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Abstract

In this note, we revisit the univariate unobserved-component (UC) model of US GDP by relaxing the traditional random-walk assumption of the permanent component. Since our general UC model is unidentified, we investigate the upper bound of the contribution of the transitory component, and find it is dominated by the permanent component.

Keywords: Unobserved-Component Model; Random Walk Assumption; Permanent and Transitory Shocks

JEL classification: C22; C49; E32

1 Introduction

Morley et al (2003) studied the equivalence of univariate unobserved-component (UC) model and the Beveridge-Nelson (BN) (1981) decomposition. They conclude that the permanent component of US GDP extracted by UC model is exactly the same as the BN trend. And the innovations of the two (permanent and transitory) components are highly negatively correlated (further discussions about this point can be found in a recent paper by Oh et al., 2008). The non-orthogonality of the two innovations is mainly caused by the random-walk

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assumption imposed on the permanent component, as shown by Nagakura (2008). In this note, we relax the random-walk assumption by allowing the permanent component to follow a general unit root process. Under our assumption, the real GDP can be decomposed into two orthogonal parts so that impulse responses to permanent and transitory shocks can be generated. Since our generalization of the random-walk assumption increases the parameter set of the UC model, the model becomes unidentified. However, we can investigate the upper bound of the contribution of the transitory component to GDP and study the dynamics of this extreme case by implementing impulse response and variance decomposition. We find that the transitory component explains less than 35% of total fluctuations in output, which is in sharp contrast to the existing literature.

2 The UC Model

Our modified UC representation takes the form,

$$y_t = g_t + c_t$$

$$g_t = \mu + g_{t-1} + \frac{\Theta_{q_1}(L)}{\Phi_{p_1}(L)}\eta_t, \;\; \eta \sim i.i.d\ N(0, \sigma_\eta^2) \tag{1}$$

$$c_t = \frac{\Theta_{q_2}(L)}{\Phi_{p_2}(L)}\varepsilon_t, \;\; \varepsilon \sim i.i.d\ N(0, \sigma_\varepsilon^2)$$

where $\{y_t\}$ is log real GDP, $\{g_t\}$ is an unobserved permanent component with a unit root (i.e., its first difference is a ARMA(p_1,q_1) process with drift μ). The unobserved transitory component $\{c_t\}$ is a stationary ARMA(p_2, q_2) process. Moreover, we assume the two innovations satisfy

$$cov(\eta_t, \varepsilon_{t\pm k}) = \{ \begin{matrix} \sigma_{\eta\varepsilon} & \text{for } k = 0 \\ 0 & \text{otherwise} \end{matrix}.$$

The parameters under interest include the mean growth rate μ, and the coefficients of the two ARMA process, $\{\Phi_{p_1}(L), \Theta_{q_1}(L), \Phi_{p_2}(L), \Theta_{q_2}(L), \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$.

Writing the model (1) more compactly gives the ARIMA representation of y_t,

$$\Phi_{p_1}(L)\Phi_{p_2}(L)\Delta y_t = \Phi_{p_1}(1)\Phi_{p_2}(1)\mu + \Phi_{p_2}(L)\Theta_{q_1}(L)\eta_t + (1-L)\Phi_{p_1}(L)\Theta_{q_2}(L)\varepsilon_t \tag{2}$$

This expression implies we can recover the parameters of the UC model by estimating the growth rate of GDP as a ARIMA process. Here we follow the strategy of Morley, et al (2003) to estimate GDP as an ARIMA(2,1,2) process[1]:

$$(1-\phi_1 L-\phi_2 L^2)\Delta y_t=(1-\phi_1-\phi_2)\mu^*+(1+\theta_1 L+\theta_2 L^2)u_t \tag{3}$$

Table 1 reports the estimated results, note that γ_j are the jth order autocovariance of MA part of ARIMA process, and μ^*, σ_u and γ_j are percentages. The data used is US quarterly real GDP from 1948:I to 2008:I.

