Explaining the Joint Behavior of Employment, Unemployment and Nonparticipation

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Abstract

This paper argues that existing matching models with unemployment as an active search and nonparticipation as an inactive search predict counterfactual results: the unemployment rate is more than two times as volatile as the employment-population ratio; only 20 percent of the actual volatility of the unemployment rate is accounted for; and the labor market variables are perfectly correlated with each other. This paper proposes a modified matching model in which workers are classified after matches take place. The modified model generates the direct transition from nonparticipation to employment with no assumption that nonparticipation is an inactive search and without adjusting the time period of the model. The model also explains the important cyclical features of the U.S. labor market.

1 Introduction

In the standard search and matching model, unemployment captures both (i)*those who have not found employment or have separated* and (ii)*those who are currently looking*

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for work. The current-period unemployment is then determined by the former, and it also determines the latter.

According to the definition of "unemployed persons" of the Current Population Survey (CPS), the unemployed are persons aged 16 years and older who had no employment during the reference week and had made specific efforts to find employment sometime during the 4-week period ending with the reference week.[1] The CPS definition demonstrates that the unemployed are those who searched but did not find employment.

Recent studies about the labor force participation which attempt to distinguish between unemployment and nonparticipation have neglected the feature of unemployment that *the unemployed are those who find no employment* by focusing only on the *search* feature of unemployment.[2] Many take one of the following classifications: "unemployment is searching and nonparticipation is not-searching" and "unemployment is an active search and nonparticipation is an inactive search."

The "searching versus not-searching" classification gives rise to the problem that no one can move directly from nonparticipation to employment. Petrongolo and Pissarides (2001) argue that the direct flows from nonparticipation to employment are due to misclassification problems, so called *time aggregate bias*: any person now have a job must have made some effort, which cannot be detected by labor force surveys. Thus several authors take this classification and adjust the time period of the model.[3] The method to adjust the time period of the model makes it hard to interpret other transition rates. Is the monthly transition rate from unemployment to employment generated by a weekly model consistent with the actual monthly transition rate? Therefore, we need more sophisticated methods to explain the direct transition from nonparticipation to employment.

The "active versus inactive search" classification helps produce the direct transition from nonparticipation to employment, but generates the counterfactual implications. In this paper, I evaluate the matching model with "active versus inactive search" classification as in Kim (2004) and Pries and Rogerson (2004) (hereafter KPR). Based on the reduced-form dynamics derived from the KPR matching model, I show that the KPR matching model predicts that: (1) the unemployment/population ratio is only one and half times more volatile than the employment/population ratio, (2) the model accounts for less than 20 percent of the actual volatility of the unemployment rate, and

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[1] See http://www.bls.gov/bls/glossary.htm

[2] Among others, see Kim (2004), Pries and Rogerson (2004), Garibaldi and Wasmer (2005), Haefke and Reiter (2006) for the Mortensen and Pissarides (1994) matching model, Veracierto (2004) for the Lucas and Prescott (1974) island model, and Tripier (2005) for the real business cycle model.

[3] See Garibaldi and Wasmer (2005) and Haefke and Reiter (2006). For example, Haefke and Reiter (2006) choose one week as a model period and match the monthly transition rates, but their job is not quite successful.

(3) perfect correlations between the labor market variables are observed.

For that reason, I modify the matching model in several ways: First, those who do not engage in job search are classified as out of the labor force. Second, the employed consist of both those who have been working and those who search and find employment. Then the transition from nonparticipation to employment occurs between two consecutive periods, and it is not necessary to assume that nonparticipants are inactive searchers. Third, although the transition rate from unemployment to employment is much higher than the transition rate from nonparticipation to employment, the conditional job-finding probability that a nonparticipant who reenters the labor market faces is much higher than that an unemployed worker faces. This leads those out of the labor force to participate in the labor market because nonparticipants have a higher job-finding probability reservation than the unemployed.

The modified model has a couple of novel features. One is that the direct transition from nonparticipation to employment is generated with no assumption that nonparticipants are inactive searchers and without adjusting the model period. Another novel feature is that, as in Cole and Rogerson (1999), the reduced-form dynamics are easily derived from the modified model, and identifying the parameters governing the model is much easier than the KPR model.

The findings can be summarized as follows. The modified model improves significantly the KPR model and accounts for the U.S. labor market. First, the modified model predicts that the unemployment rate is most volatile, and accounts for more than 60 percent of the actual volatility of the unemployment rate. Second, the employment/population ratio is highly negatively correlated with both the unemployment/population ratio and the nonparticipation rate, but the correlations are not -1. Third, the model predicts a positive correlation between the unemployment/population ratio and the nonparticipation rate, .61 which is very close to the data, .62. Finally, the unemployment/population ratio is quite negatively correlated with the vacancy rate, $-.73$ so that the model predicts the very strong Beveridge relationship. This finding is an important contribution because recent studies about the labor market fluctuations with the endogenous participation predict that unemployment is highly positively correlated with vacancy.[4]

The paper is organized as follows. Section 2 discusses and compares the different labor force classifications, and introduces matching models. Section 3 quantifies the KPR model and the modified model, and Section 4 states my conclusions.

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[4] See Tripier (2004) and Veracierto (2004).

2 The Model

The model economy is a variant of the Mortensen and Pissarides (1994) matching model which consists of workers and firms (or entrepreneurs). Both workers and firms are homogeneous. Since there is no participation decision in the standard matching model, I assume that workers can be not only employed or unemployed, but also out of the labor force.

