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An Econometric Understanding of the Monetary Policy's Dark Art*

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Abstract

The classical Taylor rules usually do not yield the same estimation error when working in a monthly or a quarterly framework. This brings us to the conclusion that there must be something that monthly Taylor rules can capture and that the quarterly one cannot: we postulate that it simply boils down to the fact that the target rate's changes are irregularly spaced in time. So as to tackle this issue, we propose to split the target rate chronicle between changes in the target and the associated durations, that is the time spending between two changes in the target rate. In this framework, we propose to consider that changes in rate can be regarded as a real monetary policy decision, whereas the duration period between two changes can be related to a "wait and see" position or some *fine tuning* problematic. To show that both these features of monetary policy do not react to the same fundamentals, we propose an econometric understanding of the Fed's reaction function using a new model derived from financial econometrics that has been proposed by Engle and Russell (2005). We propose to model the changes in target rates with a classical ordered probit and the durations with an autoregressive conditional duration model. We extracted the Fed anticipations regarding inflation and activity using some factor based method, and used these factors as explanatory variables for the changes in rates and the related durations. We show that the target rate level, the scale of the change in target rate and the associated duration do not necessarily react to the same factors and if they do, the impact can be different. This empirical result supports the idea that durations and scale of the change in target rate deserve equal attention when modeling a Central Bank reaction function.

Keywords: Taylor rule, duration models, probit models, Central Bank expectations, factor based methods.

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1 Introduction

Most of the empirical work dedicated to Taylor rules tends to ignore the key fact that the changes in target rate are both irregularly spaced and of various scales. However, some empirical literature dedicated to Taylor rules came up with some important results. For example, as presented in Fischer and Zurlinden (2004), the duration between two changes in rate provides information on the way a central bank conducts its monetary policy. The scale of the changes are also bound to convey information about - among others - the development of inflation within the economy. In this respect, the split of the target rate chronicle between durations and scales naturally allows anyone to discuss the shape of a Central Bank reaction function in a much more empirically grounded framework, regarding the sole moment of an effective change in the target rate as a monetary policy decision. The moments the Central Bank produces no move - that is the duration periods - can hardly be considered the same way. By modeling the joint distribution of the scale and durations using a model resembling of Engle and Russell (2005)'s, we show that they scarcely respond to the same macroeconomic fundamentals, supporting the idea that they both deserve equal attention when modeling a reaction function.

Since the seminal paper of Taylor (1993), the specification of rules - and particularly interest target rules - appears as a powerful approach to understand the way a central bank anchors inflation expectations. Supported by optimal target rate rules theory (see e.g. the impressive contribution of Woodford (2003) and the literature cited within), Taylor rules received an increasing attention over the past ten years. See for example Clarida et al. (1998, 2000) and the review of literature presented in Clarida et al. (1999).

Nonetheless, this empirical approach cannot handle yet some of the key facts of the Central Banks' behavior. Goodfriend (1990) proposes a rather comprehensive and surprisingly up-to-date summary of the stylized facts of monetary policy for the Federal Reserve Board. We choose to investigate three of these facts, to which we will had one more later. First, the Federal funds target is usually not adjusted immediately in response to new economic information, but until enough information has been collected. Secondly, changes in the target rate are made of jumps of 25 to 75 basis points, making of it a discrete process. Third, these changes are irregularly spaced in time. A fourth assessment emerged over the past ten years: the datasets at hand of the central bankers usually exceed those considered in the classical Taylor rules. In order to account for the Central Banks' behavior, it is necessary to find a way to incorporate the information contained in the large databases used by monetary policy makers. We investigate these four points using the Federal Reserve Board as an example.

For now, the first three considerations mentioned above led to the analysis of the central bank target rate as a discrete process, which thus entitles the estimation of models in the spirit of the classical multinomial probit or logit models (see e.g. Carstensen (2005) for an application to the European Central Bank). But most of these contributions focuses on the level of the target rate and provides little intuitions about the scale of the changes' factors. Then, the autoregressive component of the duration between two changes in the target rate has drawn attention (see e.g. Fischer (2000); Fischer and Zurlinden (2004)), driving people to assess that the Central Banks' interventions are *self exciting*. Unfortunately, using a duration time basis instead of a calendar time one makes it difficult to relate the durations to a data set that is naturally built on the latter type of time. To deal with this point, Hamilton and Jorda (2000) proposed to use an autoregressive conditional hazard model (joint with a probit model for the scale of the changes in the target rate) that makes the information incorporation easier. Their model seems to be well grounded from a prediction point of view. Nevertheless, we argue here that jumping to a duration time basis can help understand and tell what really brings about a larger change in the target rate *or* a longer duration period. At least, the articles cited above supports the idea of a strong need for models that are able to deal with the stylized facts cited earlier.

We propose here to show that simple Taylor-like rules fail to capture the fact that interest rate targeting is a double- or a triple-edge weapon: first, the rates' level convey information to the economic agents about the expected upcoming inflation and economic activity; second, the duration between two changes is an instrument on its own, linked to the idea of *interest smoothing* (see e.g. Woodford (1999)): the pace at which the central bank raises the rate can differ in response to the nature of the shocks affecting the economy; third, the scale of the change in rates works with the duration associated to smooth or sharpen the reached level. The understanding of these features underlines the fact that monetary policy - just as the rest of economic policy - is also a matter of fine tuning. With reference to the opening citation of Clarida et al. (1999), the rate level may depend on science, but the two remainders are the result of the monetary policy's *dark art*.

In this framework, we also propose to deal with the fourth critique mentioned above: in front of the growing dataset at hand for monetary policy decisions, recent developments in factor-based methods propose to sum up this huge amount of information into a few factors, often using principal component method to compute them. See e.g. Stock and Watson (1999) and the application of these methods to monetary policy in Bernanke and Boivin (2001). Such methods makes it possible to use a dataset as large as the one used by central bankers. We propose here to use another of these factor based methods that offers the advantage of relating the exogenous variables and the endogenous one when computing the factors: the *partial least squares* method.

Finally, we propose a framework that makes it possible to deal with the four features of monetary policy mentioned above: the discreteness of the process of the target rate, the importance of duration between two decisions, the accumulation of information necessary to motivate a decision of the Central Bankers and the importance of the huge data set at hand of Central Bankers. By reversing a Taylor relationship between anticipations and target rates, we derive a few leading factors that we call central bank expectations. Then, we use these factors and the shocks they are made of to explain the changes in rate and the durations of the Fed's monetary policy from July 1989 until November 2005. We compute the conditional joint distribution of the changes in target rate and of the durations associated in a framework close to that of Engle and Russell (2005), using the factors extracted as exogenous variables. We find that the factors that influences both these features are not necessarily the same, and when they are, the influence can be different. This is the main contribution of this paper: level, scale and durations are three features to consider carefully when trying to understand monetary policy.

This paper is organized as follows: in Section 2, we propose some motivations for this paper along with a method to extract the factors hidden behind Central Bank behavior. In Section 3, we modelize the target rate, separating the durations and the changes in rate periods, using a probit specification and a duration model specification. Section 4 analyses the results obtained with both the probit and the duration models and draw some important conclusion from these empirical results. Section 5 concludes. In appendices, we propose the detail of the dataset we used.

2 Some motivations and methods for Taylor rules

2.1 Motivations

Clarida et al. (2000) propose a smoothing version of the famous Taylor rule for a given Central Bank's target rate of the form[1]:

$$r_t = (1-\rho)\gamma + (1-\rho)\alpha\left(\mathbb{E}[\pi_{t+h}|\mathcal{F}_t] - \bar{\pi}\right) + (1-\rho)\beta\left(\mathbb{E}[y_{t+j}|\mathcal{F}_t]\right) + \rho r_{t-1} + \epsilon_t \quad (1)$$

where r_t is the Central Bank effective target rate at time t, $\{\alpha, \beta, \gamma, \rho\}$ some constant parameter and $\mathbb{E}[\pi_{t+h}|\mathcal{F}_t]$ and $\mathbb{E}[y_{t+j}|\mathcal{F}_t]$ are respectively the Central Bank expectation at time t of some inflation index π for a future date $t+h$ and of some real activity indicator y for a future date $t+j$. Note that $\bar{\pi}$ can either be the inflation official target by the Central Bank or some average inflation on a certain scope of calendar dates. ϵ_t is a white noise disturbance with mean 0 and variance σ^2. $\mathcal{F}_t$ denotes the filtration derived from the whole existing information at time t.

