A Microfounded Mechanism of Observed Substantial Inflation Persistence

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Abstract

Recently, it has been argued that trend inflation may be the solution to the puzzle of inflation persistence in the New Keynesian Phillips curve (NKPC). However, incorporating trend inflation into the NKPC raises another serious problem—it lacks a microfoundation. The paper presents a microfoundation for trend inflation, which indicates that trend inflation is a natural consequence of simultaneous optimization by the government and households. A purely forward-looking model is constructed based on the microfoundation presented. The model enables a unified explanation for various types of inflation. It also indicates that, if inflation is assumed to follow an autoregressive process without considering trend inflation, many measures of inflation persistence will spuriously indicate that inflation is intrinsically substantially persistent and has a backward-looking property.

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1 INTRODUCTION

A well-known and serious problem with the pure New Keynesian Phillips curve (NKPC) is that it is not consistent with the observed substantial persistence of inflation (e.g., Fuhrer and Moore 1995; Galí and Gertler 1999; Mankiw 2001). Therefore, since Galí and Gertler (1999), economists have intensely studied a modified version of the NKPC (i.e., the hybrid NKPC) that includes lagged inflation. The hybrid NKPC captures the persistent nature of inflation well, but the difficult puzzle of why rational agents behave in a partially backward-looking manner remains. Galí et al. (2005) argue that a more coherent rationale for the role of lagged inflation in the hybrid NKPC must be provided. Furthermore, Fuhrer (2006) concludes that inflation in the hybrid NKPC inherits relatively little persistence from the driving process and that a microfounded mechanism that generates substantial intrinsic persistence in inflation is required.

Recently, it has been argued that the appearance of substantial intrinsic inflation persistence is spurious due to trend inflation. Cogley and Sbordone (2005, 2006) show that, if trend inflation is incorporated into the pure NKPC, its performance on fitting actual inflation data improves considerably. They conclude that trend inflation has historically been quite volatile and that, if these fluctuations of long-run moving trend inflation are taken into account, a purely forward-looking model does a good job approximating the short-run dynamics of inflation. Woodford (2007) argues that Cogley and Sbordone (2005) present an alternative interpretation of the apparent need for lagged inflation terms in the NKPC (see also Hornstein 2007). Indeed, data from most industrial economies show that inflation is highly volatile and transitioned from high to low in the 1980s, which strongly implies the existence of trends in inflation (e.g., Stock and Watson 2006; Sbordone 2007). Ascari (2004) argues that disregarding trend inflation is very far from being an innocuous assumption and the results obtained by models log-linearized around a zero inflation steady state are misleading (see also Bakhshi et al. 2003). These studies suggest that incorporating trend inflation into the NKPC will solve the puzzle of inflation persistence. But, if we proceed further in this research direction, another serious theoretical problem arises—trend inflation lacks a microfoundation. Can trend inflation be explained as a consequence of rational agents' optimizations? Why do monetary policy makers often allow upward trends in inflation? The purpose of the paper is to explore the as yet unexplained microfoundation of trend inflation, which if discovered, will help provide a solution to the puzzle of inflation persistence in the NKPC.

An important feature of trend inflation is that it occasionally deviates markedly from zero inflation (e.g., hyperinflation or chronic inflation). Such deviations are puzzling in the NKPC because the NKPC implies that inflation stays at about a zero steady state rate. Data during the Great Inflation in the 1970s particularly perplex NKPC researchers. These deviations from zero inflation are not only inconsistent with the NKPC but also with the central banks' expected role to stabilize inflation at a low rate. Central banks sometimes look as if they are deliberately allowing continuous high rates of inflation perhaps as a result of government intervention. One of the few explanations for this behavior is to assume that governments are weak, foolish, or untruthful, and they behave irrationally from an economic point of view or are somehow forced to behave irrationally. A government may be pressured by interest groups to take an inflationary policy stance and intervene in a central bank's decision-making, and the central bank is then unable to fully commit to its policies, which generates the possibility of chronic inflation (e.g., Kydland and Prescott 1977; Barro and Gordon 1983; Rogoff 1985; Berger et al. 2000). The assumptions of ad hoc frictions or that households or firms are irrational to some extent have been used to explain hyperinflation (e.g., Cagan 1956). For example, hyperinflation can occur only if adaptive expectations or some ad hoc frictions are assumed when large budget deficits are allowed in the well-known Cagan (1956) framework (e.g., Auernheimer 1976; Evans and Yarrow 1981; Kiguel 1989). However, these explanations raise many fundamental questions. Is a government always so foolish that it obeys interest groups that represent only a part of its constituency? Why is a government so weak even though it wields great authority at will? Does a government dare to take inflationary actions even if the majority of its constituency prefers low inflation and the government itself also desires low inflation? Why do households form adaptive expectations? Is the friction used in a Cagan-type hyperinflation model well microfounded? Given these unanswered questions, neither of the aforementioned explanations is particularly compelling.

The fiscal theory of the price level (FTPL) argues that a problem with conventional inflation theory is that it almost neglects the importance of the government's borrowing behavior in inflation dynamics (e.g., Leeper 1991; Sims 1994, 1998, 2001; Woodford 1995, 2001; Cochrane 1998a, 1998b, 2005).[1] It has been argued that, if a government borrows money without limits, inflation will eventually explode (e.g., Sargent and Wallace 1981). The FTPL implies that, if a government's borrowing behavior is modeled properly, the mechanism of severely deviated inflation paths can be explained without assuming ad hoc frictions or irrationality. Most FTPL models have not, however, explicitly modeled the behavior of government in detail. Hence, some critics contend that the theory is fallacious (e.g., Kocherlakota and Phelan 1999; McCallum 2001, 2003; Buiter 2002, 2004; Niepelt 2004).

If, however, the government's borrowing behavior is modeled properly and explicitly, it would be possible to use the FTPL to explain the puzzling behavior of central banks and then present a microfoundation of trend inflation. I explore this possibility by first examining the nature of the government budget constraint in detail and then constructing a model of trend inflation that fully incorporates the government's borrowing behavior. In the model, (1) both the government and the representative household achieve simultaneous optimization, (2) the roles of the government and the central bank are explicitly separated, and (3) no ad hoc friction or irrationality is assumed. The first feature leads to the microfoundation of trend inflation.

The model indicates that trend inflation accelerates or decelerates if the time preference rates of the government and the representative household are heterogeneous. Because a government represents the median of households under a proportional representation system and the economically representative household represents the mean of households, the preferences between them are usually heterogeneous. In addition, and more importantly, even if a government behaves in a fully rational manner and understands that heterogeneous time preferences may cause high inflation, the government can barely control its own preferences by itself. The government, however, is not the only entity that cannot easily self-regulate its own preferences at will, even

[1] See also Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), and Gordon and Leeper (2002).

when these preferences may result in unfavorable consequences. Hence, an independent central bank is necessary to control the government's preferences and prevent this inflation acceleration mechanism from working.

If trend inflation is truly generated by the mechanism shown in the paper, studies that do not appropriately account for trend inflation will provide misleading conclusions. The model in the paper indicates that, if inflation is assumed to be an autoregressive process, many measures of persistence will spuriously show that inflation is substantially persistent, implying that findings of substantial intrinsic inflation persistence is a consequence of serious misspecification. The model thus will provide an essential clue to solve the puzzle of inflation persistence. Furthermore, the model enables a unified and microfounded explanation for various inflation phenomena (e.g., hyperinflation, chronic inflation, disinflation, low and stable inflation, and deflation) without assuming ad hoc frictions or irrationality.

The paper is organized as follows. In section 2, I examine the nature of the government budget constraint and construct a model of trend inflation that assumes an economically Leviathan government in which the government and the representative household achieve simultaneous optimization. With this microfoundation of trend inflation, a model of inflation is constructed in which all the agents behave in purely forward-looking manners. In section 3, the basic nature of the model is examined. The model indicates that various types of inflation are generated according to the degree of central bank independence and the difference of preferences between government and households. In section 4, I show that, if inflation is assumed to follow an autoregressive process without considering trend inflation, many measures of inflation persistence will spuriously indicate that inflation is intrinsically substantially persistent. Finally, I offer concluding remarks in section 5.

2 THE MODEL

2.1 Step 1: Optimal trend inflation[2]

2.1.1 The government

2.1.1.1 The government budget constraint

As with the FTPL, the government budget constraint is a key element in the explanation for inflation in this paper. The budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \Omega_t ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, i_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and Ω_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the

[2] The model of the optimal trend inflation in section 2 is based on the inflation model presented in Harashima (2007c). Various aspects of the model and analyses in this paper are also presented in Harashima (2004b, 2005, 2006, 2007a, 2007b).

government successively refinances the bonds by issuing new ones at each time t. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $\omega_t = \frac{\Omega_t}{P_t}$, where P_t is the price level at time t. Let also $\pi_t = \frac{\dot{P}_t}{P_t}$ be the inflation rate at time t. By dividing by P_t, the budget constraint is transformed to $\frac{\dot{B}_t}{P_t} = b_t\, i_t + g_t - x_t - \omega_t$, which is equivalent to

$$\dot{b}_t = b_t\, i_t + g_t - x_t - \omega_t - b_t \pi_t = b_t (i_t - \pi_t) + g_t - x_t - \omega_t .$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\bar{i}_t \geq E_t \int_t^{t+1} (\pi_s + r_t) ds$ at time t, where $\bar{i}_t$ is the nominal interest rate for bonds bought at t and r_t is the real interest rate in markets at t. Hence, by arbitrage, $\bar{i}_t = E_t \int_t^{t+1} (\pi_s + r_t) ds$, and if r_t is constant such that $r_t = r$ (i.e., if it is at steady state), then

$$\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r .$$

The nominal interest rate $\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between t and $t + dt$, the government's obligation to pay for the bonds' return in the future increases not by $dt(\pi_t + r)$ but by $dt\left(E_t \int_t^{t+1} \pi_s \, ds + r \right)$. If π_t is constant, then $E_t \int_t^{t+1} \pi_s \, ds = \pi_t$ and $\bar{i}_t = \pi_t + r$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at t have been issued between $t-1$ and t. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time t is the weighted sum of $\bar{i}_t$ such that

$$i_t = \int_{t-1}^{t} \bar{i}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r ,$$

where $\overline{B}_{s,t}$ is the nominal value of bonds at time t that were issued at time s. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between $t-1$ and t are not so different from each other, then approximately $i_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \, ds + r$. To be precise, if the absolute values of π_s for $t-1 < s \leq t+1$ are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r \tag{1}$$

(see Appendix 1).[3] The average nominal interest rate for the total government bonds, therefore, develops by $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds + r$. If π_t is constant, then $\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds = \pi_t$; thus, $i_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds = \pi_t$ and $i_t = \pi_t + r$ do not necessarily hold.

2.1.1.2 An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[4] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[5] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[6] For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median household under

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[3] If the absolute values of π_s for $t-1 < s \le t+1$ are very large, the weight $\dfrac{\overline{B}_{t,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ will be much larger than $\dfrac{\overline{B}_{t-1,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ when π_t is increasing. In this case, i_t will be closer to $\int_{t}^{t+1} \pi_s \, ds + r$ than $\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r$.

[4] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991; Cukierman et al. 1992).

[5] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

[6] The government behavior assumed in the FTPL reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

a proportional representation system (e.g., Downs 1957) whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[7] In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^{\ 2}} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} > 0$.[8] An economically Leviathan government

[7] It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[8] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} < 0$. However, the assumption used

therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

2.1.1.3 The optimization problem

The optimization problem of an economically Leviathan government is

$$Max\ E_0 \int_0^{\infty} u_G(g_t, x_t) \exp(-\theta_G t) dt$$

subject to the budget constraint

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \omega_t ,$$

where u_G is the constant relative risk aversion utility function of the government and θ_G is the government's rate of time preference. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in i_t in its budget constraint.

2.1.2 Households

The economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E_0 \int_0^{\infty} u_P(c_t, m_t) \exp(-\theta_P t) dt$$

subject to the budget constraint

$$\dot{a}_t = (r_t a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t) m_t] - g_t ,$$ [9]

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, σ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and

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is not an important issue here because $\frac{x_t \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2}}{\frac{\partial u_G(g_t, x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

[9] The constraint is equivalent to $\dot{a}_t = (r_t a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t) m_t] - \dot{b}_t - x_t - \omega_t + b_t(i_t - \pi_t)$.

