The Eventual Failure and Price Indeterminacy of Inflation Targeting

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Abstract

In stark contrast to the previous literature, we find that IT leads to price indeterminacy even when the central bank uses a Taylor-like feedback rule to peg the nominal interest rate. We also find that there is no mechanism with IT to determine the current inflation rate or price level. We conclude that the previous literature has either committed mathematical errors involving infinity or misused the non-explosive criterion for ruling out speculative bubbles. To avoid making errors involving infinity, we analyze inflation targeting (IT) in a typical rational-expectations, pure-exchange, general-equilibrium model where the time horizon is arbitrarily large, but finite.

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In the early 1960s, the Phillips curve was the fad in macroeconomics. However, in the late 1960s and the 1970s, changing expectations led to a breakdown in the Phillips curve. Today, the current fad in central banking is inflation targeting (IT). This paper argues that changing expectations could lead to a failure of IT in a manner similar to the breakdown in the Phillips curve.

Today many central banks follow some form of IT, including the Bank of England, the European Central Bank, and central banks in New Zealand, Canada, Australia and several developing countries. Also, Ben Bernanke, chair of the Federal Reserve Board, endorses committing the Federal Reserve to an inflation target. The definition of IT is that (i) the central bank target an inflation rate although it may have competing output-gap goals, (ii) the central bank be very committed to its stated targets and goals, and (iii) the central bank be very transparent in its goals and plans concerning monetary policy, including contingency plans. The transparency of the central bank under IT is considered very important for the formation of public expectations.

An alternative to IT is price-level targeting (PLT), where the central bank targets the price level instead of the inflation rate. IT and PLT are very similar. While IT has explicit inflation targets, we can derive implied price-level targets from those inflation targets. For example, if the central bank targets a 2% inflation rate forever and the current price level is 1.0, the implied path of price-level targets would be 1.02^t where t is the number of periods from now. Also, under PLT we can derive implied inflation targets from the explicit price-level targets. For example, if the path of price-level targets is 1.02^t, then the implied inflation targets would be 2%

each year. Under PLT, the central bank probably would communicate its intentions by announcing its implied inflation target rather than its path of price-level targets because the public is more likely to understand the former than the latter. Therefore, just because the public receives information on inflation targets does not mean the public understands that the central bank is following IT rather than PLT.

There is a subtle but very important difference between IT and PLT. This difference concerns the central bank's contingency plans; it concerns how the central bank reacts to when the actual inflation rate differs from its target. Under PLT, the price-level targets remain unchanged regardless of the past inflation rate, whereas under IT they do change. For example, assume that the current price level is 1.0 and the actual or implied inflation target is 2% per year forever. Then the actual or implied path of price-level targets under both IT and PLT would be 1.02^t. If the actual inflation rate over the first period turns out to be 3% instead of 2% , the implied path of price-level targets under IT would change to $1.03(1.02^{t-1})$ whereas under PLT, the price-level targets would remain the same. In other words, under PLT, if the price-level differs from its target, the central bank will take action to bring the price level back to the central bank's original path of price-level targets. However, under IT, the central bank is more forgiving and will respond to the higher price level by changing its implied price-level targets so to be consistent with its targeted inflation rate and the actual price level that just occurred. (Note: This paragraph does assume that output gap remains the same throughout these examples.)

In this paper, I argue that the contingency-plan difference between IT and PLT is very important to the issue about whether monetary policy can determine prices. Contrary to the previous literature, I argue that true IT cannot determine the price level of an economy when the central bank uses the nominal interest rate as its monetary instrument. [1] The stable price-level targets under PLT provides a solid anchor to the public's expectations of prices, at least if the central bank shows a strong commitment to those price-level targets as it would if it followed a strong McCallum-Woodford feedback policy rule for setting the nominal interest rate. However, under IT, the public will be unable to form expectations about future price levels because the central bank's implied targeted price levels will change whenever the actual price level differs from its target. This indeterminacy in expectations of future prices leads to an indeterminacy in the current price level.

Consider this scenario: Central banks throughout the world are following IT, but the general public is confused and thinks that the central banks are following PLT. Hence the public forms its expectations based on the principles of PLT. Initially, price levels are stable. However, over time the public will learn from experience about how the central banks handle situations where the actual inflation rate differs from its target and come to realize that the central banks are following IT instead of PLT. The public's expectations will shift as a result of this realization. If I am right that IT leads to price indeterminacy when the public understands IT, then this shift of expectations would bring instability to the price levels of the world's economies. Thus, the issue of whether or not IT determines prices could have profound implications to the world's economies.

In practice, IT is complicated by the fact that central banks following IT also have goals and objectives concerning output gap. However, economists often have to assume that "all other things remain the same." In particular, when this paper discussed the examples to illustrate the difference between IT and PLT, we assumed that output gap remained the same. In this paper,

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[1] Almost all central banks today conduct monetary policy by pegging nominal interest rates rather than by setting levels of monetary aggregates.

we want to study IT without having to worry about the complex issues of handling output gap. We can do so theoretically by using a flexible-price general equilibrium model. If the price level is perfectly flexible to move the economy immediately to equilibrium, then there never is any output gap. As a result, this paper studies IT in the context of a flexible-price general equilibrium model, not because we believe the price level is in fact that flexible, but rather so we can concentrate on the issue of price determinacy without being distracted by issues involving output gap.

A necessary though not sufficient condition for a system of equations to determine the unknowns of the system is that the number of equations be no less than the number of unknowns. Therefore, if the number of equations in a system is less than the number of unknowns, we know that we cannot determine all the values of the unknowns; in other words, some of the unknowns will be indeterminate.

Most of the previous literature on the price-determinacy of IT has concluded that IT does determine prices when the central bank follows a Taylor-like feedback rule for setting interest rates (e.g., Woodford, 2003, and Dittmar and Gavin, 2005). However, the previous studies of the price determinacy of IT have utilized infinite-horizon economic models. In his infinite-horizon model, Woodford (2003, p. 73) states, "I then have a system of two equations at each date, (1.15) and (1.21), to determine the two endogenous variables P_t and i_t ..." All that Woodford's statement means is there exists a one-to-one correspondence between the equations and the unknowns in his infinite-horizon economy. Most mathematicians should react to Woodford's statement with grave suspect as that statement is meaningless when time is infinite. Mathematicians will remember Galileo's establishing a one-to-one correspondence between the positive integers and their squares even though the set of squares is a proper subset of the positive integers.[2] Mathematicians will also remember the errors that people can make when working with infinity. For example, applying the associative law of addition with respect to the infinite sequence 1-1+1-1+1-1… would erroneously lead to the conclusion that 0=1 ().

Given that Woodford's statement is an indication of the possibility that the price-determinacy literature has been making an error involving infinity, this paper takes an approach that avoids any possibility of making logical errors involving infinity. This paper studies an economy with an arbitrarily large finite horizon and then takes the limit as that finite horizon goes to infinity. With a finite horizon, we can count the number of equations and unknowns to determine whether all the unknown price levels can be determined.

By carefully counting equations and unknowns, this paper reaches a conclusion that is in stark contrast to the existing literature. We find that the current price level cannot be determined in a finite-horizon model under IT when the central bank pegs the nominal interest rate. In the situations where we do find an equation containing the current inflation rate, we find only one such equation. That equation, which is the same whether the economy has a finite or infinite-horizon, reflects the central bank looking at the current inflation rate when it pegs the nominal interest rate. This paper argues that it is absurd to think that a mechanism where the current inflation rate <u>affects</u> the nominal interest rate is the mechanism that <u>determines</u> the price level.

The outline of the rest of this paper is as follows: Using a finite-horizon economic model, Section II presents the Fisher-Euler equation that is the basis for the price-determinacy literature. Then for a finite economy, section III analyzes PLT while Section IV analyzes "past IT." Section V extends the analysis to cover "current IT" and "expected IT". Section VI tries to anticipate some possible counterarguments and rebuts those counterarguments. Section VII invalidates the non-exploding criterion, which the previous price-determinacy literature has used

[2] See ???, p. ???.

as the basis for their analysis and conclusions. Section VIII summarizes this paper's conclusions and explains why IT does not determine the current price level.

