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canegrati, emanuele

December 2006

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Emanuele Canegrati

December 18, 2006

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Abstract

In this paper I analyse a labor market where the wage is endogenously determined according to a Right-to-Manage bargaining process between a firm and a labor union whose members are partitioned into two social groups: the old and the young. Furthermore, I exploit the Single Mindedness theory, which considers the existence of a density function which endogenously depends on leisure. I demonstrate that, when preferences of the old for leisure are higher than those of the young and when the level of productivity of the young is higher than that of the old, the young suffer from higher tax rates and gain higher level of wage rates and lower levels of leisure. Finally, since the old are more single minded than the young, they exploit their greater political power to get positive transfers from the young in a PAYG system.

JEL Classification: D71, J22, J26, J51

Keywords: bargaining models, labor unions, political economy, single mindedness

[1]DEFAP - Universita' Cattolica del Sacro Cuore - Milano; e-mail address: emanuele.canegrati@unicatt.it

1 Introduction

In recent years the interest of economists about trade unions behavior models has been gradually increasing. The earlier studies were principally oriented to the macroeconomic perspective and solely intended to explain the relationship between higher wages generated by the presence of labor unions and unemployment levels. From this point of view three models have been competitors in attempting to explain this linkage: the monopoly model, the Right to Manage (RTM) model and the efficient bargain model. The oldest monopoly model was developed by Dunlop (1944); there, the union was seen as a monopolistic seller of labor which maximized its utility function by choosing the optimal level of wage, given the firm's demand for labor. As a result, the monopoly union model entails more unemployment than would be the case under perfect labor markets and a higher level of wage rates, with respect to the competitive wage. The RTM model was first proposed by Leontief (1946) but it was only in the early 1980s with Nickell's works that it acquired its popularity. The model assumes that there exists a bargaining process between a firm and a labor union over the real wage, subject to the labor demand unilaterally chosen by the firm in maximising its profit function. In this case, the wage derived by the bargaining process is lower and the employment level is higher than that generated by the monopoly union model and the RTM solution lies on the labor demand curve. The main achievement of the RTM model is that the equilibrium wage and the level of employment depend upon the bargaining power of the involved bargainers. Finally, in the efficient bargaining model developed by McDonald and Solow (1981) the union and the firm negotiate upon both the wage and the level of employment. In this case, the quite surprising result is such that the efficient contracts lie not on the labor demand curve, since both the wage and the level of employment stand above the competitive solution. Nevertheless, these three macroeconomic models took the bargaining power as exogenous and did not take into account some relevant factors. First of all, they do not consider the distinction between "insiders ", whose preferences count and "outsiders ", whose preferences do not. Due to this distinction, the union indifference curves end up to be kinked at the point where the level of employment is equal to the membership. Secondly, the classical macroeconomic literature took the union size as exogenously given, whilst authors as Grossman started to investigate this issue considering the role of the seniority within the union and the voting mechanism which maximises the expected utility of the median worker. From the early 1990s many economists such as Nickell & Wadhwani (1990) and McDonald & Suen (1992) started to analyse, from a microeconomic perspective, the role and the determination of trade union power. This concept of trade unions, according to McDonald and Suen ([29]) is "the ability of the trade union to divide up to its advantage the rents arising from the production process given other parameters, in particular the elasticity of revenue with respect to employment ". Furthermore, an increasing number of economists began to study the impact of changes in tax rates over the wage outcome from trade union/firm bargaining. In the McDonald & Suen model, the authors attempted to measure the trade union power both in an "open union"(or utilitarian union) model and in an insider-outsider model. Unfortunately, as themselves recognized, this is not an easy task. An important contribution to the doctrine was represented by the studies on Corporatism, even though this argument is too broad to be explored in this work. In the last decade researchers have been moving toward the analysis of new fields of research, most of all in the political economy perspective. One of them is represented by the organization of workers in social groups and the political insider mechanism; according to this literature (see Gilles Saint-Paul ([14]) amongst the others) "workers may be unable to coordinate in order to form a labor union, but by voting in favor of an institution that raises they are able to collectively achieve a higher wage level exactly as if they were organized in a union. Labor market rigidities allow insiders to monopolize the market at the economy wide level even though their bargaining power may be quite reduced at a firm level ". Secondly, even the impact of unions on the voting behavior of their members represent an interesting field which may provide interesting results.

In this paper, I analyse the role of labor unions from a microeconomic perspective, exploiting the Single-Mindedness Theory. Wages are endogenously determined according to a typical RTM model between a firm and a labor union which is seen as a social institution of workers partitioned into two social groups (Young and Old). An important assumption is that the preferences of the old for leisure are greater than the preferences of the young and the level of productivity of the young is greater than the level of productivity of the old. Under these conditions, I will demonstrate that the tax rate of the young is higher than that of the old, whilst the wage rate of the old is lower than that of the young and the amount of leisure for the old is greater than the amount of leisure of the young. Since the single-mindedness of a group, which represents a proxy for the political power of that group, is captured by the density which is a monotonically increasing function with respect to leisure ([5]) I conclude that the old are more able to influence politicians and this power of influence enables them to get positive transfers in a PAYG system. Since the Government must clear the budget and cannot issue any debt, the burden of transfers is entirely carried by the young. Thus, with respect to previous work of mine, this study considers the mechanisms of labor unions, seen as an institution which represents the interests of different social groups. Again, in accordance with the SMT, the greater the ability of a single group to be focused on the minimum number of issue, the higher the probability that this group achieves its goals. The paper is organized as follows: section one introduces, section two explains the model, section three provides some numerical simulation and section four concludes.

2 The model

I consider an OLG model, where individual agents live only for two periods: the first period t represents the *present* and the second $t+1$ represents the *future*. At time t there are two generations coexisting together: the *young* and the *old*. I assume that the generation of the old was born old and had not a youth. Furthermore, the generation of the young does not have any progeny. As a consequence, the world ends at time $t+1$. Generations are unlinked, meaning that there is no possibility to leave any bequest. Individuals consume all the available income earned at a given period of time; thus, it is not possible neither to save nor to borrow money.

Then, let a population of size equal to one be partitioned into two discrete groups of workers, the *young* and the *old*, each of them endowed with a given amount of labor time (measured in hours). Thus, the space of groups is $G = \{Y, O\}$, where Y denotes the group of young workers and O the group of old workers. I will use index I to denote a social group, capital letters to indicate the group and small letters to indicate single individuals belonging to the I-th group. The size of a group does not change over time.

Each worker has to decide how to divide his time between work and leisure, denoted by l. I assume also that leisure can be employed to attend several activities, such as relaxing, taking care of family, participating in political activities and many others. Thus, leisure can be seen as a vector of N activities $l = l(l_1, l_2, ..., l_N)$, where $l_n \geq 0$.

Labor market is imperfect and this imperfection is due to the presence of a labor union which bargains with the firm the wage rate, according to a typical Right-To-Manage (RTM) model. I assume that there exists only one union and that all the workers (old and young) belong to this union, which aims to maximize its members' net-of-tax income and the level of employment. The presence of this form of market imperfection generates unemployment in equilibrium (for an introductory analysis on the effects of trade unions on the labor markets, see Oswald ([32])).

