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# Testing performance of random access memory using linear models

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**Key words:** RAM memory, linear model, analysis of covariance, deviance.

## 1. Introduction

Various discussions relating to computers comment on a reasonable extent of random access memory (RAM) increase as it is a known fact that the extension of this type of memory influences speed of computer machines. Disputes often arise as to whether half a gigabyte extension of RAM is large enough for the computers to be significantly sped up, given the complexity of present software applications. In this article, we test statistically whether such an increase speeds up computers significantly or not, using analysis of covariance as a suitable statistical tool.

We considered two sizes of the memory for the experiment, differing by half a gigabyte, and we measured how long it took an experimental computer to launch a selected software application based on the RAM size and the number of other applications already run at the time of the launch. The RAM sizes were 720 MB and 1232 MB. Significance of differences in the measured times was then tested, using analysis of covariance which took into account the number of already launched applications as the covariate.

We used an Intel-based two-chip computer running on 2,2 GHz as the experimental computer. The machine had the virtue that it could exploit different operating systems at the same time. This was important as we were able to assign different sizes of RAM memory to one operating system, while observing reaction time of the other operating system deprived of a portion of RAM memory. In other words, we monitored the reaction time of one operating system whose RAM memory capacity was being reduced by assigning it to another operating system. We carried out the experiment on a single computer. There were two reasons for that. First, no other computer capable of using two operating systems simultaneously was available at the time of the experiment. Secondly and more importantly, working with the same computer allowed us to keep the experiment as homogeneous as possible and rule out other potential factors which could arise from heterogeneous hardware and increase the variance of measured times, whereby distorting the results of the analysis.

The software application selected for the launch was a demanding programme for musicians, the already launched applications included a text editor and/or a spreadsheet programme and/or an audio – playing software.

## 2. Analysis of covariance as a linear model

It is a well-established practice to introduce a suitable mathematical model as a tool which simplifies reasonably the reality. The model we used in our computer experiment is the analysis of covariance (ANCOVA), i.e. a linear model. The analysis is a modification of the standard analysis of variance (ANOVA) in that it also tests the significance of a factor of interest, but after adjustments for the influence of another variable (also known as covariate), which „additionally“ enriches ANOVA linear model.

In our study, the factor of interest is the  $i$ th level of RAM,  $i = 1,2$ , and its potentially significant effect on the speed of computers. The influential variable, or the covariate, is the number of already launched computer programmes – we considered either zero, one, two or three open applications in the experiment, assigning these four scenarios index  $j = 1,2,3,4$ . We made ten measurements of time for each combination of  $i$  and  $j$ , thus obtaining ten replicates for each combination of  $i$  and  $j$ , or  $n = 80$  measurements altogether. We use index  $k = 1,2,\dots,10$  for the replicates. Further, we use the variable  $x_{i,j,k} = j - 1$  as the number of open programmes for the  $i$ th level of RAM, the  $j$ th scenario and the  $k$ th repetition of the experiment. To give an example,  $x_{1,1,7}$  means that when the selected application was being launched for the 7<sup>th</sup> time and the RAM memory was at its first level, there was no other software application being already processed by the computer as  $j-1$  is zero in this case. In our study,  $i = 1,2$ ,  $j = 1,2,3,4$  and  $k = 1,2,\dots,10$ .

Now, if we denote the measured launch time as  $y_{i,j,k}$ , the analysis of covariance can be described as a test which compares quality of two different classical linear models

$$E(y_{i,j,k}) = \mu_i + \alpha x_{i,j,k} \quad \text{vs.} \quad E(y_{i,j,k}) = \mu + \alpha x_{i,j,k} \quad (1)$$

