Forecasting macroeconomic variables using a structural state space model

de Silva, Ashton

RMIT University

1 September 2008

Forecasting macroeconomic variables using a structural state space model*

Ashton de Silva

School of Economics, Finance and Marketing,

RMIT University

Ashton.desilva@rmit.edu.au

September 2008

Abstract

This paper has a twofold purpose; the first is to present a small macroeconomic model in state space form, the second is to demonstrate that it produces accurate forecasts. The first of these objectives is achieved by fitting two forms of a structural state space macroeconomic model to Australian data. Both forms model short and long run relationships. Forecasts from these models are subsequently compared to a structural vector autoregressive specification. This comparison fulfills the second objective demonstrating that the state space formulation produces more accurate forecasts for a selection of macroeconomic variables.

keywords: State space, multivariate time series, macroeconomic model, forecast, SVAR.

*Comments made by Richard Heaney and George Tawadros on earlier drafts of this paper were greatly appreciated. Any errors are the sole responsibility of the author.

1 Introduction

The purpose of this paper is twofold; one, demonstrate how a state space model may be formulated to capture the character of a structural vector auto-regressive model, and two, show how this specification is useful for forecasting purposes. This formulation will be referred to as a *structural state space model*.

Capturing economic relationships using a state space specification is not new. Aoki & Havenner (1991) for instance model various US macroeconomic variables including GNP and money stock. Another example is Balke & Wohar (2002) who model low frequency movements of stock prices. A relatively recent example is Panher (2007) who considers monetary and economic identities.

One key advantage of utilising the state space approach is that inter-series relationships can be disaggregated to the latent component level. This provides a greater degree of insight which may be useful for policy analysis.

The structural state space model presented comprises two latent components for each variable. They are referred to as the permanent and transitory component. These components are specified such that they capture the long and short run relationships between variables. The specification is similar to the state space form of the Beveridge-Nelson decomposition (Morley 2002).

The state space model described in this paper nests the structural vector autoregression (SVAR) specification. Eleven variables are used, six of which are classified domestic as they can be considered to be within the sphere of influence of Australian government authorities. The remainder of the variables represent the rest of the world. As Australia is considered to be a small economy these variables are considered to be determined "outside" the Australian economy.

The structure of this paper is as follows, a brief contextual outline is presented in Section 2. In Section 3 the proposed state space approach is shown to be flexible and simple to implement. The results of a forecast exercise are presented in Section 4. In Section 5 some concluding remarks are presented.

2 Background

The number of frameworks that have been employed to model relationships between macroeconomic variables are too numerous to mention. They include: General Equilibrium models (Dixon et al. 1997); various Bayesian frameworks (Nimark

2007); and Dynamic Stochastic General Equilibrium (DSGE) specifications (Matheson 2006). Arguably, the most popular in recent times (at least in the Australian and New Zealand context) is the DSGE alternative. A technique that is also popular is the Structural Vector Auto-Regressive (SVAR) framework (Berkelmans 2005, Buncic & Melecky 2008, Fry et al. 2008).

As well as there being many frameworks, the size of these applications also vary significantly, ranging from bivariate models (Moosa 1998) to large scale formulations (Monash Model, Dixon & Rimmer 2002). A problem with large scale models is that they can be particularly difficult to implement as they require the imposition of strong economic assumptions that are often hard to verify[1].

Although the class of macroeconomic models is large and diverse, any two models may be compared using the following two principles, *degree of theoretical coherence* and *degree of empirical coherence*. In every formulation a trade-off between these two principles occur. For example: relative to the SVAR, the DSGE has more theoretical coherence, whereas, the SVAR has more empirical coherence (Pagan 2003).

As the new formulation adopts the character of a SVAR approach it must also share its qualities. That is, it has more empirical but less theoretical coherence than the DSGE approach. However, as the structural state space formulation explicitly models the permanent component, in this sense it may have more theoretical coherence than the SVAR. Therefore, the framework presented might be considered to be an improvement on the SVAR approach.

3 The State Space Model

The three key advantages[2] of the state space approach when compared to the (vector) ARMA alternative are generality, flexibility, and transparency. For example, as shown in this paper, it generalises easily to a multivariate formulation. The flexibility of the framework is demonstrated by its ability to handle data irregularities such as structural breaks. Finally, each series is decomposed into a set of latent components that are directly estimated, thus illustrating the transparent nature of this approach. The nature of these components is that they are determined before the model is fitted

[1]This problem is avoided in this paper as a small data set comprising only eleven variables has been used.

[2]For an in depth discussion regarding the advantages of a state space approach refer to Durbin & Koopman (2001, pages 51–53)

and are based on the stylised characteristics of the data. In addition, their contribution may be gauged providing valuable insight into the underlying dynamics[3].

The formulation adopted in this paper resembles the state space or unobservable component form of the Beveridge-Nelson decomposition. This specification, in its univariate form, has been employed in many instances. Perhaps the most notable are Harvey & Jaeger (1993) and Proietti (2002). The multivariate form has also been employed, applications include Morley (2002) and Sinclair (2005).

The link between the state space formulation presented in this paper and the more common multivariate time series approaches has long been established (Harvey 1989, pages 431-432). It is important, however, to appreciate that analytical equivalence does not automatically imply empirical equivalence (Hyndman 2001, Morley et al. 2003).

