The Basel II IRB approach revisited: do we use the correct model?

Varsanyi, Zoltan

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The Basel Committee on Banking Supervision published the new Basel Capital Accord in June 2004. This new regulation is becoming the basis for national regulation of financial institutions in a large part of the world.[1] However, as I show, there is a problem with the calculation of risk weights: the Basel model uses incorrect default triggers although the error caused by it is offset, to an extent, by the model's conditional default probability calculation which, in turn, is also incorrect.

The Basel capital rules are based on a one-factor model. In this model the (change in the)[2] asset value of the obligor firm is driven by a systemic factor and an idiosyncratic shock:

$$R_i = w_i X + \sqrt{1-w_i^2}\,\varepsilon_i . \tag{1}$$

Here X is the systemic factor, ε_i is the idiosyncratic shock of obligor i (both are standard normal) and w_i is the correlation with the systemic factor of the asset value of obligor i. The firm defaults if its asset value crosses a downward trigger, say, γ. This trigger level is linked to the unconditional default probability of the firm through the standard normal distribution, i.e.

$$\gamma = N^{-1}(PD_u)\,, \tag{2}$$

where $N(v)$ denotes the probability that a standard normal variable is smaller then v. Thus, using (1)-(2), the probability of default can be expressed, conditional on x, the value of the systemic factor, as:

$$p(x) = N\left(\frac{\gamma - w_i x}{\sqrt{1-w_i^2}}\right) = N\left(\frac{N^{-1}(PD_u) - w_i x}{\sqrt{1-w_i^2}}\right). \tag{3}$$

This formula is, in essence, the basis of the risk weight assigned to an exposure in the Basel IRB framework, with x=X_q, the 0.1th percentile of the distribution of x.[3] It represents the qth percentile of the portfolio loss distribution when the model's

♣ Economist, Magyar Nemzeti Bank (the central bank of Hungary); varsanyiz@mnb.hu. The views expressed here do not necessarily reflect those of the Magyar Nemzeti Bank. I thank Peter Medvegyev for helpful discussions. I also benefited much from Alberto Pambira's comments on the drafting. All the remaining errors are my own.

[1] The EU has just finalized and issued the Capital Requirements Directive, the local implementation of Basel 2. In the USA there is still debate on the implementation; according to Cornford [2005] 99% of total foreign assets and 66% of total bank sector assets can be expected to be covered with the new regulation. Discussion papers have also been issued in Asia (see HK [2004], SG [2004], AU [2005]).

[2] If we assume that the current asset value equals zero this distinction is not important.

[3] A detailed explanation of the Basel formulas can be found in BCBS [2005]. Throughout the paper I assume that the loss when an exposure defaults (LGD) equals 100%, however, this should not alter the qualitative conclusions of the paper.

assumptions are fulfilled.[4] When it is not the case the above formula is an inaccurate measure of the loss percentile and, indirectly, the risk weight.[5] However, these aspects are known and are manageable (more or less) within the modeling framework: we can apply add-ons to 'cure' non-granularity and we can use multifactor models to take into consideration correlation beyond that caused by the systemic factor.[6] At the same time, a much severe shortcoming of the Basel model can be revealed which is introduced next.

The Basel model only considers exposures at the end of the required time horizon. That is, all distributions (that of X and ε) refer to one year from the current period (day) and formulas should be interpreted accordingly: *equation (3) is based on end-of-horizon asset-values* and similarly, as is implicitly proven by (6) below, *γ, the default trigger, is derived using year-end default status based on year-end asset values* in (2). However, the asset value can reach the default trigger before the end of the period (year) – and if in such a case the asset value returns above the trigger by the end of the year the model will consider this exposure as non-defaulting, whereas, in fact, it has defaulted. An approach better reflecting the true dynamics of default events relates to the notion of exit time that comes from the stochastic processes literature and designates an event when the stochastic process reaches or crosses a trigger level (see Chhikara and Folks [1989]).[7] Let's assume that there is a stochastic process in the form:

$$dX = \mu dt + \sigma dB_t \tag{4}$$

Then, the probability that *X* drifts apart from its starting value x_0 to a distance *d* within a timeframe between t=0 and t=T is given by (Kamstra and Milevsky [2004]):

$$G(T) = N\left[\sqrt{\frac{\lambda}{T}}\left(\frac{T}{\beta}-1\right)\right] + e^{\frac{2\lambda}{\beta}} N\left[-\sqrt{\frac{\lambda}{T}}\left(1+\frac{T}{\beta}\right)\right], \tag{5}$$

where $\beta = \dfrac{d}{\mu}$ and $\lambda = \dfrac{d^2}{\sigma^2}$.