Furthermore, I introduce one of the core assumptions of the model. I assume that the old and the young are identical in every respect except three: first of all *the intrinsic value of the old workers for leisure is assumed to be greater than the same value of the young workers; secondly, the level of productivity of the young is higher than the level of productivity of the old.* That is, $\psi^o >> \psi^y$, where Greek letter *psi* denotes the intrinsic value for leisure and $z^y >> z^o$, where z indicates the productivity level of a group. Thus, the two social groups have different preferences with respect to the choice between work and leisure. This assumption is also supported by the empirical evidence[2]. Furthermore, I assume that the leisure of a group generates a positive externality to the other group; for instance, retired grandparents often provide their sons with help by babysitting their children, and carry on some useful activities in their sons' place, such as house cleaning, payment of bills and so on. Otherwise, the young helps in their spare time their parents, expecially very old parents in many activities, such as bringing them to doctors, doing shopping and so forth. Furthermore, there exist some social beliefs which consider leisure of the old as a merit good. In modern societies, individuals believe that the old deserve to retire after having spent an entire life working. In this paper I assume that $\varphi^y >> \varphi^o$, where φ denotes the magnitude of the positive externality in the worker's utility function. That is, leisure of the old provide more utility to the young than leisure of the young does to the old. Finally, I assume that each worker has a personal ideological bias for one of the two candidates, and this ideological difference generates heterogeneity among groups. The ideological bias is exogenously given.

Old workers' preferences can be represented by a quasi-linear utility function[3]. A representative young worker at time t has the following lifetime utility function:

$$U^o = c_t^o + \psi^o \log l_t^o + \varphi^o \log l_t^y \tag{1}$$

$\forall\ o \in O$

where c_t^o is the consumption at time t, l_t^o is the leisure at time t, ψ^o is a parameter representing the intrinsic preference of the old worker for leisure ($\psi^o \in [0, 1]$) and φ^o, the magnitude which leisure of the young has on the utility of the old. The old worker consumes all his income:

$$c_t^o = w^o(1 - \tau_{Lt}^o)(t_t^o - l_t^o) + b_t^o + r(S_t^o) \tag{2}$$

where w^o is the unitary wage per hour worked, τ_{Lt}^o is the tax rate on labor income, t_t^o is the total amount of time the worker has to divide between leisure and work, b_t^o is an intergenerational transfer and $r(S_t^o)$ represents the return which the old worker gains at the end of time t over an amount of money he accumulated. I assume that the intergenerational transfer is represented by a typical pay-as-you-go pension program, whilst $r(S_t^o)$ represents the quote of a mutual fund. The last day of work, the old worker withdraws the amount of money invested. Without loss of generality, I assume that the same day, he consumes all this amount of money and dies.

Similarly, the preferences of a representative young worker y are given by the following lifetime utility function:

[2] for a survey on the factors which explain the difference in preferences toward leisure among social groups, see Canegrati ([5])

[3] A quasi-linear utility function entails the non existence of the income effect

$$U^y = \ c_t^y + \psi^y \log l_t^y + \varphi^y \log l_t^o + \beta^y (c_{t+1}^y + \psi^y \log l_{t+1}^y) \tag{3}$$

$\forall y \in Y$

where c_t^y and c_{t+1}^y represent the consumption at time t and $t+1$, l_t^y and l_{t+1}^y leisure at time t and $t+1$, β^y is the time preference discount factor, ψ^y is the intrinsic preference of the young worker for leisure ($\psi^y \in [0,1]$) and φ^y represents the intrinsic preference of the young for leisure of the old ($\varphi^y \in [0,1]$).

Finally, the intrinsic value of leisure for the old worker is assumed to be much higher than the intrinsic value for the young: $\psi^o >> \psi^y$. Without loss of generality I assume that $\psi^o > \frac{1}{2}$ and $\psi^y < \frac{1}{2}$. Since the young know that at time $t+1$ will be old, their utility function includes the leisure of the next period, weighted by a discount factor $\beta^y \in [0,1]$.

The young worker's intertemporal budget constraint is given by:

$$c_t^y + \beta^y c_{t+1}^y = w_t^y (1 - \tau_{Lt}^y)(t_t^y - l_t^y) + b_t^y$$

$$+ r(S_t^y) + \beta^y (w_{t+1}^y (1 - l_{t+1}^y)(t_{t+1}^y - \tau_{Lt+1}^y) + r(S_{t+1}^y)) \tag{4}$$

Notice that the young worker's budget constraint does not contain the term which refers to the intergenerational transfer at time $t+1$, b_{t+1}^y, since at period $t+1$ there exists only generation Y and, by definition, it cannot exist any intergenerational transfer. Furthermore, I assume that $t_t^o = t_t^y = t_{t+1}^y = \bar{t}$ Furthermore, I introduce the following budget constraints:

$$r(S_t^o) = T_t^o \tag{5}$$

$$r(S_t^y) = T_t^y \tag{6}$$

$$r(S_{t+1}^y) = T_{t+1}^y \tag{7}$$

$$n^o b_t^o + n^y b_t^y + \alpha \left| n^o b_t^o \right| \left| n^y b_t^y \right| = 0 \tag{8}$$

$$(b_t^o)(b_t^y) < 0$$

Since revenues are proportional to the amount of labor supplied, the taxation entails inefficiencies, since it distorts workers' decisions on the amount of labor supplied and determines the quota of pre-funded savings.

T_t^o represents total revenues generated by the taxation of the old at time t and it is equal to $n^o \tau_{Lt}^o w^o (\bar{t} - l_t^o)$ while T_t^y the total revenues generated by the taxation of the young at time t and it is equal to $n^y \tau_{Lt}^y w^y (\bar{t} - l_t^y)$; T_{t+1}^y represents the total revenues generated by the taxation of the young at time $t+1$ and it is equal to $n^y \tau_{Lt+1}^y w^y (\bar{t} - l_{t+1}^y)$. The condition $n^o b_t^o + n^y b_t^y + \alpha \left| n^o b_t^o \right| \left| n^y b_t^y \right| = 0$ assures that an intergenerational transfer exists while the condition $(b_t^o)(b_t^y) < 0$ shows that the situation where both generations either get a positive transfer or suffer of a negative transfer is impossible to achieve.

In other words, if one generation obtains a positive transfer, the other one has to finance for it. The term $\alpha \left|n^o b_t^o\right| \left|n^y b_t^y\right|$ represents an efficiency loss which takes place via a redistribution process and can be measured by the amount of resources wasted during this process. For instance, one may think that this loss is due to the existence of bureaucracy costs or to rent grabbed by politicians. The parameter $\alpha \in [0, 1]$ represents the measure of the loss which is quadratic in the transfers.

2.1 The Government

The literature has used different formulation for the Government's objective function. A typical normative approach considers a benevolent Government which aims to maximize a Social Utility Function by choosing the optimal tax rate on labor, subject to a budget constraint where tax revenues are equal to public good expenditures. Otherwise, some authors such as Edwards and Keen considers a Leviathan model where, referring to the famous milestone paper by Brennan and Buchanan [4], they examine a Government which is concerned in part with maximizing the size of the public sector. Furthermore, the Edwards and Keen model assumes that the Government retains some degree of benevolence, perhaps because it has re-election concerns. Nevertheless, this concerns were not formally modelled.

In this paper, I provide a possible explanation to this issue, introducing a political economy model where politicians act in order to maximize the probability of being re-elected.

A public policy vector is given by:

$$\vec{q} = (\tau_{Lt}^o, \tau_{Lt}^y, b_t^o, b_t^y)$$

composed of two tax rates and two intergenerational transfers.

