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Abstract

We examine the role of tariffs levied on the imported production factor in a one-sector small open economy real business cycle model. We show that under perfect competition and constant returns-to-scale, the model may exhibit local indeterminacy and sunspots as tariff rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures. Conversely, the economy in which the government finances endogenous public spending and transfers with fixed tariff rates is immune to indeterminacy.

KeyWords : Indeterminacy, Endogenous Tariff Rate, Small Open Economy, Exogenous Government Expenditure

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1. Introduction

Benhabib and Farmer (1999, pp 390) provide five sources of indeterminacy and sunspots in macroeconomics.

> "Sunspots cannot occur in finite general equilibrium models with complete markets since their existence would violate the first welfare theorem; risk averse agents will generally prefer an allocation that doesn't fluctuate to one that does. Examples of departures from Arrow-Debreu structure that permit the existence of sunspots include (1) incomplete participation in insurance markets as in the OLG model, (2) incomplete markets due to **transactions costs** or asymmetric information, (3) increasing returns to scale in the technology, (4) market imperfections associated with fixed costs, entry costs or external effects, and (5) the use of money as a medium of exchange."

Indeterminacy means that there are an infinite number of equilibria, all of which start from the same initial condition and converge to a unique steady state. Sunspot equilibria, which can be constructed by randomizing over these (multiple) equilibria, provide an interpretation of Keynes's "animal spirits" hypothesis that economic fluctuations are driven by purely extrinsic beliefs. In this paper, I add to the literatue another mechanism for indeterminacy: tariff, which is a kind of trade taxes (or a kind of **transaction costs** in international trade).

In a one-sector neoclassical growth model with perfectly competitive markets, a constant returns-to scale technology and factor income taxes, Schmitt-Grohe and Uribe (1997, henceforth SGU) explore the mechanism that gives rise to indeterminacy and show that the model can exhibit indeterminacy if taxes rates are endogenously determined by a balanced budget rule with a pre-set level of government expenditures. The labor market effects in their model are quite similar to those of Benhabib and Farmer (1994). That is to say, the labor demand curves are upward sloping and more steeper than the labor supply curves. Guo and Harrison (2004, henceforth GH) further show that indeterminacy disappears once the goverment finances endogenous public spending and transfers with fixed income tax rates. And they illustrate that SGU's indeterminacy result depends on a balanced-budget requirement whereby the tax rate decreases with the household's taxable income. The mechanism for indeterminacy in the SGU model is through increases in aggregate employment that decrease equilibrium tax rates, and raise the after-tax return on labor.

You may ask the question whether tariffs and factor income taxes (in SGU) deliver indeterminacy in the same way. This paper gives a positive answer. Aguiar-Conraria and Wen (2005, 2006, and 2007 henceforth ACW) extend the Benhabib-Farmer model to an open economy by introducing imported foreign factors (say oil) as a third production input. They show that reliance on foreign energy has a potentially important effect on economic activity, because it destabilizes the economy by increasing the likelihood of indeterminacy, hence making fluctuations driven by self-fulfilling expectations more likely to occur. Our paper extends Schmitt-Grohe and Uribe's analysis to the open economy version of Benhabib and Farmer model (without productive externalities) by considering a different balanced-budget rule whereby constant government expenditures can be financed by the oil tariff revenue.[1] Under this type of balanced budget constraint, tariff rates are endogenously determined since the government is forced to lower the tariff rates as total output (or tax base) rises. Moreover, we show that for empirically plausible values of tariff rates (or energy taxes), indeterminacy arises in my model and the economy can exhibit an indeterminate steady state (i.e., a sink). Thus, the endogenously determined tariff rate could be a source of fiscal increasing returns and share with the labor income tax rate the similar mechanism for indeterminacy in the standard one sector growth model.[2] In

[1] For simplicity, we assume that the government doesn't impose consumption taxes on the tradable goods or factor income taxes on the production factors. Adding other taxes changes nothing as long as part of the exogenous spending must be still financed by the tariff levied on the imported factor. See Velasco (1996) for the same explanation to fiscal increasing returns induced by taxes on domestic capital.

[2] In the two sector model without "fiscal increasing returns" induced by factor income taxes, Bond, Wang and Yip (1996) and Meng and Velasco (2003) prove that distortionary factor taxation nonetheless causes indeterminacy in a

addition, this paper shows that if we allow for endogenous public spending and/or transfers financed by exogenous tariff rates, indeterminacy disappears and the model exhibits saddle-path stability, reagrdless of the existence of lump-sum transfers.

The literature on trade taxes suggests that the government can raise revenue by using the tariff instruments.[3] For example, Atolia (2006) considers an open economy model in which public investment can be financed by a tariff and income tax. Leung (1999) presents an endogenous growth model in which the tariff revenue collected from the imported production factor finances the government expenditure in a small open economy. The endogeneity of tariff rates can be traced to the paper of Ramsey (1927) and also used by Loewy (2004) and Mourmouras (1991) in a two-country open economy endogenous growth model and a small open economy OLG model respectively.

The framework that we use in section 2 is a dynamic general equilibrium model that incorporates foreign energy as a third production factor. The energy price is assumed to be set in the world markets and taken as given. Moreover, in our model economy, consumers and firms behave competitively and a government finances a flow of public spending by using energy tariffs.[4] Miguel and Manzano (2006) consider a small open economy, in which they assume that the government finances an exogenous flow of public spending by using consumption and oil taxes (or oil tariffs) and by issuing debt. Unlike Miguel and Manzano, under the assumption that government expenditures are constant and labor is indivisible, we derive the necessary and sufficient condition for the balanced budget rule to generate indeterminacy. It turns out that in order to get indeterminacy, we need the steady state tariff rate to be greater than the share ratio of capital and imported factors in the production function and less

closed-economy, endogenous growth model and a small open RBC model respectively. Does the "channel equivalence" between factor income taxes and tariffs to generate indeterminacy still hold in a small open economy two sector model? This is one issue which deserves our further research.

[3] The revenue motive behind the imposition of trade taxes is well documented. See, Kindleberger and Lindert (1978, p. 143), and Riezman and Slemrod (1987).

