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Abstract

There are several methods to convert fuzzy or stochastic LP to conventional LP models. In this simple paper we evaluate the effectiveness of three proposed methods, using a numerical example from a pure factors portfolio.

Keywords: fuzzy, stochastic, linear programming, pure factors portfolio

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1. **Introduction**

As known (Inuiguchi & Ramik, 2000), the Fuzzy Linear Programming (FLP) technique is required when ambiguity and/or vagueness exist in the coefficients of the objective function and/or in constraints, as well as in the right hand side parameters. For instance, expressions like the interest rate is "*about 3%*" are ambiguous, while expressions like the inflation is "*substantially larger than 4*" are vague since they are not sharp enough. It is also known that the stochastic variables represent randomness or chance of events.

Inuiguchi & Ramik (2000) and recently (Van Hop, 2007), have reviewed some papers that treated fuziness in constraints, in objective function, or in both. Inuiguchi & Ramik model fuzziness with possibility and necessity measures. They argue that solving a FLP problem can be easier than a Stochastic Programming (SP) problem. Van Hop (2007) also argued that the use of superiority and inferiority measures can convert effectively a FLP (or a Stochastic LP) into a deterministic one. Van Hop illustrates the effectiveness of the proposed method with some numerical examples and argues that it is superior to the complex method suggested by Luhandjula (1996).

In this short paper we present a simple financial example in order to evaluate the effectiveness of various de-fuzzification methods.

2. A two-factor example

In finance, Grinblatt & Titman (1998), pure factor portfolios are defined as portfolios with a sensitivity of one to one of its factors, and zero to the remaining factors. Let us consider the following two-factor model from Grinblatt & Titman, with the interest rate (r) and inflation rate (π) being the two factors.

An investment fund has a given portfolio *Z*, consisting of *Swaps*, *Bonds* and *Stocks*. The returns from these assets (in percentage units) are *1*, *5* and *7* respectively. Moreover, these returns are risky, not in the common sense of their variance, but rather in terms of two macroeconomic factors, the interest rate and the rate of inflation. For instance, positive or negative deviations from the equilibrium interest and/or the inflation rates lead to lower or higher returns. Assume that the beta-values of r- and π–factors are estimated to be *-2.5* and *-4.5* respectively. The fund manager must therefore design a proper hedge portfolio against inflation and interest rate, i.e. he must eliminate its sensitivity to both factors. Assume that the fund has (1), a capital of € 100,000 to invest, and (2), does not want to invest its own capital.

The fund is consulted by its investment bank to re-balancing the portfolio, by buying or selling the same three financial securities *Swaps*, *Bonds* and *Stocks*.

We will assume first that all beta-values in these securities are deterministic. Later we will alter this assumption and treat the beta-values and other parameters as chance and fuzzy values.

2. A conventional LP model

As known, in standard conventional LP models all parameters are deterministic.

(a) Deterministic parameters

(i) Enter into a five-year interest rate *Swap (S)* contract. Assume that the number of swap contract is costless and its future value has the following factor equation: $S = 5 - 5r - 3\pi$.

(ii) Enter the 30-year government *Bonds (B)* market. The investment is per 1 million € and has the following factor equation: $B = 10 - 5r - 1\pi$.

(iii) Enter the stock market, or some *Stock (K)* index. The investment is also per 1 million € and has the following factor equation: $K = 0 + 1r + 1\pi$.

Notice first that in order to neutralize the beta of r which is *-2.5* and the beta of π which is *-4.5*, we need to turn these values into *+2.5* and *+4.5* respectively, so that the new portfolio will be interest and inflation risk-free.

Let X_1 = *the number of contracts in S;* X_2 = *millions of € in B;* X_3 = *millions of € in S.*

A simple LP formulation[1] is the following:

$$
\begin{aligned}
&Max\ X_1 + 5X_2 + 7X_3 - 2.5r - 4.5\pi \\
&s.t. -5X_1 - 5X_2 + X_3 + r = 2.5 \Rightarrow 5X_1 + 5X_2 - X_3 - r = -2.5 \\
&\quad -3X_1 - X_2 + X_3 + \pi = 4.5 \Rightarrow 3X_1 + X_2 - X_3 - \pi = -4.5 \\
&\quad 1000000X_2 + 1000000X_3 = 100000\ (1), or\ 0\ (2) \\
&\quad X_1, X_2, X_3 \in [-\infty, +\infty] \\
&\quad r, \pi \geq 0
\end{aligned}
$$

Notice that the three financial variables are free. Negative values are allowed if the fund sells financial securities (called "short" in finance), and positive if it purchases.

