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# Linking Reputations: The Signaling and Feedback Effects of Umbrella Branding

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**Abstract**

This paper develops a theory of umbrella branding as a way to link the reputations of otherwise unrelated products. I show that while umbrella branding can credibly signal positive quality correlation, there are no equilibria in which umbrella branding either fully certifies high quality, or signals negative quality correlation. Finally, whenever umbrella branding signals perfect positive quality correlation, firms that already produce high quality products have stronger incentives to invest in developing further high quality products than firms that are currently inactive or produce low quality products.

**Keywords:** reputation, umbrella branding, brand extensions, quality signaling

**JEL classification codes:** L14, L15, M31

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# 1 Introduction

Umbrella branding is a widespread marketing practice that consists of selling different products under the same brand name. Some umbrella brands sell closely related products, e.g., *Dell* sells flat screen televisions, flat screen monitors and laptops. Others sell products in unrelated categories, e.g. *Virgin* sells music disks, air travel, cola drinks and financial services. Why should a firm extend a brand, that is, introduce new products under an umbrella brand with one or several existing products? One convincing explanation is that brand extensions allow firms to leverage existing brand equity.[1] The idea is that umbrella branding helps firms with strong brands to successfully introduce new products by convincing consumers that new and existing products are of similar quality. One implication is that if an umbrella branded product does not satisfy consumers, then all other products sold under the same brand will likely suffer. Conversely, individual successes may benefit several products.

The present paper analyzes these trade-offs in a model based on an information asymmetry between firms and consumers. Each product is characterized by a quality level that is observable only by firms. Product qualities are either exogeneously given, or determined upfront by the firms' investment decisions in product development. Consumers observe product performances, but these are (always) imperfect signals of quality. In this context, a product's reputation at any point in time is the belief consumers hold about its quality at that moment.

The correlation between the qualities of umbrella branded products is endogenous in my analysis. In other words, even in the baseline version of the model with exogeneously given quality levels, the qualities of the products a firm has the *option* to under the same brand need not be uniform. Uniform quality could arise due to the use of a common input for example, but in most instances it seems restrictive to rule out ex ante the possibility that a brand sells products of different qualities. This is especially true for brands such as *Virgin* that span across very dissimilar product categories. Moreover, products sold under the same brand are often manufactured by different firms. The *AT&T* brand for example is licensed to *VTech* for telephony products and to *Verbatim* for blank media. My model is general enough to be interpreted in the context of brand licensing as well as in the context of brand extensions to products developed by the brand owner itself. The essential element is that the branding decision is taken so as to maximize the expected aggregate profits from selling the different products.

Since firms can condition their branding decisions on qualities, the absence of any exogenous or technological correlation between qualities does not necessarily imply that umbrella branding has no informational value for consumers. In general the decision to umbrella brand can induce two different types of effects. First, umbrella branding leads to *feedback effects* whenever con-

[1] Classical marketing textbook references include Aaker and Keller (1990), Kapferer (1997), and Aaker (2004).

sumers believe that the qualities of the different products are correlated. The performance of each product then yields information not only about the product's own quality, but also induces consumers to update their beliefs about other products sold under the same umbrella brand. In the case of positive correlation, the success (failure) of any one of the umbrella branded products has a positive (negative) feedback effect on the other products. Second, the umbrella branding decision itself may influence the prices consumers are willing to pay for the different products; umbrella branding has *signaling effects* in this case. I analyze these effects in a two-stage game with two products sold either under separate brands or under an umbrella brand in both periods.

One key result is that, for exogenously given quality levels, the relative importance of the signaling and feedback effects may be such that positive quality correlation arises endogenously in a perfect Bayesian equilibrium. In some situations there even exist equilibria in which umbrella brands always offer products of uniform quality. In the equilibria with perfect positive correlation characterized in the paper, the signaling effects are such that firms of any quality profile could make a short term gain by using an umbrella brand; the expected long term impact of the branding decision, however, depends on actual qualities. For firms with one good and one bad product, umbrella branding means putting the future reputation of the good product at stake by inviting consumers to pool their experiences. If future profits are important, these firms will therefore prefer separate branding. For firms with two bad products, on the other hand, the branding decision's expected long term impact may be neglegible: if the consumption of a bad product is sufficiently likely to convince consumers of the product's low quality, then bad products can be expected to lose their reputations in the long term independently of the branding decision. Umbrella branding will then be attractive for firms with only bad products, since it allows them to reap short term profit gains without incurring any significant long term losses. Finally, for firms with two good products, umbrella branding is attractive not only in the short but also in the long term: thanks to feedback effects, these firms expect to consolidate their products' reputations faster under umbrella than under separate branding.

In contrast, there are no counterintuitive equilibria in which successes (failures) have negative (positive) feedback effects. If consumers expected the qualities of umbrella branded products to be negatively correlated, then firms with two bad products would benefit from positive feedback effects with a higher likelihood than firms with one or two good products. Moreover, the outside option of separate branding is always less profitable for firms with two bad products than for any other firm. These arguments imply that umbrella branding would be particularly attractive for firms with only bad products. Anticipating this, however, consumers' willingness to pay for umbrella branded products would be low. This in turn would render umbrella branding unprofitable for all firms.

Extending the baseline model where all quality levels are exogenous, I also consider a situation where producers of new goods choose their products' qualities prior to selling. Choosing high quality is associated with a fixed investment cost, privately drawn by each producer from a commonly known probability distribution. In this framework, equilibria in which umbrella branding signals perfect quality correlation continue to exist, and all share the following feature. Investing in the new product's quality is more attractive for firms with a good existing product, available for an umbrella brand, than for mono-product firms: selling good new product is more profitable if you can umbrella brand it with a good existing product and thereby benefit (in expectation) from positive feedback effects in both directions. Finally, firms that already sell a product of low quality have the lowest investment incentive. For firms that already own a brand, the option to use an umbrella brand thus renders aligning the new product's quality to that of the existing product more attractive.

Finally, I identify a number of necessary conditions for the existence of informative equilibria.[2] First, the markets for the different products must be sufficiently symmetric. Second, there must be repeated consumption of *both* products, and firms must be sufficiently patient. Third, consumers' information about product qualities must be limited at the moment the brand extension occurs. Fourth, potential quality differences must be important. These conditions are necessary to ensure that signaling and feedback effects on both products play a significant role in the branding decision, which is needed to "separate" firms with different quality profiles.

These results are in line with the empirical and experimental evidence.[3] In particular, many studies confirm that umbrella brands induce a positive correlation between quality perceptions without certifying high quality; moreover, many case studies document feedback and signaling effects of the signs predicted here. Concerning the structural conditions necessary for credible signaling, my analysis leads to several testable predictions.

**Related Literature**[4] There are two main approaches to model umbrella branding, and more generally reputation, in the economics literature. Either reputation refers to a firm's or a product's characteristics (adverse selection), or to its actions (moral hazard).[5] Brand names are important in this context as carriers of information.[6]

[2] As is standard in signaling games, there always exist uninformative ('babbling') equilibria in which umbrella branding occurs but has no impact on beliefs or prices.

[3] Section 6.2 discusses the empirical evidence in more detail; the main references in that section are Aaker and Keller (1990), Sullivan (1990), Smith and Park (1992) and Erdem (1998).

[4] Section 7.1 provides a more detailed comparison between this analysis and alternative theories.

[5] Seminal articles on the adverse selection approach to reputation are Milgrom and Roberts (1982) and Kreps and Wilson (1982). The basic framework of the moral hazard approach goes back to Klein and Leffler (1981), and Shapiro (1983). Mailath and Samuelson (2006) as well as Bar-Isaac and Tadelis (2008) present a variety of recent reputation models, some of which combine aspects of moral hazard and adverse selection.

[6] Name trading between firms has been modelled by Kreps (1990) in a moral hazard context, and by Tadelis (1999) in an adverse selection model that shares some features with my approach.

Existing theoretical analyses of umbrella branding focus on umbrella branding as a way to signal high quality. Wernerfelt (1988) considers a simple adverse selection model in which the quality correlation between umbrella branded products is endogenous. He establishes conditions for an equilibrium in which umbrella branding fully certifies the high quality of both products. This situation is sustained by pessimistic out-of-equilibrium beliefs following failures of umbrella branded products, which under some additional conditions are the only beliefs satisfying the intuitive criterion of Cho and Kreps (1987). These results hinge upon the assumptions that (i) good products never fail, and (ii) umbrella branding is more costly than the introduction of a new brand.

Choi (1998) derives a similar quality certification result in a moral hazard context.[7] In an infinite horizon model where a firm discovers a new product of given quality in every period, he finds conditions for an equilibrium in which "premium" umbrella brands only extend to high quality goods. This equilibrium is sustained by the threat of a complete breakdown of trust if consumers ever observe that a bad product was introduced under a premium brand.

This paper uses a finite horizon approach to avoid problems common to repeated-game models of reputation, in particular the high required degree of coordination between firms and consumers, and the absence of any predictions concerning the evolution of reputation over time. Moreover, in contrast to Wernerfelt (1988), the analysis does not rely on the assumptions that umbrella branding is costly and that failures are perfect signals of low quality. The modeling approach adopted in this paper leads to an important shift of focus in terms of results: while umbrella brands are guarantees of high quality in the literature described so far, I stress the role of umbrella brands as signals of quality correlation. As already mentioned, this is more consistent with the empirical and experimental evidence (in particular, Aaker and Keller (1990) and Erdem (1998)), which suggests that umbrella brands induce a positive correlation between quality perceptions without certifying high quality. Moreover, the present model permits a more complete understanding of the costs and benefits associated with umbrella branding for firms with different quality profiles.

Cabral (2000) analyzes an adverse selection model of umbrella branding in which quality correlation is *exogenous*. The main difference with respect to my model is thus that Cabral assumes that all the products a firm has the option to sell under an umbrella brand must be of the same quality. The purpose of umbrella branding is then to communicate to consumers that different products are manufactured by the same firm. Allowing for a continuum of quality levels, Cabral's main prediction is that umbrella branding signals high quality to consumers. The intuition is that for high quality firms umbrella branding will lead to a positive feedback

[7] In fact, his models features both adverse selection and moral hazard aspects; the branding problem itself, however, is subject to a moral hazard problem.

effect with a higher likelihood than for low quality firms.

The analysis in this paper is complementary to that of Cabral. The novel result is that quality correlation may arise endogenously, in which case even firms with only low quality products sometimes use umbrella brands. My findings are nonetheless consistent with Cabral's prediction that umbrella branding signals high quality: umbrella branding has a positive signaling effect on at least one of the products and increases aggregate short-term profits in the equilibria with positive quality correlation characterized in this paper.

Finally, my analysis provides a new angle on the impact of umbrella branding on quality provision. The existing literature show that umbrella branding can increase the scope for quality provision. Hakenes and Peitz (2008) find this in a finite horizon model where quality investments occur upfront as here, while Andersson (2002) and Cabral (2008) analyze infinitely repeated games where firms set qualities in every period. The mechanism that plays a crucial rule in these papers is that umbrella branding allows consumers to make inference on the basis of two performance observations and hence punish the brand even if only one product fails.[8] In a framework where the core product's quality is given by past decisions, I stress instead that firms may have strong incentives to align the new product's quality to that of their core product.

This paper is organized as follows. Section 2 describes the framework. Section 3 presents the main effects of umbrella branding on beliefs and profits; in particular, it explains the relationship between the branding strategy, the correlation of consumers' prior quality perceptions, and signaling and feedback effects. Section 4 briefly discusses equilibria without feedback effects. Section 5 discusses equilibria with feedback effects. Section 6 extends the basic model by endogenizing the quality of new products. Section 7 first discusses differences with alternative theories, and then links the main findings to the empirical evidence. All proofs are in the appendix.

## 2 Framework

There are two experience goods, each either of good or of bad quality. A good product is successful, i.e. works well, with probability $g \in (0,1)$, while a bad product is successful with probability $b \in (0,g)$. In each period of the multi-stage game with repeated consumption that will be analyzed, the performance of each product is realized anew. The products' qualities are constant over time.

At time $t = 0$, an innovator discovers a new[9] product of quality $q_n \in \{b, g\}$. The innovator

[8] In Andersson (2002) and Cabral (2008), the basic mechanism ressembles that in Bernheim and Whinston (1990)'s analysis of collusion under multimarket contact. Hakenes and Peitz (2008), using the assumption that high quality products never fail, rely on out-of-equilibrium beliefs following the failure of one or both umbrella branded products.

[9] In the baseline model described here, the terms "old" and "new" are used for illustration only. In section 6,

always has the option to (costlessly) launch a new brand to sell this product. With some exogenous probability $\alpha \in (0,1)$, however, the innovator also "meets" an incumbent with an old product of quality $q_o \in \{b,g\}$. Both the innovator and the incumbent observe $q_o$ and $q_n$. If the firms meet, then they can decide to sign a brand licensing agreement. In that case, the new product will be sold under an umbrella brand with the incumbent's old product, that is, the incumbent will extend its brand to the new product. I do not explicitly model the negotiation between the innovator and the incumbent, but simply assume that it is efficient, i.e. the parties take the umbrella branding decision so as to maximize the (expected) aggregate profits made from selling the two products. I denote by $x_{q_oq_n} \in [0,1]$ the probability that the old product of quality $q_o$ and the new product of quality $q_n$ are sold under an umbrella brand, conditional on the incumbent and the innovator having met. An umbrella branding strategy is hence a vector $(x_{gg}, x_{gb}, x_{bg}, x_{bb})$ that gives the probability of a brand extension for every possible quality profile $(q_o, q_n)$.

At time $t = 1$, consumers first observe brand names, and then take consumption decisions. After consumption all consumers observe the performances $\sigma_o \in \{F, S\}$ and $\sigma_n \in \{F, S\}$, where $F$ stands for failure and $S$ for success, of the old and the new product, respectively. Product performances are public information and distributed i.i.d. across periods. At time $t = 2$, the firms sell both products again. The following table summarizes the timing of events:

| **Timing:** | |
|---|---|
| $t = 0$ | Innovator discovers new product of quality $q_n$. |
| | If the innovator "meets" the incumbent with a product of quality $q_o$, |
| | the new product will carry the same name as the old product with probability $x_{q_oq_n}$. |
| $t = 1$ | Consumers observe brand names, and then buy both products. |
| | Consumers observe the performances $\sigma_o$ and $\sigma_n$. |
| $t = 2$ | Consumers buy both products again. |

The prior reputations of the two products at $t = 0$ are parameters of the model and denoted by $\gamma \in (0,1)$ for the new product and by $r \in (0,1)$ for the old product. Ex ante consumers hence assign probability $\gamma$ to the event that the new product is good and $r$ to the event that the old product is good. In each period, consumers then update their beliefs, i.e. the probabilities they assign to different realizations of $(q_o, q_n)$, as a function of the information they possess. At the beginning of period 1, the consumers' information set simply includes the observation of whether the products are sold under an umbrella brand or not. After first period consumption has taken

however, the new and only the new product's quality will be endogenous.

place, consumers receive two additional pieces of information, namely the performances $\sigma_o$ and $\sigma_n$.