[4] The tariff revenue in this model can also be interpreted as oil tax revenue. Hence the implication of our model is not limited to open economies with trade taxes, which means that the main conclusion in this paper also applies to domestic energy taxes. As in ACW (2005), the foreign input can also be interpreted as non-reproducible natural resources extracted domestically.

than a critical value τ^*.[5]

The intuition behind this result is easy to understand. Suppose the proverbial representative agent expects future tariff rates to increase. This implies, for any given stock of capital, future imports of foreign inputs and the marginal product of capital will be lower. This can lower the current demand for foreign inputs, thus leading to a fall in total output. Because the tariff rate is regressive with respect to the output (under the balanced budget rule with a pre-set level of government expenditures), the tariff rate today will increase, thus validating the agent's initial expectations.

My results suggest that, in general, we should either impose some restrictions on the government ability to adjust tariff rates or reduce the level of tariff rates levied on the imported factors in order to avoid aggregate instability. Consider the current high tariff rate which is prevailing in European countries (especially in year 2002). Some countries like Denmark and Netherlands, which are economies quite dependent on the imported exhaustible natural resources, can be easily pushed into destabilization.[6] I use the ACW's estimation of the imported energy share in the two countries and find that the high tariff rate on oil in the EU leads the two countries into destabilization. Similarly, the energy taxes which the EU countries have tried to impose recently also bring the potential dangers of destabilization into those countries which are economies largely dependent on non-reproducible resources. Those countries like Denmark and Netherlands should pay close attention to the control of energy taxes in order to stabilize the economy. As an optimal import tariff, energy taxes seem to be very high in these two EU countries (see Newbery (2005)). This implies that indeterminacy can easily arise in these two countries.

In the policy literature, Maskin and Newbery (1990) show that the optimal open loop tariff (the optimal tariff assuming commitment) on oil can induce dynamical inconsistency so that the importer

[5] The difference is that Miguel and Manzano only allow for exogenous tariff rates and therefore indeterminacy cannot occur in their model.

[6] Although throughout the paper, we analyze the model for the developed countries, the results also hold for the less-developed countries which productions are dependent on the imported factors.

would wish to change the time path of the tariff in the midsteam. But Bergstrom (1982) shows that tariffs on oil have an attractive property. In his paper, he shows that with international trade in oil and noncooperative consuming nations, if each nation chooses the optimal ad valorem tariff, taking the behaviors of other countries as given, and if suppliers are competitive, then the incidence of the tariffs will fall on producers and cause no distortions. Recently, in a small open economy that imports oil as a third input, Miguel and Manzano (2006) find that, in general, the government should not distort the oil price paid by firms with taxes, even when consumption of oil is considered and the government distinguishes between the taxes paid by the households and the firms. This is because the optimal tax on intermediate goods (such as oil) should be zero in order to maintain aggregate production efficiency. My paper represents the same point of view of them from another perspective. In other words, in order to avoid aggregate instability caused by endogenous tariffs, the government should not distort the oil price.

To my knowledge, the papers that study indeterminacy in open economies are Lahiri (2001), Weder (2001), Meng and Velasco (2004) and ACW (2005, 2006 and 2007). Unlike me, Lahiri, Weder and Meng and Velasco are concerned with mulptiple equilibria in the economy that uses capital and labor as inputs. The models of Lahiri and Weder depend on the unrealistic assumption of decreasing marginal costs to generate indeterminacy. Meng and Velasco analyze the effects of distortionary factor taxation in generating indeterminacy. ACW (2005, 2006 and 2007) is closer to the present paper while their model also relies on decreasing marginal costs (or increasing returns) in production. The mechanism for indeterminacy in my model is through fiscal increasing returns caused by endogenous tariffs.

In sections 3 and 4, we compare our model with Benhabib and Farmer, SGU and ACW models and find that (1) the indeterminacy condition in my model has a close correspondence with the one obtained in the increasing returns model of Benhabib and Farmer (1994); (2) if the imported factor is mainly a labor substitute, indeterminacy may not easily arise; and (3) the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. In section 5, we discuss the robustness of our indeterminacy results and in section 6, we conclude the paper.

2. An Economy with Tariffs

My paper incorporates two different formulations of the government budget constraints into a modified small open economy version of Benhabib and Farmer (1994) competitive model without production externalities. We assume that labor is indivisible (as in Hansen (1985)) and the only source of government revenue is a tariff. In particular, the balanced-budget rule consists of exogenous (and/or endogenous) government purchases (and/or transfers), and endogenous (and/or exogenous) tariff rates levied on the imported input.

2.1. *Firms*

We introduce government tariff policy into the continuous time framework of ACW without productive externalities. There is a continuum of identical competitive firms with the total number normalized to one. The single good is produced by the representative firm with a constant returns to scale Cobb-Douglas production function

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0}, \tag{1}$$

where y_t is total output, k_t is the aggregate stock of capital, n_t is the aggregate labor supply, $a_k + a_n + a_0 = 1$ and the third factor in the production, say oil (o_t), is imported.[7] Perfect competition in factor and product markets implies that factor demands are given by

[7] The third inputs can be non-reproducible natural resources.

$$w_t = a_n \frac{y_t}{n_t}, \tag{2}$$

$$r_t + \delta = a_k \frac{y_t}{k_t}, \tag{3}$$

and

$$p^o(1+\tau_t) = a_0 \frac{y_t}{o_t}, \tag{4}$$

where $(r_t + \delta)$ denotes the user cost of renting capital, w_t denotes the real wage, p^o denotes the real price of oil (the imported goods) and τ_t is the tariff rate levied on the imported oil and uniform to all firms.[8] Here we should emphasize that (1) p^o is the relative price of the foreign input in terms of the single good, which is the numeraire and tradable; and (2) the variable τ_t represents the endogenous and/or exogenous tariff rate levied on the foreign input and we require that $\tau_t \geq 0$ to rule out the existence of import subsidies.[9]

Since we assume that the economy is open to importing energy (oil), the agent can use the tradable good to buy the foreign input. The energy price is assumed to be exogenous and the foreign input is assumed to be perfectly elastically supplied.[10] These imply that the energy price, p^o, is independent of the factor demand for o_t. Hence by substituting out o_t in the production function using $o_t = a_0 \frac{y_t}{p^o(1+\tau_t)}$, we can obtain the following reduced-form production function

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{5}$$

Here the term $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ acts as the "technology coefficient" in a neoclassical growth model,

[8] $\delta \in (0,1)$ denotes the depreciation rate of capital and r_t is the rental rate of capital.