2.1 Environments

There is a continuum of infinitely-lived and risk-neutral workers with total mass equal to one. Each worker has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t \tag{1}$$

where $0 < \beta < 1$ is the discount factor and c_t is consumption which takes the following values depending upon the worker's labor market status: w_t if the worker is working, b if the worker is searching for a job, and 0 if the worker is out of the labor force.

There are also infinitely many risk-neutral firms in this economy. Each firm has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t \tag{2}$$

where $0 < \beta < 1$ is the discount factor and c_t consumption. In a certain period, firms can be active or vacant. An active firm is one that is matched with a worker and is currently producing output z, where z is assumed to follow an AR(1) process in logs:

$$\ln z' = \rho_z \ln z + \epsilon' \tag{3}$$

where ϵ follows a normal distribution with mean zero and variance σ_ϵ^2. The steady state productivity level is normalized to 1. All active firms confront exogenous separation with probability λ. A vacant firm is one that is posting a vacant position and looking for workers. All vacant firms find workers with probability q. I assume that firms pay k units of the consumption good to post a vacancy.

A constant-returns-to-scale matching function is assumed:

$$m(s, v) = \omega s^\gamma v^{1-\gamma} \tag{4}$$

where s is the number of job-searchers expressed as an efficiency unit, v the number of vacancies, γ the elasticity of the matching function, and ω a matching function parameter. The probability that a worker in the labor market finds a job, p, is given by

$$p = \frac{m}{s} = \omega\theta^{1-\gamma} \tag{5}$$

and the probability that a worker entering the labor market finds a job, f, is given by

$$f = e\frac{m}{s} = e\omega\theta^{1-\gamma} \tag{6}$$

where is θ is the vacancy/searcher ratio, and e the relative search intensity or efficiency. The worker-finding probability is

$$q = \frac{m}{v} = \omega\theta^{-\gamma} \tag{7}$$

In what follows, I introduce two different labor force classifications: one made by Kim (2004) and Pries and Rogerson (2004) and the other by Moon (2007)

2.2 Classifications

2.2.1 Unemployment Before Matches Take Place

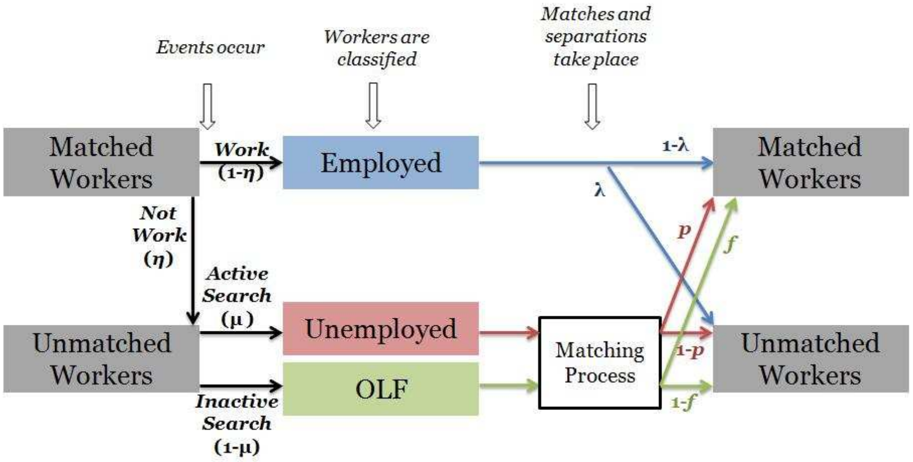
First, I describe the labor force classification that KPR make to account for U.S. labor market flows. Especially, Figure 1 presents the flow chart describing how workers move between labor force states within a given period.

At the beginning of a certain period, there are two types of workers: matched workers and unmatched workers. Matched workers have employment opportunities, but unmatched workers do not.

First, matched workers choose to work on their current jobs with probability $1 - \eta$ or not to work with probability η. Matched workers choosing not to work become unmatched workers. If a matched worker chooses to work, then (s)he is classified as *employed*. Conditional on working, workers separate with probability λ at the end of a certain period. Those who separate become unmatched, and those who do not separate remain matched in the subsequent period.

Second, unmatched workers choose to look for work *actively* with probability μ or look for work *inactively* with probability $1 - \mu$. Unmatched workers choosing active searches are classified as *unemployed*. Conditional on searching, workers find jobs with probability p. Those who find employment become matched, and those who do not find employment become unmatched at the beginning of the subsequent period. If an unmatched worker chooses an inactive search, then (s)he is classified as out-of-the-labor-

Figure 1: Flow Chart When Nonparticipation is an Inactive Search



force (*OLF*). Conditional on searching inactively, unmatched workers find employment with a different probability, say f. In the KPR model, it is assumed that p is greater than f, because those who search actively enjoy a higher job-finding probability than those who search inactively. At the end of the period, those who find employment become matched, and those who do not find employment become unmatched.

Table 1 reports the probabilities of transition between different labor force states for the U.S. labor market. The transition rate from unemployment to employment (hereafter the UE transition rate) is about 22.3 percent and the transition rate from OLF to employment about 2.8 percent. Based on this fact, Kim (2004) and Pries and Rogerson (2004) set the steady-state probability of the unemployed finding a job to 22.3 percent and the steady-state probability of the nonparticipant finding a job to 2.8 percent, respectively. Unemployed workers' search intensity being unity, the relative search intensity of nonparticipants, denoted by x, is about $1/8(= 2.8/22.3)$. The efficiency unit of job-searchers is expressed as $s = U + xO$, where U denotes the number of unemployed workers and O the number of nonparticipants.