Table 1

Maximum Likelihood Estimates For ARIMA(2,1,2)

	Estimate	Standard Error
Drift μ^*	0.8264	(0.0765)
ϕ_1	1.3638	(0.1227)
ϕ_2	-0.7616	(0.0843)
θ_1	-1.1039	(0.1319)
θ_2	0.5976	(0.1004)
σ_u	0.9068	(0.0311)
AR roots (inverted)	$0.8954 \pm 0.7151i$	
γ_0	2.1184	
γ_1	-1.4505	
γ_2	0.4915	
log likelihood	-317.2356	
Long Run Effect of u_t	1.2411	

The absence of real roots in AR part indicates that the polynomial $(1-\phi_1 L-\phi_2 L^2)$ cannot be factored further. This fact induces us to determine the form of $\Phi_{p_1}(L)$ and $\Phi_{p_2}(L)$ only in two alternative ways: $\Phi_{p_1}(L)=1$, $\Phi_{p_2}(L)=(1-\phi_1 L-\phi_2 L^2)$ or $\Phi_{p_1}(L)=\Phi_{p_2}(L)=(1-\phi_1 L-\phi_2 L^2)$.[2] Obviously, the first case is just the specification in Morley et al (2003) in which permanent component g_t is a random walk. And the second case is the one we want to discuss, in which g_t is a general ARMA(2,2) process.

Once $\Phi_{p_1}(L)$ and $\Phi_{p_2}(L)$ have been determined, we can find the form of MA polynomials $\Theta_{q_1}(L), \Theta_{q_2}(L)$. In particular, to ensure the RHS of (2) be a MA(2) process, $\Theta_{q_1}(L)$ and

[1] Oh et al (2008) also recommend this specification. They find that ARIMA (2,1,2) is preferred by the AIC and ARIMA (1,1,0) is preferred by the BIC. But one shortcoming of the latter is that it cannot capture the periodical behavior of output due to its oversimplified structure.

[2] The setting $\Phi_{p_1}(L)=(1-\phi_1 L-\phi_2 L^2)$, $\Phi_{p_2}(L)=1$ is infeasible, since this will make the order of MA part of Δy_t (the RHS of (2)) to exceed 2.

$\Theta_{q_2}(L)$ can at most take the form of $(1+\psi_1 L+\psi_2 L^2)$ and $(1+\theta L)$, respectively. Now the parameters of interest are $\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$ [3], and the representation (2) is reduced to

$$(1-\phi_1 L-\phi_2 L^2)\Delta y_t = (1-\phi_1-\phi_2)\mu + \left(1+\psi_1 L+\psi_2 L^2\right)\eta_t + (1-L)(1+\theta L)\varepsilon_t \tag{4}$$

Remember that we have estimated the autocovariances of the RHS of last equation from the data, see $\{\gamma_0, \gamma_1, \gamma_2\}$ in Table 1. Equate these moments to their counterparts in (4) and after some algebra, we get three equations about $\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$:

$$\begin{aligned}
\sigma_\eta^2 &= \frac{\gamma_0+2\gamma_1+2\gamma_2}{(1+\psi_1+\psi_2)^2} \\
\sigma_\varepsilon^2 &= \frac{-2(1+\psi_1\theta-\psi_1-\psi_2\theta)(\gamma_2-\psi_2\sigma_\eta^2)-(\theta-\psi_2)[\gamma_0-(1+\psi_1^2+\psi_2^2)\sigma_\eta^2]}{2\theta(1+\psi_1\theta-\psi_1-\psi_2\theta)-2(\theta-\psi_2)(1-\theta+\theta^2)} \\
\sigma_{\eta\varepsilon} &= \frac{\theta[\gamma_0-(1+\psi_1^2+\psi_2^2)\sigma_\eta^2]+2(1-\theta+\theta^2)(\gamma_2-\psi_2\sigma_\eta^2)}{2\theta(1+\psi_1\theta-\psi_1-\psi_2\theta)-2(\theta-\psi_2)(1-\theta+\theta^2)}
\end{aligned} \tag{5}$$

Since the MA(2) process has only three autovariances, we have six unknown parameters with just three equations. Therefore, this UC model is unidentified.