$\frac{\partial^2 u_P(c_t,m_t)}{\partial m_t^2}<0$, where $f(\cdot)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because of the assumption of an economically Leviathan government. It is also assumed that lump-sum government transfers (σ_t) equal the seigniorage (s_t), and that, although all households receive transfers from a government in equilibrium, each household takes the amount it receives as given when making decisions, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption c_t, the real investment $\dot{k}_t$, and the real government expenditure g_t such that $f(k_t)=c_t+\dot{k}_t+g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in section 2.2.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

2.1.3 The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P=u_P(c_t,m_t)\exp(-\theta_P t)+\lambda_{P,t}[r_t a_t+w_t+\sigma_t-c_t-(\pi_t+r_t)m_t-g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are:

$$\frac{\partial u_P(c_t,m_t)}{\partial c_t}\exp(-\theta_P t)=\lambda_{P,t}\,, \tag{2}$$

$$\frac{\partial u_P(c_t,m_t)}{\partial m_t}\exp(-\theta_P t)=\lambda_{P,t}(\pi_t+r_t), \tag{3}$$

$$\dot{\lambda}_{P,t}=-\lambda_{P,t}\,r_t\,, \tag{4}$$

$$\dot{a}_t = (ra_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t - g_t], \tag{5}$$

$$\lim_{t\to\infty} \lambda_{P,t}\, a_t = 0\,. \tag{6}$$

By conditions (2) and (3), $\dfrac{\dfrac{\partial u_P(c_t, m_t)}{\partial m_t}}{\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}} = \pi_t + r_t$, and by conditions (2) and (4), $-\dfrac{c_t \dfrac{\partial^2 u_P(c_t, m_t)}{\partial c_t^2}}{\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}} \dfrac{\dot{c}_t}{c_t} + \theta_P = r_t$. Hence,

$$\theta_P = r_t = r \tag{7}$$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G\, t) + \lambda_{G,t}[b_t(i_t - \pi_t) + g_t - x_t - \omega_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are:

$$\frac{\partial u_G(g_t, x_t)}{\partial g_t}\exp(-\theta_G\, t) = -\lambda_{G,t}\,, \tag{8}$$

$$\frac{\partial u_G(g_t, x_t)}{\partial x_t}\exp(-\theta_G\, t) = \lambda_{G,t}\,, \tag{9}$$

$$\dot{\lambda}_{G,t} = -\lambda_{G,t}(i_t - \pi_t), \tag{10}$$

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \omega_t\,, \tag{11}$$

$$\lim_{t\to\infty} \lambda_{G,t}\, b_t = 0\,. \tag{12}$$

Combining conditions (8), (9), and (10) and equation (1) yields the following equations: $\dfrac{g_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2}}{\dfrac{\partial u_G(g_t, x_t)}{\partial g_t}} \dfrac{\dot{g}_t}{g_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v\, dv\, ds - \pi_t$ and $-\dfrac{x_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\,2}}}{\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} + \theta_G = i_t - \pi_t =$

$r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$. Here, $\dfrac{g_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2}}{\dfrac{\partial u_G(g_t, x_t)}{\partial g_t}} \dfrac{\dot{g}_t}{g_t} = 0$ and $\dfrac{x_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\,2}}}{\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus,

$$\theta_G = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t .$$

Hence, by equation (7),

$$\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t + \theta_G - \theta_P \tag{13}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[10]

Equation (13) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

$$i_t - r = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds \neq \pi_t .$$[11]

This result might seem surprising because it has been naturally believed that $i_t = \pi_t + r$ and then $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ by equation (1). However, this is a simple misunderstanding because π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \dfrac{\dot{P}_t}{P_t}$, whereas $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ roughly indicates the average inflation rate in a period. Equation (13) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \theta_G + \theta_P$. If π_t is constant, the equations $i_t = \pi_t + r$ and $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (13) indicates that the equations $i_t = \pi_t + r$ and $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $i_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually

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[10] If and only if $\theta_G = -\dfrac{g_t - x_t - \omega_t}{b_t}$ at steady state, then the transversality condition (12) $\lim_{t \to \infty} \lambda_{G,t} \, b_t = 0$ holds. The proof is shown in Appendix 2.

[11] Note that $i_t - r = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ as equation (1) shows.

guaranteed.

2.1.4 The law of motion for trend inflation

Equation (13) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, equation (13) cannot hold in an economy in which $\theta_G \neq \theta_P$. In other words, it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. Heterogeneous time preferences ($\theta_G \neq \theta_P$) change the path of inflation and enable inflation to accelerate or decelerate.

Equation (13) implies that inflation accelerates or decelerates nonlinearly when $\theta_G \neq \theta_P$. For a sufficiently small period *dt*, π_{t+1+dt} is determined with the $\pi_s \ (t-1<s \leq t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_v \, dv \, ds = \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_v \, dv \, ds + \pi_{t+dt} - \pi_t$. A solution of the integral equation (13) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 . \tag{14}$$

Generally, the path of inflation that satisfies equation (13) for $0 \leq t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] , \tag{15}$$

where z_t is a time-dependent variable. The stream of z_t varies depending on the boundary condition, that is, past and present inflation during $-1<t\leq 0$ and the path of inflation during $0<t\leq 1$, that is set to make π_0 satisfy equation (13). However, z_t has the following important property: if π_t satisfies equation (13) for $0 \leq t$, and $-\infty<\pi_t<\infty$ for $-1<t\leq 1$, then

$$\lim_{t\to\infty} z_t = 2 . \tag{16}$$

The proof is shown in Appendix 3. Therefore, any inflation path that satisfies equation (13) for $0 \leq t$ asymptotically approaches the path of equation (14).

2.1.5 The inflation acceleration (deceleration) mechanism

Because the integral equation (13) is so different from what has been regarded as the common wisdom (i.e., $i_t = \pi_t + r$), some researchers may still be skeptical of the result. Equation (13), however, is a very natural consequence of simultaneous optimization. Rather, it would appear more surprising if heterogeneous and homogeneous preferences produced identical results. The basic mechanism behind equation (13) is that when inflation accelerates, the increase rate of the government's real obligation b_t at time t is not r but $r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ because increases in the real obligation are moved forward in time (or "moved up") by the acceleration of inflation. In this section, I explain this mechanism in more detail.

2.1.5.1 Necessity of reconciling heterogeneous discount factors

In the non-stochastic economy modeled in section 2.1 (Step 1), the sum of the government's real expenditure, the representative household's real consumption, and the real investment is equal to the real output at any time, as shown in the budget constraint of the representative household, such that

$$f(k_t) = c_t + \dot{k}_t + g_t .$$

Hence, the streams of expenditure and consumption are not determined independently of each other and should be consistent with the stream of the output. However, if the discount factors (θ_G and θ_P) are heterogeneous, there is no guarantee that the streams are consistent; that is, there is no guarantee that both transversality conditions of the government and the representative household (equations [6] and [12]) are satisfied and that both expected utilities are maximized simultaneously. For example, the expected utility of the representative household is maximized if the point $r = \theta_P$ is at steady state as usual and as equation (7) shows. However, if $\theta_G \neq \theta_P$, it is not rational for the government to stop changing its real expenditures, taxes, and borrowing at the point where $r = \theta_P$. The government's expected utility will increase by changing them even if $r = \theta_P$ because $r = \theta_P \neq \theta_G$. The government's behavior obstructs the optimization of the representative household, but it is completely rational behavior for the government. Therefore, this contradiction of discount factors should be reconciled by some mechanism to make the streams consistent (except for corner solutions) and to make both the government and the representative household able to achieve simultaneous optimization.

The easiest way to achieve a steady state in an economy when discount factors are heterogeneous is to expel the government from the market, but that is impossible. Unless a way is found that enables the government and the representative household to coexist at steady states (other than corner solutions), the economy may break down. For them to be able to coexist at steady states, the government should stop changing its real borrowing at the point where $r = \theta_P \neq \theta_G$. Hence, if there is a mechanism that penalizes the government for having θ_G unequal to θ_P and makes it refrain from changing its real borrowing at the point where $r = \theta_P \neq \theta_G$, coexistence will be possible. Equation (15) indicates that the mechanism for penalizing the government does indeed exist—the acceleration or deceleration of inflation. I explain how this mechanism works in the next section.

2.1.5.2 Moved-up real obligations

Suppose for simplicity that $\theta_G > \theta_P$. Inflation accelerates by equations (15) and (16) because of the heterogeneous discount factors. Equation (13) indicates that the government's existing real obligation b_t increases at a higher rate than r by $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds - \pi_t \, (>0)$ because $i_t = r + \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds$ as equation (1) indicates and the real obligation increases by

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \omega_t = b_t\left(r + \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds - \pi_t\right) + g_t - x_t - \omega_t$$

as shown in the real government budget constraint. This higher rate of increase in the real obligation by $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds - \pi_t$, a result of accelerating inflation, indicates the government's penalty for having $\theta_G > \theta_P$.

Note, however, that the real rate of return on investments in government bonds is always r regardless of the acceleration of inflation because

$$\int_{t}^{t+1}\left(\bar{i}_t - \pi_s\right)ds = \int_{t}^{t+1}\pi_s \, ds + r - \int_{t}^{t+1}\pi_s \, ds = r .$$

When inflation accelerates, the increased rate of the government's real obligation b_t at time t is not r, however, it is $r + \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds - \pi_t$ because increases in the real obligation are moved forward in time (or "moved up") by the acceleration of inflation. Figure 1 shows the increases in the real obligation at each time during a unit period for bonds issued at t ($\bar{B}_t$), the real value of which at time t is $\bar{b}_t$ when inflation is accelerating. Because the nominal interest rate $\bar{i}_t = r + \int_{t}^{t+1}\pi_s \, ds$ indicates that the government's nominal obligation to pay for the bonds' return in the future increases at the constant rate $r + \int_{t}^{t+1}\pi_s \, ds$ between t and $t+1$, then the real obligation expands at a time-varying rate between t and $t+1$ such that

$$r + \int_{t}^{t+1}\pi_s \, ds - \pi_v$$

owing to the accelerating rate of inflation (π_t). Clearly, the line of the increases in the real obligation should slope down to the right as shown in Figure 1. Hence, the rate of increase in the real obligation at time t is higher than r. The increases in the real obligation are moved up by the acceleration of inflation, holding the total increase in the real obligation during a unit period between t and $t+1$ to $r\,\bar{b}_t$.

However, the moved-up increases in the real obligation mean that the increases in the real obligation become smaller later in a unit period for each bond. Thus, because bonds issued between $t-1$ and t offset each other, does the rate of increase in the real obligation of total government borrowing remain r at any time? It does not, because the magnitude of the increase (i.e., the rate of increase) in moving up increases as inflation accelerates (Fig. 2). Because the rate of inflation increases by the square of time as shown in equations (14), (15), and (16), more increases in the real obligation are moved up as time passes. As a result, the increases in the real obligation cannot be indefinitely offset completely by the smaller increases in the real obligation of bonds issued in the past. Figure 3 shows the increases in the real obligation of bonds issued between $t-1$ and t at time t (i.e., the increases in the real obligation of $\overline{B}_{t-1+s,t}$ for $0 < s \leq 1$, the real value of which at time t is $\overline{b}_{t-1+s,t}$). Here, $\overline{b}_{t-1+s,t}$ is constant at steady state, $\overline{b}$. Hence, the increases in the real obligation of the total government bonds at time t is $r\overline{b}$ plus the area of triangle ABC minus the area of triangle CDE in Figure 3. The area of triangle ABC is larger than the area of triangle CDE because the magnitude of moving up increases as time passes. Thus, the rate of increase in real obligation of the total government borrowing at time t (i.e., $r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds - \pi_s$) is larger than r for any future t indefinitely. The government therefore must continue to face rates of increase in real obligation higher than r by

$$\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds \; - \pi_t$$

at all points in the future.

2.1.5.3 Optimal behaviors of the government and the representative household

The government optimally plans its streams of future real expenditures, taxes, and borrowings subject to this moved up higher rate of increase in the real obligation for a future t. The government stops changing its real expenditures, taxes, and borrowing if the rate of increase in the real obligation $r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ equals the government's time preference rate θ_G. The rate at which the real obligation increases is therefore a crucial variable in determining the government's behavior. As equations (7) and (13) show, equations

$$\theta_P = r$$

and

$$\theta_G = r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds \; - \pi_t$$

hold simultaneously at steady state. That is, both the government and the representative household can achieve simultaneous optimization. In this sense, the rate at which the real obligation increases is a crucial variable not only for the government but for the entire economy.

The mechanism by which the government stops changing its real borrowing at the point where $r = \theta_P \neq \theta_G$ implies that the government is penalized if it has a higher time preference rate than the representative household. The penalized government is obliged to refrain from changing its real borrowing at the point $\theta_P = r$ facing $\theta_G = r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$. Therefore, with the penalty

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds - \pi_t ,$$

the contradiction of discount factors between the government and the representative household is reconciled. The government is penalized by the representative household by its expectation of accelerating inflation so as to prevent the economy from breaking down.