The general specification for a N-variable system proposed is:

$$\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{\mu}_t + W\boldsymbol{\tau}_t + \Theta X_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_\varepsilon) && (3.1)\\
\boldsymbol{\mu}_{t+1} &= \boldsymbol{\delta} + \boldsymbol{\mu}_t + R_\mu \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \Sigma_\eta) && (3.2)\\
\boldsymbol{\tau}_{t+1} &= \Phi(L)\boldsymbol{\tau}_t + R_\tau \boldsymbol{\zeta}_t, \quad \boldsymbol{\zeta}_t \sim N(\mathbf{0}, \Sigma_\zeta), && (3.3)
\end{aligned}$$

where $\boldsymbol{y}_t$,$\boldsymbol{\mu}_t$ and $\boldsymbol{\tau}_t$ denote N-vectors of observations, permanent components, and temporary components at time t. Similarly $\boldsymbol{\delta}$ is also an N-vector and represents a set of constants. The term $\Phi(L)$ represents a polynomial function of the lag operator L, that is $\Phi(L) = \Phi_1 L + \Phi_2 L^2 + \ldots + \Phi_p L^p$ where $L^i y_t = y_{t-i}$. Two $N \times N$ coefficient matrices are specified, Φ and W. The Φ's are estimated subject to the roots of the polynomial function being larger than one, that is stationarity is imposed. The disturbances are assumed to be diagonal, independent and follow a Gaussian distribution. They are denoted as $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\zeta}_t$ and $\boldsymbol{\eta}_t$. The matrices R_μ and R_τ are coefficient matrices of dimension $N \times N$. For the remainder of this paper R_τ is constrained to be identity matrix (I_N). The term X_t denotes a set of exogenous variables at time t. The coefficient matrix Θ measures the influences of the exogenous variables. In the context of this paper, X_t corresponds to a set of q dummy variables.

The first equation, equation (3.1), represents the observation equation. It depicts the vector of observations being comprised of two latent components, the permanent and transitory component. The transitory component feeds into the observation equation through the coefficient matrix W.

[3]As the aim of this paper is demonstrate the forecasting advantages of this specification the estimated components are not presented. They are available upon request.

Equations (3.2) and (3.3) represent transition equations. The first of these is the permanent component which is specified to be a random walk with drift. This may also be referred to as the long run component. The second of these equations is referred to as the transitory or short run component. The influence of this component declines as the horizon increases. The temporary component is often referred to as the cyclical component. The long run character of this specification may be summarised as:

$$\lim T \to \infty \qquad \boldsymbol{y}_T = \boldsymbol{\mu}_0 + \boldsymbol{\delta}(T-2) + R_\mu \sum_{i=1}^{T-1} \boldsymbol{\eta}_{t-i} + W \sum_{i=1}^{T-1} \Phi(L)\boldsymbol{\zeta}_{T-i}. \tag{3.4}$$

Equation (3.4) shows that a set of observations at time T is determined by a set of constants $\boldsymbol{\mu}_0$, deterministic linear trends (with growth rates), stochastic trends $\sum_{i=1}^{T-1} \boldsymbol{\eta}_{t-i}$ and stochastic mean reverting processes $\sum_{i=1}^{T-1} \Phi(L)\boldsymbol{\zeta}_{T-i}$.

3.1 Adopting a SVAR characterisation

In practice, before a SVAR can be fitted, the variables must be made stationary. This is typically done by a series of transformations. In contrast, the structural state space specification models the non-stationarity component explicitly in the form of a permanent component equation.

By letting $\tilde{\boldsymbol{y}}_t$ denote an N-vector of stationary observations at time t, for $t = 1, 2, \ldots T$, a SVAR(3) may be written as:

$$B\tilde{\boldsymbol{y}}_t = \Psi_1\tilde{\boldsymbol{y}}_{t-1} + \Psi_2\tilde{\boldsymbol{y}}_{t-2} + \Psi_3\tilde{\boldsymbol{y}}_{t-3} + \boldsymbol{e}_t, \;\; \boldsymbol{e}_t \sim N(\mathbf{0}, \Sigma_e) \tag{3.5}$$

where B and Ψ_i $(i = 1, 2, 3)$ denote coefficient matrices of dimension $N \times N$. In order for B to be identifiable, there must be $\frac{N(N-1)}{2}$ restrictions imposed. Typically this is achieved by constraining B to be lower triangular. In addition, the covariance matrix Σ_e is also constrained to be diagonal.

The transitory component of the structural state space model captures the stationary dynamics of a series and therefore may be formulated to adopt the character of a SVAR specification. In particular, the characteristics of the SVAR specification denoted in equation (3.5) may be incorporated into the structural state space model in the following way:

$$\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{\mu}_t + W\boldsymbol{\tau}_t + \Theta X_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_\varepsilon), && (3.6)\\
\boldsymbol{\mu}_{t+1} &= \boldsymbol{\delta} + \boldsymbol{\mu}_t + R_\eta \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \Sigma_\eta), && (3.7)\\
\boldsymbol{\tau}_{t+1} &= \Phi_1\boldsymbol{\tau}_t + \Phi_2\boldsymbol{\tau}_{t-1} + \Phi_3\boldsymbol{\tau}_{t-2} + \boldsymbol{\zeta}_t, \qquad \boldsymbol{\zeta}_t \sim N(\mathbf{0}, \Sigma_\zeta). && (3.8)
\end{aligned}$$

The similarity between the SVAR model specified in equation (3.5) and the structural state space model (equations (3.6) to (3.8)) is revealed by equating $\boldsymbol{\tau}_t$ to $\tilde{\boldsymbol{y}}_t$, W to B and Φ to Ψ. Arguably, $\boldsymbol{\tau}_t$ can be regarded as being a proxy for $\tilde{\boldsymbol{y}}_t$.