In the expressions (1), it is assumed that  $y_{i,j,k}$ 's are independent and normally distributed variables with constant variance  $\sigma^2$ , or

$$y_{i,j,k} \sim N(Ey_{i,j,k}, \sigma^2) \quad (2)$$

The symbol  $\mu_i$  represents the effect of the  $i$ th level of RAM. The models in (1) are classical regression models of the form  $\vec{y} = X\vec{\beta} + \vec{\varepsilon}$  as the right-hand model of (1) can be re-written in the form  $E(y) = X_0\beta_0$ , where

$$E(y) = (E(y_{1,1,1}), E(y_{1,1,2}), \dots, E(y_{1,2,1}), E(y_{1,2,2}), \dots, E(y_{2,4,10}))^T,$$

$$X_0 = \begin{pmatrix} \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} \\ \bar{0} & \bar{1} & \bar{2} & \bar{3} & \bar{0} & \bar{1} & \bar{2} & \bar{3} \end{pmatrix}^T \quad (3)$$

with  $\bar{0}, \bar{1}, \bar{2}, \bar{3}$  being  $1 \times 10$  vectors of zeroes, ones, twos and threes, respectively, and

$$\beta_0 = (\mu, \alpha)^T. \quad (4)$$

The left-hand model of (1) can be re-written in the form  $E(y) = X_1 \beta_1$ , where

$$E(y) = (E(y_{1,1,1}), E(y_{1,1,2}), \dots, E(y_{1,2,1}), E(y_{1,2,2}), \dots, E(y_{2,4,10}))^T$$

$$X_1 = \begin{pmatrix} \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{1} & \bar{1} & \bar{1} & \bar{1} \\ \bar{0} & \bar{1} & \bar{2} & \bar{3} & \bar{0} & \bar{1} & \bar{2} & \bar{3} \end{pmatrix}^T \quad (5)$$

$$\beta_1 = (\mu_1, \mu_2, \alpha)^T \quad (6)$$

A standard presentation of the ANCOVA framework can be found, for instance, in [1] (page 518), in [3] (p. 179) or in [5] (p. 374). Using an appropriate test criterion, if it turns out that the left-hand model in (1) fits the data significantly better than the right-hand model, in a certain sense, it can be concluded that the factor of interest – the  $i$ th RAM level - plays a significant role. As was already mentioned, the available literature presents ANCOVA in a standard way, but the analysis can also be equivalently carried out using what is called *deviance* because the appropriate test criterion assessing the model qualities can be based on the difference in deviances of the two models.

If we have a random sample of independent and normally distributed launch times  $\bar{y} = (y_{1,1,1}, y_{1,1,2}, \dots, y_{2,4,10})$  as in (2), we may construct the joint density function  $f(\bar{y}, \bar{\eta})$  for the random vector  $\bar{y}$  with  $\bar{\eta}$  being the vector of expected values, which describes the normal distribution. If we estimate  $\bar{\eta}$  with a linear model, than the better the model fits the data, the higher the value of  $f(\bar{y}, \bar{\eta})$ , given the functional form of the normal joint density function. Thus, the maximum of  $f(\bar{y}, \bar{\eta})$ , or also the maximum of  $\log f(\bar{y}, \bar{\eta})$  denoted as  $l(\bar{y}, \bar{\eta})$ , represents the „perfect fit“ of the data. Based on this idea, the deviance is defined as  $D = 2[l(\bar{y}, \bar{b}_{\max}) - l(\bar{y}, \bar{b})]$ , where  $\bar{b}$  is the ordinary least squares estimate of the coefficients in the used linear model  $\bar{y} = X\bar{\beta} + \bar{\varepsilon}$  and  $\bar{b}_{\max}$  is the estimate which maximizes  $l(\bar{y}, \bar{\eta})$ . The smaller the deviance, the better the model  $\bar{y} = X\bar{\beta} + \bar{\varepsilon}$ . For a more profound discussion on deviance, see for instance [4].

It can be shown that if a more complex model on the left-hand side of (1), having generally  $s$  parameters, fits the data sample of size  $n$  well, its deviance has approximately the chi-squared distribution with  $n - s$  degrees of freedom, i.e.  $D \sim \chi_{n-s}^2$  approximately. It is usually assumed that the more complicated model fits the data well enough, so that this approximation holds true. Let us denote this deviance  $D_1$ . If the same is true about the simpler model with  $r$  parameters,  $r < s$ , then its deviance  $D_2 \sim \chi_{n-r}^2$  approximately.