Determining behavior on the basis of the rate at which the real obligation increases, as equation (13) indicates, is optimal for the government as well as for the representative household. This is still true even if it is assumed that a government can perceive less moved-up real obligations (i.e., perceive less penalty) in the sense of a less steep slope in Figure 1 (i.e., the government can behave on the basis of a lower i_t than equations [1] and [13] indicate). This is true because, if such a government seeks to exploit the opportunity of higher expected utility by intentionally perceiving a lower rate of increase in the real obligation, then θ_G is always higher than the rate of increase in the real obligation. Therefore, the representative household cannot achieve optimality at the point where $r = \theta_P$. To prevent this consequence, the representative household penalizes the government more heavily by expecting more rapid inflation acceleration than equation (13) shows and makes the rate of increase in the real obligation the government perceives identical to θ_G. Note here that equations (13) and (15) concern only price level changes and are unrelated to real values, and real values are thus unaffected as long as the rate of increase in the real obligation the government perceives is identical to θ_G, irrespective of how the values of i_t and the penalty are perceived by the government. Hence, the government cannot achieve the higher expected utility by intentionally perceiving less moved-up real obligations, which will only result in a more rapid acceleration of inflation. Conversely, if the representative household penalizes the government more heavily than equation (13) shows to exploit the opportunity of a higher expected utility, then θ_G is always lower than the rate of increase in the real obligation and thereby the government cannot achieve optimality. To prevent this consequence, the government changes itself to perceive less moved-up real obligations and makes the rate of increase in the real obligation identical to θ_G. Hence, the representative household cannot achieve the higher expected utility by penalizing the government more heavily, which also only results in more rapid acceleration of inflation. As a result, even if it is assumed that a government can perceive less moved-up real obligations, in the sense of a less steep slope in Figure 1, equation (13) gives both the government and the representative household the least inflation acceleration for the highest expected utilities. In this sense, the strategy profile that both the government and the representative household do not seek to exploit these opportunities is a Nash equilibrium. Both know this mechanism well and expect inflation to accelerate as equation (13) indicates when they perceive that $\theta_G > \theta_P$.

2.2 *Step 2: Elements in the model*

In Step 1, it was shown that a trend exists in inflation as a consequence of simultaneous optimization with heterogeneous time preference rates. However, merely showing the existence of the optimal trend inflation is not sufficient to state that trend inflation is microfounded. An important property of trend inflation is that it has been historically quite volatile and often breaks (e.g., Cogley and Sbordone 2005, 2006; Stock and Watson 2006; Sbordone 2007). To complete a microfoundation of trend inflation, therefore, it is necessary to show a mechanism that brings about frequent trend breaks. Equations (13) and (15) imply that, if θ_G and $\theta_P\,(=r)$ are exogenously given constants, inflation exactly follows the path of optimal trend inflation and no trend break is brought about. Conversely, trend inflation can break if θ_G, $\theta_P\,(=r)$, or both are endogenous. In Step 2, I show that θ_G is endogenous if a central bank is independent and then construct an inflation model with an endogenous θ_G in Step 3 (Section 2.3). I also examine various elements that may appear in the model in Step 3. Many of the variables and equations examined here are basically the same as those in conventional discrete-time inflation models, including the aggregate supply equation, aggregate demand equation, and the instrument rule for a central bank. Variables and equations relating to trend inflation are also examined.

2.2.1 Optimal trend inflation

The elements that represent the optimal trend inflation in the model should be consistent with equation (15). The discrete-time version of equation (15) is

$$\pi_t^T = \pi_\varphi^T + 6(\theta_G - \theta_P)\exp[z_t \ln(t-\varphi)] \tag{17}$$

and equivalently

$$\pi_{t+1}^T = \pi_t^T + 6(\theta_G - \theta_P)\{\exp[z_{t+1}\ln(t-\varphi+1)] - \exp[z_t \ln(t-\varphi)]\}, \tag{18}$$

where π_t^T is the trend component in inflation in period t and $\varphi\,(\leq t)$ is the period in which the latest shock on θ_G occurred. In section 2.2.4, I will explain that θ_G should be time-variable and shocks on θ_G play an important role in inflation dynamics. When a shock on θ_G occurs and the value of θ_G is changed in period φ, trend inflation needs to be adjusted to be consistent with the new value of θ_G for the new initial period φ. The value of z_t is determined by the mechanism explained in section 2.1.2. Equations (17) and (18) are used in the model as the trend component in inflation.

2.2.2 Aggregate supply equation

Because the pure NKPC cannot sufficiently capture the persistent nature of inflation, it is necessary to modify the pure NKPC to a variant of the NKPC, such as the hybrid NKPC. In addition, the modified NKPC in the model should be consistent with optimal trend inflation (equation [17]). Here, as in Yun (1996) and Svensson (2003), who allow firms to index prices to the average inflation rate, the following modified version of NKPC that incorporates the nature of trend inflation is used:

$$\pi_{t+1} - \pi_{t+1}^T = (1-\theta_P)\left(\pi_{t+2|t} - \pi_{t+2|t}^T\right) + \alpha_x x_{t+1|t} + \varepsilon_{t+1}, \tag{19}$$

where x_t is the output gap; $\pi_{t+1|t}$, $\pi^T_{t+1|t}$, and $x_{t+1|t}$ are the rate of inflation, rate of trend inflation, and output gaps, respectively, that are expected in period t for period $t+1$; α_x is a constant coefficient; and ε_t is an i.i.d. shock with zero mean. This trend-augmented NKPC is basically the same as the simple forward-looking model in Svensson (2003); it merely replaces the average inflation rate with the optimal trend inflation.[12] Current inflation is determined not only by expected future inflation but also by trend inflation. By iterating equation (19) forward,

$$\pi_{t+1} - \pi^T_{t+1} = \lim_{s\to\infty} (1-\theta_P)^s \left(\pi_{t+s+1|t} - \pi^T_{t+s+1|t}\right) + \alpha_x \sum_{s=1}^{\infty} (1-\theta_P)^{s-1} x_{t+s|t} + \varepsilon_{t+1},$$

and because $0 < 1-\theta_P < 1$ and it is assumed that $-\infty < \lim_{s\to\infty}\left(\pi_{t+s+1|t} - \pi^T_{t+s+1|t}\right) < \infty$, then

$$\pi_{t+1} = \pi^T_{t+1} + \alpha_x \sum_{s=1}^{\infty} (1-\theta_P)^{s-1} x_{t+s|t} + \varepsilon_{t+1}, \tag{20}$$

which indicates that inflation is a process that proceeds not around zero inflation but around trend inflation.

The presence of trend inflation, however, may make the Calvo-type exogenous price-setting assumption rather unrealistic (e.g., Ascari 2004; Bakhshi et al. 2007). Nevertheless, similar to the model in Yun (1996), if firms fully index their prices to trend inflation, it offsets the influence of trend inflation on Calvo price-setting.[13] Although some researchers may still argue that full indexation is unrealistic, it is assumed in the present paper for simplicity that firms fully index their prices to trend inflation because this indexation is completely different from the backward-looking indexation that is assumed for the hybrid HKPC in some papers (e.g., Christiano et al. 2005). Firms index their prices not to past prices but to the expected optimal trend inflation that is formed purely in a forward-looking manner as shown in section 2.1 (Step 1). Moreover, it is not the main purpose of the present paper to elaborate the microfoundation of the Calvo-type NKPC model with trend inflation but to analyze the nature of a model with microfounded trend inflation.

If θ_G is an exogenously given constant, then

$$\pi^T_{t+1|t} - (1-\theta_P)\pi^T_{t+2|t} = \theta_P \pi^T_\varphi + 6(\theta_G - \theta_P)\{\exp[z_{t+1}\ln(t-\varphi+1)] - (1-\theta_P)\exp[z_{t+2}\ln(t-\varphi+2)]\}$$

by equation (17). Similarly, if θ_G is a time-dependent endogenous variable (its mechanism will be explained in section 2.2.4), then

$$\begin{aligned}\pi^T_{t+1|t} - (1-\theta_P)\pi^T_{t+2|t} &= \theta_P \pi^T_\varphi + 6\theta_{G,t+1}\exp[z_{t+1}\ln(t-\varphi+1)] - 6(1-\theta_P)\theta_{G,t+2}\exp[z_{t+2}\ln(t-\varphi+2)] \\ &\quad - 6\theta_P\{\exp[z_{t+1}\ln(t-\varphi+1)] - (1-\theta_P)\exp[z_{t+2}\ln(t-\varphi+2)]\}\end{aligned}$$

[12] It is also similar to the model used in Sbordone (2007).
[13] See, for example, Bakhshi et al. (2007).

by equation (17) modified by replacing θ_G with $\theta_{G,t}$, where $\theta_{G,t}$ is θ_G in period *t*. Hence, the aggregate supply equation (19) is transformed to

$$\begin{aligned}\pi_{t+1} &= \theta_P \pi_\varphi^T + (1-\theta_P)\pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1} \\ &+ 6\theta_{G,t+1} \exp[z_{t+1} \ln(t-\varphi+1)] - 6(1-\theta_P)\theta_{G,t+2} \exp[z_{t+2} \ln(t-\varphi+2)] \\ &- 6\theta_P \{\exp[z_{t+1} \ln(t-\varphi+1)] - (1-\theta_P)\exp[z_{t+2} \ln(t-\varphi+2)]\}. \end{aligned} \qquad (21)$$

2.2.3 Aggregate demand equation

Similar to Clarida et al. (1999) and Svensson and Woodford (2003), the model has the structure of the New Keynesian model with both forward-looking aggregate supply and demand equations. The model uses the following forward-looking aggregate demand equation:

$$x_{t+1} = x_{t+2|t} - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} - r \right) + \eta_{t+1}, \qquad (22)$$

where i_t is the nominal interest rate, r is the real interest rate at steady state, β_r is a constant coefficient, and η_t is an i.i.d. shock with zero mean. Note that $r = \theta_P$ by equation (7). This aggregate demand equation is basically same as the one used in Svensson and Woodford (2003), and it is a variant of the one used in Clarida et al. (1999) and the simple forward-looking one used in Svensson (2003).

2.2.4 The central bank

A central bank manipulates the nominal interest rate according to the following Taylor-type instrument rule in the model:

$$i_t = \bar{\gamma} + \gamma_\pi \left(\pi_t - \pi^* \right) + \gamma_x x_t, \qquad (23)$$

where π^* is the target rate of inflation and $\bar{\gamma}$, γ_π, and γ_x are constant coefficients. As is usually assumed, $\bar{\gamma} = \pi^* + r$.

In section 2.1 (Step 1), central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, the central banks control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (21) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with equation (21), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop consistently with equation (21), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with equation (21). That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation needs to be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if θ_G is adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate according to the Taylor-type instrument rule (equation [23]). Here,

$$i_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r = \theta_G + \pi_t \tag{24}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by equations (1), (7), and (13). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (equation [24]) to

$$i_t = \theta_G + \pi_t + \psi ,$$

where $\psi > 0$, by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank maintains the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligations that is higher than θ_G by the extra rate ψ.[14] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating, the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time passes because the gap between the accelerating inflation rate and the target rate of inflation widens by equation (21) (i.e., by equation [17]) and γ_x in Taylor-type instrument rules is usually larger than unity, say 1.5. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to a rate of time preference that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is useless to try to intervene in the central bank's decision-making, the government would not again dare to attempt to raise θ_G.

Equation (17) implies that a government allows inflation to accelerate because it

[14] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects behaviors of both the government and the representative household is particularly important for price stability.

acts to maximize its expected utility based only on its own preferences. A government is not the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it still can have difficulty self-regulating its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (e.g., Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous shocks to both the government and the representative household because they are planned solely by the central bank. When a shock on θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting θ_G, π_τ, and φ in equation (17).

2.2.5 The degree of central bank independence

Because central bank independence (CBI) is an essential factor for determining inflation paths, the amount of independence of the central bank is quite important for inflation dynamics. CBI is not necessarily an unambiguous concept, however, so it is important to define it lucidly before using the concept. Legal independence may be easily defined, but the key factor that determines inflation paths is actual independence. Cukierman (2005) argues that legal independence is only one, albeit nonnegligible, factor that determines the actual independence of a central bank, and to develop a comprehensive measure of actual CBI, it is necessary to consider the entire institutional and economic structure within which the central bank operates (see also, e.g., Alesina 1988; Grilli et al. 1991; Cukierman 1992; Cukierman et al. 1992; Cukierman and Webb 1995; Jacome 2001). Although actual CBI is important, it is generally difficult to define because it relates to the entire institutional and economic structure. However, given the argument presented in section 2.2.4, the degree of CBI can be defined as the amount of control a central bank has over θ_G, and in the model, CBI is defined on the basis of this idea. The specific formulation of the degree of CBI is shown in Step 3 (section 2.3).

