Optimal Fiscal and Monetary Policy under Sectorial Heterogeneity

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Abstract

This paper characterizes optimal fiscal and monetary policy in a new keynesian model with sectorial heterogeneity in price stickiness. In particular, we (i) derive a purely quadratic welfare-based loss function from an approximation of the representative agent's utility function and (ii) provide the optimal target rule for fiscal and monetary policy. Differently from the homogeneous case, the loss function includes sectorial inflation variances instead of aggregate inflation, with weights proportional to the degree of price stickiness; and sectorial output gaps instead of aggregate output gap with equal weight in each sector. Optimal policy implies a very strong positive correlation among sectorial output gaps and some dispersion of sectorial inflation in response to shocks. Larger heterogeneity in price stickiness implies larger impact of shocks on aggregate inflation. Optimal taxes are more responsive in sectors with stickier prices.

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1 Introduction

Recent research has established some empirical facts about price rigidity. First, prices are sticky: Bils and Klenow (2004) show the average price duration is around seven months in the US and Dhyne et al (2006) found an average duration of twelve months in Europe. Second, there is substantial heterogeneity in price stickiness across different sectors.

The second fact has led researchers to incorporate heterogeneity in price stickiness in the standard models to study its positive implications. Carvalho (2006) and Nakamura and Steinsson (2007) showed that heterogeneity in price stickiness leads to larger and more persistent effects of monetary shocks. In this paper, our aim is to study the normative implications of heterogeneity. We follow the strategy of Benigno and Woodford (2003) to characterize optimal fiscal and monetary policy in a new keynesian model with heterogeneity in price stickiness and show two main results.

First, we derive the representative consumers' welfare-based quadratic loss function. Welfare depends on square deviations of all the sectorial inflations and sectorial output gaps. The weight on each sectorial inflation quadratic term is higher the greater the price rigidity in that sector.[1] Intuitively, sectors with higher price rigidities generate higher distortions in response to exogenous shocks, so their stabilization becomes more important for the policy maker. These different weights and the convexity of the loss function imply that there is an optimal level of dispersion for sectorial inflation in response to shocks. In contrast, the weights on sectorial output gap are the same across sectors, implying that they should move in the same way in response to shocks.

Second, we derive the optimal fiscal and monetary policy and show that there are key differences from the standard homogeneous price stickiness case. If the government can levy sector-specific taxes on firms, a sectorial cost-push shock leads to inflation, output and taxation movements not only in its sector, but in all the sectors of the economy. In the sector hit by the shock taxes fall to offset inflationary pressures, while in the other sectors taxes increase to keep the solvency of the government, generating inflationary pressures and preventing cross-sector output misalignment. The increase in taxation is proportional to the degree of price stickiness in the sector,

[1] Aoki (2001) shows a similar result for a economy with a flexible price and a sticky price sector and Benigno (2004) for a currency union.

because taxation responses are influenced by their impact on sectorial inflation and output gap. When we consider a pure fiscal shock instead, the increase in aggregate taxation splits among sectors the same way: the sectors with higher stickiness end up being more taxed.

We also address the case where the government must levy the same tax rate in all sectors. The optimal paths of inflation and the output remain qualitatively the same as in the sector-specific taxation case. However, because there is no sectorial instruments for the policy maker, there are more disperse optimal paths of sectorial inflations and output gaps.

In the next section, we present the model. In Section 3, we recall the results in the homogenous sticky price case. In Section 4, we show the implications of heterogeneity in price stickiness for the loss function. In Section 5, we derive optimal fiscal and monetary policy with sector-specific taxation. In Section 6, we derive optimal fiscal and monetary policy when taxation is the same in all sectors. Section 7 concludes.

2 Model

Our model departs from the standard new keynesian setup (Woodford 2003) in two ways. First, we allow for heterogeneity in price stickiness, since firms in different productive sectors may have different probabilities of updating their nominal prices. Second, we allow for sector-specific taxation.

There is a set Z of measure one of differentiated goods and respective suppliers working under monopolistic competition. These suppliers can be aggregated into a finite number of intervals or K productive sectors. Each good as well as each supplier is indexed by $z \in [0,1]$ and $k \in [1,2,...,K]$. We denote as m_k the measure obtained from the aggregation of all suppliers working under sector k, which can be understood as the relative weight of sector k, since $\sum_{k=1}^{K} m_k = 1$, where $0 < m_k < 1$. The next subsections describe the new keynesian setup with heterogeneity in price stickiness. Readers familiar with this literature can skip to Section 3.

2.1 Agents

A representative household chooses a Dixit-Stiglitz (1977) composite of differentiated consumption goods and supplies labor hours to a continuum of different types to monopolistically competitive firms (i.e., respectively, C_t and $h_{k,t}(z)$):

$$U_t \equiv E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[u\left(C_j\right) - \sum_{k=1}^{K} \int_{m_k} v\left(h_{k,t}(z)\right) dz \right], \tag{1}$$

where β is the discount factor and the utility is isoelastic;

$$u\left(C_t\right) \equiv \frac{C_t^{1-\sigma}}{1-\sigma}, \tag{2}$$

$$\sum_{k=1}^{K} \int_{m_k} v\left(h_{k,t}(z)\right) dz \equiv \sum_{k=1}^{K} \int_{m_k} \frac{\lambda}{1+\nu} h_{k,t}(z)^{1+\nu} dz, \tag{3}$$

where σ, ν and λ are all greater than zero. The terms σ and ν are, respectively, the inverse of the intertemporal elasticity of substitution of consumption and the inverse of the Frisch elasticity of labor supply, while λ is a normalizing constant.

The CES aggregate good C_t is a weighed sum of sector aggregates $C_{k,t}$:

$$C_t \equiv \left[\sum_{k=1}^{K} m_k^{1/\eta} C_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \tag{4}$$

where $\eta > 0$ is the elasticity of substitution across sectors. The sector composite consumption good $C_{k,t}$ is:

$$C_{k,t} \equiv \left[m_k^{-1/\theta} \int_{m_k} c_{k,t}\left(z\right)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}, \tag{5}$$

where $c_{k,t}\left(z\right)$ is the quantity purchased of produced good z in sector k and θ the elasticity of substitution among goods produced within each sector (independently of which sector, i.e., $\theta_k = \theta$). There is no capital, investment or liquidity services provided by money. The aggregate price index of composite consumption good produced in sector k is defined as:

$$P_{k,t} \equiv \left[m_k^{-1} \int_{m_k} p_{k,t}\left(z\right)^{1-\theta} dz \right]^{1/(1-\theta)} \tag{6}$$

and the price-level is:

$$P_t \equiv \left[\sum_{k=1}^{K} m_k P_{k,t}^{1-\eta} \right]^{1/(1-\eta)}. \tag{7}$$

At the beginning of each period t, the representative household receives a nominal tax-free gross interest rate R_{t-1}, over the stock of bonds acquired in the previous period, B_{t-1}. The flow budget constraint is:

$$P_t C_t + B_t - R_{t-1} B_{t-1} = \sum_{k=1}^{K} \int_{m_k} W_{k,t}(z) h_{k,t}(z) dz + \int_0^1 \Psi_t(z) dz, \tag{8}$$

where $\Psi_t(z)$ are profits transferred from firm z.

Firms operate a constant-returns to scale technology and are subject to a sector-specific technology factor $a_{k,t}$, that is independent among sectors and $E(a_{k,t}) = 1$, $Var(a_{k,t}) = \sigma_a^2$, all k:

$$y_{k,t}(z) = a_{k,t} h_{k,t}(z), \tag{9}$$

where $y_{k,t}(z)$ denotes the quantity produced by firm z in sector k.

As usual, α_k^{j-t} defines the probability that the price defined by firm z at period t, $p_{k,t}(z)$, will remain valid until period $t+j$. Firm z chooses a price $p_{k,t}(z)$ that maximizes the present discounted value of expected future profits:

$$\max_{\{p_{k,t}(z)\}} E_t \sum_{j=t}^{\infty} \alpha_k^{j-t} \Theta_{t,j} \left[(1 - \tau_{k,j}) \, y_{k,j}(z) p_{k,t}(z) - h_{k,j}(z) W_{k,j}(z) \right]. \tag{10}$$

The term Θ is the stochastic discount factor and τ_k denotes the proportion of firm's revenues in sector k that is taxed by the government. Within the same sector, firms are identical: they all have the same degree of market power, they face the same productive shocks and employ the same amount of differentiated labor hours. Across sectors, firms differ in terms of their productivity and are subject to different tax rates over sales revenues and different degrees of price stickiness.

Government expenses are represented by an exogenous process G_t and it will be taken as our fiscal shock. Aggregate government expenses follow the same CES characterization of household consumption:

$$G_t \equiv \left[\sum_{k=1}^{K} m_k^{1/\eta} G_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \tag{11}$$

where $G_{k,t}$ is the government consumption of sector composite good k. Government consumption of sector composite good is defined in terms of differentiated goods produces by firms within that sector, analogous to (5), where $g_{k,t}(z)$ is government consumption of good z:

$$G_{k,t} \equiv \left[m_k^{-1/\theta} \int_{m_k} g_{k,t}\left(z\right)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}. \tag{12}$$

All government revenue come from distortive taxes on firms. The government's flow budget constraint is given in a date t perspective according to

$$R_{t-1}B_{t-1}^G = B_t^G + S_t, \tag{13}$$

where B_t^G denotes the end-of-period nominal liabilities of the government in terms of the one period risk-free bond, S_t the government nominal primary surplus defined in terms of sectorial aggregates according to:

$$S_t \equiv \sum_{k=1}^{K} \tau_{k,t} P_{k,t} Y_{k,t} - P_t G_t, \tag{14}$$

where τ_k is the tax rate applied over revenues of firms in sector k, Y_k is the output of sector k. Iterating forward allow us to write the government budget constraint as:

$$W_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} C_j^{-\sigma^{-1}} s_j, \tag{15}$$

where s_t is the real value of (14) and W_t is defined as

$$W_t \equiv \frac{C_t^{-\sigma^{-1}}}{\Pi_t} R_{t-1} b_{t-1} \tag{16}$$

and b_t the real value of debt at date t, or $b_t = B_t/P_t$ and Π_t is the gross inflation rate from date $t-1$ to date t.

2.2 Competitive Equilibrium

The first-order conditions on consumer's problem imply the following demand for good z in terms of sector aggregate and for the sector aggregate in terms of aggregate consumption and relative price:

$$c_{k,t}(z) = m_k^{-1} C_{k,t} \left[\frac{P_{k,t}}{p_{k,t}(z)} \right]^{\theta}, \tag{17}$$

$$C_{k,t} = m_k C_t \left[\frac{P_t}{P_{k,t}} \right]^{\eta}. \tag{18}$$

The consumer's intertemporal problem define a unique stochastic discount factor and the transversality condition:

$$\Theta_{t,j} = \beta^{j-t} E_t \left[\frac{C_j^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_j} \right], \tag{19}$$

$$\lim_{j \to \infty} \beta^j E_t \left[C_j^{-\sigma} \right] = 0. \tag{20}$$

Sectorial real wages must satisfy:

$$\mu_{k,t} \frac{\lambda h_{k,t}(z)^{\nu}}{C_t^{-\sigma^{-1}}} = w_{k,t}(z), \tag{21}$$

where $\mu_{k,t} \geq 1$ is an exogenous sector-specific markup factor in the labor market, which is allowed to vary over time.[2]

Solving the optimization problem for the firm yields the following rule for price setting in terms of sectorial and overall aggregate variables (similarly to Benigno and Woodford (2003) and detailed in the Appendix A):

$$\frac{p_{k,t}^*(z)}{P_{k,t}} = \left[\frac{K_{k,t}}{F_{k,t}} \right]^{1/(1+\theta\nu)}, \tag{22}$$

$$K_{k,t} \equiv \frac{\theta\lambda}{\theta - 1} m_k^{-\nu} E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \mu_{k,t} \Pi_{k,j}^{\theta(\nu+1)} \left[\frac{Y_{k,j}}{a_{k,j}} \right]^{\nu+1}, \tag{23}$$

$$F_{k,t} \equiv E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} (1 - \tau_{k,j}) C_j^{-\sigma} \Pi_{k,j}^{\theta-1} p_{k,j} Y_{k,j}, \tag{24}$$

where $p_{k,t}$ stands for the relative price of sector k , or $p_{k,t} = P_{k,t}/P_t$ and $\Pi_{k,j}$ is the

[2] Benigno and Woodford (2003) introduce the same labor market disturbance in the aggregate economy. An alternative approach is undertaken by Steinsson (2003), who motivate the cost-push shock by considering the elasticity of substitution between goods stochastic. Both approaches reach the same log-linearized system. Another alternative would be to introduce stochastic shocks in the disutility of sectorial labor.

gross inflation rate from period t to $t+j$ in sector k, or $\Pi_{k,j} = P_{k,j}/P_{k,t}$. $K_{k,t}$ is the discounted sum of (constant) markups over present and future marginal costs and $F_{k,t}$ represent the discounted sum of present and future net revenues. In equilibrium, all prices set within the same sector are equivalent. The relevant difference from the homogeneous stickiness case is the definition of presence of sectorial aggregates and the sectorial relative price level term. In addition, relative prices in sector k follow:

$$p_{k,t} = \frac{\Pi_{k,t}}{\Pi_t} p_{k,t-1}$$

Following the definition of overall and sector consumption, government's demand for differentiated goods or sector aggregates can be derived in a similar fashion as household's demands, leading to demands analogous to (17) and (18):

$$g_{k,t}(z) = m_k^{-1} G_{k,t} \left[\frac{P_{k,t}}{p_{k,t}(z)} \right]^{\theta}, \tag{25}$$

$$G_{k,t} = m_k G_t \left[\frac{P_t}{P_{k,t}} \right]^{\eta}. \tag{26}$$

The market clearing conditions are: $y_{k,t}(z) = c_{k,t}(z) + g_{k,t}(z)$, all k and z; as well as $B_t = B_t^G$.

