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Efficient contests

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Abstract

In their seminal contribution Lazear and Rosen (1981) show that wages based upon rank induce the same efficient effort as incentive-based reward schemes. They also show that this equivalence result is not robust towards heterogeneity in worker ability, as long as ability is private information, as it is not possible to structure contests to simultaneously satisfy self-selection constraints and first best incentives.

This paper demonstrates that efficiency is achievable by a simple modification of the prize scheme in a mixed (heterogenous) contest. In the L&R contest, the winner's prize as well as the loser's prize are fixed in advance. In this paper I demonstrate that efficiency is restored by a modification of contest design, in which contestants choose from a menu of prizes.

Key words: Tournaments, Labor Contracts

JEL codes: J 33

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1 Introduction

Tournaments or contests are contracts that reward each agent according to her performance relative to others. In their seminal contribution Lazear and Rosen (1981), hereafter referred to as L&R, show that with risk-neutral agents wages based upon rank induce the same efficient effort as piece rate schemes. They also show that this equivalence result is not robust towards heterogeneity in worker ability, as long as ability is private information, as it is not possible to structure contests to simultaneously satisfy self-selection constraints and first best incentives. Their argument consists of two parts. First, they show that agents cannot self-sort into different leagues (where each league represents an efficient homogenous contest), since all agents prefer to participate in the "high ability" league. Second, if all types participate in the same "mixed" contest, they demonstrate that it is not possible to provide all participants with first best incentives.

This paper demonstrates that efficiency is achievable by a simple modification of the prize scheme in a mixed (heterogenous) contest. In the L&R contest, the winner's prize as well as the loser's prize are fixed in advance. I refer to this as a *standard contest*. In this paper I demonstrate that efficiency is restored if contestants choose from a *menu of prizes*, which I refer to as a

generalized contest. As in standard contests, prizes are allocated according to relative performance, hence ordinal information on performance is sufficient.

The problem identified by L&R belongs to a more general problem in optimal contract design. Designing contracts that are compatible with self-selection as well as providing efficient effort incentives are often rather complex, and typically linear only under very restrictive assumptions, see e.g. Gibbons (1997). As demonstrated below, the underlying mechanisms of the optimal contest scheme described in this paper correspond to well known mechanisms found in the contract literature.

The inefficiency of standard contests demonstrated by L&R has been discussed in several contributions in the literature, e.g. O’Keeffe et al (1984), Bhattacharaya and Guasch (1988) and Yun (1997). Unlike this literature, I analyze the case where the optimal discriminatory prize premium is non-monotonic in ability, see further discussion below.

2 The original L&R model

The L&R model is specified as follows: Two contestants (j and k) of equal abilities simultaneously invest μ_j and μ_k under strictly convex and symmetric

investment cost functions $C(\mu)$. Their respective (lifetime) outputs equal investment plus a luck component, which is $q = \mu + \varepsilon$, where ε has constant variance and zero mean and zero correlation across contestants. Gross profit equals Vq , hence first best allocation is represented by $V = C'(\mu)$.

The contestant with the largest output wins the contest and is paid the prize premium r in addition to the base wage W . Due to the luck component, the contestants' respective win probability functions are continuous in the two investment levels. If we let $G(\cdot)$ denote the CDF of the difference in luck terms $\varepsilon_j - \varepsilon_k$, and $g(\cdot)$ its density, the probability that j wins is $G(\mu_j - \mu_k)$, and that k wins, $1 - G(\mu_j - \mu_k)$. Thus, contestant j 's expected utility is:

$$W + G(\mu_j - \mu_k)r - C(\mu_j)$$

with first order condition

$$g(\mu_j - \mu_k)r - C'(\mu_j) = 0$$

In a symmetric equilibrium μ_j equals μ_k , hence the first order condition can be expressed

$$g(0)r - C'(\mu) = 0 \implies \mu[g(0)r]$$

which determines a strictly increasing investment function $\mu[g(0)r]$.

