Status Quo Bias, Multiple Priors and Uncertainty Aversion

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Abstract. Motivated by the extensive evidence about the relevance of status quo bias both in experiments and in real markets, we study this phenomenon from a decision-theoretic prospective, focusing on the case of preferences under uncertainty. We develop an axiomatic framework that takes as a primitive the preferences of the agent for each possible status quo option, and provide a characterization according to which the agent prefers her status quo act if nothing better is feasible for a given set of possible priors. We then show that, in this framework, the very presence of a status quo induces the agent to be more uncertainty averse than she would be without a status quo option. Finally, we apply the model to a financial choice problem and show that the presence of status quo bias as modeled here might induce the presence of a risk premium even with risk neutral agents.

JEL classification: D11, D81.

Keywords: Status quo bias, Ambiguity Aversion, Endowment Effect, Risk Premium.

1. Introduction

A considerable amount of recent evidence has shown that individual decision makers often attach an additional value to their default options or status quo choice - a phenomenon dubbed the *status quo bias*. Instances of this tendency have been noted in numerous experiments and in real market observations, especially for 401(k) plans, residential electrical services and car insurance.[1]

This evidence has not gone unnoticed in economic theory. In particular, non-trivial explanations of the status quo bias phenomenon have been attempted by means of reference-dependent choice models with loss aversion, as in Tversky and Kahneman (1991) and Kahneman et al. (1991). More recently, a number of papers have analyzed

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[1]See, among others, Samuelson and Zeckhauser (1988), Kahneman et al. (1991), Hartman et al. (1991), Madrian and Shea (2001) and Ameriks and Zeldes (2004).

this phenomenon from an axiomatic point of view without focusing on loss aversion. In particular, these papers take as a primitive a choice correspondence or a preference relation, and assume the presence of status quo bias at the outset, by means of behavioral postulates. In turn, they provide characterizations that embody, per force, the status quo bias phenomenon. Among these papers are Masatlioglu and Ok (2005, 2008), and Sagi (2006).[2]

In the present paper we follow this second branch of the literature, and focus on a characterization of a particular set of preference relations with a status quo bias. We depart from the earlier approaches by concentrating on the effects of the status quo bias in a specific setup, namely, that of preferences under uncertainty.[3] That is, we focus on the preferences over acts whose return depend on the realization of a state of the nature, the probability of which is *unknown* to the agent. Indeed, it is precisely in this sort of an environment that many real world examples of status quo bias are observed. For instance, pension and/or insurance plans, the choices of which are well known to be affected by this form of bias, are almost always viewed as acts whose consequences are uncertain.

To illustrate the behavior that we aim to capture, consider the preferences of an agent who currently holds a default option (act) as her status quo. Her task is to decide if she should abandon her status quo and, if so, in favor of which alternative. Suppose that all options she is comparing the status quo with have uncertain values. It is well known that the ambiguity aversion of the agent may then well kick in, thereby reducing her confidence in her ability to compare some alternatives. In this situation, it stands to reason that the status quo might be particularly relevant for the agent. On the one hand, this alternative is her default option, the most obvious candidate to prefer on the face of the difficulties about comparing the feasible alternatives. On the other hand, it may be something that the agent is "familiar" with, and hence she may feel "less worried" about making a mistake by staying with it. By contrast, the other options may be somewhat foreign and hence less attractive to her. Given this point of view, one might expect the agent to be rather cautious in moving away from her status quo option. And what does "cautious" mean in a setup with uncertainty? Following the classic works of Bewley (1986) and Gilboa and Schmeidler (1989), this may be modeled by postulating the presence of a set of prior beliefs in the mind of the agent, and requiring that she looks for an improvement for all of her prior beliefs before abandoning her status quo choice. In fact, this is exactly what we shall find by means of an axiomatic approach.

Among the numerous papers that show the importance of the status quo option in experiments, Roca et al. (2006) is of particular relevance for the present paper. In that work, the authors present the subjects with an Ellsberg-type experiment, but instead of asking them to choose directly, they first endow them with an option, and then ask them whether they wish to exchange it for another one. It is found that the subjects tend to retain their current endowment *even when this endowment is ambiguous, and*

[2]We refer to Masatlioglu and Ok (2008) for a more detailed discussion of the difference between this approach and the one based on loss aversion.

[3]The loss aversion literature already contains models of decision making under uncertainty. See, for instance, Tversky and Kahneman (1992) and Sugden (2003).

the other alternatives are not. That is, from the perspective of revealed preference, they act as if they were ambiguity seeking. This emphasizes well the relevance of the status quo bias in this environment, and shows that this phenomenon may even overcome the well-known power of ambiguity aversion. Such behavior is instead fully consistent with the theory we shall develop below.

The idea that the case of uncertainty might be of special relevance for the study of status quo bias is not new. In particular, the notion that the presence of uncertainty might make the agent "confused," and induce her to stay with her status quo act, was first proposed, to the best of our knowledge, by Bewley (1986). Bewley suggests that the presence of uncertainty might force the preference relation of the agent to become incomplete. Moreover, he emphasizes the role of the status quo in this context with the assumption of *inertia*, which says that the agent will remain with her status quo unless there is something better according to her incomplete preference relation - this postulate is very much in the same spirit of the status quo bias. The difference between the present paper and Bewley (1986) is that the latter *assumes* an incomplete ordering and inertia, and derives the characterization of the behavior involving dominance for a set of priors. By contrast, here we posit the status quo bias behaviorally, and *derive* that this must entail the presence of an underlying incomplete relation, which is, in turn, modeled as in Bewley (1986) by means of multiple priors. In this sense, one could see one of the contributions of this paper as to offering a behavioral justification for the incompleteness of the preferences of the individual suggested by Bewley (1986): an incompleteness directly connected with status quo bias. Furthermore, our analysis includes the case *without* a status quo, and addresses the issue of what will the agent prefer among many objects that are found to dominate the status quo option - something that, by nature of incomplete preferences, could not have been addressed in the previous works.

Besides Bewley (1986), the present paper has a connection also to Masatlioglu and Ok (2005). One can think of the contribution of this work as a merging of the ideas behind these two papers. Indeed, although we analyze a different primitive - we use status-quo-dependent preferences and they use status-quo-dependent choice - some of the axioms at the core of our analysis are derived from Masatlioglu and Ok (2005). But, instead of focusing on the general (ordinal) case like these authors do, we focus on the more specific model of preferences under uncertainty, and impose additional axioms that are reasonable in this framework. (Our paper stands to that of Masatlioglu and Ok as the (cardinal) utility analysis of Anscombe and Aumann (1963) stands to the (ordinal) utility analysis of Debreu (1964).) Not surprisingly, the representation that we find is related to the one found by Masatlioglu and Ok. While in our case, to abandon the status quo, the agent requires dominance over a set of priors on the state space, in theirs the agent requires dominance in an endogenous multi-utility space. Moreover, our tighter characterization, coupled with additional properties specific to our representation (like the full-dimensionality of the set of priors), allows us to draw additional interpretations of the decision maker's behavior, and especially to work out a connection with concepts like ambiguity aversion, and to apply our model to instances in which the latter paper would have less to say, due to its generality.[4] We refer to Section 2.3 and 4 for a more detailed discussion.

Our analysis takes as a primitive a system of preference relations of the agent - one for each possible status quo option, plus one for the case in which there is no status quo. We impose two sets of axioms on these preferences. First, we posit some appropriate versions of Continuity, Monotonicity, and Independence - the latter only on the decision to retain the status quo or not. Second, we use two axioms that concern the connection between the preferences of the agent with distinct status quo points. Among these, a prominent role is played by the status quo bias axiom, which states that if an option is preferred to another when it is not the status quo option, then it would be strictly preferred when it *is* the status quo.

The first contribution of the paper is to show that these axioms are satisfied by a system of preference relations if and only if it admits the following representation. The agent has a utility function u on the outcome space and a set of prior beliefs Π on the state space. When there is a status quo the agent acts as if she were "scared" of making the wrong move, and requires dominance in expected utility with respect to *all* priors in Π before moving away from her status quo. Thus, the presence of status quo bias induces the agent to act on the basis of a multi-prior decision making procedure. To rank options that both dominate the status quo in this sense, she aggregates the expected utilities for each prior in a strictly increasing fashion. This latter decision rule is also used in her choice without the status quo.

The second contribution of this paper is to show that this characterization allows us to relate two apparently distinct concepts: status quo bias and ambiguity aversion. We present three results on this matter. First, we show that, in our axiomatic framework, the presence of a status quo increases, and in most cases strictly increases, the propensity of individuals towards *hedging* - which is one of the way to express uncertainty aversion. Second, we show that the presence of a *constant* status quo strictly increases the propensity of agents to prefer constant acts - another way in which uncertainty aversion can be expressed. This, however, is not true when the status quo is a non-constant act: in this case we have a mixed effect, with the tendency to favor the acts that dominate the status quo, even if they are non-constant - all in accordance to the cited experimental results in Roca et al. (2006). Third, we show that if we require all the preferences in our system to have a constant attitude towards hedging, then these preferences *must* be uncertainty averse - and this must be the case also when there is no status quo. This is true despite the fact that we have imposed no assumptions on the attitude towards ambiguity of the agent: it is a pure consequence of status quo bias.

We then turn to show that minimal requirements of independence on the preference without status quo are sufficient to obtain much tighter characterizations. First, we establish that if we impose full independence on this preference, we can represent the system of preferences by means of a utility function u, a set of priors Π and a single prior ρ in its relative interior. The set of priors is used like before, while the single

[4]On a more technical note, our structure allows us to deal with a possibly infinite prize space, as opposed to the case of Masatlioglu and Ok (2005), in which finiteness is required.

prior is used, with the same utility function, when there is no status quo, or between alternatives that dominate it.

This last, tighter representation is however potentially too restrictive, since it imposes that the agent is ambiguity neutral without a status quo - which we know is unrealistic. We then relax the full independence of the preferences without status quo, and replace it with c-independence and uncertainty aversion, much in line with Gilboa and Schmeidler (1989). We then obtain a representation that has a utility function and two sets of priors, one in the relative interior of the other. The larger one is used to decide whether to keep or not the status quo requiring dominance for all priors - like in Bewley (1986); the smaller one is used in the other cases by comparing the minimal utility for all priors - like in Gilboa and Schmeidler (1989). This representation allows us to see more clearly how the presence of a status quo directly increases the "uncertainty" of the agent, by making her use a strictly larger set of priors.

Finally, we apply the theory developed here to the case of decision-making in a financial market. We show that, if the preferences of an agent satisfy our axioms and the status quo is to choose an unambiguous act, then risk premia must emerge in this market even if the agents are risk neutral, and ambiguity neutral without a status quo. This result, of course, cannot be obtained within the realm of the standard expected utility model. Moreover, if the agents were known to be risk averse, then our model predicts that the risk premium observed in the market would be much higher than the one predicted by the standard model. This might lead an economist who studies that market by means of the standard model to think that the risk aversion of the agent is implausibly high.[5] We find here that the status quo bias phenomenon offers a simple way of explaining this situation.

The rest of the paper is organized as follows. Section 2 presents our axiomatic framework and our first characterization theorem. Section 3 analyzes the connection of this first model with ambiguity aversion. In Section 4 we add some independence-type assumptions on the preference without status quo and present two stronger characterization theorems. Section 5 presents our application. Section 6 concludes. The proofs of the main results are relegated to the Appendix.