Definition 1 *A competitive equilibrium is a sequence of endogenous variables* $\mathcal{X}_t^{En} = \{\Pi_t,\ \Pi_{k,t},\ Y_t,\ Y_{k,t},\ F_{k,t},\ K_{k,t},\ W_t,\ P_t,\ p_{k,t},\ C_t,\ C_{k,t},\ b_t\}$*, policy variables* $\mathcal{X}_t^P = \{\tau_{k,t},\ R_t\}$ *and initial conditions* $\mathcal{X}_{t_0-1}^{In} = \{P_{t_0-1},\ p_{k,t_0-1},\ R_{t_0-1},\ b_{t_0-1}\}$ *for all* k *and* $t \geq t_0$*, that satisfy (17)-(26) and the market clearing conditions, given the exogenous processes* $\mathcal{X}_t^{Ex} = \{G_t,\ a_{k,t},\ \mu_{k,t}\}$ *, all* k*.*

2.3 Ramsey Equilibrium

Using the market-clearing conditions, we can rewrite the consumer's utility function as:

$$U_t \equiv E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(Y_t - G_t)^{1-\sigma}}{1-\sigma} - \frac{\lambda}{1+\nu} \sum_{k=1}^{K} m_k \left[\frac{Y_{k,t}}{m_k a_{k,t}} \right]^{1+\nu} \Delta_{k.j} \right], \tag{27}$$

where $\Delta_{k,t}$ is the sectorial price dispersion is (Appendix C for details):

$$\Delta_{k.t} \equiv m_k^{-1} \int_{m_k} \left[\frac{p_{k,t}(z)}{P_{k,t}} \right]^{-\theta(1+\nu)} dz. \tag{28}$$

With the model fully described, we can define the Ramsey equilibrium in this economy:

Definition 2 *In a Ramsey rational expectation equilibrium with commitment, the social planner selects a competitive equilibrium by choosing policy instruments $\mathcal{X}_t^P$, all t, in order to maximize (27).*

It is well known that, in the absence of further constraints, the solution to the Ramsey problem above implies time-inconsistency for the optimal plan.[3] In the presence of predetermined price dispersion, relative prices and debt level, the social planner would try to benefit from the forward-looking nature of price-setting decisions and attempt to reduce the real level of public debt by choosing a higher inflation rate in the first period (t_0) and then committing to a lower inflation level thereafter. As the social planner would face the same incentives at every date, the solution would imply deviating from commitment to a lower level of inflation in the first period and committing to low inflation in the future.

One possibility for obtaining a time invariant solution follows Woodford (1999), where the optimal solution with commitment is characterized from a *timeless perspective*. This approach imposes restrictions on the problem to prevent the social planner from internalizing the gains from private expectations on the evolution of inflation under commitment in the first period. In other words, consider a vector of quantities $X_t = \{F_{k,t}, K_{k,t}, W_t\}$, all k and t. A restricted Ramsey equilibrium from a timeless perspective imposes a set of preconditions on quantities so that optimization takes place also subject to the fact that X_{t_0} must take certain values. In particular, quantities X_{t_0} are chosen such as the first order conditions for the policy problem applied over t_0 are exactly the same as those applied in any date t.

Definition 3 *In a restricted Ramsey rational expectation equilibrium with commitment, the social planner uses policy instruments in order to select a competitive equilibrium that maximizes (27) subject to the additional constraint of timeless perspective $X_{t_0} = \{\bar{F}_k, \bar{K}_k, \bar{W}\}$, all k.*

[3] Stokey and Lucas (1983).

Hereafter, we study the properties of the approximated solution for the restricted Ramsey equilibrium in a multi-sector economy.

3 Revisiting the Homogeneous Price Stickiness Case

This section briefly describes the approximate solution to the Ramsey problem for the case of homogeneous price stickiness. The derivation of the welfare-based loss function and optimal fiscal and monetary policy in a homogeneous price stickiness economy is fully characterized in Benigno and Woodford (2003). We begin by the following lemma:

Lemma 1 *There is a deterministic steady state with zero inflation and positive level of public debt and tax level.*

We assume that the shocks that hit the economy are small enough that they do not lead to paths of the endogenous variables distant from their steady state levels. This is equivalent to assuming that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions.

The loss function from consumer's approximate utility function under homogenous price stickiness is presented in the following proposition:

Proposition 1 *The representative consumer's utility function can be approximated up to second-order by*

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\lambda_y y_t^2 + \lambda_\pi \pi_t^2 \right], \tag{29}$$

where $\Omega \equiv \bar{C}^{-\sigma} \bar{Y}$.

Welfare in the homogeneous price stickiness case depends only on square deviations of inflation and output gap.[4] This result rationalizes the usual loss function assumed in the literature on monetary policy evaluations. One should notice the absence of tax smoothing terms as in Barro (1979) or Bohn (1990). Policy restrictions are given

[4]The output gap here is defined as deviations from the target that results from the approximation of the loss function around an inefficient steady state.

by a new keynesian Phillips Curve and a government budget constraint, detailed in the Appendix E.

Proposition 2 *The optimal targeting rules that approximate the restricted Ramsey Equilibrium with homogenous price stickiness by maximizing (29), subject to policy restrictions are:*

$$\pi_t = \frac{\omega_\varphi}{m_\varphi}\Delta y_t - \frac{n_\varphi}{m_\varphi}\pi_{t-1}, \tag{30}$$

$$E_t\pi_{t+1} = 0. \tag{31}$$

Coefficient definitions are presented in the Appendix E. We refer to Benigno and Woodford (2003) for complete characterization of parameter definitions and proofs. The first implicit targeting rule has the form of flexible inflation targeting, similar to the lump-sum taxation case, as in Woodford (2003). As in Giannoni and Woodford (2002) the rate of change in output gap matters for adjustment of near term inflation target. The second rule implies that in the optimal equilibrium the price level should follow a random-walk.

4 Heterogenous Price Stickiness Case

In this section, we describe the log-linearized system that characterizes the heterogenous price stickiness economy and derive its loss-function.

4.1 Approximate Model

The following lemma states that there is a deterministic steady state with the same features as the one described in Section 3.

Lemma 2 *There is a deterministic symmetric steady state, characterized by zero inflation rate, uniform taxation and constant and positive level of public debt.*

Proof. Appendix B. ■

As for the approximate model equations, first-order Taylor expansion over the sectorial supply equation yields:

$$\pi_{k,t} = \kappa_k\{(\tilde{\sigma} - \eta^{-1})y_t + (\nu + \eta^{-1})y_{k,t} + \delta(\hat{\tau}_{k,t} - \hat{\tau}^*_{k,t})\} + \beta E_t\pi_{k,t+1} + u_{k,t}. \tag{32}$$

This sectorial Phillips Curve is similar to homogeneous price stickiness case, in the sense that contemporaneous inflation depends on output and expected future inflation. Differently, these are sectorial rather than aggregate relations. Moreover, there is a term that relates sectorial inflation to aggregate output. If the elasticity of substitution among different sectors is high (η^{-1} close to zero), a higher aggregate output leads to higher sectorial inflation. Tax over firms' revenues has the effect of a typical cost push shock: the higher the sectorial taxation that a firm faces, the higher the price it would charge.

The tax rate target $\tau^*_{k,t}$ is a linear function of aggregate government expenses, sectorial and aggregate productivity shocks, as well as average disturbances on the labor market and other parameters of the economy (defined in Appendix F). We interpret the term $u_{k,t}$ as a purely cost-push inefficient shock. It is a linear function of assumed stochastic disturbances on labor market and other parameters of the economy.

Sectorial Phillips Curves can be aggregated in order to yield a similar equation for the aggregate Phillips Curve[5]:

$$\pi_t = \sum_{k=1}^{K} m_k \kappa_k \{(\tilde{\sigma} - \eta^{-1}) y_t + (\nu + \eta^{-1}) y_{k,t} + \delta(\hat{\tau}_{k,t} - \hat{\tau}^*_{k,t})\} + \beta E_t \pi_{t+1} + \sum_{k=1}^{K} m_k u_{k,t}. \quad (33)$$

A first order log-linear approximation to the government budget constraint is given by:

$$\hat{b}^*_{t-1} - \tilde{b}_y y_t - \pi_t = (1-\beta)\bar{\tau} s_d^{-1} \sum_{k=1}^{K} m_k (\hat{\tau}_{k,t} - \hat{\tau}^*_{k,t}) + \beta E_t[\hat{b}^*_t - \tilde{\sigma} y_{t+1} - \pi_{t+1}] - \zeta_t \quad (34)$$

where s_d^{-1} and $\tilde{b}_y$ are constants defined in terms of parameters of the economy and steady state level variables and ζ_t is a linear function of present and future (exogenous) government expenses, aggregate productivity and aggregate wage markup shocks (Appendix F).

We introduce $\hat{b}^*_t$ as a variable that encompasses the stock of nominal debt and its nominal return. In a word, $\hat{b}^*_t$ can be interpreted as the value at maturity of public

[5] Carvalho (2006) discusses its implications.

debt, as in Ferrero (2005). The solvency of the government measured by the real value of its obligations (i.e.: $\hat{b}^*_{t-1} - \pi_t$) depends negatively on sectorial taxations and aggregate output (tax base). We also impose when solving the model that neither the present value of government assets explodes or implodes.

4.2 Welfare

In order to find the optimal policy we need an approximation of the representative consumer's welfare:

Lemma 3 *A second-order approximation of the utility function is given by*

$$\begin{aligned} U_{t_0} &= \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \hat{Y}_t + \frac{(1-\tilde{\sigma})}{2} \hat{Y}_t^2 + \tilde{\sigma} \hat{Y}_t \hat{G}_t + \\ & \quad - \sum_{k=1}^{K} m_k (1-\Phi) [\frac{\theta}{\kappa_k} \frac{\pi_{k,t}^2}{2} + \hat{Y}_{k,t} + \frac{(1+\nu)}{2} \hat{Y}_{k,t}^2 - (1+\nu) \hat{Y}_{k,t} \hat{a}_{k,t}] \} \end{aligned} \tag{35}$$

where

$$\Omega \equiv \bar{C}^{-\sigma} \bar{Y}, \tag{36}$$

$$\kappa_k \equiv \frac{(1-\alpha_k)(1-\alpha_k \beta)}{(1+\theta\nu)\alpha_k}, \tag{37}$$

and

$$\Phi = 1 - \frac{\theta - 1}{\theta} \frac{(1-\bar{\tau})}{\bar{\mu}} < 1, \tag{38}$$

Proof. *Appendix C.* ■

Φ measures the wedge between the marginal rate of substitution between consumption and leisure and marginal product of labor in the many sectors of the economy. Considering that aggregate output is given by the weighed sum of sector outputs, then it is clear that the presence of the linear term is only due to this departure from efficient level. Rotemberg and Woodford (1999) eliminate this term by assuming a distortive subsidy on firms' production level ($\bar{\tau} < 0$) financed by lump-sum taxes.