As a starting point, we reestimated Clarida et al. (2000)'s reaction function using two kinds of instrumental variables: on the base of the large dataset detailed in the appendices, we extracted the first two orthogonal factors using principal component analysis on the one hand and partial least square algorithm (in a fashion that will be detailed later) on the other hand. Both these methods are described with precision in Aguiar-Conraria (2003) and in the references within. We used these orthogonal factors as instrumental variables to estimate the following version of the reaction function proposed in equation (1):

$$r_t = \gamma + \alpha\pi_t + \beta y_t + \rho\epsilon_{t-1} + \epsilon_t \quad (2)$$

Some obvious calculations and rearrangements permit to show that the reaction functions proposed in equation (1) and (2) are identical, when assuming that the Central Bankers' expectations regarding macroeconomic variable are some kind of sticky, i.e. one can write $\mathbb{E}[\pi_{t+h-1}|\mathcal{F}_{t-1}] = \xi_1\mathbb{E}[\pi_{t+h}|\mathcal{F}_t]$ and $\mathbb{E}[y_{t+j-1}|\mathcal{F}_{t-1}] =$

[1]For the purpose of the demonstration, we slightly modified their original equation. Nevertheless, the main features remain globally unchanged.

	PCA		PLS	
	Quaterly	**Monthly**	**Quaterly**	**Monthly**
α	1.35	1.18	3.37	3.32
Standard Deviation	*0.17*	*0.12*	*0.13*	*0.07*
β	0.67	0.68	-0.28	-0.29
Standard Deviation	*0.08*	*0.04*	*0.04*	*0.02*
γ	-0.92	-0.43	-4.24	-4.11
Standard Deviation	*0.63*	*0.42*	*0.42*	*0.22*
ρ	0.89	0.84	0.60	0.84
Standard Deviation	*0.05*	*0.04*	*0.08*	*0.03*
R^2	0.87	0.87	0.96	0.97
RMSE	0.87	0.88	0.52	0.39

Table 1: Estimates for the equation (2).

$\xi_2\mathbb{E}[y_{t+h}|\mathcal{F}_t]$ for some $\{\xi_1, \xi_2\}$ close to one. When these expectations are constant over the time (which is a limiting case of stickiness), we have exactly:

$$r_t = \gamma + \alpha\pi_t + \beta y_t + \rho(r_t - \gamma - \alpha\pi_t - \beta y_t) + \epsilon_t \tag{3}$$

$$\Leftrightarrow r_t = (1-\rho)\gamma + (1-\rho)\alpha\pi_t + (1-\rho)\beta y_t + \rho r_{t-1} + \epsilon_t \tag{4}$$

This latter model presents the advantage to nest the CGG model as a special case, in this limiting case. The second advantage of this approach is an econometric one: when estimating Taylor rules, one often get autocorrelated errors, which threatens the estimation results. In our specification, we got rid of his threat by incorporating the autocorrelation within the model. The estimation of this model using PCA ans PLS instrumental variables is presented in table 2.1. At a first glance, the quarterly and the monthly estimations do not yield the same estimates and root mean square error (RMSE). This instability may be explained by the convergence pace of the estimates (given that we have the same starting date for the same dataset, the number of observations in the quarterly database is equal to third of the monthly one's). To investigate this point, we simply ran the estimations of the monthly model for periods with a length equal to the one of the quarterly database. We present the evolution of the RMSE for this sliding width in the figure 1.

Figure 1: Evolution of the RMSE for a constant number of observations (dotted line) vs. the quarterly model RMSE

Obviously, the RMSE remains unstable, and in the PCA case can even jump above the quarterly RMSE. Considering the fact that we kept the same number of observation as for the quarterly case, those divergence cannot be imputed to the convergence pace of the estimates. Our point is the following: a possible explanation for this phenomenon is linked to the structure of the data. We argue that the classical

Taylor rule approach leads to a misleading idea: the periods with no change in the target rate may not have the same fundamentals as the ones when the Fed decide to change raise or lower its target. To say things in a different manner, the determinants of the target rate level may differ from the one of the change in target rate scale and the duration associated.

In the following, we propose a methodology allowing to provide an empirical proof of this hypotheses.

2.1.1 Extracting the Central Bank expectations

In this Section, we propose a simple method to derive the unobserved factors that drive monetary policy using the PLS algorithm. Let us first start with the following motivation: relying on the assumption that there are only two variables that are driving monetary policy may be useful from a theoretical point of view, but from an empirical one, it will not hold. As noted in Bernanke and Boivin (2001), monetary policy makers have at disposal huge data sets of macroeconomic series that help to build a representative picture of the current macroeconomic context. To give an understanding of central bankers' behavior, using a data-set as close as possible from theirs does not seem to be a misleading hypothesis.

Let $\mathcal{F}_t$ be the filtration at time t, derived from Ω_t the dataset available at that time; Ω_t is a matrix $M(t,p)$ whose p columns are made of the different series observed by the Central Bank (in fact by any economic agent) until time t. Then, we have $\mathcal{F}_t = \sigma(\Omega_t)$. We can now write a forward looking Taylor rule for r_t such as, $\forall t \in \mathbb{Z}$ and for a fixed $h \in \mathbb{N}$:

$$r_t = f\left(\mathbb{E}[\pi_{t+h}, y_{t+h}|\mathcal{F}_t]\right) + \epsilon_t, \tag{5}$$

where $(\epsilon_t)_t$ is a white noise, i.e. an exogenous disturbance. The processes $(y_t)_t$ and $(\pi_t)_t$ have very little chance to be independent: thus, we cannot distinguish each of the anticipations. We make the assumption that f is linear and a more classical specification of the Taylor rule is, $\forall t \in \mathbb{Z}$:

$$r_t = \alpha_0 + \beta_1 \mathbb{E}[\pi_{t+h}, x_{t+h}|\mathcal{F}_t] + \epsilon_t, \tag{6}$$

where α_0 and $\beta_1 \in \mathbb{R}$. In the remaining of the paper, we regard $\mathbb{E}[\pi_{t+h}, x_{t+h}|\mathcal{F}_t]$ as the anticipations of the Central Bankers: these are in fact the anticipations of the variables of interest for the Fed, conditionally upon the past filtration $\mathcal{F}_t$.

Based on these assumptions, we are now able to reverse the relation (6), in order to extract the anticipations hidden behind the Fed's instrument: the Fed fund rate target. To do so, we made the following hypothesis on the chosen form of the Central Bank's anticipations: the anticipations of the Central Bank (i.e. the factors that drive monetary policy, and thus the target rate r_t) are linear combinations of the elements of the data set (Ω_t) at their disposal.

On the ground of this statement, we propose here to use one of the factor-based method developed since the seminal work of Stock and Watson (1999). Most of these methods are related to principal component analysis, and robust properties of these models have been obtained (see e.g. Forni et al. (2000)). Nevertheless, as explained in Aguiar-Conraria (2003), the principal components methods are constructed without taking into consideration any relationship between the regressors and the dependent variable. The *Partial Least Squares* (PLS) seems to provide a linear method that settles the latter problem. For references and a clear development of the method, we refer to Aguiar-Conraria (2003). The algorithm of the method is developed in the appendices, along with the detailed data set that we used. The figure 2 provides a plot for each of the underlying factors. The idea here is to find a proper way to compute a few factors relating Ω_t and r_t, that are linear combinations of the elements of Ω_t. By construction, the factors are orthogonal, allowing the simple use of OLS. Note that the data set may include current as well as lagged values of the data at hand.