II. The Fisher-Euler Equation

The price-determinacy literature primarily depends on two equations – the Fisher-Euler equation and a policy feedback rule. For a representative consumer without utility shocks, the Fisher-Euler equation states

$$\frac{U'(c_t)}{P_t} = R_t \beta E_t\left[\frac{U'(c_{t+1})}{P_{t+1}}\right] \qquad (1)$$

where β is the time discount factor. As does Carlstrom and Freust (2001), we define R_t to be the gross nominal interest rate from time t to time $t+1$. The gross nominal interest rate equals one plus the nominal interest rate. The Fisher-Euler equation (1) states that the marginal utility per "buck" today equals today's gross nominal interest rate times the expected marginal utility per "buck" tomorrow.[3]

One rational-expectations model that leads to (1) is the following: Assume a representative consumer[4] who maximizes

$$\sum_{s=t}^{T} \beta^s E_t\left[U(c_s)\right] \qquad (2)$$

subject to

$$M_t \geq P_t c_t \qquad (3)$$

[3] Carlstrom and Fuerst (2001) argue that (4) only applies for what they call CWID timing. However, for a CIA constraint where no money is held from one period to the next; (1) does apply.
[4] Just because I am using a representative consumer in this paper's model, does not mean that I condone its use. As Eagle and Domian (2005) show, having diverse consumers even with "aggregatable" utility functions gives a much more rich sense of the Pareto-efficiency involving those economies.

$$M_t + B_t = P_t y_t + B_{t-1}R_{t-1} + \left(M_{t-1} - P_{t-1}c_{t-1}\right) \tag{4}$$

where (3) and (4) hold for t=0,1,2,…T; and M_t is the money held at the beginning of the period, B_t is the amount of one-period bonds, and y_t is the representative consumer's endowment at time t. Equation (3) is a cash-in-advance (CIA) constraint and $M_{t-1} - P_{t-1}c_{t-1}$ is the amount of money held from period t-1 to period t. When the nominal interest rate is always positive, which means $R_t > 1$ for all t, the CIA constraint (3) holds with equality which implies that (4) can be written as:

$$P_t c_t + B_t = P_t y_t + B_{t-1}R_{t-1} \tag{5}$$

It is relatively elementary to maximize (2) with respect to (5) to get (1) as the first order condition.

Equation (1) can be justified in other ways as well such as including money in the utility function.[5] Also, Woodford (2003, p. 71) derives his equation (1.21), which is the same as (1) except that he does include utility shocks.

As is often done in the price-determinacy literature, assume a pure exchange economy without storage. Where Y_t is the aggregate endowment, this implies that $c_t = Y_t$. Also, for the sake of mathematical simplicity, assume that the representative consumer knows the future aggregate endowments. Then (1) can be transformed into the following:

$$\frac{U'(Y_t)}{P_t} = R_t \beta U'(Y_{t+1}) E_t\left[\frac{1}{P_{t+1}}\right] \tag{6}$$

When the central bank pegs R_t, equation (6) represents one of the mechanisms by which the current price level would be determined, if it is determined. If $E_t\left[\frac{1}{P_{t+1}}\right]$ is determined, then

[5] See Carlstrum and Fruest (2001).

(6) will determine P_t. See Eagle (2005) for an explanation how (6) reflects $E_t\left[\frac{1}{P_{t+1}}\right]$ and R_t together affecting nominal aggregate demand which then affects the price level.

III. PLT in a Finite-Horizon Economy

With a system consisting of a finite number of equations and unknowns, we know that if the number of equations is less than the number of unknowns, then it is impossible to determine values for all the unknowns. In such a situation, we say that some of the unknowns are indeterminate. However, with infinity, such simplicity is lost. For example, if we have a sequence of systems of n equations with n+1 unknowns for n=1,2,3,…; then each finite system is indeterminate. However, in the limit as n goes to infinity, both the number of equations and the number of unknowns go to infinity. In fact, in the limit there is a one-to-one correspondence of the number of equations to the number of unknowns. With an infinite number of equations and unknowns, we no longer can compare the number of equations with the number of unknowns.

Contrary to the way most economists think about such a sum, mathematicians define $\sum_{t=0}^{\infty} x_t \equiv \lim_{T\to\infty}\sum_{t}^{T} x_t$. In this paper, we take a similar approach to address the issue of the price determinacy. We study a rational expectations economy with a finite but arbitrarily large horizon. Let T be this finite horizon. We can then study the limit of this economy as T goes to infinity.

In the previous section, we discussed the Fisher-Euler equations upon which the previous literature bases its price-determinacy analysis. However, the previous literature has assumed infinite-horizon economies. We now deviate from the previous literature by assuming a finite horizon, where the last period of the economy is period T. The Fisher-Euler equation (1) and hence (6) then apply for periods 0,1,2,…,T-1. These equations do not apply for period T because there is no period T+1 for loans to be paid back. As a result, at time T there are no new loans or bonds and hence no interest rate.

We now discuss price-level targeting (PLT) so that we can see a situation where the price level is (technically) determined. Under PLT, the central bank has price-level targets for each period, which it makes transparent to the general public. Following the symbolization of Woodford (2003), define P_t^* to be the central bank's price-level target for time t. Following Bellman's Principle, we start our analysis at time T and work backwards. At time T, there is no interest rate, and therefore the central bank must set the money supply to achieve its price-level target. Under the assumption of the CIA constraint, the central bank will set $M_T = P_T^* Y_T$. This combined with $c_T=Y_T$ and the CIA constraint (3) holding with equality implies that $P_T = P_T^*$.

Assume that for periods t=0,1,…,T-1; the central bank pegs the gross nominal interest rate according to the following McCallum-Woodford feedback rule:[6]

$$R_t = \frac{U'(Y_t)P_{t+1}^*}{U'(Y_{t+1})\beta\, P_t^*}\left(E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]\right)^{-\tau} \tag{7}$$

where τ>0 is a parameter that reflects how sensitive this rule is to situations where the expected value of next year's price level differs from its target. Note that (7) is written in the spirt of Carlstrom and Fruest except that I write it in terms of the price-level targets instead of some steady state. Also, note that when $E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]$=1, we will say that the expected price level next period is "on target". Equation (7) is an "expected PLT" rule. After we discuss expected PLT, we will then study "current PLT" and "past PLT."

Substituting (7) into (6) gives:

$$\frac{P_t^*}{P_t} = E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]^{1-\tau} \tag{8}$$

which applies for t=0,1,2,…,T-1. Remembering that the central bank sets the money supply at time T to achieve $P_T = P_T^*$, (8) implies that $P_{T-1} = P_{T-1}^*$. By backwards recursion, we conclude that $P_t = P_t^*$ for t=0,1,2,…,T. This means that the price level is technically determined in this model under PLT regardless how large, but finite, T is. Hence, we conclude that in the limit as T goes to infinity, the price-level is technically determined.

In this specific model, the actual price levels for all t are determined. In general, it should be noted that for price determinacy, only P_0 needs to be determined where 0 denotes the current period. However, for P_0 to be determined in a rational expectations model under PLT, all the expected values of future prices must in some sense be determined as well.[7]

When the central bank pegs R_t, equation (8) reflects the two mechanisms by which the current price is determined. Since $E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right] = P_{t+1}^* E_t\left[\frac{1}{P_{t+1}}\right]$, (8) shows that if $E_t\left[\frac{1}{P_{t+1}}\right]$ is determined, then P_t will be determined. The "1" in the exponent of (8) reflects the mechanism of the Fisher-Euler equation (6) by which the value of $E_t\left[\frac{1}{P_{t+1}}\right]$ affects P_t. The "-τ" in the exponent of (8) reflects the mechanism in the feedback rule (7) by which the $E_t\left[\frac{1}{P_{t+1}}\right]$ affects R_t, which in turn affects P_t in the Fisher-Euler equation (6). If we take expectations at time s of (8)

[6] The current PLT version of this policy rule was initially proposed by McCallum (1981), but has more recently been discussed by Woodford (2003).