2.2.2 Unemployment After Matches Take Place

In this subsection, I introduce another way classifying workers in which they are classified after matches take place. At the beginning of each period, there are three types of workers (based on the classifications made one period before): employed, unemployed, and out of the labor force. Employed workers choose to work on their current jobs with probability $1 - \eta$ or not to work with probability η. If an employed worker chooses not to work, then (s)he is classified as *OLF*. Conditional on working, workers separate

Table 1: The Gross Labor-Force Transition Rates for the CPS, 1978-2005, Percent Per Month

(1) Unadjusted

		To		
		Working	Unemployed	Not in Labor Force
	Working	95.62	1.49	2.89
From	Unemployed	26.66	51.23	22.11
	Not in Labor Force	4.63	2.56	92.82

(2) Adjusted with the Abowd and Zellner (1985, Table 5) correction

		To		
		Working	Unemployed	Not in Labor Force
	Working	97.05	1.19	1.76
From	Unemployed	22.33	63.42	14.25
	Not in Labor Force	2.79	2.27	94.94

Source: Robert Shimer's tabulations of raw data from the CPS

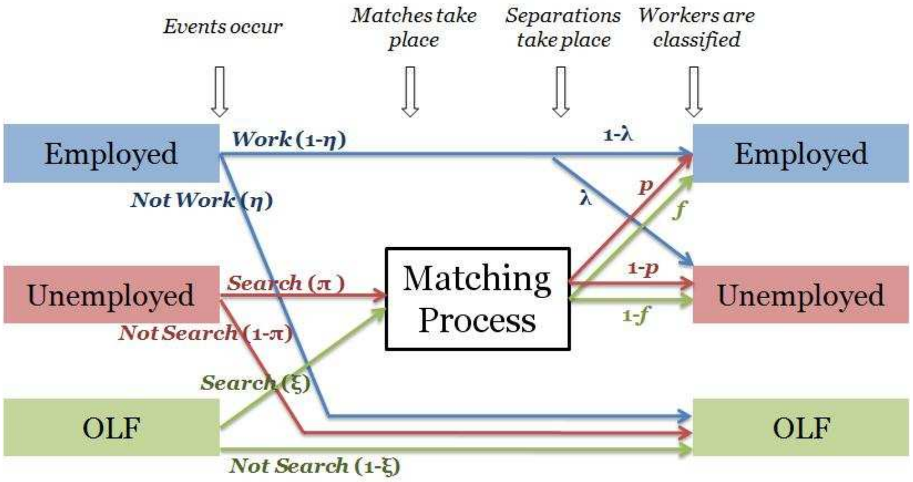
with probability λ at the end of that period. Those who separate are classified as *unemployed*, and those who do not separate are classified as *employed*.

Unemployed workers choose to look for work with probability π or not to look for work with probability $1-\pi$. If unemployed workers choose not to search, then they are classified as *OLF*. Conditional on searching, workers find a job with probability p. Those who find employment are classified as *employed*, and those who do not find employment are classified as *unemployed*.

Nonparticipants choose to look for work with probability ξ or not with probability $1-\xi$. If a nonparticipant chooses not to search, then (s)he is classified as *OLF*. Conditional on entering the labor force, nonparticipants find employment with probability f. Those who find employment are classified as *employed*, and those who do not are classified as *unemployed*. Figure 2 summarizes the worker flows.

Note that nonparticipants' job-finding probability, f, is defined as a conditional probability: conditional on participating in the labor force and searching for work, a nonparticipant finds a job with a certain specified probability. Similarly, the unemployed workers' job-finding probability, p, is also defined as a conditional probability: conditional on not being out of the labor force, an unemployed worker finds a job with probability p. The steady-state conditional job-finding probabilities are then given by

Figure 2: Flow Chart When Nonparticipation is Not Searching



about 26 percent for the unemployed and 55 percent for the nonparticipants:

$$\Pr(UE|LaborForce) = \frac{h_{UE}}{h_{UE}+h_{UU}} = \frac{22.33\%}{22.33\%+63.42\%} = 26.04\% \tag{8}$$

$$\Pr(OE|LaborForce) = \frac{h_{OE}}{h_{OE}+h_{OU}} = \frac{2.79\%}{2.79\%+2.27\%} = 55.17\% \tag{9}$$

where h_{UE} is the transition rate from unemployment to employment, for instance. If a nonparticipant decides to search, then (s)he will have about two times higher job-finding probability that an unemployed worker will. Let y denote the relative search efficiency:

$$y = \frac{\Pr(OE|LaborForce)}{\Pr(UE|LaborForce)} = 2.12 \tag{10}$$

The number of searchers is then $s = \pi U + y\xi O$, where U and O are the number of unemployed workers and the number of nonparticipants, respectively.

2.3 Equilibrium

The individual worker's and firm's problems can be formulated recursively. The state of the economy is described by (z, φ), where z is aggregate productivity and φ is the distribution of workers.

2.3.1 Unemployment Before Matches Take Place

Let $V^w(z,\varphi)$ denote the value function of a worker who works, $V^s(z,\varphi)$ the value function of a worker who searches actively, $V^o(z,\varphi)$ the value function of a worker who searches inactively, $V^M(z, \varphi)$ the value function of a matched worker, and $V^N(z, \varphi)$ the value function of an unmatched worker:

The value function of a worker who works (or an employed worker) is given by

$$V^w(z, \varphi) = w(z, \varphi) + \beta\Big\{(1-\lambda)E_z\big[V^M(z', \varphi')\big] + \lambda E_z\big[V^N(z', \varphi')\big]\Big\} \tag{11}$$

where $w(z, \varphi)$ is a Nash bargaining wage and λ the exogenous separation rate. A worker earns wages in the current period, and in the subsequent period if the match survives with probability $1 - \lambda$, then the worker becomes matched. However, if the match is dissolved exogenously with probability λ, then the worker becomes unmatched. The value functions of a matched worker and of an unmatched worker, $V^M(z, \varphi)$ and $V^N(z, \varphi)$, are given as follows

$$V^N(z, \varphi) = \mu V^s(z, \varphi) + (1-\mu)V^o(z, \varphi) \tag{12}$$

$$V^M(z, \varphi) = (1-\eta)V^w(z, \varphi) + \eta V^N(z, \varphi) \tag{13}$$

An unmatched worker who has no employment opportunities decides whether to search actively or inactively, depending on probability μ. A matched worker who has employment opportunities decides whether to work on his current job or not, depending on probability η.