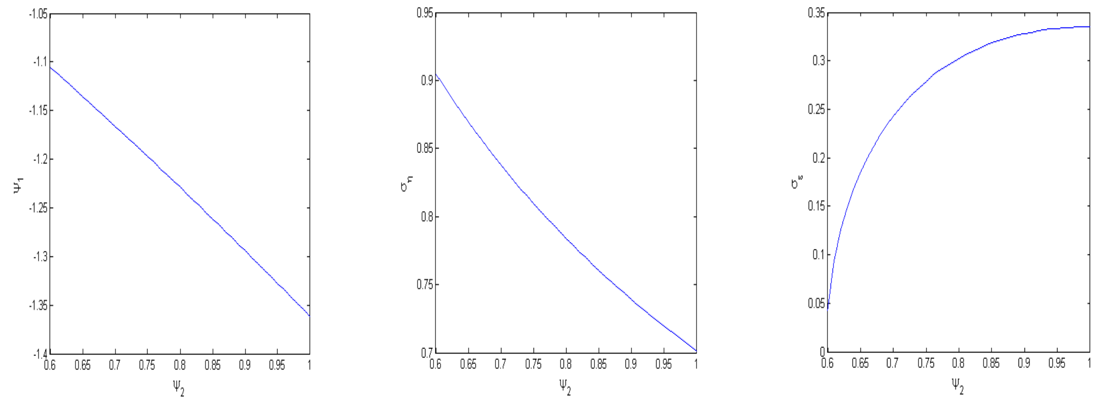
In order to obtain two structural (or orthogonal) shocks, we need to set $\sigma_{\eta\varepsilon}$ to be zero. The reader may ask whether this restriction is feasible[4], since in Morley et al (2003), when permanent component is a random walk, two innovations are always highly negative correlated. In fact, as long as the long-run effect (see the last row in Table 1) in the ARIMA representation of GDP is larger than 1, the orthogonality restriction in our modified UC model is always feasible. A formal mathematical proof can be found in the Corollary 1 of Nagukara (2008).

To learn the relationships of the unknown parameters, one method is solve three of them as functions of the other two. Unfortunately, the system (5) is nonlinear and very complicated, we cannot solve it in a closed form. So we resort to numerical method. Figure 1 below plots $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ as functions of ψ_2 when $\theta = 0$. To conserve space, we do not report cases when θ equals to other value in (-1,1), but the situations changes little. And to ensure Δg_t be invertible and σ_ε^2 always positive, ψ_2 must be in the range about 0.6 to 1.

[3] The mean growth rate μ is just the one in ARIMA representation.

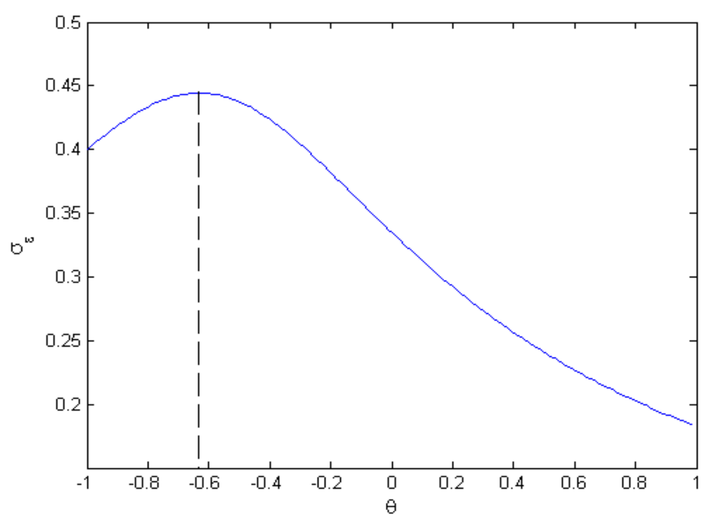
[4] Here, "feasible" means the equation system (5) always has solution when $\sigma_{\eta\varepsilon} = 0$.

