Rent Seeking Behavior and Optimal Taxation of Pollution in Shallow Lakes

Salerno, Gillian and Beard, Rodney and McDonald, Stuart

University of Alberta, University of Alberta, California Institute of Technology

2007

Rent Seeking Behavior and Optimal Taxation of Pollution in Shallow Lakes

Gillian Salerno[a] , Rodney Beard[b] and Stuart McDonald [c]

[a] *Student, Department of Mathematics, University of Alberta Edmonton AB T6G 2H1 Canada*
[b] *School of Business University of Alberta, Edmonton AB T6G 2R6 Canada*
[c] *Social and Information Systems Laboratory, California Institute of Technology, Pasadena CA 91125-9300 USA*

Abstract

In this paper we extend earlier work on the economics of shallow lakes by Mäler, Xepapadeas and de Zeeuw (2003) to the case where two communities have incommensurable preferences about lake eutrophication. In the case of incommensurable preferences interest group behavior arises, we therefore consider the case where society is divided into two interest groups and is thus unable to agree on a single management objective. In particular, the communities that share the use of the lake disagree on the relative importance of the shallow lake acting as a waste sink for phosphorus run-off as opposed to other ecosystem services. A dynamic game in which communities maximize their use of the lake results in a Nash equilibrium where the lake is in a eutrophic state when in fact the Pareto optimum would be for the lake to be in an oligotrophic state. Our paper differs from previous work by considering two communities or interest groups with different preferences for environmental services. The tax that would induce, in a noncooperative context, all of society's members to behave in such a way as to achieve a Pareto optimal outcome is derived under the assumption that a social planner does not favor one community or another. We then ask whether or not such a tax rate would in fact be implemented if each community were able to bear political pressure on the social planner and the social planner were a public representative seeking reelection. In this case both types of communities lobby to have their preferred level of tax applied based on their relative preferences for a clean lake and phosphorus loading. The effects of the lobbying on the application of the optimal tax are investigated numerically for particular values of relative preferences and the relative size of each group. The representative seeking election proposes a different tax rate in order to maximize their probability of electoral success. This problem is solved numerically assuming that the lake is in a eutrophic equilibrium. It is shown that political representatives have an incentive to propose tax rates that are insufficient to achieve a return to an oligotrophic steady-state

Key words: Pollution of shallow lakes; optimal eco-taxation; dynamic rent seeking. *JEL classification codes:* D72; H23; Q25.

1 Introduction

Certain bodies of water, and in particular shallow lakes, present a hysteresis in their response to phosphorus loading. That is, the shallow lake, subject to gradual increases in phosphorous concentration, will remain in a relatively pristine state or oligotrophic state over long periods of time until a point at which it will suddenly flip to a turbid or eutrophic state. Once the transformation has occurred, the lake remains eutrophic despite following even reductions in phosphorus loading below those levels that preceded the flip to the eutrophied state. The unique threshold point at which the lake flips between alternative basins of attraction represents society's point of indifference between the two states and it is known as a Skiba point (Wagener, 2003). For the shallow lake, the Skiba point identifies the point at which the lake changes from a clear habitat providing a high level of ecosystem services into turbid waters that contain an overabundance of aquatic plant life, which often leads to the development of toxic algal blooms.

Several authors have applied economics to resolving the eutrophication problem while taking into account the shallow lake dynamics. [1] In particular, Dechert and Brock (2000) were the first to pose the problem as a

[1] Carpenter et al. (1999) were first to integrate the dynamics of the shallow lake into economic analysis. They pose a lake dynamic equation with respect to phosphorous such that it can be used for economic analysis. Dechert and Brock (2000) first posed the problem as a dynamic game of communities each maximizing its welfare in its use of the lake and identified the presence of Skiba points when more than two communities share the use of the lake. They provide a solution to the open-loop problem, while Mäler et al. (2003) propose a tax as the optimal policy to induce a Pareto-optimal solution to the game. Grüne et al. (2005) use dynamic programming to solve the problem for the closed loop Nash equilibrium. A stochastic version of this problem has also been formulated by O'Donnell and Dechert (2004),Dechert and O'Donnell (2005) to account for the possibility that the phosphorous loading into the lake is subject to rainfall as a random shock.

dynamic game between communities sharing the use of the lake. In their model they assume that society as a whole benefits from the lake not only acting as a waste-sink for agriculture, but also providing clean water for other uses, including consumption and recreation. Hence, communities that share the lake will attribute the same relative preferences to alternative uses of the lake. This means that each community will have the same welfare function derived from the benefits of the lake.

As shown by Mäler et al. (2003) in their analysis, a tax on phosphorus loading can achieve a Pareto-optimal state for the lake when each community seeks to maximize its welfare non-cooperatively. They assume that all communities have the same relative preferences towards the benefits of the lake and hence every community has an identical welfare function.

As noted by Mäler et al., it is possible that different interest groups may not be able to agree on a common welfare function. Indeed, when communities benefit in different proportions from polluting the lake relative to other uses, it is not possible to attribute the same welfare function to all communities. In this case, each community acting to maximize its specific welfare function will constitute a new game with a different Nash equilibrium, which in turn will require a new tax to induce the Pareto-optimal level of phosphorus loading. Our paper considers this scenario: the communities that share the lake are divided into two interest groups, each with a different welfare function charactizing the group's collective preferences for the use of the lake. Each group can be thought of as a type of community where one type is predominantly agricultural and benefits more from the lake acting as a waste sink for phosphorus loading possibly leading to eutrophication, and the other type, a green community, with a higher preference for an oligotrophic lake. The green communities have a preference for an oligotrophic lake and consider that the current tax rate is too low. The n communities face an election to elect a single politician. The politician favoured by the green communities promises to implement a higher tax on phosphorus that will bring phosphorus loading down so as to reverse the lake to an oligotrophic state. The farming communities favour a politician who promises to maintain the tax at its current low level.