Cukierman (2005) also argues that laws are normally highly incomplete, leaving actual implementation open to interpretation and interference by other institutions within the public sector, and even when the law is clear and complete, there may be slippages between the letter of the law and actual practice due to imperfect compliance. This argument has the important implication that actual CBI is time-variable. Because law is incomplete and compliance with the law is imperfect, there is plenty of room for monetary policy actions that are not strictly based on a rule but are instead taken through negotiations and power struggles between a government and a central bank (e.g., Meltzer 2003; Wood 2005). Equation (23) then cannot necessarily be implemented always as initially intended by the central bank; thus, some parameter values in equation (23), particularly the target rate of inflation (π^*), need be adjusted in some cases. Consequences of confrontations will differ on each occasion, depending on the economic, social, and political conditions in each period. On some occasions, the central bank will win. On other occasions, the government will win, and sometimes they will draw. The actual degree of CBI, therefore, will fluctuate over time. The degree of fluctuation can be well described with a Markov chain.

2.3 Step 3: Models of inflation

2.3.1 Model I: A model with a completely independent central bank

Model I is a model of inflation in which the central bank is completely independent. As argued in section 2.2 (Step 2), if the central bank is completely independent, then the time preference rate of the government, that is, θ_G at t ($\theta_{G,t}$), is a time-dependent endogenous variable, whereas the target rate of the central bank (π^*) is constant. Hence, Model I consists of the following four equations:

***Aggregate supply equation*:**

$$\begin{aligned}\pi_{t+1} &= \theta_P \pi_\varphi^T + (1-\theta_P)\pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1} \\ &\quad + 6\theta_{G,t+1}\exp[z_{t+1}\ln(t-\varphi+1)] - 6(1-\theta_P)\theta_{G,t+2}\exp[z_{t+2}\ln(t-\varphi+2)] \\ &\quad - 6\theta_P\{\exp[z_{t+1}\ln(t-\varphi+1)] - (1-\theta_P)\exp[z_{t+2}\ln(t-\varphi+2)]\}. \qquad (21)\end{aligned}$$

Aggregate demand equation:

$$x_{t+1} = x_{t+2|t} - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} - r \right) + \eta_{t+1} \tag{22}$$

Instrument rule for a central bank:

$$i_t = \bar{\gamma} + \gamma_\pi \left(\pi_t - \pi^* \right) + \gamma_x x_t \tag{23}$$

Government's time preference:

$$\theta_{G,t+1} = \frac{\pi_{t+1} - \pi_\varphi^T - \alpha_x \sum_{s=1}^{\infty} (1-\theta_P)^{s-1} x_{t+s|t} - \varepsilon_{t+1}}{6 \exp\left[z_{t+1} \ln(t - \varphi + 1) \right]} + \theta_P . \tag{25}$$

The first three equations are same as those in conventional inflation models except for the terms related to trend inflation in equation (21). The fourth equation (25) is derived from equations (17) and (20), replacing θ_G with $\theta_{G,t}$. The endogenous variable $\theta_{G,t}$ is adjusted so as to satisfy equation (25). Because $\theta_{G,t}$ is fully under the control of the completely independent central bank, inflation soon stabilizes and non-zero trends disappear such that $\pi_{t+1} - \pi_\varphi^T - \alpha_x \sum_{s=1}^{\infty} (1-\theta_P)^{s-1} x_{t+s|t} - \varepsilon_{t+1} = \pi_{t+1}^T - \pi_\varphi^T = 0$. Hence, equation (25) is asymptotically reduced to the equation $\theta_{G,t} = \theta_P$ and the aggregate supply equation (21) accordingly approaches a conventional pure NKPC such that

$$\pi_{t+1} = (1 - \theta_P)\, \pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1}$$

for $\pi_\varphi^T = 0$. Model I, therefore, is not a special but a conventional inflation model, and inflation is determined by the three equations (21), (22), and (23) with $\theta_{G,t} = \theta_P$.

2.3.2 Model II: A model with a completely dependent central bank

In Model II the central bank is completely dependent. Because the government completely dominates the central bank, the target rate of inflation, that is π^* at t (π_t^*), is a time-dependent endogenous variable, whereas the time preference of government (θ_G) is constant. Model II therefore consists of the following four equations:

Trend-following inflation:

$$\pi_{t+1} = \pi_t + 6(\theta_G - \theta_P)\{\exp[z_{t+1} \ln(t - \varphi + 1)] - \exp[z_t \ln(t - \varphi)]\} - \alpha_x (1 - \theta_P)^{-1} x_t + \varepsilon_{t+1} - \varepsilon_t \tag{26}$$

Reversed aggregate supply equation:

$$x_{t+1} = x_{t+1|t} + \frac{\varepsilon_{t+1}}{\alpha_x} \tag{27}$$

***Reversed aggregate demand equation*:**

$$i_{t+1} = \frac{x_{t+2|t} - x_{t+1} + \eta_{t+1}}{\beta_r} + \pi_{t+2|t} + r + \xi_{t+1} \tag{28}$$

***Reversed instrument rule for a central bank*:**

$$\pi_t^* = -\frac{i_t - \overline{\gamma}_t - \gamma_x x_t}{\gamma_\pi} + \pi_t \tag{29}$$

where ξ_{t+1} is an i.i.d. shock with zero mean and $\overline{\gamma}_t = \pi_t^* + r$, which is a modification of the usual assumption $\overline{\gamma} = \pi^* + r$.[15]

In Model II, optimal trend inflation (equation [17] or equivalently [18]) is exogenously determined with constant θ_G. Overall inflation has to follow this exogenously determined trend inflation as indicated in equation (26), which is obtained by substituting equation (20) into equation (18). Therefore the nature of Model II is completely different from that of the conventional inflation model. Because overall inflation merely follows the trend inflation, the aggregate supply equation (21) with constant θ_G is no longer the equation that determines inflation. Instead the aggregate supply equation (21) is transformed to the reversed aggregate supply equation (27), which determines output gaps consistently with equation (21) and inflation determined by equation (26). Equation (27) is obtained by substituting equation (26) into equation (21) with a constant θ_G. Equation (27) indicates that output gaps are independent of inflation. This type of situation may typically be observed during hyperinflation. Because output gaps are determined by the reversed aggregate supply equation (27), the aggregate demand equation (22) is no longer the equation that determines output gaps. Instead, with exogenously determined inflation (equation [26]) and output gaps (equation [27]), the aggregate demand equation (22) is transformed to the reversed aggregate demand equation (28), which determines the nominal interest rate i_t consistently with equation (22) (i.e. with $i_{t+1|t} = \frac{x_{t+2|t} - x_{t+1} + \eta_{t+1}}{\beta_r} + \pi_{t+2|t} + r$). The central bank accordingly follows the nominal interest rate (equation [28]) as well as inflation (equation [26]) and output gaps (equation [27]) that the market requires. It accommodatingly adjusts the target rate of inflation consistently with these variables, and the instrument rule (equation [23]) is transformed to the reversed instrument rule for a completely dependent central bank (equation [29]).

2.3.3 Model III: A unified model

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[15] Hence, equation (29) is equivalent to $\pi_t^* = \frac{i_t - r_t - \gamma_x x_t - \gamma_\pi \pi_t}{1 - \gamma_\pi}$.

As discussed in section 2.2.5, even if a central bank is established as an independent organization, the government usually influences the central bank to some extent. Hence, in practice, a government does not fully adjust its time preference rate, and the central bank has to adjust its target rate of inflation to compensate for an insufficiently adjusted $\theta_{G,t}$. Whether it is the government or the central bank that makes the larger adjustment to its preference or target rate, respectively, will depend on the degree of CBI. If the degree of CBI is relatively high, then $\theta_{G,t}$ will receive the larger adjustment; if CBI is low, the reverse will be true. Therefore, a general inflation model that can describe all of the movements of inflation, including Models I and II as special cases, is needed. Model III is such a general model and consists of the following five equations:

***Aggregate supply equation*:**

$$\begin{aligned}\pi_{t+1} &= \theta_P \pi_\varphi^T + (1-\theta_P)\pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1} \\ &+ 6\theta_{G,t+1}\exp[z_{t+1}\ln(t-\varphi+1)] - 6(1-\theta_P)\theta_{G,t+2}\exp[z_{t+2}\ln(t-\varphi+2)] \\ &- 6\theta_P\{\exp[z_{t+1}\ln(t-\varphi+1)] - (1-\theta_P)\exp[z_{t+2}\ln(t-\varphi+2)]\}.\end{aligned} \tag{21}$$

***Aggregate demand equation*:**

$$x_{t+1} = x_{t+2|t} - \beta_r\left(i_{t+1|t} - \pi_{t+2|t} - r\right) + \eta_{t+1} \tag{22}$$

***Instrument rule for a central bank*:**

$$i_t = \overline{\gamma}_t + \gamma_\pi\left(\pi_t - \pi_t^*\right) + \gamma_x x_t \tag{30}$$

***Government's time preference*:**

$$\theta_{G,t+1} = \frac{\pi_{t+1} - \pi_\varphi^T - \alpha_x \sum_{s=1}^{\infty}(1-\theta_P)^{s-1} x_{t+s|t} - \varepsilon_{t+1}}{6\exp[z_{t+1}\ln(t-\varphi+1)]} + \theta_P . \tag{25}$$

***Degree of CBI*:**

$$\pi_t^* = \chi_t \overline{\pi}^* + (1-\chi_t)\hat{\pi}_t^* \tag{31}$$

where χ_t $(1 \geq \chi_t \geq 0)$ is a Markov chain with the stationary distribution χ^*, $\overline{\pi}^*$ is π_t^* if the central bank is completely independent (thus $\overline{\pi}^*$ is constant), and $\hat{\pi}_t^*$ is the imaginary target rate of inflation that would be set and adjusted if the central bank were completely dependent. The path of $\hat{\pi}_t^*$ is computed using Model II. Endogenous variables are π_t, x_t, i_t, $\theta_{G,t}$, and π_t^*.

The first four equations are same as those in Model I except the target rate of inflation in equation (30) is replaced with the endogenous one (π_t^*). The endogenous target rate of inflation (π^*_t) is determined by the fifth equation (31) in which the time-varying CBI is incorporated with χ_t. The variable χ_t indicates the ratio of $\overline{\pi}^*$ considered in π^*_t instead of $\hat{\pi}^*_t$. It represents how much of the influence of $\overline{\pi}^*$ remains when π^*_t is determined, that is, how firmly $\theta_{G,\,t}$ is controlled by the central bank, thus indicating the degree of CBI. As argued in section 2.2 (Step 2), the degree of CBI (χ_t) will fluctuate over time. In Model III, it fluctuates according to a Markov chain that describes the consequence of negotiations and power struggles between the government and the central bank. The mean of χ_t ($\overline{\chi}$) indicates the average degree of CBI. If $\overline{\chi} = 1$, then the central bank is completely independent, and if $\overline{\chi} = 0$, then it is completely dependent.

The time preference rate of government ($\theta_{G,\,t}$) does not necessarily approach θ_P as time passes, because $\left|\pi_{t+1} - \pi^T_{\varphi} - \alpha_x \sum_{s=1}^{\infty} (1-\theta_P)^{s-1} x_{t+s|t} - \varepsilon_{t+1}\right| = \left|\pi^T_{t+1} - \pi^T_{\varphi}\right|$ is not necessarily guaranteed to increase less than $\left|6\exp[z_{t+1}\ln(t-\varphi+1)]\right|$ as time passes in equation (25).

3 THE BASIC NATURE OF MODEL III

3.1 A unified, microfounded, and purely forward-looking model

All of the agents in Model III (i.e., households, firms, a government, and a central bank) are equally rational and optimize their objectives purely in forward-looking manners. The key difference among them affecting inflation dynamics is only the heterogeneity in their preferences. No assumptions of special and ad hoc friction or irrationality are required. With these distinguished properties, Model III can explain many essential aspects of inflation.

As $\overline{\chi}$ approaches unity, equation (31) becomes identical to $\pi^*_t = \overline{\pi}^*$, and Model III is reduced to Model I (i.e., a conventional inflation model). Inflation stabilizes around a constant target rate of inflation $\overline{\pi}^*$ set by the completely independent central bank. An example of the path of inflation when $\overline{\chi} = 1$ is shown in Figure 4. Conversely, as $\overline{\chi}$ approaches zero, equation (31) becomes identical to $\pi^*_t = \hat{\pi}^*_t$. In this case, the central bank continuously adjusts the target rate of inflation in each period to keep consistent with a constant θ_G; that is, Model III is reduced to Model II, and inflation begins to deviate greatly unless $\theta_G = \theta_P$ (Fig. 4). When $0 < \overline{\chi} < 1$, the path of inflation varies between the paths shown for the cases of $\overline{\chi} = 1$ and $\overline{\chi} = 0$ (Fig. 4).

Model III can therefore generate any type of inflation (e.g., hyperinflation, chronic inflation, disinflation, low and stable inflation, deflation, etc.) by setting various parameter values for the degree of CBI represented by the Markov chain χ_t and the difference of time preferences between government and households ($\theta_G - \theta_P$). This distinguished nature of Model III enables a unified and microfounded explanation for various types of inflation, each of which is presented in the following sections.