3.2 Two Macroeconomic Models of the Australian Economy

Both models proposed incorporate the characteristics of the SVAR model proposed by Dungey & Pagan (2000). There was no particular reason for choosing the Dungey & Pagan (2000) model except that it is the most well known Australian SVAR specification. In general the characteristics of *any* SVAR may be imposed on the framework denoted by equations (3.6) to (3.8).

Two structural state space models are presented. The first treats the permanent components as being independent. In contrast, the second allows the permanent components to be related by estimating the off-diagonal elements of R_μ[4].

The data set employed differs from Dungey & Pagan (2000) in that nominal instead of real interest rates are used. The list of variables used are presented in Table 1.

OVERSEAS	
USGDP	US real Gross Domestic Product
TOT	Terms of Trade
USR	90-day US Treasury Bill
USQ	US Q ratio
EXPT	Real Chain Weighted Exports
DOMESTIC	
AUSQ	Australian Q ratio
GDP	Gross Domestic Product (Chained Volume Measure)
GNE	Real Gross National Expenditure
INF	Annual inflation rate
A3R	3-month Bank Bill
RTWI	Real Trade Weighted Index

Table 1: Brief description of data set, see Appendix A for more details.

[4]For identification purposes the diagonal values of R_μ are constrained to equal one

Dungey & Pagan (2000) summarised the philosophy behind their model in three points. First, Australia is a small open economy, therefore it cannot influence overseas markets. Second, a variable and all its lags would only be eliminated if it could be justified. Third, some equations, like the inflation equation, reflect the findings of single equation research. Having applied these three criteria, the restrictions applied to the coefficient matrices W and Φ are summarised in Tables 2 and 3.

	Independent Variable										
	USGDP	TOT	RUS	USQ	EXPT	AUSQ	GNE	GDP	INF	CASH	RTWI
USGDP	1										
TOT	*	1									
RUS	*		1								
USQ	*		*	1							
EXPT	*	*			1						
AUSQ	*	*		*		1					
GNE		*				*	1				
GDP	*	*	*			*	*	1			
INF		*					*		1		
CASH							*		*	1	
RTWI	*	*	*	*		*	*	*	*	*	1

Table 2: Contemporaneous Restrictions: * indicates a parameter is estimated, a value of 1 is imposed for all diagonal elements and blank cells indicates no parameter is estimated, i.e., restricted to zero

	Independent Variables										
	USGDP	TOT	RUS	USQ	EXPT	AUSQ	GNE	GDP	INF	CASH	RTWI
USGDP	*		*								
TOT	*	*	*								
RUS	*		*								
USQ	*		*	*							
EXPT	*	*			*						
AUSQ	*	*		*		*	*	*	*	*	*
GNE		*				*	*	*	**	**	*
GDP	*	*			*	*	*	*	**	**	*
INF		*					*		*		*
CASH							*		*	*	*
RTWI	*	*	*	*		*	*	*	*	*	*

Table 3: Restrictions on Autoregressive matrices, * denotes all lags of the variables are present, ** denotes only the second and third lags are present. Blank cell indicates no parameter is estimated, i.e., restricted to zero.

Enforcing these restrictions means that both state space models retain the key characteristics of the SVAR outlined by Dungey & Pagan (2000). That is, the block recursive nature of the system and the ordering of the variables. Although allowing the off-diagonals of R_μ introduces a new set of relationships, these are confined to the permanent component. The two macroeconomic state space models applied take the form previously stated except $R_\zeta = I_N$ (see equations (3.6) and (3.8)).

The first and second models will be referred to hereafter as SSM1 and SSM2 (structural state space model one/two). SSM2 will model the basic form of long run association between the eleven variables.

3.3 Estimation

Before the likelihood function is presented, the general form of equations (3.1) to (3.3) is formally defined. These equations may be rewritten in the form of a two equation system as:

$$\begin{aligned} \boldsymbol{y}_t &= Z_t\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, H) && (3.9)\\ \boldsymbol{\alpha}_{t+1} &= T\boldsymbol{\alpha}_t + R\boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim N(\mathbf{0}, Q). && (3.10)\end{aligned}$$