Our paper is structured as follows. In section 2, we review the shallow lake mode. In section 3 we solve the dynamic game for the Pareto-optimal outcome induced by means of a tax on phosphorus loading and the open-loop Nah equilibrium of the lake game with two interest groups. In section 4, we explore the consequences of lobbying and rent-seeking on the Pareto-optimal tax and on the state of the lake. We show that as a result of lobbying, the optimal tax policy may not be implemented and that, due to the hysteretic nature of the shallow lake, a tax only slightly below the optimal policy may result in cumulative effects that precipitate the lake into a potentially irreversible eutrophic state. Section 5 concludes.

2 The Lake Game with Two Interest Groups

This section presents the model of a shallow lake provided by Carpenter et al. (1999). The lake equation they propose will be used as the state equation in the economic analysis that follows. In this model, the limiting factor for eutrophication is phosphorus. Lake eutrophication dynamics are based on total available phosphorus as the state variable, and phosphorus input as the control variable. Although nitrogen is also known to stimulate plant growth, phosphorus is thought to be the limiting nutrient of plant growth in many cases (Ricklefs, 1979). In addition, cyanobacteria, that are contained in eutrophied lakes, have the ability to fix nitrogen from the atmosphere, and therefore their growth will be limited by the available phosphorus (Alaouze, 1995).

When the nutrient level of the lake is low, the plants tend to be small and the water clear. Increases in nutrient loading, however, encourage the development of larger plants and of phytoplankton. These plants and the surface layer of phytoplankton create shade and turbidity, which leads to the collapse of the vegetation that does not tolerate shade. This further favours the development of phytoplankton, and can result in the emergence of toxic algal blooms, consisting of cyanobacteria, which are shade tolerant (Scheffer, 1998).

Shallow lakes are observed to be different from deep lakes because they tend to be polymictic, i.e. have a mixed water column most of the year, as opposed to deep lakes where layers of water with different temperatures form in the summer months, thus hindering the recycling of nutrients from the sediments on the bottom of the lake into the water column. In addition, the larger proportion of a shallow lake's water, compared to that of a deep lake, that is in contact with the lake bed increases the rate of recycling of nutrients into the water column. This means that more nutrients are available to consumers, including plants and algae. As a result, contrary to deep lakes, where vegetation is sparse and more present around the edges, shallow lakes are often filled with aquatic plants (Scheffer, 1998). The higher rate of recycing of nutrients into the water column tends to make shallow lakes hysteretic or irreversible in their response to phosphorus loading.

We thus consider the following initial value problem to model the lake's response to phosphorus stock and loading.

$$\frac{dP}{dt} = L(t) - sP(t) + \frac{rP^q(t)}{m^q + P^q(t)}, P(0) = P_0 \tag{1}$$

where the variables $P(t)$ and $L(t)$ are, respectively, the stock of phosphorus and the external input of phosphorus at time t. Model parameter are given as follows: s is the rate of loss of phosphorus from the stock, r is the maxiumum rate of recycling of P, m is the value of P at which recycling reaches half the maxiumum rate r and q is the parameter that provides the steepness of the recycling response to the stock of phosphorus.

To make the problem scale invariant, the following substitutions are made:[2] $P = x/m, L = ar$ and $s = br/m$. Further, by changing the time scale to tr/m, one obtains the following equations for the shallow lake dynamics:

$$\dot{x}(t) = a(t) - bx(t) + \frac{x^2}{x^2+1}, x(0) = x_0 \tag{2}$$

It can be seen that in the steady state that the external loading of phosphorus is a function of the stock of phosphorus. it may seem at first sight strange from a biological point of view to present loading as a function of the stock of phosphorus. However, the objective of policy is to manage phosphorus content in the lake and, as pointed out by Grüne et al. (2005), "[t]he management can measure the stock and can control the loading as a function of the stock".

Carpenter et al. (1999) identify three categories of lakes based on their response to increases and decreases of phosphorus input once they are eutrophied: fully reversible, hysteretic and irreversible. Our focus is an economic model of which the goal is to address eutrophication through policy aimed at mitigating phosphorus input alone. By analyzing equation 2, we find, as do Mäler et al. (2003), that for $q = 2$ and $\frac{1}{2} \leq b \leq \frac{3}{8}\sqrt{3}$, the lake displays a reversible hysteresis in its response to phosphorus loading[3]. Hence, these are the parameters that will be used to model the shallow lake that can be reversed from a eutrophic back to an oligotrophic state for the remainder of our analysis. Note that in this case, eutrophication is reversible by simple control of external phosphorus input. All that is needed to keep the lake

[2]Refer to Murray (1989) pp. 5 and 652 for a more detailed description of this technique. Carpenter et al. (1999) also make use of it in Appendix A of their article, as do Mäler et al. (2003)

[3]Refer to Mäler et al. (2003) for the derivation of these parameters.

in an oligotrophic state is to manage levels of external phosphorus loading without any requirement for more costly measures to alter the rates of phosphorus sedimentation or recycling.

3 Pareto-optimal Phosphorus Loading with Two Interest Groups

We consider that society is made up of two groups: agricultural communitites and green communities. The agricultural communities are predominantly made up of farmers who privately benefit from applying fertilizer and, by proxy, from phosphorus loading into the lake. The green communities are predominantly made up of people who, although they benefit from the application of fertilizer to crops because they consume agricultural products, have a high preference for an oligotrophic lake.