[7] For a relatively simple model, the "in some sense" may mean that $E_0[1/\pi_t]$ be determined for all t; for other more complex models, it may mean that more complex moments or expectations involving π_t will need to be determined.

for s<t, we get $E_s\left[\frac{P_t^*}{P_t}\right]=E_s\left[\frac{P_{t+1}^*}{P_{t+1}}\right]$. This can also be written as $E_s\left[\frac{1}{P_t}\right]=\frac{P_{t+1}^*}{P_t^*}E_s\left[\frac{1}{P_{t+1}}\right]$. This equation shows how we can work backwards through time to determine $E_s\left[\frac{1}{P_t}\right]$ from the value of $E_s\left[\frac{1}{P_{t+1}}\right]$.

If we solve (8) using backwards recursion, we get $\frac{P_t^*}{P_t}=E_t\left[\frac{P_T^*}{P_T}\right]^{(1-\tau)^{T-t}}$. Note that if τ=0 then P_t depends very strongly on $E_t\left[\frac{1}{P_t}\right]$ in the sense discussed by Sargent and Wallace (1975, p. ?). If τ>0, this dependence is diminished as T-t goes to infinity. The previous price-determinacy literature such as Woodford (2003, p. 82) has argued that the price level in an infinitely-lived economy is determined only if $\tau > 0$.[8] Eagle (2006a) shows that, if the public is no longer certain the central bank will meet its price-level target at time T, then a similar conclusion holds for PLT in a finite-horizon economy.

In addition to expected PLT, there are two other forms of PLT discussed in the literature – "past PLT" and "current PLT." Instead of (7), a central bank following past PLT will peg the gross nominal interest rate according to the following feedback rule:

$$R_t=\frac{U'(Y_t)P_{t+1}^*}{U'(Y_{t+1})\beta\, P_t^*}\left(\frac{P_{t-1}}{P_{t-1}^*}\right)^{\tau} \tag{9}$$

where τ>0 is a parameter that reflects how sensitive this rule is to situations where last period's price level differs from its target.

Substituting (9) into (6) gives:

$$\frac{P_t^*}{P_t} = E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]\left(\frac{P_{t-1}^*}{P_{t-1}}\right)^{-\tau} \tag{10}$$

which applies for t=0,1,2,…,T-1.

Solving for the price levels under past PLT is considerably more complex than under expected PLT. However, the price level is determined under past PLT. Since our discussion of past PLT is really just a side note, I will now just give the solution without proof.[9] Define $z(\tau,1) = \tau$ and $z(\tau,s) = \dfrac{\tau}{1 + z(\tau, s-1)}$ for s=2, 3, … . Then the solution to (10) is:

$$\frac{P_t^*}{P_t} = \left(\frac{P_{-1}^*}{P_{-1}}\right)^{(-1)^{t+1}\prod_{s=1}^{T-t} z(\tau,s)} \qquad \text{for t=0,l,2,…,T-1.}$$

Also $P_T = P_T^*$ because at time T the central bank sets the money supply so to achieve this price level. Note that if $P_{-1} = P_{-1}^*$, then the solution is that $P_t = P_t^*$ for t=0,1,…,T.

Still another form of PLT in the literature is "current PLT." Under "current PLT" the central bank uses the following "feedback rule" when it pegs the gross nominal interest rate:

$$R_t = \frac{U'(Y_t)P_{t+1}^*}{U'(Y_{t+1})\beta\, P_t^*}\left(\frac{P_t}{P_t^*}\right)^{\tau} \tag{11}$$

where τ>0 is a parameter that reflects how sensitive this rule is to situations this period's price level differs from its target.

Substituting (11) into (6) gives:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\tau} = E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right] \tag{12}$$

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[8] See Woodford (2003, p. 82).

[9] This can be proven using recursive methods.

Applying Bellman's principle and working backwards, we first get that $P_T = P_T^*$ because the central bank at time T sets the money supply so to meet its price-level target. Substituting this result into (12) applied at time t=T-1 shows that $P_{T-1} = P_{T-1}^*$. By backwards recursion, we conclude that $P_t = P_t^*$ for t=0,1,2,…,T.

Much of the previous literature that does study PLT such as Woodford (2003) uses "current PLT." However, I argue that "current PLT" does not make logical sense in a flexible-price model. The way that Woodford and others discuss the current-PLT "feedback" rule (12) is that the central bank looks at the current price level when it sets the gross nominal interest rate. For this to make sense as a feedback rule, the current price level must be predetermined when the central bank pegs the current period's gross nominal interest rate. However, if we study (6) carefully, we find that if R_t changes then P_t must change. Therefore, P_t cannot be determined prior to the central bank pegging R_t.

An advantage of a finite-horizon model is we can use very simple examples to illustrate certain points such as the point that the current price level cannot be predetermined when the central bank pegs the gross nominal interest rate. Assume T=1, Y_0=Y_1=100, $P_0^* = P_1^* = 1.0$, and β=0.99. Then the central bank at time t=1 will set the money supply equal to 100, which will cause P_1=1.0. Putting this and the other values of this example into (6) evaluated at t=0 gives $\frac{1}{P_0} = 0.99R_0$. This clearly shows that the current price level depends on the gross nominal interest rate set by the central bank. If the central bank sets R_0=1/0.99, then P_0 would equal 1.0. On the other hand if the central bank sets R_0=1.0, then P_0 would equal 1/0.99. Similarly, if the central bank sets R_0=1/0.99^2, then P_0 would equal 0.99. Since P_0 changes when R_0 changes,. the central bank cannot treat P_0 as given when it pegs R_0. More generally, for t=0,1,…,T-1, the central bank cannot treat P_t as given when it pegs R_t.

This argument that (12) cannot be viewed as a feedback rule is important to IT because "current IT" faces this same issue. If the central bank cannot treat P_t as given under current PLT when it pegs R_t, then it cannot treat π_t as given under current IT when it pegs R_t.

IV. IT in a Finite-Horizon Economy

In this section, we analyze inflation targeting (IT) in an economy with a finite horizon. By carefully counting the number of equations and unknowns, we find that the price level is not determined under IT when the central bank pegs the nominal interest rate. In some situations, we find no equation includes the current inflation rate, which means that there is no equation and hence no mechanism by which the current price level could be determined. Even when an equation does contain the current inflation rate, it represents how the current inflation rate affects the central bank setting the nominal interest rate, not a mechanism that can determine the current inflation rate, not a mechanism that can determine the current price level.

To simplify the analysis, the price-determinacy literature defines the inflation rate as $\pi_t \equiv \frac{P_t}{P_{t-1}}$. Under IT, the central bank makes its inflation targets transparent to the general public. Define π_t^* to be the central bank's target for the inflation rate at time t. Under IT, the central bank at time T will set the money supply so to achieve its targeted inflation rate π_T^*. Therefore, the central bank will set $M_T = P_{T-1}\pi_T^* Y_T$. This combined with the CIA constraint (3) holding with equality and $c_T = Y_T$ implies that:

$$\pi_T = \pi_T^* . \tag{13}$$

At time T-1, assume the central bank will set the nominal interest rate equal to

$$R_{T-1} = \frac{\pi_T^*}{\beta} \frac{U'(Y_{T-1})}{U'(Y_T)} . \tag{14}$$

Substituting this into (6) and remembering that $\pi_T \equiv P_T / P_{T-1}$ gives

$$E_t\left[\frac{1}{\pi_T}\right] = \frac{1}{\pi_T^*} \tag{15}$$

Note that (15) only involves π_T and is consistent with (13).