In order to express (35) purely in quadratic terms without the help of distortive subsidies, we derive second-order approximations for the whole set of restrictions and use the second order terms of such restrictions in order to express the discounted sum of the linear term for aggregate and sectorial outputs only in terms of quadratic endogenous variables. The following proposition shows the main result of this section: a second-order approximation to the policy problem in an environment of heterogeneity of Calvo pricing:

Proposition 3 *The representative consumer's utility function can be approximated up to second-order by*

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\lambda_{y_k} \sum_{k=1}^{K} m_k y_{k,t}^2 + \sum_{k=1}^{K} m_k \lambda_{k,\pi} \pi_{k,t}^2 \right], \tag{39}$$

where the relative weights of sectorial inflations and sectorial output gaps depend on structural parameters of the economy.

Proof. Appendix D. ■

This loss-function is different from the one derived without price heterogeneity in two ways. First, square deviations of sectorial inflations appear in the loss function in place of aggregate inflation. The greater is price stickiness in a particular sector, the greater the relative importance of that sector's inflation rate in the loss function. This generalizes Aoki's (2001) result for an economy with more than two sectors. Differently from the homogenous case, the convexity of the loss function as well as the different weights among sectors imply that, given a shock, there is an optimal sectorial inflation dispersion.

Second, the usual negative effect of aggregate output gap volatility upon welfare gives room to the sum of sectorial output gaps. Because the relative weight of each sector is the same, reaction to shocks should be, in opposition to sectorial inflation measures, the same in economies with symmetric sector sizes (same $m_k s$). Therefore, the same cross-sector weight and the convexity of the loss function will imply strong commovements of sectorial output gaps under optimal policy.

5 Optimal Policy with Sector-Specific Taxation

In this section, we find the approximate solution to the Ramsey problem described in subsection 2.3. The problem is to find the paths for the endogenous variables $\{\pi_t, \pi_{k,t}, y_t, y_{k,t}, \hat{b}_t^*, \hat{\tau}_{k,t}\}$ in order to minimize (39) subject to the k sectorial Phillips curves given in (32), government budget constraint in (34), the definitions for aggregate inflation in terms of its sectorial counterparts as well as aggregate output expressed in terms of sectorial outputs.

5.1 Policy Problem Solution

From the FOCs, it is possible to show that the shadow value of government revenue - given by the Lagrange multiplier of the government budget constraint, or M^b - follows:

$$M_t^b = E_t M_{t+1}^b \tag{40}$$

which implies, as in Barro (1979), that temporary disturbances to the level of exogenous shocks produce permanent changes in the level of public debt.

The following proposition then characterizes optimal policy.

Proposition 4 *The optimal targeting rules that approximate the restricted Ramsey equilibrium with commitment under heterogenous price stickiness by maximizing (39) subject to (34), (32) and the definitions for aggregate inflation and output are:*

$$\Delta y_{k,t} = \frac{\varphi_1}{\psi_k^\pi}\pi_{k,t} - \frac{\varphi_2}{\psi_k^\pi}\pi_{k,t-1}, \tag{41}$$

$$E_t \pi_{k,t+1} = 0, \tag{42}$$

where the coefficients are defined in Appendix G.

These targeting rules differ from the case of a homogeneous price stickiness economy. The targeting rules presented in Section 3 now hold at the sector level, so the expectation of future inflation has to be zero not only for aggregate inflation but for each individual sector. In other words, sectorial price levels follow random walks. In addition, the flexible inflation targeting rule holds at sectorial level.

5.2 Fiscal Shocks

The reaction to fiscal shocks makes these differences clear. We calibrated the parameters of the model to be close to the ones used in the literature and the median probability of adjusting prices is set to be 0.5 in all cases.[6] Figure 1 compares the optimal response of aggregate taxation, aggregate output gap, aggregate inflation and public debt to a fiscal shock in three economies with different degrees of price stickiness heterogeneity: the case with homogeneous probability of nominal adjustment, the heterogeneous case with low variance on the degree of stickiness and the heterogeneous case with high variance. The two panels on the left show that more heterogeneity leads to a higher impact of fiscal shock on inflation, as well as a higher instantaneous reaction of taxation. The intuition is that the policy maker does not care about squared deviations of aggregate inflation, but rather about how sectorial inflation rates commove. Hence, a higher level of inflation can be desirable provided it leads to commovements of sectorial inflation closer to optimal.

Benigno and Woodford (2003) show that at this level of median stickiness in a homogeneous economy the initial responses of taxation to a fiscal shock is negative. Keeping the median stickiness of the economy at the same level and making price stickiness more disperse, it is possible to obtain a positive initial response. Heterogeneity in price stickiness does not change the optimal response of aggregate output gap, as seen in the first panel to the right; only the long term levels are slightly different due to different levels of long-run aggregate taxation. The debt level is lower the higher the variance of price stickiness, which is consistent with the taxation response and solvency of public debt.

The same fiscal shock has different impacts on taxes and inflation in each sector. Figure 2 presents the optimal paths of sectorial taxes, output gaps and inflations in the economy with low variance in price stickiness. The optimal response implies all sectorial output gaps must follow exactly the same path. This requires higher inflation in the sector with lower stickiness. The initial impact on taxes is higher the stickier is the sector. Intuitively, the optimal policy takes into account the inflationary effects of higher taxation in sector with lower stickiness. Hence, taxation adjusts only moderately, as expected in the light of our discussion of the loss function.

[6] We consider three cases: a homogeneous economy, where the degree of price-stickiness is set to be .5 for each of the its three sectors; a heterogeneous case with low variance, where sectors presents a probability of nominal adjustment in each period of .2, .5 and .8; and a high variance case, with probabilities of .1, .5, and .9. All sectors have the same sizes. Risk aversion is set to 2, the inverse of Frisch elasticity is set to .47, within-sector elasticity of substitution is 10; cross-sector elasticity of substitution is 4.5; government consumption is set to 22% of GDP; with real surplus of 2%, and annual debt level over GDP of 50%. Discount factor is .99, lambda is set to .98 and 1.05 is the steady state level of gross wage markups. All shocks considered in this paper for IRFs are i.i.d..

5.3 Cost-push shocks

Figure 3 shows the impulse response functions of the same sectorial variables to a sectorial cost-push shock in the median sector. It leads to inflation, output gap and taxation movements not only in that particular sector, but in all the sectors of the economy. In the sector hit by a shock, taxation decreases substantially to offset inflationary pressures.[7] To keep the solvency of the government, other sectors must have its taxation increased, generating inflationary pressures. Optimal responses also prevent cross-sector output misalignment, as in the fiscal shock case. The increase in taxation is proportional to the degree of price stickiness in the sector, because higher taxes in a sticky sector would not impact inflation severely.

As in the fiscal shock case , optimal responses from aggregate inflation to a cost-push shock in the median sector also increase when there is more heterogeneity. The intuition here is the same as before: a higher aggregate inflation is desirable *if* combined with sectorial inflation commovements closer to optimal. In Figure 4, we point out that this fact, along with the optimal impulse responses from other aggregate variables.

5.4 Welfare Analysis

In order to check the welfare relevance of the results above, we inspected the impact of a policy maker that mistakenly disregards heterogeneity in price stickiness. Welfare losses are computed using the same parameter values of the benchmark calibration and the same distribution for price stickiness that characterizes the economies defined previously in the text. Here the shocks are assumed to follow AR(1) processes whose parameters we borrow from the empirical literature on sticky price DSGE models, listed in Table 1.[8] We refer to Schmitt-Grohé and Uribe (2004) for the methodology of welfare accounting.

Table 1 shows the difference in percentage points welfare losses when compared with optimal policy welfare levels (when the policy-maker is aware of the heterogeneity). We assume that policy maker assigns the median level of stickiness in the economy to all sectors. In this situation, taxation is uniform not because of any re-

[7] Omitted from first panel for scale reasons.

[8] As these authors do not consider sectorial estimates, we assume sectorial exogenous shocks follow independent AR(1) processes with the same degree of inertia of the original estimation and with standard deviation of $1/K$-th of the original, while K is the total number of sectors.

strictions in the number of policy instruments, but because policy maker is unable to distinguish between sectors different degree of stickiness. As shown, the impact on steady state consumption equivalent losses can be relevant: in the case of a low dispersion of sectorial price stickiness it leads to losses from .022% to .057%, depending on the parameters used. In the case of high dispersion it leads to higher losses, ranging form .201% to .489% depending on the calibration. Considering the average level of household expenses for the US economy in 2006, these welfare losses imply that each American household would consider making one annual payment from US$10 to US$27 in the first case in order to guaranty a fully informed policy maker. In the economy with large dispersion in price stickiness, this amount would be, respectively, from US$99 to US$237.

6 Optimal Policy with Uniform Taxation

In this section we restrict the model in Section 4, making the government only able to impose the same taxation over different sectors. We do that in order to distinguish which results come from the heterogenous stickiness and which come from the heterogenous taxation assumption.

6.1 Response to shocks

Figure 5 shows the effects of a fiscal shock under homogenous taxation remain qualitatively the same for aggregate variables. Aggregate inflation and public debt change their optimal paths when the degree of heterogeneity increases and aggregate taxation is roughly the same across the different economies. Hence, the conclusion reported on Section 5 regarding the aggregate endogenous variables reaction to cost-push shocks remains also the same even when we restrict government's taxation instruments.

In contrast, sectorial variables dynamics change with uniform taxation. In Figure 6, we show how a fiscal shock reflects among sectors with different price stickiness. When the government is unable to impose sector-specific taxation, sectorial inflation and output optimal paths are more spread when compared to heterogeneous taxation case. In this restricted case, the policy maker cannot tax relatively more the stickier sector where there is less inflationary pressures and higher output due to excess demand. This leads to suboptimal sectorial responses to shocks when comparing to the previous situation: sectors with higher stickiness show lower and more persistent inflationary effects and higher volatility on output gap.

The effects of a sectorial cost-push on aggregate variables leads to similar insights to those in the fiscal shock case.[9] However, uniform taxes change dramatically sectorial variables dynamics. As can be seen in Figure 7, the sector hit by the cost-push shock (that is, the sector with median stickiness) shows the usual behavior: inflation increases while output gap decreases. In order to offset these effects, optimal policy decreases taxation leading to a higher levels of public debt. Due to the lack of sectorial instrument, this taxation decrease is applied in all sectors, thus leading to deflation in the sectors not hit by the shock. Output gaps in these sectors increase as a consequence not only of lower taxes but also due to the substitution effects from the increases in prices in the sector hit by the shock. Notably, taxation under optimal policy follows a staggering adjustment to a positive level in the subsequent dates.

6.2 Welfare Analysis

In Table 2, we again borrow from the empirical literature a more realistic structure for the exogenous shocks and calculate the impact on welfare in terms of equivalent consumption in steady state of restricting the policy maker in the number of policy instruments. As Table 2 shows, when policy maker is restricted in the number of sectorial instruments, welfare losses under our benchmark calibration are small. In the case of a low dispersion in price stickiness, we observe higher losses of .001% to .003% when compared with optimal policy; while in the case of high dispersion, loss increases amount from .002% to .004% of steady state consumption. Although allowing for sectorial instruments can be relevant, welfare losses from disregarding heterogeneity are clearly more important.

7 Conclusion

The objective of this paper is to establish the normative implications of price stickiness heterogeneity, by deriving its implications for optimal fiscal and monetary policy. The welfare-based loss function derived here has key differences from the homogenous case. First, welfare is affected by the sum of sectorial inflation square deviations, which im-

[9] We present the plot with the impulse response functions in the Technical Appendix of the paper.

plies sectorial inflation dispersion in response to shocks. Second, welfare also depends on the square deviation of each sectorial output gap with the same weight. This suggests that sectorial output gap misalignments should be avoided under optimal policy paths and that there is a role for sector-specific policy instruments.

We have shown that optimal targeting rules as well as responses of inflation, output gap and public debt to shocks are different in the presence of different degrees of price stickiness. In this sense, this paper provides a word of caution on optimal policy models that disregard this heterogeneity. Beyond the theoretical considerations, realistic models for optimal fiscal and monetary policy should take into account not only aggregate but sectorial data for calibration. We believe such a framework would permit to quantitatively evaluate the implications of heterogeneity and sector specific instruments of policy on welfare.