To capture the differing preferences for these two benefits, we adopt the welfare function used by Mäler et al. (2003) and modify it to create two welfare functions that each represents the preferences of the two groups. In this scenario, the farmers attach very low importance c_1 to the ecosystem services provided by the lake, and the green communities attach a relatively high importance c_2 to ecosystem services and so $c_1 < c_2$. The total n communities previously considered can be divided into n_1 agricultural communities and n_2 green communities.
Each agricultural community i's welfare function is thus given by

$$W_i = \ln a_i - c_1 x^2, \;\; i = 1, \ldots, n_1 \tag{3}$$

while each green community j's welfare function is given by

$$W_j = \ln a_j - c_2 x^2 \;\; j = 1, \ldots, n_2 \tag{4}$$

3.1 Pareto-optimal Phosphorus Loading

A social planner acting on behalf of citizens will want to optimize social welfare. To achieve this Pareto-optimal solution, the planner needs to first find the total amount of phosphorus loading a that will maximize the sum of the communities' welfare functions:

$$\max \int_0^\infty e^{-\rho t} \left[\sum_{i=1}^{n_1} \ln a_i(t) - n_1 c_1 x^2(t) + \sum_{j=1}^{n_2} \ln a_j(t) - n_2 c_2 x^2(t) \right] dt,$$

$$\text{s.t } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1}; x(0) = x_0$$

$$\text{where } a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=2}^{n_2} a_j(t). \quad (5)$$

The current value Hamiltonian for this equation is given by:

$$H^c(t) = \sum_{i=1}^{n_1} \ln a_i(t) - c_1 n_1 x^2(t) + \sum_{j=1}^{n_2} \ln a_j(t) - c_2 n_2 x^2(t) + \lambda(t) \left[a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1} \right], \quad 0 \leq t < \infty,$$

where $\lambda(t) = e^{\rho t}\mu(t)$.

The first order conditions for the optimal control problem are as follows:

$$\frac{dH^c}{da_i(t)} = \frac{1}{a_i(t)} + \lambda(t) = 0, \quad i = 1, \ldots, n_1 \quad (6)$$

$$\frac{dH^c}{da_j(t)} = \frac{1}{a_j(t)} + \lambda(t) = 0, \quad j = 1, \ldots, n_2 \quad (7)$$

$$\frac{dH^c}{d\lambda(t)} = a(t) - bx(t) + \frac{x^2(t)}{(x^2(t)+1)^2}, \quad (8)$$

from which we also obtain the co-state equation:

$$\dot{\lambda}(t) = 2x(t)(n_1 c_1 + n_2 c_2) + \lambda(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right], \quad 0 \leq t < \infty \quad (9)$$

Together with equation (9) and the transversality conditions for equations (9) and (8), equations (6) and (7) imply that

$$\lambda(t) = -\frac{1}{a_i(t)} = -\frac{1}{a_j(t)}, \qquad i = 1, \ldots, n_1, \;\; j = 1, \ldots, n_2 \quad (10)$$

This further implies that $a_i(t) = a_j(t)$ for all $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$ and hence,

$$\lambda(t) = -\frac{(n_1+n_2)}{a(t)} \qquad \text{and} \qquad \dot{\lambda}(t) = \frac{(n_1+n_2)\dot{a}(t)}{a^2(t)}, \quad 0 \leq t < \infty \quad (11)$$

When the above two equations are substitued into the co-state equation, choosing loading to be constant over time, we obtain the following relationship between phosphorus loading and the internal phosphorus input:

$$\bar{a} = \frac{(n_1 + n_2)}{2x(t)\,(c_1 n_1 + n_2 c_2)} \left[b + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \tag{12}$$

This solution, which represents the Pareto-optimal constant loading path, can be plotted in the (x, a)-plane together with the phase plot for the steady-states of the lake when $dx/dt = 0$, given by equation (2). The intersection of the two curves provides society's optimal phosphorus loading quantity. Using the hysteretic lake value $b = 0.6$, $\rho = 0.03$, $n_1 = 2$, $n_2 = 2$, $c_1 = 0.2$ and $c_2 = 2$, the curves intersect at $(x^*, a^*) = (0.3472, 0.1007)$, as shown in Figure 1 below. Note that this point lies below the point at which the lake flips to a eutrophic state, that is, for the selected parameters, society prefers an oligotrophic lake.

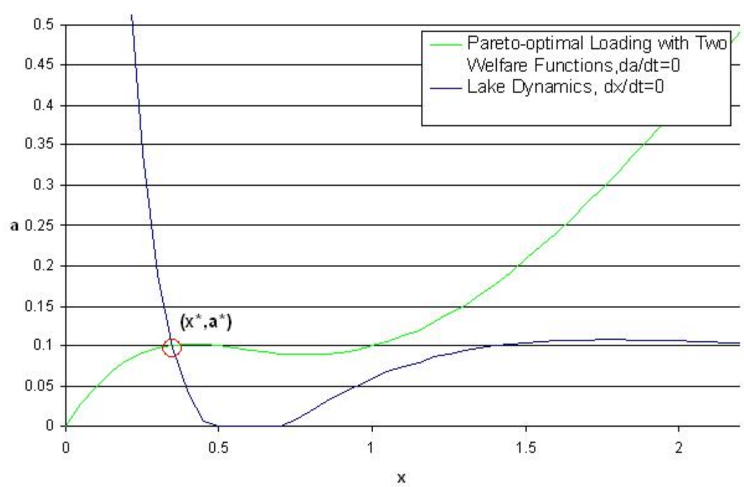

Figure 1: Pareto-optimal Loading with Two Welfare Functions

3.2 Open-loop Nash Equilibria

In the absence of management, however, each community maximizes its own utility according to its welfare function. This leads to an infinite-horizon dynamic game between the agricultural and green communities. This dynamic game is characterized by the following set of equations:

$$\max_a \int_0^\infty e^{-\rho t} \left[\ln a_i(t) - c_1 x^2(t)\right] dt, \quad i = 1, \ldots, n_1 \tag{13}$$

$$\max_a \int_0^\infty e^{-\rho t} \left[\ln a_j(t) - c_2 x^2(t)\right] dt, \quad j = 1, \ldots, n_2 \tag{14}$$

$$\text{s.t.}\dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1}, x(0) = x_0, \tag{15}$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t). \tag{16}$$