3.2 *Types of inflation*

3.2.1 Hyperinflation

Model III indicates that hyperinflation will be generated in a very short period if θ_G is unusually higher than usual and the central bank is not at all independent ($\bar{\chi} \cong 0$).[16] Faced with a an unusually high θ_G, people expect extremely high inflation and inflation explodes as equation (26) (equation [21] with $\bar{\chi} \cong 0$) indicates. What factors would contribute to a unusually higher θ_G than usual? Hyperinflation has often been observed when governments were very fragile and unstable, for example, after a defeat in war or after a revolution. Germany after WWI, Japan and Hungary after WWII, and Russia after the collapse of the Soviet Union are typical examples of hyperinflation. If a government is fragile and unstable, not only households but the government itself will anticipate that the regime may soon collapse. If the probability of the end of a regime is very high, it is likely that the government will behave very myopically (e.g., Fisher 1930; Yaari 1965). The government will not put a high value on the future, but it will struggle to survive in the present. In such a situation, it is not likely that the government will listen to the advice of a central bank, and the central bank will have little or no independence ($\bar{\chi} \cong 0$). The very fragile and unstable government's considerably myopic behavior will cause extremely high inflation expectations and then hyperinflation by equation (26). This explanation appears more natural than Cagan's (1956) hyperinflation model, in which it is suggested that hyperinflation basically occurs irrespective of the fragility or stability of the government.

The optimal trend inflation (equation [17]) implies another type of hyperinflation. Even if θ_G is not unusually high, hyperinflation will eventually be observed if no action is taken when there are relatively large positive values of $\theta_G - \theta_P$. The hyperinflation observed in some countries in South America for the past several decades (sometimes called "modern hyperinflation") may be examples of this type of hyperinflation. The situation in which relatively large positive values of $\theta_G - \theta_P$ are left as they are implies that a central bank is only somewhat independent, that is, $\bar{\chi}$ is positive but close to zero. The combination of an unusually myopic government and a dependent central bank will generate this type of hyperinflation.

Model III indicates that hyperinflation is not caused by the growth of money (i.e., not by seigniorage) but by the unusually myopic preference of a government combined with a scarcely independent central bank. This view is consistent with the conclusions of Sargent and Wallace (1973) and Fischer et al. (2002). They conclude that causation runs from inflation to money growth during hyperinflation, and that once high inflation

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[16] When θ_G is unusually high, the weight $\dfrac{\overline{B}_{t,t}}{\int_{t-1}^{t} \overline{B}_{v,t}\, dv}$ in i_t is much larger than $\dfrac{\overline{B}_{t-1,t}}{\int_{t-1}^{t} \overline{B}_{v,t}\, dv}$ and thus i_t will be closer to $\int_t^{t+1} \pi_s \, ds + r$ than $\int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \quad ds + r$. Hence, for example, the path of inflation will be closer to $\pi_t = \pi_\varphi + 2(\theta_G - \theta_P)(t - \varphi)$ rather than $\pi_t = \pi_\varphi + 6(\theta_G - \theta_P)(t - \varphi)^2$.

has been triggered, monetary policy has typically been accommodative, as equation (31) implies. The explanation is also consistent with Sargent's (1982) view that a credible change in policies, preferably embedded in legal and institutional changes, could bring a hyperinflation to an end at a very small cost. Sargent (1982) implies that the main cause of hyperinflation is the behavior of government. Model III indicates that, if the incumbent government is replaced with or changes itself into a government that has a much lower rate of time preference or if the authority to set and keep the target rate of inflation is delegated to a truly independent neutral organization that is obliged to stabilize the price level, high inflation expectations soon subside and the ongoing hyperinflation will be brought to an end at a small cost.

Model III also indicates that the mechanism of hyperinflation can be explained without any ad hoc assumption of irrationality or friction, whereas Cagan's (1956) well-known hyperinflation model needs the assumption of adaptive expectations or some ad hoc frictions if large budget deficits are allowed in the model (e.g., Auernheimer 1976; Evans and Yarrow 1981; Kiguel 1989). Model III indicates that hyperinflation is nothing more than a consequence of the various values of deep parameters (i.e., the time preference rate of the government and the degree of CBI), and no additional or special mechanism is necessary to explain it.

3.2.2 Chronic inflation

Chronic inflation occurs when relatively high rates of inflation are sustained for a relatively long period. Many industrialized countries experienced chronic inflation in the 1960s and 1970s, and this period is often called "the Great Inflation". Equation (17) implies that chronic inflation will be observed if there is a combination of sporadic periods in which $\theta_G > \theta_P$ and regular periods in which $\theta_G \cong \theta_P$. Once a positive $\theta_G - \theta_P$ is allowed (even for a short period), equation (17) implies that inflation will start to accelerate. The acceleration will stop when $\theta_G = \theta_P$ is restored. However, the higher rate of inflation and higher inflation expectations are retained because $\theta_G < \theta_P$ is necessary to decrease inflation.

Model III indicates that the combination of sporadic periods in which $\theta_G > \theta_P$ and regular periods in which $\theta_G \cong \theta_P$ is consistent with a partially independent central bank (e.g., $\overline{\chi} = 0.5$). This type of central bank cannot sufficiently control θ_G and will sometimes fail to prevent the occurrence of a situation in which $\theta_G > \theta_P$. Moreover, because of insufficient independence, the central bank usually will not be able to force the government to lower θ_G to $\theta_G < \theta_P$ even if $\theta_G < \theta_P$ is necessary to decrease inflation. As a result, the combination of a relatively more myopic government and an insufficiently independent central bank can generate chronic inflation.

Once the situation in which $\theta_G > \theta_P$ is allowed, the target rate of inflation needs to be raised by equation (31) because $\hat{\pi}_t^*$ rises as time passes when $\theta_G > \theta_P$. Clarida et al. (2000), Favero and Rovelli (2001), and Dennis (2001) conclude that the target rate of inflation in the pre-Volker era was much higher than that in the Volker-Greenspan era. Equation (31) suggests that the reason central banks at the time set high inflation targets is not because they deliberately committed the "crime" of high inflation. Instead, they were forced to raise the target rates of inflation because they were not sufficiently independent.[17]

3.2.3 Disinflation

Disinflation occurs when a high rate of inflation gradually declines to a low and stable rate of inflation, but the decline does not reach deflation. A typical episode was experienced in many industrialized countries in the 1980s after the Great Inflation. Equation (17) indicates that disinflation will be observed when the condition of $\theta_G < \theta_P$ is gradually adjusted to one in which $\theta_G = \theta_P$ as the rate of inflation declines to a low and stable rate.

Model III indicates that a truly independent central bank (i.e., high $\overline{\chi}$) is necessary for disinflation because θ_G must be gradually shifted from $\theta_G < \theta_P$ to $\theta_G = \theta_P$. A government will not be able to discipline itself to keep $\theta_G < \theta_P$ because it generally prefers the opposite condition ($\theta_G > \theta_P$). In contrast, this gradual adjustment can be easily implemented by a truly independent central bank because they can force the government to keep $\theta_G < \theta_P$, and the central bank can gradually tune the target rate of inflation as well as θ_G as inflation cools down. Eventually the rate of inflation will land softly at a low and stable rate.

High inflation before disinflation indicates that a central bank was not sufficiently independent before disinflation. Hence, a point at which a central bank abruptly becomes truly independent, that is, $\overline{\chi}$ is raised significantly, is necessary for disinflation. Taylor (2001, 2002) emphasizes the importance of changes in economic and political leadership as a cause of the Great Inflation by quoting Milton Friedman, who argued that the Great Inflation was a fundamentally political, not economic, phenomenon and that Ronald Reagan ended the Great Inflation by accepting a severe recession without bringing pressure on the Federal Reserve to reverse course. Similarly, Meltzer (2005) emphasizes the large role of political decision-making during the Great Inflation and concludes that the Federal Reserve was better able to control inflation during the administrations of Presidents Eisenhower and Reagan rather than those of Presidents Johnson, Carter, or Nixon. In other words, keeping the independence of the central bank is the key to stabilizing inflation. This view is consistent with the explanation for disinflation offered in this paper.

3.2.4 Low and stable inflation

Equation (17) indicates that, if inflation is initially low and $\theta_G = \theta_P$ is maintained, a low and stable rate of inflation will be sustained. Model III indicates that a truly independent central bank (i.e., high $\overline{\chi}$) is necessary for low and stable inflation

[17] Kydland and Prescott's (1977) and Barro and Gordon's (1983) well-known explanation for chronic inflation needs exceptionally large or successive negative supply shocks and thus needs internationally common shocks to explain the international aspect of the Great Inflation. It is hard to find such shocks in many industrialized countries during the Great Inflation. The explanation in this paper can explain the international aspect of the Great Inflation without assuming such shocks because it concerns only the attitudes of the governments and the central banks. The governments and central banks in most industrialized countries during the Great Inflation seem to have assumed common attitudes because the economic policies conducted in the United States were often imitated by other countries.

because it forces the government to keep $\theta_G = \theta_P$ completely and indefinitely.

3.2.5 Deflation

Equation (17) indicates that, if the condition in which $\theta_G < \theta_P$ continues over time, deflation will eventually occur. Nevertheless, deflation will be rarely observed because governments generally prefer $\theta_G > \theta_P$, and it is unlikely that a central bank would dare to attempt deflation and set a target rate of deflation.[18] In fact, among the industrialized countries, only Japan in the 1990s and 2000s has experienced deflation since World War II.

How can deflation occur if a government generally prefers $\theta_G > \theta_P$ and a central bank exerts itself to hold $\theta_G = \theta_P$ for a positive target rate of inflation? A huge negative exogenous shock that greatly widens the output gap may temporarily make the price level decline, but that would not necessarily be regarded as deflation because deflation means a successive decline of the price level. The possibility for deflation arises when a shock considerably raises θ_P.[19] It may rarely happen, but if θ_P becomes higher and θ_G stays constant, then it is possible for the condition $\theta_G < \theta_P$ to occur. If this condition is left unchanged, then deflation will be observed by equation (21). The higher θ_P means a lower level of consumption at steady state, and a recession as well as a deflation will generally be observed if θ_P is raised. Nevertheless, if the central bank raises θ_G so that $\theta_G \geq \theta_P$ immediately after the shock, deflation will be prevented. However, if the central bank does not respond quickly, deflation will occur.

Once deflation takes root, it is very difficult for even a truly independent central bank to control θ_G and reverse the deflation because of the zero bound of the nominal interest rate. As shown in section 2.2.4, an independent central bank controls θ_G by manipulating the nominal interest rate with the extra rate ψ. Thus, if the central bank cannot manipulate the nominal interest rate because of the zero bound, it also cannot control θ_G. The central bank may advise the government to raise its preference θ_G so far as $\theta_G > \theta_P$ to reverse the deflation, but it cannot force the government to make $\theta_G > \theta_P$.[20] Furthermore, if the deflation deepens to a point where the real interest rate is compelled to exceed the marginal productivity, the economy cannot achieve a stable equilibrium anymore. The Great Depression in the 1930s may have been such a case,

[18] In the case of the gold standard that prevailed before World War II, deflation may be observed relatively more frequently because the gold standard indicates that the target rate of inflation is zero.

[19] Since the era of Böhm-Bawerk and Fisher, the rate of time preference has been naturally regarded as time-variable. See, for example, Böhm-Bawerk (1889), Fisher (1930), and Uzawa (1968).

[20] If the time preference rates of the median and the representative households became nearly equal due to the shock that raised θ_P and $\theta_G \cong \theta_P$ is kept, deflation may continue for a long period even though the incumbent government is replaced. However, if the time preference rate of the median household is raised similarly to θ_P due to the shock, then a replacement of government would reverse deflation because the newly elected government will have the same high rate of time preference as the raised time preference rate of the median household. The election of President Franklin D. Roosevelt in 1933 may have been such a case. Nevertheless, even if deflation is reversed, the other problems caused by a raised θ_P will remain.

whereas Japan in the 1990s may have narrowly averted such a situation.[21]

Ahearne et al. (2002) argue that, to prevent deflation like the one experienced in Japan in the 1990s, both monetary and fiscal stimulus should go beyond the levels conventionally implied. For example, they argue, if the Bank of Japan had lowered short-term interest rates by a further 200 basis points at any time between 1991 and early 1995, deflation could indeed have been avoided. This view implies that there are unusual incidents behind deflation, and it seems consistent with the argument that θ_P is unusually high in cases of deflation.[22] Equation (17) suggests that, to prevent deflation, it is necessary to raise θ_G above the unusually high θ_P as soon as possible by imposing an unusually large negative extra rate ψ. Thus, deflation will be prevented if a central bank acts quickly and decisively, probably as the Federal Reserve under Chairman Greenspan attempted during the recession in the early 2000s. However, because shocks that make $\theta_G - \theta_P$ considerably negative seem to occur rarely, even a truly independent central bank may fail to respond quickly enough to such a shock owing to a lack of experience.