The solutions are: (1) $X_1 = -2.80$, $X_2 = 1.90$, $X_3 = -2.0$, $r = 0$, $\pi = 0$, $Max = 0$;

[1] Notice that in constraints, the sign of r and π is the same as the right hand side parameters. The solution is unaffected though, even if their sign is opposite to that of the right hand side parameters. Notice also that only strict equality interest rate and inflation constraints give a solution, given the unbounded financial variables. Inequalities in these two constraints lead to unbounded solution. The solution remained also unchanged if the objective function was simplified to $Max\ - 2.5r - 4.5\pi$.

(2) $X_1 = -2.75,\ X_2 = 1.875,\ X_3 = -1.875,\ r = 0,\ \pi = 0,\ Max = -6.5.$

We can check that this new portfolio is interest and inflation risk-free.

The initial portfolio had a value $V = X_1 + 5X_2 + 7X_3 - 2.5r - 4.5\pi$, with all assets already included in portfolio, i.e. $X_1, X_2, X_3 > 0$. For instance, if the fund had 4 units of S, 2 units of B and 1 unit of S, i.e. $X_1 = 4$, $X_2 = 2$, $X_3 = 1$, and $r = 4$, $\pi = 2$, the value is $V = 2$. The same portfolio with $r = 6$ and $\pi = 4$ has a $V = -12$. Thus, it is very risky to changes in the interest and inflation rates.

Let us check case (2). The portfolio that hedges the initial one, has the following value: $V' = (5 - 5r - 3\pi)(-2.75) + (10 - 5r - 1\pi)(1.875) + (1r + 1\pi)(-1.875) = 5 + 2.5r + 4.5\pi$. Thus, the fund's new portfolio has a value of $F_{new} = V + V' = (X_1 + 5X_2 + 7X_3 - 2.5r - 4.5\pi) + (5 + 2.5r + 4.5\pi) = -2.75 + 5(1.875) + 7(-1.875) + 5 = -6.5 + 5 = -1.5$, which is interest- and inflation- risk free value.

3. Chance-Constrained Programming (CCP)

In CCP the parameters of the constraints are random variables and the constraints are valid with a minimum probability.

(b_1) Chance left-hand side parameters and constraints

Let us now change the deterministic left-hand side parameters and make them expected values, independent and normally distributed random variables with the means and variances below. On the other hand, while the beta-values of the initial portfolio remain -2.5 and -4.5 in the objective function as before, the right-hand side parameters of the interest rate and inflation do not hedge perfectly. The right-hand side parameters are crispy numbers different from -2.5 and -4.5. This might be due to the fact that the left-hand side parameters are not deterministic now.

$$P\{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 - r \le -1.5\} \ge 1 - \alpha$$
$$P\{a_{21}x_1 + a_{22}x_2 + a_{23}x_3 - \pi \le -3.5\} \ge 1 - \alpha$$

$$E[a_{11}] = 5,\ E[a_{12}] = 5,\ E[a_{13}] = -1,\ E[a_{21}] = 3,\ E[a_{22}] = 1,\ E[a_{23}] = -1$$
$$\sigma^2[a_{11}] = 9,\ \sigma^2[a_{12}] = 5,\ \sigma^2[a_{13}] = 0.3,\ \sigma^2[a_{21}] = 2.5,\ \sigma^2[a_{22}] = 1.2,\ \sigma^2[a_{23}] = 0.4$$
$$\alpha = 0.05$$

For instance, the implication of the first constraint is that the probability of the expected value beta parameters (and r as well), being at most -1.5, is at least 95%. A similar interpretation is for the inflation rate constraint.