2. A first model

2.1. *The Basic Framework*

We adopt the standard Anscombe-Aumann setup. We have a (non-empty) finite set S of possible states of the world and a (non-empty) set X of consequences, which is assumed to be a compact subset of a metric space. Let $\Delta(X)$ stand for the set of all Borel probability measures (lotteries) on X. Denote by $\mathcal{F}$ the set of all acts, that is, the set of all functions from S into $\Delta(X)$. For any $p \in \Delta(X)$, with a standard abuse

[5]In fact, the observation that only an implausibly high risk would justify the risk premium in real financial markets is well known. This phenomenon is famously called the "equity premium puzzle."

of notation, we denote by $p \in \mathcal{F}$ the constant act the yields the consequence p at *every* state.

We metrize $\mathcal{F}$ by the product Prokhorov metric. Fix the symbol $\diamond$ to denote an object that does not belong to $\mathcal{F}$. We are interested in a set of complete preference relations $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$. We view $\succeq_\diamond$ as the preference relation of a rational decision maker when she has no status quo alternative, and for each $f \in \mathcal{F}$, we understand by $\succeq_f$ the preference relation that rationalizes the choices of this agent when the status quo is f. This interpretation prompts a particular restriction on such a system of preference relations, because it presumes that the status quo, if exists, *must be a feasible option.* That is, in studying choice that is rationalized by these preference relations, the status quo must always be part of the feasible choice set. Consequently, we are not able to assess the revealed preference between two acts g and h if neither is chosen when the agent's status quo is f. In other words, since $\succeq_f$ is the revealed preference from the choice when f is the status quo, and since this status quo must be feasible, then if f is the unique choice when only g and h are present, we cannot discuss the revealed ranking between g and h (when f is the status quo) - after all, the agent will never be able to express this ranking by her choice. Then, it seems reasonable to impose that this ranking coincides with the agent ranking without the status quo. We shall refer to the system of preferences that satisfy this restriction as *consistent.* More precisely:

Definition 1. A system of preference relations $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ is a *consistent system of preferences* if:

(1) for all $\tau \in \mathcal{F} \cup \{\diamond\}$, $\succeq_\tau$ is a complete preference relation on $\mathcal{F}$;
(2) for all $f \in \mathcal{F}$, if $f \succ_f g, h$, then

$$g \succeq_f h \Leftrightarrow g \succeq_\diamond h.$$

Consistent systems of preferences will act as the primitives of our analysis.

2.2. *Axioms*

The axioms that we impose on the system of preferences can be split into two groups. The first group consists of four axioms that are, respectively, the extensions of the standard axioms to our setting.

We start with a monotonicity-type property. As usual, we first define the preorder $\trianglerighteq$ on $\mathcal{F}$ as follows:

$$f \trianglerighteq g \text{ iff } f(s) \succeq_\diamond g(s) \text{ for all } s \in S.$$

That is, we have $f \trianglerighteq g$ whenever what f returns is preferred to what g returns in *every* possible state.

Axiom 1 (Weak Monotonicity)**.** For any $f, g \in \mathcal{F}$ we have:

(a) If $f \triangleright g$, then $f \succ_g g$;

(b) If $f \trianglerighteq g$ and $g \trianglerighteq f$, then, for any $h \in \mathcal{F}$

$$h \succeq_f f \ \Leftrightarrow h \succeq_g g,$$

and

$$f \succeq_h h \ \Leftrightarrow g \succeq_h h,$$

The rationale of part (a) is standard: the agent would be willing to abandon her status quo if given something better for all states. In turn, part (b) says that if the agent is indifferent between what two acts return in *all* possible states (that is, $f \trianglerighteq g \trianglerighteq f$), then their comparisons with other acts are identical. In particular, if f is "beaten" by an act h even when it is the status quo, then g should compare to h similarly; and if f is preferred to h even when h is the status quo, so must g. Notably, with this axiom we are implicitly assuming that no state is null.[6]

The core content of this axiom is to restrict the effects of the status quo to apply only to the cases in which such monotone dominance does not pertain. Naturally, one could easily imagine cases in which this latter postulate would not apply - the choice theory of Masatlioglu and Ok (2005) focuses on this case. However, our goal here is to show how status quo bias interacts with uncertainty, and therefore we focus on standard monotone preferences.

We then turn to a postulate of continuity.

Axiom 2 (Continuity)**.** For any $f, g \in \mathcal{F}$ and any $(f^n), (g^n) \in \mathcal{F}^\infty$ such that $f^n \to f$, $g^n \to g$:

(a) if $g_n \succeq_\diamond f_n$ for all n, then $g \succeq_\diamond f$;
(b) if $f \not\trianglerighteq g$ and $g^n \succeq_f f$ for all n, then $g \succeq_f f$;
(c) if $f \not\trianglerighteq g$ and $g \succeq_{f^n} f^n$ for all n, then $g \succeq_f f$.

Part (a) is a standard continuity postulate. In turn, parts (b) and (c) are the corresponding properties about the decision to keep or abandon the status quo, split into upper hemicontinuity (part (b)) and lower hemicontinuity (part (c)).

We next impose an independence property.

Axiom 3 (Independence with SQ)**.** For any $f, g, h \in \mathcal{F}$ and $\lambda \in (0, 1]$,

$$g \succeq_f f \ \Leftrightarrow \ \lambda g + (1-\lambda)h \succeq_{\lambda f + (1-\lambda)h} \lambda f + (1-\lambda)h.$$

This axiom imposes an independence-like property on one's decision about whether to keep the status quo or not. (A very similar axiom can be found in Sagi (2003)). From a normative point of view, it retains the appeal of the standard independence axiom of the Anscombe-Aumann theory. At the same time, it is restricted to the decision about whether to keep or not to keep the status quo - without imposing this linearity in

[6]This restriction is made for mere simplicity: it is standard practice to weaken the axiom to refer only to non-null states. Our results would then be identical, with the requirement that, in the representation, the set of priors Π is full-dimensional within the subset of non-null states.

general. As we shall see shortly, Axiom 3 allows for the presence of ambiguity aversion with or without a status quo, while still providing a tight structure for our analysis.

Our assumptions so far relate the preferences with either a fixed or varying status quo acts. Our second set of axioms regards the connection between the choices *with* and *without* status quo.

Axiom 4 (Status Quo Irrelevance)**.** For any $f, g, h \in \mathcal{F}$ such that $g, h \succeq_f f$, we have

$$g \succeq_f h \;\Leftrightarrow\; g \succeq_\diamond h.$$

In words, if we have two acts that both dominate the status quo, then its presence should not affect their relative rank. Put differently, this property ensures that the status quo has a distorting effect on one's preferences only when it is chosen at least against one of the elements; otherwise, it is "irrelevant" and the preferences with and without status quo should be the same. To illustrate, consider the options {*dine at a French restaurant*, *dine at an Italian restaurant*, *eat pet food*}. The idea of this axiom is that, even if the option "*eat pet food*" is the status quo, it is so clearly dominated by the other two alternatives that it will not affect one's final choice. This Axiom, although stated in different terms, merges the intuition behind Axioms SQI and D in Masatlioglu and Ok (2005), and is the preference equivalent of Axiom SQI in Masatlioglu and Ok (2008).

A caveat is called for here. In general, the presence of a status-quo option may have two distinct effects: first, it may induce a desire to hold on to the default option (the status-quo bias); second, it may induce a tendency to evaluate the alternatives in a menu relative to the status quo (the reference effect). By imposing Axiom 4, our theory is bound to focus especially on the first effect, and less on its reference effect - albeit, as we shall see, some reference effect is still present. In particular, it does not allow an undesirable status quo to act as a reference point that may affect the final choice of the agent. By contrast, in the context of the example above, a reference dependent theory would allow for the possibility that the status quo being "eat pet food" may make French cuisine more attractive than Italian cuisine. (This resonates better with intuition if we replace "eat pet food" with "eat left overs from previous night", for instance.) Moreover, it rules out the possibility that a certain status quo might alter the relative appeal of two alternatives, unless one of these alternatives is not appealing enough to "beat" the status quo. Again, this restricts the way in which a status quo alternative may act as a reference point, thereby distancing our results from interpretations that are based more on the reference effect (as in Kahneman et al. (1991)).[7]

We conclude with our main axiom.

Axiom 5 (Status Quo Bias)**.** For any $f, g \in \mathcal{F}$, if $g \succeq_f f$ or $g \succeq_\diamond f$, then $g \succ_g f$.

[7]This suggests that there is room for extending the present theory in a way that allows for status quo choices exerting reference effects. Indeed, precisely this sort of an extension is carried out by the recent work by Masatlioglu and Ok (2008). Our focus here, however, is to understand how a choice theory with status quo bias can be developed under uncertainty, so we adopt as our basic framework the simpler model in which the "reference" effects of status quo alternatives are kept to a minimum.

This axiom posits exactly the effect we aim to characterize, and hence has a special relevance for our work. It postulates that if an agent is willing to "move" to g from her status quo option f, then she would strictly prefer g if g was presented to her as her status quo point. This axiom, forcing the interpretation of the status quo as an object that exerts "attraction" toward itself, is relatively standard in the literature on status quo bias. Its formulation adapts Axiom SQB in Masatlioglu and Ok (2005), corresponds to the one imposed by Sagi (2006) in terms of preferences, and, in turn, it is closely related to the *inertia* assumption of the Knightian Uncertainty model of Bewley (1986) and its applications.

2.3. *A first characterization*

In this section we present our first characterization theorem. A final bit of notation is required for this. For any set Π of probability vectors on S and continuous function $u : X \to \mathbb{R}$, we define the real map on Π $\mathbf{U}_{\Pi,u}(f) \in \mathbb{R}^{\Pi}$ by

$$\mathbf{U}_{\Pi,u}(f)(\pi) := \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u).$$

Also, we denote by $\mathcal{D}_{\Pi,u}(f)$ the set

$$\mathcal{D}_{\Pi,u}(f) := \{g \in \mathcal{F} : \sum_{s\in S} \pi(s)\mathbb{E}_{g(s)}(u) \geq \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u), \text{ for all } \pi \in \Pi, \text{strictly for some}\}.$$

This is the set of all acts that dominate f in terms of *all* the expected utilities induced by the set Π of priors. Finally, $\mathbf{U}_{\Pi,u}(\mathcal{F})$ is define as

$$\mathbf{U}_{\Pi,u}(\mathcal{F}) := \{\mathbf{U}_{\Pi,u}(f) : f \in \mathcal{F}\}.$$

We can now state our first result.

Theorem 1. *A consistent system of preferences $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ satisfies Axioms 1-5 if, and only if, there exist a continuous function $u : X \to \mathbb{R}$, a unique, compact, $(|S| - 1)$-dimensional convex set Π of probability vectors on S, and a strictly increasing and continuous functional $\psi : \mathbf{U}_{\Pi,u}(\mathcal{F}) \to \mathbb{R}$ such that for all $f, g, h \in \mathcal{F}$:*

$$f \succeq_\diamond g \;\Leftrightarrow\; \psi(\mathbf{U}_{\Pi,u}(f)) \geq \psi(\mathbf{U}_{\Pi,u}(g)), \tag{1}$$

and

$$f \succeq_h g \;\Leftrightarrow\; \begin{array}{c} \left[f \in \mathcal{D}_{\Pi,u}(h)\ ,\ g \notin \mathcal{D}_{\Pi,u}(h)\right] \\ \text{or} \\ \left[f, g \in \mathcal{D}_{\Pi,u}(h)\ ,\ \psi(\mathbf{U}_{\Pi,u}(f)) \geq \psi(\mathbf{U}_{\Pi,u}(g))\right] \\ \text{or} \\ \left[f, g \notin \mathcal{D}_{\Pi,u}(h)\ ,\ \psi(\mathbf{U}_{\Pi,u}(f)) \geq \psi(\mathbf{U}_{\Pi,u}(g))\right]. \end{array} \tag{2}$$

Moreover, if there exist $f, g \in \mathcal{F}$ such that $f \succeq_g g$, then Π is unique and u is unique up to a positive affine transformation.