4 INFLATION PERSISTENCE

It has been argued that an important unresolved issue is a microfounded mechanism that generates substantial intrinsic inflation persistence (Galí et al., 2005; Fuhrer, 2006; Woodford, 2007). Model III offers a response to this argument, and in this section, I explain the microfounded mechanism of intrinsic inflation persistence with Model III.

4.1 Optimal trend inflation and intrinsic inflation persistence

Although all of the agents optimize their objectives purely in forward-looking manners, Model III is consistent with observations indicating that inflation possesses a backward-looking property and is substantially persistent. The key factor is the optimal trend inflation generated as a consequence of simultaneous optimization. Whether inflation is persistent ultimately hinges on the type of mean assumed when estimating persistence. Mean reversion and most measures of persistence are inversely related—basically, the more substantial the persistence, the lower the mean reversion and vice versa (e.g., Marques 2004). The trend component in inflation (equation [17]) is a mean-reverting process, and inflation itself in Model III is also a mean-reverting process because the trend component is included in equation (19) (or equivalently equation [21]). On the other hand, it is implicitly assumed that the mean of inflation is constant at steady state in autoregressive process models. Hence, if Model III is true, but inflation is assumed mistakenly to be an autoregressive process, most measures of persistence (e.g., the sum of the autoregressive coefficients) will spuriously indicate that inflation is substantially persistent and possesses a backward-looking property. From an experimental study, Sbordone (2007) concludes that, although a model has no intrinsic

[21] In the early 1930s, the ex post real interest rate in the United States was roughly 10% (e.g., Bernanke 1995), whereas that in Japan in the 1990s was generally less than 5% (e.g., Ito 2003).

[22] Harashima (2004a) estimates that θ_P in Japan rose by roughly 2% at the end of the 1980s, just before the deflation that occurred during the 1990s and 2000s.

persistence (but instead has trends), the hypothesis that there is a significant source of intrinsic persistence in inflation dynamics could not be rejected with 90% confidence. Model III therefore implies that substantial intrinsic inflation persistence is merely an illusion or a consequence of serious misspecification.

4.2 *Trend breaks*

Trend inflation has another important feature that greatly affects the nature of inflation persistence—trends in inflation often break. If a central bank is completely dependent ($\overline{\chi} = 0$) or completely independent ($\overline{\chi} = 1$), a trend may continue for long periods without any break. However, if a central bank is partially independent ($0 < \overline{\chi} < 1$), the degree of CBI will vary over time as previously argued and trend breaks will often occur. In the case where $0 < \overline{\chi} < 1$, a central bank cannot always sufficiently control θ_G. In some periods, a central bank may relatively firmly control θ_G, but in other periods, it may not. This variation in CBI makes many trend breaks occur and the path of inflation zigzag. This mechanism is modeled using a Markov chain χ_t $(0 \leq \chi_t \leq 1)$ in Model III.

An important consequence of the existence of many inflation trend breaks is that inflation appears to be substantially persistent and almost follow a random walk (e.g., Perron 1989; Lumsdaine and Papell 1997). Without breaks, trend inflation can easily be distinguished from a random walk. With many trend breaks, however, it is far less distinguishable and will often be spuriously observed as following a random walk. Levin and Piger (2004) conclude that, allowing for a break in intercept, inflation measures generally exhibit relatively low inflation persistence for many industrial economies and, evidently, substantial inflation persistence is not an inherent characteristic of industrial economies (see also Marques 2004).

4.3 *High/low degree of persistence during high/low inflation*

Many empirical studies have indicated that persistence of inflation was substantial in the 1970s and then declined in many industrialized economies. For example, Cogley and Sargent (2005) conclude that persistence of inflation increased during the 1970s, then fell in the 1980s and 1990s. Barsky (1987) and Evans and Watchel (1993) report that during the Great Inflation in the 1970s, the path of inflation looks like a random walk. On the other hand, Barsky (1987), Evans and Watchel (1993), Cogley and Sargent (2002), and Levin and Piger (2004) argue that lower persistence is observed during periods of low and stable inflation, a typical example of which is the recent low and stable inflation observed in many industrialized economies. These phenomena suggest that some mechanism exists such that substantial persistence is observed during high inflation and vice versa. Model III can offer a reasonable explanation for such a mechanism.

High rates of inflation imply that a central bank is not fully independent and cannot manipulate the nominal interest rate sufficiently to stabilize inflation. Model III indicates that the optimal trend inflation dominates a large part of the inflation path in such a case. Therefore, if an autoregressive process is mistakenly applied to high inflation as the model to estimate its persistence, then a high degree of persistence will be spuriously observed. In addition, as argued above, there will be many large-scale trend breaks in inflation according to the Markov chain χ_t in cases where $\bar{\chi}$ is neither nearly zero nor unity. If an autoregressive process is applied in such cases, it will often lead to the incorrect conclusion inflation follows a random walk. The Great Inflation in the 1970s seems to be a typical example of this kind of phenomenon.

Conversely, if the central bank is completely independent ($\bar{\chi} = 1$), the aggregate supply equation (21) is reduced to a pure NKPC such that

$$\pi_{t+1} = (1 - \theta_P)\pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1}$$

as shown in section 2.3.1, and inflation stabilizes at a low rate. Only inherited persistence from $x_{t+1|t}$ and, as Fuhrer (2006) argues, probably only a small amount of inflation persistence will therefore be observed even if an autoregressive process model is applied to capture persistence.[23] Nevertheless, a completely independent central bank ($\bar{\chi} = 1$) seems quite rare. Even though central banks are designed to be as independent as possible, governments in actuality still influence central banks to some extent; for example, a government generally keeps the power to designate the head and board members of its central bank. Cukierman (2005) argues that the economic and institutional structures within which a central bank operates affect the actual independence of the central bank, even for a given level of tightly respected legal independence. Even in industrialized economies, therefore, the average degree of actual CBI ($\bar{\chi}$) will be high but still lower than unity. Hence, Model III predicts that, even if inflation is low and stable, non-zero inflation trends remain, although their slopes are fairly mild, and small-scale trend breaks still occur. As a result, if an autoregressive model is applied, moderate to low persistence will be observed in inflation during low and stable inflation.

4.4 The hybrid NKPC and optimal trend inflation

Woodford (2007) argues that trend inflation is highly correlated with lagged inflation, so that omission of trend inflation could result in spurious positive coefficients on lagged inflation in estimates of the hybrid NKPC equation (see also Cogley and Sbordone 2006). For example, consider if a hybrid NKPC such that

$$\pi_{t+1} = \kappa\,\pi_t + (1 - \kappa)\pi_{t+2|t} + \alpha_x x_{t+1|t} + \varepsilon_{t+1}$$

is selected as a model to estimate a Phillips curve, although the true data generation mechanism is the trend-augmented NKPC (equation [19]). Then the value of the parameter κ will be calculated as the one that on average satisfies $\kappa(\pi_{t+2|t} - \pi_t) = \pi^T_{t+2|t} - \pi^T_{t+1|t} - \theta_P(\pi^T_{t+2|t} - \pi_{t+2|t})$ for the sample period because the estimated value of κ must be consistent with equation (19). Here, $\theta_P(\pi^T_{t+2|t} - \pi_{t+2|t})$ will be negligibly smaller than $\pi^T_{t+2|t} - \pi^T_{t+1|t}$ for sufficiently small θ_P; thus,

[23] If a univariate AR model is used, moderate to low persistence may be observed due to autocorrelations in x_t.

$$\pi_{t+2|t} - \pi_t \cong \frac{\pi^T_{t+2|t} - \pi^T_{t+1|t}}{\kappa}.$$

If trend inflation moves steadily during two periods from t to $t+2$, as equation (17) indicates, and it occasionally changes its direction upward or downward due to trend breaks, then on average $\pi^T_{t+2|t} - \pi^T_t = \left(\pi^T_{t+2|t} - \pi^T_{t+1|t}\right) + \left(\pi^T_{t+1|t} - \pi^T_t\right) \cong 2\left(\pi^T_{t+2|t} - \pi^T_{t+1|t}\right)$. As equation (20) indicates, inflation is a process proceeding around trend inflation in Model III and thus on average $\pi_{t+2|t} - \pi_t \cong \pi^T_{t+2|t} - \pi^T_t$ and

$$\pi_{t+2|t} - \pi_t \cong 2\left(\pi^T_{t+2|t} - \pi^T_{t+1|t}\right),$$

which means

$$\kappa \cong 0.5.$$

Model III therefore predicts that a statistically significant value of κ in the hybrid NKPC will spuriously be estimated to be about 0.5. Most empirical estimates of κ indicate that the value of κ is neither nearly zero nor nearly unity but that it is between 0.3 and 0.6 (e.g., Galí and Gertler 1999; Galí et al. 2001, 2003; Jondeau and Le Bihan 2005; Rudd and Whelan 2006, 2007; Kurmann 2007) and is statistically significant. Particularly when other estimation methods than GMM (the generalized method of moments) are used to estimate hybrid NKPC equations, the values of κ are often computed to be nearly 0.5 (e.g., Jondeau and Le Bihan 2005; Kurmann 2007). These estimates are consistent with the argument that trend inflation is playing an important role in inflation dynamics, as Model III indicates.

5 CONCLUDING REMARKS

Recently it has been argued that the puzzle of persistence in the NKPC will be solved if trend inflation is well incorporated into the model. But, incorporating trend inflation raises another serious theoretical problem—trend inflation lacks a microfoundation. In this paper, I tackled this problem and presented a microfoundation for trend inflation. On the basis of this microfoundation, I constructed an inflation model, in which both the government and the representative household achieve simultaneous optimization. The model indicates that, without an independent central bank, inflation accelerates or decelerates if the time preference rates of the government and the representative household are heterogeneous. Because a government represents the median of households under a proportional representation system and the economically representative household represents the mean of households, the preferences between them are usually heterogeneous. More importantly, a government can barely control its own preferences even if it is fully rational. If a government is left without some neutral organization to help control inflation, the risk of considerable acceleration of inflation exists. A truly independent central bank is therefore necessary to rein in inflation. As the average degree of CBI ($\bar{\chi}$) approaches unity, the model is reduced to a conventional inflation model, and inflation stabilizes. Conversely, as $\overline{\chi}$ approaches zero, inflation begins to deviate greatly depending on the difference of preferences between government and households. When $0 < \overline{\chi} < 1$, the path of inflation proceeds between those in the cases of $\overline{\chi} = 1$ and $\overline{\chi} = 0$.

All of the agents in the model behave in purely forward-looking manners. However, the model indicates that inflation spuriously appears to have a backward-looking property and to be substantially persistent for a very simple reason. If inflation is assumed to be an autoregressive process even though there is a trend, many measures of persistence (e.g., the sum of the autoregressive coefficients) will spuriously indicate that inflation is substantially persistent. In addition, trends in inflation will often break because the degree of CBI will vary over time according to the consequences of successive negotiations or power struggles between the government and the central bank. The existence of many trend breaks makes inflation appear to be substantially persistent and sometimes to appear like it is following a random walk. The model therefore implies that substantial intrinsic inflation persistence is merely an illusion or a consequence of serious misspecification.

Trend inflation has been ignored in models of inflation because of its lack of microfoundation. Now that a microfoundation has been presented, trend inflation should no longer be ignored as a source of observed substantial inflation persistence. The model presented here not only solves the puzzle of inflation persistence, but it enables a unified and microfounded explanation for various types of inflation by setting various parameter values for the degree of CBI and the difference of preferences between the government and the representative household. Even though the model can explain many essential aspects of inflation, it needs no assumptions of special and ad hoc friction or irrationality to do so. All of the agents in the model (i.e., households, firms, a government, and a central bank) are equally rational and optimize their objectives purely in forward-looking manners even though their preferences are not necessarily identical.