Hence the prize premium r multiplied by the density of the noise term $g(0)$ corresponds to the incentive power of the classic piece rate reward scheme. The density of the noise term $g(0)$ is crucial; due to the noise term an increase in the level of investment increases the player's win probability; with a stronger effect the more dense the noise distribution is. If the noise distribution is very dense, the pure strategy equilibrium breaks down, as demonstrated in L&R, see also Nalebuff and Stiglitz (1983). Observe that as the variance in the noise term approaches zero, the contest approaches an all pay auction, in which equilibrium achieved is in mixed strategies.

Since $\mu[g(0)r]$ is strictly increasing, first best allocation is achievable by a proper choice of prize premium. With efficient allocation, wealth is maximized, and the correspondence to optimal piece rate schemes is established.¹

The next section introduces type heterogeneity.

¹As shown by L&R the equivalence result does not hold when risk aversion is introduced. Observe that the risk structure of a contest differs in two respects: the down side is limited since the worst case scenario is to win the loser's prize - that is good news for risk averse agents. The bad news is that there is no possible realization between the winner's and the loser's prizes. L&R shows that the ranking of the two schemes is indecisive and dependent on the specification of the utility function.

3 A contest with type heterogeneity

Denote by θ the single agent's ability. I assume that ability is private information² and continuously distributed with symmetric probability density $f(\theta)$. With no loss of generalization I assume the support is $[\underline{\theta}, \bar{\theta}]$.

The investment cost function of an agent with ability θ is denoted by $C(\mu; \theta)$. I assume that the agent's total investment cost $C(\mu; \theta)$, as well as her marginal investment cost $C'(\mu; \theta)$, are strictly decreasing in θ .

In first best, the marginal value of investment V equals the agent's marginal cost

$$V = C'(\mu; \theta) \implies \mu(\theta) \tag{1}$$

which defines a strictly increasing first best investment path $\mu(\theta)$.

As in L&R, first best incentives are unattainable within at standard contest. To see this, consider a contest with base wage W and prize premium r , and assume to the contrary that first best incentives are achievable. The expected prize obtained by type θ_j , conditioned on her opponent investing

²If abilities were common knowledge a set of handicaps adjusting for differences in incentive power due to the type distribution effect can be derived. Optimal handicapping has received a lot of attention in the literature. See for example the discussion in Che and Gale (2003) on R&D contests, Szymanski's (2003) survey on sporting contests and Tsoulouhas et al (2007) analysis of CEO contests. However, as pointed out by McLaughlin (1988): "The real problem with tournaments with heterogeneous contestants arises if the contestant's types cannot be identified".

according to the first best rule $\mu(\theta)$, can be expressed

$$W + \int_{\underline{\theta}}^{\bar{\theta}} G(\mu_j - \mu(\theta))f(\theta)r d\theta$$

Differentiating with respect to μ_j yields the incentive power of the scheme

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu_j - \mu(\theta))f(\theta)r d\theta \tag{2}$$

which can be approximated as follows³

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu_j - \mu(\theta))f(\theta)r d\theta \approx f(\theta_j)r$$

Hence, with a fixed prize premium r the incentive power of the contest is increasing in type density $f(\theta_j)$. To provide intuition for this specific property, observe that with type heterogeneity agents compete *locally* in the following sense: on the margin, the agent's gross benefit from a small increase in her investment level is proportional to the density of types investing at that specific level; that is, the density of opponents of exactly her own type. Clearly, given her ability, she beats inferior types, as she loses to superior

³The approximation is exact as the spread of $G(\cdot)$ converges to zero.

types. Thus, facing an opponent of unknown ability, she invests *as if* her opponent were of her own type.

First best requires a marginal benefit of investment equal to the marginal social value V . Thus, unless θ is uniformly distributed, the incentive power of the standard contest fluctuates with type density, incompatible with first best (1).

From the literature on price discrimination, it is well known that the principal may extract more surplus by introducing a menu of contracts and let participants self-select, which is referred to as second degree price discrimination. Let me first investigate whether this approach is feasible in the present setting. First best incentives would require that each type chooses a contract providing her with a prize premium r that is inversely proportional to her own type density $f(\theta)$, that is

$$r(\theta) = \frac{V}{f(\theta)} \tag{3}$$

This yields an *inverse* relationship between density $f(\theta)$ and prize premium $r(\theta)$.