What is more, the use of these methods seems to reduce the *data vintage problem* that arose over the past years: macroeconomic data are often subject to revisions: using initial as well as revised data seems to lead people to "*mix apple and oranges*", as asserted in Kishor and Koenig (2005). The previous paper by Bernanke and Boivin (2001) concludes with the fact that data revision is not that much a problem when handling with huge data sets and factor-based methods, which is what we are about to do here.

Figure 2: Each of these plots present the evolution of the first four factors extracted from the dataset using the PLS algorithm (dotted line), jointly with the evolution of the Central Bank target rate (plain line). Both the factors and the target rate where scaled for the graphics in order to make them comparable.

In this respect, the Central Bank's expectations can be rewritten using $\{f_{1,t}, f_{2,t}, ..., f_{j,t}\}$ the j factors extracted from the data set. Then, from equation (6), we get:

$$r_t \quad = \alpha_0 + \sum_{i=1}^{j} \alpha_i f_{i,t} + \epsilon_t \tag{7}$$

What we are actually looking for is a proper model for the changes in the target rate and the durations associated. The proposed model is grounded on the Taylor rule proposed in equation (7) : once we propose a model for the target rate level, it is easy to infer a model for the changes in the target rate. With the computed factors, one can derive a model for $\Delta r_t = r_t - r_{t-1}$, i.e. the scale of the current change in the Fed's target rate. If the former change occurred at time t_j and the current at time t_k, we can then rewrite the model proposed in the equation (7) this way:

$$r_{t_k} - r_{t_j} \quad = \sum_{i=1}^{j} \alpha_i \left(f_{i,t_k} - f_{i,t_j}\right) + \epsilon_{t_k} - \epsilon_{t_j}. \tag{8}$$

If the white noise $(\epsilon_t)_t$ has a Gaussian distribution function, then the random variable $(\epsilon_{t_k} - \epsilon_{t_j})$ is still Gaussian. Thus, the model we propose assumes that the changes in the target rate are led by the changes in the level of the factors. What is more, these variations of the factors can be rewritten as a sum of the changes for each time interval, for $i = 1, ..., j$:

$$\left(f_{i,t_k} - f_{i,t_j}\right) = \quad \left(f_{i,t_k} - f_{i,t_{k-1}}\right) + \left(f_{i,t_{k-1}} - f_{i,t_{k-2}}\right) + \ldots + \left(f_{i,t_{j+1}} - f_{i,t_j}\right) \tag{9}$$

$$= \Delta f_{i,t_k} + \Delta f_{i,t_{k-1}} + \ldots + \Delta f_{i,t_{j+1}} \tag{10}$$

with $\Delta f_{i,t_k} = f_{i,t_k} - f_{i,t_{k-1}}$. The changes in each of the factors on a duration time basis can be interpreted as a sum of shocks, namely the $\{\Delta f_{i,t_k}\}$ as shown in equation (10). From an economic point of view, this fact tends to provide empirical evidence of one of the stylized facts underlined in Goodfriend (1990): each change in the Fed's target rate is the result of an accumulation of shocks in the variables that drive monetary policy. Monetary policy makers seem to wait until the accumulation of shocks is sufficient to justify their decision of changing the target rate[2]. This way of writing the variations of the shocks provides a fertile way to explain both the changes in target rate moment and the duration between two decisions. In the next section, we derive a model for estimating the joint probability distribution function of the target changes and the durations, using a model close to the one developed in Engle and Russell (2005) and relating the scale changes and the durations associated to the estimated factors.

Before skipping to this model, some economic thoughts must be put forward here. The fact that we use a change in time, moving from a calendar time to a *decision* time is something deeper than what it actually seems. In the framework that we propose, we only consider as *decision* the realization of an effective change in the monetary policy. The no-change points are often linked to a "wait and see" strategy. To our mind, the fact that the Fed fund rate remains unchanged has two possible meanings: first, it can mean that the governors are waiting for the effects of their

[2]Here is the quote taken from Goodfriend (1990): "*The federal funds target has not been adjusted at irregular intervals to new information. Rather, the target has been adjusted at irregular intervals only after sufficient information has been accumulated to trigger a target change.*"

previous change in target rate to come out: the behavior adjustment of the economic agent can take time to show; Secondly, it can also mean that the economic context cannot be easily understood and that the governors prefer to postpone their decision to change the target rate level.

This latter hypothesis can be related to the literature dedicated to the analysis of monetary policy facing uncertainty. This uncertainty is difficult to modelize: it can arise from different facts. For example, the temporary breaking of the well-known relationships between economic indicators can bring about a policy puzzle: central bankers should wait and see how things are going before taking regretful decisions. Another example of these problems related to economic uncertainty is the following: the volatility of the most watched indicators can be a serious problem. Some strong movement in these leading indicators can be spurious information about the future of the economy, because of the importance of the noisy part of the signal conveyed during high volatility periods.

These points are not handled in this paper: we assume the expectations of central bankers are given, and our main task is to relate these factors to joint probability distribution function of changes in rate and durations associated. Once again, we extracted some factors whose variations are able to explain the scale of the changes in the target rate. We will use the same factors to try to explain the durations between two changes, in order to show that they do not react to same economic fundamentals.

In the next part, we present a model for the joint process of the changes in target rate and the durations, based on the extracted factors presented above.

3 A model explaining the changes in the target rate and the durations

The aim of this section is to provide a background to modelize the Fed target rate, separating the durations and the changes-in-rate periods. We will use the factors computed as presented above. Let Δr_k be the k^{th} change in target rate; let τ_k be the duration associated with the k^{th} change in rate. As proposed earlier, let $\mathcal{F}_k$ be the filtration derived from Ω_k, the dataset at hand at time k. Thus, when Central Bankers have at their disposal a dataset made of p different series, we have: $\mathcal{F}_k = \sigma\left(\Omega_{j,s}, \forall s \leq k, \forall j = 1, ..., p\right)$. The purpose of the proposed method is to compute the joint distribution $f(\Delta r_k, \tau_k|\mathcal{F}_k)$, i.e. of the duration and the change in rate process, according to the filtration $\mathcal{F}_k$. Note that we do not use anymore t to indicate the time, but k to underline the fact that we jumped from a calendar time to a *duration time*. Without any loss of generality and in the spirit of Engle (2000), it is easy to show that the following equation holds, using simple Bayes's rule:

$$f(\Delta r_k, \tau_k|\mathcal{F}_k) = g(\Delta r_k|\tau_k, \mathcal{F}_k)q(\tau_k|\mathcal{F}_k), \tag{11}$$

where $g(.)$ denotes the probability density function associated with the rates changes $(\Delta r_k)_k$ and $q(.)$ the probability density of the duration process $(\tau_k)_k$, both conditionally upon the filtration. This conditional model provides a fertile framework for the understanding of the Fed monetary policy over the past decades.

We need to specify a model for the changes in rate and a model for the duration process. The latter is directly inspired from Engle and Russell (1994): Fischer (2000) and Fischer and Zurlinden (2004) put forward the fact that the duration process presents a strong autoregressive component and thus led us to the use of an *Autoregressive Conditional Duration Model* as it was first developed in the article of Engle and Russell (1994). Note that this model is taken from Econometrics of Finance and applied here to an economic problem. It seems to be quite natural to modelize the change in target process $(\Delta r_k)_k$ using an ordered probit model: in such a framework, we will show that it is easy to recover specification *à la Taylor* for the latent process of the model from equation (8). The combination of both the ACD and the ordered probit model can be seen as a special case of the ACD-ACM model developed in Engle and Russell (2005).

The remaining of this section is organized as follow : first, we present the simple use of a probit model for the changes in rate. Then, we discuss how to use duration model, and how to introduce the exogenous variables - that is the factors - within this model.