An agent whose preferences satisfy Axioms 1-5 can thus be thought of as one with a utility function u over the prize space X, and a set Π of priors on the state space S (the latter can be thought of a set of possible models of the world in the mind of the agent). In order to decide whether to retain her status quo or not, the agent acts as "pessimist." If there is nothing better than her status quo option for *all* the models she considers, she gets conservative and prefers to keep her status quo option. Otherwise, she aggregates her evaluation in each of these models by means of some strictly increasing function ψ. And, it is this very rule that she uses in the absence of status quo option. (This is a consequence of our postulates on the relation of the preferences with and without status quo - Axioms 4).

To illustrate the representation in Theorem 1, consider the case in which there are two states of the world and the agent is facing the acts f, g and h, whose utility returns are depicted in Figure 1. (Here, $I(f, \pi)$ represents the set of elements with the same expected return of f, when this is computed using the prior π.) We assume that ψ is such that the agent's preference without a status quo are the maximization of the expected utility u computed using some prior ρ in the relative interior of Π. (In Section 4 we will discuss when this is the case). From the graph it is easy to see that, if there is no status quo act, g would be the best alternative in $\{f, g, h\}$. By contrast, consider the case in which f is the status quo. Let π_1 and π_2 be the extreme points of Π. It is then easy to see from the graph that, if f is the status quo, the agent would instead prefer h to both f and g. For, g has a lower return than f when the expected value is computed using prior π_1, and therefore the agent does not want to move there from the status quo f. Therefore, although g has a higher return than h for the prior ρ, f being the status quo option induces the decision maker to prefer $h \in \mathcal{D}_{\Pi,u}(f)$. Again, it is "as if" the agent wanted to be "sure" not to make the wrong move from her status quo choice f. Of course, she has no qualms of moving to h in this regard, for h is better than f for *all* of her priors. This is not the case for g, which is better than f for some, but not for every prior belief in Π. Indeed, if h were not available, then the agent would remain with f instead of moving to g, hence the status quo bias phenomenon.

As we discussed in the Introduction, Theorem 1 is connected to the Knightian Uncertainty model of Bewley (1986) and the status quo bias model of Masatlioglu and Ok (2005). In brief, our analysis comes down to combining the intuition behind the Masatlioglu-Ok choice-with-status-quo model with the Anscombe-Aumann preference-under-uncertainty model. Just like in Bewley (1986), the decision of whether to abandon the status quo or not is modeled by requiring dominance for a set of priors. But unlike it, the analysis here derives from assumptions on the relation of the preferences with different status quibus and not from incompleteness of preferences. Rather, we show that status quo bias *implies* the presence of an underlying *incomplete* preference relation that governs the choice of abandoning the status quo, and that satisfies the properties suggested by Bewley (1986). From this point of view, one might see our contribution as offering a behavioral justification for the incompleteness of preferences suggested by Bewley - a justification based on status quo bias. Moreover, this approach allows us to model in a unitary manner the preferences without status quo, or between elements both dominating the status quo - something that, structurally, cannot be done in a setup with incomplete preferences.

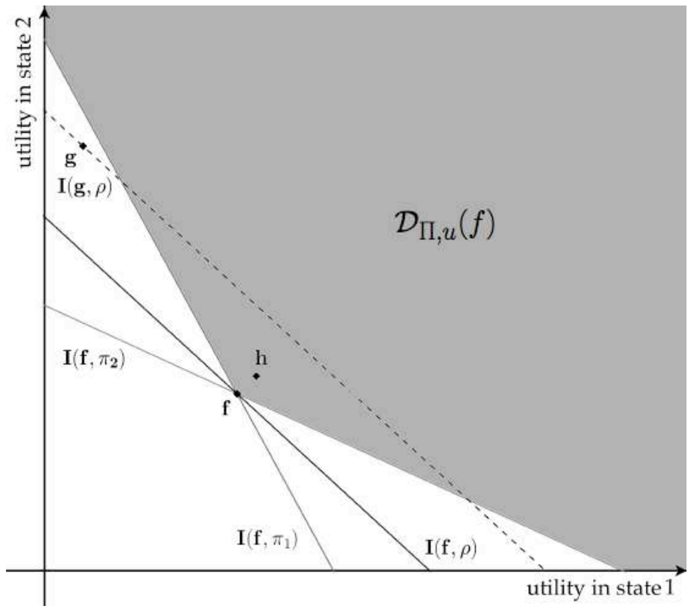

Figure 1

The connection with Masatlioglu and Ok (2005) is also very strong. In fact, Theorem 1 amounts to translating their work from the analysis of choice in a general setup, to the analysis of a system of preferences under uncertainty. In fact, the axiomatic structure on the relation between preference with different status quibus (Axioms 4, 5) is derived from theirs. (An even more similar version can be found in Masatlioglu and Ok (2008).) Having said this, our focus on a setup of uncertainty allows us to extend their analysis to an infinite set, and obtain a unique (cardinal) utility function and a set of priors, as opposed to a set of (ordinal) utility functions. Furthermore, our representation has an additional feature: namely, the full-dimensionality of the set of priors. This may seem at first of only technical interest, but, in fact, it has rather strong consequences for the potential applications of the model. For example, this property guarantees that, as long as Axioms 1-5 are satisfied, the agent's behavior must entail a gap between her Willingness to Pay and her Willingness to Accept, which is not necessarily true in the model of Masatlioglu and Ok (2005). As we shall see in Section 4, thanks to this property, and to its tight characterization, we are able to apply our theory to areas in which the choice theories of the previous papers on status quo bias lack predictive power. Furthermore, this property allows us to identify relation of the characterized model with ambiguity averse decision making, as we shall see in the next session.

3. Status quo bias, Ambiguity Aversion and coherent preferences

The representation described in Theorem 1, based on status quo bias, has a component reminiscent of the models of Gilboa and Schmeidler (1989) or Maccheroni et al. (2006), based on uncertainty aversion, or of the robust control preferences (Hansen and Sargent (2000), Hansen and Sargent (2001), Strzalecki (2007)). In all of these models, the agent makes her choice by using a set of priors with a strong degree of "pessimism." This allows their models to incorporate uncertainty aversion within the realm of decision making. By contrast, in our model multiplicity of priors arises due to the status quo bias phenomenon. This suggests that there might be a connection between the traits of uncertainty aversion and status quo bias. This section is devoted to the investigation of this matter.

We attack the problem from three different perspectives. First, we go back to the original notion of *uncertainty aversion* introduced by Gilboa and Schmeidler (1989), which is a preference for hedging, and investigate whether the presence of a status quo induces such tendency. Second, we consider the alternative notion of *ambiguity aversion* introduced by Ghirardato and Marinacci (2002), which allows us to make comparative statements on the strengths of one's ambiguity attitude. We will study if, and how, the presence of a status quo modifies this attitude. Third, we investigate the consequences of imposing a coherence requirement on one's preferences in terms of like or dislike of hedging.

3.1. *Status quo bias and hedging*

We now turn to investigate the relation of status quo bias and uncertainty aversion in the sense of preference for hedging. The following definition is due to Gilboa and Schmeidler (1989).

Definition 2. A preference relation $\succeq$ on $\mathcal{F}$ is *uncertainty averse* if for any $f, g \in \mathcal{F}$,

$$f \sim g \Rightarrow \tfrac{1}{2}f + \tfrac{1}{2}g \succeq f.$$

It is *uncertainty loving* if for any $f, g \in \mathcal{F}$,

$$f \sim g \Rightarrow f \succeq \tfrac{1}{2}f + \tfrac{1}{2}g.$$

It is *uncertainty neutral* if for any $f, g \in \mathcal{F}$,

$$f \sim g \Rightarrow f \sim \tfrac{1}{2}f + \tfrac{1}{2}g.$$

In order to understand how the presence of a status quo might affect the attitude towards hedging, we introduce a comparative notion of such attitude.

Definition 3. Consider two preference relations $\succeq_1, \succeq_2$ on $\mathcal{F}$. We say that $\succeq_1$ is *more willing to hedge* than $\succeq_2$ if, for every $f, g \in \mathcal{F}$ and $\alpha \in (0,1)$ with $f \succeq_2 g$, $f \succeq_1 g$ we have

$$\alpha f + (1-\alpha)g \succeq_2 g \quad \Rightarrow \alpha f + (1-\alpha)g \succeq_1 g.$$

We say that $\succeq_1$ is *strictly more willing to hedge* than $\succeq_2$ if it is more willing to hedge than $\succeq_2$ and the converse is not true.

The rationale of this comparative notion lies in the fact that uncertainty aversion implies a weak preference for hedging - a mixture between two alternatives is better then the worst of the two. (In fact, uncertainty aversion is stronger then this.) Now, if we consider two preference relations, we wish to say that the first is (weakly) more uncertainty averse then the second if it prefers to hedge whenever the other does. That is, if the two preferences agree on how to rank two acts, then whenever the second preference likes a mixture of those better then one of the acts, so must the first preference - because if the second preference liked to hedge there, the first must like it as well; hence Definition 3.

Proposition 1. *Consider a consistent system of preferences* $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ *that satisfies Axioms 1-5. Then, for any* $f \in \mathcal{F}$, $\succeq_f$ *is more willing to hedge than* $\succeq_\diamond$.

Within the context of Theorem 1, therefore, the presence of a status quo option induces an agent to be *more* willing to hedge than she would have been without a status quo option. In particular, notice that we do not assume that the agent has any preference for hedging in any occasion: potentially, she could be ambiguity lover, and dislike hedging. Notice, moreover, that this proposition "has a bite": it is easy to show that if $\succeq_\diamond$ is uncertainty neutral or uncertainty lover, then for any $f \in \text{int}(\mathcal{F})$, $\succeq_f$ is *strictly* more willing to hedge than $\succeq_\diamond$.

3.2. *Status quo bias and preference for constant acts*

An alternative way to define the notion of ambiguity aversion is suggested by Ghirardato and Marinacci (2002), and it is based, instead on one's willingness to hedge, on one's preference for constant acts.[8]

Definition 4. Consider two preference relations $\succeq_1$, $\succeq_2$ on $\mathcal{F}$. We say that $\succeq_2$ is *more ambiguity averse* than $\succeq_1$ if, for any $f \in \mathcal{F}$ and $p \in \Delta(X)$,

$$p \succeq_1 f \Rightarrow p \succeq_2 f.$$

and

$$p \succ_1 f \Rightarrow p \succ_2 f.$$

We say $\succeq_2$ is *strictly more ambiguity averse* than $\succeq_1$ if it is more ambiguity averse than $\succeq_1$ and the converse is not true.

The following result extends Proposition 1 to the context of comparison of one's attitude towards ambiguity with and without a status quo option.

Proposition 2. *Consider a consistent system of preferences* $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ *that satisfies Axioms 1-5. Then, for any* $p \in \Delta(X)$, $\succeq_p$ *is strictly more ambiguity averse than* $\succeq_\diamond$.