As known from the literature, this approach is appropriate if preferences satisfy "single crossing". However, unless $f(\theta)$ is monotone, which is not

standard, (3) creates non-monotonicity in $r(\theta)$ which complicates contest design. To see this, let us consider the contestants' incentives to reveal their true ability in a setting where each of them individually and independently choose prizes r and W from a prize menu. Assume initially that first best incentives are achievable, and denote by $P_\theta(\mu^*)$ the equilibrium win probability of a contestant of type θ who invests μ^* . Her expected utility can then be written

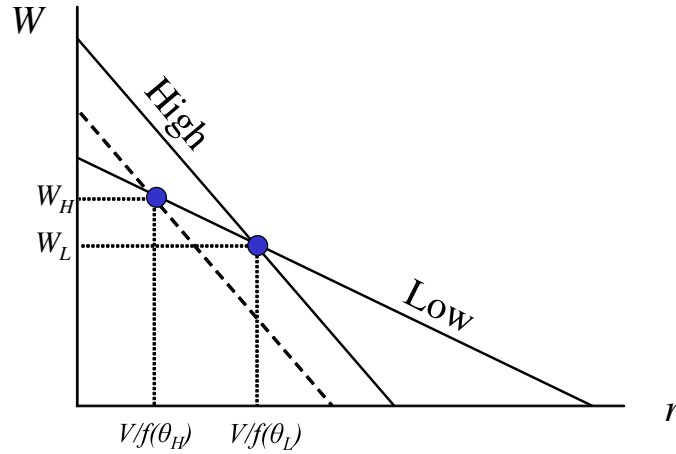
$$U_\theta = W + P_\theta(\mu^*)r - C(\mu^*; \theta)$$

The marginal rate of substitution between base wage W and prize premium r reflects her win-probability

$$\frac{dW}{dr} \Big|_U = -P_\theta(\mu^*) \tag{4}$$

Consider now two types, $\theta_H > \theta_L$, and assume $f(\theta_H) > f(\theta_L)$. Then, first best incentives requires that the two contestants choose prize premia $r_H = V/f(\theta_H)$ and $r_L = V/f(\theta_L)$ respectively, where $r_H < r_L$ since $f(\theta_H) > f(\theta_L)$. Furthermore, in first best equilibrium the high type's win probability P_H exceeds the win probability of the low type, P_L , since the high type has a higher investment level, $\mu(\theta_H) > \mu(\theta_L)$. The two types' respective marginal

rates of substitution are depicted in figure 1, where it follows from (4) that the high type's MRS crosses the low type's MRS from above.



Efficiency requires, ref (3), that type θ_L prefers $V/f(\theta_L)$ and that the higher type θ_H prefers the smaller prize premium $V/f(\theta_H)$. Furthermore, denote by W_L the base wage in the L contract and W_H the base wage in the H contract. For type θ_L to prefer the L contract, the base wage in the H contract must not exceed W_H as depicted in the figure. However, if L is indifferent between the two contracts, type H strictly prefers the L contract, since H's indifference curves are steeper than L's. Accordingly, any contract that induces the low ability contestant to choose the high prize premium provides the high type with incentive to mimic the low type.

Hence, to satisfy single crossing, higher types must be provided with higher prize premia, incompatible with (3). This restriction can be satisfied

by a slight modification of the scheme. The modification is to let the prize distribution be conditioned on all contestants' menu choices. This idea is illustrated in figure 2.