[9] If tariff rates are exogenous, $\tau_t = \tau$ holds for all t.

[10] The model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models have been used widely to study the business-cycle effects of oil price shocks. This literature includes Finn (2000), Rotemberg and Woodford (1996), Wei (2003), ACW (2005, 2006, and 2007).

which is inversely related to the foreign factor price and τ_t. In this reduced-form production function, the "effective returns to scale" is measured by

$$\frac{a_k + a_n}{1 - a_0} = 1. \tag{6}$$

2.2. *Households*

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time and maximizes the intertemporal utility function

$$\int_0^{\infty} e^{-\rho t}(\log c_t - bn_t)dt,\ b > 0, \tag{7}$$

where c_t and n_t are the individual household's consumption and hours worked, and $\rho \in (0, 1)$ is the subjective discount rate. We assume that there are no intrinsic uncertainties present in the model.

The budget constraint of the representative agent is given by

$$\dot{k}_t = r_t k_t + w_t n_t - c_t + T_t,\ k_0 > 0 \text{ given}, \tag{8}$$

where $\dot{k}_t$ denotes net investment and $T_t \geq 0$ is the lump-sum transfers.

The first order conditions for the household's problem are

$$\frac{1}{c_t} = \Lambda_t, \tag{9}$$

$$b = \Lambda_t w_t, \tag{10}$$

$$\dot{\Lambda}_t = (\rho - r_t)\Lambda_t, \tag{11}$$

where Λ_t denotes the marginal utility of income.

2.3. *Government*

The government in my model chooses the tariff/transfer policy $\{\tau_t, T_t\}$, and balances its budget in each period. At each point in time, the budget constraint of the government can be stated as follows

$$p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)} = G_t + T_t, \tag{12}$$

where $G_t \geq 0$ represents government expenditures. Finally, market clearing requires that aggregate demand equal aggregate supply

$$c_t + G_t + \dot{k}_t + \delta k_t + o_t p^o = y_t. \tag{13}$$

Note that the international trade balance is always zero. Foreigners are paid in goods. This is clear in the above equation, according to which domestic production is divided between consumption, investment, imports and government expenditures ($c_t + i_t + p^o o_t + G_t = y_t$, $i_t = \dot{k}_t + \delta k_t$). So part of what is produced domestically is used to pay for the imports. This is the interpretation of Finn (2000), Wei (2003) and ACW (2006).

2.4. *Analysis of the model dynamics*

As in GH (2004), we assume that tariff revenues can be either consumed by the government (i.e. $G_t \geq 0$ for all t) or returned to households as transfers (i.e., $T_t \geq 0$, for all t). And it is easy to verify that the economy in which the government finances endogenous public spending and/or transfers with fixed tariff rates is immune to indeterminacy. That is due to the following proposition.

Proposition 1. *If the tariff rate is exogenous, production doesn't exhibit increasing returns to scale since A_t term is a constant for all t. (In this case, government expenditures are endogenous under the balanced budget rule.) Therefore, the economy exhibits saddle path stability, regardless of the existence of lump-sum transfers.*

GH prove that under perfect competition and constant returns-to-scale, if the government finances endogenous public spending and transfers with fixed income tax rates, a one-sector real business cycle model exhibits determinacy, regardless of the existence of lump-sum transfers. In our tariff model, we have the same result. Once we fix the tariff rate (or oil tax rate) like Miguel and Manzano (2006), the model doesn't display increasing returns to scale, so indeterminacy cannot arise.

To remain comparable with SGU's analysis, we focus on the cases where the government either consumes all tariff revenues (i.e., $T_t = 0$) or transfers the revenue to the household in a lump-sum way (i.e., $G_t = 0$). In the following sections, we **only** discuss the case where $T_t = 0$ holds for all t. Under this specific assumption, we replace the consumption with $\frac{1}{\Lambda_t}$ and transform wage rate and rental rate into functions of capital and labor, then the equilibrium conditions can be reduced to the following five equations

$$b = \Lambda_t a_n A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1}, \tag{14}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - a_k A_t k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}}, \tag{15}$$

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau_t})y_t - \delta k_t - \frac{1}{\Lambda_t} - G_t, \tag{16}$$

$$G_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}, \tag{17}$$

and

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{18}$$

First we claim that for a given tariff rate (i.e., $\tau_t = \tau$, for all t), the dynamical system possesses a unique interior steady state.

Lemma 2. *The dynamical system possesses a unique interior steady state when the government consumes all tariff revenues and the tariff rate is exogenously given, i.e., $\tau_t = \tau$, for all t. (In this case, $A_t = A(\tau)$ holds for all t.)*

Proof. To find such a steady state, set $\dot{\Lambda}_t$ in (15) equal to zero. We can solve the capital/labor ratio in the steady state, which is *not independent* of the tariff rate. $(\frac{k}{n})_{ss} = (\frac{\rho+\delta}{a_k A})^{\frac{1-a_0}{-a_n}}$ is unique for the given tariff rate. Equation (14) can be solved for a unique and positive value of Λ in the steady state, i.e., $\Lambda_{ss} = \frac{b}{a_n A}(\frac{\rho+\delta}{a_k A})^{a_k/a_n}$. Using this value of Λ, the government budget constraint (17), and the fact that in the steady state $\dot{k}_t = 0$, we can write the market-clearing condition (16) as

$$(1-a_0)k_{ss}[A(\frac{k}{n})_{ss}^{\frac{-a_n}{1-a_0}} - \delta] = \frac{a_n A}{b}(\frac{\rho+\delta}{a_k A})^{-a_k/a_n}. \tag{(kss)}$$

Since $(\frac{k}{n})_{ss}$ is known given the tariff rate, we can find that k_{ss} (the steady state value of the capital stock) is unique and positive. Because both the capital stock and the capital/labor ratio are positive and unique in the steady state, n_{ss} (the steady state value of the labor supply) is also positive and unique. Finally, the steady state level of government purchases given by (17) is also unique and can be written as

$$G_{ss} = \frac{\tau}{1+\tau} a_0 A k_{ss} (\frac{k}{n})_{ss}^{-\frac{a_n}{1-a_0}}, \tag{(g)}$$

where k_{ss} is the solution to (kss) equation. It is clear that G_{ss} is continuous in τ.