The models presented in the previous two sections are attained by setting:

$$Z_t = \begin{bmatrix} I & B & O_N & O_N & X_t \end{bmatrix}, \qquad \boldsymbol{\alpha}_t = \begin{bmatrix} \boldsymbol{\mu}_t \\ \boldsymbol{\tau}_t \\ \boldsymbol{\tau}_{t-1} \\ \boldsymbol{\tau}_{t-2} \\ \Theta \end{bmatrix}$$

$$T = \begin{bmatrix} I_N & O_N & O_N & O_N & O_{N,q} \\ O_N & \Phi_1 & \Phi_2 & \Phi_3 & O_{N,q} \\ O_N & I_N & O_N & O_N & O_{N,q} \\ O_N & O_N & I_N & O_N & O_{N,q} \\ O_N & O_N & O_N & O_N & I_q \end{bmatrix}, \qquad R = \begin{bmatrix} R_\tau & O_N & O_N & O_N & O_{N,q} \\ O_N & I_N & O_N & O_N & O_{N,q} \\ O_N & I_N & O_N & O_N & O_{N,q} \\ O_N & O_N & O_N & O_N & O_{N,q} \\ O_{q,N} & O_{q,N} & O_{q,N} & O_{q,N} & O_q \end{bmatrix}$$

$$\boldsymbol{\nu}_t = \begin{bmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\zeta}_t \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_q \end{bmatrix},$$

where I_k and O_k denote the identity and null matrices of dimension $k \times k$ respectively. The terms $\mathbf{0}_k$ and $O_{i,j}$ denote a k-vector of zeros and a null matrix of dimension $i \times j$.

The estimation of this framework is straightforward. The likelihood to be maximised is

$$\log L(\Lambda) = \frac{NT}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log |F_t| - \frac{1}{2} \sum_{t=1}^{T} \boldsymbol{\nu}_t' F_t^{-1} \boldsymbol{\nu}_t'. \tag{3.11}$$

The likelihood is calculated using the Kalman Filter. The prediction equations are given by

$$\boldsymbol{a}_{t+1} = T\boldsymbol{a}_{t|t}, \tag{3.12}$$

$$P_{t+1} = TP_{t|t}T' + RQR'. \tag{3.13}$$

The updating equations are

$$\boldsymbol{a}_{t|t} = \boldsymbol{a}_t + P_t Z' F_t^{-1} \boldsymbol{\nu}_t, \tag{3.14}$$

$$P_{t|t} = P_t - P_t Z' F_1^{-1} Z P_t', \tag{3.15}$$

where

$$\boldsymbol{\nu}_t = \boldsymbol{y}_t - \boldsymbol{d} - Z\boldsymbol{\alpha}_t, \tag{3.16}$$

$$F_t = ZP_tZ' + H. \tag{3.17}$$

Before the estimation procedure can be employed, a set of initial conditions need to be determined. These initial conditions specify seed values for the states ($\boldsymbol{\alpha}_0 = [\boldsymbol{\mu}_0, \boldsymbol{\tau}_0, \boldsymbol{\tau}_0, \boldsymbol{\tau}_0, \Theta_0]$) and their variances P_0 and Q_0. Starting values for the coefficients (W, Φ, $\boldsymbol{\delta}$ and R) also need to be determined.

Initial values for $\boldsymbol{\mu}$, $\boldsymbol{\tau}$ and $\boldsymbol{\delta}$ are determined simultaneously by running a linear regression model. Specifically, the first ten observations of each variable (y_i, $i = 1, .., N$) are regressed against a constant and a time trend. The initial value for the permanent state is set to be the constant. Similarly, the estimated linear time trend coefficient is used as the starting value for δ. Finally, the initial values of the temporary state[5] are set to be the median of the residuals.

As the estimation process is initiated with a diffuse prior, P is set to be an identity matrix with dimensions of $4N+3$, multiplied by a large number. Before this estimation

[5]Three initial values are required as an VAR(3) is being fitted

procedure is conducted however, the variances relating to Θ are constrained to be zero, thus Θ is time invariant.

Two other second moment matrices need to be given starting values. These are H and Q. The seed values for H correspond to the variance of each series. The structure that is imposed on P is also imposed on Q. The first N leading diagonals are set to be the variance of each series. The following $3N$ leading diagonals are set to be the variance of the first difference. Finally, the remaining three diagonals are set to be zero.

According to Table 2, only a subset of elements are non-zero. These non-zero elements are given a starting value of 0.8 as this was found to work well in practice. Each Φ was set to be diagonal. The diagonal values were determined by fitting an AR(3) to each variable.

As the exogenous variables are yet to be identified, the starting values for Θ will be discussed briefly in the next section. The final set of coefficients that require starting values are those in R_μ. As no prior information is available on what these values might be, the matrix is initially set to be an identity matrix.

3.4 Specification of exogenous variables

The model is fitted to a data set of quarterly observations spanning 20 years. These observations are non seasonal. The earliest observation corresponds to the first quarter of 1985. Observations in years 2005 and 2006 are retained for an out-of-sample forecast comparison. Plots of each variable is presented in appendix B.

Examination of the eleven variables suggests that three dummy variables is appropriate. Each of these indicator variables relate to a specific domestic macroeconomic variable. The first is a double pulse dummy which is assigned to AUSQ. This dummy variable captures the extraordinary growth observed in the second and third quarters of 1987. The variable is given a value of 1 for these quarters and zero elsewhere. The A3R variable exhibits a shift in the mean post 1992. Therefore the variable is specified to be one pre-1992 and zero elsewhere. A similar story is also evident for inflation. An inspection of the series reveals the Australian economy experienced considerably higher growth rates in the period 1985 to 1992, quarter 2. As such, a dummy variable is specified to capture this phenomenon.