APPENDIX

1 The condition for approximately identical weights

If b_t is constant (e.g., if it is at steady state), the weights for $t-1<s\le t$ are $\dfrac{\overline{B}_{s,t}}{\int_{t-1}^{t}\overline{B}_{v,t}\,dv}=\dfrac{1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv}{\int_{t-1}^{t}\left[1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}$; thus,

$$i_t=r+\int_{t-1}^{t}\int_s^{s+1}\pi_v\,dv\left(\frac{1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv}{\int_{t-1}^{t}\left[1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}\right)ds=r+$$

$$\int_{t-1}^{t}\left[\int_s^{s+1}\pi_v\,dv-\frac{\int_s^{s+1}\pi_v\,dv\left\{\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv\right\}}{1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}\right]ds\cdot$$

Here, the absolute value of $\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds$ $-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv$ is $|\pi_{s1}-\pi_{s2}|$ at the maximum, where π_{s1} is the largest and π_{s2} is the smallest of π_s for $t-1<s\le t+1$, and the absolute value of $1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds$ is $|1+\pi_{s1}|$ at the minimum if $\pi_{s1}\le -1$ or if $\pi_{s2}<-1<\pi_{s1}$ and $|1+\pi_{s1}|<|1+\pi_{s2}|$, otherwise it is $|1+\pi_{s2}|$ at the minimum. In addition, $\pi_{s2}\le\int_s^{s+1}\pi_v\,dv\le\pi_{s1}$. Hence, the absolute value of

$$\frac{\int_s^{s+1}\pi_v\,dv\left\{\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv\right\}}{1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}$$

is $\dfrac{|\pi_{s1}||\pi_{s1}-\pi_{s2}|}{|1+\pi_{s1}|}$ or $\dfrac{|\pi_{s1}||\pi_{s1}-\pi_{s2}|}{|1+\pi_{s2}|}$ at the maximum, and it is negligibly smaller than the absolute value of $\int_s^{s+1}\pi_v\,dv$ if the absolute values of π_s for $t-1<s\le t+1$ are sufficiently smaller than unity because $\pi_{s2}\le\int_s^{s+1}\pi_v\,dv\le\pi_{s1}$; thus, approximately $R_t=\int_{t-1}^{t}\int_s^{s+1}\pi_v\,dv\,ds+r$.

2 The transversality condition

By equations (7) and (13), $i_t=\int_{t-1}^{t}\int_s^{s+1}\pi_v\,dv\,ds+r=\theta_G+\pi_t$; thus, $i_t-\pi_t=\theta_G$ at steady state. Substituting the equation $i_t-\pi_t=\theta_G$ and equation (13) into conditions (4) and (5) and solving both differential equations yield the equation

$\lambda_{G,t}\, b_t = -\exp\left[\left(g_t - x_t - \omega_t\right)\int \frac{1}{b_t} dt + C^{\#}\right]$ at steady state, where $C^{\#}$ is a certain constant. Therefore, it is necessary to satisfy $g_t - x_t - \omega_t < 0$ and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$ for the transversality condition (6) to hold.

Here, by condition (5), $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - \omega_t}{b_t}$ at steady state. Hence, if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - \omega_t}{b_t} = 0$ at steady state, then b_t is constant; thus, $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$. Thereby, the transversality condition holds. However, if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - \omega_t}{b_t} < 0$ at steady state, then b_t diminishes to zero and transversality condition (6) cannot hold because $g_t - x_t - \omega_t < 0$. If $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - \omega_t}{b_t} > 0$ at steady state, then $\lim_{t\to\infty} \frac{\dot{b}_t}{b_t} = \theta_G$; thus, b_t increases as time passes and $\lim_{t\to\infty} \int \frac{1}{b_t} dt = \frac{C^{\#\#}}{\theta_G}$, where $C^{\#\#}$ is a certain constant. Therefore, transversality condition (6) also cannot hold.

3 *Proof of equation* (16)

(Step 1)

An inflation stream π_t^A moves on the path $\pi_t^A = \pi_0^A + 6(\theta_G - \theta_P)t^2$; thus, $\pi_t^A = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v^A \, dv\, ds - (\theta_G - \theta_P)$ for any *t*. Another inflation stream π_t^B is different from π_t^A but satisfies $\pi_t^B = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v^B \, dv\, ds - (\theta_G - \theta_P)$ for $0 \le t$. Suppose that, during the period $-1 < t \le 1$, π_t^B is different from π_t^A only in the two periods, that is, in the period between $t = -1$ and $t = -1 + dt$ by $\hat{\pi}^C_{initial,-1+dt} = \int_{-1}^{-1+dt} \left(\pi_s^B - \pi_s^A\right) ds \neq 0$ and in the period between $t = 1 - dt$ and $t = 1$ by $\hat{\pi}^C_{initial,1} = \int_{1-dt}^{1} \left(\pi_s^B - \pi_s^A\right) ds \ \left(= -\hat{\pi}^C_{initial,-1+dt}\right)$ for a sufficiently small period *dt*. In other periods during $-1 < t \le 1$, $\pi_t^A = \pi_t^B$ and thus $\pi_0^A = \pi_0^B$, $\pi_{dt}^A = \pi_{dt}^B$, … , $\pi_{ndt}^A = \pi_{ndt}^B$ for $ndt < 1$, where $n = 1, 2, 3, \cdots$.

In the period between $t = 1$ and $t = 1 + dt$ that is adjacent to the period $-1 < t \le 1$, π_t^B must be different from π_t^A to achieve $\pi_{dt}^A = \pi_{dt}^B = \int_{-1+dt}^{dt}\int_{s}^{s+1} \pi_v^A \, dv\, ds - (\theta_G - \theta_P)$ $= \int_{-1+dt}^{dt}\int_{s}^{s+1} \pi_v^B \, dv\, ds - (\theta_G - \theta_P)$ because the difference between π_t^B and π_t^A in the period between $t = -1$ and $t = -1 + dt$ is no longer relevant to π_{dt}^B; thus, the difference between π_t^B and π_t^A in the period between $t = 1 - dt$ and $t = 1$ by $\hat{\pi}^C_{initial,1} = \int_{1-dt}^{1} \left(\pi_s^B - \pi_s^A\right) ds \ \left(= -\hat{\pi}^C_{initial,-1+dt}\right)$ must be compensated for in other periods and not in the period between $t = -1$ and $t = -1 + dt$. Because π_t^B in the period during

$-1+dt<t\leq 1$ is already fixed, the only way to compensate for it is to add $-\hat{\pi}^C_{initial,1}=-\int_{1-dt}^{1}\left(\pi^B_s-\pi^A_s\right)ds\ \left(=\hat{\pi}^C_{initial,-1+dt}\right)$ in the newly added period between $t=1$ and $t=1+dt$. Thereby, π^B_t is different from π^A_t in the period between $t=1$ and $t=1+dt$ by $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}=-\hat{\pi}^C_{initial,1}=-\int_{1-dt}^{1}\left(\pi^B_s-\pi^A_s\right)ds=\hat{\pi}^C_{initial,-1+dt}$ due to $\hat{\pi}^C_{initial,1}$.

Successively, in the next period between $t=1+dt$ and $t=1+2dt$, π^B_t must also be different from π^A_t to achieve $\pi^A_{2dt}=\pi^B_{2dt}\ =\int_{-1+2dt}^{2dt}\int_s^{s+1}\pi^A_v\,dv\,ds-\left(\theta_G-\theta_P\right)$ $=\int_{-1+2dt}^{2dt}\int_s^{s+1}\pi^B_v\,dv\,ds-\left(\theta_G-\theta_P\right)$ due to the differences between π^B_t and π^A_t in the periods between $t=1-dt$ and $t=1$ and between $t=1$ and $t=1+dt$. The difference in the period between $t=1-dt$ and $t=1$ originating in $\hat{\pi}^C_{initial,1}$ makes $\int_0^{dt}\int_s^{s+1}\pi^B_v\,dv\,ds$ and $\int_{dt}^{2dt}\int_s^{s+1}\pi^B_v\,dv\,ds$ be different from both $\int_0^{dt}\int_s^{s+1}\pi^A_v\,dv\,ds$ and $\int_{dt}^{2dt}\int_s^{s+1}\pi^A_v\,dv\,ds$ by $-\hat{\pi}^C_{initial,1}=\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$. Hence, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+2dt}$ (i.e., the difference in the period between $t=1+dt$ and $t=1+2dt$ originating in $\hat{\pi}^C_{initial,1}$) is $-2\,\hat{\pi}^C_{initial,1}=2\,\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$.

Similarly, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+3dt}=3\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, and thus $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+ndt}=n\,\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$ for $1<t<2-dt$. In the period between $t=2-dt$ and $t=2$, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+ndt}$ begins to decline and is zero after $t=3$. However, similarly $\hat{\pi}^C_{\hat{\pi}^C_{initial,1+dt},1+dt+ndt}$ starts at $t=1$, begins to decline in the period between $t=2$ and $t=2+dt$, and is zero after $t=3+dt$. Successively, new and similar $\hat{\pi}^C_{\hat{\pi}^C_{initial,1+dt},1+2dt+ndt}$, $\hat{\pi}^C_{\hat{\pi}^C_{initial,+2dt},1+3dt+ndt}$, …, and so forth start and become zero.

(Step 2)

As a result, the total difference between π^B_t and π^A_t in the period between $t=1+(n-1)dt$ and $t=1+ndt$ originating in the initial difference $\hat{\pi}^C_{initial,-1+dt}$ is $\overline{\pi}^C_{\pi^C_{initial,-1+dt},1+ndt}=\hat{\pi}^C_{\pi^C_{initial,1},1+ndt}+\hat{\pi}^C_{\pi^C_{initial,1+dt},1+ndt}+\hat{\pi}^C_{\pi^C_{initial,1+2dt},1+ndt}+\cdots+\hat{\pi}^C_{\pi^C_{initial,1+(n-1)dt},1+ndt}$ $=\sum_{v=1}^{n}(n-v+1)\,\hat{\pi}^C_{\pi^C_{initial,1+(v-1)dt},1+vdt}$, where $\pi^C_{initial,t}=\int_{t-dt}^{t}\left(\pi^B_s-\pi^A_s\right)ds$ for $0<n\,dt<1$. Here, $\hat{\pi}^C_{\pi^C_{initial,1+dt},1+2dt}=-\hat{\pi}^C_{\pi^C_{initial,1},1+dt}$, $\hat{\pi}^C_{\pi^C_{initial,1+2dt},1+3dt}=-\hat{\pi}^C_{\pi^C_{initial,1+dt},1+2dt}$, $\hat{\pi}^C_{\pi^C_{initial,1+3dt},1+4dt}=-\hat{\pi}^C_{\pi^C_{initial,1+2dt},1+3dt}$ and so forth; that is, $\hat{\pi}^C_{\pi^C_{initial,1+ndt},1+(n+1)dt}$ and $\hat{\pi}^C_{\pi^C_{initial,1+(n+1)dt},1+(n+2)dt}$ cancel each other out in turn; thus, $\sum_{v=1}^{n}(n-v+1)\,\hat{\pi}^C_{\pi^C_{initial,1+(v-1)dt},1+vdt}$ is $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, 0, $-\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, $-\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, 0, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}$, … for n = 1,2,3,4,5,6,7,8, …, respectively.

Because of the alternate cancelation of $\hat{\pi}^C_{\pi^C_{initial,1+ndt},1+(n+1)dt}$, $\overline{\pi}^C_{\pi^C_{initial,-1+dt},1+ndt}$ $=\sum_{v=1}^{n}(n-v+1)\,\hat{\pi}^C_{\pi^C_{initial,1+(v-1))dt},1+vdt}$ grows less rapidly than $\hat{\pi}^C_{\hat{\pi}^C_{initial,1},1+dt}\sum_{v=1}^{n}(n-v+1)$

$= \hat{\pi}^{C}_{\hat{\pi}^{C}_{initial,\ 1},1+dt} \dfrac{n(n+1)}{2}$; thus, $\bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},1+ndt} < \pi^{C}_{\hat{\pi}^{C}_{initial,1},1+dt} \dfrac{n(n+1)}{2}$ and $-\hat{\pi}^{C}_{\hat{\pi}^{C}_{initial,\ 1},1+dt} \dfrac{n(n+1)}{2}$ $< \bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},1+ndt}$. Hence, $\bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},t} < \hat{\pi}^{C}_{\hat{\pi}^{C}_{initial,1},1+dt} \dfrac{\left(\dfrac{t-1}{dt}\right)^2}{2} = \dfrac{(t-1)^2}{2} C_0$ and $-\dfrac{(t-1)^2}{2} C_0$ $< \bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},t}$ for sufficiently large $t(>1)$ where C_0 is a constant.