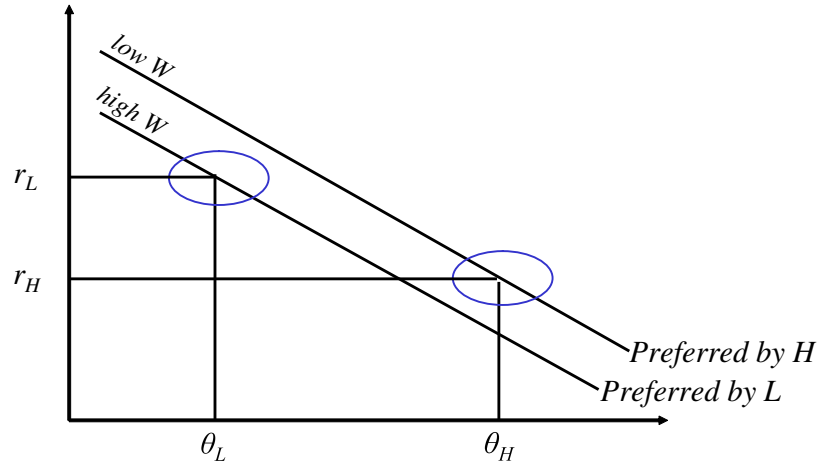


Figure 2

The two downward sloping paths, with corresponding base wages, illustrate two alternative contracts. The upper path is combined with a low base wage, and the lower path associated with a high base wage. Each path gives the realized prize premium, *conditioned on opponent's type θ* which is revealed by the opponent's choice of contract. Observe that the upper path *dominates* the lower path in the following sense: the upper path, for any given opponent type, yields a strictly higher prize premium than the lower path. Thus, with proper specifications of base wages, high types would prefer the upper path, whereas low types would prefer the lower path.

Consider now the incentive power of the scheme. Despite the described dominance property, a low type's incentives may in equilibrium be stronger than a high type's. As discussed above, on the margin, the single agent is concerned about the density of types around her own type. Thus she calculates the prize premium obtained conditioned on beating an opponent of her own type. Since the two paths are downward sloping, type θ_L can be provided with stronger incentives than a high type θ_H . In the figure this is indicated by the ellipses.

To provide more intuition, sorting in this example requires that high ability types (with a high win probability) obtain higher prize premia than low ability types. This explains the dominance in paths. Consider now, as an illustration, a standard single peak ability distribution: due to low density of types in the low ability tail, in a standard contest low ability contestants have weak incentive power. To restore efficiency, their stake in the contest must be strengthened - hence they must be provided with a larger prize premium. Since their ability is low and it is likely that they will lose, they are also provided with a high losing prize. To prevent high ability types from mimicking low ability types, the contract designed for low ability types yields a low prize premium if a contestant (who pretends to be of low ability) beats a

more able opponent. The point is that a low ability type does not suffer much from a low prize premium conditioned on beating a more able contestant, as it is unlikely that she actually will beat this superior opponent. A high ability contestant however, is "punished" in the sense that, by mimicking low types, she can only obtain a high prize premium by facing an opponent who claims to be a low type - which again is highly unlikely. Observe that the expected prize premium of the low type, *conditional on winning*, exceeds its unconditional expectation, a kind of "reversed winner's curse". Hence low ability types are induced to "bid" aggressively.

The next section proves the existence of a contest design that provides agents with first best incentives in a setting with dual information asymmetry. Let me first add some comments regarding the literature. This paper is not the first to deal with this specific sorting problem in contests. As mentioned, O'Keeffe et al (1984), Bhattacharaya and Guasch (1988) and Yun (1997) all address similar problems. However their models are formulated such that the non-monotonicity problem described above is avoided. O'Keeffe et al (1984) consider a model of self-selection which divides contestants into separate high and low ability leagues, where self-selection is achieved by increasing the prize spread in the high-ability league (which prevents climbing of low ability

contestants) and reducing the prize spread of the low ability league (which prevents slumming of high ability contestants into the low ability league). To restore effort incentives in the low ability league, the degree of monitoring precision in this league must increase (which increases the marginal return from effort).

Bhattacharaya and Guasch (1988) show that efficiency is restored through self-selection among wage contracts, where each contestant is compared with the output of an agent with the *lowest* efficient investment level. I comment below on the motivation behind this specific ranking mechanism.

Yun (1997), addressing the first part of the L&R inefficiency result regarding self-selection into homogenous leagues, considers two-prize standard contests with multiple agents, where the proportion of agents paid the low prize is endogenously determined. He demonstrates that by varying what he refers to as the "penalizing rule" - the proportion of low prizes in each contest - he establishes a sorting device where each (discrete) type will self-sort into her own league.

The sorting mechanism used in these contributions, basically that high ability contestants have a stronger preference for high prize premia than low ability contestants, cannot support full efficiency. The reason is that

the mechanism is compatible with first best incentives *only if* the incentive problem is *one-sided* in the sense that the optimal incentive power is monotonically increasing in type. However, this assumption is not compatible with standard ability distributions.