Having identified the exogenous variables, the starting values for Θ can now be determined. In all three cases the linear trend equation that was outlined earlier is modified to include the dummy variable. The estimated coefficient is then used as the starting value for Θ.

4 Forecast comparison

The forecasting accuracy of the two structural state space models are compared by conducting a roll-out forecasting comparison. As indicated in an earlier section the observations corresponding to years 2005 and 2006 were withheld. These observations are now employed in a roll-out forecasting exercise. The maximum horizon length is eight quarters.

The first batch of forecasts spans eight horizons and was generated by fitting the SVAR, SSM1 and SSM2 to data ranging 1985 quarter 1 to 2004 quarter 4. The second batch of forecasts spans seven horizons and was generated by fitting the three alternatives to data ranging 1985 quarter 1 to 2005 quarter 1[6]. This incremental procedure was repeated eight times. The final prediction was a one step ahead forecast corresponding to quarter 4 of 2006.

The measure used to compare the forecast accuracy is the mean absolute scaled error (or MASE for short, Hyndman & Koehler 2006). This measure is chosen as it is numerically robust, unlike conventional measures such as the mean absolute percentage error (MAPE) and root mean square error (RMSE). Furthermore, the measure is unitless and as such can be averaged across series. For an h-step head forecast the MASE is:

$$\text{MASE(h)} = \frac{|e_{T+h}|}{\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}$$

As indicated above, the MASE is calculated by dividing the absolute forecast error by the average of the absolute within sample first difference.

Three aspects of forecast accuracy will be examined. The first will consider the accuracy across the entire data set. The second will focus on each variable separately. The last will compare the approaches across horizons.

As part of the proceeding forecast evaluation formal hypothesis tests are conducted. The test used is the Wilcoxen test. This test is a non-parametric test and therefore is a function of order rather than magnitude. The motivation for using this test is that at longer horizons the sample size is very small[7].

[6]Before the models were fitted each variable was standardised by dividing through by its standard deviation

[7]Performing a Wilcoxen test is in keeping with Diebold & Mariano (1995, page 255)

4.1 Results

The relative forecasting accuracy of the state space models outlined earlier are assessed in this section. The comparison begins by analysing overall forecasting accuracy.

	Overall			Domestic Variables		
Statistic	SVAR	SSM1	SSM2	SVAR	SSM1	SSM2
Min.	0.000	0.000	0.001	0.001	0.000	0.001
1st Qu.	0.013	0.030	0.020	0.016	0.016	0.013
Median	0.036	0.063	0.047	0.043	0.035	0.026
3rd Qu.	0.090	0.134	0.125	0.107	0.064	0.049
Max.	1.089	0.673	0.698	1.089	0.336	0.335
Mean	0.090	0.109	0.097	0.134	0.056	0.045

Table 4: Five Number Summary and Mean of Forecast Accuracy by Model

Table 4 displays a statistical summary of the overall forecasting accuracy for each of the three approaches. By construction, all the scores are strictly non-negative. As each raw score measures forecast error, a relatively smaller value is desirable.

The statistical measures used to summarise forecasting performance are the minimum, 1^{st} quartile, median, 3^{rd} quartile, maximum and mean. Table 4 is separated into two sections. The first is a general overview, the second considers a subset corresponding to the *domestic* variables $\{$AUSQ, GNE, GDP, INF, A3R, RTWI$\}$.

For the domestic subset each statistic (excluding the minimum) indicates that the state space models are noticeably more accurate. This is especially true for SSM2 where long run interrelationships are explicitly modeled. This conclusion is best illustrated by a comparison of the means, which show that the state space models have inaccuracy measures less than half of that of the SVAR.

A similar analysis of the forecasting accuracy at the overall level does not indicate the same conclusions, however the maximum values seem to indicate that the state space models are more robust. Furthermore, as shown shortly, these results are skewed by two poor performances and therefore not indicative of the true overall performance. In addition, as the model is designed for an Australian context, it is the variables which are within Australia's sphere of control which are of primary focus. It is these *domestic* variables that the state space models are better at predicting.

The second comparison examines forecasting accuracy on a variable by variable basis. Positive values indicate that the state space models are more accurate. The Box and Whisker plots show that in eight instances the state space alternatives

Variable	SSM1	SSM2
USGDP	1.000	1.000
TOT	1.000	1.000
RUS	1.000	1.000
USQ	1.000	0.972
EXP	1.000	0.999
AUSQ	1.000	1.000
GDP	1.000	0.951
GNE	1.000	0.960
INF	$>$0.001	$>$0.001
A3R	0.020	0.004
RTWI	$>$0.001	$>$0.001

Table 5: P-values relating to a one-sided Wilcoxon test for each variable, the test being: $H_0 : SSM_{MASE} \geq SVAR_{MASE}$ v $H_1 : SSM_{MASE} < SVAR_{MASE}$

performed at least as well as the SVAR alternative. In particular, the state space approaches performed noticeably better for the variables: USQ, EXPT, INF, A3R and RTWI.