Finally, the Government is committed to clear the budget constraint; this means that it cannot transfer more resources than those collected by taxing individuals at every period of time. Thus, I assume that the Budget Surplus (Deficit) must be equal to zero. Since the Government cannot issue bonds to collect more financial resources and can only rely on taxation, the increase in a social group's welfare entails the decrease in the welfare of the other social group, since the latter has to pay for the transfer to the former.

2.1.1 The union

Denoting by n the total members of the union which coincide with the population (in the sense that I assume that every citizen belongs to the union[4]), denoting by n^o the number of old workers, by n^y the number of young workers, by $w^o(1-\tau^o_{Lt})$ the net-of-tax wage rate of the old, and by $w^y(1-\tau^y_{Lt})$ the net-of-tax wage rate of the young, the expected utility of a representative member is:

$$U(w,l) = n^o U(w^o(1-\tau^o_{Lt})) + n^y U(w^y(1-\tau^y_{Lt})) \tag{9}$$

and I assume that the workers are risk neutral, so that
$U(w^i(1-\tau^i_{Lt})) = w^i(1-\tau^i_{Lt})$
Thus, the objective function of the union can be written as follows:

$$U(w,l) = L^o w^o(1-\tau^o_{Lt}) + L^y w^y(1-\tau^y_{Lt}) \tag{10}$$

where L^o is the total level of old workers' employment and L^y is the total level of young workers' employment.

2.1.2 The firm problem

I assume that the firm maximizes the following objective function:

$$\Pi(w, L^o, L^y) = R(L^o) + R(L^y) - \sum_{I=y,o} w^I(1+\tau^I_{Lt})L^I \tag{11}$$

where $R(L^o)$ and $R(L^y)$ represent the total revenues generated by the old and the young, respectively. Furthermore, I assume that the firm is a price-taker, selling the product at a price p, which I normalize to unity. The production function is represented by $y(L^o, L^y) = (z^o L^o - (L^o)^2) + (z^y L^y - (L^y)^2)$, where z represents a productivity parameter and I assume that $z^y > z^o$. Notice that $R(0) = 0$, that $\frac{\partial y}{\partial L} > 0, \forall L < \frac{z}{2}$ and that $\frac{\partial^2 y}{\partial^2 L} = -2 < 0$ (y is a concave function).

Thus, the firm's objective function is given by:

$$\Pi(w^o, w^y, L^o, L^y) = (z^o L^o - (L^o)^2) + (z^y L^y - (L^y)^2) - w^o L^o(1+\tau^o_{Lt}) - w^y L^y(1+\tau^y_{Lt}) \tag{12}$$

2.1.3 A five-stage game

I consider a non cooperative five-stage game among two candidates, a trade union representing the two social groups and a corporation. In the first stage the two candidates announce their fiscal policy vector, by choosing the optimal level of labor tax rate τ_{Lt} and the optimal transfers. In the second, elections take place. In the third, individuals choose the optimal

[4]This assumption is fundamental to assure that the union utility function is correctly specified; otherwise, if we assume that not every worker belong to the union, it could be possible that the employment level n exceed membership m. In this case, the correct specification for the utility function of the union would be $U(w,l) = n^o U(w^o(1-\tau^o_{Lt})) + n^y U(w^y(1-\tau^y_{Lt}))) \max[0, n^o - m^o] - U(w^y(1-\tau^y_{Lt})) \max[0, n^y - m^y]$

level of leisure, taken the policy vector chosen by the Government as given. Thus, multiplying the labour chosen by the representative worker by the numerosity of the group, we obtain the Labour Offer Function. The other two stages are typical of the classic Nickell's Right-to-Manage model. In the fourth stage the firm independently bargains over the wages with the labor union, given the pre-determined fiscal policy vector chosen by Government in the first stage; thus, the optimal wage rate is determined by a bargaining process between the trade union and the corporation and the resulting outcome derives by an asymmetric Nash bargaining. The Nash bargaining maximand is a weighted function Ω of the firm and union objective functions and can be written as follows:

$$\Omega_t = \lambda \log \left[U(w, L_t^o, L_t^y)\right] + (1-\lambda) \log \left[\Pi(w^o, w^y, L^o, L^y) - \bar{\imath}\right] \tag{13}$$

where the parameter λ denotes the relative bargaining power of the trade union ($0 \leq \lambda \leq 1$) and $\bar{\imath}$ the fall back position of the firm. Without loss of generality I impose that $\bar{\imath} = 0$.

Finally, in the fifth stage the firm sets its output, given the pre-determined wage choices from the fourth stage, as to maximize profits. The assumption that the LOF is determined before the LDF means that workers have a sort of advantage of first move, or in another words, they act as a Stackelber leader. This assumption is strong since usually one is prone to think that workers have not decision power over the level of work. What I argue here is that this could be true in the Short Run, where the macroeconomic, legislative and political environments are given and thus firms decide according to efficiency criterion. But in the Long Run these environments change: new labor laws are passed by the new governments and if we assume that a law is a synthesis of policies that politicians are committed to undertake and, as we assumed, politicians commit only to policies which are more likely to be voted, the result is that labor policies which affect the level of employment are chosen by society. Thus, the firm is assumed to have a complete control over the level of employment but only in the Short Run; it observes the economic, political and normative environments and then derives the LDF bargaining wages with the labor union.

To solve the model I use the backward induction.

I start to solve the model from the *fifth stage* by deriving the optimal labor demand of the firm, obtained by the maximization of the profit function with respect to the labor:

$$\Gamma \equiv \Pi(w^o, w^y, L^o, L^y) = (z^o L^o - (L^o)^2) + (z^y L^y - (L^y)^2) - w^o L^o (1 + \tau_{Lt}^o) - w^y L^y (1 + \tau_{Lt}^y) \tag{14}$$

Solving Γ, we obtain the optimal labor demand functions for the corporate:

$$L_t^{y*} = L_t^y(w^y, \tau_{Lt}^y, z^y) = \frac{z^y - w^y(1+\tau_{Lt}^y)}{2} \tag{15}$$

$$L_t^{o*} = L_t^o(w^o, \tau_{Lt}^o, z^o) = \frac{z^o - w^o(1+\tau_{Lt}^o)}{2} \tag{16}$$

Substituting (15) and (16) into (14) I provide an expression for the Indirect Profit Function:

$$\Pi^*(w^o, w^y, L_t^{o*}, L_t^{y*}) = \Pi(w^o, w^y, L_t^y(w^y, \tau_{Lt}^y, z^y), L_t^o(w^o, \tau_{Lt}^o, z^o)) =$$

$$\sum_{I=O,Y} \left((z^I \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) - \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right)^2 \right)$$

$$- \sum_{I=O,Y} w^I(1+\tau_{Lt}^I) \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) \tag{17}$$

Furthermore, substituting (15) and (16) into (10), we obtain an expression for the Indirect Utility Function of the union:

$$U(w,l) = \sum_{I=O,Y} \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) w^I(1-\tau_{Lt}^I) \tag{18}$$

In the *fourth stage* of the game the firm and the union bargain over the optimal wage rate, which is determined by the resolution of the following problem:

$$\max_{w^o, w^y} \Omega = \lambda \log \left[U_t(L_t^{y*}, L_t^{o*}, w^o, w^y) \right] +$$

$$(1-\lambda) \log \left[\Pi_t(w^o, w^y, L_t^{o*}, L_t^{y*}) \right]$$

$$s.t.\ R'(L_t^o) - w^o(1+\tau_{L_t^o}) = 0$$

$$R'(L^y) - w^y(1+\tau_{L^y}) = 0$$

The constraints impose that the bargain's outcome must stand on the firm's labor demand functions.