It follows from (kss) and (g) that when τ is equal to zero, G_{ss} is also equal to zero because k_{ss} is in this case positive and finite. If the tariff rate is exogenous, we can prove that there exists a unique tariff rate that maximizes G_{ss}. It is $\tau_m = \frac{a_n}{a_0}$. ■

Secondly, for a given level of government expenditures, there is a (steady-state) Laffer curve-type relationship between the tax rate and tax revenue, which means that the number of steady state tariff rates that generate enough revenue to finance the pre-set level of government purchases will be general either 0 or 2. I prove it in the following lemma.

Lemma 3. *When tariff rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures, the steady state in the dynamical system which consists of (14)-(18) may exist and the number of steady state tariff rates (τ_{ss}) that generate enough revenue to finance the pre-set level of government purchases will be general either 0 or 2. If two steady states exist in the model, we only focus on the steady state associated with the low steady state tariff rate since the steady state associated with the high steady state tariff rate is always deteminate.*

Proof. We derive the steady state values of these variables $\frac{k}{n} = (\frac{\rho+\delta}{a_k A(\tau_{ss})})^{\frac{1-a_0}{-a_n}}$, $\Lambda = \frac{b}{a_n A(\tau_{ss})}(\frac{\rho+\delta}{a_k A(\tau_{ss})})^{\frac{a_k}{a_n}}$ and $k = \frac{\frac{a_n A(\tau_{ss})}{b}(\frac{\rho+\delta}{a_k A(\tau_{ss})})^{-\frac{a_k}{a_n}}}{[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$, where $A(\tau_{ss})$ denotes the steady state value of A as τ_t is equal to its steady state value τ_{ss}. We also find that in the steady state, $G = \frac{\tau_{ss}}{(1+\tau_{ss})^{\frac{a_n+a_0}{a_n}}}$constant$= F(\tau_{ss})$ holds and the constant is $\frac{(\frac{a_0}{p^o})^{\frac{a_0}{a_n}} a_0(\rho+\delta)a_n(\frac{\rho+\delta}{a_k})^{-\frac{a_k}{a_n}}}{a_k b[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$. It is clear that $F(\tau_{ss})$ is non-monotone and the number of positive steady state tariff rates that generate enough revenue to finance a pre-set level of government purchases will be general either 0 or 2.

Insert the figure

The second interesting finding is that if τ is exogenous as we assume in the above lemma, $\frac{\partial G_{ss}}{\partial \tau} = 0$ implies that there exists a unique exogenous tariff rate that maximizes G_{ss}. It is $\frac{a_n}{a_0}$. This is due to the fact that G_{ss} is equal to $\frac{\tau}{(1+\tau)^{\frac{a_n+a_0}{a_n}}}$constant. ∎

In the third step, we show that when the government expenditures are exogenous, the tariff rate is countercyclical with respect to the tax base or the output under the balanced budget rule. The next proposition is the key to indeterminacy in my model.

Proposition 4. *If the government expenditure is exogenous, the tariff rate is regressive with respect to the tax base ($p^o o_t$), or the output under the balanced budget rule, i.e. $\frac{\partial \tau_t}{\partial y_t} < 0$.*[11] *The regressive (countercyclical) tariff rate ($\frac{\partial \tau_t}{\partial y_t} < 0$) can induce increasing returns to scale with respect to capital and labor.*

Proof. $p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)} = G$ implies that $\frac{\partial \tau_t}{\partial y_t} < 0$. Consider the log-linearization of the following three equations around the steady state $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$, $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, and $y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}$.[12] It is easy to verify that $\hat{y}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t$, which means that production exhibits increasing returns to scale with respect to capital and labor, i.e., $\frac{a_k+a_n}{1-a_0(1+\tau_{ss})} > 1$. Thus, an endogenous tariff rate could be a source of fiscal increasing returns. ∎

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[11] *This relation doesn't violate the evidence of a negative relationship between tariffs and growth, especially among the world's rich countries like those in EU, which is documented by Dejong and Ripoll (2005).*

[12] Log-linearizing the equation $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ around the steady state, we have $(G - a_0 y_{ss})\hat{\tau}_t = a_0 y_{ss}\hat{y}_t$ and $G = \frac{\tau_{ss} a_0 y_{ss}}{(1+\tau_{ss})}$, where $\hat{\tau}_t$ and $\hat{y}_t$ denote the log deviations of τ_t and y_t from their respective steady states (i.e., τ_{ss} and y_{ss}). Combining these two equations yields $\hat{y}_t = -\frac{1}{1+\tau_{ss}}\hat{\tau}_t$. $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ implies that $\hat{A}_t = -\frac{\tau_{ss}\hat{\tau}_t}{\frac{1-a_0}{a_0}(1+\tau_{ss})}$ and log-linearizing the production function implies that $\hat{y}_t = \hat{A}_t + \frac{a_k}{1-a_0}\hat{k}_t + \frac{a_n}{1-a_0}\hat{n}_t$. It is straightforward to see that $\hat{y}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t$.

GH illustrate that under perfect competition and constant returns-to-scale, Schmitt-Grohé and Uribe's indeterminacy result depends on a balanced-budget requirement whereby the tax rate decreases with the household's taxable income. In our model, we get a similar result that requires the countercyclical rate to generate indeterminacy.[13]

To facilitate the analysis of the model dynamics, we consider the log linear approximation of the equilibrium conditions around the steady state. Let λ_t, $\hat{k}_t$, $\hat{\tau}_t$ and $\hat{n}_t$ denote the log deviations of Λ_t, k_t, τ_t and n_t from their respective steady states. The log linearized equilibrium conditions then are

$$0 = \lambda_t - \frac{\tau_{ss}\hat{\tau}_t}{\frac{1-a_0}{a_0}(1+\tau_{ss})} + \frac{a_k}{1-a_0}(\hat{k}_t - \hat{n}_t), \tag{19}$$

$$\dot{\lambda}_t = (\rho+\delta)[\frac{a_n}{1-a_0}(\hat{k}_t - \hat{n}_t) + \frac{\tau_{ss}\hat{\tau}_t}{\frac{1-a_0}{a_0}(1+\tau_{ss})}], \tag{20}$$

$$\dot{\hat{k}}_t = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta]\hat{k}_t + \frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}\hat{n}_t + [-\delta + \frac{(1-a_0)}{a_k}(\rho+\delta)]\lambda_t, \tag{21}$$

and

$$\hat{y}_t = -\frac{1}{1+\tau_{ss}}\hat{\tau}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t. \tag{22}$$

Combining (19) and (22) yields

$$\hat{n}_t = \frac{\lambda_t}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}} + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\hat{k}_t.$$

[13] We think that the progressive tariff rate may make the economy against the sunspots in ACW model.