At time t=0,1,2,…,T-2; assume the central bank follows the following feedback rule when it pegs the gross nominal interest rate:

$$R_t = \frac{U'(Y_t)}{U'(Y_{t+1})} \frac{\pi_{t+1}^*}{\beta} \left(\frac{\pi_{t-1}}{\pi_{t-1}^*}\right)^{\tau} \tag{16}$$

where $\tau > 0$ is the sensitivity of this feedback rule to when the past inflation rate differs from the central bank's targeted inflation rate. Equation (12) is called "past inflation targeting." We will discuss "current IT" and "expected IT" in the next section.

Substituting (16) into (6) gives

$$E_t\left[\frac{\pi_{t+1}^*}{\pi_{t+1}}\right] = \left(\frac{\pi_{t-1}^*}{\pi_{t-1}}\right)^{\tau} \tag{17}$$

which applies for t=0,1,2,…,T-2. Please note that if we had made (16) and (17) apply to time t=T-1, we would then have been inconsistent with (13) whenever π_{T-1} differs from π_{T-1}^*. That is why for t=T-1, we assume that (14) and (15) apply instead of (16) and (17).[10]

We are now ready to count equations and unknowns. Our system of equations consists of (17), (13), and (15). Equation (17) represents T-1 equations since (17) applies to t=0,1,2,…,T-2.

Adding equations (13) and (15) to these T-1 equations from (17) gives T+1 equations. The unknowns in these T-1 equations are π_t or expected values involving π_t for t=0,1,…,T; which means that at a minimum[11] there are T+1 unknowns. Therefore, we do have an equal number of equations and unknowns (if the minimum number of unknowns applies), which is a necessary though not sufficient condition for those unknowns being determined.

When we look closer, however, we find that two of these equations, (13) and (15), only involve π_T or its target and hence neither (13) nor (15) can determine anything other than π_T. This implies that there are only T-1 equations that are available to determine unknowns that do not involve π_T; these T-1 equations are (17) for t=0,1,…,T-2. Since there are only T-1 equations to determine the T unknowns other than π_T, we are unable to determine all the unknowns other than π_T. In fact, π_0 is indeterminate and since $\pi_0 = P_0 / P_{-1}$, this means that P_0 is indeterminate. In summary, IT leads to price indeterminacy in a finite-horizon economy.

The only equation containing π_0 is (17) evaluated at t=1, which is given below:

$$E_t\left[\frac{\pi_2^*}{\pi_2}\right] = \left(\frac{\pi_0^*}{\pi_0}\right)^{\tau} \tag{18}$$

Note that if T<3, then even (18) does not apply since (17) only applies for t=0,1,…,T-2. Hence, if T<3, there is no equation containing π_0 and therefore no mechanism to determine π_0 or P_0.

Remember that we derived (17) by substituting the "past IT" rule, equation (16), into (6). Since (6) evaluated for t=0 does not contain π_0, the π_0 in (18) must come from (16) evaluated for t=0, which is

[10] An alternative formulation is to assume that (16) and (17) do apply to time t=T-1 instead of (14) and (15), but that the central bank then targets sets the money supply at time T to be consistent with (17). This change will not, however, affect the price indeterminacy of IT.

$$R_1 = \frac{U'(Y_1)}{U'(Y_2)} \frac{\pi_1^*}{\beta} \left(\frac{\pi_0}{\pi_0^*} \right)^{\tau} \tag{19}$$

However, (19) represents how the current inflation rate <u>affects</u> how the central bank pegs next period's nominal interest rate. Unless you believe in time travel, such a relationship cannot represent the mechanism that <u>determines</u> the current inflation rate.

Even with an infinite horizon, (19) is the only equation containing the current inflation rate. If we agree that (19) does not represent a mechanism by which the current inflation rate and hence the current price level can be determined, we then must conclude that regardless whether the horizon is finite or infinite, IT does not determine the current price level.

Because this paper's conclusion is so at odds with the previous literature on the price determinacy of IT, we next discuss some specific examples to help increase the likelihood that readers will realize the price indeterminacy of IT in finite-horizon economies. For these examples, assume that $\pi_t^* = \pi_{-1} = 1.02$ for all t. Also, assume that $P_{-1}=1.0$. We will now look at specific values for the finite horizon T:

First assume that T=1. Including the inflation rate at time T=1, we have two unknowns -- π_0 and π_1. Since T-2=-1, (13) does not apply at all. Therefore, only equations (13) and (15) apply. Equation (13) implies that $\pi_1 = \pi_1^* = 1.02$. This conclusion is consistent with equation (15). Since (13) and (15) only involve π_1, there is no equation that determines π_0. Since $\pi_0 = P_0/P_1$, this means there is no equation to determine P_0. Hence, the current price level is indeterminate. This indeterminacy means there are an uncountable infinite number of solutions to this system of equations. One such example is $P_0=1.02$, $P_1=1.02^2$. A second such example is

[11] We use the word "minimum" here because there more be more than one unknown moment or expected value involving π_t for each t.

P_0=1.0 P_1=1.02. A third such example is P_0=1.05, P_1=1.02(1.05). Again, there is no equation to determine the current price level.

Next, assume that T=2. Equation (13) implies that $\pi_2 = \pi_2^*$=1.02 which is consistent with equation (15). Equation (17) applied to t=0 states that $E_0\left[\frac{\pi_1^*}{\pi_1}\right] = \left(\frac{\pi_{-1}^*}{\pi_{-1}}\right)^{\tau}$. Since we assumed that $\pi_{-1} = \pi_{-1}^*$=1.02, this implies that $E_0\left[\frac{1}{\pi_1}\right] = \frac{1}{1.02}$. Thus, this expected value of $1/\pi_1$ is determined. However, once again, there is no equation to determine π_0 or P_0, which means the current price level is again indeterminate.

Next, assume that T=3. Equation (13) implies that $\pi_3 = \pi_3^*$=1.02, which is consistent with (15). Equation (17) applied for t=0 gives $E_0\left[\frac{\pi_1^*}{\pi_1}\right] = \left(\frac{\pi_{-1}^*}{\pi_{-1}}\right)^{\tau}$ which implies that $E_0\left[\frac{1}{\pi_1}\right] = \frac{1}{1.02}$. Equation (17) applied for t=1 gives (18), which includes two unknowns, $E_1\left[\frac{1}{\pi_2}\right]$ and π_0. Since (17) does not apply for t>1, (18) is the only equation to determine these unknowns, but it impossible for (18) to do so by itself. Hence, these two unknowns are indeterminate. Since π_0 is indeterminate so is P_0.

V. "Current IT", and Expected IT

The previous section showed that price indeterminacy results when the central bank follows a past-IT feedback rule as it pegs the nominal interest rate. This section shows that the price indeterminacy also occurs under "current IT" and expected IT. First, we will look at expected IT, where the central bank sets the gross nominal interest rate according to the following feedback rule for t=0,1,…,T-1:

$$R_t = \frac{U'(Y_t)}{U'(Y_{t+1})}\frac{\pi^*_{t+1}}{\beta}\left(E_t\left[\frac{\pi^*_{t+1}}{\pi_{t+1}}\right]\right)^{-\tau} \tag{20}$$

where τ>0 is the sensitivity of the gross nominal interest rate to when the expected inflation rate differs from its target.

Substituting (20) into (6) gives $1 = \left(E_t\left[\frac{\pi^*_{t+1}}{\pi_{t+1}}\right]\right)^{1-\tau}$, which implies that for t=0,1,…,T-1:

$$E_t\left[\frac{1}{\pi_{t+1}}\right] = \frac{1}{\pi^*_{t+1}} \tag{21}$$

The system of equations under current IT consists of (13) for time t=T and (21) for times t=0,1,2,…,T-1. This is a total of T+1 equations to determine the T+1 unknowns, which are π_t for t=0,1,2,…,T. However, both (13) and (21) for t=T-1 only include π_T. Therefore, there are only T-1 equations available to determine the T unknowns other than π_T. These T-1 equations are (21) for t=0,1,2,…,T-2. The T unknowns other than π_T are π_t for t=0,1,2,…,T. Therefore, there are two few equations to determine the unknowns of this system other than π_T.