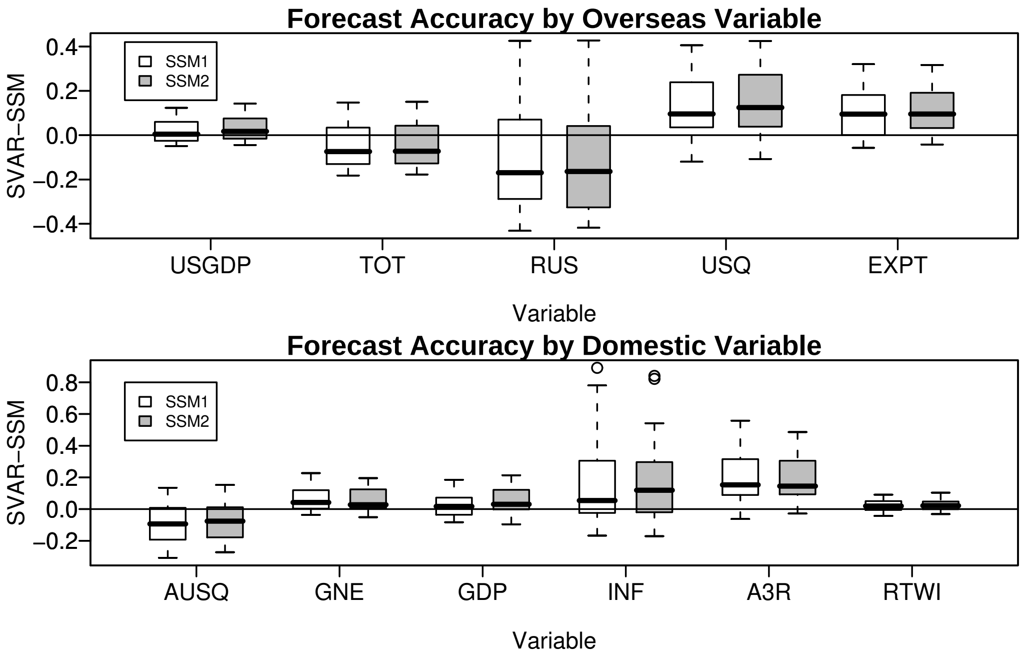

Figure 1: Forecast Comparison by Model.

Table 5 formally evaluates the differences between the state space models and the SVAR alternative. The test employed was the Wilcoxon-test which is a non-parametric test with the null corresponding to the case when the state space forecasts are greater than or equal to the the SVAR alternative.

The results presented in Table 5 indicate that the state space approaches performed significantly better for three variables, inflation, Australian three month treasury rate and real trade weighted index.

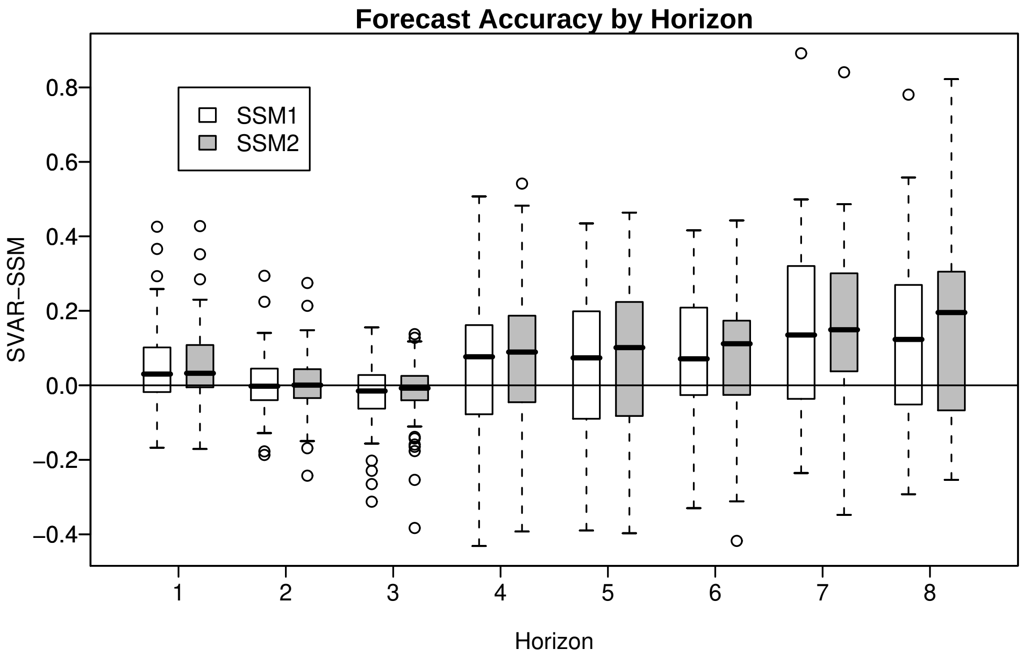

Figure 2: Forecast Comparison by Horizon (Domestic subset only).

Figure 2 displays the relative forecast accuracy for over horizons one to eight for the domestic subset only. Careful examination of this figure suggests that as the horizon increases the state space alternative becomes relatively more accurate. This is especially true for the SSM2 which models the long run interrelationships.