3.1 The Probit Specification

Engle and Russell (2005) propose to characterize the dynamic of the price change through a Markov chain, using dynamic conditional transition matrices. We propose here to use one of the special cases of the ACM model proposed in Engle and Russell (2005), that is an ordered probit model, in the spirit of Hausman et al. (1992). This type of econometric model is widely used for the analysis of transaction data, and the reader can have an insight into the use of such models in Gerhard (2001). We apply these methods to the econometrics of monetary policy in the following manner.

Let Ω_k be the set of exogenous variables at disposal at time k. Let $\mathcal{G}_k$ be the filtration derived from (Ω_k, τ_k), that is $\mathcal{G}_k = \sigma(\Omega_s, \tau_s, \forall s \leq k)$. The essence of the ordered probit analysis is the assumption that the observed rate changes Δr_k are related to the random variable $(r_k^*)_k$, which is unobserved. Hausman et al. (1992) propose a nice presentation of these models, along with numerous references. From section 2, we specify the latent variable using equation (8):

$$\Delta r_k^* \quad = \textstyle\sum_{i=1}^{u} \alpha_i \Delta f_{i,k} + \Delta \epsilon_k \tag{12}$$

where $\Delta r_k^* = r_k^* - r_{k-1}^*$, $\Delta f_{i,k} = f_{i,k} - f_{i,k-1}$ and $\Delta \epsilon_k = \epsilon_k - \epsilon_{k-1}$ is a Gaussian random variable mean 0 and variance σ^2. We rewrite equation (12) the following way:

$$\Delta r_k^* = \alpha \Delta f_k' + \Delta \epsilon_k, \tag{13}$$

where α is an $M(1 \times u)$ matrix containing the coefficients corresponding to the α_i in the equation (12); Δf_k is an $M(1 \times u)$ matrix containing the series of each of the changes in the factors for time k, on a duration time basis. Time k is the moment of the target rate change and time $k-1$ is the date of the former one. We also need to characterize the thresholds this latent variable has to cross for the observed variable to go through an actual change. This can be represented in the following manner:

$$\Delta r_k = \begin{cases} s_1 \text{ if } \Delta r_k^* \in A_1 \\ s_2 \text{ if } \Delta r_k^* \in A_2 \\ \vdots \\ s_m \text{ if } \Delta r_k^* \in A_m \end{cases}, \tag{14}$$

where the sets A_j form a partition of the state space $\mathcal{S}^*$ of Δr_k^*, i.e. $\mathcal{S}^* = \cup_{j=1}^m A_j$. In this approach we have:

$$s_j = \{-1, -0.75, -0.5, -0.25, 0.25, 0.5, 0.75, 1\}. \tag{15}$$

Equation (14) can be rewritten using real thresholds denoted $(\nu_i)_{i \in [1,m]}$ instead of a simple partition of $\mathcal{S}^*$:

$$\Delta r_k = \begin{cases} s_1 \text{ if } \Delta r_k^* \leq \nu_1 \\ s_2 \text{ if } \nu_1 \leq \Delta r_k^* \leq \nu_2 \\ \vdots \\ s_{m-1} \text{ if } \nu_{m-2} \leq \Delta r_k^* \leq \nu_{m-1} \\ s_m \text{ if } \nu_{m-1} \leq \Delta r_k^* \end{cases}. \tag{16}$$

From equation (3), it can be easily seen that for m states, the estimation only requires the inference of $m-1$ thresholds. Knowing that $(\Delta \epsilon_{t_k})_{t_k}$ is a centered Gaussian process with variance σ^2, we can write the probability that $\Delta r_k = s_i$ conditionally upon the corresponding filtration using the probit assumption:

$$\begin{aligned} P(\Delta r_k = s_i | \mathcal{G}_k) &= \Phi\left(\frac{\nu_1 - \alpha \Delta f_k'}{\sigma}\right) \text{ if } i = 1 && (17) \\ &= \Phi\left(\frac{\nu_i - \alpha \Delta f_k'}{\sigma}\right) - \Phi\left(\frac{\nu_{i-1} - \alpha \Delta f_k'}{\sigma}\right) \text{ if } 1 < i < m && (18) \\ &= 1 - \Phi\left(\frac{\nu_{m-1} - \alpha \Delta f_k'}{\sigma}\right) \text{ if } i = m, && (19) \end{aligned}$$

where Φ denotes the standard Gaussian cumulative distribution function. Knowing this, it is straightforward to obtain the log-likelihood function associated to (Δr_k) required for the estimation of the parameters:

$$\begin{aligned} logL = \sum_{k=1}^{K} \Bigg[& \mathbb{1}_{\Delta r_k = s_1} log\left(\Phi\left(\frac{\nu_1 - \alpha \Delta f_k'}{\sigma}\right)\right) && (20) \\ & + \sum_{i=2}^{m-1} \mathbb{1}_{\Delta r_k = s_i} log\left(\Phi\left(\frac{\nu_i - \alpha \Delta f_k'}{\sigma}\right) - \Phi\left(\frac{\nu_{i-1} - \alpha \Delta f_k'}{\sigma}\right)\right) && (21) \\ & + \mathbb{1}_{\Delta r_k = s_m} log\left(1 - \Phi\left(\frac{\nu_{m-1} - \alpha \Delta f_k'}{\sigma}\right)\right)\Bigg], && (22) \end{aligned}$$

where K is the actual number of changes in target rate, that is the total number of observations we have at hand, on a duration time basis. One can obtain maximum likelihood estimates by numerically maximizing the latter expression with respect to the unknown parameters. Note that we chose to ignore the fact that the distribution of Δr_k is taken conditionally upon the current duration. We skipped the τ_k during the presentation of the probit model in order to make the presentation clearer. In the part dedicated to the estimation of the parameters, the latent variable is augmented by the current duration in the following manner:

$$\Delta r_k^* = \alpha \Delta f_k' + \beta \tau_k + \Delta \epsilon_k \tag{23}$$

where $\beta \in \mathbb{R}$ and using the same assumption we made for equation (13) about $\Delta \epsilon_k$. The next section deals with the task of specifying a model for the duration process.

3.2 The duration model specification

Bauwens and Giot (1998) provide a detailed presentation of duration models specifications for ACD models. The usual specification for the observed duration τ_k is the following:

$$\tau_k = \Psi_k \epsilon_k, \tag{24}$$

where $(\epsilon_k)_k$ is a white noise process defined on $\mathbb{R}_*^+$, with $\mathbb{E}[\epsilon_k] = \mu$ and Ψ_k a process defined below. Thus, using the previous notations, we have $\mathbb{E}[\tau_k|\mathcal{G}_{k-1}] = \mu \Psi_k$. It is worth mentioning that μ is not a parameter to estimate, but a function of the parameters of the distribution function of the ϵ_k. Note that one can normalize the random variable ϵ_k by dividing them by their expectation: then we obtain a new sequence of noises with mean equal to 1. From now on, we use the same framework as Engle and Russell (1995). The equations (24)-(27) specify the autoregressive model called ACD(1,1) for the duration process $(\tau_k)_k$, defined conditionally on the filtration:

$$\Psi_k = \omega_0 + \omega_1 \tau_{k-1} + \omega_2 \Psi_{k-1}, \tag{25}$$
$$\text{with } \omega_0 > 0, \omega_1 > 0, \omega_2 > 0 \tag{26}$$
$$\text{and } \omega_1 + \omega_2 < 1 \tag{27}$$

where τ_{k-1} is the last duration observed and Ψ_{k-1} the last expectation of the duration process computed. To ensure the positivity of the conditional durations, we impose the restrictions specified in equation (26). To ensure the existence of the conditional mean of duration, we also have to assume the condition presented in equation (27) (see Engle and Russell (1995)).

As proposed in Engle and Russell (1995), a tractable extension of their model is obtained adding some exogenous variables, denoted here $\Delta f_{i,k}$. Then, the model becomes:

$$\tau_k = \Psi_k \epsilon_k, \tag{28}$$
$$\Psi_k = \omega_0 + \omega_1 \tau_{k-1} + \omega_2 \Psi_{k-1} + \alpha \Delta f_k. \tag{29}$$

We will discuss the implications for the estimation procedure of adding these exogenous variables in the section dedicated to the empirical results.