The value function of a worker who searchers actively (or an unemployed worker) is given by

$$V^s(z, \varphi) = b + \beta\Big\{p(z, \varphi)E_z\big[V^M(z', \varphi')\big] + \big(1 - p(z, \varphi)\big)E_z\big[V^N(z', \varphi')\big]\Big\} \tag{14}$$

where $p(z, \varphi)$ is the job-finding probability. A worker who looks for work actively receives unemployment insurance benefits b in the current period, and in the subsequent period finds a job with probability $p(z, \varphi)$. If a worker finds a job, then (s)he becomes matched. Otherwise, (s)he becomes unmatched.

The value function of a worker who searches inactively (or a nonparticipant) is given by

$$V^o(z, \varphi) = \beta\Big\{f(z, \varphi)E_z\big[V^M(z', \varphi')\big] + \big(1 - f(z, \varphi)\big)E_z\big[V^N(z', \varphi')\big]\Big\} \tag{15}$$

where $f(z, \varphi)$ is the job-finding probability given by $xp(z, \varphi)$, where $x < 1$ captures the relative search intensity. A worker who looks for work inactively finds a job with probability $f(z, \varphi)$. If a worker finds a job, then (s)he becomes matched. Otherwise, (s)he becomes unmatched.

Let $J(z, \varphi)$ denote the value function of a firm matched with a worker. The value function of this matched firm is given by

$$J(z, \varphi) \quad = \quad z - w(z, \varphi) + \beta(1 - \lambda)E_z\left[(1 - \eta)J(z', \varphi')\right] \tag{16}$$

where z is output, $w(z, \varphi)$ a Nash bargaining wage, and the remaining term the discounted expected values of the match weighted by the probability that the match survives, $1 - \lambda$.

The equilibrium number of job vacancies is determined by the following free-entry condition which states that vacancies earn zero profits:

$$k \quad = \quad \beta q(z, \varphi)E_z\left[(1 - \eta)J(z', \varphi')\right] \tag{17}$$

where k is the job posting cost, and $q(z, \varphi)$ the worker-finding probability.

Let $S(z, \varphi)$ denote the match surplus between a worker and a firm. The match surplus is defined to be the difference in the sum of the payoffs of the worker and the firm:

$$S(z, \varphi) = V^w(z, \varphi) - V^o(z, \varphi) + J(z, \varphi) \tag{18}$$

Note that the threat point of the worker is the value from being out of the labor force. A worker who breaks up the match is then out of the labor force, but never becomes a job-seeker because (s)he cannot find a better wage through a search in this framework.

The wage is derived by assuming that fixed fractions of the surplus accrue to the worker and the firm. The total match surplus is shared in accordance with the following Nash product:

$$w(z, \varphi) = \arg\max\left(V^w(z, \varphi) - V^o(z, \varphi)\right)^{\alpha} J(z, \varphi)^{1-\alpha} \tag{19}$$

where α is the worker's bargaining power, which is set to equal the elasticity of the matching function with respect to search, $\gamma = \alpha$, so that the Hosios (1990) rule is satisfied. Following Hall (2005), I assume that wages are rigid over the business cycle so that they are given by $w(z, \varphi) = w(z^*, \varphi^*)$ for all z and φ, where z^* is the steady state productivity, φ^* the steady state distribution of workers, and $w(z^*, \varphi^*)$ the Nash bargaining wage at (z^*, φ^*).

Finally, the evolution of the aggregate state is described by the function $T(z, \varphi)$, where for each (z, φ) this function specifies a distribution over the next period's values of the state variables.

The *recursive equilibrium* is a list of value functions, job- and worker-finding probabilities and wages such that:

1. Taking the probabilities and the wages as given, workers and firms solve their value functions (11)-(16),
2. The free-entry condition (17) is satisfied,
3. Wages are determined by Nash bargaining (19), and
4. For each (z, φ), decisions generate a distribution over the next period's state that is consistent with the distribution given by $T(z, \varphi)$.

2.3.2 Unemployment After Matches Take Place

I describe the equilibrium of the model when workers are classified after matches take place. Except for the worker's value functions and the job-finding probabilities, other functions are the same as ones defined in the previous section.

Let $V^w(z, \varphi)$ denote the value function of a worker who works, $V_p^s(z, \varphi)$ the value function of a worker who was classified as unemployed in the previous period and who searches in the current period, $V_f^s(z, \varphi)$ the value function of a worker who was classified as OLF in the previous period and who searches in the current period, and $V^o(z, \varphi)$ the value function of a worker who does not search in the current period.