Since (21) applies for t=0,1,2,…,T-1; we should note that there is no equation that contains π_0. Therefore, regardless how large T is, there is no equation and hence no mechanism by which either π_0 or P_0 could be determined. Therefore, the current price level is indeterminate under expected IT.

Next, consider "current IT," which is the form of IT that is most commonly studied in the previous price-determinacy literature (See for example, Woodford, 2003). Under "current IT", the central bank sets the gross nominal interest for periods t=0,1,2,…,T-2, equal to

$$R_t = \frac{U'(Y_t)}{U'(Y_{t+1})} \frac{\pi_{t+1}^*}{\beta} \left(\frac{\pi_t}{\pi_t^*} \right)^{\tau} \tag{22}$$

Substituting (22) into (6) gives

$$E_t \left[\frac{\pi_{t+1}^*}{\pi_{t+1}} \right] = \left(\frac{\pi_t^*}{\pi_t} \right)^{\tau} \tag{23}$$

where τ>0 is the sensitivity of the gross nominal interest rate to how the current inflation rate compares to its target. Equation (23) applies for t=0,1,2,…,T-2 since (22) applies for t=0,1,2,…,T-2. We are assuming that the central bank sets R_{T-1} according to (14) which means (15) applies for t=T-1 in order to be consistent with the central bank setting M_T^S so that $\pi_T = \pi_T^*$. Note that (23) and hence (22) applied to t=T-1 would not have this consistency when $\pi_{T-1} \neq \pi_{T-1}^*$.

We are now ready to count equations and unknowns. The system of equations consists of (13), (15), and (23) where (23) is for t=0,1,…,T-2. Since (13) and (15) only involve π_T or its target, again it is useful to count the unknowns other than π_T and equations that are available to help determine the unknowns other than π_T.

The unknowns other than π_T are π_t for t=0,1,2,…,T-1. Therefore, other than π_T there are T unknowns. The only equations available to determine these unknowns other than π_T are (23) for t=0,1,2,…,T-2. Therefore, there are only T-1 equations available to determine the T unknowns other than π_T. Therefore, even under current IT, the price level is indeterminate.

The only equation that contains the current inflation rate is (23) evaluated at t=0 which is $E_t \left[\frac{\pi_1^*}{\pi_1} \right] = \left(\frac{\pi_0^*}{\pi_0} \right)^{\tau}$. Even with an infinite-horizon economy, this still would be the only equation containing the current inflation rate. Remember that (23) was derived by substituting (22) into

(6). Since (6) evaluated at t=0 does not include the current inflation rate, the π_0 in (23) must come from (22) evaluated at t=0, which is

$$R_0 = \frac{U'(Y_0)}{U'(Y_1)}\frac{\pi_1^*}{\beta}\left(\frac{\pi_0}{\pi_0^*}\right)^{\tau} \tag{24}$$

The previous literature discussing "current IT" (e.g., Woodford, 2003 describes equation (24) as a feedback rule by which the central bank looks at the current inflation rate when it sets the current gross nominal interest rate. However, equation (24) cannot be the mechanism by which the current inflation rate and hence the current price level be determined because (24) is a process by which the price level <u>affects</u> an action, not a process by which demand and supply and equilibrating forces <u>affect</u> the price level.

This absurdity of "current IT" is even greater because "current IT" does not make sense in a flexible-price model for the same reasons that "current PLT" does not make sense. When we earlier discussed "current PLT", we noted that (6) implies that P_t depends on the value of R_t. If R_t increases (decreases), then P_t must decrease (increase). Since $\pi_t \equiv P_t / P_{t-1}$, this also means that π_t depends on the value of R_t. If R_t increases (decreases), then π_t must decrease (increase). Therefore, the central bank cannot take π_t as predetermined when it pegs R_t. Hence, (22) does not make sense as a feedback rule, which means that "current IT" does not make sense.

VI. Rebuttal to Possible Counterarguments

Many economists involved in the price-determinacy literature at this point of the paper are likely to have reasons for which they are unwilling to accept that IT leads to price indeterminacy. In this section, I try to anticipate and rebuff those counterarguments.

One counterargument that many economists will level against this paper is that they will argue against assuming a finite-horizon economy. My rebuttal is that we are able to assume an arbitrarily large finite horizon and we are able to study the limit as that finite horizon goes to infinity. The alternative is to assume an infinite-horizon economy, where we are unable to compare an infinite number of equations to an infinite number of unknowns, and where it is very easy to make mathematical errors concerning infinity. Also, as this paper shows in the next section, the non-explosive criterion that the price-determinacy literature uses to analyze infinite-horizon economies is invalid.

Furthermore, even if we do consider infinite-horizon economies, the only equation that includes π_0 is where the central bank looks at π_0 when it sets the nominal interest rate. Since the direction of causality in that equation is from π_0 to the nominal interest rate, this equation cannot be the basis for the determination of π_0 even in an economy with an infinite horizon.

A second possible counterargument some economists might level against this paper is that I did not conform enough to the previous price-determinacy literature. While the Fisher-Euler equations (1) and (6) and the feedback rules for the central bank setting the nominal interest rates are similar to those in the previous literature, this paper did not address the steady-state solution, nor did it do log-linear approximation. The reason is that this paper did not need to do so. Because this paper (i) states the feedback rules in terms of the targeted inflation rates rather than the steady-state inflation rate and (ii) is willing to work with expected values of the reciprocal of inflation rates (or price levels under PLT); there is no need to work with steady-state solutions or with log-linear approximations. In other words, this paper has discovered innovations that are more general and more precise than the previous literature. Economists should consider these innovations as positive contributions to the literature, not as reasons to reject this paper's arguments.

A third possible counterargument that I anticipate might be stated as, "The author neglected to consider the fact that last year, the Fisher-Euler equation applied for time t = -1. When we consider that, then we do have enough equations to determine the unknowns. When we go back far enough, we will also have other equations the contain π_0." This counterargument confuses the determinacy of an expected value involving π_0 with the determinacy of the realized value of π_0. Going back in time does not affect the price-indeterminacy of inflation targeting. While it is true that by going back to time t = -1 does result in $E_{-1}\left[\frac{1}{\pi_0}\right]$ being "determined" in a certain sense, that does not help determine the actual realized value of π_0.

In all three types of IT – past IT, current IT, and expected IT – the $E_{-1}\left[\frac{1}{\pi_0}\right]$ is mathematically "determined." By equations (17), (23) and (21) respectively, $E_{-1}\left[\frac{1}{\pi_0}\right]$ equals $\frac{1}{\pi_0^*}\left(\frac{\pi_{-2}^*}{\pi_{-2}}\right)$ under past IT, $\frac{1}{\pi_0^*}\left(\frac{\pi_{-1}^*}{\pi_{-1}}\right)^{\tau}$ under current IT, and $\frac{1}{\pi_0^*}$ under expected IT. However, just because $E_{-1}\left[\frac{1}{\pi_0}\right]$ is in some sense "determined" by the equations does not mean that the realized value of π_0 is determined.

Once again, we can use the simplicity allowed in a finite horizon model to demonstrate that π_0 and hence P_0 is not determined even though $E_{-1}\left[\frac{1}{\pi_0}\right]$ is in some sense "determined".