Using this expression to eliminate the $\hat{n}_t$in (20) and (21) results in the following system of differential equations

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{\hat{k}}_t \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \begin{bmatrix} \lambda_t \\ \hat{k}_t \end{bmatrix} \text{ and } J = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}, \tag{23}$$

where $\mathbf{J}_{11} = -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}}$, $\mathbf{J}_{22} = (\rho+\delta)\frac{1-a_0}{a_k - a_0\tau_{ss}} - \delta$, $\mathbf{J}_{12} = (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k - a_0\tau_{ss}}$ and $\mathbf{J}_{21} = (\rho+\delta)\frac{(1-a_0)}{a_k}\frac{1-(\tau_{ss}+1)a_0}{a_k - a_0\tau_{ss}} - \delta$. We can then compute the Jacobian matrix of the dynamical system (23) evaluated at the steady state. The trace and the determinant of the Jacobian are stated as follows

$$trace(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta, \tag{24}$$

$$\det(J) = \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}\{\delta(a_n + a_0\tau_{ss}) - \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}[a_n(1-a_0) - a_0\tau_{ss}\frac{1-a_0}{a_k}(1 - a_0(1+\tau_{ss}))]\}. \tag{25}$$

Proposition 5. *The necessary and sufficient condition for the indeterminacy of the equilibrium is* $\mathbf{J}_{11} + \mathbf{J}_{22} = trace(\mathbf{J}) < 0 < \mathbf{J}_{22}\mathbf{J}_{11} - \mathbf{J}_{12}\mathbf{J}_{21} = \det(\mathbf{J})$*, or,* $\frac{a_k}{a_0} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0) - \delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0) + \delta a_0 a_k]}$.

Notice that since the dynamical system contains one predetermined variable, k_t, the equilibrium is indeterminate if and only if both eigenvalues of the Jacobian matrix have negative real parts. It is equivalent to requiring that the determinant be positive and the trace negative. It is easy to verify that, $trace(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta < 0$ if and only if $\tau_{ss} > \frac{a_k}{a_0}$. If the trace condition is satisfied, the term $\frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}$ on the right side of the determinant is negative. $det(J) > 0$ if and only if $G(\tau_{ss}) = [\frac{(\rho+\delta)a_0^2(1-a_0)}{a_k} + \delta a_0^2]\tau_{ss}^2 - \tau_{ss}[\frac{(\rho+\delta)a_0(1-a_0)^2}{a_k} + \delta a_0(a_k - a_n)] + [(\rho+\delta)a_n(1-a_0) - \delta a_n a_k] < 0$. It is easy to show $G(\frac{a_k}{a_0}) = 0$ and $G(0) > 0$. Then the necessary and sufficient condition for the equilibrium indeterminacy is equivalent to $G < 0$, or, $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$ where $\tau^* = \frac{[(\rho+\delta)a_n(1-a_0) - \delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0) + \delta a_0 a_k]} > \frac{a_k}{a_0}$.

A sufficient condition for the set of tariff rates satisfying the necessary and sufficient condition to be nonempty is that the labor share is larger than the capital share (i.e., $a_n > a_k$). For steady-state tariff rates smaller than $\frac{a_k}{a_0}$ or greater than τ^*, the determinant of J is negative and therefore the equilibrium is locally determinate. The thing that should be pointed out is that SGU show that the revenue maximizing tax rate is the least upper bound of the set of taxes rate for which the rational expectations equilibrium is indeterminate, but this property doesn't hold in our case.

The intuition behind the indeterminacy result is quite straightforward. Suppose that agents expect future tariff rates to increase. This implies that, for any given capital stock, future oil imports and the rate of return on capital will be lower (the latter is due to the fact that the marginal product of capital is increasing in the oil input). The decrease in the expected rate of return on capital, in turn, lowers the current oil demand, leading the current output decrease. Since the tariff rate is countercyclical $\frac{\partial \tau_t}{\partial y_t} < 0$, budget balance can cause the current tariff rate to increase, thus validating agents' initial expectations. (For certain choices of the parameter values, namely those satisfying $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$, the expectation of an above steady state tariff rate in the next period leads to an increase in tariff rates today that is larger than the one expected for next period.)

To help understand the intuition, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

$$\frac{c_{t+1}}{c_t} = \beta(1 - \delta + a_k \frac{y_{t+1}}{k_{t+1}}) = \beta[1 - \delta + (1 + \tau_{t+1})^{-\frac{a_0}{1-a_0}} r_{t+1}^{bt}], \tag{26}$$

where β denotes the discount factor, $r_{t+1}^{bt} = a_k(\frac{a_0}{p_0})^{\frac{a_0}{1-a_0}} k_{t+1}^{\frac{a_k}{1-a_0}-1} n_{t+1}^{\frac{a_n}{1-a_0}}$ the before-tariff return on capital and τ_{t+1} the tariff rate in period $(t+1)$. Households' optimistic expectations that lead to higher investment raise the left hand side of this equation, but result in a lower before-tariff return on capital r_{t+1}^{bt} due to the diminishing marginal products. The countercyclical tariff rate can increase the right hand side of the equation, thus validating the initial optimistic expectations.

Capital accumulation is crucial in generating indeterminacy in this economy. One can easily show that in the absence of capital accumulation, the equilibrium is determinate. The reason is that without capital, (14) becomes $b/\Lambda_t = a_n(\frac{a_0}{p^o(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, (16) becomes $1/\Lambda_t = (1 - \frac{a_0}{1+\tau_t})y_t - G$, and (17) becomes $G/n_t = \tau_t(\frac{1}{p^o})^{\frac{a_0}{1-a_0}}(\frac{a_0}{1+\tau_t})^{\frac{1}{a_n}}$. These three equations yield locally unique solutions for τ_t, n_t, and Λ_t.