Proposition 3. *Consider a consistent system of preferences* $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ *that satisfies Axioms 1-5. Then, for any non-constant* $f \in \text{int}(\mathcal{F})$, *there exist* $g \in \mathcal{F}\backslash\{f\}$ *and* $q \in \Delta(X)$ *such that*

$$g \succ_f q \text{ and } q \succ_\diamond g.$$

Therefore, when the agent has a status quo option in the form of a constant act, she becomes strictly more ambiguity averse than she was before. In particular, notice that

[8]This definition is connected to comparative uncertainty aversion if preferences are continuous and satisfy c-independence. But as this latter property is not presumed here, a separate treatment of comparative willingness to hedge and preferences for constant acts is necessary. And, as we shall see, we will obtain different results.

this is *strict*; the presence of a status quo bias *must* have an effect on her behavior and on her ambiguity aversion. By contrast, when her status quo is a non-constant act, then the presence of a status quo induces a conflicting effect. In particular, Proposition 3 states that the agent might end up choosing a *more ambiguous return* - even if that act is not the status quo. This will be the case for the elements dominating the status quo, which, even if ambiguous, will be preferred to unambiguous acts that do not dominate the status quo option. It might be worth noting that this finding conforms with the empirical results of Roca et al. (2006), which show that agents may tend to act as ambiguity *lovers* when they have an ambiguous status quo.

3.3. *Rationalization by coherent preferences and ambiguity aversion*

In what follows we say that a preference is *coherent* if it has a coherent attitude towards uncertainty: if it is ever uncertainty averse or lover, then it should always be (weakly) so. Formally:

Definition 5. A preference relation $\succeq$ on $\mathcal{F}$ is *coherent* if the following holds:

(1) if there exist $f, g \in \mathcal{F}$ such that $\frac{1}{2}f + \frac{1}{2}g \succ f \sim g$, then we have $h \sim l \succ \frac{1}{2}h + \frac{1}{2}l$ for *no* $h, l \in \mathcal{F}$.
(2) if there exist $f, g \in \mathcal{F}$ such that $f \sim g \succ \frac{1}{2}f + \frac{1}{2}g$, then we have $\frac{1}{2}h + \frac{1}{2}l \succ h \sim l$ for *no* $h, l \in \mathcal{F}$.

We are interested here in the case in which our preference relations $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ are coherent. Notice that by this we do *not* require that they are coherent between each other. That is, $\succeq_f$ can be strictly uncertainty loving for some status quo act f, while $\succeq_\diamond$ or $\succeq_g$ for any $g \neq f$, can be strictly ambiguity averse. We require only that the agent has the same attitude *fixing* the status quo; she is free to have a different attitudes with different status quo options.

We are now ready to state our main result in this section.

Proposition 4. *Consider a coherent system of preferences $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ that satisfies Axioms 1-5. Then, the following statements are equivalent:*

(1) $\succeq_f$ *is coherent for all $f \in \mathcal{F} \cup \{\diamond\}$;*
(2) $\succeq_\diamond$ *and* $\succeq_f$ *are coherent for some $f \in \text{int}(\mathcal{F})$;*
(3) $\succeq_f$ *is uncertainty averse for all $f \in \mathcal{F} \cup \{\diamond\}$;*
(4) $\succeq_\diamond$ *is uncertainty averse.*

The postulates we have imposed on the system of preference relations, therefore, imply that whenever we require that each of the preferences is coherent, then each of them must be uncertainty averse - and, obviously, vice-versa. And further, this equivalence is true even if we only require that only $\succeq_\diamond$ is uncertainty averse, or only $\succeq_\diamond$ and one of the preferences is coherent for some status quo in the interior: each of these statements are equivalent to saying that *all* preferences are coherent, and *all* ambiguity averse. Therefore, if we consider coherence a desirable requirement - for at least one status quo and $\succeq_\diamond$ - under the postulates of our model ambiguity aversion *must* be entailed for all preferences in our system, including $\succeq_\diamond$.

4. Stronger characterizations: some independence

4.1. *Adding linearity without the status quo*

In our analysis thus far we have imposed no independence-type postulates on the preference without a status quo option. We now turn to investigate how their introduction would affect our characterization. A natural first candidate is, of course, full independence.

Axiom 6 (Independence without the SQ)**.** For any $f, g, h \in \mathcal{F}$ and $0 < \lambda \leq 1$,

$$f \succ_\diamond g \text{ iff } \lambda f + (1-\lambda)h \succeq_\diamond \lambda g + (1-\lambda)h$$

It turns out that this postulate allows us to obtain a much neater characterization.

Theorem 2. *A consistent system of preferences $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ satisfies Axioms 1-5 if, and only if, there exist a continuous function $u : X \to \mathbb{R}$, a unique, compact, $(|S|-1)$-dimensional convex set Π of probability vectors on S and a probability vector ρ in the relative interior of Π, such that*

$$f \succeq_\diamond g \;\Leftrightarrow\; \sum_{s\in S} \rho(s)\mathbb{E}_{f(s)}(u) \geq \sum_{s\in S} \rho(s)\mathbb{E}_{g(s)}(u), \tag{3}$$

and

$$f \succeq_h g \;\Leftrightarrow\; \begin{array}{c} \big[f \in \mathcal{D}_{\Pi,u}(h)\ ,\ g \notin \mathcal{D}_{\Pi,u}(h)\big] \\ \text{or} \\ \big[f, g \in \mathcal{D}_{\Pi,u}(h)\ ,\ \sum_{s\in S} \rho(s)\mathbb{E}_{f(s)}(u) \geq \sum_{s\in S} \rho(s)\mathbb{E}_{g(s)}(u)\big] \\ \text{or} \\ \big[f, g \notin \mathcal{D}_{\Pi,u}(h)\ ,\ \sum_{s\in S} \rho(s)\mathbb{E}_{f(s)}(u) \geq \sum_{s\in S} \rho(s)\mathbb{E}_{g(s)}(u)\big]. \end{array} \tag{4}$$

Moreover, if there exist $f, g \in \mathcal{F}$ such that $f \succeq_g g$, then Π and ρ are unique and u is unique up to a positive affine transformation.

The agent whose preferences satisfy Axioms 1-6 can thus be thought of as one with a utility function u over the prize space, a prior ρ, and a set Π of priors on S that are "around" ρ (in the sense that ρ is in the relative interior of Π).[9] This agent's preferences will then be as follows: when there is no status quo, she prefers the act that maximizes the subjective expected utilities computed using the prior ρ and the utility u - just like a standard Savagean agent. When she has a status quo act, however, she acts as if she

[9] Alternatively, standard results ensure that we could write this result with a measure μ over Π with full support, and replace the prior ρ with the resultant of this measure.

were "scared" of making the wrong move. In that case, she considers a set Π of priors, and before abandoning her status quo she requires dominance in expected utility for every one of these priors. If there are no acts dominating the status quo for each $\pi \in \Pi$, then she gets conservative and prefers her status quo. Otherwise, she chooses among the acts that dominate her status quo - and only among them - and prefers those that yield the highest expected utility relative to her *original belief* ρ.

There are two features of this characterization that are worth emphasizing. First, the agent is guaranteed to use the same utility function in both the choice about keeping the status quo, and the choice without any. Second, the single prior ρ of the agent is found to lie in the relative interior of the full-dimensional set Π. This requirement, which again appears to be technical, guarantees that the presence of a status quo makes our agent *strictly* more ambiguity averse in *every occasion*. In a way, we have an agent who, when she has a status quo, is expanding her set of priors in all possible directions - she is more pessimistic in all possible sense. Once again, this is the feature that guarantees that the results on the relation with ambiguity aversion that we have found in the previous section would apply to this case as well.

4.2. *A generalization to Ambiguity Aversion*

The representation we have obtained with Theorem 2 implies that the agent *must* be ambiguity neutral without a status quo: this is a direct consequence of independence of the preference without a status quo option, Axiom 6. This, however, restricts the presence of uncertainty aversion to the case in which there is a status quo option: in fact, while our previous result *entails* that a status quo must induce uncertainty aversion, at the same time it forces this uncertainty aversion to be limited to this case. The goal of this section is to relax this requirement and to allow for the (more realistic) presence of an ambiguity averse behavior even without a status quo option. We should therefore expect to obtain a representation in which the agent is uncertainty averse in general, and even more when she has a status quo. And this is precisely what we find.

In order to allow for a presence of uncertainty aversion, we need first of all to weaken the independence axiom: following a standard approach in the literature, we replace it with the c-independence and uncertainty aversion axioms of Gilboa and Schmeidler (1989) - both imposed only on the preference without a status quo.

Axiom 7 (c-Affinity)**.** For any $f, g \in \mathcal{F}$, $p \in \Delta(X)$ and $\lambda \in (0,1]$,

$$f \succeq_\diamond g \;\text{ iff }\; \lambda f + (1-\lambda)p \succeq_\diamond \lambda g + (1-\lambda)p.$$

Axiom 8 (Uncertainty Aversion without SQ)**.** $\succeq_\diamond$ is uncertainty averse.

By replacing independence (Axiom 9) with the weaker c-independence (Axiom 7) above, and adding uncertainty aversion without the status quo (Axiom 8), we obtain the following representation.

Theorem 3. *Consider a consistent system of preferences* $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$. *The following three statements are equivalent:*

(i) $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ *satisfies Axioms 1-5, 7 and 8;*
(ii) $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ *satisfies Axioms 1-5, 7 and* $\succeq_\diamond$ *and* $\succeq_f$ *are coherent for some* $f \in \mathrm{int}(\mathcal{F})$*;*
(iii) *there exist a continuous function* $u : X \to \mathbb{R}$, *a unique, compact,* $(|S|-1)$*-dimensional convex set* Π *of probability vectors on* S *and a convex, compact convex set* $\hat{\Pi}$ *of probability vectors on* S *such that* $\hat{\Pi} \subset \mathrm{ri}(\Pi)$, *such that*

$$f \succeq_\diamond g \;\Leftrightarrow\; \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u) \geq \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{g(s)}(u), \tag{5}$$

and

$$\begin{aligned} f \succeq_h g \;\Leftrightarrow\; & \big[f \in \mathcal{D}_{\Pi,u}(h)\ ,\ g \notin \mathcal{D}_{\Pi,u}(h)\big] \\ & \text{or} \\ & \big[f, g \in \mathcal{D}_{\Pi,u}(h)\ ,\ \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u) \geq \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{g(s)}(u)\big] \\ & \text{or} \\ & \big[f, g \notin \mathcal{D}_{\Pi,u}(h)\ ,\ \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u) \geq \min_{\pi\in\hat{\Pi}} \sum_{s\in S} \pi(s)\mathbb{E}_{g(s)}(u)\big]. \end{aligned} \tag{6}$$

Moreover, if there exist $f, g \in \mathcal{F}$ *such that* $f \succeq_g g$, *then* Π *and* $\hat{\Pi}$ *are unique and* u *is unique up to a positive affine transformation.*

The interpretation of this representation is very similar to that of Theorem 2. In fact, Theorem 3 extends Theorem 2 just like Gilboa and Schmeidler (1989) extends the classical Anscombe-Aumann representation theorem. Here, our agent has two sets of priors, instead of a set and a single one. The first set, larger, she uses to decide whether to keep the status quo or not; and the second, smaller and in the relative interior of the first, she uses in the case of status quo option, or between elements both dominating the status quo. This relative position of the two sets leads immediately to the results discussed in Section 3: the presence of a status quo implies that the set of priors get *strictly larger*, which we well know leads to strictly more uncertainty aversion. And just like in the previous case, the latter being in the relative interior of the former means that there is an increase in uncertainty aversion in all "directions": the agent enlarges the possibilities she considers - her pessimism - in all possible dimensions.