The first stochastic constraint is now formulated as:

$$P\{(5x_1 + 5x_2 - 1x_3 - r) \le -1.5\} \ge 1 - \alpha = F\left(\frac{-1.5 - (5x_1 + 5x_2 - 1x_3 - r)}{\sqrt{9x_1^2 + 5x_2^2 + 0.3x_3^2}}\right).$$

F is the cumulative density function of the standard normal distribution. If $F(K_\alpha)$ is the standard normal value such that $F(K_\alpha) = 1 - \alpha$, then the above constraint reduces to:

$$\left\{\frac{-1.5 - (5x_1 + 5x_2 - 1x_3 - r)}{\sqrt{9x_1^2 + 5x_2^2 + 0.3x_3^2}}\right\} \ge (K_\alpha)$$

And finally, given $\alpha = 0.05$, the constraint is simplified to:

$$-1.5 \ge 1.645\sqrt{9x_1^2 + 5x_2^2 + 0.3x_3^2} + (5x_1 + 5x_2 - 1x_3 - r)$$

Similarly, the second constraint is simplified to:

$$-3.5 \ge 1.645\sqrt{2.5x_1^2 + 1.2x_2^2 + 0.4x_3^2} + (3x_1 + x_2 - x_3 - \pi).$$

(b_2) Chance right-hand side parameters and constraints

In this case we assume that only the right-hand side parameters are normally distributed with the following means and variances:

$$P\{5x_1 + 5x_2 - 1x_3 - r \le b_1\} \ge 0.10$$
$$P\{3x_1 + 1x_2 - 1x_3 - \pi \le b_2\} \ge 0.10$$

$$E[b_1] = -3.5,\ E[b_2] = -5.5$$
$$\sigma^2[b_1] = 1.4,\ \sigma^2[b_2] = 1.6$$

For instance, the first constraint (interest rate) implies that the probability of the crispy parameters, being at most *-3.5*, is at least *10%*. A similarly interpretation is for the inflation constraint.

The first stochastic constraint is now formulated as:

$$P\left\{\frac{(5x_1 + 5x_2 - 1x_3 - r) - (-)3.5}{\sqrt{1.4}}\right\} \ge 0.10$$

This stochastic constraint reduces to the following deterministic one:

$$\frac{(5x_1+5x_2-1x_3-r)+3.5}{\sqrt{1.4}} \le K_{0.10} \Rightarrow (5x_1+5x_2-1x_3-r) \le -3.5+1.285\sqrt{1.4}$$

and similarly for the inflation constraint: $(3x_1+x_2-x_3-\pi) \le -5.5+1.285\sqrt{1.6}$.

Notice that, precisely as in the deterministic model previously, we keep the sign of r and π and the sign of the right hand side parameters the same. The CCP model[2] is as follows:

$$\begin{aligned}
&Max\ X_1+5X_2+7X_3-2.5r-4.5\pi \\
&s.t.\ \ 5X_1+5X_2-X_3-r+1.645\sqrt{9X_1^2+5X_2^2+0.3X_3^2} \le -1.5 \\
&\quad 3X_1+X_2-X_3-\pi+1.645\sqrt{2.5X_1^2+1.2X_2^2+0.4X_3^2} \le -3.5 \\
&\quad 5X_1+5X_2-X_3-r \le -3.5+1.285\sqrt{1.4} \\
&\quad 3X_1+X_2-X_3-\pi \le -5.5+1.285\sqrt{1.6} \\
&\quad 1000000X_2+1000000X_3 = 100000\ (1), or\ 0\ (2) \\
&\quad X_1, X_2, X_3 \in (-2.8, +2.8) \\
&\quad r, \pi \ge 0
\end{aligned}$$

Notice that the unbounded financial variables are now bounded in order to obtain a solution. As bounds, we used the highest absolute values found from the deterministic model.

The solutions are: (1) X_1 = *-1.7658*, X_2 = *-2.70*, X_3 = *2.80*, r = *0*, π = *0*, *Max* = *4.33;*
(2) X_1 = *-1.7812*, X_2 = *-2.80*, X_3 = *2.80*, r = *0*, π = *0*, *Max* = *3.82*

The solutions were efficient too, but close to their bounds.

4. A Possibilistic LP (PLP) model

Let us now make our left-hand side beta parameters ambiguous.

(c) Symmetric triangular fuzzy beta-values

The ambiguity of estimated beta values can be restricted by a symmetric triangular fuzzy number, determined by a center a_i^c and a spread w_{a_i}, represented as: $A_i = \langle \alpha_i^c, w_{a_i} \rangle$. For instance, the beta-estimate of r in *Swaps* can be restricted by a fuzzy number A_{11} with the following membership function:

[2] We changed all signs in constraints by multiplying all variables by -1. The problem is in fact nonlinear and can be solved, using for instance Lingo, or it can be converted to a separable LP.