We propose a numerical case based on ACW (2005) model without increasing returns: $a_k = 0.09$, $a_n = 0.7$, and $a_o = 0.21$.[14] We choose $\delta = 0.1$ for the depreciation rate and $\rho = 0.04$ for the discount rate. These values are taken from SGU. Assuming that the steady state tariff rate in the country for oil import is $\tau_{ss} = 0.6$, then the two roots of the matrix J are $-0.2250 \pm 1.5714i$.[15] Indeterminacy arises due to the presence of endogneous tariff rates.[16] The high tariff rate (0.6) is consistent with the empirical data in those EU countries (especially in year 2002).

In a closely related paper, Chen and Zhang (2008) introduce intrinsic uncertainty in the form of exogenous productivity and government purchases shocks into the discrete time version of this model and investigate the propagation mechanism of sunspot and fundamental shocks under three kinds of assumptions about their correlation. As in SGU, we calibrate the model and find that (1) under indeterminacy, the impulse responses of tariff rates, output, and hours to sunspot, technology and government purchases shocks are hump-shaped and highly persistent; and (2) neither the first-order serial correlations, the contemporaneous correlations with output, nor the standard deviation relative to output of tariffs, output, hours, and consumption is affected by the relative volatility of the sunspot shock or its correlation with the technology shock. Therefore it validates the equivalence between factor income taxes and tariffs, in the sense that they share similar propagation mechanisms

[14] In this numerical case, following ACW (2005), we set a_0 to be 0.21 which is the cost share of foreign inputs in domestic production in Netherlands.

[15] $\frac{\text{import tariff}}{\text{import price}} = \frac{15.6\$/bbl}{26\$/bbl} = 0.6$ is the optimal tariff rate of oil from Newbery (2005, section 3.1).

[16] One issue should be pointed out. Consumption taxes, which are a significant source of tax revenue in the European countries, are ignored. In fact, it can be shown that if the only source of government revenues is a tax on consumption then the rational expectations equilibrium under a balanced-budget rule implemented through an endogenous consumption tax rate is locally determinate.

of sunspot and fundamental shocks.[17]

The conclusions in the model also hold for the energy taxes. As we see, the energy taxes as the optimal tariff argument are relatively high in some European countries.[18] For instance, oil is heavily taxed in Denmark, the effective tax rate on domestic fuels exceeds 0.8. It will push the Denmark's economy into destabilizing easily. (Assuming that the time unit is a quarter, the capital share is 0.1 (i.e., $a_k = 0.1$), the labor share is 0.7 (i.e., $a_n = 0.7$), the depreciation rate is 0.1 (i.e., $\delta = 0.1$), the discount rate is 0.04 (i.e., $\rho = 0.04$) and the steady state oil tax rate is $\tau_{ss} = 0.8$, indeterminacy arises since two roots are $0.1667 \pm 1.1213i$).[19]

3. Comparison with Benhabib-Farmer Model

In this section, we show that there exists a close correspondence between the indeterminacy condition of the model with endogenous tariff rates and constant government purchases presented in my paper and that of the increasing returns model of Benhabib and Farmer (1994). That is, the necessary condition for local indeterminacy is that the "equilibrium labor demand schedule" can be upward sloping and steeper than the labor supply schedule. Unlike the Benhabib and Farmer (1994) model, my model doesn't rely on explicit increasing returns in production to make the equilibrium labor demand be upward sloping. In fact, the equilibrium labor demand in my model can be upward sloping because increases in the aggregate employment are accompanied by decreases in the tariff rate. The after-tariff labor demand function can be written (in log deviations around the steady state) as

$$\hat{w_t} = \frac{a_k}{1-a_0}\hat{k_t} - \frac{a_k}{1-a_0}\hat{n_t} - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau_t}, \tag{27}$$

[17] The difference is that in the calibration exercise, all parameter values are taken from ACW (2005).
[18] The energy tax revenue is overwhelmingly oil tax revenue in some EU countries, see Newbery (2005).
[19] The parameter values of a_k and a_n are taken from ACW (2005). The values of δ and ρ are taken from SGU.

where $\hat{w}_t = \hat{w}_t^{bt} - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau}_t$ denotes the log deviation of the after-tariff wage rate from the steady state.[20] The firm's labor demand schedule is decreasing in $\hat{n}_t$. However, using the balanced-budget equation (22) to eliminate $\hat{\tau}_t$ yields the following expression for the equilibrium labor demand schedule:

$$\hat{w}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{-(a_k - a_0\tau_{ss})}{1-a_0(1+\tau_{ss})}\hat{n}_t. \tag{28}$$

As $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$, the equilibrium labor demand function is upward sloping since $\frac{-(a_k-a_0\tau_{ss})}{1-a_0(1+\tau_{ss})} > 0$. Because in our case $\hat{w}_t = \hat{c}_t$, the aggregate labor supply is infinitely elastic (for a given tariff rate and marginal utility of income), the labor demand schedule will be steeper than the labor supply schedule whenever $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$. It is worth noting that our economy can be easily shown to be equivalent to SGU model since in both cases, the price-to-cost markup is countercyclical with respect to the output, which in turn gives rise to indeterminacy (see the appendix).

4. Comparison With SGU and WAC models

SGU prove that within a standard neoclassical growth model, a balanced budget rule can make expectations of higher tax rates self fulfilling if the fiscal authority relies on changes in labor income taxes to eliminate the short run fiscal imbalances. People will naturally think if the import factor is a labor substitute, the endogenous tariff rate levied on the imported oil will make indeterminacy arise more easily. Although in the above sections, we follow ACW to assume that the imported factor is mainly a substitue for capital, we can not eliminate the possibility that imported factor is a substitute for labor.

We get the following proposition:

[20] Here we should emphasize that $\hat{w}_t^{bt}$ denotes the log deviation of the before-tariff wage rate from the steady state.