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8 Appendix A - The Firms' Problem

Noting that $\theta > 1$, FOC from firms' optimization problem is given by:

$$E_t \sum_{j=t}^{\infty} \alpha_k^{j-t} \Theta_{t,j} \frac{\partial \Psi_j \left(p_{k,t} \left(z \right), \ . \right)}{\partial p_{k,t} \left(z \right)} = 0.$$

Taking derivatives and isolating terms $p_{k,t}(z)/P_{k,t}$, yields:

$$\frac{p_{k,t}(z)}{P_{k,t}}^{1+\theta\nu} = \frac{\frac{\theta\lambda}{\theta-1} m_k^{-\nu} E_t \sum_{j=t}^{\infty} \left(\alpha_k \beta \right)^{j-t} \mu_{k,j}^w \frac{P_{k,j}}{P_{k,t}}^{\theta(\nu+1)} \frac{Y_{k,j}}{a_{k,j}}^{\nu+1}}{E_t \sum_{j=t}^{\infty} \left(\alpha_k \beta \right)^{j-t} \left(1 - \tau_{k,j} \right) C_j^{-\sigma-1} \frac{P_{k,j}}{P_{k,t}}^{\theta-1} p_{k,j} Y_{k,j}} \tag{43}$$

9 Appendix B - Steady State

The government budget constraint in steady state is given by:

$$(1-\beta)\bar{b}^* = \sum_{k=1}^{K} \bar{\tau}_k \bar{Y}_k - \bar{G}. \tag{44}$$

Assuming debt and government expenses are non-zero in steady state imply $\bar{\tau}_k > 0$, for some k. Also, given $p_{k,-1} = 1$ and zero inflation, all k. It is clear that , $\bar{Y}_k = m_k \bar{Y}$, which imply (44) becomes

$$(1-\beta)\bar{b}^* + \bar{G} = \bar{\tau}\bar{Y}, \tag{45}$$

where $\bar{\tau} = \sum_{k=1}^{K} m_k \bar{\tau}_k$. From the firms maximization problem:

$$\bar{K}_k = \bar{F}_k.$$

Using definitions for both terms:

$$\frac{\theta\lambda}{\theta-1} \bar{\mu}_k^w m_k^{-\nu} \bar{Y}_k^{\nu} = (1-\bar{\tau}_k) \left(\bar{C} \right)^{-\sigma}, \tag{46}$$

which implies that sectorial tax rate is given by

$$\bar{\tau}_k = 1 - \frac{\theta\lambda}{\theta-1} \bar{\mu}_k^w \left(\bar{C} \right)^{\sigma} \bar{Y}^{\nu}, \tag{47}$$

which only depends of aggregate variables and sector specific parameter $\bar{\mu}_k^w$. We assume that steady state wage markup is the same across sectors, that is $\bar{\mu}_k^w = \bar{\mu}^w$, all k. In this case, steady state distortive tax rates are the same across sectors, that is

$$\bar{\tau}_k = \bar{\tau}, \forall k \tag{48}$$

Once one considers an always-possible normalization $\bar{Y} = 1$, the only restriction made is that the level of consumption over GDP should not be too high in order to tax rates to be positive. Equations

$$\frac{\theta\lambda}{\theta - 1}\bar{\mu}^w \bar{Y}^\nu = (1 - \bar{\tau})\left(\bar{Y} - \bar{G}\right)^{-\sigma} \tag{49}$$

and (45) define the aggregate output level in steady state as well as the aggregate tax rate, as in Benigno and Woodford (2003).

Define the set of commitments $X_t = \{K_{k,t},\ F_{k,t},\ W_t\}$, all k, and let X_0 be the set of initial commitments that make policy optimal form a timeless perspective. The centralized policy maker chooses a sequence of $\mathcal{X}_t = \{\Pi_t,\ \Pi_{k,t},\ Y_t,\ Y_{k,t},\ F_{k,t},\ K_{k,t},\ W_t,\ \Delta_{k,t},\ \tau_{k,t},\ b_t^*,\ p_{k,t}\}$, all k, for $t \geq t_0$ in order to maximize the representative consumer's utility subject to the constraints given in the main text and taking as given the initial commitments X_0 and the initial conditions $\mathcal{I}_{-1} = \{b_{-1}^*,\ \Delta_{k,-1},\ p_{k,-1}\}$ for every k and $t \geq t_0$. In order to impose constant commitments $X_0 = \bar{X}$ we consider additional restrictions such as the first order conditions for the problem in $t = t_0$ are equivalent to the first order conditions for a generic $t > 0$. Consider the set of Lagrange multipliers corresponding to equations in the main text. In order to complete the proof, we need to show that first order conditions for the indicated steady state are satisfied for time-invariant Lagrange multipliers. After taking FOCs from maximization problem, it is possible to show that the system of steady state variables and time-invariant multipliers is just-identified. Complete proof is given in the Technical Appendix.

10 Appendix C - Second Order Approximation to Utility Function

10.1 Second Order Approximation of Utility Function

We start with a second order Taylor expansion of the representative consumer's welfare function where ξ_t refers to the full vector of random disturbances, as in Benigno and Woodford (2003). We start by working with $u(Y_t, \xi_t)$. Define hereafter, for any variable X_t, $\tilde{X}_t \equiv \frac{X_t - \bar{X}}{\bar{X}}$, $\hat{X}_t \equiv \log \frac{X_t}{\bar{X}}$

It is know that the following relation holds up to second order:

$$\tilde{X}_t \simeq \hat{X}_t + \frac{1}{2}\hat{X}_t^2. \tag{50}$$

Given the functional form assumed, we have:

$$u(Y_t, \xi_t) = \bar{C}^{-\sigma}\bar{Y}[\tilde{Y}_t - \frac{\sigma}{2}\frac{\bar{Y}}{\bar{C}}\tilde{Y}_t^2 + \sigma\frac{\bar{Y}}{\bar{C}}\tilde{Y}_t\tilde{G}_t] + tips + O_p^3, \tag{51}$$

where $\tilde{G}_t$ represents the absolute deviation *over GDP*. Defining $s_C = \frac{\bar{C}}{\bar{Y}}$, yields

$$u(Y_t, \xi_t) = \bar{C}^{-\sigma}\bar{Y}[\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2(1 - \sigma s_C^{-1}) + \sigma s_C^{-1}\hat{Y}_t\hat{G}_t] + tips + O_p^3. \tag{52}$$

A second order Taylor expansion of $v(Y_{k,t}, \xi_t)\,\Delta_{k.t}$ around steady state values yield

$$\begin{aligned} v(Y_{k,t}, \xi_t)\,\Delta_{k.t} \;=\;& v\left(\bar{Y}_k, \bar{\xi}\right)\tilde{\Delta}_{k.t} + v_{Y_k}\left(\bar{Y}_k, \bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t} + \frac{1}{2}\hat{Y}_{k,t}^2) + \\ & + \frac{1}{2}v_{Y_kY_k}\left(\bar{Y}_k, \bar{\xi}\right)\bar{Y}_k^2(\hat{Y}_{k,t}^2) + v_{Y_k}\left(\bar{Y}_k, \bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t})\tilde{\Delta}_{k.t} + \\ & + v_{Y_k\xi}\left(\bar{Y}_k, \bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t}\hat{a}_{k,t}) + v_{\xi}\left(\bar{Y}_k, \bar{\xi}\right)\tilde{\Delta}_{k.t}(\hat{a}_{k,t}) + \\ & + tips + O_p^3. \end{aligned} \tag{53}$$

Using the definition for $\Delta_{k,t}$ one can show that $\tilde{\Delta}_{k,t}$ is a term of second order. In this sense, interactions between $\tilde{\Delta}_{k,t}$ and $\hat{a}_{k,t}$ or $\tilde{\Delta}_{k,t}$ and $\hat{Y}_{k,t}$ can be ignored up to second order. Hence, expression (53) simplifies to

$$v\left(Y_{k,t},\xi_{t}\right)\Delta_{k.t}=\lambda\left[\frac{\bar{Y}_{k,t}}{m_{k}}\right]^{1+\nu}\{\frac{\hat{\Delta}_{k.t}}{1+\nu}+\hat{Y}_{k,t}+\frac{1+\nu}{2}\hat{Y}_{k,t}^{2}-(1+\nu)\hat{Y}_{k,t}\hat{a}_{k,t}\}+tips+O_{p}^{3}, \quad (54)$$

once one notice that $\hat{\Delta}_{k,t}^{2}$ is of higher order than O_{p}^{2}. Using a second order Taylor expansion over the law of motion for sectorial price dispersion given by

$$\Delta_{k.t}=\alpha_{k}\Pi_{k,t}^{\theta(1+\nu)}\Delta_{k.t-1}+(1-\alpha_{k})\left(\frac{1-\alpha_{k}\Pi_{k,t}^{\theta-1}}{(1-\alpha_{k})}\right)^{\frac{\theta(1+\nu)}{\theta-1}} \quad (55)$$

yields

$$\hat{\Delta}_{k.t}=\alpha_{k}\hat{\Delta}_{k.t-1}+\frac{1}{2}\frac{\alpha_{k}}{(1-\alpha_{k})}\theta(1+\nu)(1+\theta\nu)\pi_{k,t}^{2}+O_{p}^{3}, \quad (56)$$

once we used the relation $\hat{\Pi}_{k,t}=\pi_{k,t}+\frac{1}{2}\pi_{k,t}^{2}$, where $\pi_{k,t}$ is the percent variation of sectorial price level $\pi_{k,t}=\log P_{k,t}/P_{k,t-1}$. Iterating backwards yields

$$\hat{\Delta}_{k.t}=\alpha_{k}^{t-1}\hat{\Delta}_{k.-1}+\frac{1}{2}\frac{\alpha_{k}}{(1-\alpha_{k})}\theta(1+\nu)(1+\theta\nu)\sum_{j=0}^{t}\alpha_{k}^{t-j}\pi_{k,j}^{2}+O_{p}^{3}. \quad (57)$$

Here we consider the sectorial price dispersion in the remote past as a "term independent of policy". Further considering that it is possible to change positions of sums over t and k on (54), and re-ordering the terms:

$$\sum_{t=0}^{\infty}\beta^{t}\hat{\Delta}_{k.t}=\frac{1}{2}\frac{\alpha_{k}}{(1-\alpha_{k})(1-\alpha_{k}\beta)}\theta(1+\nu)(1+\theta\nu)\sum_{t=0}^{\infty}\beta^{t}\pi_{k,t}^{2}+tips+O_{p}^{3}. \quad (58)$$

Substituting (58) over (54) yields

$$\begin{aligned}v\left(Y_{k,t},\xi_{t}\right)\Delta_{k.t}=\lambda\left[\frac{\bar{Y}_{k,t}}{m_{k}}\right]^{1+\nu}\{\frac{1}{2}\frac{\alpha_{k}\theta(1+\theta\nu)}{(1-\alpha_{k})(1-\alpha_{k}\beta)}\pi_{k,t}^{2}+\hat{Y}_{k,t}+ \\ +\frac{1+\nu}{2}\hat{Y}_{k,t}^{2}-(1+\nu)\hat{Y}_{k,t}\hat{a}_{k,t}\}+tips+O_{p}^{3}.\end{aligned}$$

This way, we can approximate the representative consumer utility up to second order by the following expression:

$$
\begin{aligned}
U_{t_0} \quad = \quad & \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \hat{Y}_t + \frac{(1-\tilde{\sigma})}{2} \hat{Y}_t^2 + \tilde{\sigma} \hat{Y}_t \hat{G}_t + \\
& - \sum_{k=1}^{K} m_k (1-\Phi) [\frac{\theta}{\kappa_k} \frac{\pi_{k,t}^2}{2} + \hat{Y}_{k,t} + \frac{1+\nu}{2} \hat{Y}_{k,t}^2 + \\
& -(1+\nu) \hat{Y}_{k,t} \hat{a}_{k,t}] \} + tips + O_p^3,
\end{aligned}
\tag{59}
$$

where

$$
\Omega \equiv \bar{C}^{-\sigma} \bar{Y}, \tag{60}
$$

$$
\kappa_k \equiv \frac{(1-\alpha_k)(1-\alpha_k \beta)}{(1+\theta\nu)\alpha_k}, \tag{61}
$$

$$
\tilde{\sigma} \equiv \sigma s_C^{-1} \tag{62}
$$

and

$$
(1-\Phi) \equiv \frac{\theta-1}{\theta} \frac{(1-\bar{\tau})}{\bar{\mu}^w}. \tag{63}
$$