To simplify, I will focus on the limit case where $\alpha$, the probability of a "match" between the innovator and the incumbent, goes to zero. Having the opportunity to umbrella brand is then a measure zero event. The key implication is that if consumers observe separate brands rather than an umbrella brand, then they will not draw any inferences about qualities from this observation. Hence, the *absence* of a brand extension does not lead consumers to revise their prior beliefs $\gamma$ and $r$.[10] The remark at the end of this section briefly summarizes an overlapping generations model with a continuum of firms that shares this feature.

For all consumers, the consumption of a well functioning product procures a utility of one while consumption of a failing product procures no utility. A consumer's willingness to pay for a product with reputation $\mu$ is thus

$$w(\mu) \equiv \mu g + (1-\mu)b.$$

In every period the firms make take-it-or-leave-it offers to consumers, and therefore optimally set each product's price equal to consumers' willingness to pay for that product.[11]

The production costs of both products are zero. The markets for the different products can be asymmetric however, i.e. firms may attach different weights to different products at different times. I denote the share of the market for the new product in total sales by $\beta \in (0,1)$. Moreover, the weight attached to second period profits is denoted by $\delta \in (0,1)$. Normalizing the total number of units sold (of the two products in the two periods) to one, consumers then buy $(1-\delta)(1-\beta)$ units of the old product in period 1, $(1-\delta)\beta$ units of the new product in period 1, $\delta(1-\beta)$ units of the old product in period 2, and $\delta\beta$ units of the new product in period 2.

Attention will be restricted to equilibria in which umbrella branding indeed occurs with positive probability, i.e. $x_{q_o,q_n} > 0$ for some $(q_o, q_n)$, which implies that there will be no need to deal with out-of-equilibrium beliefs. I will hence use the concept of Bayesian equilibrium:

**Definition 1** *A (Bayesian) equilibrium consists of a belief system and an umbrella branding*

[10] If $\alpha > 0$, then the beliefs assigned to mono-product brands would have to take into account the possibility that a sellers was matched but decided in favor of separate brands. For example, under separate branding the first period reputation of an old product would be

$$\frac{(1-\alpha)r + \alpha\left[r\gamma(1-x_{gg}) + r(1-\gamma)(1-x_{gb})\right]}{(1-\alpha) + \alpha\left[r\gamma(1-x_{gg}) + r(1-\gamma)(1-x_{gb}) + (1-r)\gamma(1-x_{bg}) + (1-r)(1-\gamma)(1-x_{bb})\right]}.$$

instead of simply $r$. Since all beliefs (and hence profits) are continuous in $\alpha$ at $\alpha = 0$, however, all results in this paper would continue to hold for strictly positive but small $\alpha$.

[11] Tadelis (1999) uses a similar simplification in his model of name trading with a continuum of firms and consumers by supposing that consumers are on the long side of the market. Cabral (2000) as well as Hakenes and Peitz (2004) use the same assumptions in the context of umbrella branding.

*strategy* $(x_{gg}, x_{gb}, x_{bg}, x_{bb})$ *such that (i) the branding strategy maximizes aggregate profits given beliefs, and (ii) beliefs are Bayesian consistent given the branding strategy.*

**Remark** It is easy to reinterpret this model without referring to any outside innovator. Consider an economy with overlapping generations of firms that live for at least three periods. In each period, there is a continuum of incumbent sellers and a continuum of new entrants. Each firm produces one product that it sells under the same brand name in every period of its lifetime. In addition, one randomly chosen incumbent with two more periods left to live discovers a new product. This firm can then either extend its existing brand to the new product or introduce a new brand. Since there is a continuum of firms with two more periods to live, a brand extension is a measure zero event, and the observation that a brand is not extended has no relevant informational content for consumers. Similarly, consumers cannot distinguish between unknown brands of entrants and unknown brands of incumbents. Cabral (2000) uses a similar set-up.

# 3 Signaling and Feedback Effects

## 3.1 The Impact of Umbrella Branding on Beliefs

Umbrella branding may induce two types of effects on the consumers' beliefs. First, the decision to umbrella brand by itself may serve as a quality signal. For the new product, for example, such a *signaling effect* occurs whenever consumers believe that a new product introduced under an established brand is either more or less likely to be of high quality than new products carrying unknown brand names. These signaling effects are relevant both for first and second period beliefs.

Second, umbrella branding may lead to *feedback effects* if consumers believe that the qualities of the umbrella branded products are correlated. The success or failure of one product, say the new product, then induced consumers to revise their beliefs not only about the new product itself but also about the old product. Since these feedback effects are linked to performance observations that first occur at the end of period one, feedback effects are only relevant for the analysis of second period beliefs.

**Period 1** To analyze the short term signaling effects of umbrella branding, I compare consumers' beliefs at the beginning of the first period under umbrella branding and under separate branding. Since a match between the incumbent and the outside innovator is a measure zero event, consumers confronted with two separate brands do not revise their priors $r$ and $\gamma$.

Confronted with an umbrella brand instead, consumers possibly revise their beliefs about both products' qualities. Whenever brand extensions happen with positive probability in equilibrium, all beliefs can be obtained by Bayesian updating. I denote by $\mu_{q_o q_n}$ the probability consumers assign to the quality profile $(q_o, q_n)$ if they observe an umbrella brand; for example:[12]

$$\mu_{gg} = \frac{r\gamma x_{gg}}{r\gamma x_{gg} + r(1-\gamma)x_{gb} + (1-r)\gamma x_{bg} + (1-r)(1-\gamma)x_{bb}}.$$

To simplify notations, I will denote the aggregate belief that the old product of the umbrella brand is good by

$$\mu^o = \mu_{gg} + \mu_{gb},$$

and the belief that the new product is good by

$$\mu^n = \mu_{gg} + \mu_{bg}.$$

Umbrella branding then has a positive signaling effect on the new product if and only if it improves the consumers' belief about the quality of the new product, i.e. whenever

$$\mu^n > \gamma.$$

This condition is satisfied whenever, given the prior $r$ about the quality of the old product, a brand extension is more likely to involve a good rather than a bad new product:

$$rx_{gg} + (1-r)x_{bg} > rx_{gb} + (1-r)x_{bb}.$$

Similarly, the signaling effect on the core product is positive if and only if

$$\mu^o > r,$$

which is equivalent to

$$\gamma x_{gg} + (1-\gamma)x_{gb} > \gamma x_{bg} + (1-\gamma)x_{bb}.$$

**Period 2** After first period consumption, consumers observe the performances of both products. They then update their beliefs so as to take into account these additional pieces of information in the second period.

Under separate branding, the updating of beliefs is "standard" in the sense that consumers only take into account each product's own performance. If the old product succeeds in the first

[12] The probabilities consumers assign to the quality profiles other than $(g, g)$ are

$$\begin{aligned}
\mu_{gb} &= \frac{r(1-\gamma)x_{gb}}{r\gamma x_{gg} + r(1-\gamma)x_{gb} + (1-r)\gamma x_{bg} + (1-r)(1-\gamma)x_{bb}}, \\
\mu_{bg} &= \frac{(1-r)\gamma x_{bg}}{r\gamma x_{gg} + r(1-\gamma)x_{gb} + (1-r)\gamma x_{bg} + (1-r)(1-\gamma)x_{bb}}, \text{ and} \\
\mu_{bb} &= \frac{(1-r)(1-\gamma)x_{bb}}{r\gamma x_{gg} + r(1-\gamma)x_{gb} + (1-r)\gamma x_{bg} + (1-r)(1-\gamma)x_{bb}}.
\end{aligned}$$

period, for example, consumers update their belief that the old product is of high quality from $r$ to

$$\lambda_S(r) \equiv \frac{rg}{rg + (1-r)b}.$$

Similarly, if the new product fails, consumers revise their belief that it is of high quality from $\gamma$ to

$$\lambda_F(\gamma) \equiv \frac{\gamma(1-g)}{\gamma(1-g) + (1-\gamma)(1-b)}.$$

The updated belief after a failure of the old product is then $\lambda_F(r)$, and the updated belief following a success of the new product is $\lambda_S(\gamma)$.

Under umbrella branding, consumers update their first period beliefs $\mu^o$ and $\mu^n$ not only taking into account each product's own performance, but possibly also the brand's overall performance. Consider consumers' belief about the old product after a success of this product for example. If consumers ignored the new product's performance, then their second period belief would simply be $\lambda_S(\mu^o)$. Now suppose that the new product was successful, too, and that consumers indeed use this additional information. In that case, the belief consumers hold about the old product "before" taking account of the old product's own performance, is no longer $\mu^o$ but instead

$$\mu_S^o = \frac{\mu_{gg}g + \mu_{gb}b}{\left(\mu_{gg} + \mu_{bg}\right)g + (\mu_{gb} + \mu_{bb})b}.$$

The "final" belief consumers assign to the core product being good if both products were successful in the first period is then simply

$$\lambda_S(\mu_S^o) = \frac{\mu_{gg}g^2 + \mu_{gb}gb}{\mu_{gg}g^2 + \mu_{gb}gb + \mu_{bg}bg + \mu_{bb}b^2}.$$

The success of the new product has a positive *feedback effect* on the old product if and only if

$$\mu_S^o > \mu^o.$$

The beliefs consumers hold about the quality of the new product, or after observing different performance outcomes, can be obtained using the same hypothetical two-step procedure. First, the observation of product $-i$'s performance $\sigma_{-i}$ has a feedback effect on the belief about product $i$'s quality ($i \neq -i$), captured by the revision from $\mu^i$ to $\mu^i_{\sigma_{-i}}$.[13] Second, consumers update their

[13] In the cases ignored so far, the beliefs after the first step of the revision are

$$\begin{aligned}
\mu_F^o &= \frac{\mu_{gg}(1-g) + \mu_{gb}(1-b)}{\left(\mu_{gg} + \mu_{bg}\right)(1-g) + (\mu_{gb} + \mu_{bb})(1-b)}, \\
\mu_S^n &= \frac{\mu_{gg}g + \mu_{bg}b}{\left(\mu_{gg} + \mu_{gb}\right)g + (\mu_{bg} + \mu_{bb})b}, \text{ and} \\
\mu_F^n &= \frac{\mu_{gg}(1-g) + \mu_{bg}(1-b)}{\left(\mu_{gg} + \mu_{gb}\right)(1-g) + (\mu_{bg} + \mu_{bb})(1-b)}.
\end{aligned}$$

belief to take into account product $i$'s own performance $\sigma_i$, which amounts to a revision from $\mu^i_{\sigma_{-i}}$ to $\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}})$. The following table summarizes second period beliefs as a function of the pieces of information (umbrella versus separate branding, and the products' performances) consumers have:

| | $S$ of old, $S$ of new | $S$ of old, $F$ of new | $F$ of old, $S$ of new | $F$ of old, $F$ of new |
|---|---|---|---|---|
| umbrella brand | $\lambda_S(\mu^o_S), \lambda_S(\mu^n_S)$ | $\lambda_S(\mu^o_F), \lambda_F(\mu^n_S)$ | $\lambda_F(\mu^o_S), \lambda_S(\mu^n_F)$ | $\lambda_F(\mu^o_F), \lambda_F(\mu^n_F)$ |
| separate brands | $\lambda_S(r), \lambda_S(\gamma)$ | $\lambda_S(r), \lambda_F(\gamma)$ | $\lambda_F(r), \lambda_S(\gamma)$ | $\lambda_F(r), \lambda_F(\gamma)$ |

After any realization of performances, the effect of umbrella branding on the second period belief about each product can then be decomposed into two effects. Following two successes, for example, the impact of umbrella branding on the old product's reputation is

$$\lambda_S(\mu^o_S) - \lambda_S(r) = [\lambda_S(\mu^o_S) - \lambda_S(\mu^o)] + [\lambda_S(\mu^o) - \lambda_S(r)].$$

The first term between brackets is positive if and only if the success of the new product has a positive feedback effect on the old product. The second term is positive if and only if umbrella branding has a positive signaling effect on the old product.

**Quality Correlation and Feedback Effects** Calculating the correlation coefficient between the (prior) quality perceptions of umbrella branded products yields

$$\rho = \frac{\mu_{gg}\mu_{bb} - \mu_{gb}\mu_{bg}}{\sqrt{(\mu_{gg} + \mu_{gb}) - (\mu_{gg} + \mu_{gb})^2}\sqrt{(\mu_{gg} + \mu_{bg}) - (\mu_{gg} + \mu_{bg})^2}}. \tag{1}$$

This correlation is positive if and only if

$$\mu_{gg}\mu_{bb} > \mu_{gb}\mu_{bg}.$$

Relying on this simply condition, it is straightforward to check that for any $i \in \{o, n\}$,

$$\begin{aligned} \mu^i_S > \mu^i > \mu^i_F \quad &\text{if and only if} \quad \rho > 0, \\ \mu^i_S = \mu^i = \mu^i_F \quad &\text{if and only if} \quad \rho = 0, \\ \mu^i_S < \mu^i < \mu^i_F \quad &\text{if and only if} \quad \rho < 0. \end{aligned} \tag{2}$$

If consumers believe that the qualities of umbrella branded products are positively correlated ($\rho > 0$), then successes have positive feedback effects. Conversely, if $\rho < 0$, then the success of any one of the products has a negative feedback effect on the other product's reputation. For $\rho = 0$, there are no feedback effects at all.