Proposition 1. *If we assume that the imported factor is mainly a labor substitute instead of a capital substitute, which means that we fix a_k at a given level (say, $a_k = 0.3$) and let a_0 vary in the interval $(0, 1 - a_k - a_n)$,*[21] *indeterminacy may not easily arise under the labor substitute assumption.*

Proof. First I give you an example. Let a_0 be 0.2 as we did in the numerical case, and a_n be 0.5. The necessary and sufficient condition becomes $\frac{a_k}{a_0} = 1.5 < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]} \approx 2.5$. Consider the case which we analyzed before (the imported factor is a capital substitute, $a_k = 0.1$, $a_n = 0.7$, and $a_0 = 0.2$). The necessary and sufficient condition is $\frac{a_k}{a_0} = 0.5 < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]} \approx 3.5$. Indeterminacy won't easily arise under the labor substitute assumption since tariff rates cannot be that high (more than 150%). A simple proof can be stated as follows. Consider an economy with capital share (a) and labor share ($1-a$). When we introduce into the model the foreign input with share b as a labor substitute, the indeterminacy region becomes $\frac{a}{b} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)(1-a-b)(1-b)-\delta(1-a-b)a]}{[(\rho+\delta)b(1-b)+\delta ba]}$. When we introduce into the model the foreign input with share b as a capital substitute, the indeterminacy region becomes $\frac{a-b}{b} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)(1-a)(1-b)-\delta(1-a)(a-b)]}{[(\rho+\delta)b(1-b)+\delta b(a-b)]}$. It is clear that the lower bound of the region under the labor substitute assumption is larger than the one obtained under the capital substitute assumption. ■

From this proposition, we find that although tariffs share the similar mechanism for indeterminacy with factor income taxes, they have different implications in generating indeterminacy. That is, the "equivalence" relationship between them only holds through fiscal increasing returns by endogenizing rates and making the government spending exogenous. ACW find that if the imported factor is a substitute for labor, a larger oil share (a_0) implies a smaller threshold value of the production externality although the reduction of externality is less dramatic. Here we find that for the same oil share (a_0), under the labor substitute assumption, the threshold value of the (steady state) tariff rate needed to generate indeterminacy (i.e., the lower bound of the indeterminacy region) can be

[21] *Under the capital substitute assumption, we fix a_n at a given level and let a_0 vary in the interval $(0, 1 - a_k - a_n)$.*

larger than the one obtained under the capital substitute assumption. In the numerical calibration example of Chen and Zhang (2008), we assume that the time unit is a quarter, the capital share is 0.09 (i.e., $a_k = 0.09$), the labor share is 0.7 (i.e., $a_n = 0.7$), the depreciation rate is 0.025 (i.e., $\delta = 0.025$), the discount rate in the discrete time model is 0.99 (i.e., $\beta = 0.99$) and the steady state oil tax rate is 0.6 (i.e., $\tau_{ss} = 0.6$).[22] Then we find that indeterminacy arises in this case. We can easily prove that indeterminacy disappears if we assume that the imported oil is a labor substitute. Since $a_k = 0.3$, $a_0 = 0.21$, and $a_n = 0.49$ imply that indeterminacy would require the steady state tariff rate be at least 10/7, the empirical tariff rate isn't in this region.

WAC show that heavy reliance on imported energy can have a significant effect on economic instability in the presence of increasing returns to scale: the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. We have the similar proposition below:

Proposition 2. *We fix a_n at a given level (say, $a_n = 0.7$), which implies that the imported input is a capital substitute (i.e., $a_k + a_0 = (1 - a_n)$ is fixed). The larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. Because the lower bound of the indeterminacy region $\frac{a_k}{a_0} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]}$ decreases as a_0 increases, indeterminacy is easier to arise in the range of empirical tariff rates the larger a_0 is.*

As a_0 decreases, the minimum tariff rate that generates indeterminacy increases (given that a_n is fixed). Some intuition for this result can be shown by considering the equilibrium condition in the labor market. Suppose that expectations of a future tariff increase shift the labor supply schedule up (since the firm will import more oil today to produce more output). Because the slope of the labor demand schedule is equal to $-\frac{a_k}{1-a_0} = -\frac{1}{1+\frac{a_n}{a_k}}$, the smaller a_k is (given that a_n is fixed), the larger

[22] The country in this example is Netherland and we assume that the foreign input is a capital substitute.

the decline in employment (since the slope of the labor demand schedule decreases in absolute value as a_k decreases). Consequently, the increase in the tariff rate required to bring about budget balance is larger the smaller a_k is or the larger a_0 is, and hence stationary sunspot equilibria become more likely the larger a_0 is.

5. Robustness

In this section, we briefly discuss the robustness of indeterminacy in the economy with endogenously determined tariff rates. Specifically, we allow for income-elastic government spending and more general preferences.[23] First, we conclude that the more procyclical government expenditures are, the less likely it is that indeterminacy arises. That is because $\frac{G_t}{Y_t} = \frac{a_0}{1+\frac{1}{\tau_t}}$ (the budget balance) implies that the more countercyclical government expenditures are, the larger the required change in tariff rates necessary to balance the budget for a given change in output, the more likely it is that indeterminacy arises. Secondly, we claim that the larger the share of public expenditures financed by tariffs, the more likely it is that indeterminacy arises. In a recent paper of Zhang (2008b), I show that if $T_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ is exogenous (i.e., $G_t = 0$—the public expenditures financed by tariffs are zero), indeterminacy still could arise but the lower bound of the indeterminacy region increases. Thirdly, we can consider the general period utility function $U(c_t, n_t) = \log c_t - \frac{n_t^{1+\gamma}}{1+\gamma}, \gamma \geq 0$, which implies less than perfectly elastic aggregate labor supply. In this case, the Frisch elasticity of labor supply with respect to wage is equal to $1/\gamma$. Like SGU, when we introduce a nontaxed sector (such as home production) into the model, the balanced-budget rule may induce indeterminacy under realistic tariff rates and labor supply elasticities. And indeterminacy is more likely the higher the Frisch labor supply elasticity.

[23] As Jess Benhabib suggested to me, tariffs and factor income taxes are very close to each other in nature. Tariffs can inherit many good characteristics of factor income taxes in the SGU model. So the method that I may use to analyze how **public debt** and **predetermined tax rates** affect indeterminacy can be similar to the one that SGU used.

6. Conclusion

We explore the "channel equivalence" between factor income taxes and tariffs to generate indeterminacy. The channel is through fiscal increasing returns by endogenizing rates and making the government spending exogenous. We show that, in the presence of fiscal increasing returns caused by endogenous tariffs, it is easy for indeterminacy to occur in small open economies that import foreign energy and take as given the international energy price. The required steady state tariff rates can be empirically realistic. An implication of our paper is that economies largely dependent on non-reproducible natural resources may be vulnerable to sunspot fluctuations if the government finances public spending with endogenous energy taxes.

One future task is to see under what circumstances, tariffs and capital income taxes are equivalent to generating indeterminacy since the essential element for indeterminacy in the SGU model is the endogenous labor income tax rate.