Setting up a current value Hamiltonian and solving first order conditions yields the following co-state equations:

$$\dot{a}_i(t) = 2{a_i}^2(t) n_1 c_1 x(t) - n_1 a_i(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], \quad i = 1, \ldots, n_1$$

and

$$\dot{a}_j(t) = 2{a_j}^2(t) n_2 c_2 x(t) - n_2 a_j(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], \quad j = 1, \ldots, n_2$$

When aggregated, these two equations can be used to derive the following equation describing the dynamics of total phosphorus input:

$$\dot{a}(t) = 2x(t) \left[c_1 \sum_{i=1}^{n_1} {a_i}^2(t) + \sum_{j=1}^{n_2} {a_j}^2(t)\right] - a(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], 0 \leq t < \infty \tag{17}$$

Solving for constant phosphorus loading, $da/dt = 0$, one obtains the steady-state open-loop Nash equilibrium for total loading $\tilde{a}$:

$$\tilde{a} = \frac{1}{2x(t)} \left[\frac{n_1}{c_1} + \frac{n_2}{c_2}\right] \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]. \tag{18}$$

Again, this solution is plotted in the (x, a)-plane together with the phase plot for the steady-states of the lake when $\dot{x} = 0$. The intersection of the two curves gives the Nash equilibrium phosphorus loading solutions. Using the hysteretic lake value $b = 0.6$, $\rho = 0.03$ and $n_1 = 2$, $n_2 = 2$, $c_1 = 0.2$ and $c_2 = 2$, the curves intersect at $(0.4485, 0.1016)$, $(0.7402, 0.0902)$ and $(3.1832, 0.9976)$, as show in Figure 2. Point $(0.7402, 0.0902)$ is an unstable skiba point, i.e., a small variation in loading will cause the equilibrium to shift to either the lower equilibrium at $(0.4485, 0.1016)$ or the higher equilibrium at $(0.7402, 0.0902)$. Note that both of these points are above the point at which the lake flips to a eutrophic state. This means that when the green communities and agricultural communities do not cooperate, the lake will be in a eutrophic state.

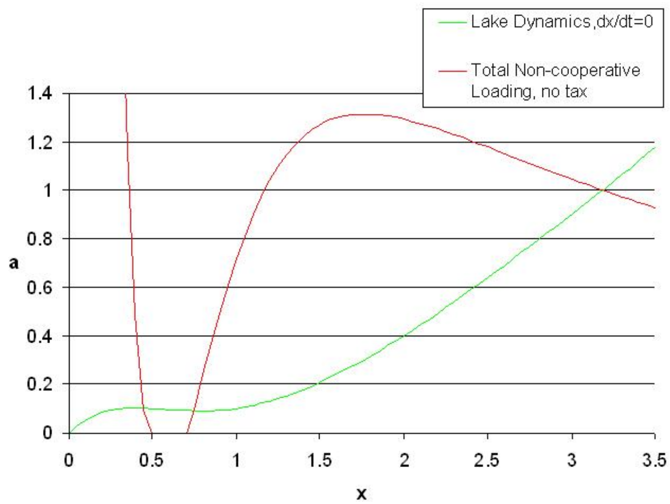

Figure 2: Nash Equilibrium Loading with Two Welfare Functions

3.3 Optimal Taxation with Two Interest Groups

A social planner will therefore want to find the tax rate that will achieve the optimal social welfare outcome derived in section 3.1 without the need for direct management of phosphorus loading. The effect of the tax will be to modify each community's welfare function and induce each one to modify its phosphorus loading accordingly. With the tax, the agricultural and green communities will each

$$\max_{a_i} \int_0^\infty e^{-\rho t} \left[\ln a_i(t) - \tau(t) a_i(t) - c_1 x^2(t) \right] dt, \quad i = 1, \ldots, n_1$$

and

$$\max_{a_j} \int_0^\infty e^{-\rho t} \left[\ln a_j(t) - \tau(t)a_j(t) - c_2 x^2(t)\right] dt, \quad j = 1, \ldots, n_2 \tag{19}$$

$$\text{s.t.}\dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1}, \quad x(0) = x_0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t)$$

Setting up a current value Hamiltonian and solving first order conditions for each of these two problems yields:

$$\tau(t) = \frac{1}{a_i(t)} + \lambda_i(t) = \frac{1}{a_j(t)} + \lambda_j(t), \qquad \forall i = 1, \ldots, n_1,\ j = 1, \ldots, n_2 \tag{20}$$

This implies that the following holds:

$$(n_1 + n_2)\tau(t) = \sum_{i=1}^{n_1} \frac{1}{a_i(t)} + \sum_{j=1}^{n_2} \frac{1}{a_j(t)} + \sum_{i=1}^{n_1} \lambda_i(t) + \sum_{j=1}^{n_2} \lambda_j(t)$$

From Section 3.1, the first two terms on the right hand side are equal to $-(n_1 + n_2)\lambda(t)$, and so the optimal rate of taxation can be expressed as follows:

$$\tau(t) = -\lambda(t) + \frac{1}{(n_1 + n_2)} \left[\sum_{i=1}^{n_1} \lambda_i(t) + \sum_{j=1}^{n_2} \lambda_j(t)\right] \tag{21}$$

Note that this is in parallel with Mäler et al.'s result for the single welfare function case. This result implies that the optimal ecotax must bridge the gap between the aggregate shadow cost of phosphorus loading and each community's private cost, expressed in terms of each community's shadow price.