Assume that $\pi_{-2} = \pi^*_{-2}$ and $\pi_{-1} = \pi^*_{-1}$. Then $E_{-1}\left[\frac{1}{\pi_0}\right] = \frac{1}{\pi^*_0}$ regardless whether the central bank follows past IT, current IT, or expected IT. Let us further assume that $\pi^*_0 = 1.02$ so that $E_{-1}\left[\frac{1}{\pi_0}\right] = \frac{1}{1.02}$. To see that $E_{-1}\left[\frac{1}{\pi_0}\right] = \frac{1}{1.02}$ does not determine π_0, consider the possibility that there are five possible values for π_0: 0.98/1.02, 0.99/1.02, 1/1.02, 1.01/1.02, and 1.02/1.02. If these five possible values are equally likely, then the average of these solutions would be $E_{-1}\left[\frac{1}{\pi_0}\right] = \frac{1}{1.02}$. Thus, this example shows how multiple solutions could exist, yet $E_{-1}\left[\frac{1}{\pi_0}\right]$ could still equal a constant "predetermined" value. Hopefully, this example makes it clear that just because an equation "determines" $E_{-1}\left[\frac{1}{\pi_0}\right]$ does not mean that π_0 is determined.

Technically speaking, this is more complicated than the above paragraph indicates because the expectation of an indeterminate variable does not exist. If a model does not determine a variable X_t at time t, then there is no $E_{t-1}[X_t]$. With regard to IT, if π_0 is not determined, then there are no probabilities associated with all the possible solutions. Even if the number of solutions was finite, there would be no mechanism to assign probabilities to the different solutions. Hence, if the value of π_0 is indeterminate, then $E_{-1}\left[\frac{1}{\pi_0}\right]$ cannot really exist. We should realize that when we write down $E_t\left[\frac{1}{P_{t+1}}\right]$ in the Fisher-Euler equation, we were assuming that expected value existed. When we do make that assumption, we need to later go back and check to see if that assumption really is true. If P_{t+1} is not determined at time t+1 under any realization, then that previous expectations of P_{t+1} could not have existed.

VII. The Inapplicability of the Non-Explosive Criterion

The price-determinacy literature has relied on what McCallum calls the "non-explosive criterion" as the basis for its claim that IT determines the current price level. The non-explosive criterion, which came from Sargent (1979) and Blanchard and Kahn (1982), was developed to eliminate speculative bubbles in rational expectations models. While it does eliminate speculative bubbles, it also eliminates other explosive price behavior that is caused by fundamental variables rather than speculation. This is bad enough, but the price-determinacy literature has taken this to its absurd extreme by making the fundamental variables endogenous in such a way to "cause" explosive behavior for all but one solution. By applying the non-explosive criterion, the previous price-determinacy literature has claimed that the only legitimate solution is the unique non-exploding solution.

McCallum (1999) criticizes the non-explosive criterion, which he refers to as the stability criterion: "…one important objective of dynamic economic analysis … will often be to determine the conditions under which a system will be dynamically stable and unstable. … To the extent, then, that this objective of analysis is important, the stability criterion is inherently unsuitable." This paper's critique of the non-explosive criterion is related to McCallum's critique.

In a different paper, I write more about the problems with the non-explosive criterion. For this paper, I will just show examples where the non-explosive criterion does not apply. Since the non-explosive criterion does not universally apply, these examples meet my burden of proof to dispose of the non-explosive criterion as a legitimate argument that prices are determined under IT.

The simplest example is a one-dimensional trajectory. Imagine a ship traveling in a frictionless one-dimensional space, without any fuel, and with an unknown speed. The equations describing its motion are:

$$x_t = x_{t-1} + (x_1 - x_0) \tag{25}$$

where x_t is the location of the ship at time t and T is the time horizon, which could be finite or infinite. Equation (25) applies for t=2,3,…,T so there are T-1 equations. The ship's current location x_0 is known, but its speed is not. Therefore, the unknowns are x_t for t=1,2,…,T so there are T unknowns. If T is finite, then the T-1 equations are less than the T unknowns. As a result the system is indeterminate; the unknowns are indeterminate. Because we do not know the speed at which the ship is traveling, we cannot determine its future locations.

Now, let us consider T being infinite. I consider it obvious to physicists, mathematicians, and economists that even with an infinite time horizon the system is indeterminate; if we do not know the speed that the ship is traveling, then we cannot determine the future locations of the ship. In order for me to show that the non-explosive criterion is logically defective, all I need to show is one example where the non-explosive criterion leads to an incorrect conclusion. Therefore, assume that the targeted path for this ship over time is given by $x_t^* = x_0 + s^* \cdot t$ where x_t^* is the targeted location of the ship at time t and $s*$ is the desired speed for the ship. Define $\left| x_t - x_t^* \right|$ to be the measure of the targeting error. Of course, since the ship has no fuel, the captain of the ship has no control over whether or not the ship will meet this target. All the captain can do is to formulate a target and just hope the ship will meet it.

Sargent's (1979) version of the non-exploding criterion is to assume that the solution is bounded. If we define the solution in terms of the measurement error, we would conclude that all solutions of $\left|x_t - x_t^*\right|$ are unbounded except the one solution where $\left|x_t - x_t^*\right|$=0 which is where $x_t = x_t^*$ for all t. If we used Sargent's version of the non-explosive criterion to insist that the only legitimate solution is this unique bounded solution, we would conclude that the ship will be forever on target even though we do not know the ship's current speed and we have no means by which to affect the speed of the ship. Such a conclusion is absurd. This therefore shows that Sargent's bounded assumption does not universally apply.

McCallum (1999) also includes the restrictions of Blanchard and Kahn (1982) as part of the non-explosive criterion. Blanchard and Kahn do allow unbounded solutions, but they restrict how fast something can increase or decrease. In essence their restrictions only allow less-than exponential growth which means that these restrictions rule out speculative bubbles.

We will now show that the Blanchard and Kahn restrictions do not universally apply. Again imagine a ship traveling through a one-dimension frictional space, but now this ship has a cruise control that changes the speed according to the following:

$$\frac{s_t}{s_{t-1}} = \left(\frac{s_{t-1}}{s^*}\right)^{\tau} \tag{26}$$

where s_t is the speed of the vehicle at time t and s^* is the vehicle's targeted speed, which we assume to be constant. Since, we assume that the current speed (s_0) is unknown, but that the cruise control will observe s_0 and use it to set s_1 according to (26). Therefore, (26) applies for t=1, 2, 3,…,T where T can be either a finite horizon or infinity. Therefore, there are T equations in our system. The unknowns are s_t for t=0,1,2,…,T; which means there are T+1 unknowns.. If T is finite, then since there are only T equations and T+1 unknowns, the system is indeterminate. We cannot determine the speeds of the ship through time. In summary, if we do not know the current speed, we then cannot determine the future speeds of this ship with its cruise control. (However, if τ>0, the limit of the speed as T goes to infinity would equal s^*.)

Now let us consider T as infinity. Again, it should be obvious to physicists, mathematicians, and economists that if we do not know the current speed, then will be unable to determine future speeds through equation (26). We will now show that applying the Blanchard and Kahn (1982) restrictions to this example does result in a contradiction to this obvious statement. To do so, note that we can rewrite (26) as $\frac{s_t}{s^*} = \left(\frac{s_{t-1}}{s^*}\right)^{1+\tau}$, which we could solve backwards recursively to get $\frac{s_{t-1}}{s^*} = \left(\frac{s_0}{s^*}\right)^{(1+\tau)^{t-1}}$. Substituting this into (26) gives:

$$\frac{s_t}{s_{t-1}} = \left(\frac{s_0}{s^*}\right)^{(1+\tau)^t} / s^* \tag{27}$$

Note that $\frac{s_t}{s_{t-1}}$ is a measure of acceleration. If τ< 0 then the ship will decelerate when $s_0 > s^*$ and accelerate when $s_0 < s^*$. While this is how a normal cruise control works, there would be an infinite number of solutions that meet the Blanchard and Kahn restrictions.