In Bhattacharaya and Guasch the non-monotonicity problem is avoided by assuming that each contestant competes with a threshold represented by the investment level of a hypothetical agent with the lowest possible ability (consequently the density of the noise term is strictly decreasing, since each contestant's investment exceeds the threshold). However, competing against a threshold requires that performance relative to the threshold is measurable, which is more information demanding - thus their contribution is incompatible with one of the appealing aspects of contests - that ordinal information is sufficient. In O'Keeffe et al and in Yun, the problem is avoided by introducing new elements (for instance by manipulating the noise term as in O-Keeffe et al).

The next section characterizes formally the modified contest scheme in the two-player case, and discusses briefly generalizations to n players. The final section concludes.

4 The generalized contest

The model is based on the following time structure: in stage one two contestants enter, and pay an entry fee to the principal. After entering, in stage two, contestants learn their abilities. In stage three they simultaneously and independently choose winning and losing prizes from a prize menu and in stage four they compete in the contest.

The motivation behind the assumption that contestants learn their abilities after entry (and after the entry fee is sunk), is to maintain first best effort as the optimal benchmark, thus focusing on the design problem discussed in L&R and subsequent literature. If contestants knew their abilities in advance, the standard trade off between rent extraction and efficiency arises.⁴ Since contestants are symmetric ex ante, the entry fee is determined such that agents are on their respective participation constraints, and optimal design corresponds to the first best benchmark.

In the model it is convenient, as is common in the literature, to represent self-selection by letting the contestants report their abilities. Formally the reporting stage and the contest stage go as follows: first contestants j and

⁴If contestants knew their types before sinking the entry cost, the problem corresponds to the design of optimal contracts under asymmetric information. An equivalence result demonstrating that ordinal comparison is sufficient for optimal contracts can be shown.

k report their respective abilities $\widehat{\theta}_j, \widehat{\theta}_k$. As in L&R, contestants compete in investment levels μ_j, μ_k . If j loses the contest she is paid the base wage $W(\widehat{\theta}_j)$, and if she wins she receives the additional prize premium $r(\widehat{\theta}_j, \widehat{\theta}_k)$, which depends on her own announcement $\widehat{\theta}_j$ as well as on her opponent's report $\widehat{\theta}_k$. Observe that $r(\widehat{\theta}_j, \widehat{\theta}_k)$ may differ from $r(\widehat{\theta}_k, \widehat{\theta}_j)$, the prize premium k obtains in case she is deemed the winner. Finally, contestant j does not observe k 's report, and vice versa.

Worker j 's utility as a function of her type θ_j and report $\widehat{\theta}_j$ is

$$U(\theta_j, \widehat{\theta}_j) = \max_{\mu} \left[W_2(\widehat{\theta}_j) + \int_{\underline{\theta}}^{\bar{\theta}} G(\mu - \mu(\theta)) r(\widehat{\theta}_j, \theta) f(\theta) d\theta - C(\mu; \theta_j) \right] \quad (5)$$

where $\mu(\theta_j)$ denotes the optimal investment for type θ_j measured in a truth-telling equilibrium. As the object function is strictly concave in μ , $\mu(\theta_j)$ is continuous and strictly increasing in θ_j .

Consider the sorting conditions. From the envelope theorem it follows that

$$\frac{dU(\theta_j, \widehat{\theta}_j)}{d\widehat{\theta}_j} = W_2'(\widehat{\theta}_j) + \int_{\underline{\theta}}^{\bar{\theta}} G(\mu(\theta_j) - \mu(\theta)) r_1(\widehat{\theta}_j, \theta) f(\theta) d\theta = 0 \quad (6)$$

Differentiating (6) with respect to type θ_j yields

$$\frac{dU^2(\theta_j, \hat{\theta}_j)}{d\hat{\theta}_j d\theta_j} = \int_{\underline{\theta}}^{\bar{\theta}} g(\mu(\theta_j) - \mu(\theta)) r_1(\hat{\theta}_j, \theta) f(\theta) d\theta \frac{d\mu(\theta_j)}{d\theta_j} \quad (7)$$

Sorting requires that (7) is positive; the marginal benefit of a higher announcement $\hat{\theta}$ is increasing in type θ . A sufficient (but not necessary) condition for (7) to be positive for all $\hat{\theta}, \theta$ is that $r_1(\hat{\theta}_j, \theta)$ is positive for all θ . If this holds the standard single crossing condition is satisfied, and a separating contract exists.