10.2 Second Order Approximation to AS Equation

The starting point is the expression for the sectorial non-linear Phillips Curve, given by:

$$
\left(\frac{1-\alpha_k \Pi_{k,t}^{\theta-1}}{(1-\alpha_k)} \right)^{\frac{1+\theta\nu}{\theta-1}} = \frac{F_{k,t}}{K_{k,t}}. \tag{64}
$$

We define $V_{k,t}$ as

$$
V_{k,t} = \frac{1-\alpha_k \Pi_{k,t}^{\theta-1}}{(1-\alpha_k)}. \tag{65}
$$

Using a second order Taylor expansion on $\hat{V}_{k,t}$:

$$\hat{V}_{k.t} = -\frac{\alpha_k(\theta-1)}{(1-\alpha_k)}\left[\pi_{k,t} + \frac{1}{2}\frac{(\theta-1)}{(1-\alpha_k)}\pi_{k,t}^2\right] + O_p^3. \tag{66}$$

Considering the expression for $K_{k,t}$ define $\Pi_{k,t,s} = P_{k,s}/P_{k,t}$, where $s \geq t$ is some date in the future and $P_{k,t}$ the aggregate price level in sector k in period t. We use a second order Taylor expansion:

$$\tilde{K}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{k}_{k,j} + \frac{1}{2}\hat{k}_{k,j}^2\} + O_p^3, \tag{67}$$

where the term $\hat{k}_{k,t}$ can be defined as $\hat{k}_{k,j} = \theta(1+\nu)\pi_{k,t,j} + (1+\nu)\hat{Y}_{k,j} - (1+\nu)\hat{a}_{k,j}$

Taking the expression in the text for $F_{k,t}$ given by (24), we define the net revenue factor as $\Gamma_{k,t} \equiv 1-\tau_{k,t}$, and taking second-order Taylor expansion:

$$\tilde{F}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{f}_{k,j} + \frac{1}{2}\hat{f}_{k,j}^2\} + O_p^3. \tag{68}$$

where we define $\hat{f}_{k,j} = \hat{\Gamma}_{k,j} - \sigma\hat{C}_j + \hat{Y}_{k,j} + \hat{p}_{k,j} + (\theta-1)\,\pi_{k,t,j}$.

Using $\tilde{F}_{k,t}$, $\tilde{K}_{k,t}$, as well as $\hat{V}_{k.t}$, $\hat{F}_{k,t}$ and $\hat{K}_{k,t}$,after some algebra, we get:

$$\begin{aligned}\left[\frac{1+\theta\nu}{\theta-1}\right]\hat{V}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{[z_{k,j} - (1+\theta\nu)\pi_{k,t,j}] + \\ + \frac{1}{2}[z_{k,j} - (1+\theta\nu)\pi_{k,t,j}][\hat{X}_{k,j} + [(\theta-1)+\theta(1+\nu)]\pi_{k,t,j}]\} \\ -\frac{1}{2}\left[\frac{1+\theta\nu}{\theta-1}\right]\hat{V}_{k,t}(1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{X}_{k,j} + [(\theta-1)+\theta(1+\nu)]\pi_{k,t,j}\} + O_p^3,\end{aligned}$$

where

$$\hat{X}_{k,j} \equiv \hat{\Gamma}_{k,j} - \sigma\hat{C}_j + (2+\nu)\hat{Y}_{k,j} + \hat{p}_{k,j} - (1+\nu)\hat{a}_{k,j} + \hat{\mu}_{k,t}^w, \tag{69}$$

$$\hat{f}_{k,j} - \hat{k}_{k,j} = z_{k,j} - (1+\theta\nu)\pi_{k,t,j} \tag{70}$$

and

$$z_{k,j} = \hat{\Gamma}_{k,j} - \sigma \hat{C}_j - \nu \hat{Y}_{k,j} + \hat{p}_{k,j} + (1+\nu)\hat{a}_{k,j} - \hat{\mu}^w_{k,t}. \tag{71}$$

Define

$$Z_{k,t} \equiv E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \left\{ \left[\hat{X}_{k,j} + [(\theta-1) + \theta(1+\nu)]\pi_{k,t,j} \right] \right\} \tag{72}$$

We can replace in the expression above and after some algebra we get:

$$\frac{(1+\theta\nu)}{(\theta-1)(1-\beta\alpha_k)} \hat{V}_{k,t}(\pi_{k,t+1}) = (\pi_{k,t+1}) E_t \sum_{j=t+1}^{\infty} (\alpha_k \beta)^{j-t-1} \left\{ z_{k,j} - (1+\theta\nu)(\pi_{k,t,j}) \right\} + O_p^3 \tag{73}$$

We can use the definition for $\hat{V}_{k,t}$ and replace above, also ignoring the terms O_p^3 or of higher order:

$$\begin{aligned}
&- \kappa_k^{-1} \left[\pi_{k,t} + \frac{1}{2} \frac{(\theta-1)}{(1-\alpha_k)} \pi_{k,t}^2 - \alpha_k \beta E_t \pi_{k,t+1} - \frac{1}{2} \frac{(\theta-1)}{(1-\alpha_k)} \alpha_k \beta E_t \pi_{k,t+1}^2 \right] = \\
&\qquad z_{k,t} + \frac{1}{2} z_{k,t} \hat{X}_{k,t} - (1+\theta\nu) \frac{\alpha_k \beta}{(1-\alpha_k \beta)} E_t \pi_{k,t+1} + \\
&\qquad - \frac{1}{2} [(\theta-1) + \theta(1+\nu)] \frac{\beta}{\kappa_k} E_t \pi_{k,t+1}^2 + \\
&\qquad - \frac{1}{2} (1+\theta\nu)(\alpha_k \beta) E_t [\pi_{k,t+1} Z_{k,t+1}] + \\
&\qquad + \frac{1}{2} \frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)} \left[\pi_{k,t} Z_{k,t} - \alpha_k \beta E_t [\pi_{k,t+1} Z_{k,t+1}] \right] + O_p^3,
\end{aligned}$$

where we have defined κ_k elsewhere.

Further simplification yields

$$
\begin{aligned}
-\kappa_k^{-1}\pi_{k,t} - \frac{1}{2}\kappa_k^{-1}\frac{(\theta-1)}{(1-\alpha_k)}\pi_{k,t}^2 - \frac{1}{2}\frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)}\pi_{k,t}Z_{k,t} \\
= z_{k,t} + \frac{1}{2}z_{k,t}\hat{X}_{k,t} - \kappa_k^{-1}\beta E_t\pi_{k,t+1} \\
- \frac{1}{2}\kappa_k^{-1}\{\frac{(\theta-1)}{(1-\alpha_k)} + \theta(1+\nu)\}\beta E_t\pi_{k,t+1}^2 \\
- \frac{1}{2}\frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)}\beta E_t[\pi_{k,t+1}Z_{k,t+1}] + O_p^3.
\end{aligned}
$$

Multiplying both sides for $-\kappa_k$ allow us to write above expression as

$$
\mathcal{V}_{k,t} = -\kappa_k\{z_{k,t} + \frac{1}{2}z_{k,t}\hat{X}_{k,t}\} + \frac{\theta(1+\nu)}{2}\pi_{k,t}^2 + \beta E_t\mathcal{V}_{k,t+1} + O_p^3, \tag{74}
$$

where:

$$
\mathcal{V}_{k,t} = \pi_{k,t} + \frac{1}{2}\{\frac{(\theta-1)}{(1-\alpha_k)} + \theta(1+\nu)\}\pi_{k,t}^2 + \frac{1}{2}\frac{\kappa_k\alpha_k}{(1-\alpha_k)}[\pi_{k,t}Z_{k,t}]. \tag{75}
$$

A second order Taylor expansion of $\log(1-\tau_{k,t})$ yields

$$
\log(1-\tau_{k,t}) = \log(1-\bar{\tau}) - \frac{\bar{\tau}}{1-\bar{\tau}}\tilde{\tau}_{k,t} - \frac{1}{2}\frac{\bar{\tau}^2}{(1-\bar{\tau})^2}\tilde{\tau}_{k,t}^2 + O_p^3,
$$

which can be recasted as

$$
\hat{\Gamma}_{k,t} = -\delta\hat{\tau}_{k,t} - \frac{\delta}{(1-\bar{\tau})}\frac{1}{2}\hat{\tau}_{k,t}^2 + O_p^3.
$$

Log-approximation on consumption as a function of aggregate output and government expenses yields:

$$
\hat{C}_t = s_C^{-1}\hat{Y}_t - s_C^{-1}\hat{G}_t + \frac{1}{2}s_C^{-1}(1-s_C^{-1})\hat{Y}_t^2 - \frac{1}{2}s_C^{-1}(1+s_C^{-1})\hat{G}_t^2 + s_C^{-2}\hat{Y}_t\hat{G}_t + O_p^3 \tag{76}
$$

Using both results, one can be generally express (74) as

$$\mathcal{V}_{k,t} = E_{t_0} \sum_{j=t}^{\infty} \beta^{j-t} \{ -\kappa_k [z_{k,t} + \frac{1}{2} z_{k,t} \hat{X}_{k,t}] + \frac{\theta(1+\nu)}{2} \pi_{k,t}^2 \} + tips + O_p^3. \tag{77}$$

One could finally note that a first order approximation to (77) yields the known Phillips Curve of the form:

$$\begin{aligned} \pi_{k,t} &= \kappa_k \{ (\tilde{\sigma} - \eta^{-1}) \hat{Y}_t + (\nu + \eta^{-1}) \hat{Y}_{k,t} + \delta \hat{\tau}_{k,t} \\ &\quad - \tilde{\sigma} \hat{G}_t - (1+\nu) \hat{a}_{k,t} + \hat{\mu}_{k,t}^w \} + \beta E_t \pi_{k,t+1} + O_p^2. \end{aligned} \tag{78}$$

10.3 Second Order Approximation to the Budget Constraint

We approximate the intertemporal government budget restriction by a second order Taylor expansion. Taking the definitions of the intertemporal government budget constraint and primary surplus and making a second-order approximation, we get:

$$\tilde{W}_t = (1-\beta) E_t \sum_{j=t}^{\infty} \beta^{j-t} \{ -\sigma \tilde{C}_t + \tilde{s}_t + \frac{1}{2} \sigma (\sigma + 1) \tilde{C}_t^2 - \sigma \tilde{C}_t \tilde{s}_t \} + O_p^3. \tag{79}$$

It is also easy to show that $\hat{W}_t = \hat{b}_{t-1}^* - \sigma \hat{C}_t - \pi_t$ and $\tilde{W} = \hat{W} + \frac{1}{2} \hat{W} + O_p^3$. Then, we can re-write $\tilde{W}_t$ as:

$$\tilde{W}_t = \hat{b}_{t-1}^* - \sigma \hat{C}_t - \pi_t + \frac{1}{2} (\hat{b}_{t-1}^* - \sigma \hat{C}_t - \pi_t)^2 + O_p^3. \tag{80}$$

The approximation to the primary surplus is

$$s_d \tilde{s}_t = \sum_{k=1}^{K} m_k \bar{\tau} [(\hat{\tau}_k + \hat{p}_{k,t} + \hat{Y}_{k,t}) + \frac{1}{2} (\hat{\tau}_k + \hat{p}_{k,t} + \hat{Y}_{k,t})^2] - \hat{G}_t - \frac{1}{2} \hat{G}_t^2 + O_p^3, \tag{81}$$

where $s_d \equiv \frac{\bar{s}}{\bar{Y}}$ and $\bar{s} = \sum_{k=1}^{K} \bar{\tau} \bar{Y}_k - \bar{G} = \bar{\tau} \bar{Y} - \bar{G}$.

Hence, the second order approximation for the intertemporal budget constraint can be obtained from the above expressions. One can notice that a first order approximation yields:

$$\hat{b}^*_{t-1} - \tilde{\sigma}(\hat{Y}_t - \hat{G}_t) - \pi_t =$$
$$(1-\beta)E_t \sum_{j=t}^{\infty} \beta^{j-t} \{ s_d^{-1} \sum_{k=1}^{K} m_k \bar{\tau} [\hat{\tau}_k + \hat{p}_{k,t} + \hat{Y}_{k,t}] +$$
$$+ (\tilde{\sigma} - s_d^{-1})\hat{G}_t - \tilde{\sigma}\hat{Y}_t \} + tips + O_p^2$$

where $\hat{p}_{k,t}$ is a function of sectorial and overall outputs.