Then, substituting (17) and (18) into the maximand we obtain:

$$\max_{w^o, w^y} \Omega_t = \lambda \log \left[\sum_{I=O,Y} \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) w^I(1-\tau_{Lt}^I) \right] +$$

$$(1-\lambda) \log \left[\sum_{I=O,Y} \left(z^I \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) - \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right)^2 \right) - \sum_{I=O,Y} w^I(1+\tau_{Lt}^I) \left(\frac{z^I - w^I(1+\tau_{Lt}^I)}{2} \right) \right]$$

Solving this equation maximization problem is not an easy task. Anyway, one may easily note that Ω_t may be written as $\lambda \log(U(z^o, w^o, \tau^o) + U(z^y, w^y, \tau^y)) + (1-\lambda) \log(\Pi(z^o, w^o, \tau^o) + \Pi(z^y, w^y, \tau^y))$ and that

$$U\left(z^o, w^o, \tau^o\left|_{w^o=w^y, z^o=z^y, \tau^o=\tau^y}\right.\right) = U\left(z^y, w^y, \tau^y\right) \forall y$$

$$U\left(z^y, w^y, \tau^y\left|_{w^y=w^o, z^y=z^o, \tau^y=\tau^o}\right.\right) = U\left(z^o, w^o, \tau^o\right) \forall o$$

$$\Pi\left(z^o, w^o, \tau^o\left|_{w^o=w^y, z^o=z^y, \tau^o=\tau^y}\right.\right) = \Pi\left(z^y, w^y, \tau^y\right) \forall y$$

$$\Pi\left(z^y, w^y, \tau^y\left|_{w^y=w^o, z^y=z^o, \tau^y=\tau^o}\right.\right) = \Pi\left(z^o, w^o, \tau^o\right) \forall o$$

Substituting and solving the problem we obtain the optimal wage rate:

$$w^{o*} = \frac{z^o\lambda}{2(1+\tau^o)} \tag{19}$$

$$w^{y*} = \frac{z^y\lambda}{2(1+\tau^y)} \tag{20}$$

It can be seen that the optimal wage is increasing both in the level of productivity and in the bargaining power of the union. The second result underlines how a powerful union is able to obtain higher levels of wage in a easier manner.
At this point some considerations are useful. If the bargain is (Pareto) efficient the optimal solution (w^{I*}, L_t^{I*}) must stands over the so called contract curve (CC), which represents the locus of tangency points between a union's indifference curve and a firm's isoprofit curve defined by the following condition:
$(V(w) - \bar{u})/V'(w) = w\left(1 + \tau_{Lt}\right) - R'(L_t)$
(see *equation 3* in MacDonald & Solow, 1981).

Unfortunately, it can be demonstrated (see Manning, 1987) that the MacDonald & Solow efficient bargain solution can be only achieved under very strict conditions; in other words the efficient solution holds if and only if the measure of the union's influence over employment is exactly equal to the union's influence over wage. Whenever this condition is not verified, the bargaining outcome produces an inefficient result in which the equilibrium (w_t^*, L_t^*) stays leftwards with respect to the CC. In a RTM model the measure of the union's influence over employment is exactly equal to zero and the union's influence over the wage is less than one (or equal to one in the monopoly union special case). As a consequence, an RTM model leads to an inefficient equilibrium.

Finally, substituting (19) and (20) into (15) and (16) we get an expression for the optimal labor demand functions:

$$L^o(w^{o*}) = \frac{z^o\left(1 - \frac{\lambda}{2}\right)}{2} \tag{21}$$

$$L^y(w^{y*}) = \frac{z^y\left(1 - \frac{\lambda}{2}\right)}{2} \tag{22}$$

We can see that the optimal wage rate is higher than the wage rate derived in a labor market characterized by perfect competition and absence of unions (in other words when $\Pi_t^I = 0$ for every I and $\lambda = 0$) and the employment level is lower than the competitive level. Thus, the RTM solution stands on the labor demand curve, but in a point where the wage rate is higher and the employment level is lower than the competitive solution case. We can conclude that the wage bargaining determines a Pareto inefficient solution, which is not represented by a point where the isoprofit curve of the firm is tangent to the union indifference curve.

Proposition 1 *The optimal wage deriving from a RTM bargaining between the firm and the labor union, w^*, increases with respect to an increase in the union bargaining power λ and in the productivity factor z, whilst it decreases with respect to an increase in the marginal tax rate τ_{Lt}^I.*

Proof: (see Appendix B).

Proposition 2 *in a RTM model the existence of a labor union entails an equilibrium characterized by an higher wage rate and a lower level of employment with respect to model where the firm has the exclusive power to determine both of the variables.*

Proof: It can be seen that the co-domain of the wage function $w = f(\lambda)$ is between 0, when $\lambda = 0$ and $\frac{z}{2(1+\tau)}$, when $\lambda = 1$. Note also that the co-domain of the employment function $L = f(\lambda)$ is between $\frac{z}{2}$, when $\lambda = 0$ and $\frac{z}{4}$, when $\lambda = 1$.
In the *third stage*, a representative old worker solves the following optimization problem:

$$\max U^o = c_t^o + \psi^o \log l_t^o + \varphi^o \log l_t^y$$

$$s.t.\ c_t^o = \left(\frac{z^o\lambda}{2(1+\tau^o)}\right)(1-\tau_{Lt}^o)(\bar{t} - l_t^o) + b_t^o + r(S_t^o)$$

Solving with respect to l_t^o I obtain an expression for the optimal leisure:

$$l_t^{o*} = \frac{\psi^o}{(\frac{z^o\lambda}{2(1+\tau^o)})(1-\tau_{Lt}^o)} \tag{23}$$

I do the same for the representative young worker:

$$\max\ U^y = c_t^y + \psi^y \log l_t^y + \varphi^y \log l_t^o + \beta^y(c_{t+1}^y + \psi^y \log l_{t+1}^y)$$

$$s.t.\ c_t^y + \beta^y c_{t+1}^y = \left(\frac{z^y\lambda}{2(1+\tau^y)}\right)(1-\tau_{Lt}^y)(\bar{t} - l_t^y) + b_t^y + r(S_t^y)$$

$$+\beta^y\left(\left(\frac{z^y\lambda}{2(1+\tau^y)}\right)(1-\tau^y_{Lt})(\bar{t}-l^y_{t+1})(1-\tau^y_{Lt+1})+r(S^y_{t+1})\right)$$

$$l^{y*}_t=\frac{\psi^y}{(\frac{z^y\lambda}{2(1+\tau^y)})(1-\tau^y_{Lt})} \tag{24}$$