5. Application: Risk premium with risk neutral agents

We now turn to apply our representations to the analysis of a simple financial choice. Our objective is to show that the decision making model captured by our Axioms entails the presence of risk premia in a financial market with *risk neutral* participants. As it is well known, if a market consists only of risk neutral standard expected utility maximizing agents, a risk premium cannot possibly arise. By contrast, we will show below that this is no longer the case when agents have a status quo bias, even though they might be *ambiguity neutral without a status quo.* We will argue that it is the very presence of a status quo option that lead to the emergence of such risk premia. To do so, we will make use of the results in Theorem 2, the relevant model for such enterprise.

Consider an economy in which there is one representative agent, a government and a firm. There are two possible states of the world: s_1, the "good state", and s_2, the "bad state." The government issues a bond, which is traded for the price p_b and yields, with certainty, $\$B$. The firm can issue a stock, priced at price p_{st}, which yields payoffs $\$M$ and $\$m$, respectively, in the two states of the world, where $m < B < M$. The representative agent can choose whether to buy a stock, a bond, or not to invest. To keep the analysis simple, we assume that only one of these three options can be taken.

More formally, define $S := \{s_1, s_2\}$ and $X := \mathbb{R}$. We focus on the preferences on three acts: buy the stock, st; buy the bond, b; keep the money uninvested, ni. For any given M, B, m, p_{st}, p_b, define these acts as: $b(s) := B - p_b$ for all $s \in S$; $ni(s) = 0$ for all $s \in S$; $st(s_1) := M - p_{st}$ and $st(s_2) := m - p_{st}$. Assume that the preferences of the agent in this setup satisfies Axioms 1-5, 7-8. Then, there exist a utility function u on X, a single prior ρ, and a full dimensional set of priors Π that rationalize her preferences in the manner described in Theorem 2. Define the agent's utility function on X as simply $u(x) := x$, which guarantees her risk neutrality. Moreover, assume that her status quo or default option is *not to invest in the market.*[10]

We also assume that in the economy there are market analysts who agree on a probability distribution $\hat{\pi}$ on S (on the basis of, say, a suitable regression analysis), with full support. Let us impose, again for simplicity, that $M \cdot \hat{\pi}(s_1) + m \cdot \hat{\pi}(s_2) = B$, that is, the expected payoff of the stock and of the bond are the same with respect to the probability distribution declared by the analysts. This information is common knowledge in the market, and we assume, for illustrative purposes, that $\rho = \hat{\pi}$. (That is, without a status quo, the agent uses the prior suggested by the analysts.) Finally, consider the set Π of priors, which we know to be not a singleton, and define $\underline{\pi}$ as the prior in Π which assigns the smallest probability to state s_1. Again for illustrative purposes, assume that $B > p_b > M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$.[11] We will now analyze the choice of the agent in this environment.

First of all, since $B > p_b$, buying the bond is certainly better then leaving the money uninvested. Therefore, the agent would never choose to leave the money uninvested if the bond was available. Notice also that, if the stock has the same price as the bond, the agent would *not* buy it. This is because, if $p_{st} = p_b$, we have $p_{st} = p_b > M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$, which means that the expected return of buying the stock computed using prior $\underline{\pi}$ is negative, hence worse than the default option of leaving the money uninvested. Therefore, since the stock is worse than the status quo for at least one prior in the set Π, our model prescribes that it will not be chosen by the agent. Instead, she would then buy the bond.

[10]This seems to be the most natural choice in this environment. Note, however, that we would get the same result if the status quo were to invest in the bond.

[11]Notice that it is always possible to find p_b that satisfies this condition, since the set Π of priors is full-dimensional and $\hat{\pi}$ lies in the relative interior of Π.

This implies that if the stock is traded in the market, its price must be below that of the bond, and in particular, below $M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$. If this is the case, then the agent buys only the stock (and not the bond). The implication of this is that the stock must be priced below the price of the bond for it to be sold - although the bond and the stock have the same expected payoff according to the market analysts and "according to the agent" when she has no status quo.

Let us now compute the risk premium in this economy. First, notice in this case that the risk-free rate is equal to the expected return of the bond, that is $r_b := \frac{B-p_b}{p_b}$. At the same time, the expected return of the stock according to the market is

$$r_s := \frac{M\hat{\pi}(s_1) + m\hat{\pi}(s_2) - p_t}{p_t} = \frac{B - p_t}{p_t}.$$

Since $p_s < p_b$, we then have $r_s > r_b$. This means that there is *positive risk premium* in this economy, *even though the agent is risk neutral.* Obviously, such a case cannot materialize with "standard" expected utility maximizing agents.

Notably, a risk premium emerges here in a situation where the status quo "do not invest" is never chosen, whatever the price of the stock and the set of priors are. This underlines an important feature of our model. The presence of the status quo might affect the final choice of a decision maker even though it is not itself chosen. Hence, we might have a role for the status quo "do not invest" also in more realistic situations, in which many people do actually choose to invest. Of course, the same qualitative effect would have been found if the status quo were buying the bond. Also, notice that this risk premium is larger the "larger" the set of priors is. In particular, it depends on the worst possible prior in the set Π. The more "pessimistic" the agent is, that is, the more she is scared of moving away from the status quo to an uncertain alternative, the higher the risk premium will be in the market.

Moreover, notice that, if an external observer (economist) studied this market, but disregarded the role of the status quo by using the standard expected utility model, she would erroneously deduce that the agent is risk averse. In particular, it is easy to see that if the agent were really risk averse, then the risk premium would be even higher, owing both to risk aversion and status quo bias. Consequently, if the external observer disregarded the role of the latter, then she would attribute to the agent a much higher, possibly implausible, level of risk aversion. This situation is not unfamiliar to the macro-finance literature, where extremely high levels of risk aversion are required to justify the risk premium observed in financial markets; this is dubbed the *equity premium puzzle.* We see here that our choice theory provides an easy solution to this puzzle by deriving it as a behavioral consequence of the status quo bias phenomenon.

This result is similar to the ones found by assuming ambiguity aversion in the sense of Gilboa and Schmeidler (1989), or robustness in the sense of Hansen and Sargent (2000, 2001), or incompleteness in the sense of Bewley (1986).[12] Indeed, the models developed

[12]For the case of ambiguity aversion, see Chen and Epstein (2002) and Epstein and Wang (1994). For the case of robust control, see Barillas et al. (2007). Notice, however, that the models based on ambiguity aversion do not offer a term of comparison. This is not the case here, since in our characterization, as well as in that of robustness, a unique prior is singled out as a natural term of comparison.

in these papers would also yield a positive risk premium in this environment with risk neutral agents.[13] And, just as it is here, this is due to the presence of a set of priors, and of a certain degree of pessimism. Our contribution is to show that this explanation could be motivated by a completely different set of postulates. In particular, we show the phenomenon may be caused by the status quo bias of the agents, even if the agent were ambiguity neutral without a status quo, thereby further strengthening the argument in favor of a multi-prior explanation of the observed risk premia.

Finally, it is worth mentioning that our model can be similarly used to show that the presence of a status quo might induce the existence of a range of prices for which agents take neither a long nor a short position on a stock. This is similar to the results found by Dow and da Costa Werlang (1992) - albeit, again, it would be obtained here by means of the status quo bias phenomenon instead of ambiguity aversion.

6. Conclusions

In this paper we have developed, axiomatically, three revealed preference models that characterize a system of preference relations under uncertainty with multiple status quibus. In all of them, the agent acts as if she had one von-Neumann-Morgenstern utility function and multiple priors. She needs the weak dominance for all of her priors (in the sense of expected utility), and strict for at least one, in order to prefer something to her status quo option. This model combines the classical Knightian Uncertainty model with the status quo bias phenomenon, and allows us to draw a connection between status quo bias and ambiguity aversion. In addition, this model may be applied to the study of investment decisions and might offer an insight to phenomena like the equity premium puzzle.

Among the possible extensions of the model we have presented here, a natural candidate would be a theory that is able to derive *endogenously* the status quo alternatives, without requiring these to be observable at the outset. However, it may not be best to inquire into such an extension in our specific context. First, the problem of choice of the status quo element is inherently dynamic, hence a dynamic model, as opposed to the static one developed here, would presumably be required. Second, and more important, the status quo bias case is only one among the many possible cases of reference dependence, and among them is possibly the one in which the reference point is more likely to be observable. If we are interested in a theory of endogenous reference point, it might thus be better not to focus on the special case of the status quo bias, but rather on a more generic situation of reference dependence.[14]

[13]As we have seen, although the same risk premium may arise, but the behavior might be different. In particular, concerning the choice between elements that both dominate the status quo, our characterization is such that the comparison is done according to a unique prior, while both ambiguity aversion and robustness would induce the use of multiple priors in this case as well. If we used incomplete preferences a' la Bewley (1986), given the connections between these two works, we would have something very similar. But as we have argued before, incomplete preference would have a (structural) limited ability in helping us to model behavior, since they would not tell us what would be chosen if two elements were to dominate the status quo.

[14]See Ok et al. (2007) for more on this matter.

Appendix: Proofs

Proof of Theorem 1

We consider first the "only if" part of the assertion. Let $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ be a consistent system of preferences that satisfies Axioms 1-5. For any $f \in \mathcal{F}$ and preorder $\succeq$ on $\mathcal{F}$, denote by $U_\succ$ the strict upper contour set of f with respect to $\succeq$, that is, $U_\succ(f) := \{g \in \mathcal{F} : g \succ f\}$. Moreover, for any probabiliy vector π on S and functional $u : X \to \mathbb{R}$, define $U_{\pi,u} : \mathcal{F} \to \mathbb{R}$ as

$$U_{\pi,u}(f) := \sum_{s\in S} \pi(s)\mathbb{E}_{f(s)}(u),$$

where $\mathbb{E}_{f(s)}(u)$ denotes the expected value of u with respect to the Borel probability measure $f(s)$ on X.

Claim 1. *There exists a partial order $\succeq$ such that $\succeq_\diamond$ completes $\succeq$ and*

$$f \succeq_h g \quad \Leftrightarrow \quad \begin{array}{c} \big[f \succ h \ , \ g \not\succ h\big] \\ \text{or} \\ \big[f, g \succ h \ , \ f \succeq_\diamond g\big] \\ \text{or} \\ \big[f, g \not\succ h \ , \ f \succeq_\diamond g\big]. \end{array} \tag{7}$$

Proof. Standard practice would allow us to translate Lemma 1 in Masatlioglu and Ok (2005) to this case, simply defining the system of preference relations $\{\succeq_\diamond, \{\succeq_f\}_{f\in\mathcal{F}}\}$ as the only consistent one that rationalizes the choice correspondence that is the primitive of their analysis. Following the Lemma, we then obtain one partial order $\succ$ and one complete order $\succeq^*$ that rationalizes the choice without status quo. But clearly this must coincide with $\succeq_\diamond$, hence the result. □

It is easy to see that the partial order $\succeq$ here must be of the following form:

$$f \succ g \Leftrightarrow f \succ_g g.$$

Moreover, since $\succeq_\diamond$ is continuous, standard arguments and Axiom 2 guarantee that there exist a continuous function $v : \mathcal{F} \to \mathrm{R}$ that represents $\succeq_\diamond$.