Lemma 1 *Consider prize premium functions $r(\hat{\theta}, \theta) > 0$ defined on $[\underline{\theta}, \bar{\theta}] * [\underline{\theta}, \bar{\theta}]$ with $r_1(\hat{\theta}, \theta) \geq 0$ for all $\theta, \hat{\theta}$. Then there exists a loser prize function $W(\hat{\theta})$ consistent with truth-telling.*

First best efficiency requires that

$$V = C'(\mu_j; \theta_j)$$

and we refer to $\mu^*(\theta)$ as first best investment which is strictly increasing in θ .

From the first order condition of the agent's maximization problem we

can derive the following: if there exists a prize premium $r(\theta_j, \theta)$ satisfying lemma 1 such that

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta))r(\theta_j, \theta)f(\theta)d\theta = C'(\mu^*(\theta_j); \theta_j) \equiv V \quad \text{all } \theta_j \quad (8)$$

then the generalized contest yields first best incentives.

Due to lemma 1 it is sufficient to prove that there exists a set of profiles $r(\theta_j, \theta)$ which induce each type θ_j to invest at the efficient level $\mu^*(\theta_j)$. The main result, Proposition 1 below, states that first best efficiency is achievable in a contest. The mechanism is to offer prize premia functions $r(\theta_j, \theta)$ that are i) increasing in own announcement θ_j and ii) decreasing in the opponent's announcement θ .

The main result, which is proved in an appendix, can be stated:

Proposition 1 *First best incentives are achievable in a generalized contest.*

Proof. See appendix ■

The proposition characterizes prize premium functions providing first best incentives in a two-player contest. Generalizing the result to a multi-agent setting is straightforward. As the probability of winning the contest increases in ability, separation is feasible in a contest which remunerates the

overall winner with a prize premium, and pays the remaining contestants their respective loser prizes⁵. Let me provide a sketch of this procedure. Denote by $\mathbf{F}(\theta)$ the probability that the highest competing type is θ , and by $\mathbf{G}(\mu_j - \mu_{(-)}^*(\theta))$ the probability that the contestant beats all of her opponents (with types drawn below θ according to the type distribution $F(\cdot)$, truncated at θ). Then, the utility of contestant j can be expressed:

$$U(\theta_j, \hat{\theta}_j) = \max_{\mu_j} \left[W_2(\hat{\theta}_j) + \int_{\underline{\theta}}^{\bar{\theta}} \mathbf{G}(\mu_j - \mu_{(-)}^*(\theta)) r(\hat{\theta}_j, \theta) d\mathbf{F}(\theta) - C(\mu_j; \theta_j) \right]$$

with the first order condition (in a truth-telling equilibrium)

$$\int_{\underline{\theta}}^{\bar{\theta}} \mathbf{g}(\mu^*(\theta_j) - \mu_{(-)}^*(\theta)) r(\theta_j, \theta) d\mathbf{F}(\theta) d\theta = C'(\mu^*(\theta_j); \theta_j)$$

(where \mathbf{g} denotes the density of \mathbf{G}). The proof of proposition 2 can now be replicated.

⁵Remunerating the overall winner is sufficient.

5 Concluding remarks

The paper demonstrates how a generalized contest can be designed to support efficient allocation under dual information asymmetry, where ability is private information and output is observable with noise. High ability contestants' incentives are restored by providing them with a large upside, a high prize premium and a low loser's prize. Low ability contestants' incentives are restored by awarding them a high prize premium if winning *conditioned* on being challenged by low ability opponents. The latter restriction ensures that the contract is unattractive to high ability types.