### 3.2 The Impact of Umbrella Branding on Profits

Given beliefs, one can analyze the firms' incentive to use an umbrella brand by considering the marginal impact of umbrella branding on the expected aggregate profits. I denote by $\Delta_1$ the difference between the first period profits under umbrella branding and under separate branding:

$$\Delta_1 = (1-\beta)\left[w(\mu^o) - w(r)\right] + \beta\left[w(\mu^n) - w(\gamma)\right].$$

Similarly, $\Delta_2(\sigma_o, \sigma_n)$ denotes the (ex post) difference in second period profits between umbrella and separate branding, following the performance profile $(\sigma_o, \sigma_n)$:

$$\Delta_2(\sigma_o, \sigma_n) = (1-\beta)\left[w\left(\lambda_{\sigma_o}(\mu^o_{\sigma_n})\right) - w\left(\lambda_{\sigma_o}(r)\right)\right] + \beta\left[w\left(\lambda_{\sigma_n}(\mu^n_{\sigma_o})\right) - w\left(\lambda_{\sigma_n}(\gamma)\right)\right].$$

For example,

$$\Delta_2(S, F) = (1-\beta)\left[w\left(\lambda_S(\mu^o_F)\right) - w\left(\lambda_S(r)\right)\right] + \beta\left[w\left(\lambda_F(\mu^n_S)\right) - w\left(\lambda_F(\gamma)\right)\right].$$

Finally, $\Delta(q_o, q_n)$ denotes the total difference in expected profits between umbrella and separate branding, which is equal to the weighted sum of the first period and expected second period profit differences:

$$\Delta(q_o, q_n) = (1-\delta)\Delta_1 + \delta E[\Delta_2(\sigma_o, \sigma_n) \mid q_o, q_n], \tag{3}$$

where

$$\begin{aligned} E[\Delta_2(\sigma_o, \sigma_n) \quad \mid \quad q_o, q_n] &= q_o q_n \Delta_2(S,S) + q_o(1-q_n)\Delta_2(S,F) \\ &\quad + (1-q_o)q_n\Delta_2(F,S) + (1-q_o)(1-q_n)\Delta_2(F,F). \end{aligned}$$

The branding strategy $(x_{gg}, x_{gb}, x_{bg}, x_{bb})$ is optimal, i.e. maximizes aggregate profits, if and only if for all $(q_o, q_n)$,

$$x_{q_o q_n} = \begin{cases} 1 & \text{if } \Delta(q_o, q_n) > 0, \\ \text{any } \varepsilon \in [0,1] & \text{if } \Delta(q_o, q_n) = 0, \\ 0 & \text{if } \Delta(q_o, q_n) < 0. \end{cases} \tag{4}$$

**The Umbrella Branding Strategy and Quality Correlation** Let me now link the branding strategy to the correlation of consumers' quality perceptions. It is easy to check that

$$\rho > 0 \quad \text{if and only if} \quad x_{gg}x_{bb} > x_{gb}x_{bg}. \tag{5}$$

Note also that perfect positive correlation (i.e. $\rho = 1$) obtains for any branding strategy such that $x_{gb} = x_{bg} = 0$ but $x_{gg}, x_{bb} > 0$. Perfect negative correlation (i.e. $\rho = -1$), on the other hand, obtains for any branding strategy such that umbrella branding always involves products of opposite qualities, i.e. if $x_{gg} = x_{bb} = 0$ but $x_{gb}, x_{bg} > 0$.

**The Profit Impact of Signaling and Feedback Effects** To illustrate the impact of the signaling and feedback effects on profits, note first that in

$$\begin{aligned}\Delta_1 &= (1-\beta) \underbrace{[w(\mu^o) - w(r)]}_{\text{short run price impact of the } \textbf{signaling effect on the old product}} \\ &+\beta \underbrace{[w(\mu^n) - w(\gamma)]}_{\text{short run price impact of the } \textbf{signaling effect on the new product}},\end{aligned}$$

the first term is positive if and only if the signaling effect on the core product is positive, and the second term is positive if and only if the signaling effect on the extension product is positive.

The expected second period profit difference can be decomposed as follows:

$$\begin{aligned}E[\Delta_2(\sigma_o,\sigma_n) \mid q_o, q_n] = \\ &(1-\beta) \underbrace{E\left[w\left(\lambda_{\sigma_o}(\mu^o_{\sigma_n})\right) - w\left(\lambda_{\sigma_o}(\mu^o)\right) \mid q_o, q_n\right]}_{\text{expected price impact of the } \textbf{feedback effect on the old product}} \\ &+(1-\beta) \underbrace{E\left[w\left(\lambda_{\sigma_o}(\mu^o)\right) - w\left(\lambda_{\sigma_o}(r)\right) \mid q_o\right]}_{\text{expected long term price impact of the } \textbf{signaling effect on the old product}} \\ &+\beta \underbrace{E\left[w\left(\lambda_{\sigma_n}(\mu^n_{\sigma_o})\right) - w\left(\lambda_{\sigma_n}(\mu^n)\right) \mid q_o, q_n\right]}_{\text{expected price impact of the } \textbf{feedback effect on the new product}} \\ &+\beta \underbrace{E\left[w\left(\lambda_{\sigma_n}(\mu^n)\right) - w\left(\lambda_{\sigma_n}(\gamma)\right) \mid q_n\right]}_{\text{expected long term price impact of the } \textbf{signaling effect on the new product}}\end{aligned}$$

Since $\lambda_S(\cdot)$ and $\lambda_F(\cdot)$ are both increasing, the "expected long term price impact of the signaling effect on the old product" is positive if and only if $\mu^o > r$. Similarly, for the new product, the expected long term price impact of the signaling effect is positive if and only if $\mu^n > \gamma$. The signs of these effects are hence independent of the concerned products' quality; their magnitudes, however, generally vary with quality.

As already explained in the section on beliefs, given the sign of the correlation coefficient $\rho$, it is easy to assess the signs of the two feedback effects ex post. For $\rho > 0$, for example, a success of the extension product has a positive feedback effect on the (price of the) core product. The following lemma shows that knowledge of the quality of product $-i$ is sufficient to also assess the "*expected* price impact of the feedback effect on product $i$":

**Lemma 1** • *For $\rho > 0$, the expected price impact of the feedback effect on product $i \in \{o, n\}$ is positive if and only if product $-i \neq i$ is good. Formally, for any $q_o \in \{b, g\}$:*

$$E\left[w\left(\lambda_{\sigma_o}(\mu^o_{\sigma_n})\right) - w\left(\lambda_{\sigma_o}(\mu^o)\right) \mid q_o, q_n\right] > 0 \textit{ if and only if } q_n = g,$$

*and for any $q_n \in \{b, g\}$:*

$$E\left[w\left(\lambda_{\sigma_n}(\mu^n_{\sigma_o})\right) - w\left(\lambda_{\sigma_n}(\mu^n)\right) \mid q_o, q_n\right] > 0 \textit{ if and only if } q_o = g.$$

- *For $\rho < 0$, the expected price impact of the feedback effect on product $i \in \{o, n\}$ is positive if and only if product $-i \neq i$ is bad.*

In some instances, lemma 1 will prove helpful in evaluating the total impact of umbrella branding on profits: the lemma implies for example that if (i) $\rho > 0$ and (ii) both signaling effects are non-negative, then $\Delta(gg) > 0$.

# 4 Equilibria without Feedback Effects

There are potentially two kinds of equilibria without feedback effects (i.e. such that $\rho = 0$): 'babbling' equilibria in which umbrella branding does not affect prices at all, and equilibria in which it allows consumers to *fully* infer the quality of (at least) one of the products. In the latter case, since consumers never revise their belief about one of the products after the beginning of the first period, performances cannot induce any feedback effects (in either direction).

### 'Babbling' Equilibria

**Proposition 1** *There always exist 'babbling' equilibria in which umbrella branding occurs but has no effect on beliefs.*

*Moreover,* ***only*** *babbling equilibria exist if - ceteris paribus - one of the following conditions is satisfied:*

1. *The markets for the old and for the new product are too asymmetric, i.e. $\beta$ is too close to* 0 *or to* 1.

2. *The consumers' prior about one of the products is already very accurate, i.e. $r$ is too close to* 1 *or to* 0*, or $\gamma$ is too close to* 1 *or to* 0.

3. *Firms are too impatient, i.e. $\delta$ is too close to* 0.

4. *Quality differences are too small, i.e. $b$ is too close to $g$.*

The existence of babbling equilibria is standard in signaling games. If, for example, the firms choose an umbrella brand whenever the opportunity arises, then brands reveal no information about qualities. This in turn implies that the firms are indeed indifferent to the choice between umbrella branding and separate branding.

Moreover, for umbrella branding to credibly affect beliefs and thus be more than just babbling, firms with products of different qualities must have different incentives to influence beliefs. This requires that profits are sufficiently responsive to the performances of both products. A prerequisite for this is of course that beliefs are sufficiently responsive to performances.

It is easy to see why this cannot be true in the last two situations treated in the proposition. For $b$ too close to $g$, beliefs hardly respond to performance observations. If $\delta$ is too close to 0, on the other hand, then intertemporal profits are not at all responsive to performances, as future profits do not receive any weight.

If consumers are already very well-informed about the quality of one of the products (situation 2.), then (i) umbrella branding cannot affect the price of that product in either the short or the long run, and (ii) beliefs almost do not respond to that product's performance. This means that the signaling effect on the other product will drive the branding decision. Since the sign of that effect is the same for all quality profiles, however, there is no way to induce certain types to umbrella brand with a higher probability than other types, which would be necessary to create such a signaling effect in the first place.

Finally, if markets are very asymmetric, then branding decisions almost exclusively depend on the profit impact on one single product. Suppose for example that the market for the extension product is very small compared to that for the core product. Then, the performance of the extension product mainly matters because of its feedback effect on the core product. The core product's performance, on the other hand, is mainly important with respect to its impact on the *size* of the signaling effect on the core product. Performances thus have two instead of four relevant effects on profits, which, as shown in the proof of proposition 1, does not suffice to sustain a non-babbling equilibrium.

Hence, for non-babbling equilibria to exist, it is necessary that (i) firms sufficiently care about the profits made from selling both the old and the new product, as well as about future profits, (ii) consumers are relatively ill-informed about both products' qualities at the moment of the brand extension, and (iii) potential quality differences are significant.

### Equilibria with Full Revelation

**Proposition 2** *There does not exist any equilibrium in which umbrella branding fully reveals the high quality of the old and/or the new product.*

Firms would clearly like consumers to believe that both products are of high quality. However, even firms with one or several bad products want consumers to hold such beliefs, and the certification provided by umbrella branding would actually be more valuable for such firms than for those with good products: since under separate branding consumers are more likely to revise their beliefs downwards when products are bad rather than good, the "outside option" of separate branding is less attractive the higher the number of bad products. As a result, there cannot be any equilibrium such that only good products are sold under umbrella brands. By the same reasoning, umbrella branding cannot be used to certify the quality of only one of the products on offer. Note that this result relies on the fact that performances are always imperfect signals of qualities. Consumers who are completely convinced that a product is good therefore do not revise their beliefs downwards even after observing a failure.

For the sake of completeness, note that there may exist non-babbling equilibria without feedback effects in which the decision to umbrella brand reveals the bad quality of one product while signaling that the other product is good with a high probability:

**Proposition 3** *There may exist mixed strategy equilibria such that* $\rho = 0$ *but umbrella branding affects beliefs. Any such equilibrium must be of the form* $x_{gg} = 0$*,* $\min[x_{gb}, x_{bg}] = 0$*,* $\max[x_{gb}, x_{bg}] > 0$*, and* $x_{bb} \in (0, \max[x_{gb}, x_{bg}])$*.*

# 5 Equilibria with Feedback Effects

There are two kinds of equilibria with feedback effects. First, equilibria with positive quality correlation (i.e. such that $\rho > 0$) in which the success (failure) of any one of the products has a positive (negative) feedback effect on the other product. Second, equilibria with negative correlation (i.e. such that $\rho < 0$) in which failures (successes) have positive (negative) feedback effects. In this section, I first discuss some general characteristics of equilibria with positive quality correlation, and then show that equilibria with perfect quality correlation exist in some situations. Next, it is shown that there are no equilibria with negative quality correlation.

## 5.1 Equilibria with Positive Quality Correlation

### 5.1.1 General Discussion

In any equilibrium with positive quality correlation, i.e. such that $\rho > 0$, umbrella branding must be profitable for firms with products of uniform quality. Hence, the following two conditions must be met:

$$\Delta(g, g) \geq 0, \text{ and } \Delta(b, b) \geq 0. \tag{6}$$

Intuitively, the condition $\Delta(g, g) \geq 0$ is easy to satisfy. If $\rho > 0$ in equilibrium, then feedback effects have a positive expected long term impact for firms with only good products (see lemma 1). Hence, as long as signaling effects are not "too" adverse, umbrella branding is profitable for firms with two good products.

It seems more difficult to induce firms with two bad products to umbrella brand, since the expected total impact of the feedback effects is negative in that case. For umbrella branding to be attractive nonetheless, its signaling effect on at least one of the products must therefore be positive. *Thus, in any equilibrium such that* $\rho > 0$*, umbrella branding must have a positive signaling effect on the core product and/or the extension product.*

For positive quality correlation to arise, it is also necessary that (at least some) firms with products of differing qualities do not have any strict incentive to use umbrella brands, i.e. that

$$\Delta(g, b) \leq 0, \text{ or/and } \Delta(b, g) \leq 0. \tag{7}$$

Intuitively, it seems difficult to reconcile any one of these conditions with $\Delta(b, b) \geq 0$: failures have negative feedback effects here, and failures are most likely when both products are bad. To explain why firms with products of differing qualities may nevertheless have lower incentives to use umbrella brands than firms with only bad products, one needs to consider the *sizes* of these expected negative feedback effects. To fix ideas, let me focus on candidate equilibria in which $x_{gb} < 1$, which requires that $\Delta(g, b) \leq 0$. In any such equilibrium, it must be that

$$\Delta(b, b) \geq \Delta(g, b). \tag{8}$$

This condition is equivalent to

$$E\left[\Delta_2(\sigma_o, \sigma_n) \mid b, b\right] \geq E\left[\Delta_2(\sigma_o, \sigma_n) \mid g, b\right],$$

or

$$\begin{aligned} & bE\left[\Delta_2(S, \sigma_n) \mid q_n = b\right] + (1-b)E\left[\Delta_2(F, \sigma_n) \mid q_n = b\right] \\ \geq \quad & gE\left[\Delta_2(S, \sigma_n) \mid q_n = b\right] + (1-g)E\left[\Delta_2(F, \sigma_n) \mid q_n = b\right]. \end{aligned}$$

Clearly, the latter condition holds if and only if

$$E\left[\Delta_2(F, \sigma_n) \mid q_n = b\right] \geq E\left[\Delta_2(S, \sigma_n) \mid q_n = b\right]. \tag{9}$$

Hence, given the new product is bad, umbrella branding must be less attractive in comparison to separate branding if the old succeeds than if it fails. How is this possible, given that a success of the old product has a positive feedback effect on the new product under umbrella branding?[14] If $q_n = b$, then the new product is likely to fail, which induces a negative feedback effect under umbrella branding. The key point is that this negative feedback effect from the new on the old product may be stronger if the old product succeeds than if it fails. If the old product's failure already gives consumers a strong indication of low quality, then the new product's failure just provides some additional evidence pointing into the same direction. If the old product succeeds, however, then the negative feedback effect may severely damage the old product's reputation. Intuitively, this can be the case if consumers expect a strong quality correlation and a failures are very strong indications of low quality, while successes leave open the possibility that the product is bad. Separate branding may then be attractive for mixed quality firms because it allows them to "protect" the reputation of the good product.

[14] Any success (failure) also has a positive (negative) direct effect on the product concerned itself. The branding decision can have an impact on the size, but not the sign, of this effect. My discussion in the main text focuses on feedback effects instead, since these are key to understanding endogenous quality correlation.