The value function of a worker who works is given by

$$\begin{aligned} V^w(z, \varphi) &= w(z, \varphi) + \beta(1-\lambda)E_z\left[\eta V^o(z', \varphi') + (1-\eta)V^w(z', \varphi')\right] \\ &\quad + \beta\lambda E_z\left[\pi V_p^s(z', \varphi') + (1-\pi)V^o(z', \varphi')\right] \end{aligned} \tag{20}$$

where $w(z, \varphi)$ is a Nash bargaining wage and λ the exogenous separation rate. A worker earns wages in the current period, and in the subsequent period if the match survives with probability $1 - \lambda$, then the worker will have to decide whether to continue or terminate the match, depending on probability η. However, if the match is dissolved exogenously with probability λ, then the worker will decide whether to search with probability π.

The value function of a worker who was classified as unemployed in the previous period and searches in the current period is given by

$$\begin{aligned} V_p^s(z, \varphi) &= b + \beta p(z, \varphi)E_z\left[\eta V^o(z', \varphi') + (1-\eta)V^w(z', \varphi')\right] \\ &\quad + \beta(1 - p(z, \varphi))E_z\left[\pi V_p^s(z', \varphi') + (1-\pi)V^o(z', \varphi')\right] \end{aligned} \tag{21}$$

where $p(z, \varphi)$ is the job-finding probability. A worker who looks for work receives unemployment insurance benefits b in the current period, and in the subsequent period finds a job with probability $p(z, \varphi)$. If a worker finds a job, then (s)he will choose to work on that job or not, depending on probability η. Otherwise, (s)he will have to decide whether to search again or not, with probability π.

The value function of a worker who was classified as OLF in the previous period and searches in the current period is given by

$$\begin{aligned} V_f^s(z, \varphi) &= \beta f(z, \varphi) E_z \left[\eta V^o(z', \varphi') + (1-\eta) V^w(z', \varphi')\right] \\ &\quad + \beta(1 - f(z, \varphi)) E_z \left[\pi V_p^s(z', \varphi') + (1-\pi) V^o(z', \varphi')\right] \end{aligned} \tag{22}$$

where $f(z, \varphi)$ is the job-finding probability with $f(z, \varphi) = yp(z, \varphi)$. Note that a worker who was out of the labor force in the last period and is looking for work in the current period does not receive unemployment insurance benefits.

Finally, the value function of a worker who does not search is given by

$$V^o(z, \varphi) = \beta E_z \left[\xi V_f^s(z', \varphi') + (1-\xi) V^o(z', \varphi')\right] \tag{23}$$

A worker does not search in the current period decides whether to search or not in the subsequent period with probability ξ.

The *recursive equilibrium* is a list of value functions, job- and worker-finding probabilities and wages such that:

1. Taking the probabilities and the wages as given, workers and firms solve their value functions (20)-(23) and (16),
2. The free-entry condition (17) is satisfied,
3. Wages are determined by Nash bargaining (19), and
4. For each (z, φ), decisions generate a distribution over the next period's state that is consistent with the distribution given by $T(z, \varphi)$.

2.4 Reduced-Form Labor Market Dynamics

Following Cole and Rogerson (1999), I characterize the implications of the models for the time series. For simplicity, the population size is normalized to unity. Let E_t, U_t and O_t denote the number of employed workers (employment/population ratio), the number of unemployed workers (unemployment/population ratio) and the number of nonparticipants (nonparticipation rate), respectively.

2.4.1 Unemployment Before Matches Take Place

From the KPR model and their classification, the following system of equations of the reduced-form labor market dynamics can be derived. Time t matched workers, M_t, and unmatched workers, N_t, are given as follows:

$$\begin{aligned} M_t &= (1-\lambda)E_{t-1} + p_{t-1}U_{t-1} + f_{t-1}O_{t-1} \\ N_t &= \lambda E_{t-1} + (1-p_{t-1})U_{t-1} + (1-f_{t-1})O_{t-1} \end{aligned}$$

Time t employment/population ratio, unemployment/population ratio and nonparticipation rate can be expressed in terms of time t matched workers and unmatched workers:

$$\begin{aligned} E_t &= (1-\eta)M_t && (24) \\ U_t &= \mu\left(\eta M_t + N_t\right) && (25) \\ O_t &= (1-\mu)\left(\eta M_t + N_t\right) && (26) \end{aligned}$$

Note that the nonparticipation rate depends on the job-finding probabilities, p_{t-1} and f_{t-1}. Using $M_t + N_t = 1$, we have the law of motion for the unemployment/population ratio expressed in terms of E_t:

$$U_t = \mu\left(1 - E_t\right) \tag{27}$$

which implies that $Corr(E, U) = -1$ and $Std(U) = \mu Std(E)$. The relative standard deviation of the unemployment/population ratio, expressed as a ratio to the standard deviation of the employment/population ratio, is simply μ. We also find the relationship between the unemployment/poplation ratio and the nonparticipation rate:

$$O_t = \left(\frac{1-\mu}{\mu}\right) U_t \tag{28}$$

which implies that $Corr(U, O) = 1$ and $Std(O) = \left(\frac{1-\mu}{\mu}\right) Std(U)$. The relative standard deviation of the nonparticipation rate, expressed as a ratio to the standard deviation of the employment/population ratio, is $1 - \mu$. In addition, over the business cycle frequencies, the HP-filtered series of $\log U_t$ and $\log O_t$ move together.

Suppose, for example, that in a steady state the unemployment/population ratio is 4 percent and the nonparticipation rate 36 percent. The probability of an active job search, μ, is given by .1 (4 percent divided by 40 percent). Then, the relative standard deviation of the unemployment/population ratio is .1 and the relative standard deviation of the nonparticipation rate is .9. Therefore, the model predicts that the nonparticipation rate has nine times more variations than unemployment/population ratio does.