Figure 1 —The relationship between $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ and ψ_2 when $\theta = 0$



One thing worth noting in Figure 1 is that $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ are monotonic functions of ψ_2, and this feature does not change for different θ. Furthermore, the standard deviation of transitory shock ε_t reaches its maximum when ψ_2 approaches to 1. Since σ_ε is a continuous function of ψ_2 and θ, without loss of generality, we fix $\psi_2 = 1$ for different θ to find the largest transitory component (in terms of variance) in our modified UC model. Figure 2 plots σ_ε against θ, when $\psi_2 = 1$.

Figure 2 — The maximum of σ_ε for different θ in (-1,1)



From the figure, we can see σ_ε obtains its maximum of 0.4442 at $\theta = -0.63$. In the next section, we will study the dynamic features of the two components under such extreme parameter values (i.e., under the upper bound of the transitory component) and compare the results with those obtained by using the Blanchard-Quah (1989) decomposition.

3 Dynamics

The largest possible variance of the transitory component $\{c_t\}$ has standard deviation 0.4442 when setting $\theta = -0.63$ and $\psi_2 = 1$. The remaining parameters ψ_1 and σ_η can be solved directly from the equation system (5). In particular, we have $\psi_1 = -1.2612$ and $\sigma_\eta = 0.6059$.[5] Since both BQ (1989)[6] and our UC model implement orthogonal decomposition with a general unit-root permanent component, we can use impulse responses and variance decomposition to compare our results with theirs. To ensure consistency (i.e., GDP in the bivariate BQ decomposition must also follow a ARIMA(2,1,2) process), we estimate a 2-variable VAR system with GDP growth and unemployment rate as a vector ARMA(1,1) process[7].

Figure 3 plots the impulse responses of GDP to a one-standard-deviation permanent and transitory shock respectively.[8] In particular, under the permanent shock η_t (the left graph), output in our UC model has a larger and periodic response compared with that obtained by the BQ method. The maximum response reaches its peak after six quarters. The long run effect of the permanent shock is also significantly larger (about 1.1), while under the BQ decomposition this value is only about 0.6. Under the transitory shock ε_t (the right graph), output movement in our model dies out quickly, while under the BQ decomposition the response is much larger and more persistent.

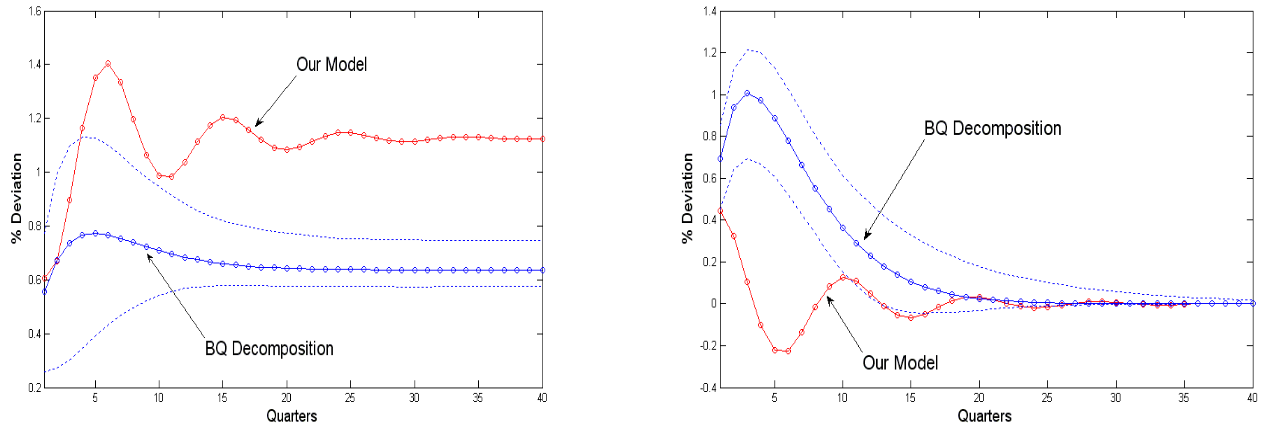
[5] The parameters $\{\sigma_\varepsilon, \sigma_\eta, \psi_1\}$ are statistically significant, we calculate their t statistics by Monte Carlo simulation, but not report here.