### 5.1.2 Pure Strategy Equilibria with Perfect Quality Correlation

The following proposition shows that for $g$ sufficiently close to 1, there always exists a non-empty set of values of the other parameters such that in equilibrium umbrella branding signals that products are of uniform quality:

**Proposition 4** *There exist thresholds* $\bar{b}(r,\gamma,\beta) \in (0,1)$*,* $\underline{\delta}(r,\gamma,\beta,b) \in (0,1)$ *and* $\bar{\delta}(r,\gamma,\beta,b) \in (\underline{\delta}(\cdot),1)$ *such that for* $g$ *sufficiently close to* 1 *a pure strategy equilibrium with* $\rho = 1$ *exists if*

**i)** *the parameters* $r$*,* $\gamma$ *and* $\beta$ *are such that, given the branding strategy* $(1,0,0,1)$*, the decision to umbrella brand increases short term profits:*

$$\begin{aligned} \Delta_1 &> 0 \\ &\leftrightarrow \\ \frac{r\gamma}{r\gamma+(1-r)(1-\gamma)} &> (1-\beta)r+\beta\gamma. \end{aligned}$$

**ii)** *quality differences are sufficiently large:*

$$b \in \left(0, \bar{b}(r,\gamma,\beta)\right), \text{ and}$$

**iii)** *firms care both about present and future profits:*

$$\underline{\delta}(r,\gamma,\beta,b) < \delta < \bar{\delta}(r,\gamma,\beta,b).$$

The intuition for this result is as follows. Suppose that consumers indeed believe that umbrella brands sell products of uniform quality. Then, umbrella branding is a risky decision for an incumbent-innovator pair with one good and one bad product. In the likely scenario that the good product succeeds and the bad product fails, the failure of the bad product has a negative feedback effect on the good product. If failure is a strong indication of bad quality (i.e. if $g$ is close to 1), then this negative effect dominates the positive feedback effect arising from the good product's success. In the limit, for $g$ equal to 1, consumers will always conclude that both products are bad. Under separate branding, on the other hand, a failure only destroys the reputation of the failing product itself, not that of the other product. Therefore, $\Delta_2(F,S)$ and $\Delta_2(S,F)$ are negative for large enough $g$.

If both products are bad, on the other hand, two failures are (relatively) more likely. For sufficiently large $g$, two failures (almost fully) convince consumers that both products are of bad quality, both under umbrella and under separate branding. Indeed, as $g$ approaches 1, the ex post profit difference completely vanishes: $\lim_{g\to1}\Delta_2(F,F) = 0$. Hence, there is not much to lose in the long run for firms with two bad products. The driving force behind the branding incentives of a $(b, b)$-type will then be signaling effects. In particular, if $\Delta_1$ is sufficiently large, the $(b, b)$-type will want to umbrella brand to exploit short-term profit opportunities, in spite of possible negative feedback effects in the future. In other words, if both products are bad but the prior beliefs $r$ and $\gamma$ are high enough so that $\Delta_1 > 0$,[15] then it is optimal to umbrella brand to maximally exploit the existing reputation in the short run. Firms with products of different qualities, on the other hand, are more willing to forego short term profit gains in order to preserve and continue building the good reputation of one of their products.

The condition that $g$ is close to 1 ensure that failures are sufficiently strong indications of low quality. The remaining three conditions in proposition 4 then play the following roles. Condition ii) ensures that bad products are sufficiently likely to fail. Hence, firms with products of differing qualities are indeed very likely to experience one success and one failure, and lose profits in the long term by opting for an umbrella brand. For firms with two bad products, on the other hand, two failures are sufficiently more likely, so that the long term effect of umbrella branding is negligible. Condition i) guarantees that $\Delta_1 > 0$, so that for sufficiently low discount factors, umbrella branding is nonetheless profitable for firms with two bad products. However, for too low discount factors, umbrella branding would also be profitable for firms with products of differing qualities; condition iii) is therefore needed to ensure that the discount factor lies in an intermediate range.

Finally, for firms with two good products, umbrella branding is very profitable. For $g$ close to 1, two successes are almost certain for these firms. Hence, umbrella branding not only leads to a short term gain, but also allows firms with two good products to consolidate the reputations of their products faster than otherwise.

**Numerical Example** Figure 1 illustrates equilibria with perfect quality correlation. Figure 1a) shows for which values of $r$ and $b$ perfect correlation equilibria exist if $g$ is almost equal to 1 ($g = 0.99999999999999$), $\gamma = 0.4$, and the different markets are symmetric, i.e. $\beta = \frac{1}{2}$ and $\delta = \frac{1}{2}$. In this figure, umbrella branding is profitable for the $(q_o, q_n)$-type if and only if $r$ lies above the indifference curve along which $\Delta(q_o, q_n) = 0$. Moreover, umbrella branding increases profits in the first period if and only if $r$ lies above the threshold $\underline{r}(0.4, 0.5) = 0.6$ for which $\Delta_1 = 0$.[16] I can observe first that for any $r > 0.6$, it is indeed optimal for the $(g, g)$-type to umbrella brand. This is intuitive. For $g$ close to 1, two successes in the first period are almost certain if both products are good, and successes have positive feedback effects under umbrella branding. As long as signaling effects are not too negative, umbrella branding is therefore profitable.

Consider the indifference curve defined by $\Delta(b, b) = 0$ now. If bad products are likely to fail,

[15] It can be shown that $\Delta_1 > 0$ here if, given $\gamma$ and $\beta$, the prior $r$ is sufficiently high.

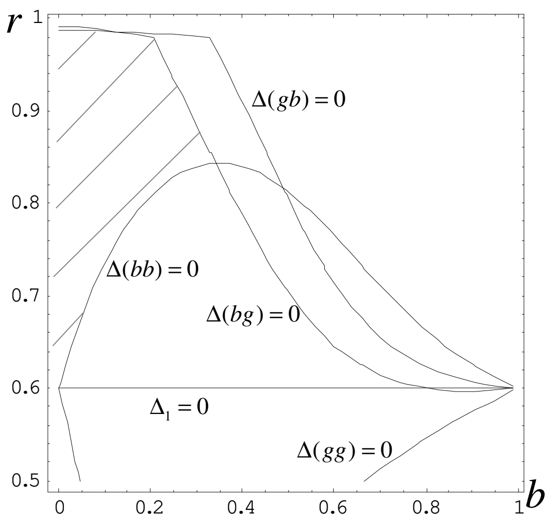
[16] In fact, whenever $\beta = \frac{1}{2}$, the strategy is pure and such that $\rho > 0$, then $\Delta_1 > 0$ if and only if $r > 1 - \gamma$.

i.e. $b$ is small, then the $(b, b)$-type is likely to experience two failures. Since $\lim_{g \to 1} \Delta_2(F, F) = 0$, umbrella branding is then profitable for firms with two bad products whenever $\Delta_1 > 0$. As $b$ rises, it becomes more probable that one product will fail and one succeed, which renders umbrella branding less attractive. If $r$ is not sufficiently high so that a large $\Delta_1$ can offset this expected loss,[17] then firms with two bad products no longer want to umbrella brand. Finally, for sufficiently large $b$, umbrella branding is more profitable again, since two successes become a likely scenario. Moreover, as $b$ approaches $g$, failures induce less and less updating.

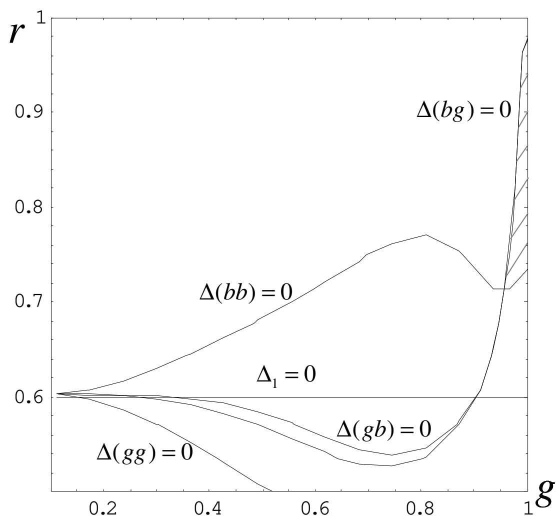
For firms with one good and one bad product, on the other hand, umbrella branding is unprofitable if the bad product is likely to fail, i.e. $b$ is small. As explained above, the negative feedback effect of a failure of the bad product would lead to a profit loss in the second period. For small $b$, this second period effect is so strong that even if umbrella branding increases profits in the first period, it may still lower the total expected profits for firms with one good and one bad product. As $b$ rises however, umbrella branding becomes more and more profitable for any given $r$. Conversely, for any given $b$, umbrella branding is profitable only if $r$ is sufficiently large. Equilibria with perfect correlation then exist for $b$ and $g$ in the highlighted area that lies above the indifference curve defined by $\Delta\left(b, b\right) = 0$ but below the lower envelope of the indifference curves corresponding to $\Delta\left(b, g\right) = 0$ and $\Delta\left(g, b\right) = 0$.

Figure 1b) depicts the same indifference curves, except that the value of $b$ is now fixed at 0.1 while $g$ varies. For any $(q_o, q_n)$, umbrella branding is again profitable if and only if $r$ lies above the indifference curve defined by $\Delta(q_o, q_n) = 0$. Thus, perfect correlation equilibria only exist if $g$ is sufficiently large in this example. This suggests that the focus on the case where $g$ is close to 1 is justified. Figure 1c) shows for which combinations of $b$ and $\beta$ perfect correlation equilibria exist: as expected in the light of the discussion about market symmetry in section 4.1, equilibria only exist for $\beta$ in an intermediate range. In figure 1d), umbrella branding is profitable for the type $(q_o, q_n)$-type if and only if $r$ lies below the indifference curve defined by $\Delta(q_o, q_n)$. The figure shows that ceteris paribus equilibria only exist for $\delta$ in an intermediate range, as required in condition (iii) of proposition 4.

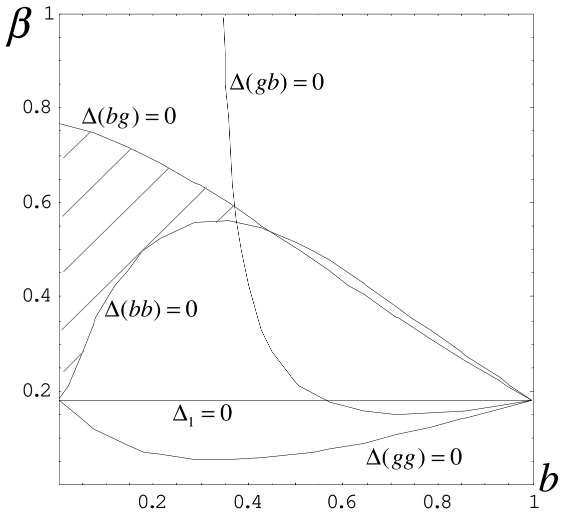
[17] In this example, the first period profit difference $\Delta_1$ is increasing in $r$, but this need not always be the case.



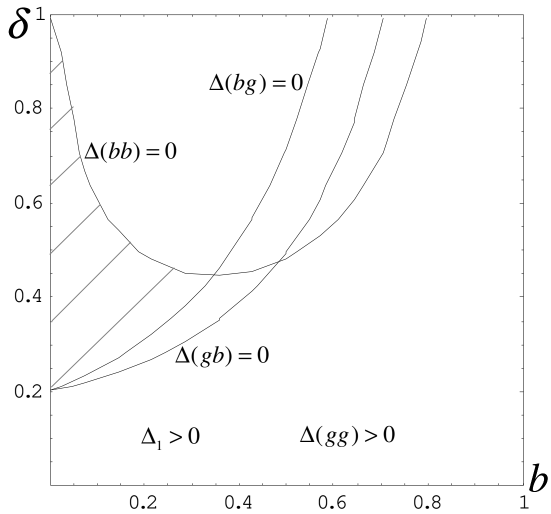

1a): $\gamma = 0.4$, $g \simeq 1$, $\beta = \delta = 0.5$




1b): $\gamma = 0.4$, $b = 0.1$, $\beta = \delta = 0.5$




1c): $r = 0.8$, $\gamma = 0.4$, $g \simeq 1$, $\delta = 0.5$




1d): $r = 0.8$, $\gamma = 0.4$, $g \simeq 1$, $\beta = 0.5$

Figure 1: Pure Strategy Equilibria with Perfect Quality Correlation

### 5.1.3 Mixed Strategy Equilibria with Perfect Quality Correlation

For $g$ sufficiently close to 1, there may also exist mixed strategy equilibria such that $\rho = 1$. In any such equilibrium, $x_{gg} = 1$ however. This is true because in the proof of proposition 4, I did not rely on the purity of the equilibrium strategy to show either that $\Delta_1 > 0$ is a necessary condition for an equilibrium with perfect correlation, or that $\Delta_1 > 0$ implies $\lim_{g \to 1} \Delta(g, g) > 0$. Any mixed equilibrium strategy must thus be of the form $(1, 0, 0, x_{bb})$ where $x_{bb} \in (0, 1)$. This result has a similar flavor to the finding of Cabral (2000) that, under the assumption of exogenous perfect quality correlation, umbrella branding incentives increase in quality.

The conditions under which a mixed strategy equilibrium of the form $(1, 0, 0, x_{bb})$ indeed exists are then the following. First, the necessary condition $\Delta_1 > 0$ is satisfied if and only if $r$ exceeds some lower bound that now depends not only on $\gamma$, $\beta$, but also on $x_{bb}$. If moreover $b$ lies below some upper bound that depends on $r, \gamma, \beta$, and $x_{bb}$, then there exists a unique discount factor such that $\Delta(b, b) = 0$ and all the other equilibrium conditions are satisfied.

Ceteris paribus, the decision to umbrella brand signals higher quality the lower $x_{bb}$. This implies that for any mixed strategy $(1, 0, 0, x_{bb})$, the indifference curves $\Delta(q_o, q_n) = 0$ must lie below those depicted in figure 1a) for the pure strategy $(1, 0, 0, 1)$. Moreover, mixed strategy equilibria can only occur for $(b, r)$ that lie *on* the indifference curve $\Delta(b, b) = 0$. Therefore, even if $r < 0.6$ in the numerical example, there can exist (mixed strategy) equilibria with perfect quality correlation provided that $b$ is sufficiently low. For $r$ so high that no pure strategy equilibrium exists, on the other hand, there cannot be any mixed strategy equilibrium either. Note also that equilibria in which $x_{bb}$ is very small so that umbrella branding almost fully reveals high quality only exist for very small $r$ in the example analyzed here.

Finally, note that for some parameter values, there may also exist (pure and mixed strategy) equilibria with *imperfect* positive quality corrlation.