To be able to estimate the parameters by maximum likelihood method, one must impose a specification for the distribution function of the noise process $(\epsilon_k)_k$. In the literature authors proposed that $(\epsilon_k)_k$ follows an exponential, a Weibull, a Burr or a generalized Gamma noise probability distribution function. However, we must remain cautious here: as put forward by Meitz and Teräsvirta (2004), if the true distribution of the errors is of the last three distributions, then the estimates will be consistent. However, if not, one will have to deal with Quasi Maximum Likelihood estimates, and each distribution cited before but the exponential one will not produce consistent estimates: these distributions do not belong to the linear exponential family. Here, we only need to produce consistent estimates, regardless of the true distribution of the errors. Thus, we assume that the $(\epsilon_k)_k$ is an exponential white noise process.

The model chosen for the duration process $(\tau_k)_k$ is therefore the following:

$$\begin{cases} \tau_k = \Psi_k \epsilon_k \\ \Psi_k = \omega_0 + \omega_1 \tau_{k-1} + \omega_2 \Psi_{k-1} + \alpha \Delta f_k \\ \omega_1, \omega_2 \geq 0 \text{ and } \omega_0 > 0 \\ \omega_1 + \omega_2 < 1 \end{cases}, \tag{30}$$

where $(\epsilon_k)_k$ is an exponential disturbance, with expectation equal to λ. Here we assume that $\lambda = 1$, so $\mathbb{E}[\tau_k|\mathcal{G}_{k-1}] = \Psi_k$, with $\Psi_k \in \mathcal{G}_{k-1}$. Then, we derive the associated log-likelihood of the model:

$$\begin{cases} L(\tau, \Psi) = \sum_{k=1}^{K} l_k = -\sum_{k=1}^{K} log(\Psi_k) + \frac{\tau_k}{\Psi_k} \\ \Psi_k = \omega_0 + \omega_1 \tau_{k-1} + \omega_2 \Psi_{k-1} + \alpha \Delta f_k \\ \omega_1, \omega_2 \geq 0 \text{ and } \omega_0 > 0 \\ \omega_1 + \omega_2 < 1 \end{cases}. \tag{31}$$

We provide estimates by numerically maximizing the log-likelihood function presented in equation (31).

4 Empirical Results

We detail here the dataset we used, along with the results of the estimations.

We used a large data set of macroeconomic variables extracted from Bloomberg's database: it includes 176 series of data. To build the factors, we chose to use the current value of each variable, along with its first to third lagged values. The reason why we made this choice was that it enabled us to work in a framework close to the quarterly database commonly used in the literature tackling those issues (see e.g. Taylor (1993) and Clarida et al. (1998)). We exclusively used monthly data because it is in line with the huge number of figures available on this monthly basis. What is more, Central Bank decisions are often made on a monthly basis.

We designed the data base we used so as to mimic as close as possible Stock and Watson (1999)'s. Knowing that some of the series present a non stationary behavior, we had to differentiate some of them. Note that we often used a twelve-month difference to obtain stationary and informative series. One major argument to do so is that many series are known among central bankers to be watched using these twelve-month differences. A good example of such a series is the *Consumer Price*

Index (*CPI*). Let CPI_t be the level of the *CPI* index reached at time t. Then the 12-month difference is the following:

$$\frac{CPI_t - CPI_{t-12}}{CPI_{t-12}} \times 100. \tag{32}$$

What is more, many of these series taken in first difference are really close to white noise: in this respect, it does not seem to provide any useful information for our purpose. We present in the appendix the list of variables that we used to compute the factors $f_{i,t}, i = 1, ..., j$, along with the differences we chose for each of the series.

As explained in Section 2, we computed the factors using the *PLS* algorithm. No well-known criterion exists to decide on the number of factors that should be used in the proposed framework. We decided to keep four of them for the following reasons: classical Taylor rules - that is models for the target rate *level* - are commonly known to dwell on two factors. We are trying to seek for two additional effects: a scale of the change in target rate effect and a duration effect. In this respect, we chose to maintain four of these factors. Note that there may be more underlying factors that may provide information for both durations and changes in target rate. Nevertheless, our point here is only to show that some factors that are usually neglected must be taken into account from now on.

Finally we have at disposal 91 observations of changes in target rate along with the duration associated and the changes in factor on a duration time basis. This is the dataset that is about to be used in the estimations.

4.1 Dealing with correlation

Even if the factors - taken in level - are instantaneously uncorrelated, the changes in the factors can be correlated. This can be shown very easily as follows:

$$\begin{aligned} corr(\Delta f_{i,t}, \Delta f_{j,t}) &= corr(f_{i,t} - f_{i,t-1}, f_{j,t} - f_{j,t-1}) \quad (33) \\ &= corr(f_{i,t}, f_{j,t-1}) - corr(f_{i,t-1}, f_{j,t}) \quad (34) \end{aligned}$$

which is not necessarily equal to 0. This problem is amplified by the change in time, skipping from $\Delta f_{i,t}$ to $\Delta f_{i,k}$, using the former notations. Before starting the estimation procedures, we have to examine the correlation matrix of the changes in factors, using a duration time basis. The estimated matrix is the following:

$$\begin{pmatrix} 1.00 & -0.60 & -0.43 & -0.03 \\ - & \textit{(-7.10)} & \textit{(-4.46)} & \textit{(-0.24)} \\ -0.60 & 1.00 & 0.16 & -0.52 \\ \textit{(-7.10)} & - & \textit{(1.58)} & \textit{(-5.73)} \\ -0.43 & 0.16 & 1.00 & 0.21 \\ \textit{(-4.46)} & \textit{(1.58)} & - & \textit{(2.09)} \\ -0.03 & -0.52 & 0.21 & 1.00 \\ \textit{(-0.24)} & \textit{(-5.73)} & \textit{(2.09)} & - \end{pmatrix}$$

Most of the coefficients of the correlation matrix are significative at the 5% risk level (the *t-stat* is given below each figure between brackets). This correlation problem can at least brings about unstable estimates, depending on the numbers of $\Delta f_{i,k}$

introduced in the proposed models. This simply means that we will have to be extremely careful when examining the estimation results. But this correlation will not necessarily jeopardize our results: given that we are working on a duration time basis, there is no evidence that this correlation is not spurious. Our point of view is that our framework makes the meaning of those correlations unclear. In this respect, we will produce alternative estimates of our models, using a corrected data set, based on this correlation. To do so, we will simply use for factor 2 to 4 the residuals of the linear regression of each of these three factors on the preceeding ones. For example, we used as $\Delta f_{2,k}$ the residuals of the following regression equation estimated by OLS: $\Delta f_{2,k} = \gamma f_{1,k} + \nu_t$, with the classical assumptions used when dealing with ordinary least square. By doing so, we obtain orthogonal exogeneous variables that should be harmless for the estimation procedure. By comparing the first results with those obtained with these new factors, we will be able to maintain some vigilance upon the effect of this correlation problem. The correlation matrix of the corrected factors is the following:

$$\begin{pmatrix} 1 & \text{-1.65E-16} & \text{2.44E-17} & \text{6.93E-17} \\ (0.00) & (0.00) & (0.00) & (0.00) \\ \text{-1.65E-16} & 1 & \text{5.30E-17} & \text{3.673317e-17} \\ (0.00) & (0.00) & (0.00) & (0.00) \\ \text{2.44E-17} & \text{5.30E-17} & 1 & \text{-2.08E-17} \\ (0.00) & (0.00) & (0.00) & (0.00) \\ \text{6.93E-17} & \text{3.67E-17} & \text{-2.08E-17} & 1 \end{pmatrix}$$

Note that from now on, we will call these orthogonal factors *corrected factors*. The rest of this section is organized as follows: first, we present the result of the estimation of the probit model proposed in the former section. Then, we comment the results of the estimation of the duration model proposed earlier.