$\mu_{A_{11}}(x) = max\left(0, 1 - \frac{|x-5|}{3}\right)$. Thus, the center is *5* (again all values are multiplied by *-1*), its upper value is *8* and its lower value is *2*. Similar fuzzy numbers exist *Bonds* and *Stocks* and for π as well.

Assume that the symmetric triangular fuzzy beta values are the following:

$$A_{11} = \langle 5, 3 \rangle,\ A_{12} = \langle 5, 2 \rangle,\ A_{13} = \langle -1, -0.2 \rangle,\ A_{21} = \langle 3, 1.5 \rangle,\ A_{22} = \langle 1, 0.4 \rangle,\ A_{23} = \langle -1, -0.5 \rangle$$

Of course one can make the right-hand side parameters ambiguous as well, depending upon the possibilistic beta values in the left hand sides. We assume that the *certainty* degrees of interest rate constraint being at most *-3.5* and of inflation constraint being at most *-5.5* are not less than 90%. We also assume that the possibility degrees of interest rate constraint being at least *-1.5* and of inflation constraint being at least -*3.5* is not less than 80%.

Inuiguchi & Ramik, (2000) used possibility and/or necessity measures to de-fuzzify a fuzzy LP.

The possibility measures measure to what extent it is <u>possible</u> that the possibilistic beta values, restricted by the possibility distribution $\mu_{A,}$ are at least or at most some certain values.

Given two fuzzy sets, *A* and *B*, and a possibility distribution μ_A of a possibilistic variable α, the possibility measure is defined as:

$$\Pi_A(B) = \sup_r min(\mu_A(x), \mu_B(x)).$$

If $B = (-\infty, g]$, i.e. *B* is a crisp (non-fuzzy) set of real numbers not larger than *g*, the possibility index is defined as:

$$Pos(a \leq g) = \Pi_A((-\infty, g]) = sup\{\mu_A(x) | x \leq g\}$$

If $B = [g, +\infty)$, the possibility index is defined as:

$$Pos(a \geq g) = \Pi_A([g, +\infty)) = sup\{\mu_A(x) | r \geq g\}$$

The necessity measures measure to what extent it is <u>certain</u> that the possibilistic beta values, restricted by the possibility distribution $\mu_{A,}$ are at least or at most some certain values.

The necessity measures and the necessity index are similarly defined as:

$$N_A(B) = \inf_r max(1 - \mu_A(x), \mu_B(x))$$

$$Nes(a \le g) = N_A((-\infty,\ g]) = 1 - sup\{\mu_A(x) | x \succ g\}$$
$$Nes(a \ge g) = N_A([g,\ +\infty)) = 1 - sup\{\mu_A(x) | x \prec g\}$$

Inuiguchi & Ramik (2000) show that the classical portfolio model can be regarded as a PLP problem with independent possibilistic variables and that is equivalent to a Stochastic LP problem with unknown correlation coefficients between normal random variables. We will now follow Inuiguchi & Ramik and formulate the necessity and possibility constraints.

Given the symmetric triangular fuzzy beta-values, the PLP model is:

$$\begin{aligned}
&Max\ X_1 + 5X_2 + 7X_3 - 2.5r - 4.5\pi \\
&s.t.\ \ 5X_1 + 5X_2 - X_3 - r + 0.9(3X_1 + 2X_2 - 0.2X_3) \ge -3.5 \\
&\quad\ \ 3X_1 + X_2 - X_3 - \pi + 0.9(1.5X_1 + 0.4X_2 - 0.5X_3) \ge -5.5 \\
&\quad\ \ 5X_1 + 5X_2 - X_3 - r + 0.8(3X_1 + 2X_2 - 0.2X_3) \le -1.5 \\
&\quad\ \ 3X_1 + X_2 - X_3 - \pi + 0.8(1.5X_1 + 0.4X_2 - 0.5X_3) \le -3.5 \\
&\quad\ \ 1000000X_2 + 1000000X_3 = 100000\ (1),\ or\ 0\ (2) \\
&\quad\ \ X_1, X_2, X_3 \in [-\infty, +\infty] \\
&\quad\ \ r, \pi \ge 0
\end{aligned}$$

Notice that the three financial variables are unbounded.

