

We can write equation **(AII.2)** as

$$p = a_n[I - (1+r)A]^{-1} wI \qquad \text{(AII.5)}$$

wI is a diagonal matrix with the wage rate on its major diagonal. We replace the wage rate for each sector by its value of the marginal product $p_i \, \partial x_i/\partial L$ and call that matrix ***W*** so that our equation **(AII.2)** becomes

$$p = a_n[I - (1+r)A]^{-1} W \qquad \text{(AII.6)}$$

Now it is evident that the elements of $a_n[I - (1+r)A]^{-1}$ must be the marginal labour values as in **(AII.3)** to cancel out with the marginal productivities of ***W*** to yield the price vector ***p***.

The vector of marginal labour values can also be represented as a power series. Equation (AII.3) can be written as

$$v = \left[\partial L/\partial x_1,\ldots,\partial L/\partial x_{n-1}\right] = a_n\left[I - (1+r)A\right]^{-1} = a_n + (1+r)a_n A + (1+r)^2 a_n A^2 + \ldots \quad \text{(AII.7)}$$

in which the elements $ra_n A$, $r^2 a_n A^2$, … represent surplus labour.

The relationship between the marginal labour values and the rate of profit is a purely technical one and therefore the marginal labour values or equivalently dated labour should be interpreted as embodied labour.

References

Jevons, W. Stanley (1871)

The Theory of Political Economy, 5th edition, New York: Augustus M. Kelley; 1965.

Pasinetti, Luigi L. (1977)

Lectures on the Theory of Production, London and Basingstoke: Macmillan; 1977.

Schumpeter, Joseph A. (1943)

Capitalism, Socialism and Democracy, London: Unwin University Books; 1943.

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