4.2 Estimation of the probit models

The estimated model proposed in Section 3 is the following:

$$\Delta r_k * = \alpha_1 \Delta f_{1,k} + \alpha_2 \Delta f_{2,k} + \alpha_3 \Delta f_{3,k} + \alpha_4 \Delta f_{4,k} + \beta \tau_k + \Delta \epsilon_k \tag{35}$$

where r_k* is the unobserved latent process as defined in Section 3; $\Delta f_{i,k}$ is the change in the i^{th} factor on a duration time basis; τ_k is duration observed at time k and $\Delta \epsilon_k$ a Gaussian noise of mean 0 and variance σ^2. For this model, we provide five different estimations: four of them only include one of the four factors as an explanatory variable at a time; the last one includes the four factors at the same time. We used a similar approach with the corrected factors. Table 2 provides the estimation for the model using the uncorrected factors and the table 3 provides the estimation of the model using the corrected factors. For each table, t-stats are given between brackets below each estimated parameters.

Only two factors seem to have a clear influence on the changes in target rate process: factors 1 and 4, as both corrected and uncorrected factors clearly show.

	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5
α_1	0.0081	-	-	-	0.01895
	(4.44)				(6.38)
α_2	-	-0.0050	-	-	0.0254
		(-1.59)			(4.38)
α_3	-	-	-0.0043	-	0.0140
			(-0.39)		(1.05)
α_4	-	-	-	0.0430	0.1386
				(2.30)	(4.79)
β	-0.0221	-0.0091	0.0141	0.0097	0.0271
	(-0.65)	(-0.26)	(0.42)	(0.29)	(0.70)
Residual deviance	341.353	358.923	361.3073	356.1339	311.7363
AIC	371.353	388.923	391.3073	386.1339	347.7363

Table 2: Estimates of the first probit model

	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5
α_1	0.0081	-	-	-	0.0091
	(4.44)				(4.79)
α_2	-	0.0052	-	-	0.0067
		(1.36)			(1.70)
α_3	-	-	0.0222	-	0.0275
			(1.74)		(2.10)
α_4	-	-	-	0.1224	0.1386
				(4.40)	(4.79)
β	-0.0221	0.0270	0.0001	0.0553	0.0271
	(-0.65)	(0.78)	(0.00)	(1.60)	(0.70)
Residual deviance	341.353	359.6135	358.42	341.6864	311.7363
AIC	371.353	389.6135	388.42	371.6864	347.7363

Table 3: Estimates of the probit model with corrected factors

The last result for this first model is linked to the inclusion of the duration in the conditional probability to observe a change in target rate. The results are unanimous: there is no linear link of the duration level on the scale of the change. This is surprising and central for our analysis: as asserted in the first section, it may mean that both duration and scale of the change are independent instruments of the monetary policy of the Fed. This seems to reinforce the founding principle of our work: both the scales of the changes in target rate and the durations between two decisions are central bank instruments, and deserve in this respect all the attention of traditional instruments.

4.3 Estimation of the duration models

We detail first the estimation procedure, and then the results of the estimation of the duration model proposed in Section 3. Here again, we computed our estimations both with corrected and uncorrected factors.

The estimation procedure of the ACD models are far less known than the one of the probit models. Two elements have to be discussed here: first, the constraint of positivity of the conditional mean of the duration process; secondly, the estimation of the Fisher information matrix, in the case of maximum likelihood method and quasi maximum likelihood method.

The duration process is a made of positive values. The estimation of such a process requires that we check the positivity of the conditional duration at each step of the maximisation procedure. As explained in the section 2, when no exogenous variables are added, this requires:

$$\frac{\omega_0}{1-\omega_1-\omega_2} > 0 \tag{36}$$

which naturally requires: $\omega_0 > 0$ and $0 < \omega_1 + \omega_2 < 1$. Adding exogenous variables such as the changes in factors that can be negative requires to closely watch the positivity of the conditional duration process estimated. We perform the estimation using Metropolis Hastings algorithm: at each step of the maximization process of the log-likelihood, we add the constraint of having a positive estimated process. What is more, the use of such MCMC estimation method makes it possible to find the global maximum, regardless of the existence of local maxima. We found that the introduction of the changes in the factors on a duration time basis makes the classical estimation procedures (such as Fisher scoring) unstable. The use of this MCMC method ensures the significativeness of the results, without proving the strict concavity of the log-likelihood with the ACD augmented with exogenous variables.

Once the estimates are computed, the Fisher information matrix must be obtained. Engle and Russell (1995) propose to compute it directly. Knowing that $\mathbb{E}[\frac{\tau_k}{\psi_k}] = 1$, it is easy to see that:

$$I(\theta) = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{1}{\psi_k^2}\frac{\partial\psi_k}{\partial\theta}\frac{\partial\psi_k}{\partial\theta'}\right], \tag{37}$$

where $\theta = (\omega_0, \omega_1, \omega_2, \alpha_i, i = \{1, 2, ..., u\})$. One has to note that this estimation of the Fisher information matrix is asymptotically the same as the BHHH one (see Berndt et al. (1974)), which is the one used when performing quasi-maximum likelihood estimation. Nevertheless, we performed our estimations of the variance of the estimates using both these methods and found similar results. Given that we only have a hundred observations at hand (on a duration time basis), we preferred to use the first method, computing the Fisher information matrix as proposed in equation (37).

Now we comment the results obtained and we conclude.

4.3.1 Results of the estimation of the duration model

The estimated duration model is the following:

$$\begin{cases} \tau_k = \psi_k \epsilon_k \\ \psi_k = \omega_0 + \omega_1\tau_{k-1} + \omega_2\psi_{k-1} + \alpha_1\Delta f_{1,k} + \alpha_2\Delta f_{2,k} + \alpha_3\Delta f_{3,k} + \alpha_4\Delta f_{4,k} \end{cases} \tag{38}$$

where Ψ_k is the expected duration at time k as computed by the model, $\Delta f_{i,k}$ is the change in the i^{th} factor on a duration time basis and $\Delta\epsilon_k$ an exponential random variable with mean equal to 1.

Following the previous method developed for the probit model, we computed several estimations including, first, only one of the exogenous variables at a time and then the whole variables at the same time. This enables us to ensure the results obtained. We provide the results of our estimations for the duration model using uncorrected factors in table 4 and corrected factors in table 5. For each table, t-stats are given between brackets below each estimated parameters.

	Eq. 1	**Eq. 2**	**Eq. 3**	**Eq. 4**	**Eq. 5**	**Eq. 6**
ω_0	1.0270	1.1313	1.2169	1.0297	1.1749	0.9332
	(1.98)	(2.05)	(2.35)	(2.15)	(1.89)	(2.23)
ω_1	0.4357	0.4517	0.3510	0.4323	0.3273	1.5986
	(3.34)	(3.26)	(2.38)	(2.93)	(2.94)	(5.04)
ω_2	0.1957	0.1378	0.1977	0.1974	0.2529	0.0586
	(2.41)	(2.68)	(2.52)	(2.31)	(2.64)	(2.06)
α_1	-	0.3052	-	-	-	0.0218
		(3.07)				(0.51)
α_2	-	-	-0.3094	-	-	-0.5410
			(-2.95)			(-2.46)
α_3	-	-	-	0.0315	-	0.1035
				(0.83)		(1.61)
α_4	-	-	-	-	0.1674	-0.3805
					(2.76)	(-3.15)
Residual deviance	1.08	1.04	1.00	1.08	1.04	0.93
Log-likelihood	-183.6878	-182.5036	-180.9156	-183.6683	-183.0971	-179.6989