We now characterize the incomplete preference relation $\succeq$. In order to do this, define $\succeq'$ as $\succ' := \succ$ and $\sim' := \{(f, g) \in \mathcal{F}^2 : f(s) \sim_\diamond g(s) \text{for all s} \in S\}$.

Claim 2. *$\succeq'$ is a preference relation.*

Proof. Notice that all we have to show is that $\succeq'$ satisfies transitivity. Notice also that it is an incomplete relation that is derived by adding elements to the incomplete preference relation $\succ$: all we have to show, then, is that adding this additional relations still keeps the transitivity. Since the relations we are adding are all of indifference, and the original ones are all strict, we have to show only two things. First, that if $f \succ' g$ and $g \sim' h$, then $f \succ h$, hence $f \succ' h$: to see it, notice that $f \succ' h$ must derive from $f \succ h$; but then Axiom 1 tells us that $f \succ h$, hence $f \succ' h$. Same goes for showing that $f \succ' g$ and $f \sim' h$, then $h \succ g$, and this proves transitivity. □

Now, it is immediate to see that $\succ' \neq \emptyset$ (from the fact that $\succ_\diamond \neq 0$ and Axiom 1), that $\succeq'$ satisfies Monotonicity (from Axiom 1), Upper and Lower Hemicontinuity (from Axiom 2), Independence (from Axiom 3). We shall now show that $\succeq'$ satisfies the Partial Completeness Axiom. That is, if $p \in \mathcal{F}$ is the constant act returning $p \in \Delta(X)$ in all states, the induced preference relation $\bar{\succeq}$ on $\Delta(X)^2$ defined by $p \bar{\succeq} q \Leftrightarrow p \succeq' q$ for all $p, q \in \Delta(X)$, is complete and $\bar{\succ} \neq \emptyset$.

Claim 3. *$\succeq'$ satisfies the Partial Completeness Axiom. Moreover, $\bar{\succ} \neq \emptyset$.*

Proof. Take $p, q \in \Delta(X)$. Notice that, since $\succeq_\diamond$ is complete, then either $p \succ_\diamond q$, or $q \succ_\diamond p$ or $p \sim_\diamond q$. If we are in one of the first two cases, then Axiom 2 and the representation of $\succeq_\diamond$ tell us that we have $p \succ' q$ or $q \succ' p$. Else, if $p \sim_\diamond q$, then by definition of $\succeq'$ we have $p \sim' q$. Hence, $\succeq'$ is complete on unambiguous acts, and so $\bar{\succeq}$ is complete. Finally, notice that since $\succ_\diamond \neq \emptyset$, then there exists $p, q \in X$ such that $p \succ_\diamond q$, which means that $\bar{\succ} \neq \emptyset$. □

We then can apply Bewley's Expected Utility Therem to characterize $\succeq'$.

Claim 4. *There exists a unique nonempty convex compact set Π of probability vectors on S and a continuous utility function $u : X \to \mathbb{R}$ such that, for all $f, g \in \mathcal{F}$,*

$$f \succeq' g \Leftrightarrow U_{\pi,u}(f) \geq U_{\pi,u}(g) \quad \textit{for all } \pi \in \Pi.$$

Proof. Notice that $\succ'$ satisfies all requirements of Bewley's Expected Utility Theorem, applying which proves the result. □

We have therefore a characterization of the preference relation $\succeq$. Recall the definition of $\mathcal{D}_{\Pi,u}(f)$ for $f \in \mathcal{F}$, $\mathcal{D}_{\Pi,u}(f) := \{g \in \mathcal{F} : U_{\pi,u}(g) \geq U_{\pi,u}(f)$ for all $\pi \in \Pi$ strictly for some$\}$. Notice that, given the characterization of our preference relation, then $U_{\succ}(f) = \mathcal{D}_{\Pi,u}(f)$ for all $f \in \mathcal{F}$. By using this last result, our characterization of $\succeq_\diamond$ and Lemma 1, we get to the characterization in the Theorem.

We will now prove that the set Π is full dimensional in the simplex of $\mathbb{R}^{|S|}$, i.e. $\dim(\Pi) = |S| - 1$. The following notation is required. Given $u : X \to \mathbb{R}$, define $v : \Delta(X) \to \mathbb{R}$ as $v(x) := \mathbb{E}_x(u)$.

Claim 5. $\dim(\Pi) = |S| - 1$.

Proof. Say, by contradiction, that this is not the case, which means, since Π is a subset of the simplex of $\mathbb{R}^{|S|}$, that $\dim(\Pi) < |S| - 1$. Notice that this implies that there does not exist $|S|$ linearly independent element in Π, which in turn means that there exists $x \in \mathbb{R}^{|S|}$, $x \neq 0$ such that $x \cdot \pi = 0$ for all $\pi \in \Pi$ [15]. This implies that for all $\lambda \in \mathbb{R}$, then $\lambda x \cdot \pi = 0$ for each $\pi \in \Pi$. Notice that without loss of generality we can say that $x_s \in [-1, 1]$ and that $u(X) = [-1, 1]$. Now, consider two acts $f, g \in \mathcal{F}$ such that $v(f(s)) = x_s$ and $v(g(s)) = \frac{1}{2} x_s$ for all $s \in S$. Since, $x \neq 0$, then $f \neq g$. Since $x \cdot \pi = 0$ for all $\pi \in \Pi$, notice that we must have $U_{\pi,u}(f) = 0 = U_{\pi,u}(g)$. Following the characterization of $\succeq'$ in Claim 4, this implies $f \sim' g$. But this clearly contradicts the definition of $\succeq'$. □

We now only need to prove that there exist a (point-wise) continuous function $\psi : \mathbf{U}_{\Pi,u}(\mathcal{F}) \to \mathbb{R}$ that is maximized in the choice without status quo. First of all, notice that, since $\succeq_\diamond$ completes $\succeq'$ and the latter satisfies standard monotonicity and partial completeness, so does the former. Moreover, notice that, by standard arguments, there exists a function $h : \mathbb{R}^S \to \mathbb{R}$ such that

$$v(f) = h((u(f(s))_{s \in S}).$$

Moreover, continuity of v and u guarantee continuity of h, and monotonicity of $\succeq_\diamond$ guarantee that h is strictly increasing. Now, notice further that we can define another (continuous) function $m : \mathbb{R}^S \to \mathbf{U}_{\Pi,u}(\mathcal{F})$ which is an injection: this is possible thanks to the full dimensionality of Π. Now define $\psi : \mathbf{U}_{\Pi,u}(\mathcal{F}) \to \mathbb{R}$ as $\psi(x) := h(m^{-1}(x))$, which is well-defined since m is an injection. Now notice that we must that for any $f \in \mathcal{F}$, $\psi(\mathbf{U}_{\Pi,u}(f)) = h(m^{-1}(\mathbf{U}_{\Pi,u}(f))) = h((u(f(s))_{s \in S}) = v(f)$. Also, notice that if we have f, g such that $\mathbf{U}_{\Pi,u}(f) > \mathbf{U}_{\Pi,u}(g)$, then we must also have that $m^{-1}(\mathbf{U}_{\Pi,u}(f)) = (u(f(s)))_{s \in S} > (u(g(s)))_{s \in S} = m^{-1}(\mathbf{U}_{\Pi,u}(f))$ (again also thanks to the full dimensionality of Π), which,

[15] The reason for this is that, if there were at least $|S|$ linearly independent vectors in Π, then the system of equations $x \cdot \pi = 0$ for all $\pi \in \Pi$ will admit only one solution, $x = 0$. But since this is not the case, standard arguments in linear algebra points out that there is at least another non-zero solution.

since h is strictly increasing, implies $\psi(\mathbf{U}_{\Pi,u}(f)) > \psi(\mathbf{U}_{\Pi,u}(g))$. We can then conclude that ψ is strictly increasing.

We only need to show that ψ is (point-wise) continuous. To prove it, consider a sequence $(x_n) \in \mathbf{U}_{\Pi,u}(\mathcal{F})^\infty$, $x \in \mathbf{U}_{\Pi,u}(\mathcal{F})$ such that $x_n \to x$ point-wise. We need to show that $\psi(x_n) \to \psi(x)$. Now, notice that the image of ψ must be compact by compactness of $\mathcal{F}$ and continuity of h. Consider therefore a convergent subsequence $\psi(x_m)$. Also, consider a sequence $(f_m) \in \mathcal{F}^\infty$ such that $x_m = \mathbf{U}_{\Pi,u}(f_m)$. Since $\mathcal{F}$ is compact, there exist a convergent subsequence $f_k \to f$ for some $f \in \mathcal{F}$. But notice that, for continuity of u, we must have $u((f_k(s))_{s\in S} \to u((f(s))_{s\in S}$ point-wise. But then, we must have $\mathbf{U}_{\Pi,u}(f) = x$, and by continuity of h and the definition of ψ we are done.

This concludes the proof of the "only if" direction. The uniqueness properties of our representation are directly inherited from those in Bewley (1986), the representation of which we have used to prove our result. We now turn to the if direction. The proof of Axioms 1, 3, 4 is either standard or trivial: we are then left with Axioms 2 (Continuity) and Axiom 5 (Status Quo Bias). As for Continuity, continuity of u and ψ and standard arguments guarantee part (a). Now consider part (b). Take $(g^n) \in \mathcal{F}^\infty$, $g^n \to g$, $g^n \succeq_f f$ for all n, and such that we do not have $f \trianglerighteq g$. First notice that, if we had $g \trianglerighteq f$, then Axiom 1 would guarantee $g \succeq_f f$, and we would be done. Assume this is not the case, i.e. we do not have $g \trianglerighteq f$. Notice that, for all n, $g^n \succeq_f f$ implies $U_{\pi,u}(g^n) \geq U_{\pi,u}(f)$ for all $\pi \in \Pi$, strictly for some. Hence, $g^n \to g$ implies $U_{\pi,u}(g) \geq U_{\pi,u}(f)$ for all $\pi \in \Pi$ for continuity of u. Now, if this inequality holds strictly for some $\pi \in \Pi$, then $g \succeq_f f$ and we are done. We will now show that this must be the case. Say not, hence we have $U_{\pi,u}(g) = U_{\pi,u}(f)$ for all $\pi \in \Pi$. This means that $\sum_{s\in S} \pi_s(v(g(s) - v(f(s)) = 0$ for all $\pi \in \Pi$. But this cannot happen because $v(g(s)) - v(f(s)) \neq 0$ (since we do not have $f \trianglerighteq g$ and $g \trianglerighteq f$) and $\dim(\Pi) = |S| - 1$ implies that the only $z \in \mathbb{R}^{|S|}$ such that $\sum_{s\in S} z_s\pi_s = 0$ for all $\pi \in \Pi$ is $z = 0$. For part (c), consider any $f, g \in \mathcal{F}$, $(f^n) \in \mathcal{F}^\infty$, $f^n \to f$, $g \succ_{f^n} f^n$ for all n, and we do not have $f \trianglerighteq g$. Notice again that if we had $g \trianglerighteq f$ the claim would follow from Axiom 1, so assume this is not the case. Then, we must have $U_{\pi,u}(g) \geq U_{\pi,u}(f^n)$ for all n and for all $\pi \in \Pi$, strictly for some, which implies $U_{\pi,u}(g) \geq U_{\pi,u}(f)$ for all $\pi \in \Pi$. Again, if there exist a $\pi \in \Pi$ such that the inequality holds strictly we are done, since it means that $y \in c(\{x, y\}, x)$. As we did for part (b), we shall now prove that this must be the case: say not, then we have $\sum_{s\in S} \pi_s(v(g(s) - v(f(s)) = 0$ for all $\pi \in \Pi$, and that both $f \trianglerighteq g$ and $g \trianglerighteq f$ are false. We have already shown that this is a contradiction.