10.4 Aggregate and Sectorial Output Relation

Sectorial demand expressed is $p_{k,t}^{\eta} = m_k Y_t / Y_{k,t}$ and log-linearized as

$$\hat{p}_{k,t} = \eta^{-1}(\hat{Y}_t - \hat{Y}_{k,t}), \tag{82}$$

which establishes an exact (inverse) relation between sector relative price and sector relative product. Also, using $p_{k,t} = \frac{\Pi_{k,t}}{\Pi_t} p_{k,t-1}$ and $\Pi_t^{1-\eta} \equiv \sum_{k=1}^{K} m_k \left(\Pi_{k,t} p_{k,t-1}\right)^{1-\eta}$ one gets

$$Y_t^{(\eta-1)/\eta} = \sum_{k=1}^{K} m_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta}, \tag{83}$$

which relates aggregate and sectorial outputs. Log linearization of (83) yields

$$\hat{Y}_t + \frac{1}{2}(1-\eta^{-1})\hat{Y}_t^2 = \sum_{k=1}^{K} m_k \hat{Y}_{k,t} + \frac{1}{2}(1-\eta^{-1}) \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 + O_p^3. \tag{84}$$

11 Appendix D - Elimination of Linear Terms

11.1 Matrix Notation

We invite the reader to check out the complete version is available in the Technical Appendix. We start by defining

$$x_t' = \left[\begin{array}{cccccccccc} \hat{Y}_t & \hat{Y}_{1,t} & ... & \hat{Y}_{K,t} & \pi_{1,t} & ... & \pi_{K,t} & \hat{\tau}_{1,t} & ... & \hat{\tau}_{K,t} \end{array} \right], \tag{85}$$

$$\xi_t' = \left[\begin{array}{cccccccc} \hat{G}_t & \hat{a}_{1,t} & ... & \hat{a}_{K,t} & \hat{\mu}_{1,t}^w & ... & \hat{\mu}_{K,t}^w \end{array} \right]. \tag{86}$$

For notational convenience, we also define the following terms: $\upsilon \equiv 1+\nu$, $\omega_\eta \equiv 1-\eta^{-1}$, $\chi \equiv \nu+\eta^{-1}$, $\tilde{\sigma} \equiv \sigma s_C^{-1}$, $\varsigma \equiv \tilde{\sigma}-\eta^{-1}$, $\delta \equiv \frac{\bar{\tau}}{1-\bar{\tau}}$ and $-\omega_C \equiv 1-s_C^{-1}$.

Using the definitions above, expression (59) can be written in matrix notation as

$$U_{t_0} = \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ A_x' x_t - \frac{1}{2} x_t' A_{xx} x_t - x_t' A_\xi \xi_t \} + tips + O_p^3 \tag{87}$$

where A_x, A_{xx}, and A_ξ are, respectively, $(3K+1)\times 1$, $(3K+1)\times(3K+1)$ and $(3K+1)\times(2K+1)$ matrices.

The Sectorial Phillips Curve expressed in (77) can also be written in matrix notation. We start by substituting expressions for $\hat{p}_{k,t}$ into definitions for $z_{k,t}$ and $\hat{X}_{k,t}$, underlined in (71) and (69). Our aim is to separate quadratic and linear terms. Quadratic and linear terms of random disturbances are placed into *tips*. After some manipulation one obtains:

$$\mathcal{V}_{k,t_0} = E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} \{ C_{x,k}' x_t + \frac{1}{2} x_t' C_{xx,k} x_t + x_t' C_{\xi,k} \xi_t \} + tips + O_p^3 \tag{88}$$

for a generic sector k. As in (87), matrices $C_{x,k}$, $C_{xx,k}$, and $C_{\xi,k}$ have, respectively, dimension $(3K+1)\times 1$, $(3K+1)\times(3K+1)$ and $(3K+1)\times(2K+1)$.

The government budget constraint can also be simplified in matrix notation. Taking expression given in (79), we eliminate references for $\hat{p}_{k,t}$, and replace $\hat{C}_t$ and $\tilde{s}_t$ for their expressions in terms of endogenous variables x_t and exogenous processes ξ_t. Grouping linear and quadratic terms, yields:

$$\tilde{\mathcal{W}}_{t_0} = (1-\beta) E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} \{ B_x' x_t + \frac{1}{2} x_t' B_{xx} x_t + x_t' B_\xi \xi_t \} + tips + O_p^3 \tag{89}$$

where, as in (87) and (88), matrices B_x, B_{xx}, and B_ξ are, respectively, of dimensions $(3K+1)\times 1$, $(3K+1)\times(3K+1)$ and $(3K+1)\times(K+1)$.

Finally, (84) can be expressed in matrix notation as

$$0 = \sum_{j=t}^{\infty} \beta^{j-t} \{ H_x' x_t + \frac{1}{2} x_t' H_{xx} x_t \} + O_p^3 \tag{90}$$

where we have used the fact that the definition for aggregate output in terms of its sectorial counterparts expressed in (84) is valid at all dates. Matrices H_x and H_{xx} have, respectively, dimension $(3K+1)\times 1$ and $(3K+1)\times(3K+1)$.

11.2 Elimination of Linear Terms

In order to eliminate linear terms in (87), we need to find a set a multipliers ϑ_C^1, ..., ϑ_C^K, ϑ_B, ϑ_H, such as

$$\vartheta_C^1 C_x^{1\prime} + ... + \vartheta_C^K C_x^{K\prime} + \vartheta_B B_x' + \vartheta_H H_x' = A_x' \tag{91}$$

By solving the linear system of equations, one gets the following set of solution: $\vartheta_B = -\frac{\Phi}{\Upsilon}, \vartheta_H = 1 - \Xi\frac{\Phi}{\Upsilon}$ and, for every k, $\vartheta_C^k = \frac{m_k(1-\bar{\tau})}{\kappa_k}\frac{\Phi}{\Upsilon}$, where we have used the fact that $\bar{\tau} = \bar{\tau}_k$, all k, and defined: $\Upsilon \equiv (\varsigma+\chi)(1-\bar{\tau}) + \tilde{\sigma}s_d - \bar{\tau}$ and $\Xi \equiv \varsigma(1-\bar{\tau}) + \tilde{\sigma}s_d - \bar{\tau}\eta^{-1}$.

Hence, using relations (87), (88),(89), (90), and (91) one can write:

$$\begin{aligned} E_{t_0}\sum_{j=t_0}^{\infty}\beta^{j-t_0}A_x' x_t &= E_{t_0}\sum_{j=t_0}^{\infty}\beta^{j-t_0}[\sum_{k=1}^{K}\vartheta_C^k C_x^{k\prime} + \vartheta_B B_x' + \vartheta_H H_x']x_t \\ &= -E_{t_0}\sum_{j=t_0}^{\infty}\beta^{j-t_0}\{\frac{1}{2}x_t' D_{xx} x_t + x_t' D_\xi \xi_t\} + \sum_{k=1}^{K}\vartheta_C^k \mathcal{V}_{k,t_0} + \frac{\vartheta_B \tilde{W}_{t_0}}{(1-\beta)}, \end{aligned} \tag{92}$$

where $D_{xx} = \sum_{k=1}^{K}\vartheta_C^k C_{xx,k} + \vartheta_B B_{xx} + \vartheta_H H_{xx}$ and $D_\xi = \sum_{k=1}^{K}\vartheta_C^k C_\xi^k + \vartheta_B B_\xi$.

We use this last relations in order to rewrite (87)

$$U_{t_0} = -\Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{\frac{1}{2}x_t' Q_{xx} x_t + x_t' Q_\xi \xi_t\} + T_{t_0} + tips + O_p^3, \tag{93}$$

where

$$T_{t_0} = \Omega\{\sum_{k=1}^{K}\vartheta_C^k \mathcal{V}_{k,t_0} + \frac{\vartheta_B \tilde{\mathcal{W}}_{t_0}}{(1-\beta)}\}, \tag{94}$$

is a vector of predetermined variables. Definitions of Q_{xx} and Q_{ξ} in terms of parameters of the economy defined in the Technical Appendix. As in Benigno and Woodford (2003) and Ferrero (2005), references to sector tax rates have been eliminated. Only references to sectorial inflation measures, sectorial and aggregate outputs remain, which imply (93) can be simplified further by getting rid-off tax rates references and by separating terms referring to sectorial and overall outputs from references to sectorial inflation. Proceeding in such fashion yields

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ x'_{y,t} \tilde{Q}_y x_{y,t} + 2x'_{y,t} \tilde{Q}_{\xi} \xi_t + x'_{\pi,t} \tilde{Q}_{\pi} x_{\pi,t} \} + T_{t_0} + tips + O_p^3, \quad (95)$$

where $x_{y,t}$ is a $K+1 \times 1$ vector containing only references to aggregate and sectorial outputs measures, and $x_{\pi,t}$ is a $K \times 1$ vector containing only sectorial inflation measures and $\tilde{Q}_y$, $\tilde{Q}_{\xi}$ and $\tilde{Q}_{\pi}$ are matrices of coefficients. From (95), we now focus on the term:

$$x'_{y,t} \tilde{Q}_y x_{y,t} = q_y Y_t^2 + \sum_{k=1}^{K} m_k q_{y_k} Y_{k,t}^2 + 2 \sum_{k=1}^{K} m_k q_{y,y_k} Y_t Y_{k,t}, \quad (96)$$

where q terms are combinations of the parameters of the economy defined in the Technical Appendix. Under the assumption that wage markups is steady state as well as markups over marginal costs are the same across sectors ($\bar{\mu}_k^w = \bar{\mu}^w$ and $\theta_k = \theta$) , q coefficients are all independent of k. We use the following lemmas in order to simplify (96) further:

Lemma 4 *The following expression relating sum of sectorial output variances and covariances of sectorial outputs and aggregate output is of third order.*

$$\hat{Y}_t \sum_{k=1}^{K} m_k \hat{Y}_{k,t} - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3 \quad (97)$$

Proof. Technical Appendix. ■

Lemma 5 *The following expression is, at least, of second order:*

$$\hat{Y}_t - \sum_{k=1}^{K} m_k \hat{Y}_{k,t} = O_p^2. \quad (98)$$

Proof. Follows directly from (84). ■

Lemma 6 *The following expression holds:*

$$[\hat{Y}_t - \sum_{k=1}^{K} m_k Y_{k,t}]\hat{G}_t = O_p^3. \tag{99}$$

Proof. From proposition above plus the fact that all exogenous processes are O_p^1. ■

Lemma 7 *The following expression is of third order:*

$$\hat{Y}_t^2 - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3. \tag{100}$$

Proof. Technical Appendix. ■

From (96), and using (97) and (100) one gets:

$$x'_{y,t}\tilde{Q}_y x_{y,t} = \lambda_{y_k} \sum_{k=1}^{K} m_k Y_{k,t}^2 + O_p^3, \tag{101}$$

where

$$\lambda_{y_k} = q_{y_k} + 2q_{y,y_k} + q_y. \tag{102}$$

From (95), we focus on the term:

$$x'_{y,t}\tilde{Q}_\xi \xi_t = q_{yG}\hat{Y}_t\hat{G}_t + q_{y_kG}\sum_{k=1}^{K} m_k Y_{k,t}\hat{G}_t + \sum_{k=1}^{K} m_k \hat{Y}_{k,t}[q_{y_k a_k}\hat{a}_{k,t} + q_{y_k \mu_k}\hat{\mu}_{k,t}]. \tag{103}$$

where q- coefficients are defined in the Technical Appendix. Using (99) we get:

$$x'_{y,t}\tilde{Q}_\xi \xi_t = \sum_{k=1}^{K} m_k Y_{k,t}[q'_{y_kG}\hat{G}_t + q_{y_k a_k}\hat{a}_{k,t} + q_{y_k \mu_k}\hat{\mu}_{k,t}] + O_p^3, \tag{104}$$

where

$$q'_{y_kG} = q_{yG} + q_{y_kG}.$$

Replacing (104) and (101) over (95) yields the expression for the second order approximation for the utility function:

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{\lambda_{y_k} \sum_{k=1}^{K} m_k y_{k,t}^2 + \sum_{k=1}^{K} m_k \lambda_{k,\pi} \pi_{k,t}^2\} + T_{t_0} + tips + O_p^3, \quad (105)$$

where

$$y_{k,t} = \hat{Y}_{k,t} - \hat{Y}_{k,t}^* \quad (106)$$

and

$$-\hat{Y}_{k,t}^* = \lambda_{y_k}^{-1}[(q_{yG} + q_{y_k G})\hat{G}_t + q_{y_k a_k} \hat{a}_{k,t} + q_{y_k \mu_k} \hat{\mu}_{k,t}] \quad (107)$$

all k, and, most importantly, λ_{y_k} and $\lambda_{k,\pi}$ provide the weigh of each of these terms in the welfare-based criteria. Besides the complete definition of such terms, the Technical Appendix addresses the conditions for concavity.