However, if τ> 0, then the speed will accelerate when $s_0 > s^*$ and decelerate when $s_0 < s^*$. This is the opposite of how a cruise control. However, as seen in (27), if τ> 0 the acceleration increases exponentially when $s_0 > s^*$ and decreases exponentially when $s_0 < s^*$. Therefore, when τ> 0, there is only one solution that meets the Blanchard and Kahn restrictions and that is where s_0=s^*. Therefore, if the Blanchard and Kahn (1982) restrictions were universally applicable, then that would mean that we should design cruise controls to speed up when a vehicle goes too fast and to slow down when a vehicle goes too slow. It would also mean that we would not have to know the initial speed to determine the speed the vehicle travels. Both conclusions are absurd, which means that the Blanchard and Kahn restrictions are not universally applicable.

In order that the length of this paper be finite, I need to ask that the reader properly distribute burdens of proof according to normal academic standards. I have met my burden of proof by showing that the non-explosive criterion is not universally applicable. Therefore, in this debate, those who have based their conclusions on the non-explosive criterion now have the burden of proof to show that the non-explosive criterion does apply to their analysis. The previous price-determinacy literature has not done that. For example, not only does Woodford (2003) fail to meet that burden of proof, but he lacks any citation of why he assumes his solution must be bounded. The situation has been made worse by Woodford's refusal to respond to my requests for information on this issue. I have repeatedly tried contacting Woodford by email, by phone, and by formal letter, asking him to tell me his basis for assuming his solution is bounded. Over the last three years, he has never responded to my requests for this information.

That some journals have been letting specific literatures determine whether a paper is accepted or rejected has caused these journals to stray from normal burdens of proof. For example, one journal rejected a critique[12] I have written on Woodford (2003) because the referee stated that there did exist some literature not cited by Woodford that did justify Woodford assuming that his solution was bounded. The literature the referee cited included Lucas (1978, *Econometrica*), Calin, Chen, Cosimano and Himonas (2005 *Econometrica*), and Altug and Labadie (1994, Chapter 5). In fact, none of this literature applies since the assumptions of those models differ substantially from Woodford's model. In particular, Lucas (1978) did not even have money in his economy. Neither did Calin, Chen, Cosimano and Himonas (2005) as their work was an extension of Lucas (1978). While Altug and Labadie (1994, Chapter 5) did include money, they assumed upper bonds to the money supply. Those upper bounds to the money supply do not apply for Woodford's model when the central bank pegs the nominal interest rate rather than setting the money supply.

By accepting the referee's rejection recommendation and refusing to respond to my rebuttal letter, this journal has shifted an important burden of proof from Woodford to me the challenger. Rather than requiring Woodford to cite the basis for his assuming a bounded solution and to prove that it does apply to his model, the journal in effect expected me to show that there did not exist any literature that supports Woodford making this bounded assumption. Such an expectation of a challenger in essence helps protect Woodford from legitimate logical challenges. In economics as in any academic discipline, we want to encourage critical thinking. To do so, someone reading an article that does not make logical sense should be able to logically challenge the steps of a theory by reviewing the cited basis for those steps in the paper; the challenger should not be required to read all possible literature that is relevant to the theory before making such a logical challenge. All this means is that journals need to follow what should be considered the normal academic distribution of burdens of proof. To do that, the journals need to police the referees to make sure that their recommendations are not based on an improper distribution of burden of proofs.

VIII. Conclusions and Reflections

The previous literature on the price-determinacy of IT such as Woodford (2003) has used the non-explosive criterion to conclude that IT determines the current price level. However, this paper shows that the non-explosive criterion does not apply universally. In particular, if we

[12] See Eagle (2005c)

applied the non-explosive criterion to a cruise control, then we would reach the absurd conclusion that the control should cause the vehicle to accelerate when it is going too fast and to slow down when it is going too slow.

Because the non-explosive criterion does not universally apply, and because the previous price-determinacy literature such as Woodford (2003) has not formally justified why the non-explosive criterion applies to IT, this paper challenges that literature to complete its burden of proof, to show that the non-explosive criterion does apply to their analysis. In the meantime, this paper has avoided logical errors involving infinity by analyzing IT in an economy with an arbitrarily large but finite time horizon. In stark contrast to the previous literature, when the central bank pegs the nominal interest rate, carefully counting the equations and unknowns reveals that IT does not determine the current price level when the time horizon is finite.

One advantage of working with a finite horizon is that not only can the horizon be arbitrarily large, but it can also be small, which enables us to think about and analyze relatively simple concrete examples. This simplicity has enabled us to verify by example that the current price level is indeterminate under IT. This simplicity also has allowed us to think more clearly about how IT works. By doing so, we found that the only equation under IT that contains the current inflation is one which reflects the central bank looking at the current inflation rate when it sets the nominal interest rate. Since it is absurd and totally against price theory to argue that a mechanism by which the current inflation rate affects the nominal interest rate is the mechanism by which the current inflation rate is determined, this adds to this paper's argument that IT does not determine the current price level. However, even with an infinite time horizon, that equation is still the only equation containing the current inflation rate. Therefore, this argument against the price determinacy of IT applies to infinite-horizon models as well as to finite-horizon models.

At this point, this paper has presented its logical arguments that IT does not determine the current price level. Remaining to do is to give some intuitive understanding why. To do so, let us use the simplicity of a finite-horizon model to help us to think more clearly about the mechanisms by which the current price level may be determined under a McCallum-Woodford PLT feedback rule. Under PLT, the Fisher-Euler equation (6) is the mechanism by which P_t is determined from R_t and $E_t\left[\frac{1}{P_{t+1}}\right]$. Since R_t itself under the feedback rule (7) also is affected by $E_t\left[\frac{1}{P_{t+1}}\right]$, equations (6) and (7) are the mechanisms by which $E_s\left[\frac{1}{P_t}\right]$ is determined from $E_s\left[\frac{1}{P_{t+1}}\right]$ in a backwards fashion.

Under IT, $E_t\left[\frac{1}{P_{t+1}}\right]$ is not determined, because the central bank is pursuing a inflation target not a price-level target. The reason that $E_t\left[\frac{1}{P_{t+1}}\right]$ is not determined is because the central bank will respond to changes in P_t not by returning to the original implied price-level target path, but by changing its price-level-target path to be consistent with the actual level of P_t. In other words, if the future price level is not anchored, if $E_t\left[\frac{1}{P_{t+1}}\right]$ is not determined, then the Fisher-Euler equation (6) cannot determine P_t.

An interesting mental exercise that can help the reader understand why IT leads to price indeterminacy is to look at a particular hybrid IT-PLT regime. Assume that the central bank targets the inflation rate for periods t=0,1,2,…,T and at time T, the central bank also pursues a price-level target announced at time t=0. Label this time T price-level target as P_T^*. For simplicity, assume aggregate output is constant, which means that the Fisher-Euler equation (6) can be rewritten as

$$\frac{1}{P_t} = R_t \beta E_t \left[\frac{1}{P_{t+1}} \right] \tag{28}$$

which applies for t=0,1,…,T-1. Also, for simplicity, assume the central bank sets the nominal interest rate equal to

$$R_t = \pi_{t+1}^* / \beta \tag{29}$$

for t=0,1,…,T-1.

Applying Belman's principle and working backwards, we start at time T where the central bank sets M_T so that $P_T = P_T^*$. At time t=T-1, the central bank sets $R_{T-1} = \pi_T^* / \beta$. Substituting this and $P_T = P_T^*$ into (28) and solving for P_{T-1} gives $P_{T-1} = P_T^* / \pi_T^*$. By backwards recursion, we conclude that

$$P_t = \frac{P_T^*}{\prod_{s=t+1}^{T} \pi_s^*}$$

This then shows that not only is P_0 determined but so is P_t for t=0,1,2,…,T.

To make this more concrete and less abstract, assume T=3, β = 0.99, π_t^* = 1.02 for t=0,1,2,and 3. Also assume that $P_T^* = P_3^* = 1.02^3$. If you work backwards using Bellman's principle, you should conclude that $P_3 = 1.02^3$, $P_2 = 1.02^2$, $P_1 = 1.02$, and $P_0 = 1.00$.