In such cases, it follows from the general statistical theory that  $D_2 - D_1 \sim \chi_{s-r}^2$  approximately. Based on these findings, the test criterion that is used to compare the two models in (1) is

$$T = \frac{(D_2 - D_1)/(s - r)}{D_1/(n - s)} \quad (7)$$

If the simpler model isn't significantly worse than the more complex model, which is the null hypothesis, the test criterion (7) has approximately the  $F$  distribution with  $s - r$  and  $n - s$  degrees of freedom. High values of  $T$  suggest that the more complex model is significantly better, i.e. that the factor of interest has a significant effect, leading to rejection of the null hypothesis. For linear models (see again [4]), the deviance  $D$  is

$$\begin{aligned} &= 2[-(1/2)n \log(2\pi\sigma^2)] - 2[-(1/2\sigma^2) \sum_k (y_k - x_k^T b)^2 - (1/2)n \log(2\pi\sigma^2)] \\ &= (1/\sigma^2)(y^T y - b^T X^T y). \end{aligned} \quad (8)$$

### 3. RAM case study

The tables 1 and 2 contain times of launching the selected application based on how many other programmes were already keeping the computer busy, and on RAM size. The times are measured in seconds.

a) RAM size = 1232 MB

No. of open applications

0	2,3	2,4	2,4	2,3	2,3	2,4	2,1	2,3	2,1	2,2
1	2,6	2,6	2,7	2,8	2,6	2,7	2,6	2,7	2,6	2,4
2	3,0	2,8	3,0	3,1	3,0	3,0	2,9	3,1	3,2	3,1
3	3,3	3,2	3,3	3,2	3,1	3,2	3,3	3,5	3,4	3,5

Tab 1 Times for launching the application

b) RAM size = 720 MB

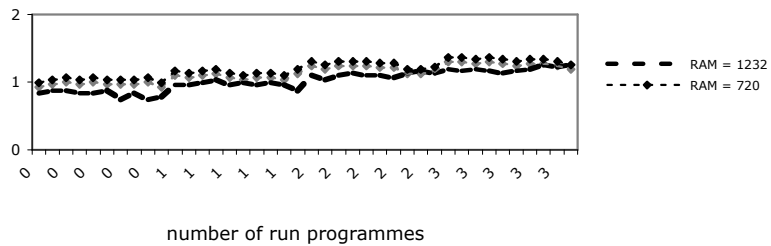
No. of open applications

0	2,7	2,8	2,9	2,8	2,9	2,8	2,8	2,8	2,9	2,7
1	3,2	3,1	3,2	3,3	3,1	3,0	3,1	3,1	3,0	3,3
2	3,7	3,5	3,7	3,7	3,7	3,6	3,6	3,3	3,3	3,4
3	3,9	3,9	3,8	3,9	3,8	3,7	3,8	3,8	3,7	3,5

Tab 2 Times for launching the application

In order for the analysis to be valid, it is necessary to check whether  $y_{i,j,k}$  are normally distributed and have constant variance for a given value of  $x_{i,j,k}$ , that is for a given  $j$ . Standard tests show that lognormal distribution is more appropriate for the data, therefore we perform the analysis on the logarithmic scale, or by using  $\log(y_{i,j,k})$ , to bring the data closer to normal distribution.

The following graph shows series of  $y_{i,j,k}$  on the logarithmic scale for the two sizes of RAM. Both series seem to follow a linear trend and have similar slopes. This suggests that the approach in (1) is appropriate for the analysis.



Sample variances of  $y$  on the logarithmic scale are

$j$	0	1	2	3
$Var$	0,013	0,009	0,009	0,006

Tab 3 Sample variances of  $y$

The variances do not seem to differ much, so we assume that they are constant to keep the analysis simple. Slight differences in variances do not seem to play an important role in these statistical settings anyway.