12 Appendix E - Definitions of Homogeneous Case

The typical policy restrictions are given by a new keynesian Phillips Curve and a government budget constraint, respectively:

$$\pi_t = \kappa[(\tilde{\sigma} + \nu)y_t + \delta(\hat{\tau}_t - \hat{\tau}_t^*)] + \beta E_t \pi_{t+1} + u_t \quad (108)$$

$$\hat{b}_{t-1}^* - \tilde{\sigma} y_t - \pi_t = (1-\beta)[(\hat{\tau}_t - \hat{\tau}_t^*) + b_y y_t] + \beta E_t[\hat{b}_t^* - \tilde{\sigma} y_{t+1} - \pi_{t+1}] + \zeta_t \quad (109)$$

where $\hat{b}_t^*$ is defined as the debt at maturity at date t, or $\hat{b}_t^* = \hat{b}_t + \hat{R}_t$, where $\hat{R}_t$ is the gross interest rate. Other variables are defined according to the notation of our model given in Section 2: $\tilde{\sigma} \equiv \sigma s_C^{-1}$, $s_C^{-1} \equiv \bar{Y}/\bar{C}$, $b_y \equiv \bar{\tau}\bar{Y}/(\bar{\tau}\bar{Y} - \bar{G}) - \tilde{\sigma}$, and $\delta \equiv \bar{\tau}/(1-\bar{\tau})$. Hat-variables are defined as steady state levels. The shock terms such as ζ_t and u_t are linear functions of aggregate government expenses, productivity and wage markup shocks. Finally, $\hat{\tau}_t^*$ is the *tax rate target,* also defined as a linear combination of exogenous shocks.[10]

Definitions of the coefficient of the optimal targeting rules are given, in terms parameter in the model presented at Section 2, as

$$\omega_\varphi = -\lambda_\pi^{-1}[(1-\beta)s_d^{-1}(1-\bar{\tau})\kappa^{-1}+1] \tag{110}$$

$$n_\varphi = -\lambda_y^{-1}\tilde{\sigma} \tag{111}$$

$$m_\varphi = -\lambda_y^{-1}[(s_d^{-1}(1-\bar{\tau})+b_y)(1-\beta)+\tilde{\sigma}] \tag{112}$$

13 Appendix F - Log-linear Approximation of Restrictions

13.1 Definition of Target Variables

Explicitly using the assumption that sector specific tax rates as well as wage markups in steady state are the same across sectors, we can define the target level of aggregate output using (107):

$$-\hat{Y}_t^* = \lambda_{y_k}^{-1}[(q_{yG}+q_{y_kG})\hat{G}_t + q_{y_k a_k}\hat{a}_t + q_{y_k\mu_k}\hat{\mu}_t] \tag{113}$$

where q- coefficients are defined in terms of the structural parameters of the economy and $\hat{a}_t$ and $\hat{\mu}_t$ are respectively defined as: $\hat{a}_t = \sum_{k=1}^K m_k\hat{a}_{k,t}$ and $\hat{\mu}_t^w = \sum_{k=1}^K m_k\hat{\mu}_{k,t}^w$.

13.2 Aggregate supply and cost-push disturbance term

Adding and subtracting, respectively, the terms referring to overall and sectorial output targets with the appropriate coefficients yield over first order approximation of AS equation yields

$$\pi_{k,t} = \kappa_k\{(\tilde{\sigma}-\eta^{-1})y_t + (\nu+\eta^{-1})y_{k,t} + \delta(\hat{\tau}_{k,t}-\hat{\tau}_{k,t}^*)\} + \beta E_t\pi_{k,t+1} + u_{k,t}, \tag{114}$$

[10] Details in Benigno and Woodford (2003).

for every k, where the definition for the cost-push term $u_{k,t}$ is a function of sectorial wage markup shocks:

$$u_{k,t} = \kappa_k[1 - (\nu + \eta^{-1})\lambda_{y_k}^{-1} q_{y_k\mu_k}]\hat{\mu}_{k,t}^w \tag{115}$$

and

$$\begin{aligned} -\delta\hat{\tau}_{k,t}^* &= -[(\tilde{\sigma} + \nu)\lambda_{y_k}^{-1}(q_{yG} + q_{y_kG}) + \tilde{\sigma}]\hat{G}_t - (\tilde{\sigma} - \eta^{-1})\lambda_{y_k}^{-1} q_{y_k\mu_k}\hat{\mu}_t^w \\ &\quad -(\tilde{\sigma} - \eta^{-1})\lambda_{y_k}^{-1} q_{y_k a_k}\hat{a}_t - [(\nu + \eta^{-1})\lambda_{y_k}^{-1} q_{y_k a_k} + (1 + \nu)]\hat{a}_{k,t}. \end{aligned} \tag{116}$$

can be understood as the target level for distortive taxation in sector k and q- coefficients are defined in terms of the structural parameters of the economy. Averaging across sectors allows us to determine the generalized aggregate first order approximation for the AS equation in (33), similar to Carvalho (2006).

13.3 Budget Constraint and fiscal disturbance term

We start by taking a first order approximation to expression (79), yielding

$$\hat{b}_{t-1}^* - \tilde{\sigma}(\hat{Y}_t - \hat{G}_t) - \pi_t = (1-\beta)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{b_y\hat{Y}_t + \bar{\tau}s_d^{-1}\sum_{k=1}^{K} m_k[\hat{\tau}_k + \omega_\eta\hat{Y}_{k,t}] + b_G\hat{G}_t\}, \tag{117}$$

where we have defined for convenience the terms s_d^{-1}, b_y and b_G, respectively, as $s_d^{-1} \equiv \frac{\bar{Y}}{\bar{\tau}\bar{Y} - \bar{G}}$, $b_y \equiv s_d^{-1}\bar{\tau}\eta^{-1} - \tilde{\sigma}$, and $b_G \equiv \tilde{\sigma} - s_d^{-1}$. Expression (117) can be written in recursive terms. Using the definition for aggregate output in terms of sectorial outputs and the definitions for target variables given in (107) and (116), we get:

$$\hat{b}_{t-1}^* - \tilde{b}_y y_t - \pi_t + \zeta_t = (1-\beta)\bar{\tau}s_d^{-1}\sum_{k=1}^{K} m_k(\hat{\tau}_{k,t} - \hat{\tau}_{k,t}^*) + \beta E_t[\hat{b}_t^* - \tilde{\sigma}y_{t+1} - \pi_{t+1}], \tag{118}$$

where $\tilde{b}_y \equiv \tilde{\sigma} + (1-\beta)(b_y + \bar{\tau}\omega_\eta s_d^{-1})$ and

$$\zeta_t = \omega_1^G\hat{G}_t + \omega_1^a\hat{a}_t + \omega_1^\mu\hat{\mu}_t^w - \omega_2^G E_t\hat{G}_{t+1} - \omega_2^a E_t\hat{a}_{t+1} - \omega_2^\mu E_t\hat{\mu}_{t+1}^w, \tag{119}$$

where ω_1^G, ω_2^G, ω_1^a, ω_2^a, ω_1^μ and ω_2^μ are defined in terms of the structural parameters of the economy.

13.4 Aggregate and Sectorial Output Relation

First order approximation to (84) combined with the redefinition in terms of deviation from aggregate and sectorial output targets, yields

$$y_t = \sum_{k=1}^{K} m_k y_{k,t}. \tag{120}$$

14 Appendix G - Optimal Solution with Commitment

For simplicity, define: $\check{\tau}_{k,t} \equiv \hat{\tau}_{k,t} - \hat{\tau}^*_{k,t}$. Setting up the Lagrangian, one can establish the following set of FOCs, where M_t^x denotes the multiplier of equation referred tovariable x and where the last line correspond to the preconditions that allow the problem to be valid for all $t \geq 0$. FOCs with respect to $\pi_{t,k}$, π_t, $\check{\tau}_{k,t}$, y_t, $y_{k,t}$ and b_t^* are, respectively, given by:

$$\lambda_{\pi,k}\pi_{t,k} + M_{k,t}^\pi - M_{k,t-1}^\pi = M_t^\pi, \tag{121}$$

$$M_t^\pi = M_t^b - M_{t-1}^b, \tag{122}$$

$$M_{k,t}^\pi = -M_t^b \frac{(1-\bar{\tau})(1-\beta)}{\kappa_k} s_d^{-1}, \tag{123}$$

$$-\sum_{k=1}^{K} m_k M_{k,t}^\pi \kappa_k(\tilde{\sigma} - \eta^{-1}) - M_t^b \tilde{b}_y + M_{t-1}^b \tilde{\sigma} + M_t^y = 0, \tag{124}$$

$$\lambda_{y_k} y_{k,t} - M_{k,t}^\pi[\kappa_k(\nu + \eta^{-1})] - M_t^y = 0, \tag{125}$$

$$M_t^b = E_t M_{t+1}^b, \tag{126}$$

plus the problem's constraints. Substituting (122) and (123) into (121) yields the law of motion to sectorial inflation in terms of debt Lagrange Multiplier M_t^b:

$$\pi_{k,t} = \psi_k^\pi (M_t^b - M_{t-1}^b), \tag{127}$$

where

$$\psi_k^\pi \equiv \lambda_{\pi,k}^{-1} \left[1 + \frac{(1-\beta)(1-\bar{\tau}) s_d^{-1}}{\kappa_k} \right].$$

From (124),

$$M_t^y = \tilde{\Phi}_1 M_t^b - \tilde{\Phi}_2 M_{t-1}^b, \tag{128}$$

where $\tilde{\Phi}_1 = \tilde{b}_y - (1-\bar{\tau})(1-\beta) s_d^{-1}(\tilde{\sigma} - \eta^{-1})$ and $\tilde{\Phi}_2 = \tilde{\sigma}$. Taking (125), replacing for $M_{k,t}^\pi$ from (123) and isolating for $y_{k,t}$ yields

$$y_{k,t} = \varphi_1 M_t^b - \varphi_2 M_{t-1}^b, \tag{129}$$

where

$$\varphi_1 \equiv \lambda_{y_k}^{-1} [\tilde{\Phi}_1 - (1-\bar{\tau})(1-\beta) s_d^{-1}(\nu + \eta^{-1})],$$

$$\varphi_2 \equiv \lambda_{y_k}^{-1} \tilde{\Phi}_2.$$

Summing up across sectors yields the aggregate output in terms of debt Lagrange Multiplier:

$$y_t = \Sigma_1 M_t^b - \Sigma_2 M_{t-1}^b, \tag{130}$$

where we defined coefficients Σ_1 and Σ_2, respectively as $\Sigma_1 \equiv \varphi_1$ and $\Sigma_2 \equiv \varphi_2$.