7. Appendix:

We summarize the equilibrium conditions of the model with a balanced-budget rule, endogenous tariff rates, and constant government purchases presented in this paper. Consider the discrete time case of a tariff. The balanced-budget rule is given by

$$G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}.$$

The following equilibrium conditions hold for all t,

$$U_c(c_t, n_t) = \theta_t,$$

$$U_n(c_t, n_t) = w_t \theta_t,$$

$$\mathsf{Y}_t = c_t + k_{t+1} - (1-\delta)k_t,$$

and

$$1 = \beta \frac{\theta_{t+1}}{\theta_t}(1 - \delta + r_{t+1}),$$

where θ_t is the Lagrangian multiplier of the budget constaint of the agent. In my model, disposable income, Y_t, is given by

$$\mathsf{Y}_t = (1 - a_0)y_t = y_t - p^0 o_t - G,$$

G represents a fixed cost that ensures that firms do not make pure profits in the long run (given that the foreign firms take away their payments). The after-tariff wage rate w_t, and the after-tariff rental rate r_t are given by

$$r_t^{bt} = a_k (\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} = \mu_t r_t,$$

and

$$w_t^{bt} = a_n (\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} = \mu_t w_t,$$

where r_t^{bt} and w_t^{bt} denote the before-tariff rental rate and before-tariff wage rate respectively. In the balanced budget model, μ_t represents the wedge between marginal product and after tariff factor prices introduced by endogenous tariffs. It is easy to verify that the markup μ_t is countercyclical with respect to y_t since

$$\mu_t = (1+\tau_t)^{\frac{a_0}{1-a_0}} = (1 - \frac{1-a_0}{a_0}\frac{G}{\mathsf{Y}_t})^{-\frac{a_0}{1-a_0}} = \mu(\frac{G}{\mathsf{Y}_t}).$$

References

[1] Atolia M., 2006, Tax Evasion, Public Capital, and Welfare Effects of a Tariff Reform (Revised: March, 2006), Florida State University

[2] Benhabib, J., Farmer, R.E., 1994. Indeterminacy and increasing returns. Journal of Economic Theory 63, 19–41.

[3] Benhabib, J., Farmer, R.E., 1999. Indeterminacy and sunspots in macroeconomics. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1A. North-Holland, New York, pp. 387–448

[4] Bergstrom, T.C., 1982. On capturing oil rents with a national excise tax, American Economic Review, March, 72, 194-201.

[5] Bond, E., Wang P., Yip C. K., 1996. A general two-sector model of endogenous growth model with human and physical capital: balanced growth and transitional dynamics. Journal of Economic Theory 68, 149-173.

[6] Chen Y., Zhang Y., 2008. Tariff and Equilibrium Indeterminacy–A Note. Manuscript, Shandong University and New York University. http://mpra.ub.uni-muenchen.de/10044/

[7] De Miguel C., Manzano B., 2006. Optimal oil taxation in a small open economy. Review of Economic Dynamics 9, 438–454

[8] Dejong D. N., Ripoll M., 2006. Tariffs and growth: An empirical Exploration of Contingent Relationship, The Review of Economics and Statistics, Vol. 88(4), 625-640, October.

[9] Finn, M., 2000. Perfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit, and Banking, 32, 400-417.

[10] Guo, J.T., Harrison, S.G., 2004. Balanced-Budget Rules and Macroeconomic (In)stability, Journal of Economic Theory, Vol. 119, No. 2, December, 357-363

[11] Hansen, G., 1985. Indivisible labor and the business cycles, Journal of Monetary Economics 16:309- 327.

[12] Kindleberger, C., Lindert P., 1978. International economics (Irwin, Homewood).

[13] Lahiri, A., 2001. Growth and equilibrium indeterminacy: the role of capital mobility. Economic Theory 17. 197-208.

[14] Leung, C. Ka Yui, 1999. Income Tax, Property Tax, and Tariff in a Small Open Economy. Review of International Economics, 7(3), 541-554.

[15] Loewy M. B , 2004. Optimal tariffs, optimal taxes and economic development, Journal of International Trade & Economic Development, Volume 13, Number 4 / December 2004

[16] Maskin, E., Newbery, D., 1990. Disadvantageous oil tariffs and dynamic consistency. American Economic Review, March, 143-156.

[17] Meng, Q., Velasco A., 2004. Market Imperfections and the Instability of Open Economies, Journal of International Economics, Vol. 64 (2).

[18] Mourmouras, A., 1991. Infant governments and the fiscal role of tariffs, inflation, and reserve requirements, Journal of International Economics 31, 271 – 90.

[19] Newbery, D., 2005. Why Tax Energy? Towards a More Rational Policy, The Energy Journal, Vol. 26, No. 3.

[20] Ramsey, F. P., 1927. A contribution to the theory of taxation, Economic Journal, 37, 47 – 61.

[21] Riezman, R., Slemrod J., 1987. Tariffs and collection costs, Weltwirtschaftliches Archiv. 123, no. 3, 545-550.

[22] Rotemberg, J., Woodford, M., 1996. Imperfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit, and Banking, 28, 549-577.

[23] Schmitt-Grohe S., Uribe M., 1997 (October). Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability. Journal of Political Economy, 105, 976-1000.

[24] Velasco, A., 1996. Animal spirits, capital repatriation and investment. Journal of International Money and Finance 15, 221– 238.

[25] Weder, M., 2001. Indeterminacy in the small open economy Ramsey growth model. Journal of Economic Theory 98, 339-356

[26] Wen, Y., Aguiar-Conraria L., 2007. Understanding the Large Negative Impact of Oil Shocks, Journal of Money, Credit, and Banking, 39, 925-944.

[27] Wen, Y., Aguiar-Conraria L., 2005. Foreign trade and equilibrium indeterminacy, Federal Reserve bank of St. Louis, 2005-041a

[28] Wen, Y., Aguiar-Conraria L., 2006. Oil dependence and economic Instability, Federal Reserve bank of St. Louis.

[29] Wei, C., 2003. Energy, the stock market, and the putty-clay investment model, American Economic Review, 93, 311-323.

[30] Zhang, Y., 2008a. Tariff and Equilibrium Indeterminacy–(I), Manuscript, New York University.

[31] Zhang, Y., 2008b. Tariff and Equilibrium Indeterminacy–(II), Manuscript, New York University.