The KPR matching model, therefore, gives the following implications: (1) the correlations between the employment/population ratio and the unemployment/population ratio, and between the employment/population ratio and the nonparticipation rate are -1; (2) the correlation between the unemployment/population ratio and the nonparticipation rate is 1; (3) the relative standard deviation of the unemployment/population, expressed as a ratio to the standard deviation of the employment/population ratio, is μ in levels; and (4) the relative standard deviation of the nonparticipation rate is $1-\mu$ in levels. Note that the relative standard deviations of the unemployment/population ratio and the nonparticipation rate are irrelevant to η.

2.4.2 Unemployment After Matches Take Place

The model-implied times series of the employment/population ratio, the unemployment/population ratio and the nonparticipation rate are as follows:

$$\begin{aligned} E_t &= (1-\eta)(1-\lambda)E_{t-1} + \pi p_{t-1}U_{t-1} + \xi f_{t-1}O_{t-1} && (29) \\ U_t &= (1-\eta)\lambda E_{t-1} + \pi(1-p_{t-1})U_{t-1} + \xi(1-f_{t-1})O_{t-1} && (30) \\ O_t &= \eta E_{t-1} + (1-\pi)U_{t-1} + (1-\xi)O_{t-1} && (31) \end{aligned}$$

Note that the nonparticipation rate is independent of the job-finding probabilities, p_{t-1} and f_{t-1}. Recall that in the KPR matching model, the nonparticipation rate is a function of the job-finding probabilities. The KPR matching model predicts the linear relationships between the employment/population ratio, the unemployment/population ratio and the nonparticipation rate, which imply the perfect correlations. The modified model, however, does not give any linear relationship between the variables, and no perfect correlations are derived.

Another advantage of the modified model is that the UE and OE transition rates, πp and ξf, are directly identified from the data. In particular, the OE transition rate, ξf, consists of two parts: the probability of a nonparticipant entering the labor force (ξ), and the job-finding probability conditional on participation in the labor force (f). Although we observe that the UE transition rate is higher than the OE transition rate, this does not necessarily imply that p is greater than f. Suppose that is p less than f. If the probability of remaining unemployed, π, is much higher than the probability of entering the labor force, ξ, then a higher UE transition rate can be observed.

Table 2: Calibration to the U.S. Labor Market

Parameters	Notation	KPR	Modified
Discount factor	β	.9967	.9967
Workers' bargaining power	α	.5	.5
Matching function elasticity	γ	.5	.5
Unemployed income	b	.28	.264
Prob. of an unemployed worker remaining in the labor force	π	-	.8575
Prob. of a nonparticipant entering the labor force	ξ	-	.0506
Unemployed workers' conditional job-finding probability	p	.2233	.2604
Nonparticipants' conditional job-finding probability	f	.0279	.5517
Prob. of a matched worker leaving the labor force	η	-	.0187
Prob. of an employed worker being laid off	λ	.0302	.0117
Worker-finding probability	q	.5	.5
Vacancy/searcher ratio	θ	.4466	.5209
Matching function constant	ω	.3342	.3609
Job-posting cost	k	4.2672	4.9879
Persistence of productivity shock	ρ_z	.97	.97
Standard deviation of shock	σ_ϵ	.01	.01

3 Quantitative Analysis

3.1 Calibration

I calibrate the models to the U.S. data and evaluate them quantitatively. The models operate at a monthly frequency, and therefore the discount factor is set to $\beta = .9967$, equivalent to an annual interest rate of 4 percent. Workers' bargaining power (α) and the matching function elasticity with respect to search (γ) are set to .5. Following Shimer (2005), the level of unemployment benefits (or the replacement ratio) is set to 40 percent of the steady-state wage.[5]

Following Andolfatto (1996), I set the worker-finding probability to .5 which is consistent with an average vacancy duration of about 45 days.[6] Accordingly, the steady-state vacancy/searcher ratio (θ^*) and matching function parameter (ω) are pinned down.

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[5] The level of unemployment insurance benefits is about .28 for both cases.

[6] Andolfatto (1996) sets the worker-finding probability to .9 at quarterly frequencies. At monthly frequencies, it is about .5

That is, $\theta^* = p^*/q^*$ and $\omega = p^*\theta^{*\gamma-1}$. Finally, the persistence parameter of the productivity shock, ρ_z, is set to .97. The standard deviations of the innovation to the productivity shock, σ_ϵ, is set to 1 percent.

3.1.1 Unemployment Before Matches Take Place

The steady-state probability that an unemployed worker finds a job, p^*, is set to the UE transition rate, 22.33 percent, and the steady-state probability that a nonparticipant finds a job, f^*, is set to the OE transition rate, 2.79 percent. The relative search intensity is given by $x = f^*/p^* < 1$. The probabilities of a worker being laid off and of an unmatched worker searching for a job are calculated from the steady-state condition of the reduced-form labor market dynamics:

$$\lambda = \frac{p^*U^* + f^*O^*}{E^*} \tag{32}$$

$$\mu = \frac{U^*}{U^* + O^*} \tag{33}$$

where E^*, U^* and O^* are the steady-state employment/population ratio, unemployment/population ratio and nonparticipation rate, respectively. Since the relative standard deviations are independent of η, η is set to 0.