(Step 3)

Suppose another situation in which π_t^A and π_t^B have the same properties as shown in (Step 1), but they are initially different in many other periods during $-1 < t \le 1$. Each initial difference between π_t^A and π_t^B in each short period *dt* during the period $-1 < t \le 1$ has the same nature as $\bar{\pi}^{C}_{\pi^{C}_{-1+dt},1+ndt}$ shown above; thus, if $-\infty < \pi_t < \infty$ for $-1 < t \le 1$, π_t^B can be expressed as π_t^A plus the sum of differences that originated in these initial differences such that $\pi_t^B = \pi_t^A + C_1\tau_{1,t}\exp[\tau_{2,t}\ln(t-1)]$ for $t(>1)$, where C_1 is a constant, $\tau_{1,t}$ and $\tau_{2,t}$ are time-dependent variables, and $\tau_{1,t}$ takes the value of only 1 or –1. Because $\bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},t} < \dfrac{(t-1)^2}{2} C_0$ and $-\dfrac{(t-1)^2}{2} C_0 < \bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},t}$ for sufficiently large $t(>1)$ as shown in (Step 2) and the other initial differences have the same nature as $\bar{\pi}^{C}_{\pi^{C}_{initial,-1+dt},1+ndt}$, then $\tau_{2,t} < 2$ for sufficiently large $t(>1)$. Here, let $\pi^B = \pi_0^A + 6(\theta_G - \theta_P)\exp[z_t \ln(t)]$ for $t(\ge 0)$. Then, $\pi^B = \pi_0^A + 6(\theta_G - \theta_P)\exp[z_t \ln(t)]$ $= \pi_0^A + 6(\theta_G - \theta_P)t^2 + C_1\tau_{1,t}\exp[\tau_{2,t}\ln(t-1)]$; thus, $z_t - 2 = \ln\left[1 + \dfrac{C_1\tau_{1,t}\exp[\tau_{2,t}\ln(t-1)]}{6(\theta_G - \theta_P)t^2}\right]$ for $t(>1)$. Because $\tau_{2,t} < 2$ for sufficiently large $t(>1)$, then $\lim_{t\to\infty} z_t = 2$. ■

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Figure 1 ***Increases in the real obligation of $\bar{b}_t$ when inflation is accelerating***

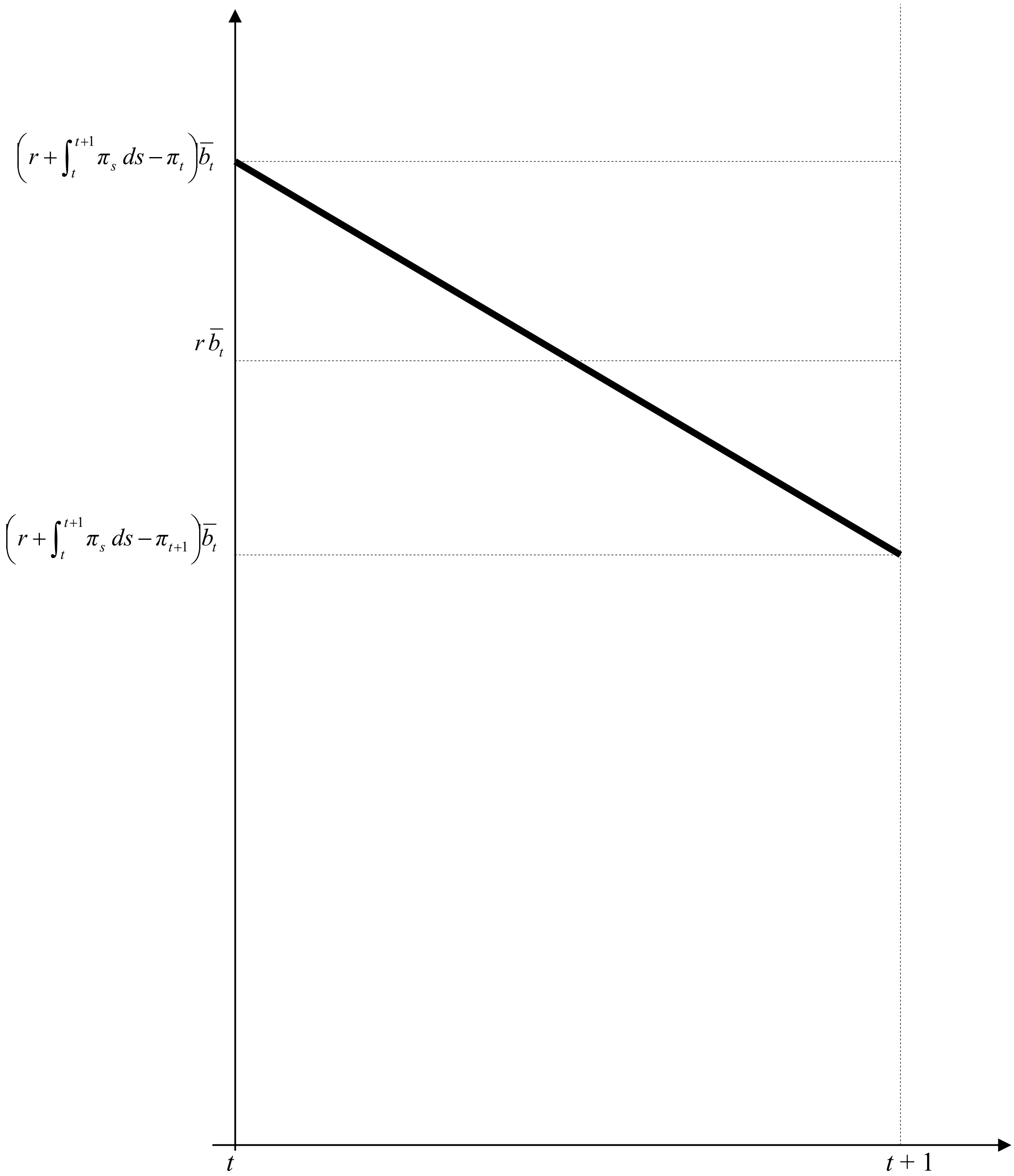

Figure 2 ***Increases in the real obligation of*** $\bar{b}_t$, $\bar{b}_{t+1}$, ***and*** $\bar{b}_{t+2}$ ***when inflation is accelerating***

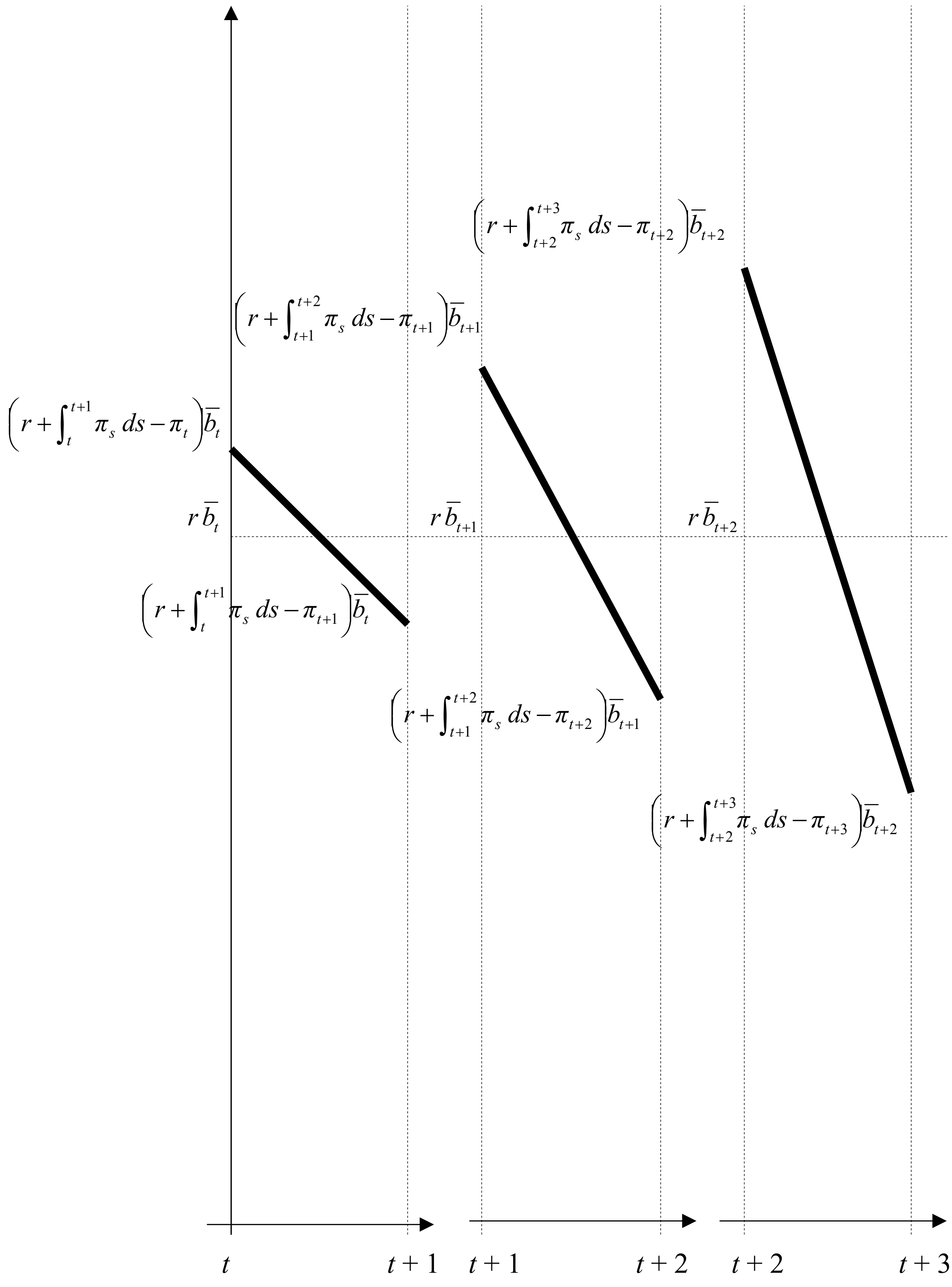

Figure 3 ***Increases in the real obligation of*** $\bar{b}_{t-1+s,t}$ $(0 < s \leq 1)$ ***at time t when inflation is accelerating***

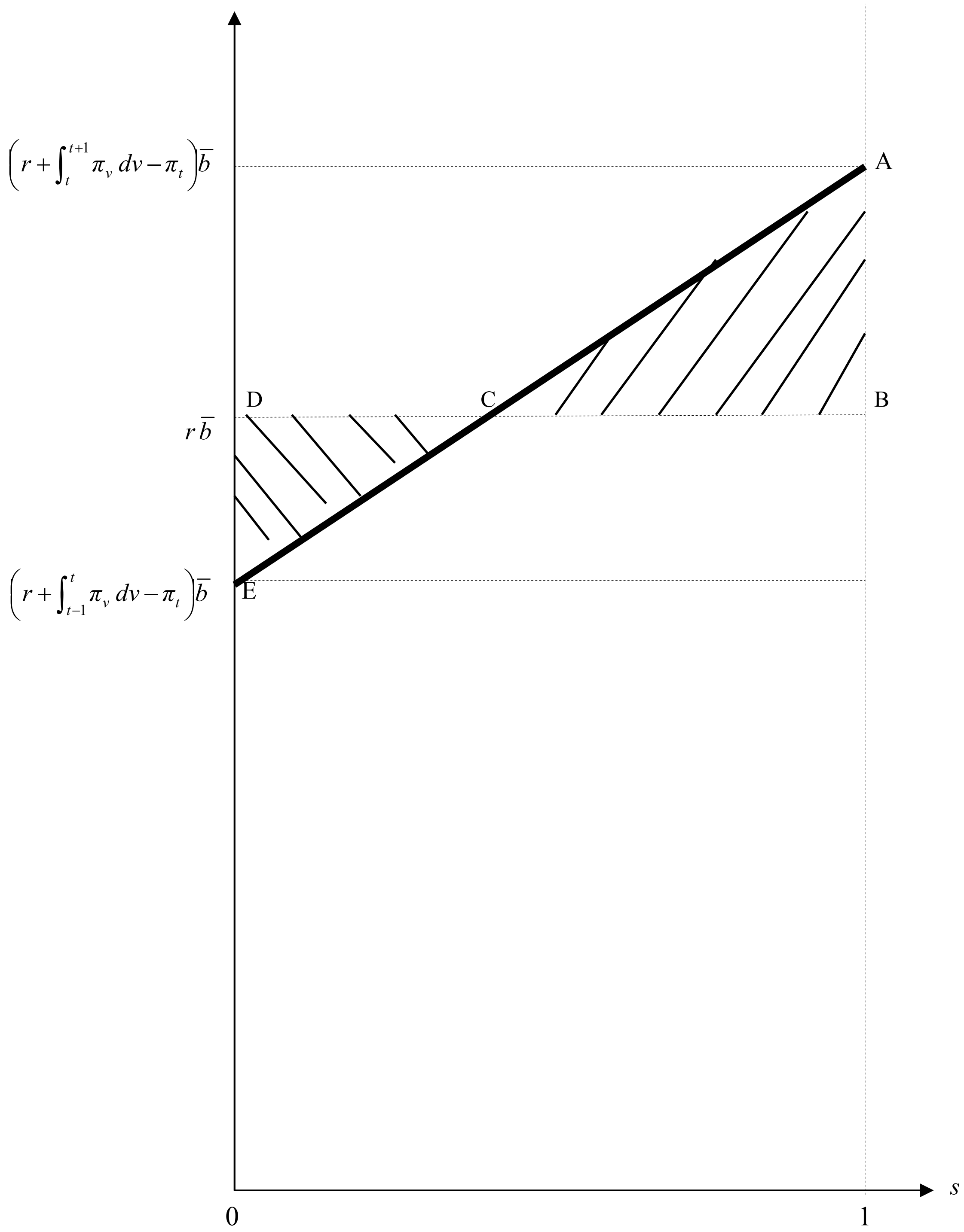

Figure 4 ***Examples of inflation paths based on Model III***