Table 4: Estimates of the duration model with uncorrected factors

	Eq. 1	**Eq. 2**	**Eq. 3**	**Eq. 4**	**Eq. 5**	**Eq. 6**
ω_0	1.0270	1.1313	1.9872	1.0565	0.6538	0.9127
	(1.98)	(2.05)	(2.21)	(2.63)	(3.08)	(3.44)
ω_1	0.4357	0.4517	0.0007	0.4270	0.7236	0.6062
	(3.34)	(3.26)	(2.04)	(2.91)	(2.95)	(4.75)
ω_2	0.1957	0.1378	0.2559	0.1920	0.1387	0.1590
	(2.41)	(2.68)	(2.41)	(2.13)	(2.76)	(2.21)
α_1	-	0.3052	-	-	-	0.3090
		(3.07)				(2.99)
α_2	-	-	-0.3548	-	-	-0.2351
			(-3.15)			(-3.10)
α_3	-	-	-	0.0681	-	0.0447
				(0.54)		(0.44)
α_4	-	-	-	-	-0.3909	-0.384
					(-2.83)	(-3.03)
Residual deviance	1.08	1.04	1.02	1.07	1.00	0.93
Log-likelihood	-183.6878	-182.5036	-180.9519	-183.5982	-182.3291	-179.7086

Table 5: Estimates of the duration model with corrected factors

Several points are worth being commented upon. As asserted in Fischer and Zurlinden (2004), there is evidence that the American Central Bankers' interventions *are* self-exciting. In each of our estimations, we found that the parameters of the autoregressive process are significative: this means that when the current duration is short (long), the next one will be also of a short (long) kind. Those results give some ground to the use of an ACD model to modelize the duration process.

The analysis of the estimation of the exogenous variables are of great interest. First, when the probit model only found two significant variables (factors 1 and 4), the ACD model led to three factors: 1, 2 and 4. The sign of the estimates conveys an important message for the analysis of monetary policy. Unlike with factors 2 and 4, a positive shock in factor 1 is supposed to lead to a greater duration. On the contrary, a negative shock of this factor 1 seems to make the Central Bank react quicker. A positive shock in factors 2 and 4 will lead to a shorter duration between two decisions to change the level of the target rate.

4.4 Results summary

The comparison of the results obtained with both the probit and ACD models enables us to formulate some stylized facts for the Fed's reaction function. An increase of factor 1 seems to bring about a sharp raise of the target rate, linked to a longer duration. Secondly, an increase of factor 2 is linked to a shorter duration but it does not seem to have any effect on the scale. Finally, an increase of factor 4 may bring about a rate increase and a shorter duration. These results are summed up in the following table:

	Factor 1	Factor 2	Factor 3	Factor 4
Result of a positive shock on the scale	positive	-	-	positive
Result of a positive shock on the duration	positive	negative	-	negative

Finally, the latter table seems to give shape to our work: distinguishing the scale and the duration of the target rate process seems to be a fertile way to understand monetary policy. Each economic shock can have a different effect on the reaction function of Central Bankers. The factors that move duration and scale processes can be different, and, what is more, do not necessary correspond to the ones that drives the target rate level. In the case where they are the same, the effect of these factors on each of these processes can be different. To our mind, this fact is of particular importance for anyone that would want to understand or to predict the future of monetary policy.

5 Conclusion

The main findings of this paper can be summarized as follow. The factors affecting the movement of the target rate, the scale of the changes in target rate and the associated durations are not necessary the same. This could help understand the interrogation raised by the instability of the estimation on a monthly and quarterly basis asserted at the beginning of this paper. This instability seems to be linked to the fact that when estimating a classical Taylor rule, we are trying to account for three possibly independent processes with a single model, based on a single equation. So, trying to give shape to the joint probability distribution function of the duration process and the change in target rate process seems to be a fertile way to understand monetary policy.

As emphasized in the introduction part, Clarida et al. (1999)'s article starts from a quote of Alan S. Blinder, Princeton's professor of Economics and vice chairman of the Board of Governors of the Federal Reserve System from 1994 to 1996. The experience he drew from both these positions was pondered in the following sentence: "*Having looked at monetary policy from both sides now, I can testify that central banking in practice is as much art and science. Nonetheless, while practicing this dark art, I have always found the science quite useful*". Using theory as guidelines is essential; anyway, empirical work can help find and accommodate new features of the reality.

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6 Appendices

6.1 Presentation of the data set

We present hereafter the data set we used to compute the factors. The data were transformed as follow: 1 is for no change in the data; 2 is for a first order difference; 3 is for the difference between the current value and its value 12 monthes before. We also define the type of the data: i is for *index* type data, mom is for *month-over-month* data and yoy for *year-over-year* data.

	Series	Bloomberg ticker	Type	Transformation
Personal Income				
1	Total Personal income yoy	PITLYOY Index	yoy	1
2	Total Personal income mom	PITLCHNG Index	mom	1
3	Disposable Income	PIDSDI Index	i	3
4	Personal outlays	PIDSSO Index	i	3
5	Personal saving	PIDSS Index	i	1
6	Disposable Per Capita	PIDSDPC Index	i	3
7	Saving rates	PIDSDPS Index	i	1
8	Interest paid	PIDSINT Index	i	2
9	Disposable chained	PIDSDCWT Index	i	3
10	Per Capita chained	PIDSDPCW Index	i	3
11	Total compensation	PIWGTOTL Index	i	3
12	Wages and salaries	PIWGWAGE Index	i	3
13	Wages private	PIWGPRIV Index	i	3
14	Wages government	PIWGGOVT Index	i	3
15	Other labor income	PIOCOLI Index	i	3
16	Proprietors	PIOCPROP Index	i	3
17	Farm	PIOCFRM Index	i	1
18	Non-farm	PIOCNFRM Index	i	3
19	Rental	PIOCRENT Index	i	1
20	Personal dividend	PIOCDIV Index	i	3
21	Personal interest	PIOCINT Index	i	3
22	Transfer	PIOCTRAN Index	i	3
23	Transfer payment for old age	PIOCTRHL Index	i	3
24	Government unemployment	PIOCUNEM Index	i	1
25	Private debt % income	.PRDBT%GD Index	i	1
Personal Expenditure				
26	Personal exp. Total current dollar mom	PCE CRCH Index	mom	1
27	Personal exp. Total current dollar yoy	PCE YOY$ Index	yoy	1
28	Expenditure durable	PCE DRBL Index	i	3
29	Expenditure non-durable	PCE NDRB Index	i	3
30	Expenditure services	PCE SRV Index	i	3
31	Expenditure chained mom	PCE CHNC Index	mom	1
32	Chained type mom	PCE DEFM Index	mom	1
33	Chained type yoy	PCE DEFY Index	yoy	1
34	Chained type annual percentage change	PCE DEFA Index	i	1
35	Chained type durable	PCE DRBD Index	i	3
36	Chained type non-durable	PCE NDRD Index	i	3
37	Chained type Service	PCE SRVD Index	i	3
38	Chained type core mom	PCE CMOM Index	mom	1
39	Chained type core yoy	PCE CYOY Index	yoy	1