3.1.2 Unemployment After Matches Take Place

The probability that an unemployed worker remains in the labor force is set to $\pi = h_{UE} + h_{UU}$. The steady-state probability that an unemployed worker finds a job is $p^* = h_{UE}/(h_{UE} + h_{UU})$. The probability that a nonparticipant enters the labor force is set to $\xi = h_{OE} + h_{OU}$. The steady-state probability that a nonparticipant finds a job is $f^* = h_{OE}/(h_{OE} + h_{OU})$. The probabilities of a worker leaving the labor force and of being laid off are calculated from the steady-state condition of the reduced-form labor market dynamics:

$$\eta = \frac{\xi O^* - (1-\pi)U^*}{E^*} \tag{34}$$

$$\lambda = \frac{(1-\pi+\pi p^*)U^* - \xi(1-f^*)O^*}{(1-\eta)E^*} \tag{35}$$

The relative search efficiency is given by $y = f^*/p^* > 1$. All parameters are summarized in Table 2.

Table 3: Descriptive Statistics for the CPS, 1978-2005

(1) Level Variables

	E	U	O	u
Mean (%)	61.67	4.06	34.28	6.19
Standard deviation (%)	1.90	.87	1.23	1.41
	(1.00)	(.46)	(.65)	(.74)
Correlation				
E	1.00	-.86	-.93	-.89
U		1.00	.61	1.00
O			1.00	.66

(2) Cyclical Variables

	$\ln E$	$\ln U$	$\ln O$	$\ln u$
Standard deviation (%)	1.21	12.99	.72	13.22
	(1.00)	(10.75)	(.60)	(10.94)
Correlation				
$\ln E$	1.00	-.96	-.78	-.96
$\ln U$		1.00	.62	1.00
$\ln O$			1.00	.64

Sample period: Jan.1978-Dec.2005. The seasonally adjusted BLS series are used: employment/population ratio (E), unemployment/population ratio (U), nonparticipation rate (O), and unemployment rate ($u = U/(E{+}U)$) for those aged 16 years and over as provided by the BLS. The relative standard deviations, as expressed a ratio to the standard deviation of employment, are given in the parentheses. Cyclical variables are obtained by a Hodrick-Prescott filter with smoothing parameter 900,000.

3.2 Simulations

The time period of the models is 336 months, and the associated statistics are obtained by 100 simulations. I report both level and cyclical variables. The cyclical variables are obtained by a Hodrick-Prescott filter with smoothing parameter 900,000. The main purpose of the paper is to investigate to what extent both models are able to explain the volatilities of the employment/population ratio, the unemployment/population ratio and the nonparticipation rate. Table 3 describes the U.S. data.

3.2.1 Unemployment Before Matches Take Place

In this subsection I evaluate the KPR model numerically in which unemployment is an active search and nonparticipation is an inactive search. I investigate to what extent the model is able to explain the volatilities of the labor market variables and the correlations

Table 4: Model Results

(1) Level Variables

	U.S. data	KPR Model	Modified Model
Standard deviations			
$Std(E)$	1.90%	1.15%	.58%
$Std(U)/Std(E)$	.46	.11	.44
$Std(O)/Std(E)$	.65	.89	.62
$Std(u)/Std(E)$	.74	.25	.72
$Std(V)/Std(E)$	-	.34	.48
Correlations			
$Corr(E,U)$	-.86	-1.00	-.92
$Corr(E,O)$	-.93	-1.00	-.96
$Corr(U,O)$	.61	1.00	.78
$Corr(U,V)$	-	-.63	-.83

(2) Cyclical Variables

	U.S. data	KPR Model	Modified Model
Standard deviations			
$Std(E)$	1.21%	1.09%	.55%
$Std(U)/Std(E)$	10.75	1.60	7.85
$Std(O)/Std(E)$	.60	1.60	1.06
$Std(u)/Std(E)$	10.94	2.44	8.21
$Std(V)/Std(E)$	-	7.44	10.60
Correlations			
$Corr(E,U)$	-.96	-1.00	-.88
$Corr(E,O)$	-.78	-1.00	-.91
$Corr(U,O)$	.62	1.00	.61
$Corr(U,V)$	-.91	-.44	-.73

Averages over 100 simulations of length 336 months. u denotes the unemployment rate. Cyclical variables are obtained by an HP filter with smoothing parameter 900,000. The seasonally adjusted monthly help-wanted advertising index V is constructed by the Conference Board.

between them.

Table 4 (KPR Model) shows the same results as what the reduced-form analysis implies. The relative standard deviation of the unemployment/population ratio, expressed as the ratio to the standard deviation of the employment/population ratio, is 8 time greater than that of the nonparticipation rate in level variables. For cyclical variables, the relative volatility of the unemployment/populatin ratio, 1.6, is the same as one of the nonparticipation rate. Therefore, we find that the model accounts for only 20 percent of the actual volatility of the unemployment rate.

The correlations of the employment/population ratio with both the unemployment/population ratio and the nonparticipation rate are -1 and the correlation between the unemployment/population ratio and the nonparticipation rate is 1. The model predicts a weak negative relationship between the unemployment/population ratio and the vacancy rate with correlation of $-.44$ which is less than the half of the actual relationship of the U.S. data, $-.91$.

3.2.2 Unemployment After Matches Take Place

I return to the modified model in which nonparticipation is not searching and unemployment captures both those who search but do not find employment and those who work but separate. Table 4 (Modified Model) shows the moments of the model-generated data.

The modified model gives better results than the KPR model does. The relative volatilities of the unemployment/population ratio and the nonparticipation rate are 7.85 and 1.06, respectively. Compared with the KPR model, the relative volatility of the unemployment/population ratio increases about five times, and the relative volatility of the nonparticipation rate decreases one-third. The modified model predicts much more volatile unemployment rate than the KPR model does, so that it accounts for three-quarters of the actual volatility of the unemployment rate.