In the Basel model the change of the asset value of the firm is assumed to follow a standard Brownian motion (cf. (1)), so that – in each point in time – it has standard normal distribution. This implies a model in the form of (4) where μ=0 and σ=1. With these parameter values and substituting γ for *d* (5) can be written as:

$$G(T) = 2N\left[-\frac{\gamma}{\sqrt{T}}\right] \tag{6}$$

Function *G*(*T*) gives the probability that the asset value reaches the default trigger (γ) in *T*, at the latest.[8] It can be seen immediately that if we set the length of the period to

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[4] E.g. elements in the portfolio are correlated only through the systemic factor. More on this issue can be found in Gordy [2002].

[5] For example, the lack of granularity results in an underestimation of risk by the formula; similarly, when idiosyncratic shocks are correlated risks are again underestimated.

[6] See, for example, Pykhtin [2004].

[7] Alternatively, the notion of exit time can be approached by examining the maximum of a Wiener process in a time interval. The results must be the same.

[8] It is worth mentioning that in reality *year-end* defaults cannot even be measured since at the end of the year we count all defaults occurred *over* the whole year.

one (year) the probability calculated using (6) is exactly the double of what (2) gives. *This results in a higher default trigger in the Basel model than in the 'true' model* (alternatively, this higher γ is needed for the Basel model to result in the 'true', 'observed' PD_u, see (2)). Everything else equal, this would result in an overstatement of the required loss percentile. *However, in (3) the model applies an incorrect formula again, by considering the asset-value at the end of the horizon (this time, conditional on the systemic factor).* This formula corresponds to the end-of-horizon approach of the Basel model and, as we saw above, results in lower default probabilities then the exit-time model. This is an important, new insight of the paper into the Basel model.

The logic of loss-percentile (and risk weight) calculation can be summarized in 3 steps (see Table 1).

Table 1: Steps in the calculation of the percentiles of the loss distribution and the relation of the corresponding quantities in the Basel vs. the exit-time model

Step	Action	Basel-model	Relation	'True' model (exit-time)
1	Observed (measured) unconditional expected default probability	PD_u	=	PD_u
2	Default trigger (γ), 1 year	$\gamma_B=N^{-1}(PD_u)$	>	$\gamma_G=N^{-1}(PD_u/2)$
3	Percentile of the distribution of unconditional probability	$N((\gamma - wX_q)/\varphi)$	<=> (?)	? (simulation)

In the first step we measure the expected value of the *unconditional* default probability from the sample. In the second step we substitute it, within the framework of the Basel model, directly into the inverse of the standard normal distribution (instead of dividing it by 2) to arrive at γ. However, the correct default trigger is calculated using (6) and is shown in the last column belonging to Step 2 ($N^{-1}(PD_u/2)$). Using these different default triggers in the two approaches ensures that the unconditional default probabilities will be identical.[9] The question is then whether in Step 3 the Basel transformation ensures that the loss distributions – or at least their 99.9^{th} percentile – coincide.

In general, it is a question to what extent these two errors offset each other; in what follows, I examine it for PD_u=20%. The calculations are complicated by the fact that – to my knowledge – there is no closed form solution to be applied in the 'true' case, so one has to simulate the necessary asset-value processes. The difficulty can immediately be seen from (3): whereas in the Basel approach the required percentile of the loss distribution is calculated by conditioning on the systemic factor at the end of the horizon, in a 'continuous time' approach one cannot use the conditioning technique.