Since the conditions for the analysis seem to be valid, we now test the significance of the RAM size increase from 720 MB to 1232 MB.

For the model  $E(y_{i,j,k}) = \mu + \alpha x_{i,j,k}$ , ordinary least squares give the vector of coefficient estimates  $b_2 = (0,938, 0,113)^T$ . For the model  $E(y_{i,j,k}) = \mu_i + \alpha x_{i,j,k}$ , the least squares result in the vector  $b_1 = (1,023, 0,852, 0,113)^T$ . Now, using the test criterion (7), in which we substitute the expression (8) for the  $D$ 's, we get for  $s = 3$  and  $r = 2$

$$T = \frac{(D_2 - D_1)/(s-r)}{D_1/(n-s)} = \frac{(b_1^T X_1^T y - b_2^T X_2^T y)/1}{(y^T y - b_1^T X_1^T y)/77} = \frac{0,5861}{0,121} = 4,8438.$$

Assuming a five per cent nivel of test  $\alpha$ , the critical value for the test is  $F_{1,77}(\alpha = 0,05) = 3,965$ . We see that the test criterion exceeds the critical value, which means that we reject the null hypothesis. In other words, the more complex model seems to be significantly better than the simpler model, suggesting that the larger RAM memory does make a difference in term of the computer speed.

#### 4. Some final remarks

Comparing the test criterion and the critical value, we conclude that half a gigabyte increase in random access memory does seem to have a significant effect on computer performance, statistically speaking. However, certain caution is appropriate. First, we carried out the test on a single computer. Other computers with similar hardware can react differently, although the difference should not be striking given the current similarity in their architecture. Second, the rejection of the hypothesis is not particularly convincing and nivel of test at one per cent would not lead to the rejection. Therefore, even though the considered RAM increase undoubtedly has a positive impact on the computer (as any other increase in RAM, after all), it still might be reasonable to improve the memory size to even a greater extent if one wants to be sure that a significant improvement in the speed of the computer has been achieved.

#### Resumé

The article aims to show results of a statistical test by means of which it was observed whether half a gigabyte increase of RAM memory from 720 MB to 1232 MB influenced significantly computer speed at the time of a software application launch. Analysis of covariance was used as the statistical tool, in a modification based on the results of the generalized linear models theory. The selected extent of the RAM increase is not coincidental, it is related to numerous discussions on the practical usability of such a memory extension. Given the limited resources for the experiment, concrete software applications were chosen for an experimental computer to work with, none the less similar results can be expected with other applications as well due to the deterministic character of computers. The same argument can be used for half a gigabyte RAM changeover between two levels different than 720 MB and 1232 MB. The analysis of

covariance showed that the above-mentioned RAM extension does have a significant impact on the computer speed, although for a clearer approval of such a computer improvement, it would have been better if the test had given a bit more convincing numerical result.

## Resumé

Cílem příspěvku je předložit výsledky statistického testu, kterým se zjišťovalo, zda zvýšení paměti RAM o půl gigabajtů z úrovně 720 MB na úroveň 1232 MB ovlivňuje významně rychlost počítačů při spouštění konkrétních softwarových aplikací. Jako statistický nástroj byla použita analýza kovariance, a to ve verzi vycházející z teorie zobecněných lineárních modelů. Zvolený stupeň navýšení paměti RAM není náhodný, ale vychází z četných diskuzí ohledně praktické využitelnosti takového navýšení. Vzhledem k omezeným možnostem experimentu byly vybrány konkrétní aplikace, s nimiž experimentální počítač pracoval, nicméně podobné výsledky lze očekávat i u jiných programů vzhledem k deterministickému charakteru počítačů. Stejný argument lze uplatnit, pokud jde o půlgigabajtový přechod paměti RAM mezi jinými úrovněmi než 720 MB a 1232 MB. Analýza kovariance ukázala významný dopad uvedeného rozšíření paměti RAM, i když pro jednoznačnější odsouhlasení takového vylepšení počítače by bývalo vhodnější, kdyby z testu vzešly o něco přesvědčivější numerické závěry.

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