### 5.2 Inexistence of Equilibria with Negative Quality Correlation

Whenever $\rho < 0$, successes (failures) have negative (positive) feedback effects. This is clearly counter-intuitive. Moreover, the available empirical evidence strongly indicates the opposite. Erdem (1998) for example estimates a correlation coefficient of 0.882 between the prior perceptions consumers have about the qualities of toothbrushes and toothpastes sold under umbrella brands.[18]

To exclude equilibria with negative quality correlation, I first rule out situations in which the umbrella branded products are always of opposite qualities:

**Proposition 5** *There does not exist any equilibrium in which $\rho = -1$.*

The inuition behind this result is the following. In the case of perfect negative correlation, umbrella branding convinces consumers that one product is good and one product is bad, without them knowing which product is the good one. If the firms indeed opt for an umbrella brand, the success of product $i$ then has a positive direct impact on consumers' belief about $i$'s quality that is exactly *equal* to its negative feedback effect on consumers' belief about product $-i$. Under separate branding, on the other hand, successes have positive direct effects but no negative

[18] See section 6.1 for more empirical evidence.

feedback effects. This implies that firms with two bad products have a higher incentive to umbrella brand than either firms with a good old and a bad new product and/or firms with a bad old and good new product.[19] In other words, umbrella branding is particularly attractive for firms with two bad products, since (i) the outside option of separate branding is less attractive for the $(b, b)$-type than for any other type, and (ii) successes, which are more likely to be experienced by firms with one good product, induce large negative feedback effects under umbrella branding.

Therefore it seems impossible to keep firms with two bad products from umbrella branding in any candidate equilibrium with negative quality correlation. I am able to show, however, that for $x_{bb}$ too large (in particular for $x_{bb} = 1$), there does not exist any equilibrium such that $\rho < 0$ either. The reason is that signaling effects clearly decrease in $x_{bb}$; and it is impossible to offset this stigmatization of umbrella brands by means of a high $x_{gg}$ without inducing a positive quality correlation. If $x_{bb} \geq \max[x_{gb}, x_{bg}]$, then the stigmatization becomes so strong that umbrella branding would lower both first and second period prices (after any history of performances) and therefore be unprofitable for all firms. Thus:

**Lemma 2** *There does not exist any equilibrium such that $\rho < 0$ and*

$$x_{bb} \geq \max[x_{gb}, x_{bg}].$$

Lemma 2 implies that $x_{bb} < 1$ is a necessary condition for an equilibrium with negative quality correlation. Since by lemma 1 the expected price impacts of the feedback effects are positive if both products are bad and $\rho < 0$, this requires that at least one of the signaling effects must be negative in equilibrium, otherwise $\Delta(b, b) > 0$. For at least one of the signaling effects to be negative in turn, $x_{gg}$ must not be too large:

**Lemma 3** *There does not exist any equilibrium such that $\rho < 0$ and*

$$x_{gg} \geq \min\left[x_{bg} - \frac{1-\gamma}{\gamma}\left(x_{gb} - x_{bb}\right), x_{gb} - \frac{1-r}{r}\left(x_{bg} - x_{bb}\right)\right].$$

Lemma 3 implies that there cannot be any equilibrium in which $x_{gg} = 1$: by lemma 2, $x_{bb} < \max[x_{gb}, x_{bg}]$ is a necessary condition for an equilibrium with negative correlation, which implies that the threshold of $x_{gg}$ in lemma 3 cannot exceed 1. The following corollary therefore follows directly from proposition 5, lemma 2 and lemma 3:

**Corollary 1** *There does not exist any pure strategy equilibrium in which $\rho < 0$.*

These analytical results leave open the possibility that, for some parameter values, there may exist mixed strategy equilibria with negative quality correlation in which $x_{bb} \in (0, 1)$. On

[19] The relative importance of the different products, i.e. $\beta$, determines whether $\Delta(b, b) > \Delta(g, b)$ or $\Delta(b, b) > \Delta(b, g)$ or both. For $\beta = \frac{1}{2}$, both inequalities hold.

the basis of numerical simulations, however, I am confident that such mixed strategy equilibria do not exist either.

## 6 Extension: Endogenous Choice of the New Product's Quality

This section considers a model extension that endogenizes the new product's quality. The key difference with respect to the previous setup is that before launching the new product, the innovator must decide whether to make a one-time investment to ensure that $q_n = g$; absent the investment, $q_n = b$. The cost $c$ of the quality-improving investment is a random draw from the uniform distribution with support $[0, \beta(g-b)]$, where the upper bound, $\beta(g-b)$, is chosen so as to equal the profit (and welfare) gain from investment under symmetric information.

The timing is such that the innovator[20] finds out whether umbrella branding with the incumbent's product is feasible or not *before* the investment decision stage.In case umbrella branding is an option, the innovator and the incumbent make the investment and branding decisions simultaneously so as to maximize expected aggregate profits, given $q_o$ and the realized investment cost $c$, both known to both firms.[21] If umbrella branding is not an option, then the innovator makes the investment decision so as to maximize the expected profit from selling the new product only. Consumers cannot observe investment decision and product qualities, but are aware of the distribution of the investment cost.

In this more complex game, a strategy consists of (i) an investment decision rule, as a function of the cost realization $c$, in case the innovator remains unmatched, and (ii) a rule determining both the investment and the umbrella branding decision, as a function of $c$ and $q_o$, in case of a match. A (Bayesian) equilibrium is again a situation in which the strategy maximizes expected aggregate profits, and consumer beliefs are Bayesian consistent with the strategy.

Since the investment cost is random, any strategy induces three investment probabilities: one for an innovator that is not matched with an incumbent, one for an innovator matched with an incumbent of quality $q_o = g$, and one for an innovator matched with an incumbent of quality $q_o = b$. I will denote these probabilities by $i$, $i_g$, and $i_b$, respectively. As before, perfect positive quality correlation means that $\mu_{gg}, \mu_{bb} > 0$ while $\mu_{gb} = \mu_{bg} = 0$, where these beliefs now depend on the anticipated joint investment/branding decision.

**Proposition 6** *Suppose an equilibrium in which umbrella branding signals perfect quality cor-*

[20] Along the lines of the remark concluding section 2, the model could again be reinterpreted as one where a single firm with two products makes both decisions.

[21] Results would remain unchanged if the matched firms made the investment decision prior to the umbrella branding decision.

*relation exists. Then,*

$$i_b \leq i \leq i_g,$$

*and* $0 < i < 1$ *in this equilibrium.*

Proposition 6 shows that in any equilibrium with perfect positive quality correlation, if it exists, having the option to use an umbrella brand affects the optimal investment decision. In particular, the umbrella branding option introduces a tendency to align the new product's quality to that of the existing product. In fact, $i_b < i$ whenever firms with $q_o = b$ that do not invest *strictly* prefer separate branding. Similarly, $i < i_g$ whenever firms with a good old product that invest *strictly* prefer umbrella branding.

The intuition is straightforward. Treating perfect quality correlation as a given, firms with the option to umbrella brand have to decide between (i) selecting quality $q_n = q_o$ and umbrella branding, or (ii) $q_n \neq q_o$ and separate branding. Moroever, in equilibrium the branding decision has to remain optimal when the (simultaneous) investment decision is taken as given.[22] Firm(s) that can umbrella brand a good old product then have a higher investment incentive than a stand-along innovator, because umbrella branding increases the value of the investment beyond what a stand-along innovator could expect. For firms with a bad old product, the situation is exactly the reverse: the option to umbrella brand, chosen only if the investment is not made, increases the expected relative value of having two bad products.

Given the existence result for equilibria with perfect quality correlation in the baseline model (see proposition 4), it is not suprising that in the extended model there exist parameter values for which perfect quality correlation arises in equibrium; in particular, this is again the case if failures are sufficiently strong indications of low quality, i.e., for $g$ close enough to 1. Consider the following example:

**Numerical Example** For the parameter values $r = 0.65$, $\delta = \beta = 0.5$, $g = 0.999999$, and $b = 0.2$, there exists a (Bayesian) equilibrium, in which the firms' strategy is as follows:

- If $q_o = g$, then invest in the quality of the new product *and* umbrella brand if $c < \widehat{c}_g \simeq 0.27$; otherwise, do not invest *and* sell under separate brands.
- If $q_o = b$, then invest in the quality of the new product *and* sell under separate brands if $c < \widehat{c}_b \simeq 0.14$; otherwise, do not invest *and* sell under an umbrella brand.
- If you do not have the option to umbrella brand, then invest in the quality of the new product if $c < \widehat{c} \simeq 0.15$.

[22] If, given the investment decision, firms had an incentive to opt for a different branding decision ex post, then their (joint investment and branding) strategy would clearly not be optimal ex ante.

The corresponding equilibrium investment probabilities are $i_g \simeq 0.68$, $i \simeq 0.37$, and $i_b \simeq 0.35$. In expectation, firms that have an umbrella branding option invest more often than stand-alone firms:

$$ri_g + (1-r)\,i_b \simeq 0.56 > i \simeq 0.37.$$

Finally, for these parameter values, this is not only the unique pure strategy equilibrium with $\rho = 1$, but also the only pure strategy equilibrium *tout court*.

# 7 Discussion

## 7.1 Comparison with Alternative Theories of Umbrella Branding

It is useful to compare the proposed model to alternative theories of umbrella branding in order to explain the roles played by different assumptions, and to argue for what kinds of products my approach seems most appropriate.

Wernerfelt (1988) was the first to show that umbrella branding can serve as a perfect signal of product quality. He assumes (i) that umbrella branding is more costly than separate branding, and (ii) that failures are perfect signals of low quality. His main result is that under some conditions the unique equilibrium that survives the intuitive criterion of Cho and Kreps (1987) is such that a firm uses an umbrella brand if and only if all products are of high quality. Using the same two assumptions, Hakenes and Peitz (2008) analyze a game where a monopolist selects not only the branding strategy but also both products' qualities before consumers make any purchases. They find that both equilibria with positive and with negative quality correlation may exist.[23] Applying a forward induction argument, they argue however that the unique pure strategy equilibrium is one where firms invest in the high quality of both products and use an umbrella brand.[24]

My analysis highlights that if one of this literature's assumptions, namely that failures are perfect indications of low quality, is weakened only slightly, then umbrella branding can no longer fully reveal high quality (proposition 2). Umbrella branding can signal positive quality correlation however. The empirical literature confirms this prediction. In an empirical study of the branding of toothpaste and toothbrushes based on scanner data, Erdem (1998) finds that (i) consumers indeed remain uncertain about quality (here, the cavity-fighting ability) even after repeated consumption, and (ii) "*...though consumers perceive quality levels of umbrella brands as correlated across product categories, which makes it easier for strong umbrella brands to introduce*

[23] This result applies to the basic model in which products are symmetric. In the asymmetric case, the authors restrict attention to equilibria with non-negative correlation.

[24] Hakenes and Peitz (2008) also show that applying Pareto-dominance as an equilibrium selection criterion would lead to the same result, even if umbrella branding is cost neutral.

*new products, umbrella branding is not a guarantee for successful extensions*". A key contribution of my analysis to the existing literature is hence to move the emphasis from guaranteeing high quality to signaling positive quality correlation. Moreover, I show that whenever umbrella branding signals positive quality correlation, firms that already sell high quality products have higher inventives to invest in new products' quality than firms without any existing products, and - to an even larger extent - than firms that already sell low quality products.

To refine the equilibrium, and thus justify the focus on situations in which unbrella brands reveal high quality, Wernerfelt (1988) relies on the assumption that umbrella branding is costly. The motivation for this assumption is that umbrella brands may "confuse" consumers by diluting the brand's identity in the product space.[25] While this argument is probably relevant for products that consumers strongly identify with their category (such as *Tempo* handkerchiefs in Germany), there are also a number of reasons to believe that - absent signaling considerations - umbrella branding may be less costly than the introduction of a new brand. First of all, I would expect "direct" development costs (for package design etc.) to be lower for umbrella branded products. Second, as Pepall and Richards (2002) argue, extensions may also be a way to fully exploit a brand's intrinsic value, e.g., the status it confers to consumers. Consumers may then buy branded products even if they expect these to be of relatively low quality. Grossman and Shapiro (1988) point out that this explains why consumers are willing to pay for counterfeited Gucci or Louis Vuitton bags. Third, if firms must invest in advertising simply to make consumers aware of the existence of a new brand, an issue ignored in my analytical model, then there are clearly high costs associated with separate branding. These points suggest that umbrella branding may well be equally or even less costly than the use of separate brands. In a recent book addressed to managers, Aaker (2004, p.213/214) even states that "*The development of a new brand (or the continued support of an existing separate brand) is expensive and difficult. ...Thus, a separate brand should be developed or supported only when a compelling need can be demonstrated.*".

Cabral (2000) considers a (continuous) model in which umbrella branding is cost neutral, and performances are always imperfect signals of quality, as in my analysis; however, he assumes exogenous (perfect) quality correlation. The structure of his model is also simpler: consumers only buy the extension product at $t = 1$, and only the core product at $t = 2$.[26] Therefore, umbrella branding has at most three different effects: a signaling effect on the new product (called "direct reputation effect" by Cabral), a "signaling effect" on the old product, and a feedback effect on the core product (called "feedback reputation effect").

It is easy to see why there could not be any (pure strategy) equilibrium with *endogenous*

[25] See for example Loken & John (1993).

[26] Wernerfelt's (1988) model has the same structure. In Hakenes & Peitz (2004), on the other hand, consumers buy both products in each period.

quality correlation in the game considered by Cabral. First, if consumers do not buy the extension product at $t = 2$, then the magnitude of the price impact of the signaling effect on the new product does not depend on $q_n$. Second, if consumers do not buy the core product at $t = 1$, neither the profit impact of the signaling effect on the core product nor that of the feedback effect on the core product depend on $q_o$. For any strategy such that $\rho > 0$, this implies that $\Delta(g, g) = \Delta(b, g) > \Delta(g, b) = \Delta(b, b)$, which directly leads to a contradiction in the case of pure strategies.[27] To endogenize quality correlation, I therefore need to extend Cabral's set-up.

Choi (1998) analyzes an infinitely repeated game in which in every period a monopolist discovers a new product that is of either high or low quality, and then introduces this product under either a premium or a no-name brand. Choi shows that there may exist an equilibrium in which the premium brand consistently provides high quality, assuming that a single consumption allows consumers to perfectly learn a good's quality.

Like Choi, I consider a monopolist's decision to assign products to brands. Whereas Choi relies on coordination in an infinitely repeated game to sustain consistent brands, I however show that even in a finite horizon game umbrella brands may provide uniform quality. I believe that my analysis is more pertinent in the context of goods for which consumers remain - at least to some degree - uncertain about product quality even after repeated consumption. The reason is that with imperfect monitoring the bootstrap mechanism used by Choi would require a high degree of coordination between consumers and the firm. In particular, consumer trust would have to break down temporarily if the incumbent is unlucky and a high quality product sold under the premium brand fails.[28]

Hence, my theory seems to be more appropriate for products in categories where quality is difficult to evaluate for consumers, such as high-tech products, some food categories or drugs. The example of dental care products (toothpaste and toothbrushes) analyzed empirically by Erdem (1998) also falls into this group. Choi's theory, on the other hand, seems more appropriate for pure 'taste products' such as Classic Coke which was extended to Diet Coke and Cherry Coke among others. Finally, neither theory describes brand extensions in industries such as fashion, cosmetics, and accessories, in a satisfactory way. The analysis of branding strategies in such markets, where consumption decision may derive from the desire to belong to a certain group or differentiate yourself from other groups, remains an interesting topic for future research.