This sorting problem is rather common and well known from the incentive contract literature. For reference, consider the following classic information problem within incentive contract design: in the design of incentive schemes one often aims at enhancing the incentive power *locally*, for instance by providing the agent with a bonus for accomplishing a task or fulfilling a target. Clearly, setting the target too low or too high yields poor incentives as the bonus is either almost certain or unattainable. Hence, adjusting the target is a classic information problem when ability is private information. To elaborate a bit further on this, consider the piecewise linear "kinked" incentive scheme analyzed in Weitsman (1976), see also Holmström (1982), which

goes as follows: initially the single agent is presented with a tentative reward scheme consisting of a strictly increasing linear function of observed output. Thereafter, the agent self-selects her final reward scheme which is piecewise linear with one kink. The essential point is that this final reward function is strictly below the tentative reward function everywhere except one single point - the self selected kink. Clearly the agent is best off choosing a kink that corresponds exactly with the output level which is optimal given her inherent ability and given the initial tentative scheme. Thus, the sorting condition is satisfied since the agent (relative to the tentative scheme) is punished by choosing a low kink (that is by mimicking low types) as well as by choosing a high kink (mimicking high types).

The self selected "kink" as sorting device corresponds logically to the mechanism yielding self-sorting in the optimal contest derived in this paper, where contestants choose from a menu of prizes. Since the principal only has access to ordinal information, the reward can neither be conditioned on observed output, nor on the bid (investment) itself. Yet the reward can be conditioned on the opponents' types, as this information is revealed through their self selection of contest prizes. Mimicking inferior types is then avoided, as we will see, by "punishing" agents for beating superior opponents.

6 Appendix

Proof. Proposition 1

I first prove the existence of a prize premium function supporting first best efficiency given that the truth telling constraint is weakly satisfied. Hence the prize premium depends only on the opponent's announcement, $r(\theta_j, \theta) = r^c(\theta)$. Let $h(\theta) \equiv r^c(\theta)f(\theta)$. Then, efficiency requires the existence of a function $h(\theta)$ such that

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta))h(\theta)d\theta = V \text{ all } \theta_j \in [\underline{\theta}, \bar{\theta}] \quad (9)$$

To prove the existence of a function $h(\theta)$ define $h_1(\theta)$ and $h_0(\theta)$ such that

$$h_1(\theta_i) = h_0(\theta_i) + V - V_0(\theta_i) \quad (10)$$

where

$$V_0(\theta_i) = \int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_i) - \mu^*(\theta))h_0(\theta)d\theta$$

It follows that (10) is continuous and maps a closed, bounded and convex function $h(\theta)$ into itself. Hence a fix point exists.

Secondly, to prove the existence of prize premia functions $r(\theta_j, \theta)$ strictly

increasing in the first argument (hence the sorting constraint is strictly satisfied), and supporting efficiency: consider an arbitrary type θ_j^n and assume $h(\theta)$ satisfies (9). Construct a new function $h^n(\theta)$ which is "tilted around" $h(\theta_j^n)$, that is $h^n(\theta) - h(\theta) > (<)0$ if $\theta < (>)\theta_j^n$, where $|h^n(\theta) - h(\theta)|$ is strictly increasing in $|\theta - \theta_j^n|$, and such that

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j^n) - \mu^*(\theta))h^n(\theta)d\theta = V$$

Observe that an increase in θ_j moves the probability mass $g(\mu^*(\theta_j) - \mu^*(\theta))$ to the right, and since $g(\cdot)$ is unimodal and strictly increasing (decreasing) below (above) zero, it follows that

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta))h^n(\theta)d\theta > V \quad \text{for all } \theta_j < \theta_j^n \quad (11)$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta))h^n(\theta)d\theta < V \quad \text{for all } \theta_j > \theta_j^n$$

Consider then an arbitrary type $\theta_j^m < \theta_j^n$. Due to (11) we can establish the

existence of a function $h^m(\theta)$ where $h^m(\theta) < h^n(\theta)$ for all θ and

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\mu^*(\theta_j^m) - \mu^*(\theta))h^m(\theta)d\theta = V \quad (12)$$

As this holds for any pairs (θ_j^n, θ_j^m) it follows that we can draw for each type θ_j a type specific prize premium denoted $r(\theta_j, \theta)$ such that $r(\theta_j^m, \theta) < r(\theta_j^n, \theta)$ whenever $\theta_j^m < \theta_j^n$, hence strict truth-telling is established. ■

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