3.4 Private Equilibrium with Constant Tax Rate

As noted by Mäler et al., it is not practical to implement a time-variable tax and a constant tax is preferable, that is, a tax such that $\dot{\tau} = 0$. Using this condition and combining with equation (21) leads to the following equation:

$$\dot{\lambda}(t) = \frac{1}{(n_1 + n_2)} \left[\sum_{i=1}^{n_1} \dot{\lambda}_i(t) + \sum_{j=1}^{n_2} \dot{\lambda}_j(t)\right], \quad 0 \le t < \infty \tag{22}$$

From the Hamiltonian's first order conditions for this problem, the following two co-state equations can be derived:

$$\dot{\lambda}_i(t) = 2c_1 x(t) + \lambda_i(t)\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], \quad i = 1, \ldots, n_1 \tag{23}$$

and

$$\dot{\lambda}_j(t) = 2c_2 x(t) + \lambda_j(t)\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], \quad j = 1, \ldots, n_2 \tag{24}$$

Substituting (21), (23) and (24) back into (22), we obtain:

$$\dot{\lambda}(t) = 2x(t)\frac{(c_1 n_1 + c_2 n_2)}{n_1 + n_2} + (\lambda(t) + \tau(t))\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right], \tag{25}$$

Recall equations (11) from Section 3.1:

$$\lambda(t) = -\frac{(n_1 + n_2)}{a(t)} \quad \text{and} \quad \dot{\lambda}(t) = \frac{(n_1 + n_2)\dot{a}(t)}{a^2(t)}, 0 \leq t < \infty. \tag{26}$$

Substituting equations (11) and (26) into (21), we obtain the steady-state optimal constant tax that will achieve the Pareto-optimal amount of phosphorus loading when each community acts to maximize its welfare non-cooperatively:

$$\tau^* = \frac{(n_1 + n_2 - 1)}{a^*} \tag{27}$$

This aggregate Pareto-optimal amount of phosphorus loading is determined by substituting equations (11) and (26) into (22, and with a constant tax over time:

$$\dot{a}(t) = \frac{a^2(t)}{(n_1 + n_2)^2}\left[\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right][a(t)\tau^* - (n_1 + n_2)] - 2x(t)(n_1 c_1 + n_2 c_2)\right] \tag{28}$$

Assuming constant loading, i.e. $\dot{a}(t) = 0$ and solving for a^*, we obtain the Nash-equilibrium phosphorus loading with constant tax:

$$a^* = \frac{\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]}{2x(t)\frac{(c_1 n_1 + c_2 n_2)}{(n_1+n_2)^2} + \frac{\tau^*}{(n_1+n_2)}\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]} \tag{29}$$

The plot of the solution is overlayed onto the Pareto-optimal steady-state loading curve from section 3.1 and shown in Figure 3. Note that when the optimal tax is applied, the Nash equilibrium loading intersects the lake dynamics in exactly the same point $(x^*, a^*) = (0.3472, 0.1007)$ as the Pareto-optimal loading. Moreover, there is now only one Nash equilibrium, and it is oligotrophic in accordance with society's preferences. This is not surprising given that the objective of the tax is to bring phosphorus loading to the same level as that which would be achieved under optimal management.

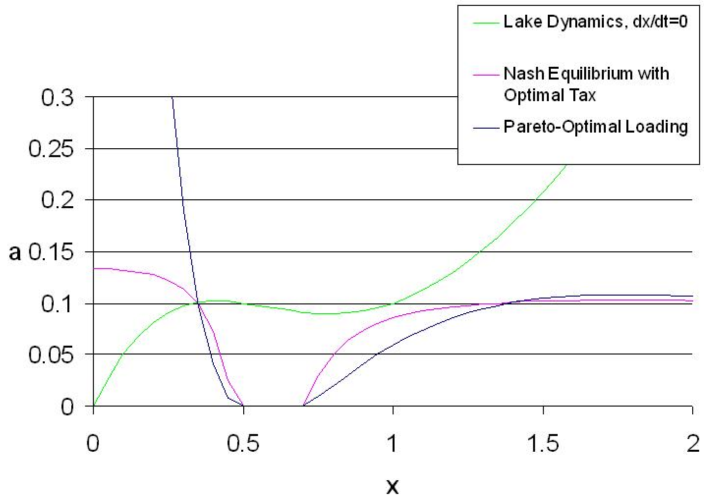

Figure 3: Nash Equilibrium Loading with Two Welfare Functions and Tax

4 The Impact of Rent-seeking and Political Ambition on the Optimal Tax

Consider the case where the lake is in a eutrophic state in spite of a current tax on phosphorus loading. The green communities have a preference for an oligotrophic lake and consider that the current tax rate is too low. The n communities face an election to elect a single politician. The politician favoured by the green communities promises to implement a higher tax on phosphorus that will bring phosphorus loading down so as to reverse the lake to an oligotrophic state. The farming communities favour a politician who promises to maintain the tax at its current low level.

Figure 4 shows the Nash equilibrium loading with a low tax, $\tau = 1$, together with the curve of the Nash equilibrium loading with the optimal tax $\tau^* = 29.78$, as given by equation (27). Note that in the context of a low tax, the high eutrophic Nash equilibrium could prevail, whereas in the case of the optimal tax, only one Nash equilibrium is possible.

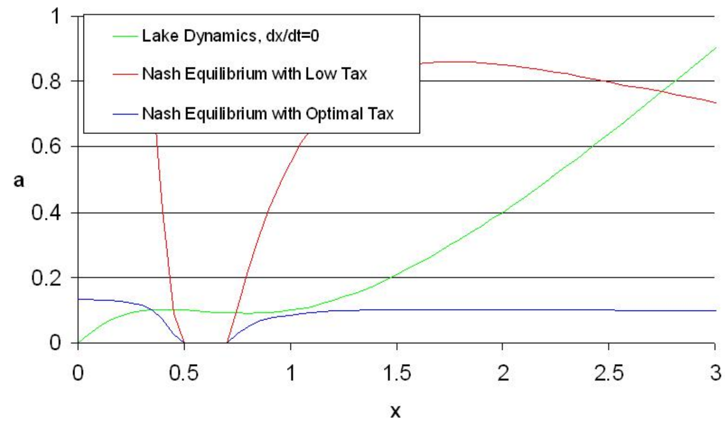

Figure 4: Nash Equilibrium Loading Low Tax and Optimal Tax

4.1 Lobbying to Influence Policy Outcomes

We now apply the Tullock model of rent-seeking (Tullock, 1980) to obtain the probability of each group's lobbying efforts being successful at having their desired policy applied. The simplest, linear, form of the model is applied where the expected payoff of each player is defined as the ratio of his investment divided by the sum of the investments of all the players, multiplied by the reward if he wins. Here, the investments are the lobbying efforts and the rewards are the communities' net benefit of having their desired policy applied, that is, their welfare function with their preferred tax rate. Note that the probability of having their preferred policy implemented is synonymous with the probability of having their preferred politician elected.