[27] This reasoning easily extends to the case where quality is a continuous variable.

[28] Alternatively, trust would need to break down for ever with some probability. Whereas such relatively complex punishments seem appropriate in models of collusion between a small number of firms, it is more difficult to imagine the play of such equilibria when there is a large number of anonymous and possibly short-lived consumers.

### 7.2 Empirical Evidence

The available evidence strongly supports the prediction that consumers' perceptions of the qualities of different products sold under the same umbrella brand are positively correlated. Aaker and Keller (1990) provide experimental evidence that the perceived quality of a branded product positively affects the expected quality of hypothetical brand extensions. Erdem (1998) estimates a structural model of umbrella branding on scanner panel data about toothpaste and toothbrush purchases. The dataset comprises information about both a number of umbrella branded products and products sold under separate brands. The underlying structural model is similar to the one I propose: consumers have prior beliefs about individual products' qualities, which they update over time as they accumulate experience. Erdem estimates a correlation between prior quality perceptions, corresponding to my parameter $\rho$, of 0.882.

If quality correlation is indeed positive, then umbrella branding may help firms with high reputation products to successfully introduce new products. Indeed, there is plenty of empirical evidence showing that products introduced as brand extensions are on average more successful than newly branded products.[29] In a recent study based on survey data about a large range of consumer products, Smith and Park (1992) confirm that umbrella branding contributes favorably to market share and advertising efficiency. However, "*revenue and advertising cost differentials diminish considerably after a product's introductory period*"; thus, umbrella branding helps initially thanks to signaling effects, but as consumers accumulate information over time, branding loses importance. Moreover, they find that "*the brand extension-new brand differential in the revenue component of cash flow widens as brand strength increases*". Interpreting brand strength as the reputation $r$ of the core product,[30] this finding is consistent with the model proposed here: as can be easily checked, the signaling effect on the extension product, i.e. $\mu^n - \gamma$, increases in $r$ in any equilibrium characterized by positive quality correlation.[31]

In some cases the decision to umbrella brand was also found to have a positive effect on the consumers' willingness to pay for the old product. Sullivan (1990) shows that when in 1988 Jaguar launched its first new model in 17 years, the old Jaguar models experienced significant increases in demand. Furthermore, Balachander and Ghose's (2003) analysis of scanner panel data suggests that the advertising of brand extensions produces significant reciprocal spillover effects on core products. This is consistent with my model, which predicts that umbrella branding has a positive signaling effect on the extension and/or the core product in any equilibrium

[29] See for example Claycamp & Liddy (1969) for an early reference.

[30] In Smith and Park (1992), consumers are asked to evaluate the strength of the brand as a whole, rather than of individual products.

[31] The same holds in the extended model in section 6. There, $(\mu^n - i)$ is increasing in $r$ in any equilibrium with perfect positive quality correlation.

with positive quality correlation.

Another important feature of equilibria with positive quality correlation is that the success (failure) of any one of the products has a positive (negative) feedback effect on the other product sold under the same umbrella brand. Sullivan (1990) provides an example of the negative feedbacks effects failures can have. She shows that the alleged sudden-acceleration defect of the Audi 5000 model resulted in a significant demand drop for the Audi 4000 model. Conversely, the success of Apple iPods in recent years seems to considerably boost the demand for Apple computers. The Economist (January 12th, 2006) even states that "*the 'halo' effect from the iPod remains Apple's most effective means of boosting sales of its computers...In 2005 the iPod helped the company to increase its share of the personal computer market from 3% to 4%.*"

My analysis shows that if consumers have very optimistic or pessimistic prior beliefs about one of the products, then umbrella branding cannot credibly signal quality correlation (see proposition 1). Smith and Park (1992) find that the effect of brand extensions (on market share and advertising efficiency) is smaller when consumers' knowledge of the extension product is high than when it is low. Since well-informed consumers have priors close to either 0 or 1, this is consistent with my result.

A testable prediction of my analysis is that the successful leveraging of reputation from one product to another is more difficult if markets are very asymmetric (see proposition 1). This would be the case for example if a brand used for products of mass consumption were extended to a niche category. I am not aware of any existing empirical study addressing this issue. My analysis also underlines the importance of repeated future consumption of both products for the successful signaling of quality correlation (see the discussion in section 7.1 and point 3. of proposition 1). This suggests that extensions can only be successful if both the core and the extension product are expected to survive for a sufficiently long time.

# A Appendix

**Proof of Lemma 1:** Let $i \neq -i \in \{o, n\}$. First note that since prices are linear in beliefs,

$$E\left[w\left(\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}})\right) - w\left(\lambda_{\sigma_i}(\mu^i)\right) \mid q_o, q_n\right] > 0$$

is equivalent to

$$E\left[\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}}) - \lambda_{\sigma_i}(\mu^i) \mid q_o, q_n\right] > 0\ .$$

As explained in section 3.1, $\lambda_{\sigma_i}(\mu^i)$ is the probability consumers would assign to product $i$ being good given only the following two pieces of information: first, the fact that product $i$ is umbrella branded, and second, its performance $\sigma_i$:

$$\lambda_{\sigma_i}(\mu^i) = \Pr\{q_i = g \mid \sigma_i, \text{ umbrella branding}\}\ .$$

The belief $\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}})$ is the probability consumers assign to product $i$ being good given not only $\sigma_i$ and the observation of an umbrella brand but also $\sigma_{-i}$:

$$\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}}) = \Pr\{q_i = g \mid \sigma_i, \sigma_{-i}, \text{ umbrella branding}\}\ .$$

Since $\sigma_{-i} \in \{S, F\}$, the following equality then directly follows from Bayes' rule:

$$\begin{aligned} \lambda_{\sigma_i}(\mu^i) &= \Pr\{\sigma_{-i} = S \mid \sigma_i, \text{ umbrella branding}\}\, \lambda_{\sigma_i}(\mu^i_S) \\ &\quad + (1 - \Pr\{\sigma_{-i} = S \mid \sigma_i, \text{ umbrella branding}\})\lambda_{\sigma_i}(\mu^i_F). \end{aligned} \tag{10}$$

As the belief that a product is good is a probability, any conditional probability assigned to a product's success must trivially lie between $b$ and $g$. Therefore:[32]

$$\Pr\{\sigma_{-i} = S \mid \sigma_i, \text{ umbrella branding}\} \in (b, g)\ . \tag{11}$$

Now consider the case $q_{-i} = g$. Since $g > \Pr\{\sigma_{-i} = S \mid \sigma_i, \text{ umbrella branding}\}$, it is a straightforward implication of the equality in (10) that for any $\sigma_i \in \{S, F\}$:

$$g\lambda_{\sigma_i}(\mu^i_S) + (1-g)\lambda_{\sigma_i}(\mu^i_F) > \lambda_{\sigma_i}(\mu^i) \quad \text{if and only if} \quad \lambda_{\sigma_i}(\mu^i_S) > \lambda_{\sigma_i}(\mu^i_F). \tag{12}$$

As $\lambda_{\sigma_i}(\cdot)$ is strictly increasing, $\lambda_{\sigma_i}(\mu^i_S) > \lambda_{\sigma_i}(\mu^i_F)$ if and only if $\mu^i_S > \mu^i_F$, which is the case whenever successes have positive feedback effects, i.e. for $\rho > 0$.

The total "expected feedback effect" can be decomposed as follows:

$$\begin{aligned} E\left[\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}}) - \lambda_{\sigma_i}(\mu^{-i}) \mid q_o, q_n\right] &= q_i\left[q_{-i}\lambda_S(\mu^i_S) + (1-q_{-i})\lambda_S(\mu^i_F) - \lambda_S(\mu^i)\right] \\ +(1-q_i)\left[q_{-i}\lambda_F(\mu^i_S) + (1-q_{-i})\lambda_F(\mu^i_F) - \lambda_F(\mu^i)\right]. \end{aligned}$$

[32] The *strict* inclusion in the set $(b, g)$ follows from the simple fact that $\rho \neq 0$ is incompatible with umbrella branding fully revealing the quality of one (or both) of the products to consumers.

The finding in (12) directly implies that if quality correlation is positive, so that $\mu^i_S > \mu^i_F$, then both terms between square brackets are positive. Hence, if $\rho > 0$, then

$$E\left[\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}}) - \lambda_{\sigma_i}(\mu^{-i}) \mid q_i, q_{-i} = g\right] > 0. \tag{13}$$

If quality correlation is negative on the other hand, so that $\mu^i_S < \mu^i_F$, then

$$E\left[\lambda_{\sigma_i}(\mu^i_{\sigma_{-i}}) - \lambda_{\sigma_i}(\mu^{-i}) \mid q_i, q_{-i} = g\right] < 0. \tag{14}$$

Conversely, (10) and (11) imply that for $q_{-i} = b$ the inequalities in (13) and (14) are reversed. Q.E.D.

**Proof of Proposition 1:** I first show that babbling equilibria always exist. Suppose that the equilibrium branding strategy is such that $x_{gg} = x_{gb} = x_{bg} = x_{bb} > 0$. Then, the branding decision does not have any signaling effects, i.e. $\mu^o = r$ and $\mu^n = \gamma$. Moreover, $\rho = 0$ and umbrella branding does not induce any feedback effects either. Consequently, prices are unaffected by the branding decision, so that $\Delta(q_o, q_n) = 0$ for any $(q_o, q_n)$. The branding strategy is therefore optimal.

I now show that if one of conditions 1. to 4. of proposition 1 is satisfied, then any equilibrium must be a babbling equilibrium:

1. First consider the limit case $\beta = 0$, in which the decision to umbrella brand is fully driven by its impact on the expected profits made from selling the core product. This expected profit impact can be decomposed into two terms corresponding to the signaling effect and the feedback effects on the old product, respectively. Moreover, the sign of the signaling effect is independent of qualities. The sign of the expected impact of the feedback effect on the old product depends on the quality of the new product however: by lemma 1, it is positive if and only if $q_n = g$. Now consider any candidate equilibrium such that $\rho > 0$. First, it is easy to see that the signaling effect must be positive in any such equilibrium. If it were negative, then for firms with a bad new product umbrella branding would reduce profits, since both signaling and feedback effects would reduce expected profits. If $x_{gb} = x_{bb} = 0$ however, then umbrella branding does not signal any quality correlation. Therefore suppose that the signaling effect is positive. In that case, both signaling and feedback effects increase expected profits for firms with a good new product. This implies that $x_{gg} = x_{bg} = 1$ in any such equilibrium. For the signaling effect to be indeed positive, it must then be that $x_{gb} > x_{bb}$. This leads to a contradiction of the initial assumption $\rho > 0$, since - given that good new products are always umbrella branded - positive correlation would require that bad new products are more likely to be under an umbrella brand with other bad rather than good products.

Hence, for $\beta = 0$, there does not exist any equilibrium such that $\rho > 0$. Using the same approach, I can rule out all equilibria such that $\rho \neq 0$ for $\beta = 1$ or $\beta = 0$. Since profits are smooth in $\beta$, it follows from these results that equilibria with feedback effects do not exist for $\beta$ "too" close to 0 or 1 either.

'Non-babbling' equilibria such that $\rho = 0$ are also impossible. Since there are no feedback effects for $\rho = 0$, whether firms want to umbrella brand or not will only depend on the signaling effect on one of the products, the sign of which is independent of qualities.

2. If consumers' prior about one of the products is already perfectly accurate, then umbrella branding induces no feedback effects. Suppose, for example, that $r = 1$. Then, the new product's performance cannot have any feedback effect on the old product under umbrella branding, since consumers remain convinced that the old product is good no matter what happens. Similarly, as the old product's performance has no direct effect on the belief about the old product itself, it cannot have any feedback effect on the new product either: $\mu^n = \mu^n_S = \mu^n_F \left(= \mu_{gg}\right)$. Umbrella branding has no signaling effect on the old product either, since consumers are already convinced of its quality. Branding incentives are hence driven by the signaling effect on the new product. The branding incentives of different types $(q_o, q_n)$ are thus fully aligned, and umbrella branding cannot credibly signal any quality information. As profits are smooth in $r$, it follows that for any strategy and values of the other parameters, there exists a threshold of $r$ above which the performance of the old product does not affect beliefs sufficiently for a non-babbling equilibrium to exist. I can use the same line of reasoning to rule out 'non-babbling' equilibria for $r$ close to 0, or $\gamma$ close to 1 or to 0.

3. If the weight attached to future profits approaches 0, then short term signaling effects completely drive the firm's decision to umbrella brand or not:

$$\lim_{\delta \to 0} \Delta(q_o, q_n) = \Delta_1.$$

In the limit, the incentives to umbrella brand are hence completely independent of qualities. By continuity, this implies that for $\delta$ sufficiently close to 0, only babbling equilibria can exist.[33]

4. If $b$ were equal to $g$, then performance would no longer yield any information about quality to consumers. Formally, this would mean that for any initial belief $\mu$, $\lambda_S(\mu) = \lambda_F(\mu) = \mu$. Hence,

$$\lim_{b \to g} \Delta(q_o, q_n) = \Delta_1,$$

which is independent of $(q_o, q_n)$. It then follows from continuity that for $b$ sufficiently close to $g$, only 'babbling' equilibria can exist.

Q.E.D.

[33] Whenever $\Delta_1 \neq 0$, the umbrella branding incentives of the different types of the incumbent are completely aligned for all $\delta$ below some strictly positive threshold. There may not exist any such strictly positive threshold of $\delta$ if $\Delta_1 = 0$ and one signaling effect is strictly positive, however. This case is neglected here because generically it does not occur.

**Proof of Proposition 2:** Suppose (in negation) that there is an equilibrium in which only firms with good old products extend their brands, i.e. $x_{bg}= x_{bb}= 0$, but $x_{gg}> 0$ and/or $x_{gb}> 0$. Since even good products can fail (i.e., $g < 1$), consumers then believe that the old product is good with probability 1 in both periods. Moreover, as $q_o$ is fully revealed, the old product's performance does not have any feedback effect on the new product. Aggregate profits under umbrella branding are therefore independent of the old product's true quality.

For any given $q_n$, if there is a difference between $\Delta(g, q_n)$ and $\Delta(b, q_n)$, then this must be due to a profits difference under separate branding. Since profits under separate branding are always lower the higher the number of bad products however, for any $q_n \in \{b, g\}$:

$$\Delta(b, q_n) > \Delta(g, q_n).$$

This implies, however, that $x_{bg}\geq x_{gg}$ and $x_{bb}\geq x_{gb}$, which contradicts the initial assumption. By the same reasoning, it is easy to rule out equilibria in which umbrella branding fully reveals the new product's quality. Q.E.D.

**Proof of Proposition 3:** I first show that the four conditions $x_{gg}= 0$, $\min[x_{gb}, x_{bg}] = 0$, and $\max[x_{gb}, x_{bg}] > 0$ must be satisfied in any non-babbling equilibrium such that $\rho = 0$.