[6] In their paper, BQ decompose GDP based on a structural bivariate VAR system of (ΔGDP, Unemployment rate). They just identify the model by imposing a long run restriction on transitory component.

[7] We implement this estimation by RATS 7.0 . One interesting thing is the VARMA(1,1) specification reduces the estimation error greatly and all the parameters are highly significant.

[8] The dotted line is the 95% error band calculated by Monte Carlo simulation with 5000 samples.

Figure 3—Impulse Responses of GDP to Different Shocks



a. Response to Permanent Shock

b. Response to Transitory Shock

To see the relative importance of two shocks to the total variance of GDP, Table 2 reports the variance decomposition, i.e., the proportion of fluctuations due to transitory shock ε_t in different forecasting horizons.

Table 2—Variance Decompositions in Different Models

Percentage of Forecasting Error Due to Transitory Shock		
Horizon (Quarters)	Our Model ($\psi_2 = -1, \theta = -0.62$)	BQ Decomposition
1	34.96	61.06 (25.92, 91.38)
2	27.17	64.11 (29.16, 93.24)
3	16.29	62.04 (27.50, 93.23)
4	9.88	59.96 (26.40, 92.21)
8	4.26	54.63 (25.94, 81.71)
12	3.28	50.56 (26.08, 70.12)
40	0.98	27.00 (13.85, 38.95)

The numbers in parentheses are 95% confidence intervals. Even through these error bands of the BQ decomposition are large, contribution of transitory shocks to GDP are significant lower in our model even compared with the lower bound of the BQ decomposition (except for the first period). That is, our model attributes most fluctuations of output to permanent shock; the transitory component is not important.

To see what may have caused these discrepancies in the two different approaches, we compare the data generating processes of output implied by the two methods. Since we estimate the bivariate system of BQ decomposition as a VARMA(1,1) process, the growth rate of GDP can be recovered as an ARMA(2,2) process. Table 3 below (in comparison with Table 1) lists the implied parameters under the VARMA (asterisk indicates the value is significantly different from the univariate ARMA(2,2) used in the UC model). Clearly, these different values implied by the VARMA (1,1) and the univariate ARMA (2,2) induce a much smaller long run effect. This explains why the permanent shock in the BQ decomposition has smaller long run effect than what we obtain in the UC model.[9]

Table 3—ARIMA(2,1,2) implied by VARMA(1,1)

AR Part		MA Part			Long Run Effect	Log
ϕ_1	ϕ_2	θ_1	θ_2	σ	of Innovation	Likelihood
1.4863	-0.5564^*	-1.1969	0.2461^*	0.9149	0.7193	-317.8866

4 Conclusions

This note has re-examined the UC method of decomposition of GDP by relaxing the random-walk assumption made in the existing literature. Based on this generalization, we are able to decompose GDP into two orthogonal components (permanent and transitory). This allows us to conduct impulse response analysis and variance decompositions. We find that the permanent component explains the bulk of GDP fluctuations, in sharp contrast to the conclusion reached by Blanchard and Quah (1989).

5 References

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[9]This point can be easily seen from a spectrum perspective: the spectrum of growth rate of GDP shares the same value with growth rate of permanent component at zero frequency, and this value is just the squared long run effect multiplying the variance of innovation in ARIMA process.

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