Now change the model so that it is pure IT, by assuming the central bank no longer sets the money supply at time T so that $P_T = P_T^*$. Instead, the central bank sets the money supply at time T so that $\pi_T = \pi_T^*$. As explained before, the current price level no longer is determined. We can verify this for our T=3 example with $\beta = 0.99$, $\pi_t^* = 1.02$ for t=0,1,2,and 3. Working backwards, we start at time t=3, where the central bank sets the money supply so that $\pi_3 = 1.02$. Working backwards, at time t=2, the central bank will peg $R_2 = \pi_2^* / \beta = 1.02/0.99$. Substituting this into (28) gives $\frac{1}{P_2} = 1.02 E_2\left[\frac{1}{P_3}\right]$. Rearranging and noting that $\pi_3 \equiv P_3 / P_2$, this implies that $\frac{1}{1.02} = E_2\left[\frac{1}{1.02}\right]$, which is always true. Next, work backwards to time t=1, when the central bank will peg $R_1 = \pi_1^* / \beta = 1.02/0.99$. Substituting this into (28) gives $\frac{1}{P_1} = 1.02 E_1\left[\frac{1}{P_2}\right]$. Rearranging and noting that $\pi_2 \equiv P_2 / P_1$, this implies that $\frac{1}{1.02} = E_1\left[\frac{1}{\pi_2}\right]$. Finally, work backwards to time t=0, when the central bank will peg $R_0 = \pi_0^* / \beta = 1.02/0.99$. Substituting this into (28) gives $\frac{1}{P_0} = 1.02 E_0\left[\frac{1}{P_1}\right]$, which we can rearrange and get $\frac{1}{1.02} = E_0\left[\frac{1}{\pi_1}\right]$. That's it! There is no equation that contains π_0. I am sure you will confirm that the system is indeterminate.

This example demonstrates that for a finite-horizon economy, a hybrid IT-PLT does determine prices, whereas when we replace the price-level target at time T with an inflation target, the current price level becomes indeterminate. The reason is that the Fisher-Euler equation is a mechanism by which P_t is determined from R_t and $E_t\left[\frac{1}{P_{t+1}}\right]$. If a future price level is anchored down such as $P_T = P_T^*$, then applying Bellman's principle and working backwards through the Fisher-Euler equation and the central-bank's feedback rule for setting the nominal interest does eventually lead us to determining P_0. However, if there is no anchoring of an expectation involving a future price level, then that mechanism does not work. The reason IT does not determine the price-level is because the central bank is not committed to a particular targeted price level in the future; instead it is only committed to targeted inflation rate. Hence, there is nothing anchoring down a future price level regardless whether the economy has a finite or an infinite time horizon. Since there is no anchoring of the future price level under IT even in an infinite economy, this paper then concludes that the current price level is indeterminate under IT in an economy with a infinite horizon as well as a finite horizon.

That the current price level is undetermined under IT, but determined under PLT, means that if the public erroneously thought the central bank was pursuing PLT, the current price level would be determined. However, just as the Keynesian economic systems fell apart in the late 1960s and 1970s when people learned how that system worked and adjusted their expectations accordingly (See Lucas, 1976), people will eventually learn from experience that the central bank is pursuing IT rather than PLT. When they do so, then the economies of the world will likely experience great price instability because of the price indeterminacy of IT.

Such a doomsday prediction may be mitigated somewhat because if IT fails, central banks are likely to switch to something like PLT. As mentioned before, IT followed by PLT does determine prices. However, only if the central bank's future price-level target or targets are known by people in advance will IT determine price levels. Since the central bank will probably formulate its price-level targets anew when it abandons IT, the indeterminacy of IT remains, and the risk of this doomsday prediction remains.

Many may think this paper is a recommendation that the central bank target price levels rather than IT. It is not. While PLT under a McCallum-Woodford feedback rule for pegging the nominal interest rate does determine prices, Eagle and Domian (2005) present a strong theoretical argument in favor of nominal-income targeting. They argue that nominal-income targeting better handles real-output contingencies than does PLT.

Readers should note that this paper is based on rational expectations. Since rational expectations is facilitated by economic agents understanding what the central bank is doing; I do fully endorse the transparency of the central bank. That transparency is enhanced by the central bank being clear about what it is targeting and being committed to that target. However, what this paper and papers like Eagle and Domian 92005) show is that we still are learning and probably have a great deal more to learn about monetary economics. Thus, until the central bank understands what is best for it to do in all contingencies, the central bank needs to be flexible to change its targets based on its growing understanding of monetary economics and its growing understanding of what it should do in different contingencies.

References:

Altug, Sumru & Pamela Labadie (1994). *Dynamic Choice and Asset Markets* (Academic Press).

Blanchard, Olivier J. & Charles M. Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48(#5):1305-1312.

Calin, Ovidue, Yu Chen, Thomas Cosimano, and Alex Himonas, (2005), "Solving Asset Pricing Models when the Price-Dividend Function is Analytic," *Econometrica* 73(#3):961-982.

Carlstrom, Charles T. & Fuerst, Timothy S. (2001). "Timing and real indeterminacy in monetary models," *Journal of Monetary Economics*. 47(#2):285-298.

Clegg, Brian (2003), *A Brief History of infinity: the Quest to Think the Unthinkable* (Carroll & Graf – New York)

Dittmar, Robert D. and William T. Gavin (2005), "Inflation-Targeting, Price-Path Targeting and Indeterminacy," Economic Letters 88:336-342.

Eagle, David (2005a). "Price Indeterminacy Reinvented: Pegging Interest Rates While Targeting Prices, Inflation, or Nominal Income," Macroeconomics 0501028, EconWPA database, http://ideas.repec.org/p/wpa/wuwpma/0501028.html, accessed on November 16, 2006.

__________ (2005b). "The Inflation Dynamics of Pegging Interest Rates," Macroeconomics 0502029, EconWPA database, http://ideas.repec.org/p/wpa/wuwpma/0502029.html, accessed on November 16, 2006.

__________ (2005c). "Multiple Critiques of Woodford's Model of a Cashless Economy," Macroeconomics 0504028, EconWPA database, http://ideas.repec.org/p/wpa/wuwpma/0504028.html, accessed on November 16, 2006.

__________ (2006). "Revealing the Naked Truth about Price Determinacy and Infinite-Horizon Rational Expectations," Working Paper, http://www.cbpa.ewu.edu/~deagle/banking/NonExplosive.doc, accessed on November 22, 2006.

Eagle, David & Dale Domian, 2005. "Sounding the Alarm on Inflation Indexing and Strict Inflation Targeting," working paper, http://www.cbpa.ewu.edu/~deagle/banking/SOundingWEApaper.pdf, accessed on November 16, 2006.

Eagle, David & Elizabeth Murff, 2005. "Logical Pitfalls of Assuming Bounded Solutions to Expectational Difference Equations," GE, Growth, Math methods 0501002, EconWPA database, , accessed on November 16, 2006.

Lucas, Robert E. (1976). "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series on Public Policy* 1, 19-46.

Lucas, Robert (1978), "Asset Prices in an Exchange Economy," *Econometrica* 46(#6):1429-1445.

McCallum, Bennett T. (1981). "Price Level Determinacy with an Inteerst Rate Policy Rule and Rational Expectations," *Journal of Monetary Economics* 8:319-329.

McCallum, Bennett (1999). "Role of the Minimal State Variable Criterion in Rational Expectations Models," *International Tax and Public Finance*, 6:621–639.

Sargent, Thomas J. (1979). *Macroeconomic Theory – Economic Theory, Econometrics, and Mathematical Economics* (Academic Press – New York).

Sargent, Thomas J. and Neil Wallace (1975). "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy* 83:241-254.

Woodford, Michael (2003). *Interest and Prices – A Foundation of a Theory of Monetary Policy* (Princeton University of Press – Princeton, New Jersey).