First, suppose (in negation) that $x_{gg}> 0$. Then, whenever $x_{bb}= 0$, and the equilibrium strategy is such that $\rho = 0$, umbrella branding certifies the quality of at least one of the products. By proposition 2, this is impossible. If $x_{bb}> 0$, on the other hand, then for $\rho = 0$ to be true, it must be that $x_{q_oq_n}> 0$ for all $(q_o, q_n)$. However, as is easy to see, this is only possible in a babbling equilibrium.

Second, suppose that $\min[x_{gb}, x_{bg}] > 0$. Then, the necessary condition $x_{gg}= 0$ implies that $\rho < 0$, a contradiction.

Third, suppose that $\max[x_{gb}, x_{bg}] = 0$. Then, since it must also be that $x_{gg}= 0$, umbrella branding certifies the bad quality of both products (I ignore situations in which umbrella branding never happens on the equilibrium path). Therefore, umbrella branding is clearly unprofitable for all firms.

I hence know that it must be that $x_{gg}= 0$, $\min[x_{gb}, x_{bg}] = 0$ and $\max[x_{gb}, x_{bg}] > 0$ in any non-babbling equilibrium such that $\rho = 0$. Umbrella branding thus certifies the bad quality of either the old or the new product. If $x_{bb}$ were equal to 0, then umbrella branding would at the same time certify the high quality of the other product, which is impossible by proposition 2. Hence, it must be that $x_{bb}> 0$.

For $x_{bb}> 0$ to be optimal, it must be that $\Delta(b, b) \geq 0$. Since umbrella branding does not lead to any feedback effects here, at least one of the signaling effects must then be positive, otherwise umbrella branding would never be profitable. Given that $x_{gg}=\min[x_{gb}, x_{bg}] = 0$, at least one of the signaling effects is positive if and only if $x_{bb}< \max[x_{gb}, x_{bg}]$. Hence, $x_{bb}< \max[x_{gb}, x_{bg}]$ is a necessary equilibrium condition.

To show that there indeed exist equilibria of the form $x_{gg}= 0$, $\min[x_{gb}, x_{bg}] = 0$, $\max[x_{gb}, x_{bg}] > 0$, and $x_{bb}\in (0, \max[x_{gb}, x_{bg}])$, I provide a numerical example. Consider the mixed strategy $(0, 0, 1, 0.1)$.

Then, for $\gamma = 0.4$, $g = 0.98$, $b = 0.4$, $\beta = \delta = 0.5$, and $r = \widehat{r} \simeq 0.423$, this mixed strategy is indeed optimal since $\Delta(gg) \simeq -0.154 < 0$, $\Delta(gb) \simeq -0.208 < 0$, $\Delta(gb) \simeq 0.0544 > 0$, and $\Delta(bb) = 0$. Q.E.D.

**Proof of Proposition 4:** The proof will proceed as follows. First I show that in the limit case $g = 1$ (while all other parameters remain strictly between 0 and 1), a pure strategy equilibrium such that $\rho = 1$ exists if and only if $\Delta_1 > 0$, $b \leq \overline{b}(r, \gamma, \beta)$, and $\underline{\delta}(r, \gamma, \beta, b) \leq \delta \leq \overline{\delta}(r, \gamma, \beta, b)$, where $\overline{b}(r, \gamma, \beta) \in (0, 1)$, $\underline{\delta}(r, \gamma, \beta, b) \in (0, 1)$ and $\overline{\delta}(r, \gamma, \beta, b) \in [\underline{\delta}(\cdot), 1)$. Since profits are smooth in $g$, the statement made in the proposition will follow from this result.

Suppose the incumbent plays the strategy $\left(x_{gg}, x_{gb}, x_{bg}, x_{bb}\right) = (1, 0, 0, 1)$ in equilibrium. Then, $\mu^o = \mu^n$, $\mu^o_S = \mu^n_S$, and $\mu^o_F = \mu^n_F$. To simplify notations, I define $\mu \equiv \mu^o = \mu^n$, $\mu_S \equiv \mu^o_S = \mu^n_S$, and $\mu_F \equiv \mu^o_F = \mu^n_F$.

It is straightforward that, if good products never fail (i.e. if $g = 1$), then a single failure suffices to convince consumers of bad quality:

$$\lim_{g \to 1} \lambda_F(\widetilde{\mu}) = 0 \text{ for any } \widetilde{\mu} \in [0, 1).$$

Moreover, since consumers expect umbrella brands to always sell products of uniform quality, the failure of a single umbrella branded product suffices to convince consumers that both products are bad:

$$\lim_{g \to 1} \mu_F = 0.$$

Hence, under an umbrella brand, the willingness to pay in period 2 is zero whenever (at least) one of the products fails in period 1:

$$\lim_{g \to 1} \lambda_F(\mu_S) = \lim_{g \to 1} \lambda_S(\mu_F) = \lim_{g \to 1} \lambda_F(\mu_F) = 0.$$

These observations, and the linearity of $w(\cdot)$, imply that

$$\lim_{g \to 1} \Delta_2(S, S) = (1 - b) \left( \begin{array}{c} (1 - \beta) \left[\lim_{g \to 1} \lambda_S(\mu_S) - \lim_{g \to 1} \lambda_S(r)\right] \\ +\beta \left[\lim_{g \to 1} \lambda_S(\mu_S) - \lim_{g \to 1} \lambda_S(\gamma)\right] \end{array} \right), \tag{15}$$

$$\lim_{g \to 1} \Delta_2(S, F) = (1 - b)(1 - \beta) \left[- \lim_{g \to 1} \lambda_S(r)\right] < 0, \tag{16}$$

$$\lim_{g \to 1} \Delta_2(F, S) = (1 - b)\beta \left[- \lim_{g \to 1} \lambda_S(\gamma)\right] < 0, \tag{17}$$

$$\lim_{g \to 1} \Delta_2(F, F) = 0. \tag{18}$$

The expected marginal impacts of umbrella branding on second period profits are

$$\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,g\right] = \lim_{g\to 1} \Delta_2(SS), \tag{19}$$

$$\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,b\right] = b\lim_{g\to 1} \Delta_2(SS) + (1-b)\lim_{g\to 1} \Delta_2(SF), \tag{20}$$

$$\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,g\right] = b\lim_{g\to 1} \Delta_2(SS) + (1-b)\lim_{g\to 1} \Delta_2(FS), \tag{21}$$

$$\begin{aligned}\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,b\right] &= b\left[b\lim_{g\to 1} \Delta_2(SS) + (1-b)\lim_{g\to 1} \Delta_2(SF)\right] \\ &\quad +(1-b)\left[b\lim_{g\to 1} \Delta_2(FS) + (1-b)\lim_{g\to 1} \Delta_2(FF)\right].\end{aligned} \tag{22}$$

An equilibrium with perfect quality correlation then exists if and only if the following four inequalities are satisfied, so that the branding strategy $(1,0,0,1)$ is indeed optimal:

$$\lim_{g\to 1} \Delta(g,g) = (1-\delta)\Delta_1 + \delta\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,g\right] \geq 0, \tag{23}$$

$$\lim_{g\to 1} \Delta(g,b) = (1-\delta)\Delta_1 + \delta\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,b\right] \leq 0, \tag{24}$$

$$\lim_{g\to 1} \Delta(b,g) = (1-\delta)\Delta_1 + \delta\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,g\right] \leq 0, \tag{25}$$

$$\lim_{g\to 1} \Delta(b,b) = (1-\delta)\Delta_1 + \delta\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,b\right] \geq 0. \tag{26}$$

To derive sufficient conditions under which (23) to (26) are indeed satisfied, I prove the following five statements step by step:

**Step 1** In equilibrium, it must be that $\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,b\right] < 0$ and $\Delta_1 > 0$.

**Step 2** If $\Delta_1 > 0$, then equilibrium condition (23) is satisfied.

**Step 3** If $\Delta_1 > 0$, then there exists a threshold $\bar{b}(r,\gamma,\beta) \in (0,1)$ such that

$$\lim_{g\to 1} \Delta(b,b) > \max\left[\lim_{g\to 1} \Delta(g,b), \lim_{g\to 1} \Delta(b,g)\right]$$

if and only if $b < \bar{b}(r,\gamma,\beta)$.

**Step 4** If $\Delta_1 > 0$ and $b < \bar{b}(r,\gamma,\beta)$, then there exist thresholds $\underline{\delta}(r,\gamma,\beta,b) \in (0,1)$ and $\bar{\delta}(r,\gamma,\beta,b) \in [\underline{\delta}(\cdot),1)$ such that the equilibrium conditions (24), (25) and (26) are simultaneously satisfied if and only if $\delta \in [\underline{\delta}(r,\gamma,\beta,b), \bar{\delta}(r,\gamma,\beta,b)]$.

**Step 1:** For any $q_o \in \{b,g\}$, $E[\Delta_2(\sigma_o,\sigma_n) \mid q_o,b]$ is equal to the following weighted sum:

$$E[\Delta_2(\sigma_o,\sigma_n) \mid q_o,b] = q_o E\left[\Delta_2(S,\sigma_n) \mid q_n = b\right] + (1-q_o) E\left[\Delta_2(F,\sigma_n) \mid q_n = b\right]. \tag{27}$$

In any equilibrium with perfect positive quality correlation, umbrella branding must be more profitable for firms with two bad products than for firms with a good old and bad new product. Clearly, for this to be true, it must be that

$$E\left[\Delta_2(S,\sigma_n) \mid q_n = b\right] \leq E\left[\Delta_2(F,\sigma_n) \mid q_n = b\right]. \tag{28}$$

Now consider the limit case $g = 1$. Since

$$\lim_{g\to 1} E\left[\Delta_2(F,\sigma_n) \mid q_n = b\right] = b \underbrace{\lim_{g\to 1} \Delta_2(FS)}_{<0} + (1-b) \underbrace{\lim_{g\to 1} \Delta_2(FF)}_{=0} < 0,$$

(28) implies that for any $q_o$, all terms in $\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid q_o, b\right]$ are negative. Hence,

$$\lim_{g\to 1} E\left[\Delta_2(\sigma_o,\sigma_n) \mid b, b\right] < 0.$$

For $(b,b)$-firms to be willing to use umbrella brands nevertheless, i.e. for equilibrium condition (26) to hold for some discount factors, it is then necessary that

$$\Delta_1 > 0,$$

which here is equivalent to the condition

$$\mu = \frac{r\gamma}{r\gamma + (1-r)(1-\gamma)} > (1-\beta)r + \beta\gamma. \tag{29}$$

**Step 2:** Given the strategy $(1,0,0,1)$,

$$\Delta_2(S,S) = w\left(\lambda_S(\mu_S)\right) - (1-\beta)w\left(\lambda_S(r)\right) - \beta w\left(\lambda_S(\gamma)\right),$$

which can be rewritten as

$$\begin{aligned}\frac{\Delta_2(S,S)}{(g-b)} \quad &= \quad \left[\lambda_S(\mu_S) - \lambda_S(\mu)\right] + \left[\lambda_S(\mu) - \lambda_S\left((1-\beta)\, r + \beta\gamma\right)\right] \\ &\quad + \left[\lambda_S\left((1-\beta)\, r + \beta\gamma\right) - (1-\beta)\lambda_S(r) - \beta\lambda_S(\gamma)\right].\end{aligned}$$

The first of the terms in this expression is positive because $\rho > 0$ implies that $\mu_S > \mu$. The last term is positive because $\lambda_S(\cdot)$ is a concave function. Finally, the second term is positive whenever $\Delta_1 > 0$ (see (29)). Hence, if $\Delta_1 > 0$ then $\Delta_2(S,S) > 0$.

In the limit case $g = 1$, firms with two good products are certain to experience two successes. Hence, whenever $\Delta_1 > 0$, then also

$$\lim_{g\to 1} \Delta(g,g) = (1-\delta)\Delta_1 + \delta \underbrace{\lim_{g\to 1} \Delta_2(S,S)}_{>0} > 0,$$

i.e. equilibrium condition (23) is satisfied.

**Step 3**: I now show that there exists a threshold $\bar{b}(r,\gamma,\beta) \in (0,1)$ such that

$$\lim_{g\to 1} \Delta(b,b) > \max\left[\lim_{g\to 1} \Delta(g,b), \lim_{g\to 1} \Delta(b,g)\right] \tag{30}$$

if and only if $b < \bar{b}(r,\gamma,\beta)$. First, $\lim_{g\to 1}\Delta(b,b) > \lim_{g\to 1}\Delta(g,b)$ if and only if

$$\lim_{g\to 1} E[\Delta_2(\sigma_o,\sigma_n) \mid b,b] > \lim_{g\to 1} E[\Delta_2(\sigma_o,\sigma_n) \mid g,b], \tag{31}$$

Substituting for the expected profit differences and simplifying, this condition rewrites as

$$b \lim_{g\to 1} \lambda_S(\mu_S) < (1-\beta) \lim_{g\to 1} \lambda_S(r),$$

which is equivalent to

$$b\frac{\mu}{\mu+(1-\mu)b^2} < (1-\beta)\frac{r}{r+(1-r)b},$$

or

$$K(b) \equiv b^2\left[\mu(1-\beta r)-(1-\beta)r\right] + br\mu - (1-\beta)r\mu < 0.$$

It is easy to see that $K(0) < 0$. Moreover, $\Delta_1 > 0$ implies that $K(1) > 0$. Since $K(b)$ describes a parabola, there then exists a unique $\bar{b}_1(r,\gamma,\beta) \in (0,1)$ such that $K(b) < 0$ if and only if $b < \bar{b}_1(r,\gamma,\beta)$. Using the same line of reasoning, it is easy to show that there exists a $\bar{b}_2(r,\gamma,\beta) \in (0,1)$ such that

$$\lim_{g\to 1} E[\Delta_2(\sigma_o,\sigma_n) \mid b,b] > \lim_{g\to 1} E[\Delta_2(\sigma_o,\sigma_n) \mid b,g] \tag{32}$$

if and only if $b < \bar{b}_2(r,\gamma,\beta)$. Defining

$$\bar{b}(r,\gamma,\beta) \equiv \min\left[\bar{b}_1(r,\gamma,\beta), \bar{b}_2(r,\gamma,\beta)\right],$$

it follows that condition (30) is satisfied if and only if $b < \bar{b}(r,\gamma,\beta)$.

**Step 4**: Steps 1 and 3 establish that if $b < \bar{b}(r,\gamma,\beta)$, then

$$\max\left[E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,b\right], E\left[\Delta_2(\sigma_o,\sigma_n) \mid g,b\right]\right] < E\left[\Delta_2(\sigma_o,\sigma_n) \mid b,b\right] < 0.$$

It is straightforward to see that if these inequalities hold and moreover $\Delta_1 > 0$, then there exists a non-empty range $\left[\underline{\delta}(r,\gamma,\beta,b), \bar{\delta}(r,\gamma,\beta,b)\right] \in (0,1)$ such that the equilibrium conditions (24), (25) and (26) are simultaneously satisfied for any $\delta \in \left[\underline{\delta}(r,\gamma,\beta,b), \bar{\delta}(r,\gamma,\beta,b)\right]$.[34] Moreover, for $\delta \in \left(\underline{\delta}(r,\gamma,\beta,b), \bar{\delta}(r,\gamma,\beta,b)\right)$, the three equilibrium conditions (24), (25) and (26) are satisfied with strict inequalities.