The solutions are: (1) $X_1 = -1.401,\ X_2 = 0.928,\ X_3 = -0.828,\ r = 0,\ \pi = 0,\ Max = -2.56$
(2) $X_1 = -1.4644,\ X_2 = 0.9744,\ X_3 = -0.9744,\ r = 0,\ \pi = 0,\ Max = -3.41$

5. Van Hop's FLP model

Let us now make our left-and right-hand side parameters fuzzy.

(d) Fuzzy parameters

Assume the following symmetric triangular type, fuzzy random parameters.

The interest rate fuzzy parameters are:

$$\left(\tilde{\tilde{A}}_1, \tilde{\tilde{b}}_1\right) = \begin{cases} (\tilde{A}_{1,w1}, \tilde{b}_{1,w1}) = \left[\{\tilde{5}.0\ \ \tilde{5}.0\ \ -\tilde{1}.0\}, \{-\tilde{2}.5\}\right] \\ (\tilde{A}_{1,w2}, \tilde{b}_{1,w2}) = \left[\{\tilde{4}.5\ \ \tilde{5}.0\ \ -\tilde{1}.1\}, \{-\tilde{2}.7\}\right] \end{cases},\ \text{with}\ \ p(w_1) = 0.65,\ p(w_2) = 0.35$$

Similarly, the inflation rate fuzzy parameters are:

$$\left(\tilde{\tilde{A}}_2, \tilde{\tilde{b}}_2\right) = \begin{cases} (\tilde{A}_{2,w1}, \tilde{b}_{2,w1}) = \left[\{\tilde{3}.0\ \ \tilde{1}.0\ \ -\tilde{1}.0\}, \{-\tilde{4}.5\}\right] \\ (\tilde{A}_{2,w2}, \tilde{b}_{2,w2}) = \left[\{\tilde{2}.0\ \ \tilde{0}.7\ \ -\tilde{0}.6\}, \{-\tilde{4}.1\}\right] \end{cases},\ \text{with}\ \ p(w_1) = 0.65,\ p(w_2) = 0.35$$

In order to be consistent with the symmetric triangular fuzzy beta-values in (c) above, we keep the same spreads. Thus, we have the following fuzzy numbers:

$$\mu_{A_{11},1} = \mu_{A_{11},2} = 3,\ \mu_{A_{12},1} = \mu_{A_{12},2} = 2,\ \mu_{A_{13},1} = \mu_{A_{13},2} = -0.2$$
$$\mu_{A_{21},1} = \mu_{A_{21},2} = 1.5,\ \mu_{A_{22},1} = \mu_{A_{22},2} = 0.4,\ \mu_{A_{23},1} = \mu_{A_{23},2} = -0.5$$

Notice that the first rows of the fuzzy numbers $(\tilde{5}\ \tilde{5}\ -\tilde{1}),(-\tilde{2}.5)$ and $(\tilde{3}\ \tilde{1}-\tilde{1}),(-\tilde{4}.5)$ are exactly equal to the deterministic (crisp) values. In addition, the right-hand side fuzzy numbers are restricted by the following membership functions:

$$\mu_B(r) = max(0,1-|r+2.5|),\ with\ -3.5 \le r \le -1.5$$
$$\mu_B(\pi) = max(0,1-|\pi+4.5|),\ with\ -5.5 \le \pi \le -3.5\ .$$

Van Hop (2007) formulated a FLP model, using superiority and inferiority measures.

Given two fuzzy numbers, $\tilde{A} = (u,a,b)$, $\tilde{B} = (v,c,d)$ where, (u,v) = central values and $(a,b,c,d \in R)$, i.e. the left and right spreads respectively, and if $\tilde{A} \le \tilde{B}$,

the superiority of $\tilde{B}$ over $\tilde{A}$ is defined as: $Sup(\tilde{B},\tilde{A}) = v-u+\dfrac{d-b}{2}$,

and the inferiority of $\tilde{A}$ to $\tilde{B}$ is defined as: $Inf(\tilde{A},\tilde{B}) = v-u+\dfrac{a-c}{2}$.