Horizon	SSM1	SSM2
1	0.066	0.008
2	0.175	0.061
3	0.037	0.006
4	0.040	0.001
5	0.071	0.015
6	0.036	0.011
7	0.042	0.014
8	0.094	0.031

Table 6: P-values relating to a one-sided Wilcoxon test for each horizon (domestic subset only), the test being: $H_0 : SSM_{MASE} \geq SVAR_{MASE}$ v $H_1 : SSM_{MASE} < SVAR_{MASE}$

Using the Wilcoxon-test to verify the pattern observed in the box and whisker plots confirms the better performance of the state space approach. According to Table 6, at the 10% significance level only once can the null not be rejected. That is, across all horizons for the SSM2, and all but the second horizon for SSM1, the forecasts from the state space alternative are superior.

5 Conclusion

The purpose of writing this paper was twofold, one to illustrate how the state space approach can be formed to represent a small open macroeconomic model and two, demonstrate that forecasts from this adaption are accurate. On both accounts this paper has satisfied these objectives.

In particular, the first objective was achieved (in part) by showing how the character of a SVAR specification may be incorporated into a structural state space model. Furthermore, long run (inter) relationships were explicitly modeled and showed to have a positive effect on forecast accuracy at longer horizons. Also, the advantages of the state space model were presented, these being generality, flexibility and transparency.

Improvements in forecast accuracy have also been demonstrated. This was particularly true for the domestic subset. In general the Box and Whisker plots of the previous section demonstrate that the structural state space model performed at least as good as the SVAR on eight occasions.

In summary, the evidence provided suggests that the structural state space approach is a useful forecasting tool that can yield significant improvements when compared to a standard alternative.

References

Aoki, M. & Havenner, A. (1991), 'State space modeling of multiple time series', *Econometric Reviews* **10**, 1–59.

Balke, N. S. & Wohar, M. E. (2002), 'Low-frequency movements in stock prices: a state-space decomposition', *The Review of Economics and Statistics* **84**(4), 649–667.

Berkelmans, L. (2005), Credit and monetary policy: An australian SVAR, Technical report, Reserve Bank of Australia.

Buncic, D. & Melecky, M. (2008), 'An estimated new keynesian policy model for australia', *The Economic Record* **84**(264), 1–16.

Diebold, F. X. & Mariano, R. S. (1995), 'Comparing Predictive Accuracy', *Journal of Business and Economic Statistics* **5**, 253–263.

Dixon, P., Parmenter, B., Sutton, J. & Vincent, D. (1997), *ORANI: A Multisectoral Model of the Australian Economy*, North Holland.

Dixon, P. & Rimmer, M. (2002), *Dynamic General Equilibriun Modelling For Forecasting And Policy*, North Holland.

Dungey, M. & Pagan, A. (2000), 'A structural VAR of the Australian economy', *The Economic Record* **76**, 321–342.

Durbin, J. & Koopman, S. (2001), *Time Series Analysis by State Space Method*, Oxford.

Fry, R., Hocking, J. & Martin, V. (2008), 'The role of portfolio shoocks in a structural autoregressive model of the australian economy', *The Economic Record* **84**(264), 17–33.

Harvey, A. C. (1989), *Forecasting, structural time series models and the Kalman filter*, Cambridge University Press, Cambridge.

Harvey, A. & Jaeger, A. (1993), 'Detrending, stylised facts and the business cycle', *Journal of Applied Econometrics* **8**, 231–247.

Hyndman, R. J. (2001), 'It's time to move from 'what' to 'why'', *International Journal of Forecasting* **17**, 567–570.

Hyndman, R. & Koehler, A. B. (2006), 'Another look at measures of forecast accuracy', *International Journal of Forecasting* **22**, 679–688.

Matheson, T. (2006), Assessing the fit of small open economy dsges, Technical report, Reserve Bank of New Zealand.

Moosa, I. (1998), 'An investigation into the cyclical behaviour of output, money, stock prices and interest rates', *Applied Economic Letters* **5**, 235–238.

Morley, J. C. (2002), 'A state-space approach to calculating the Beveridge-Nelson decomposition', *Economics Letters* **75**, 123–127.

Morley, J. C., Nelson, C. R. & Zivot, E. (2003), 'Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different?', *The Review of Economics and Statistics* **75**(2), 235–243.

Nimark, K. (2007), A structural model of australia as a small open economy, Technical report, Reserve Bank of Australia.

Pagan, A. (2003), An examination of some tools for macro-econometric model building, *in* 'METU Lecture at ERC Conference VII, Ankara'.

Panher, G. S. (2007), 'Modelling and controlling monetary and economic identities with constrained state space models', *International Statistical Reveiw* **75**, 150–169.

Proietti, T. (2002), Some reflections on trend-cycle decompositions with correlated components, Working Paper, Department of Economics, European University Institute.

Sinclair, T. (2005), Permanent and transitory movements in output and unemployment: Okun's law persists, Working Paper, Department of Economics, Washington University.

A Description of Data used

USGDP Real Gross Domestic Product of USA. Data Source: Bureau of Economic Analysis {http://www.bea.gov/}, issue date 20/12/2007.

TOT Terms of Trade (AUS). Data Source: Australian Bureau of Statistics, Catalogue 5206, Table 1.

USR 3 month US Treasury Bill. Data Source: Federal Reserve. Identifier: H15/H15/RIFSGFSM03_N.M. This series was calculated by averaging the monthly observations for each quarter.