Finally, it is relevant to notice that under commitment, optimal solution imply that policy is conducted in such a way that:

$$E_t \pi_{k,t+1} = 0, \tag{131}$$

every k. In order to see this, we take leads in (127), apply expectation and use relation (126). In its turn, (131) for every k imply the same behavior for aggregate inflation, or:

$$E_t \pi_{t+1} = 0. \tag{132}$$

Also, for very k, (127) and (129) imply

$$\Delta y_{k,t} = \frac{\varphi_1}{\psi_k^\pi} \pi_{k,t} - \frac{\varphi_2}{\psi_k^\pi} \pi_{k,t-1} \tag{133}$$

and the aggregate relation

$$\Delta y_t = \frac{\Sigma_1}{\psi^\pi} \pi_t - \frac{\Sigma_2}{\psi^\pi} \pi_{t-1}, \tag{134}$$

where

$$\psi^\pi \equiv \sum_{k=1}^{K} m_k \psi_k^\pi.$$

15 Figures and Tables

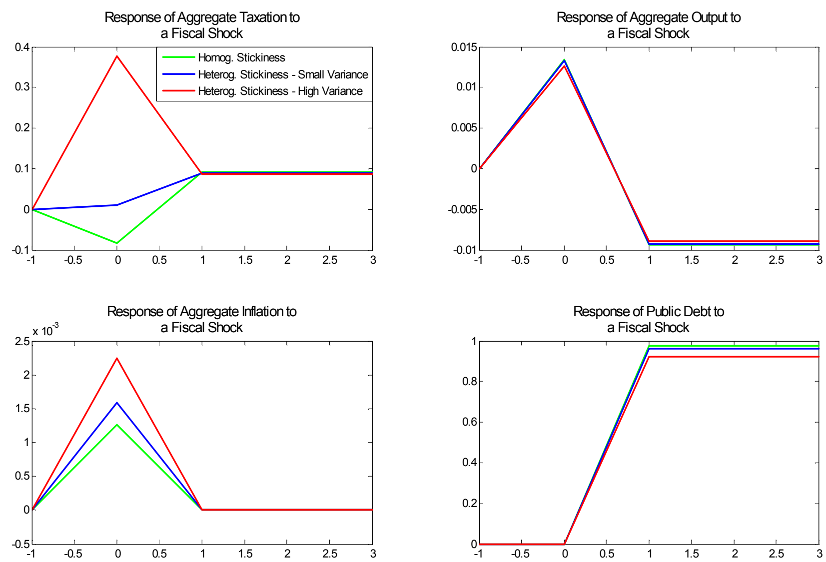

Figure 1: Effects of a Fiscal Shock on Aggregate Variables.

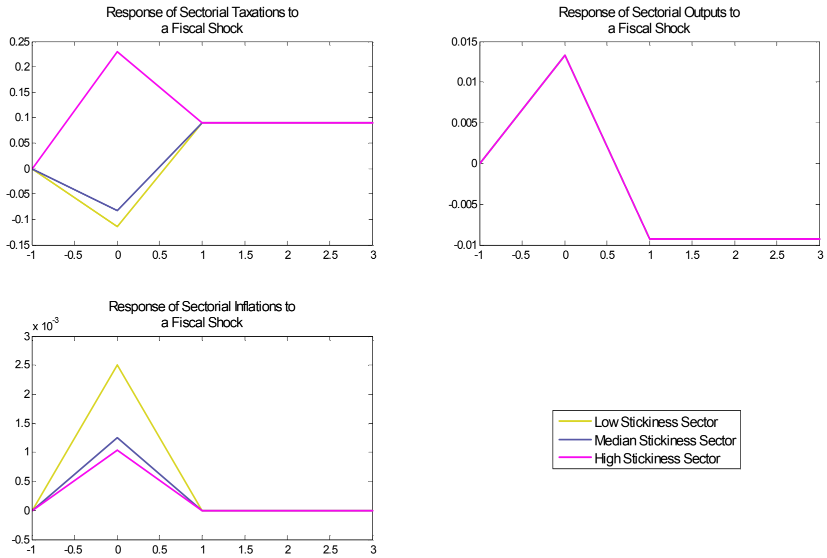

Figure 2: Effects of a Fiscal Shock on Sectorial Variables

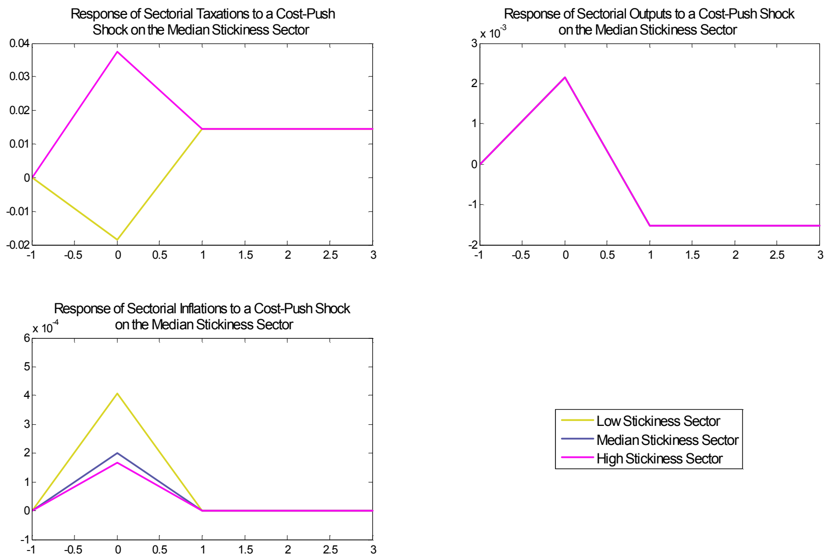

Figure 3: Effects of a Cost-Push Shock in Median Sector on Sectorial Variables.

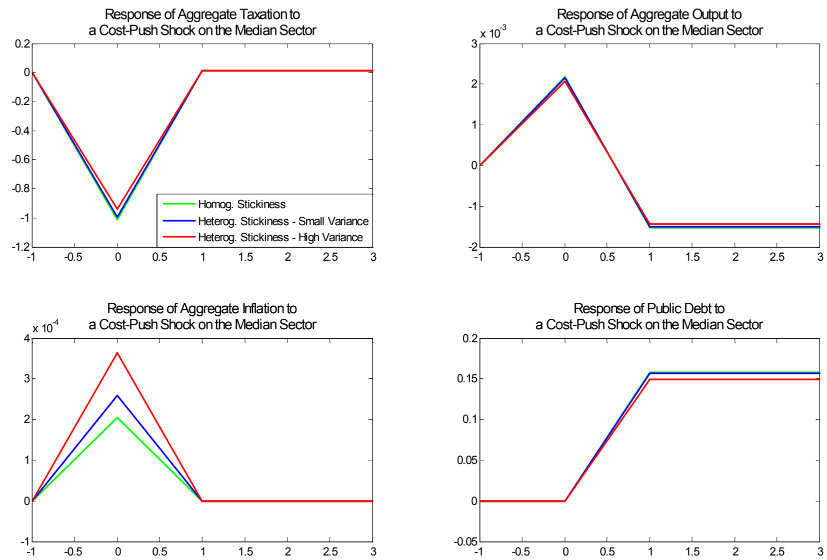

Figure 4: Effects of a Cost-Push Shock in the Median Sector on Aggregate Variables.

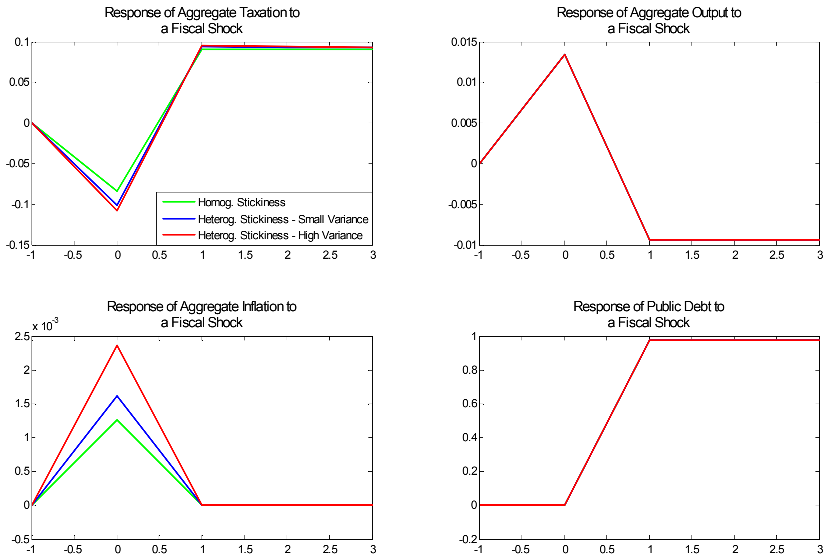

Figure 5: Effects of a Fiscal Shock on Aggregate Variables: Homogeneous Taxation Case.

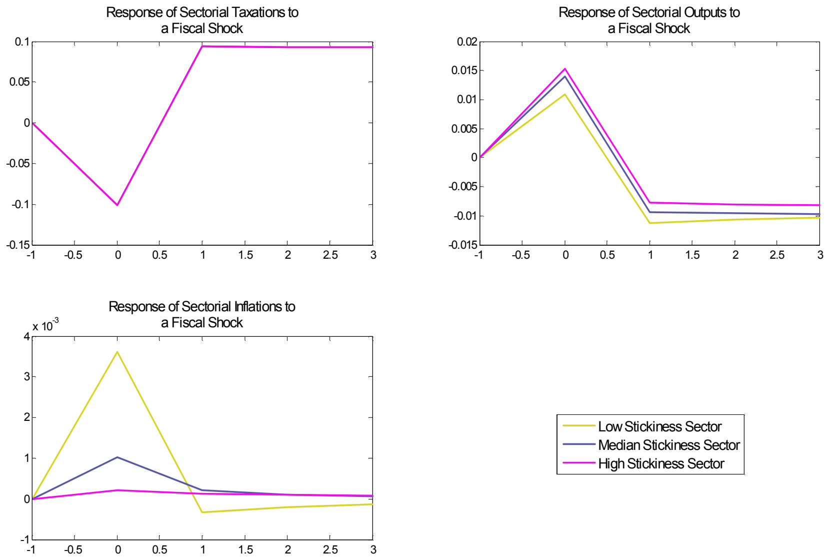

Figure 6: Effects of a Fiscal Shock on Sectorial Variables: Homogeneous Taxation Case.

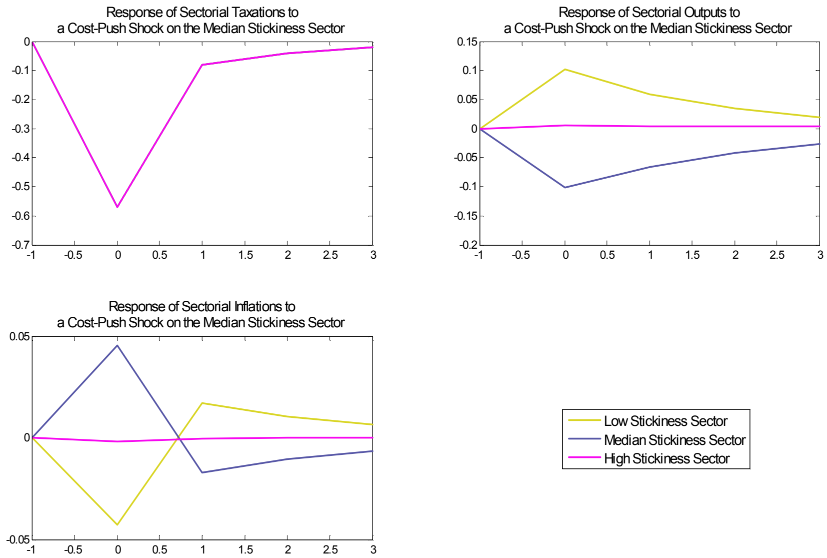

Figure 7: Effects of a Cost-Push Shock in the Median Stickiness Sector on Sectorial Variables: Homogeneous Taxation Case.

Reference for Parameter Estimation	Low Variance Heterogeneous Economy	High Variance Heterogeneous Economy
Smets and Wouters (2007) US: 1966QI-2004QIV	0.0223%	0.2051%
Smets and Wouters (2005)* US: 1974QI-2002QII	0.0567%	0.4895%
Smets and Wouters (2005)* Euro Area: 1974QI-2002QII	0.0523%	0.4449%
Justiniano, Primiceri and Tambalotti (2008)** US: 1954QIII-2004QIV	0.0380%	0.3475%

*Wage markups are estimated as random noises instead of AR(1)s.

** Only neutral technology shock considered.

Table 1: Welfare losses under misperception of heterogeneity in price stickiness (% difference from 1st best in steady state consumption level)

Reference for Parameter Estimation	Low Variance Heterogeneous Economy	High Variance Heterogeneous Economy
Smets and Wouters (2007) US: 1966QI-2004QIV	0.0014%	0.0023%
Smets and Wouters (2005)* US: 1974QI-2002QII	0.0007%	0.0034%
Smets and Wouters (2005)* Euro Area: 1974QI-2002QII	0.0007%	0.0031%
Justiniano, Primiceri and Tambalotti (2008)** US: 1954QIII-2004QIV	0.0027%	0.0043%

*Wage markups are estimated as random noises instead of AR(1)s.

** Only neutral technology shock considered.

Table 2: Welfare losses under homogeneous taxation (% difference from 1st best in steady state consumption level).