Table 6: Presentation of the data set

	Series	Bloomberg ticker	Type	Transformation
Employment statistics				
40	Unemployment total sa	USURTOT Index	i	1
41	Unemployment number sa	USUETOT Index	i	1
42	Unemployment both sa	USURBTHS Index	i	1
43	Civilian non labor sa	USNLTOT Index	i	3
44	Payroll non farm sa	USNATOTL Index	i	2
45	Payroll farm sa	USAGTOT Index	i	2
46	Employment net change sa	USEMNCHG Index	mom	1
47	Full time	USEMFULL Index	i	3
48	Part time economic	USEMPTER Index	i	3
49	Part time slack	USEMPTSW Index	i	3
50	Part time noneconomic	USEMPTNE Index	i	3
51	Part time self-employed	USNASELF Index	i	3
52	Ratio employment/population sa	USERTOT Index	i	1
53	Participation sa	PRUSTOT Index	i	2
54	Civilian labor force sa	USLFTOT Index	i	2
55	Civilian non institutionnal population	USCPTOT Index	i	2
56	Duration of unemployment	USDUMEAN Index	i	1
57	Job leavers	USJLJOBL Index	i	1
58	Job leavers % of work force	USJLLVR% Index	i	1
59	Non Farm Payroll Net change sa	NFP TCH Index	mom	1
60	Non Farm Payroll yoy level change	NFP TYCH Index	yoy	1
61	Non Farm Payroll private sa	NFP P Index	i	2
62	Non Farm Payroll good produce sa	NFP GP Index	i	2
63	Non Farm Payroll construction sa	USECTOT Index	i	2
64	Non Farm Payroll manufacturing sa	USMMMANU Index	i	2
65	Non Farm Payroll durable goods sa	USEDTOT Index	i	2
66	Non Farm Payroll non durable goods sa	USENTOT Index	i	2
67	Non Farm Payroll service sa	NFP SP Index	i	2
68	Non Farm Payroll commerce total sa	USRTTOT Index	i	2
69	Non Farm Payroll transport sa	USETTOT Index	i	2
70	Non Farm Payroll utilities sa	NFP UTLS Index	i	2
71	Non Farm Payroll information sa	USEITOTS Index	i	2
72	Non Farm Payroll finance sa	USEFTOT Index	i	2
73	Non Farm Payroll services sa	USESTOT Index	i	2
74	Non Farm Payroll education and health sa	USEETOTS Index	i	2
75	Non Farm Payroll leisure sa	USEHTOTS Index	i	2
76	Non Farm Payroll other services sa	USEOTOTS Index	i	2
77	Non Farm Payroll government sa	USEGTOT Index	i	2
78	Average weekly hours private sa	USWHTOT Index	i	1
79	Average weekly hours good producing sa	USWHGPSA Index	i	1
80	Average weekly hours mining sa	USWHMINS Index	i	1
81	Average weekly hours manufacturing sa	USWHMANS Index	i	1
82	Average weekly hours overtime sa	USWHMNOS Index	i	1
83	Average weekly hours service providing sa	USWHSPS Index	i	1
84	Average weekly hours durable goods sa	USWDTOT Index	i	1
85	Average weekly hours non durable goods sa	USWNTOT Index	i	1
86	Aggregage weekly hours	USAWTOT Index	i	1
87	Average hourly earnings mom	USHETOT% Index	mom	1
88	Average hourly earnings yoy	USHEYOY Index	yoy	1
89	Average hourly earnings good producing	USHEGPSA Index	i	3
90	Average hourly earnings non durable goods	USHENDRB Index	i	3
91	Average hourly earnings total sa	USWETOTA Index	i	3
92	Claims	INJCJC Index	i	3
93	conf board job plentiful	CONCJOBP Index	i	1
94	conf board job not plentiful	CONCJOBN Index	i	1
95	conf board job hard to get	CONCJOBH Index	i	1
Retail sales				
96	Total domestic cars	SAARDCAR Index	i	3
97	Manufacturing and trade total	MTSL Index	i	3
98	Wholesale total	MWSLTOT Index	i	3
99	Wholesale durable goods	MWSLDRBL Index	i	3
100	Wholesale non durable goods	MWSLNDRB Index	i	3
101	Advance retail sales yoy	RSTAYOY Index	yoy	1
Housing				
102	New home sales median	NHSLAVSL Index	i	3
103	New home sales house for sales	NHSLNFS Index	i	3
104	Total starts new home residential	NHSPSTOT Index	i	3
105	Housing starts North east	NHSPSNE Index	i	3
106	Housing starts Mid West	NHSPSMW Index	i	3
107	Housing starts South	NHSPSSO Index	i	3
108	Authorized starts North east	NHSPANE Index	i	3
109	Authorized starts Mid West	NHSPAMW Index	i	3
110	Authorized starts South	NHSPASO Index	i	3
111	Authorized starts West	NHSPAWE Index	i	3
112	Existing houses sales	EHSLSL Index	i	3
113	Total starts autorized	NHSPATOT Index	i	3

Table 7: Presentation of the data set

	Series	Bloomberg ticker	Type	Transformation
US confidence				
114	University of Michigan Sentiment	CONSSENT Index	i	1
115	University of Michigan expectation	CONSEXP Index	i	1
116	University of Michigan current	CONSCURR Index	i	1
117	Conference board confidence	CONCCONF Index	i	1
118	Conference board situation	CONCPSIT Index	i	1
119	Conference board expectations	CONCEXP Index	i	1
120	Leading indicator yoy	LEI YOY Index	yoy	1
121	Leading indicator mom	LEI CHNG Index	mom	1
Financial variables				
122	US Treasury bonds and notes	FRNTUSBN Index	i	1
123	Debt and credit yearly change	CCOSYOY Index	yoy	1
124	S&P 500	SPX Index	i	3
125	NYSE	NYA Index	i	3
126	Stock Prices	LEI STKP Index	i	3
Industrial production				
127	Industrial production	IP Index	i	3
128	Industrial production consumer goods	IPTLCG Index	i	3
129	Industrial production durable consumer goods	ICGDDCGS Index	i	3
130	Industrial production non durable consumer goods	IPNDTOTL Index	i	3
131	Industrial production manufacturing	IPMG Index	i	3
132	Industrial production business equipment	IPEQBUS Index	i	3
133	Industrial production materials	IPTLMATS Index	i	3
134	Industrial production durable good materials	IGMDDRBL Index	i	3
135	Industrial production non durable good materials	IGMNNOND Index	i	3
136	Industrial production business equipment	IPEQBUS Index	i	3
137	Industrial production High Tech	IPXHTOTL Index	i	3
138	Industrial production Automobile	IPXVTOTL Index	i	3
139	Industrial production Mining	IPMUMNG Index	i	3
140	Industrial production utilities	IPMUUTIL Index	i	3
141	Capacity utilisation rate total industry	CPTICHNG Index	i	1
142	Capacity utilisation rate manufacturing total	CPMFTOT Index	i	1
143	Capacity utilisation rate durable manufacturing	CPDMTOT Index	i	1
144	Capacity utilisation rate High Tech	IPSACXTT Index	i	1
145	Capacity utilisation rate mining	CPMN Index	i	1
146	Capacity utilisation rate utilities	CUTLTOT Index	i	1
147	Purchasing managers' index	NAPMPMI Index	i	1
Price indexes				
148	NAPM Price indexes	NAPMPRIC Index	i	1
149	CPI Services	CPSSTOT Index	i	3
150	Consumer price index total	CPI INDX Index	i	3
151	Consumer price index less food and energy	CPUPXYOY Index	i	3
152	Consumer price commodity	CPCATOT Index	i	3
153	Consumer price index commodity food	CPCACXFB Index	i	3
154	Consumer price durable goods	CPCADUR Index	i	3
155	Consumer price food and beverage	CPSFTOT Index	i	3
156	Consumer price housing	CPSHTOT Index	i	3
157	Consumer price transportation	CPSTTOT Index	i	3
158	Consumer price medical care	CPUMTOT Index	i	3
159	Producer price finished goods	PPI INDX Index	i	3
160	Producer price crude materials	PPICTOTL Index	i	3
161	Producer price intermediate materials	PPIITOTL Index	i	3
162	Producer price energy	PPCMENER Index	i	3
Interest rates and money				
163	Money supply M1	M1 Index	i	3
164	Money supply M2	M2 Index	i	3
165	Money supply M3	M3 Index	i	3
166	PhiliFed Index	OUTFGAF Index	i	1
Orders and unfilled orders				
167	NAPM New orders index	NAPMNEWO Index	i	1
168	New orders total	TMNOTOT Index	i	3
169	New orders durable goods	DGNOTOT Index	i	3
170	New orders non durable goods	NDGNTOT Index	i	3
171	Unfilled orders total	TMUOTOT Index	i	3
Inventories				
171	Manufacturing and trade inventories	MTIB Index	i	3
172	NAPM inventory index	NAPMINV Index	i	1
173	Sales/inventories Index business	MGT2TB Index	i	3
174	Sales/inventories Index manufacturing	MGT2MA Index	i	3
175	Sales/inventories Index retail	MGT2RE Index	i	3
Governement spending				
176	Budget % GDP	FDDSGDP Index	i	1

Table 8: Presentation of the data set