USQ US Q ratio. Calculated by averaging the monthly S&P500 index and then dividing it by the USA Implicit Price Deflator. The IPD was downloaded from the Bureau of Economic Analysis.

EXPT Real exports of Goods and Services. Source: Datastream, Code: AUOCFEGSD.

AUSQ Australian Q ratio, calculated in the same way as USQ. Date Source: Austalian Bureau of Statistics (ASX200) and Reserve Bank of Australia (IPD).

GDP Chain Weighted Volume Gross Domestic Product of Australia. Source ABS, Table 5206.0.

GNE Australian Gross National Expenditure. Source: Datastream

INF Consumer Price Index (all groups), source DataStrean

A3R 3-month Bank Bill. Average of monthly observations. Source: RBA

RTWI Real Trade Weighted Index, source RBA

B Plots of the Data

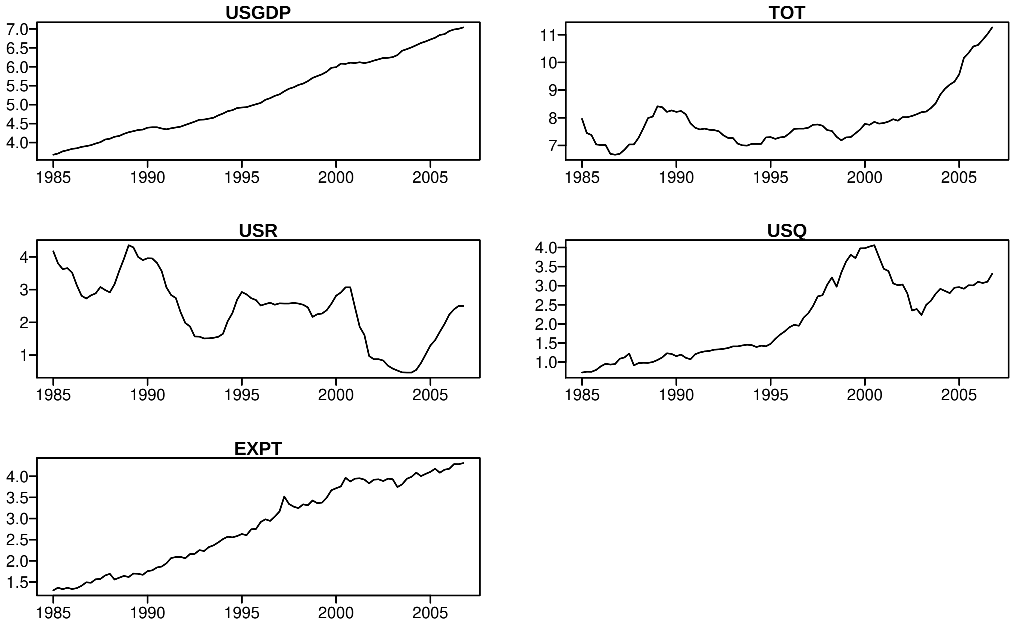

Figure 3: Plots of overseas.

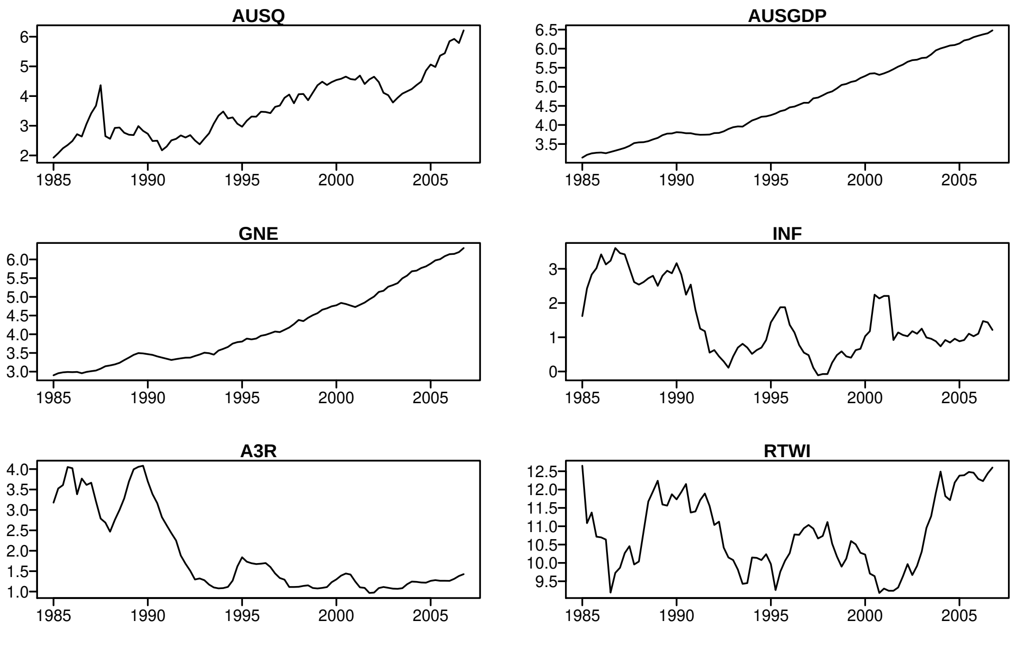

Figure 4: Plots of domestic variables.