Substituting (23) and (24) into (1) and (3) I obtain an expression for the IUF of the old and the young:

$$V^o_t=\bar{t}\left(\frac{z^o\lambda}{2(1+\tau^o)}\right)(1-\tau^o_{Lt})-\psi^o+b^o_t+r(S^o_t)+\psi^o\log\psi^o$$

$$-\psi^o\log\left(\frac{z^o\lambda}{2(1+\tau^o)}\right)-\psi^o\log(1-\tau^o_{Lt})$$

$$+\varphi^o\log\psi^y-\varphi^o\log\left(\frac{z^y\lambda}{2(1+\tau^y)}\right)-\varphi^o\log(1-\tau^y_{Lt}) \tag{25}$$

$$V^y_t=\bar{t}\left(\frac{z^y\lambda}{2(1+\tau^y_{Lt})}\right)(1-\tau^y_{Lt})-\psi^y+b^y_t+r(S^y_t)+$$

$$+\beta^y\left(\bar{t}\left(\frac{z^y\lambda}{2(1+\tau^y_{Lt+1})}\right)(1-\tau^y_{Lt+1})-\psi^y+r(S^y_{t+1})\right)+$$

$$+\varphi^y\log\psi^o-\varphi^y\log\left(\frac{z^o\lambda}{2(1+\tau^o)}\right)-\varphi^y\log(1-\tau^o_{Lt})+\beta^y\psi^y\log\left(\frac{2\psi^y(1+\tau^y_{t+1})}{z^y\lambda(1-\tau^y_{t+1})}\right) \tag{26}$$

In the *second stage* of the game elections take place. It is easy to verify that the elections' outcome is a tie. The proof arises from the resolution of the first stage, where it will be demonstrated that in equilibrium, both parties choose an identical policy vector.

In the *first stage*, the two candidates choose their policy vectors. They face exactly the same optimization problem and maximize their share of votes or, equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.

$$\max\ \pi^A=\frac{1}{2}+\sum_{I=\{o,y\}}n^Is^I[V^i(\vec{q}^A)-V^i(\vec{q}^B)]$$

$$s.t.\ T_1\equiv r(S^o_t)=T^o_t$$

$$T_2\equiv r(S^y_t)=T^y_t$$

$$T_3\equiv r(S^y_{t+1})=T^y_{t+1}$$

$$T_4\equiv n^ob^o_t+n^yb^y_t+\alpha\,|n^ob^o_t|\,|n^yb^y_t|=0$$

$$T_5\equiv b^o_tb^y_t<0$$

In the Appendix C I provide a complete resolution to the problem.

Proposition 3 *In equilibrium both candidates' policy vectors converge to the same platform; that is* $\vec{q}^A = \vec{q}^B = \vec{q}^*$

Proof: $\vec{q}^*$ represents the policy which captures the highest number of swing voters. Suppose instead there exists other two policies $\vec{q}'$ and $\vec{q}''$; in moving from $\vec{q}^*$ to $\vec{q}'$ (or $\vec{q}''$) a candidate loses more swing voters than those it is able to gain. Thus, suppose a starting point where candidate A chooses $\vec{q}'$ and candidate B chooses $\vec{q}''$ such that by choosing $\vec{q}'$ and $\vec{q}''$ the elections outcome is a tie. If one candidate moved toward $\vec{q}^*$, it would be able to gain more swing voters than those it loses and thus, it would win the elections. So, choosing any policy but $\vec{q}^*$ cannot be an optimal answer. The only one policy which represents a Nash Equilibrium is $\vec{q}^*$ since it is the intersection between the optimal answers of the two candidates and no one candidate has an incentive to deviate. Since each candidate maximizes its share of votes, in equilibrium the two candidates receive both one half of votes; if one candidate should receive less than one half of votes it would always have the possibility to adopt the platform chosen by the other candidate and get the same number of votes.

Corollary 1 *The utility levels reached by workers are the same; that is:* $V^i(q^A) = V^i(q^B)$

Proposition 4 *The marginal tax rates are positive both for the young and for the old, but the marginal tax rate of the young is higher than the marginal tax rate of the old; that is* $\tau^y_{Lt}(\lambda, \psi^y, z^y) > \tau^o_{Lt}(\lambda, \psi^o, z^o)$.

Proof: simulations performed with *Mathematica*; results available upon request to the author.

The comparative statics shows that: $\frac{\partial \tau^I_{Lt}}{\partial \lambda} < 0$, $\frac{\partial \tau^I_{Lt}}{\partial \psi^I} < 0$, $\frac{\partial \tau^I_{Lt}}{\partial z^I} < 0$, $\frac{\partial \tau^I_{Lt}}{\partial n^I} < 0$, $\frac{\partial \tau^I_{Lt}}{\partial n^{-I}} > 0$, $\frac{\partial \tau^I_{Lt}}{\partial s^I} < 0$, $\frac{\partial \tau^I_{Lt}}{\partial s^{-I}} > 0$, $\frac{\partial \tau^I_{Lt}}{\partial \varphi^{-I}} > 0$. [5]

These results show how an increase in the numerosity and the density of a group reduce the tax rate of the other group, whilst an increase in the numerosity and density of the group entails an increase in the tax rate of the the other group.

Proposition 5 *The old offer a lower supply of labor than the young due to the difference between* l^o_t *and* l^y_t.

Proof: Evidence provided by simulation with *Mathematica*. Results are shown in Appendix C.

[5] These results were obtained via numerical simulations. Results are available upon request to the author.

Corollary 2 *The old workers are more single minded than the young ($s^o > s^y$).*

Proof: $\tau^o_{Lt} < \tau^y_{Lt} = 0$ and $\psi^o >> \psi^y$ but eventually $l^{o*}_t > l^{y*}_t$. Since the density is a positive function of l $s^o = s(l^o_t) > s^y = s(l^y_t)$.

Proposition 6 *There exists Social Security transfers from the young to the old. That is: $b^o_t > 0$ and $b^y_t < 0$.*

Proof: From the first order conditions with respect to b^o_t and b^y_t, it is: $\frac{s^o}{s^y} = \frac{1-\alpha n^y b^y_t}{1-\alpha n^o b^o_t}$. From Corollary 2, $s^o = s(l^o_t) > s^{ly} = s(l^y_t)$ it must be $1-\alpha n^y b^y_t > 1-\alpha n^o b^o_t$ for the workers. Since $\alpha n^o b^o_t > \alpha n^y b^y_t$, under conditions $b^o_t b^y_t < 0$, and α, n^o, n^y it must be that $b^o_t > 0$ and $b^y_t < 0$. The equilibrium levels of the transfers between the young and the old are the following:

$$b^y_t = \frac{1-\sqrt{\frac{s^o}{s^y}}}{\alpha n^y} \tag{27}$$

$$b^o_t = \frac{1-\sqrt{\frac{s^y}{s^o}}}{\alpha^l n^o} \tag{28}$$

$$b^y_{t+1} = 0 \tag{29}$$

Given the budget constraint: $n^o b^o_t = \frac{-n^y b^y_t}{1-\alpha n^y b^y_t}$ taking into account the equilibrium conditions $\frac{s^o}{s^y} = \frac{1-\alpha n^y b^y_t}{1-\alpha n^o b^o_t}$, it is $\frac{s^o}{s^y} = \frac{1-\alpha n^y b^y_t}{\alpha n^y \frac{b^y_t}{1-\alpha n^y b^y_t}+1} = (1-\alpha n^y b^y_t)^2$. Solving with respect to b^y_t and b^o_t we obtain the optimal values. Furthermore, since at time $t+1$ only the young generation exists, there does not exist any intergenerational transfer, by definition. Notice that when densities of both groups are the same, transfers are equal to zero; that is if $s^o = s^y$, then $b^o = b^y = 0$.

Proposition 7 *A transfer in the I-th group decreases with an increase in the amount of resources distorted by government and with an increase in the density of the other group, whilst it increases with an increase in the density of his own group.*

Proof: Calculating the total differentials, we obtain: $\frac{\partial b^I_t}{\partial \alpha} < 0$, $\frac{\partial b^I_t}{\partial s^I} > 0$, $\frac{\partial b^I_t}{\partial s^{-I}} < 0$.