Each group applies lobbying effort to influence the policy outcome. The green communities lobby in favour of a high tax and the farmers lobby for a low tax on phosphorus loading. The lobbying effort of each agricultural community i is denoted by l_i and the lobbying effort of each green community j is denoted by m_j. The probability of the agricultural communities having their preferred policy applied is given by the following contest success function:

$$P_f = \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j}, \tag{30}$$

while the probability of the green communities having their preferred policy applied is given by:

$$P_g = \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \tag{31}$$

The expected payoff of each community is the probability of having its preferred policy applied, plus the probability of it not being applied, minus its own initial lobbying investment. The problem thus becomes for each community to maximize its expected return by applying the correct amount of lobbying effort, i.e.

$$\begin{aligned} \max_l E\left\{\Pi_F\right\} = & \quad \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2\right) \\ + & \quad \left(1 - \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\right) \left(\ln a_{H\,\tau} - c_1 x_{H\,\tau}^2\right) - l_i \end{aligned} \tag{32}$$

and

$$\begin{aligned} \max_m E\left\{\Pi_G\right\} = & \quad \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \left(\ln a_{H\,\tau} - c_2 x_{H\,\tau}^2\right) \\ + & \quad \left(1 - \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\right) \left(\ln a_{L\,\tau} - c_2 x_{L\,\tau}^2\right) - m_j \end{aligned} \tag{33}$$

where τ is the tax applied to phosphorus loading and the indices $L\tau$ and $H\tau$ denote actions and states of the world associated with the low tax and the high tax, respectively. The lobbying effort of farmers in favor of a low tax and greens in favour of a a high tax are denoted by l and m, respectively.

To find the optimum lobbying efforts l_i^* and m_j^*, one finds the values of l_i and m_j for which the first derivative is equal to zero, i.e. $dE\left\{\Pi_F\right\}/dl_i = 0$ and $dE\left\{\Pi_G\right\}/dm_j = 0$. (These values will maximize the expected pay-offs provided the profit functions are concave, i.e. if their second derivatives are negative).

For simplicity, we assume symmetry amongst members of the same community that is, they contribute an equal amount of lobbying effort, so that $\sum_{i=1}^{n_1} l_i = n_1 l$ and $\sum_{j=1}^{n_2} m_j = n_2 m$. Equations (32) and (33) can then be expressed as:

$$\max_l E\left\{\Pi_F\right\} = \frac{n_1 l}{n_1 l + n_2 m}\left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau}\right)$$

$$+\left(\frac{1-n_1 l}{n_1 l + n_2 m}\right)\left(\ln a_{H\,\tau} - c_1 x^2_{H\,\tau}\right) - l \tag{34}$$

and

$$\max_m E\left\{\Pi_G\right\} = \frac{n_2 m}{n_1 l + n_2 m\left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau}\right)} \tag{35}$$

$$+\left(\frac{1-n_2 m}{n_1 l + n_2 m}\right)\left(\ln a_{L\,\tau} - c_2 x^2_{L\,\tau}\right) - m \tag{36}$$

The first order conditions for the game described by equations (34) and (36) give the following two second-order polynomials:

$$n_1 n_2 m\left(\ln a_{L\,\tau} - c_1 {x^2}_{L\,\tau} - \ln a_{H\,\tau} + c_1 {x^2}_{H\,\tau}\right) = \left(n_1 l + n_2 m\right)^2 \tag{37}$$

and

$$n_2 n_1 l\left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + x^2_{L\,\tau}\right) = \left(n_1 l + n_2 m\right)^2, \tag{38}$$

Solving for the positive real root in equations (37) and(38), gives the optimal level of lobbying expenditure for the farming and green communities:

$$l = \frac{-n_2 m}{n_1} + \frac{1}{n_1}\sqrt{n_1 n_2 m\left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau}\right)} \tag{39}$$

and

$$m = \frac{-n_1 l}{n_2} + \frac{1}{n_2}\sqrt{n_2 n_1 l\left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau}\right)} \tag{40}$$

We note from equations (39) and (40) that l and m are dependent on each other and that the following implied relationship between the lobbying efforts of the two communities can be derived:

$$l = m\frac{\left(\ln a_{l\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau}\right)}{\left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau}\right)}. \tag{41}$$

By substituting this equation into equations (39) and (40), we can express the respective optimal lobbying effort l^* and m^* of farmers and 'greens', as:

$$l^* = \frac{n_1 n_2 \left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right)^2 \left(\ln a_{h\,\tau} - c_2 x_{H\,\tau}^2 - \ln a_{L\,\tau} + c_2 x_{L\,\tau}^2\right)}{\left[n_1 \left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right) + n_2 \left(\ln a_{H\,\tau} - c_2 x_{H\,\tau}^2 - \ln a_{L\,\tau} + c_2 x_{L\,\tau}^2\right)\right]^2} \tag{42}$$

and

$$m^* = \frac{n_1 n_2 \left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right) \left(\ln a_{H\,\tau} - c_2 x_{H\,\tau}^2 - \ln a_{L\,\tau} + c_2 x_{L\,\tau}^2\right)^2}{\left[n_1 \left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right) + n_2 \left(\ln a_{h\,\tau} - c_2 x_{H\,\tau}^2 - \ln a_{L\,\tau} + c_2 x_{L\,\tau}^2\right)\right]^2} \tag{43}$$

In the following section, we use these results to study the impact of the lobbying efforts on the optimal tax policy derived in section 3.4 and depicted in Figure 3.