To conclude note that, as long as $r,\gamma,b,\beta \in (0,1)$,[35] the equilibrium profits from selling each of the products are smooth in $g$. The statement made in proposition 1 then directly follows from the results established in steps 1 to 4. Q.E.D.

[34] It is obvious that the range of $\delta$ for which an equilibrium exists is always included in $(0,1)$ here: since $\delta$ is the *share* of profits accruing to the second period, $\lim_{g\to 1}\Delta(q_o,q_n)$ can achieve any value between $\lim_{g\to 1}\Delta_2(q_o,q_n)$ and $\Delta_1$ by letting $\delta$ vary between 1 and 0.

[35] If $r$ were equal to 1, for example, then beliefs may not be smooth: While for $g=1$, it would not be clear what beliefs consumers should hold following a failure (of either one of the products), for $g$ almost equal to 1, they would always continue to believe that both products are good (even after observing two failures).

**Proof of Proposition 5:** The correlation coefficient $\rho$ is equal to $-1$ whenever $x_{gg}= x_{bb}= 0$ but $x_{gb}, x_{bg}> 0$. Such a strategy indeed maximizes aggregate profits if $\Delta(b,g), \Delta(g,b) \geq 0$ but $\Delta(g,g),\ \Delta(b,b) \leq 0$. Hence, the following two conditions must hold in any equilibrium with perfect negative quality correlation:

$$\Delta(g,b) \geq \Delta(b,b), \tag{33}$$

$$\Delta(b,g) \geq \Delta(b,b). \tag{34}$$

Since the first period impact of umbrella branding is independent of $(q_o, q_n)$, condition (33) is equivalent to

$$E\left[\Delta_2(\sigma_o, \sigma_n) \mid g, b\right] \geq E\left[\Delta_2(\sigma_o, \sigma_n) \mid b, b\right], \tag{35}$$

Condition (35) is indeed satisfied, i.e. the $(g,b)$-type has higher incentives to umbrella brand than the $(b,b)$-type, if and only if, given $q_n= b$, the expected impact of a success of the old product on the profit difference between umbrella and separate branding is positive:

$$E\left[\Delta_2(S, \sigma_n) \mid q_n= b\right] - E\left[\Delta_2(F, \sigma_n) \mid q_n= b\right] \geq 0. \tag{36}$$

I now show that this condition is always violated if $\beta \geq \frac{1}{2}$. First, note that, since prices are linear in beliefs, condition (36) can be rewritten as follows:

$$\begin{aligned} 0 \leq b & \left[(1-\beta)\underbrace{\left[\lambda_S(\mu_S^o) - \lambda_F(\mu_S^o)\right]}_{>0 \text{ (direct effect)}} + \beta\underbrace{\left[\lambda_S(\mu_S^n) - \lambda_S(\mu_F^n)\right]}_{<0 \text{ (feedback effect)}}\right] \\ &+ (1-b)\left[(1-\beta)\underbrace{\left[\lambda_S(\mu_F^o) - \lambda_F(\mu_F^o)\right]}_{>0 \text{ (direct effect)}} + \beta\underbrace{\left[\lambda_F(\mu_S^n) - \lambda_F(\mu_F^n)\right]}_{<0 \text{ (feedback effect)}}\right] \\ &- (1-\beta)\underbrace{\left[\lambda_S(r) - \lambda_F(r)\right]}_{>0 \text{ (direct effect under separate branding)}}. \end{aligned} \tag{37}$$

Both in the case of a success or of a failure of the new product, which occur with probabilities $b$ and $(1-b)$ respectively, the success of the old product has two different effects under umbrella branding: first, a positive direct effect on the belief consumers hold about the old product itself, and second, a negative feedback effect on consumers' belief about the new product. Under separate branding, the success of the old product only has a positive direct effect, whose size is independent of the new product's performance.

Next, note that in any equilibrium such that $\rho = -1$ consumers must be convinced that the umbrella brand sells one bad and one good product. This means that for any realization of performances, the probability consumers assign to the new product being good must be the "complement" of the probability they assign to the old product being good. Formally, for any $(\sigma_o, \sigma_n)$:

$$\lambda_{\sigma_o}(\mu_{\sigma_n}^o) + \lambda_{\sigma_n}(\mu_{\sigma_o}^n) = 1. \tag{38}$$

From this it follows that (given the new product's performance) the positive (direct) effect a success of the old product has on the old product itself is exactly offset by its negative (feedback) effect on the new product: since for any $\sigma_n$,

$$\lambda_S(\mu^o_{\sigma_n}) + \lambda_{\sigma_n}(\mu^n_S) = 1 = \lambda_F(\mu^o_{\sigma_n}) + \lambda_{\sigma_n}(\mu^n_F),$$

it is always true that

$$\left[\lambda_S(\mu^o_{\sigma_n}) - \lambda_F(\mu^o_{\sigma_n})\right] = -\left[\lambda_{\sigma_n}(\mu^n_S) - \lambda_{\sigma_n}(\mu^n_F)\right]. \tag{39}$$

Substituting $-\left[\lambda_S(\mu^o_{\sigma_n}) - \lambda_F(\mu^o_{\sigma_n})\right]$ for $\left[\lambda_{\sigma_n}(\mu^n_S) - \lambda_{\sigma_n}(\mu^n_F)\right]$, I can then simplify condition (37) to

$$\begin{aligned} 0 \quad &\leq b(1-2\beta) \underbrace{\left[\lambda_S(\mu^o_S) - \lambda_F(\mu^o_S)\right]}_{>0} + (1-b)(1-2\beta)\underbrace{\left[\lambda_S(\mu^o_F) - \lambda_F(\mu^o_F)\right]}_{>0} \\ &\quad -(1-\beta)\underbrace{\left[\lambda_S(r) - \lambda_F(r)\right]}_{>0}. \end{aligned} \tag{40}$$

This condition is clearly violated for any $\beta \geq \frac{1}{2}$. First, if the firm attaches the same or more weight to the profits made on the new product, the negative feedback effect of a success of the old product outweighs its positive direct effect, hence the first two terms in (40) are negative. Second, successes always increase profits under separate branding, so that the last term in (40) is negative for any $\beta$.

Using the same line of reasoning, it is easy to show that for any $\beta \leq \frac{1}{2}$, firms with a bad old and a good new product would prefer separate to umbrella branding, i.e. condition (34) would be violated.

I can conclude that the necessary conditions (33) and (34) are never simultaneously satisfied, and no equilibrium such that $\rho = -1$ exists. Q.E.D.

**Proof of Lemma 2:** Suppose that $\rho < 0$ in equilibrium. Then, $\mu^o_F > \mu^o > \mu^o_S$ and $\mu^n_F > \mu^n > \mu^n_S$. In the following, I will show that $\mu^o_F < r$ if $x_{bb} \geq x_{gb}$, and $\mu^n_F < \gamma$ if $x_{bb} \geq x_{bg}$. From this, it then follows that for $x_{bb} \geq \max\left[x_{gb}, x_{bg}\right]$ umbrella branding is unprofitable, because it deteriorates the beliefs about both products in all circumstances.

First consider

$$\mu^o_F = \frac{r\gamma x_{gg}(1-g) + r(1-\gamma)x_{gb}(1-b)}{\left[r\gamma x_{gg} + (1-r)\gamma x_{bg}\right](1-g) + \left[r(1-\gamma)x_{gb} + (1-r)(1-\gamma)x_{bb}\right](1-b)}.$$

It is easy to check that the belief $\mu^o_F$ is increasing in $x_{gg}$. For any $x_{bb} > 0$, the correlation $\rho$ is negative if and only if

$$x_{gg} < \frac{x_{gb}x_{bg}}{x_{bb}}.$$

Hence,

$$\mu^o_F < \frac{r\gamma \frac{x_{gb}x_{bg}}{x_{bb}}(1-g) + r(1-\gamma)x_{gb}(1-b)}{\left[r\gamma \frac{x_{gb}x_{bg}}{x_{bb}} + (1-r)\gamma x_{bg}\right](1-g) + \left[r(1-\gamma)x_{gb} + (1-r)(1-\gamma)x_{bb}\right](1-b)},$$

which simplifies to

$$\begin{aligned}\mu_F^o &< \frac{x_{gb}r\left[\gamma x_{bg}(1-g)+r(1-\gamma)x_{bb}(1-b)\right]}{x_{gb}r\left[\gamma x_{bg}(1-g)+r(1-\gamma)x_{bb}(1-b)\right]+x_{bb}(1-r)\left[\gamma x_{bg}(1-g)+r(1-\gamma)x_{bb}(1-b)\right]}\\ &= \frac{x_{gb}r}{x_{gb}r+x_{bb}(1-r)}.\end{aligned}$$

If $x_{bb}\geq x_{gb}$, then

$$\frac{x_{gb}r}{x_{gb}r+x_{bb}(1-r)}\leq r.$$

I can thus conclude that if $\rho<0$ and $x_{bb}\geq x_{gb}$, then

$$\mu_F^o< r.$$

Using the same line of reasoning, it is easy to show that if $\rho<0$ and $x_{bb}\geq x_{bg}$, then

$$\mu_F^n< \gamma.$$

If $\rho<0$, then it follows from these findings that also $\mu_S^o,\mu^o< r$ and $\mu_S^n,\mu^n< \gamma$. Therefore, $\Delta_1< 0$ and $\Delta_2(\sigma_o,\sigma_n)<0$ for any $(\sigma_o,\sigma_n)$, so that $\Delta(q_o,q_n)<0$ for any $(q_o,q_n)$, which contradicts $\rho<0$. Q.E.D.

**Proof of Lemma 3:** The condition $x_{gg}\geq x_{bg}-\frac{1-\gamma}{\gamma}\left(x_{gb}-x_{bb}\right)$ is equivalent to $\mu^o\geq r$, and the condition $x_{gg}\geq x_{gb}-\frac{1-r}{r}\left(x_{bg}-x_{bb}\right)$ is equivalent to $\mu^n\geq\gamma$. Hence, if both conditions are satisfied, then both signaling effects are non-negative. Suppose that $\rho<0$ in equilibrium. Then, by lemma 1, the total impact of feedback effects on expected profits is positive if both products are bad. Therefore, whenever $\rho<0$ and both signaling effects are non-negative, $\Delta(b,b)>0$. By lemma 2, however, there cannot be any negative correlation equilibrium such that $x_{bb}=1$. Q.E.D.

**Proof of Proposition 6:** Suppose that there exists at least one equilibrium that generates perfect positive quality correlation between umbrella branded products. In what follows, let me denote by $\pi^U\left(q_o,q_n\right)$ and $\pi^S\left(q_o,q_n\right)$, respectively, expected aggregate profits gross of investment costs under umbrella branding and under separate branding, given $q_o$, and $q_n$ as determined by the investment decision. In line with previous notations, use $\Delta\left(q_o,q_n\right)=\pi^U\left(q_o,q_n\right)-\pi^S\left(q_o,q_n\right)$. Note that in any equilibrium with perfect quality correlation, it must be that $\Delta\left(g,g\right),\Delta\left(b,b\right)\geq 0$ and $\Delta\left(g,b\right),\Delta\left(b,g\right)\leq 0$: the branding decision must be optimal for any given investment decision, otherwise the firm's strategy (that determines investment and branding jointly) could not be optimal in the first place.

Consider now the investment decision, taking as given consumers' beliefs and that the firms use umbrella branding only in conjunction with investment decisions that induce uniform quality. Consider an unmatched innovator first. Investing in high quality is profitable for this firm if and only if the expected increase in second-period profits exceeds the investment cost, i.e., whenever

$$c\leq\Omega\equiv\beta\delta\left[w\left(\lambda_S\left(i\right)\right)-w\left(\lambda_F\left(i\right)\right)\right].$$

Given my distributional assumptions, in equilibrium the following condition implictly defines $i$ :

$$i = \frac{\delta \left[ w\left(\lambda_S\left(i\right)\right) - w\left(\lambda_F\left(i\right)\right)\right]}{g-b}. \tag{41}$$

Clearly, $i = 1$ cannot be a solution. $i = 0$ solves (41), but cannot arise in an equilibrium with $\rho = 1$ nonetheless. If $i = 0$, then $\Delta(b, g) > \Delta(b, b)$: (i) since consumers consider an individually branded new product to be bad no matter what, separate branding would give the same profit for the two quality profiles, but (ii) under umbrella branding the expected profit is evidently higher for the $(b, g)$-profile than for the $(b, b)$-profile. Hence, whenever $\Delta(b, g) \leq 0$, then $\Delta(b, b) < 0$, which violates a necessary equilibrium condition. Therefore, in any equilibrium with $\rho = 1$, it must be that $i \in (0, 1)$.[36]

Next consider firms with an old product of quality $q_o = g$. In any equilibrium with perfect quality correlation, these firms will either invest and umbrella brand, or alternatively not invest and use separate brands. Therefore, making the investment increases expected aggregate profits if and only if

$$\begin{aligned}
\pi^S(g, b) &\leq \pi^U(g, g) - c \\
&\Leftrightarrow \\
c &\leq \underbrace{\left[\pi^U(g, g) - \pi^S(g, g)\right]}_{=\Delta(g,g)} + \underbrace{\pi^S(g, g) - \pi^S(g, b)}_{=\Omega}
\end{aligned}$$

Since for $\rho = 1$ to arise it is necessary that $\Delta(g, g) \geq 0$, we can conlude that $i_g \geq i$. Moreover, whenever $\Delta(g, g) > 0$ (as is the case in such equilibria for $g$ close to 1), then $i_g > i$.

Now consider firms that have the option to umbrella brand the new product with an existing product of low quality. In any equilibrium with $\rho = 1$, these firms will either not invest and umbrella brand, or invest and opt for separate brands. Investing in the high quality if therefore profitable if and only if

$$\begin{aligned}
\pi^U(b, b) &\leq \pi^S(b, g) - c \\
&\Leftrightarrow \\
c &\leq -\underbrace{\left[\pi^U(b, b) - \pi^S(b, b)\right]}_{=\Delta(b,b)} + \underbrace{\pi^S(b, g) - \pi^S(b, b)}_{=\Omega}.
\end{aligned}$$

Since $\Delta(b, b) \geq 0$ is a necessary condition for an equilibrium with $\rho = 1$, the right-hand side of this expression lies (weakly) below $\Omega$. Hence, $i^b \leq i$. Q.E.D.

[36] A strictly positive solution of (41) indeed exists if $\delta(g-b)^2 > (1-b)b$, which is hence a necessary condition for an equilibrium with perfect quality correlation here.

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