Proposition 5 makes sense and spouses the SMT: the higher the homogeneity among a group, the higher the power of influence of that group on the Government and the higher the transfer that the group gets.

Proposition 8 *The optimal Lagrange multipliers assume the following values:*

$$\lambda^* = \sqrt{s^o s^y} \tag{30}$$

Proof: $\lambda = \frac{n^o s^o}{n^o - n^o n^y \alpha b_t^y} = \frac{s^o}{1 - n^y \alpha b_t^y} = \frac{s^y}{1 - n^o \alpha b_t^o}$

Substituting the optimal intergenerational transfers value we obtain:λ^*.

The Lagrange multiplier has a political meaning since it represents the increase in the probability of winning for a candidate, if it had an additional dollar available to spend on redistribution.

3 Numerical Simulations

Appendix C shows the result of numerical simulations performed with *Mathematica 5.2*. The parameters matrix is a 12x11, whilst the outocome matrix is 11x11. As a premise, I would like to underline that these simulations are not intended to provide numbers perfectly adherent to the real world but only to numerically demonstrate what it is stated in the propositions of the model. I start to analyse what I will call the *status quo* situation given by the parameters vector:

$$\{n^o, n^y, s^o, s^y, z^o, z^y, \lambda, \psi^o, \psi^y, \varphi^o, \varphi^y, t\} = \{0.5, 0.5, 0.006, 0.004, 0.2, 0.5, 0.45, 0.4, 0.25, 1, 0.8, 40\}$$

The results vector confirms the propositions of the model. The tax rate of the young is equal to 0.814 whilst the tax rate of the old is equal to 0.621, confirming what it is stated in Proposition 4. The wage rate of the young is nearly double than the wage of the old, whilst the leisure of the old is much more greater than the leisure of the young. The Demand of Labour is higher for the young but so is also the level of unemployment. The total unemployment is equal to 9.856.

1. *An increse in the bargaining power of the labor union.* All the other variables being equal, an increase in the bargaining power of the labor union provokes a general decrease in the taxation of both groups. Indeed, the tax rate of the old fells from 0.621396 when λ is equal to 0.45 to 0.609347 when λ is equal to 0.5 and 0.594811 when λ is equal to 0.55. The same holds for the tax rate of the young which fells from 0.814036 to 0.809401 and 0.804147 rispectivly. Otherwise, the wage rate increases for both groups but this generates an increase in the labor supply, since leisure shrinks from 21.6772 to 18.9865 ($\lambda = 0.5$) and 16.7486 ($\lambda = 0.55$) for the group of the young and from 38.0672 to 32.9571 ($\lambda = 0.5$) and 28.6252 ($\lambda = 0.55$) for the group of the old. As a consequence, the unemployment increases in both groups (and so does the overall level of unemployment (13.7657 $\lambda = 0.5$ with 17.0593 $\lambda = 0.55$). This simple exercise confirms the assumption that if the union does not have any control on the level of employment but it aims only to get an higher wage, the labor market suffers with a reduction in the level of employment.

2. *An increase in the level of productivity.* A change in the level of productivity of one group generates a decrease in the tax rate of that group (whilst the tax rate of the other group remains equal) and an increase in the wage rate. The supply of labor increases for the old, but since it is not matched by and adeguate increase in the labor demand, this generates an increase in the unemployment level for the old group. In the numerical simulation I analyse both an increase in the level of productivity of the old (from 0.2 to 0.25) and of the young (from 0.5 to 0.7).

3. *An increase in the preferences of a group for his own leisure* In this case, the tax rate of that group whose preferences for leisure have increased decreases and so does the wage rate. The level of leisure increases as well whilst the total level of unemployment of the old and the overall level of umeployment fells.

4. *An increase in the preferences of a group for leisure of the other group.* An increase in the preferences of the old for leisure of the young entails a decrease in the tax rate, an increase in the wage and an increase in the labour supply by the group of the old. Since the demand for labor remains steady, this means that the overall level of unempoloyment increases. Otherwise, an in increase in the preferences of the young for leisure of the old resolves in an increase in the wage rate, a reduction in the wage and a reduction of the supply of labor and a slight decrease in the oveall unemployment.

5. *An increase in the numerosity of a group* An increase in the numerosity of a group entails a general decrease in the tax rate and an icnrease in the wage of that group whilsts it provokes an increase in the tax rate and a decrease in the wage of the other group. The level of leisure decreases for the group whose numerosity has grown whilst it increase for the other group. Otherwise, the level of unemployment increases due to the increase of the numerosity of the group whilst in decreases in the other group. Anyway the overall effect on the employment of society is uncertain. If the increase happens in the group of the old (in the simulation the percentage of the old increases from 0.5 to 0.55 whilst the percentage of the young shrinks from 0.5 to 0.45) the overall level of unemployment increases, whilst the vice-versa holds if the increases happens in the group of the old (in this case the percentage of the old decreases from 0.5 to 0.49 whilst the percentage of the young surges from 0.5 to 0.51. This exercise shows the danger of an increase in the population of the old that modern societies are experimenting. Since the old become more powerful from a political point of view, they succed to obtain lower levels of taxation and higher. Otherwise, the young are less numerous and they have to

carry the burden of the entire social security system: they pay more taxes, the have low levels of wages and they have also to pay for the pension transfers.

4 Conclusions

In this model, I applied the Single-Mindedness theory to the labor market, where wages are endogenously determined according to a Right-to-Manage bargaining model. I analysed a society composed by two groups of workers (the old and the young) which belong to a labor union and a firm. I assumed also that the preferences of the old for leisure are greater than the preferences of the young and that the level of productivity of the young is greater that the level of productivity of the old. Under these conditions, I demonstrated that the tax rate of the young is greater than that of the old whilst the wage rate of the old is lower than that of the young and the amount of leisure for the old is greater than the amount of leisure of the young. Since the single-mindedness of a group, which represents a proxy for the political power of that group, is captured by the density which is a monotonically increasing function with respect to leisure, I conclude that the old have a great power of influence onto politicians. This power enables them to get positive transfers in a PAYG system, whose burden is entirely carried by the young. Thus, with respect to the previous work, this study consider the mechanisms of labor unions, seen as an institution representing the interests of different social groups. Again, according to the Single-Mindedness theory, the greater the ability of a single group to be oriented toward the minimum number of issue, the higher the probability that this group achieves its goals. Nevertheless, this work does not consider some aspects which would deserve to be analysed. First of all, it would be interesting to consider more in details, the mechanisms which are undertaken by unions to take their decisions (i.e. voting process, elections and so forth). Furthermore, since an imperfect labor market entails unemployment in equilibrium, it would be interesting to add some new social groups (i.e. the unemployed) to analyse the impact of labor unions on excluded workers. I hope this suggestions will find a place in future researches.

5 Appendix A

In this Appendix I provide a complete resolution for the Second Order Conditions. First of all, notice that $\Omega(w^o, w^y)$ is a strictly concave function if:

$$D^2\Omega(w^o, w^y) = \begin{pmatrix} \Omega_{oo(w^o,w^y)} & \Omega_{oy(w^o,w^y)} \\ \Omega_{yo(w^o,w^y)} & \Omega_{yy(w^o,w^y)} \end{pmatrix}$$

is negative definite for all (w^o, w^y).