4.2 Probability of the Optimal Tax Being Implemented

In the following section, we use these results to study the impact of the lobbying efforts on the optimal tax policy derived in section 3.4 and depicted in Figure 3. Recall that in our scenario, the lake is in a eutrophic state in spite of an existing tax on phosphorus loading. Either the current state of the lake reflects the preferences of all of the communities around the lake or the tax is too low to keep it in an oligotrophic state. Knowing that the Pareto-optimal state of the lake is as shown in Section 3.1, a benevolent politician promises if he is elected to implement the optimal tax rate.

What is the likelihood of the optimal tax policy being implemented, that is, of this politician being elected, given the relative preferences of the green communities and farming communities and their resulting lobbying efforts? To address this question we study the sensitivity of the contest success function to different parameter values.

The following constant values are used in the Nash equilibrium with tax equation (29) and evaluated for values of x between 0 and 3.5.

- $b = 0.6$ - this is the phosphorus recycling value that gave rise to a hysteresis in the lake dynamics.

- $c_1 = 0.2$ - denotes the farming communities' low relative preference for lake ecosystem services.
- $c_2 = 2$ - thus denoting the green communities' high relative preference for a clean lake.
- $L\,\tau = 1$ - is selected as the current taxation that results in high phosphorus loading and thus a eutrophic state of the lake.

The dynamic socially optimal equilibrium level of phosphorus loading is given by the intersection of the optimal a^* equation (12) and the lake dynamics equation (2), as depicted in Figure 1. The optimal tax is such that the dynamic non-cooperative equilibrium intersects the lake dynamics equation for the same optimal (x^*, a^*) coordinates, as shown in Figure 3.

Varying values of n_1 and n_2 results in different (x^*, a^*) coordinates and affects the amount of lobbying applied by the different communities to obtain their desired outcome with respect to the proposed tax increase versus keeping the current low tax. This in turn affects the probability of the benevolent politician being elected and thus of the optimal tax policy being implemented. Substituting these values back into the equations derived earlier in the chapter, namely, equations (42), (43), (30) and (31) gives us the lobbying efforts of the green and agricultural communities as well as the probabilities of the optimal tax policy being implemented.

The results are summarized in Table 1 below.

Table 1: Summary of Results.

	n_1=5, n_2=1	n_1=4, n_2=1	n_1=3, n_2=1	n_1=2, n_2=2
P_i	1	0.0021	0.0492	0.0511
P_g	0	0.9979	0.9508	0.9488
(x^*,a^*)	(0.3932,0.102)	(0.3934,0.102)	(0.3795,0.1018)	(0.3472,0.1007)
τ^*	37.74	33.82	29.44	29.7792

We interpret these results as follows. For the selected preference ratios, even with as little as 1 in 6 communities with a high preference for a clean lake, the Pareto-optimal outcome is for a level of phosphorus loading that results in an oligotrophic lake. And yet, for these same preferences, when the ratio of n_2 to n_1 is less than or equal to one to five, the probability of the green politician being elected is 0.

On the other hand, when n_2 to n_1 is one to four or greater, the probability of the politician being elected increases to very high levels, i.e. relatively close to 1. This means that a relatively small proportion of the population can gain enough power to influence policy when their preferences are strong enough. This result can be attributed to the amount of lobbying effort that is expended when communities attach a relatively high value to ecosystem services. Moreover, as the proportion of green communities increases, the tax rate required to bring the lake back to oligotrophic levels is lower, which also explains a lower lobbying effort against a higher tax by farming communities.

The reason for this result can be seen by comparing the elctoral probabilities tho the steady-state level of phosphorus x^* and the level of loading a^*. As the composition of the comunities varies in terms of their preferences, i.e. as n_1 and $n-2$ change the steady-state changes, however in a highly non-linear manner. As a result of this the election probabilities also change. Notice that as the level of pollution in the lake increases the electoral chances of a green politician increase compared to a pro-agricultural politician.

In the next section we discuss computational issues associated with determining the tax rate in a politco-economic equilibrium along with numerical results.

4.3 Determining the tax policy in a politico-economic equilibrium

An ambitious politician will want to implement the policy that will ensure that he is elected. To do this he will propose a tax so as to maximize the probability of being elected, that is he will solve the following:

$$\max_{\tau} P_g = \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j}$$

This problem cannot be solved analytically however a numerical solution is possible.We solve this problem numerically using a generalized reduced gradient method. The generalized reduced gradient method generalizes Wolfe's reduced gradient method (not to be confused with gradient descent) to allow for non-linear constraints and arbitray bounds. The reduced gradient methiood is discussed briefly in Judd (Judd, 1998, p.126-127).We describe the method here briefly but in somewhat more detail than Judd. Consider the following optimization problem

$$\max f(x), x \in [L, U] \text{ and } h(x) = 0 \tag{44}$$

The algorithm also works in the unconstrained case, i.e. there is no $h(x)$. The generalized reduced gradient method works by partitioning x

into basic and non-basic variables $x = (v, w)$. Binding constraints are used to eliminate v. Consequently, a reduced form of the maximization problem is defined in which

$$\max F(w) = f(v(w), w) \text{ subject to } h(v(w), w) = 0, v \in (L_v, U_v) \tag{45}$$

A reduced gradient is then computed as follows

$$\nabla_w F(w)) = \nabla_w f(v(w), w) - \nabla_v h(v(w), w)^{-1} \nabla_w h(v(w), w) h(v(w), w) \tag{46}$$

For the case where x is a vector a search direction d also needs to be determined from the gradient Lasdon et al. (1978). For a maximization problem if the gradient is positive d will be positive and if it is negtive d will be negative. In our case $x = \tau$ which is a scalar and is unconstrained so that the problem reduces to one of solving

$$\max_{\alpha} F(\tau_0 + \alpha d), \alpha > 0 \tag{47}$$

for given initial $\tau = \tau_0$ and solving for step-size α using a quasi-Newton's method where $d = \frac{df(\tau)}{d\tau}$. This is done to ensure that the Wolfe conditions for linesearch are fulfilled[4]. The derivative d is evaluated here using finite differences. Reduced gradient methods are both robust and efficient. In principle any non-linear solver could have been used. So for example Newton's method could have been used for optimization directly, However this would have required computation of partial derivatives. In practice these methods also employ step-size and search direction techniques so that for this problem one method or another has little advantage in terms of efficiency.