As for Status Quo Bias, suppose that we have $g \succeq_f f$. If $f = g$ the claim is trivial. For $f \neq g$, we want to show that $g \succ_g f$. Assume, by contradiction, that $f \succeq_g g$. This implies that $f \in \mathcal{D}_{\Pi,u}(g)$; at the same time, $g \succeq_f f$ implies $g \in \mathcal{D}_{\Pi,u}(f)$. But this is clearly impossible given the definition of $\mathcal{D}_{\Pi,u}$, since $f \neq g$. Finally, consider $f, g, \in \mathcal{F}$ such that $g \succeq_\diamond f$. We need to show that $g \succ_g f$. Say, by contradiction, that $f \succeq_g g$. Notice that this implies $f \in \mathcal{D}_{\Pi,u}(g)$, hence $U_{\pi,u}(f) \geq U_{\pi,u}(g)$ for all $\pi \in \Pi$, and $U_{\bar{\pi},u}(f) > U_{\bar{\pi},u}(g)$ for some $\bar{\pi} \in \Pi$. Since ψ is strictly increasing, this implies $f \succ_\diamond g$, a contradiction.

This concludes the proof of Theorem 1. *Q.E.D.*

Proof of Proposition 1

For any $\succeq \subseteq \mathcal{F} \times \mathcal{F}$, $f \in \mathcal{F}$, define UCS$(\succeq, f)$ as the upper contour set of $\succeq$ at f. Consider a generic $h \in \mathcal{F}$, $\alpha \in (0, 1)$, $f, g \in \mathcal{F}$ such that $f \succeq_\diamond g$, $f \succeq_h g$. Notice that we need to show that $\alpha f + (1 - \alpha)g \in UCS(\succeq_\diamond, g)$ implies $\alpha f + (1 - \alpha)g \in UCS(\succeq_h, g)$. Say first that $g \in U_h$. Since $f \succeq_h g$, by construction of $\succeq_h$ we must have $f \in U_h$. By Theorem 1 and by construction of $\succeq_h$, we know that $UCS(\succeq_h, g) = UCS(\succeq_\diamond, g) \cap (D_{\Pi,u}(h) \cup \{h\})$ for some u, Π constructed appropriately. But notice that we must have $U_h = D_{\Pi,u}(h)$, and therefore $f, g \in D_{\Pi,u}(h)$. The latter set being convex, in turns, implies $\alpha f + (1 - \alpha)g \in D_{\Pi,u}(h)$. But then, $\alpha f + (1 - \alpha)g \in UCS(\succeq_\diamond, g)$ implies $\alpha f + (1 - \alpha)g \in UCS(\succeq_\diamond, g) \cap D_{\Pi,u}(h)$ as sought. *Q.E.D.*

Proof of Proposition 2 and 3

To prove Proposition 2, consider u, Π, ψ as defined in Theorem 1. For any $\succeq \subseteq \mathcal{F} \times \mathcal{F}$, $f \in \mathcal{F}$, define UCS$(\succeq, f)$ as the upper contour set of $\succeq$ at f. Consider any $f, h \in \mathcal{F}$, $x \in X$ such that $x \in UCS(\succeq_\diamond, f)$. We need to show that we have $x \in UCS(\succeq_h, f)$ if h is a constant act. Say first that we have $f \notin D_{\Pi,u}(h)$. Then, if $x \in D_{\Pi,u}(h)$, then $x \succ_h f$ by construction of $\succeq_h$; if $x \notin D_{\Pi,u}(h)$, since we have imposed that $\succeq_h$ and $\succeq_\diamond$ must agree among the elements that are never chosen when h is present, then again we must have $x \succeq_h f$. If $f \in D_{\Pi,u}(h)$, then by Theorem 1 and construction of $\succeq_h$, we know $UCS(\succeq_h, f) = UCS(\succeq_\diamond, f) \cap D_{\Pi,u}(h)$. So, we only need to show that $x \in D_{\Pi,u}(h)$ and we are done: since both x and h are constant acts, we only need to show $\mathbb{E}_x[u] > \mathbb{E}_h[u]$. Now, say by contradiction that we have $\mathbb{E}_x[u] \leq \mathbb{E}_h[u]$. Since we have $f \in D_{\Pi,u}(h)$, then it must be that $U_{\pi,u}(f) \geq U_{\pi,u}(h)$ for all $\pi \in \Pi$, strictly for some. At the same time, since we have assumed $\mathbb{E}_x[u] \leq \mathbb{E}_h[u]$, we must have $U_{\pi,u}(f) \geq U_{\pi,u}(h) \geq U_{\pi,u}(x)$ for all $\pi \in \Pi$, where the first inequality is strict for some $\pi \in \Pi$. But this means that we must have $\psi(\mathbb{U}_{\Pi,u}(f)) > \psi(\mathbb{U}_{\Pi,u}(x))$, since ψ is strictly increasing. But this contradicts $x \succeq_\diamond f$, proving the symmetric part of the comparative ambiguity aversion. The proof of the asymmetric part is a trivial consequence. For any $f, h \in \mathcal{F}$, $x \in X$, we have just argued that, if $x \succeq_\diamond f$ and $f \in D_{\Pi,u}(h)$, then $x \in D_{\Pi,u}(h)$. But Theorem 1 shows that $\succeq_h$ and $\succeq_\diamond$ agree among the elements within $D_{\Pi,u}(f)$, and we have assumed that they agree within the elements that are both outside of it: therefore we have the asymmetric part as well.

To prove Proposition 3, take $h \in \text{int}(\mathcal{F})$, non-constant. For any $\epsilon > 0$, consider, if it exists, any act f_ϵ such that $u(f_\epsilon(s)) - u(h(s)) = \epsilon$ (the existence of such f_ϵ for ϵ small enough is guaranteed by the fact that $h \in \text{int}(\mathcal{F})$). Notice that we must obviously have $f_{\bar{\epsilon}} \succ_h h$ by monotonicity. Now consider the constant-equivalent of f_ϵ and h under $\succeq_\diamond$, i.e. $x_\epsilon, h \in X$ such that $x_\epsilon \sim_\diamond f$ and $y \sim_\diamond h$ (the existence of which is guaranteed by the continuity of $\succeq_\diamond$). We will now argue that, for ϵ small enough, $x_\epsilon \notin D_{\Pi,u}(h)$. Notice first of all that $y \notin D_{\Pi,u}(h)$, since we have $y \sim_\diamond h$, and hence $\psi(\mathbb{U}_{\Pi,u}(h)) = \psi(\mathbb{U}_{\Pi,u}(y))$, which in turns implies, since h is not-constant, y constant and Π full dimensional, that there must be $\pi \in \Pi$ such that $U_{\pi,u}(h) > U_{\pi,u}(y)$.[16] Therefore, $y \notin D_{\Pi,u}(h)$. Now notice that $D_{\Pi,u}(h) \cup \{h\}$ must be an open set, and therefore, since $u(x_\epsilon(s)) \to u(y(s))$ for all $s \in S$ as $\epsilon \to 0$, we must have that $x_{\bar{\epsilon}} \notin D_{\Pi,u}(h)$ for some $\bar{\epsilon}$ small enough. But then, we have $f_{\bar{\epsilon}}, h \in \mathcal{F}$, $x_{\bar{\epsilon}} \in X$ such that $x_{\bar{\epsilon}} \sim_\diamond f_{\bar{\epsilon}}$ but $f_{\bar{\epsilon}} \succ_h h \succ_h x_{\bar{\epsilon}}$, giving us the desired result. *Q.E.D.*

Proof of Proposition 4

To show (1) $\Rightarrow$ (3), notice that, for any $f \in \mathcal{F}$, the upper-contour set of $\succeq_f$ at f is strictly convex (since anything that is preferred to f must be return a higher expected utility for a full-dimensional set of priors). This means that the preference must be strictly convex there, which in turns means, by coherence, that it must be weakly convex everywhere, leading immediately to (3). The reverse, (3) $\Rightarrow$ (1), is trivial, since any uncertainty averse preference is coherent. To prove (3) $\Rightarrow$ (4) it is also trivial: if $\succeq_\diamond$ is uncertainty averse, we are done. We only need to show (4) $\Rightarrow$ (3). Notice that, by Theorem 1, it is immediate to see that, for any $f \in \mathcal{F}$, we must have UCS$(\succeq_f, f) =$ UCS$(\succeq_\diamond, f) \cap D_{\Pi,u}(f)$ (where Π and u are chosen as in Theorem 1, $UCS(\succeq, f)$ is defined as in the proof of Proposition 1). Notice that, since $D_{\Pi,u}(f)$ is clearly convex and UCS$(\succeq_\diamond, f)$ is convex by (4), then UCS$(\succeq_f, f)$ is convex, which immediately leads to the result.

We now turn to prove the equivalency of (2). (1) $\Rightarrow$ (2) is trivial. To conclude the proof, we now show that (2) $\Rightarrow$ (4). Assume (2) and say, by means of contradiction, that (4) does not hold, which means that there are $f, g \in \mathcal{F}$ s.t. $f \sim_\diamond g \succ_\diamond \frac{1}{2}f + \frac{1}{2}g$. Since $\succeq_\diamond$ is coherent, then it must be uncertainty loving. By same arguments used above, it is trivial to show that if for some $f \in \text{int}(\mathcal{F})$ s.t. $\succeq_f$ is coherent, then $\succeq_f$ must be uncertainty averse. Since $\succeq_\diamond$ is uncertainty loving and $\succeq_f$ is uncertainty averse, then they must be uncertainty neutral when restricted to all the convex subsets of $\mathcal{F}$ in which they coincide. Define by

[16]The reason is, since h is not-constant, y constant and Π full dimensional, $\psi(\mathbb{U}_{\Pi,u}(h)) = \psi(\mathbb{U}_{\Pi,u}(y))$ can be true only if there are $\pi, \pi' \in \Pi$ such that $U_{\pi,u}(h) > U_{\pi,u}(y)$ and $U_{\pi',u}(h) < U_{\pi',u}(y)$.

$\text{ext}(\Pi)$ the set of extreme points of Π, and define the sets $A := \{g \in \mathcal{F} : g \succeq_f f\}$, $B_0 := \{g \in \mathcal{F} : f \succeq_\diamond g\}$ and, for all $\pi \in \text{ext}(\Pi)$, $B_\pi = \{g \in \mathcal{F} : U_{\pi,u}(g) > U_{\pi,u}(f)$ and $U_{\pi',u}(f) > U_{\pi',u}(g)$ for some $\pi' \in \Pi\}$. Notice that all these sets are clearly convex, open, and full-dimensional, since $f \in \text{int}(\mathcal{F})$. Moreover, since ϕ is strictly increasing and Π is full dimensional, then for any $\pi \in \text{ext}(\Pi)$, B_π must have a full dimensional intersection with B_0.