The correlation between the unemployment/population ratio and the nonparticipation rate (.61) is very close to the actual number (.62). Moreover the Beveridge relationship between the unemployment/population ratio and the vacancy rate is much stronger in the modified model than in the KPR model, and is also very close to the actual data.

In spite of all the advantages, however, the model predicts that the nonparticipation rate is highly negatively correlated with the employment/population ratio than what the data shows.

Table 5: Correlations of the UE transition rate with π and ξ, CPS

	$Corr(h_{UE}, \pi)$	$Corr(h_{UE}, \xi)$
Feb 1978 - Jun 1985	-.61	.50
Nov 1985 - Dec 1993	-.39	.42
Oct 1995 - Dec 2005	-.24	.32

h_{UE} is the UE transition rate. All variables are detrended by an HP filter with smoothing parameter 900,000.

3.3 Discussion

I calibrate π and ξ to the transition rates adjusted with the method that Abowd and Zellner (1985) propose because there are serious misclassification problems with the gross flows data derived from the Current Population Survey (CPS). I apply their adjustment under the assumption that these biases are time-invariant, as in Abraham and Shimer (2002) and Garibaldi and Wasmer (2005). The assumption is necessary because Abowd and Zellner (1985) use only 6 years data, January 1977 - December 1982.

I find that π is 77.9 percent and ξ is 7.2 percent using the unadjusted transition rates while π and ξ are 85.8 percent and 5.1 percent, respectively, using the adjusted transition rate. The Abowd and Zellner (1985) correction increases the UU transition rate, but decreases the UE transition rate and the transition rates from OLF to the labor force. The differences then are about 8 percent for π and 2 percent for ξ. Since the nonparticipation rate depends on $(1-\pi)U$ and $(1-\xi)O$, the coefficients play an important role in explaining the cyclical behavior of the labor market variables. If a more recent correction measure was available, then I could evaluate the model better.

One may argue that the probabilities, π and ξ, are not constant over time, but are moving with the labor market conditions. I look at the correlations of the UE transition rate with π and ξ. Since the UE transition rate, π and ξ have some missing observations, I split the entire sample into 3 sub-samples: February 1978 - June 1985, November 1985 - December 1993 and October 1995 - December 2005. The correlations of the UE transition rate which captures the labor market conditions with π and ξ are a bit strong for the first sub-sample, whereas for the most recent and longest sub-sample, Oct 1995 - Dec 2005, the correlations are very weak, -.24 and .32, respectively.

4 Concluding Remarks

This paper argues that existing matching models with unemployment as an active search and nonparticipation as an inactive search predict counterfactual business-cycle results.

This paper proposes a modified matching model in which workers are classified after matches take place and the distinction between unemployment and nonparticipation becomes clear. That is, those who do not engage in job search are classified as out of the labor force, and those who search and find no employment are classified as unemployed, consistent with the definition of the CPS.

As a result, the modified model generates the direct transition from nonparticipation to employment with no assumption that nonparticipation is an inactive search and without adjusting the time period of the model. In addition, some parameters which governing the model is easily identified by the reduced-form dynamics derived from the model.

The modified model accounts for the U.S. labor market much better than the the existing models do. It explains three-quarters of the actual volatility of the unemployment rate, realistic contemporaneous correlations between the labor market variables, and the Beveridge relationship.

References

Abraham, Katherine G. and Robert Shimer. "Changes in Unemployment Duration and Labor Force Attachment." in Alan Krueger and Robert Solow, eds., *The Roaring Nineties*. New York: Russell Sage Foundation, 2002, pp. 367-420.

Abowd, John M. and Arnold Zellner. "Estimating Gross Labor-Force Flows." *Journal of Business & Economic Statistics*, July 1985, 3(3), pp. 254-283.

Andolfatto, David. "Business Cycles and Labor-Market Search." *American Economic Review*, March 1996, 86(1), pp. 112-132.

Cole, Harold L. and Richard Rogerson. "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?" *International Economic Review*, November 1999, 40(4), pp. 933-959.

Garibaldi, Pietro and Etienne Wasmer. "Equilibrium Search Unemployment, Endogenous Participation and Labor Market Flows." *Journal of the European Economics Association*, June 2005, 3(4), pp. 851-882.

Hall, Robert E. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, March 2005, 95(1), pp. 50-65.

Haefke, Christian and Michael Reiter. "Endogenous Labor Market Participatioin and the Business Cycle." Working paper, Universitat Pompeu Fabra, March 2006.

Kim, Sun-Bin. "Unemployment Insurance Policy with Endogenous Labor Force Participation." Working paper, Concordia University, August 2004.

Lucas, Robert E. and Edward C. Prescott. "Equilibrium Search and Unemployment." *Journal of Economic Theory*, February 1974, 7(2), pp. 188-209.

Moon, Weh-Sol. "Individual Labor Market Transitions and Aggregate Dynamics." Ph.D. Dissertation, University of Rochester, 2007.

Mortensen, Dale and Christopher A. Pissarides. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, July 1994, 61(3), pp. 397-415.

Petrongolo, Barbara and Christopher A. Pissarides. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, June 2001, 39(2), pp. 390-431.

Pries, Michael and Richard Rogerson. "Search Frictions and Labor Market Participation." Working paper, University of Maryland, November 2004.

Shimer, Robert. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, March 2005, 95(1), pp. 25-49.

Tripier, Fabien. "Can the Labor Market Search Model Explain the Fluctuations of Allocations of Time." *Economic Modelling*, January 2004, 21(1), pp. 131-146.

Veracierto, Marcelo. "On the Cyclical Behavior of Employment, Unemployment, and Labor Force Participation." Working paper, Federal Reserve Bank of Chicago, March 2004.