After solving we find that for $n_1 = 2$ and $n_2 = 2$, to maximize his probability of being elected, the benevolent politician would have to set the tax rate at $\tau = 11.40$. This tax increases the probability of election to one, that is, by proposing this tax he is certain of being elected. Unfortunately, this tax will result in an insufficient reduction in phosphorus loading levels and the lake will remain in its eutrophic state. Recall that the skiba point is at $(x_{F1}, a_{F1}) = (0.4084, 0.1021)$, c.f. Mäler et al. (2003). For $n_1 = 2$ and $n_2 = 2$, the optimal levels of phosphorus are $(x^*, a^*) = (0.3472, 0.1007)$, which denotes an oligotrophic state of the lake. To achieve the optimal level of phosphorus loading, the required tax rate is $\tau^* = 29.78$. Therefore

[4]For a discussion of the Wolfe conditions see (Bartholomew-Biggs, 2005, pp.58-59)

a proposed tax policy of $\tau = 11.40$ would be far inferior to that required to achieve the socially desirable level of phosphorus loading. This may be an example that illustrates the observation by Lee (1985) that "political objectives can be realized by establishing "acceptable" pollution standards and many of them have little to do with protecting the environment."

5 Conclusion

In summary, we have found that lobbying and the composition of the electorate have an effect on the implementation of the socially optimal tax policy. When a portion of the communities have a strong preference for a clean lake, as little as one fifth, the probability of the politician being elected increases to very high levels, i.e. relatively close to 1. This is an interesting result because it implies that the number of green communities need not be very high, only sufficiently high, for the environmental policy to have a very high chance of being implemented.

A perhaps more interesting result is that by proposing a tax level below the one required to bring the lake back to a socially optimal oligotrophic state, a politician can ensure that he is elected. This shows that political ambition can indeed prevent socially desirable policy from being implemented. It would also be interesting to discover whether this result is time-consistent, i.e., whether this kind of political behaviour is sustainable in the face of environmetnal change and responses of the lake communities to this change.

Some work in this direction has been carried out by Dockner and Wagener (2006) who study Markov-perfect Nash equilibria for the lake game and Kossioris et al. (2008), who have derived a feedback Nash equilibrium for a shallow lake. However, as far as we are aware, no attempt has been made to find an optimal time-consistent and political acceptable ecotax for shallow lake pollution. Such a tax (if one exists) would remove the possibility of a rent-seeking game such as the one described in our paper and would have the potential to be an important contribution to this literature. We will leave this as an area for future research.

References

C. Alaouze. A short run economic analysis of the eutrophication problem of the barwon and darling rivers in new south wales. Technical report, Discussion Paper, The University of New South Wales, 1995.

M. Bartholomew-Biggs. *Nonlinear Optimization with Financial Applications*. Springer, 2005.

S. Carpenter, D. Ludwig, and W. Brock. Management of eutrophication for lakes subject to potentially irreversible change. *Ecological Applications*, 9 (3):751–771, 1999.

W. D. Dechert and W. A. Brock. The lake game, 2000. URL `http://www.ssc.wisc.edu/econ/archive/wp2003-24.pdf`.

W. D. Dechert and S. I. O'Donnell. The stochastic lake game: A numerical solution. *Journal of Economic Dynamics and Control*, 30:1569–1587, 2005.

E. J. Dockner and F. O. Wagener. Markov-perfect nash equilibria in models with a single capital stock, 2006.

L. Grüne, M. Kato, and W. Semmler. Solving ecological management problems using dynamic programming. *Journal of Economic Behavior and Organization*, 57(4):448–473, 2005.

K. L. Judd. *Numerical Methods in Economics*. The MIT Press, 1998.

G. Kossioris, M. Plexousakis, A. Xepapadeas, A. de Zeeuw, and K.-G. Mäler. Feedback nash equilibria for non-linear differential games in pollution control. *Journal of Economic Dynamics and Control*, 32:1312–1331, 2008.

L. S. Lasdon, A. D. Waren, A. Jain, and M. Ratner. Design and testing of a generalized reduced gradient code for nonlinear programming. *ACM Trans. Math. Softw.*, 4(1):34–50, 1978.

D. R. Lee. Rent-seeking and its implications for pollution taxation. *Southern Economic Journal*, 51(3):731–744, 1985.

K.-G. Mäler, A. Xepapadeas, and A. D. Zeeuw. The economics of shallow lakes. *Environmental and Resource Economics*, 26:603–624, 2003.

J. D. Murray. *Mathematical Biology*. Biomathematics. Springer-Verlag, 1989.

S. I. O'Donnell and W. D. Dechert. A stochastic lake game, 2004. URL `http://panda.econ.uh.edu/research/stochastic.lake.ps`.

R. E. Ricklefs. *Ecology*. Chiron Press, 2nd edition, 1979.

M. Scheffer. *Ecology of Shallow Lakes*. Chapman & Hall, 1998.

G. Tullock. Efficient rent seeking. In J. M. Buchanan, R. D. Tollison, and G. Tullock, editors, *Toward a Theory of the Rent-Seeking Society*, pages 97–112. Texax A & M University Press, 1980.

F. Wagener. Skiba points and heteroclinic bifurcations, with applications to the shallow lake system. *Journal of Economic Dynamics & Control*, 27: 1533–1561, 2003.