For any preference relation $\hat{\succeq}$ on $\mathcal{F}$ and any set $A \subseteq \mathcal{F}$, define $\hat{\succeq}|_A$ the restriction of $\hat{\succeq}$ on A. Now notice that by Theorem 1, $\succeq_f$ and $\succeq_\diamond$ must coincide when restricted to all those sets, and, as we have argued, they must therefore be linear. This means that we can find a set of priors $\rho_A, \{\rho_{B_\pi}\}_{\pi\in\Pi}$ such that $\succeq_\diamond |_i = \succeq_f |_i$ are represented by $U_{\rho_i,u}$ for $i \in \{A\} \cup \{B_0\} \cup \{B_\pi\}_{\pi\in\text{ext}(\Pi)}$. (To prove this, simply notice that we can extend linearly $\succeq_\diamond |_{B_i}$ and then use the standard Anscombe-Aumann results to prove the existence of these priors - this is standard practice). But also notice that, for every $\pi \in \text{ext}(\Pi)$, $\rho_{B_\pi} = \rho_{B_0}$, since has we have argued B_π and B_0 must have a full-dimensional intersection, and therefore the uniqueness of the prior in the representation forces them to be equal. But then, we have $\rho_{B_\pi} = \rho_{B_0}$ for all $\pi \in \text{ext}(\Pi)$. If we the define $B := (\cup_{\pi\in\text{ext}(\Pi)} B_\pi) \cup B_0$, we have therefore a prior $\rho = \rho_{B_0}$ on S such that $\succeq_\diamond |_B = \succeq_f |_B$ are represented by $U_{\rho,u}$. Let us now notice that also the set A is convex and full dimensional, and that we must have $\succeq_\diamond |_A = \succeq_f |_A$. Using the same arguments used above, it is trivial to show that there exists a prior ρ' on S such that the preferences restricted on A are both represented by $U_{\rho',u}$. We now argue that we must have $\rho = \rho'$. To see why, notice that we must have $\text{cl}(B) \cup A = \mathcal{F}$ (where by $\text{cl}(\cdot)$ we understand the closure of a set). Say that $\rho' \neq \rho$. Geometrically, this means that the preferences $\succeq_\diamond$ of the agent will be such that her indifferent curve will have kinks at many points in $A \cap \text{cl}(B)$, and this will render them strictly convex on some interval (around some kinks), strictly concave on others (around others). But we have argued that $\succeq_\diamond$ must satisfy ambiguity love, which means that its indifference curves cannot be concave anywhere. Hence $\rho' = \rho$. But then, $\succeq_\diamond$ must be ambiguity neutral everywhere, a contradiction of our initial assumption that there are $f, g \in \mathcal{F}$ s.t. $f \sim_\diamond g \succ_\diamond \frac{1}{2}f + \frac{1}{2}g$. *Q.E.D.*

Proof of Theorem 2 and 3

In what follows we will offer only a complete proof of Theorem 3, and not of Theorem 2: it is trivial to notice that the latter is nothing but a special case of the former, from which it can be proved using standard methods.

Notice first that Proposition 4 implies the equivalence between (1) and (2). We then only need to show the equivalence between (1) and (3). The proof of (3) ⇒ (1) is trivial given Theorem 1. To prove (1) ⇒ (3), follow the steps in the proof of Theorem 1 to define $\succeq$ and to characterize it. Notice that since $\succeq_\diamond$ completes $\succeq$ and that the latter is complete on constant acts, then they must agree on those. Moreover, since $\succeq$ satisfies standard monotonicity, then so must do $\succeq_\diamond$. Given Axioms 7 and 8, $\succeq_\diamond$ satisfies all the Axioms in Gilboa and Schmeidler (1989), which prove that there exist a closed and convex set of priors $\hat{\Pi}$ and a continuous function $v : X \to \mathbb{R}$ such that $\succeq_\diamond$ is represented by

$$V(f) = \min_{\pi\in\hat{\Pi}} U_{\pi,v}(g).$$

We now only need to prove that we have v is a positive affine transformation of u, and that $\hat{\Pi} \subset \text{ri}(\Pi)$ to conclude the proof. Notice that the uniqueness properties of u and Π derive from Theorem 1 and those of $\hat{\Pi}$ derive from the uniqueness in Gilboa and Schmeidler (1989) (since if we have $f \succeq_g g$ for some $f, g \in \mathcal{F}$, then we must also have $f \succ_\diamond g$, hence $\succeq_\diamond$ is non-degenerate).

Claim 6. *v is a positive affine transformation of u.*

Proof. Notice that, since $\succeq$ is complete on constant acts (Claim 3), then we must have that $\succeq$ and $\succeq_\diamond$ agree on those, which immediately gives the desired equality. □

Claim 7. $\hat{\Pi} \subseteq \Pi$.

Proof. Following Ghirardato et al. (2004), define as $\succeq_\diamond^*$ the unambiguously preferred preference relation from $\succeq_\diamond$, that is, the largest restriction of $\succeq_\diamond$ that satisfies vNM independence. Since $\succeq_\diamond^*$ is a monotone, affine and continuous preference relation, then it must have representation a' la Bewley, with a compact and convex set of priors $\tilde{\Pi}$ and utility u (which is the same utility used to represent $\succeq_\diamond$ since $\succeq_\diamond^*$ coincides with $\succeq_\diamond$ on constant acts). As shown by Ghirardato et al. (2004), we must have $\tilde{\Pi} = \hat{\Pi}$. Now, notice that, since $\succeq$ is also an affine restriction of $\succeq_\diamond$, then we must have that $\succeq_\diamond^*$ (weakly) completes $\succeq$ (since $\succeq_\diamond^*$ is defined as the largest affine restriction). But then, it is immediate to see that, since the utility is the same, we must have $\tilde{\Pi} \subseteq \Pi$, and therefore $\hat{\Pi} \subseteq \Pi$. □

Claim 8. $\hat{\Pi} \subset \mathrm{ri}(\Pi)$.

Proof. By contradiction, say that there exist $\rho \in \hat{\Pi}$ such that ρ is a boundary point of Π relative to aff(Π). Since Π is a convex subset of $\mathbb{R}^{|S|}$, then consider a non-trivial supporting hyperplane of Π at ρ whose norm vector $\bar{H}$ is such that $\bar{H} \in [-1, 1]$.[17] Now, notice that if $\alpha > 0$, there exists a $\lambda \in (0, 1)$ such that $\rho \cdot (\lambda\bar{H} + (1 - \lambda)[-1, \ldots, -1]^T) = 0$. Define H as $H := \lambda\bar{H} + (1 - \lambda)[-1, \ldots, -1]^T$.[18] So, we have H such that $\rho \cdot H = 0$, $\pi \cdot H \geq 0$ for all $\pi \in \Pi$, and $\bar{\pi} \cdot H > 0$ for some $\bar{\pi} \in \Pi$. Notice that we can assume, without loss of generality, that $u(X) = [-1, 1]$, and therefore we could find an act $h \in \mathcal{F}$ such that $\mathbb{E}_{h(s)}[u] = H_s$ for all $s \in S$. Also, define g a constant act that return utility zero in every state, that is, $\mathbb{E}_{g(s)}[u] = 0$ for all $s \in S$. Notice that, since we have $\pi \cdot H \geq 0$ for all $\pi \in \Pi$, and $\bar{\pi} \cdot H > 0$ for some $\bar{\pi} \in \Pi$, then we must have $h \succ g$. Notice also that, since $\hat{\Pi} \subseteq \Pi$, then we have $\pi \cdot H \geq 0$ for all $\pi \in \hat{\Pi}$. This means that, since $\rho \cdot H = 0$, then $\rho \cdot H = \min_{\pi \in \hat{\Pi}} \pi \cdot H$, which in turn implies $h \sim_\diamond g$. But this contradicts the fact that $\succeq_\diamond$ completes $\succ$, since we had $h \succ g$. □

This concluses the proof of Theorem 3. *Q.E.D.*

References

Ameriks, J. and S. P. Zeldes (2004): "How Do Household Portfolio Shares Vary with Age?" Discussion Paper, Columbia University.

Anscombe, F. J. and R. J. Aumann (1963): "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, 34, 199–205.

Barillas, F., L. Hansen, and T. J. Sargent (2007): "Doubts or Variability?" Mimeo, New York University.

Bewley, T. F. (1986): "Knightian Decision Theory: Part I," *Cowles Foundation discussion paper*, 807.

Chen, Z. and L. Epstein (2002): "Ambiguity, Risk and Asset Returns in continuous time," *Econometrica*, 70, 1403–1443.

Debreu, G. (1964): "Contuinity Properties of Paretian Utility," *International Economic Review*, 5, 285–293.

Dow, J. and S. R. da Costa Werlang (1992): "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio," *Econometrica*, 60, 197–204.

[17]The existence of it is guaranteed by convexity of Π and the fact that rint(Π) $\neq \emptyset$.

[18]If $\alpha < 0$ take the convex combination with $[1, \ldots, 1]^T$ instead of $[-1, \ldots, -1]^T$ and define H accordingly.

Epstein, L. G. and T. Wang (1994): "Intertemporal Asset Pricing under Knightian Uncertainty," *Econometrica*, 62, 283–322.

Ghirardato, P., F. Maccheroni, and M. Marinacci (2004): "Differentiating ambiguity and ambiguity attitude," *Journal of Economic Theory*, 118, 133–173.

Ghirardato, P. and M. Marinacci (2002): "Ambiguity Made Precise: A Comparative Foundation," *Journal of Economic Theory*, 102, 251–289.

Gilboa, I. and D. Schmeidler (1989): "Maxmin expected utility with a non-unique prior," *Journal of Mathematical Economics*, 18, 141–153.

Hansen, L. P. and T. J. Sargent (2000): "Wanting robustness in macroeconomics," Mimeo, New York University.

——— (2001): "Robust control and model uncertainty," *American Economic Review*, 91, 60–66.

Hartman, R., M. Doane, and C.-K. Woo (1991): "Consumer Rationality and the Status Quo'," *Quarterly Journal of Economics*, 106, 141–162.

Kahneman, D., J. Knetsch, and R. Thaler (1991): "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias," *Journal of Economic Perspectives*, 5, 193–206.

Maccheroni, F., M. Marinacci, and A. Rustichini (2006): "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica*, 74, 1447–1498.

Madrian, B. C. and D. F. Shea (2001): "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior," *Quarterly Journal of Economics*, 107, 1149–1187.

Masatlioglu, Y. and E. A. Ok (2005): "Rational Choice with status quo bias," *Journal of Economic Theory*, 121, 1–29.

——— (2008): "A Canonical Choice Model with Initial Endowment," Mimeo, New York University.

Ok, E. A., P. Ortoleva, and G. Riella (2007): "Rational Choice with Endogeneously Determined Reference Points," Mimeo, New York University.

Roca, M., R. M. Hogarth, and A. J. Maule (2006): "Ambiguity seeking as a result of the status quo bias," *Journal of Risk and Uncertainty*, 32, 175–194.

Sagi, J. (2003): "Anchored Preference Relations," University of California.

——— (2006): "Anchored Preference Relations," *Journal of Economic Theory*, 130, 283–295.

Samuelson, W. and R. Zeckhauser (1988): "Status quo bias in decision making," *Journal of Risk and Uncertainty*, 1, 7–59.

Strzalecki, T. (2007): "Axiomatic Foundations of Multiplier Preferences," Northwestern University.

Sugden, R. (2003): "Reference-dependent subjective expected utility," *Journal of Economic Theory*, 111, 172–191.

Tversky, A. and D. Kahneman (1991): "Loss Aversion in Riskless Choice: A Reference-Dependent Model," *Quarterly Journal of Economics*, 106, 1039–1061.

——— (1992): "Advances in prospect theory: cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323.