This is true if and only if

$$\left|\Omega_{oo(w^o,w^y)}\right| < 0 \text{ and } \begin{vmatrix} \Omega_{oo(w^o,w^y)} & \Omega_{oy(w^o,w^y)} \\ \Omega_{yo(w^o,w^y)} & \Omega_{yy(w^o,w^y)} \end{vmatrix} > 0$$

or equivalently, if and only if

$$\Omega_{oo(w^o,w^y)} < 0 \;\text{ and } \Omega_{oo(w^o,w^{ly})}\Omega_{yy(w^o,w^y)} - [\Omega_{oy(w^o,w^y)}]^2 > 0$$

It is easy to verify that:

$$\frac{\partial^2\Omega_t}{\partial^2 w^o} = \lambda(\theta_1\,(\overbrace{\frac{\partial^2(\log(U^o(w^o,\tau^o_{Lt},b^o_t)))}{\partial^2 w^o}}^{<0}) + (1-\theta_1-\theta_2)\,(\overbrace{\frac{\partial^2(\log(U^l(w^o,w^y,\tau^o_{Lt},\tau^y_{Lt},\delta^o,\delta^y)))}{\partial^2 w^o}}^{<0})) +$$

$$(1-\lambda)\,(\overbrace{\frac{\partial^2\log\Pi(w^o,w^y,L^y_t(w^y,\tau^y_{Lt}),L^o_t(w^o,\tau^o_{Lt}))}{\partial^2 w^o}}^{<0}) < 0$$

The same holds for

$$\frac{\partial^2\Omega_t}{\partial^2 w^y} = \lambda(\theta_1\,(\overbrace{\frac{\partial^2(\log(U^y(w^{ly},\tau^y_{Lt},b^{ly}_t)))}{\partial^2 w^y}}^{<0}) + (1-\theta_1-\theta_2)\,(\overbrace{\frac{\partial^2(\log(U^l(w^o,w^y,\tau^o_{Lt},\tau^y_{Lt},\delta^o,\delta^y)))}{\partial^2 w^y}}^{<0})) +$$

$$(1-\lambda)\,(\overbrace{\frac{\partial^2\log\Pi(w^o,w^y,L^y_t(w^y,\tau^y_{Lt}),L^o_t(w^o,\tau^o_{Lt}))}{\partial^2 w^y}}^{<0}) < 0$$

Finally

$$\frac{\partial^2\Omega_t}{\partial w^o\partial w^y} = \frac{\partial^2\Omega_t}{\partial w^y\partial w^o} = \lambda(\theta_1\,(\overbrace{\frac{\partial^2(\log(U^o(w^o,\tau^o_{Lt},b^o_t)))}{\partial w^o}\partial w^y}^{>0}) +$$

$$(1-\theta_1-\theta_2)\,(\overbrace{\frac{\partial^2(\log(U^l(w^o,w^y,\tau^o_{Lt},\tau^y_{Lt},\delta^o,\delta^y)))}{\partial w^o}\partial w^y}^{>0})) +$$

$$(1-\lambda)\,(\overbrace{\frac{\partial^2\log\Pi(w^o,w^y,L^y_t(w^y,\tau^y_{Lt}),L^o_t(w^o,\tau^o_{Lt}))}{\partial w^o}\partial w^y}^{>0}) > 0$$

6 Appendix B

In this appendix I provide a demonstration of **Proposition 1**. I derive the optimal wage rate, $w^{I*} = \frac{z^I \lambda}{2(1+\tau^I)}$ with respect to λ and I obtain:

$$\frac{\partial w^{I*}}{\partial \lambda} = \frac{z}{2(1+\tau^I_{Lt})}$$

which is easy to verify, is always greater than zero.
Secondly, I derive the optimal wage rate with respect to the parameter z:

$$\frac{\partial w^{I*}}{\partial z} = \frac{\lambda}{2(1+\tau^I_{Lt})}$$

which again is always greater than zero.
Finally, I derive the optimal wage rate with respect to the marginal tax rate:

$$\frac{\partial w^{I*}}{\partial \tau^I_{Lt}} = -\frac{z\lambda}{2(1+x)^2}$$

which is always negative.

7 Appendix C

In this Appendix I provide a complete resolution to the candidates' problem. The two candidates face exactly the same optimization problem; they maximize their share of votes or, equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.

$$\max\ \pi^A = \frac{1}{2} + \sum_{I=\{o,y\}} n^I s^I [V^i(\vec{q}^A) - V^i(\vec{q}^B)]$$

$$s.t.\ T_1 \equiv r(S^o_t) = T^o_t$$

$$T_2 \equiv r(S^y_t) = T^y_t$$

$$T_3 \equiv r(S^y_{t+1}) = T^y_{t+1}$$

$$T_4 \equiv n^o b^o_t + n^y b^y_t + \alpha\, |n^y b^y_t|\, |n^o b^o_t| = 0$$

$$T_5 \equiv b^o_t b^y_t < 0$$

where: $s^I = s^I(l(\tau_{Lt}, w))$
I substitute T_1, T_2 and T_3 into the IUF of individuals and I write the Lagrangian function:

$$L = \frac{1}{2} + \sum_{I=\{o,y\}} n^I s^I [V^i(\vec{q}^A) - V^i(\vec{q}^B)] + \lambda\,(T_4)$$

$$T_t^o = n^o \left(\bar{t}\tau_{Lt}^o w^o - \frac{\tau_{Lt}^o \psi^o}{(1-\tau_{Lt}^o)} \right) \tag{31}$$

$$T_t^{ly} = n^y \left(\bar{t}\tau_{Lt}^y w^y - \frac{\tau_{Lt}^y \psi^y}{(1-\tau_{Lt}^y)} \right) \tag{32}$$

$$T_{t+1}^y = n^y \left(\bar{t}\tau_{Lt+1}^y w^y - \frac{\tau_{Lt+1}^y \psi^y}{(1-\tau_{Lt+1}^y)} \right) \tag{33}$$

Substituting $w^{I*} = \frac{z^I \lambda}{2(1+\tau^I)}$ into equation (31), (32) and (33) we obtain:

$$T_t^o = n^o \left(\bar{t}\tau_{Lt}^o (\frac{z^o \lambda}{2(1+\tau_{Lt}^o)}) - \frac{\tau_{Lt}^o \psi^o}{(1-\tau_{Lt}^o)} \right) \tag{34}$$

$$T_t^y = n^y \left(\bar{t}\tau_{Lt}^y (\frac{z^y \lambda}{2(1+\tau_{Lt}^y)}) - \frac{\tau_{Lt}^y \psi^y}{(1-\tau_{Lt}^y)} \right) \tag{35}$$

$$T_{t+1}^y = n^y \left(\bar{t}\tau_{Lt+1}^y (\frac{z^y \lambda}{2(1+\tau_{Lt+1}^y)}) - \frac{\tau_{Lt+1}^y \psi^{y\prime}}{(1-\tau_{Lt+1}^y)} \right) \tag{36}$$

I write the First Order Conditions:

(1) $\frac{\partial L}{\partial \tau_{Lt}^o} = n^o s^o \frac{\partial V^o}{\partial \tau_{Lt}^o} + n^y s^y \frac{\partial V^y}{\partial \tau_{Lt}^o} = 0$
(2) $\frac{\partial L}{\partial \tau_{Lt}^y} = n^y s^y \frac{\partial V^y}{\partial \tau_L^y} + n^o s^o \frac{\partial V^o}{\partial \tau_{Lt}^y} = 0$
(3) $\frac{\partial L}{\partial b_t^o} = n^o s^o = \mu(n^o - n^o n^y \alpha^l b^o)$
(4) $\frac{\partial L}{\partial b_t^y} = n^y s^y = \mu(n^y - n^y n^o \alpha^l b^o)$
(5) $\sum_{I=LO,LY} (n^i b_t^i) + \ \alpha^l \left| n^{lo} b_t^{lo} \right| \left| n^{ly} b_t^{ly} \right| = 0$
$\mu \geq 0$

And solving we obtain:
(1) $s^o n^o (-\frac{tz^o\lambda + \frac{2(1+\tau^o)(3\tau^o-1)\psi^o}{(\tau^o-1)^2}}{2(1+\tau^{o2})}) + s^y n^y (\frac{2\varphi^y}{1-\tau^{2o}}) = 0$
(2) $s^y n^y (-\frac{tz^y\lambda + \frac{2(1+\tau^y)(3\tau^y-1)\psi^y}{(\tau^y-1)^2}}{2(1+\tau^{y2})}) + s^o n^o (\frac{2\varphi^o}{1-\tau^{2y}}) = 0$
(3) $\frac{\partial L}{\partial b_t^o} = n^o s^o = \mu(n^o - n^o n^y \alpha^l b^y)$
(4) $\frac{\partial L}{\partial b_t^y} = n^y s^y = \mu(n^y - n^y n^o \alpha^l b^o)$
(5) $\sum_{I=O,Y} (n^i b_t^i) + \ \alpha^l \left| n^o b_t^o \right| \left| n^y b_t^y \right| = 0$
$\mu \geq 0$

Solving (1) and (2) we derive the optimal tax rates (one of the two roots is not accettable):

$$\tau_{Lt}^{o*} = \frac{tn^o s^o z^o \lambda - 2n^o s^o \psi^o + 2\sqrt{2}\sqrt{2n^{2y}s^{2y}\varphi^{2y} - tn^{2o}s^{2o}z^o\lambda\psi^o + 4n^o n^y s^o s^y \varphi^y \psi^o + 2n^{2o}s^{2o}\psi^{2o}}}{tn^o s^o z^o \lambda + 4n^y s^y \varphi^y + 6n^o s^o \psi^o}$$

$$s.t. \left\{ t, n^o, s^o, z^o, n^y, s^y, \lambda, \varphi^y, \psi^o \in \Re^7 \right|$$

$$2n^{2y}s^{2y}\varphi^{2y} - tn^{2o}s^{2o}z^o\lambda\psi^o + 4n^o n^y s^o s^y \varphi^y \psi^o + 2n^{2o}s^{2o}\psi^{2o} \geq 0, tn^o s^o z^o \lambda + 4n^y s^y \varphi^y + 6n^o s^o \psi^o \neq 0$$

$$\tau_{Lt}^{y*} = \frac{tn^y s^y z^y \lambda - 2n^y s^y \psi^y + 2\sqrt{2}\sqrt{2n^{2o}s^{2o}\varphi^{2o} - tn^{2y}s^{2y}z^y\lambda\psi^y + 4n^y n^o s^y s^o \varphi^o \psi^y + 2n^{2y}s^{2y}\psi^{2y}}}{tn^y s^y z^y \lambda + 4n^o s^o \varphi^o + 6n^y s^y \psi^y}$$

$$s.t. \left\{ t, n^y, s^y, z^y, n^o, s^o, \lambda, \varphi^o, \psi^y \in \Re^7 \right|$$

$$2n^{2o}s^{2o}\varphi^{2o} - tn^{2y}s^{2y}z^y\lambda\psi^y + 4n^y n^o s^y s^o \varphi^o \psi^y + 2n^{2y}s^{2y}\psi^{2y} \geq 0, tn^y s^y z^y \lambda + 4n^o s^o \varphi^o + 6n^y s^y \psi^y \neq 0$$

The parameters matrix is a 12x11:
$n^o = \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.55, 0.49\}$
$ny = \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.45, 0.51\}$
$s^o = \{0.006, 0.006, 0.006, 0.006, 0.006, 0.006, 0.006, 0.006, 0.006, 0.006, 0.006\}$
$s^y = \{0.004, 0.004, 0.004, 0.004, 0.004, 0.004, 0.004, 0.004, 0.004, 0.004, 0.004\}$
$z^o = \{0.2, 0.2, 0.2, 0.25, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2\}$
$z^y = \{0.5, 0.5, 0.5, 0.5, 0.7, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\}$
$\lambda = (0.45, 0.5, 0.55, 0.45, 0.45, 0.45, 0.45, 0.45, 0.45, 0.45, 0.45\}$
$\psi^o = \{0.4, 0.4, 0.4, 0.4, 0.4, 0.45, 0.4, 0.4, 0.4, 0.4, 0.4\}$
$\psi^y = \{0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.3, 0.25, 0.25, 0.25, 0.25\}$
$\varphi^o = \{0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.85, 0.8, 0.8\}$
$\varphi^y = \{1, 1, 1, 1, 1, 1, 1, 0.95, 1, 1, 1\}$
$t = \{40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40\}$
The outcomes matrix is a 11x11:
$\tau^o_{Lt} = \{0.621396, 0.609347, 0.594811, 0.590644, 0.621396, 0.595993, 0.621396, 0.621396, 0.604559, 0.546086, 0.633736\}$
$\tau^y_{Lt} = \{0.814036, 0.809401, 0.804147, 0.814036, 0.793924, 0.814036, 0.781714, 0.825828, 0.814036, 0.849237, 0.80554\}$
$w^o = \{0.0277539, 0.0310685, 0.0344869, 0.035363, 0.0277539, 0.0281956, 0.0277539, 0.0277539, 0.0280451, 0.0291058, 0.0275442\}$
$w^y = \{0.0620164, 0.0690836, 0.0762133, 0.0620164, 0.0877964, 0.0620164, 0.0631415, 0.0616159, 0.0620164, 0.0608359, 0.0623082\}$
$l^o = \{38.0672, 32.9571, 28.6252, 27.6318, 38.0672, 39.5041, 38.0672, 38.0672, 36.068, 30.2766, 39.6493\}$
$l^y = \{21.6772, 18.9865, 16.7486, 21.6772, 13.8177, 21.6772, 21.7661, 23.2954, 21.6772, 27.2574, 20.6331\}$
$L^o = \{0.0775, 0.075, 0.0725, 0.096875, 0.0775, 0.0775, 0.0775, 0.0775, 0.0775, 0.0775, 0.0775\}$
$L^y = \{0.19375, 0.1875, 0.18125, 0.19375, 0.27125, 0.19375, 0.19375, 0.19375, 0.19375, 0.19375, 0.19375\}$
$U^o = \{0.8889, 3.44647, 5.61491, 6.0872, 0.8889, 0.170451, 0.8889, 0.8889, 1.8885, 5.27035, 0.0943629\}$
$U^y = \{8.96765, 10.3193, 11.4444, 8.96765, 12.8199, 8.96765, 8.92321, 8.15856, 8.96765, 5.54041, 9.68337\}$
$U^{o+y} = \{9.85655, 13.7657, 17.0593, 15.0548, 13.7088, 9.1381, 9.81211, 9.04746, 10.8561, 10.8108, 9.77773\}$

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