Similarly, given two triangular fuzzy random variables $\tilde{\tilde{A}} \le \tilde{\tilde{B}}$, the superiority of $\tilde{\tilde{B}}$ over $\tilde{\tilde{A}}$ is defined as:

$$Sup(\tilde{\tilde{B}},\tilde{\tilde{A}}) = v(w)-u(w)+\frac{d(w)-b(w)}{2},$$

and the inferiority of $\tilde{\tilde{A}}$ to $\tilde{\tilde{B}}$ is defined as: $Inf(\tilde{\tilde{A}},\tilde{\tilde{B}}) = v(w)-u(w)+\dfrac{a(w)-c(w)}{2}$.

Following Van Hop, the corresponding LP model is:

$$Max\ \ x_1+5x_2+7x_3-2.5r-4.5\pi-0.65\left(\sum_{j=1}^{2}\lambda_{j1}^{sup}+\sum_{j=1}^{2}\lambda_{j1}^{inf}\right)-0.35\left(\sum_{j=1}^{2}\lambda_{j2}^{sup}+\sum_{j=1}^{2}\lambda_{j2}^{inf}\right)$$

$s.t.$

The interest rate constraints: $\tilde{A}_1 x - r \ge \tilde{b}_1$ are linearised as:

$$5.0x_1 + 5.0x_2 - 1.0x_3 - r + 2.5 + \frac{3x_1 + 2x_2 - 0.2x_3 - 1}{2} = \lambda_{11}^{sup}$$

$$4.5x_1 + 5.0x_2 - 1.1x_3 - r + 2.7 + \frac{3x_1 + 2x_2 - 0.2x_3 - 1}{2} = \lambda_{12}^{sup}$$

$$5.0x_1 + 5.0x_2 - 1.0x_3 - r + 2.5 + \frac{-3x_1 - 2x_2 + 0.2x_3 + 1}{2} = \lambda_{11}^{inf}$$

$$4.5x_1 + 5.0x_2 - 1.1x_3 - r + 2.7 + \frac{-3x_1 - 2x_2 + 0.2x_3 + 1}{2} = \lambda_{12}^{inf}$$

The inflation constraints: $\tilde{A}_2 x - \pi \geq \tilde{b}_2$ are similarly linearised as:

$$3.0x_1 + 1.0x_2 - 1.0x_3 - \pi + 4.5 + \frac{1.5x_1 + 0.4x_2 - 0.5x_3 - 1}{2} = \lambda_{21}^{sup}$$

$$2.0x_1 + 0.7x_2 - 0.6x_3 - \pi + 4.1 + \frac{1.5x_1 + 0.4x_2 - 0.5x_3 - 1}{2} = \lambda_{22}^{sup}$$

$$3.0x_1 + 1.0x_2 - 1.0x_3 - \pi + 4.5 + \frac{-1.5x_1 - 0.4x_2 + 0.5x_3 + 1}{2} = \lambda_{21}^{inf}$$

$$2.0x_1 + 0.7x_2 - 0.6x_3 - \pi + 4.1 + \frac{-1.5x_1 - 0.4x_2 + 0.5x_3 + 1}{2} = \lambda_{22}^{inf}$$

In addition we have the non-negativity bounds, $\lambda_{j1}^{sup} = \lambda_{j2}^{sup} = \lambda_{j1}^{inf} = \lambda_{j1}^{inf} \geq 0, j = 1,2$

The third constraint, the unbounded financial variables and the non-negativity bounds of interest rate and inflation remain unchanged as in the deterministic model.

The solutions are: (1) $X_1 = 1.4190, X_2 = --1.4714, x_3 = 1.5714, r = 0, \pi = 0$
(2) $X_1 = 1.4333, X_2 = -1.50, X_3 = 1.50, r = 0, \pi = 0$

As we can see, Van Hop's formulation leads to an optimal solution as well, despite the fact that no bounds are required in the financial assets.

Conclusions

Apart from the CLP that requires upper and lower bounds in the financial assets, when short selling is allowed, both the possibility and necessity constraints in the PLP model and the superiority and inferiority constraints in Van Hop's FLP model lead to efficient and optimal solutions. Both these methods are strong candidates to the complex model suggested more than a decade ago by Luhandjula (1996).

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