Two simulations were carried out, with the following parameters:

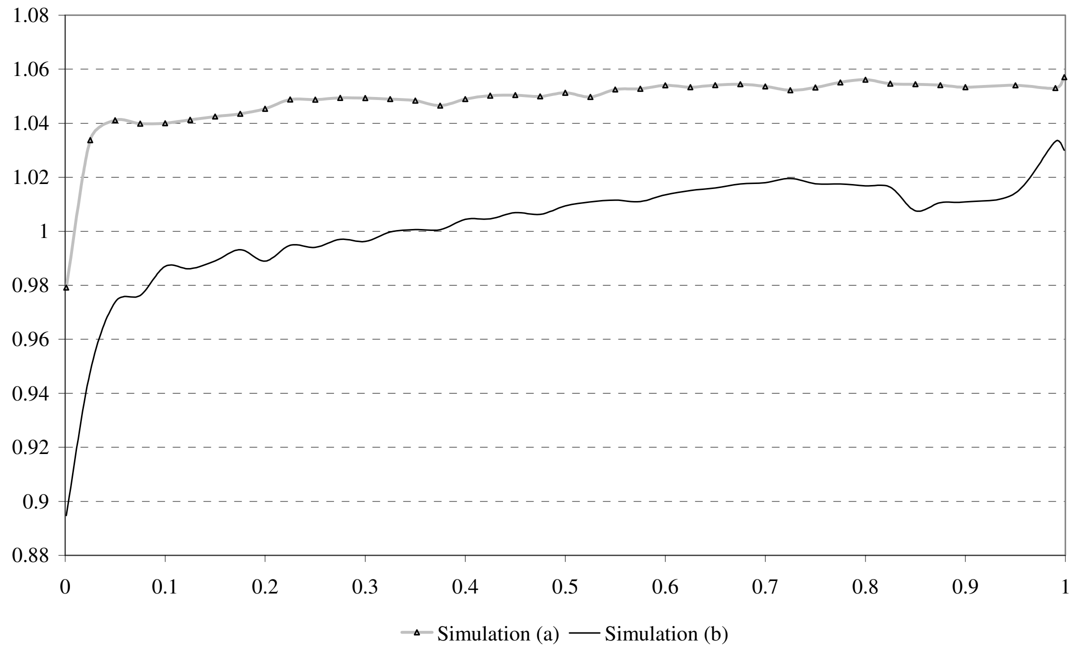
- PD_u (unconditional – observed – default probability): 20%
- γ (default trigger): N^{-1}(10%) – cf. (2.3)
- Δt (time interval between two default-checks): (a) 0.0025 and (b) 0.0005
- Number of years simulated: 1

[9] This argument shows that the Basel model doesn't use even 'virtual' firms and asset values (and should not be interpreted so): rather, it uses those formulas from a purely technical reason.

- Number of exposures in the bucket: 5000
- Number of sample paths: (a) 45000 and (b) 15000
- Basel conditional PD: 59.63%

I carried out the simulation twice.[10] In both cases I calculated default rates by dividing the number of defaults in the sample by the sample size (5000). This is justified by the fact that in the place of defaulting exposures I didn't put new exposures into the portfolio, so the problem of default probability measurement is not present here.[11] In case (a) I set Δt to 0.0025 which means evaluating default status 400 times a year and the number of sample paths was 45000. In case (b) I decreased Δt to 0.0005, i.e. I evaluated default status 2000 times a year (about every 4.5 hours); this slowed down the program so much that I decreased the number of sample paths to 15000.
The means of the default rates, which correspond to the unconditional expected PD, were almost exactly 19% in case (a) which is quite close to the unconditional PD of 20%; in case (b) I got – as expected – a closer approximation, 19.8%. Figure 1 shows the proportion of the Basel loss to losses from the two simulations for different percentiles.

Figure 1: The ratio of the Basel loss to losses from the two simulations for different percentiles, PD_u=20%



It can be seen from Figure 1 that in Simulation (b) the simulated losses are higher than the Basel losses up to around the 35th percentile. Although the exact numerical results may change from simulation to simulation and across different parameter sets it seems that the percentiles in the Basel and exit-time models do not coincide.

To conclude, in this paper I derived and demonstrated through a simulation that the Basel model calculates portfolio loss percentiles inaccurately. The size and sign of the error is difficult to assess and may vary across different parameter sets.

[10] The reason for this low number is that the program runs very long.
[11] The calculation of default rates from historical data is a problematic point related to the regulation and the model behind it.

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