

# MPRA

Munich Personal RePEc Archive

## **Partnership Dissolution and Proprietary Information**

Li, Jianpei

University of International Business and Economics

January 2009

Online at <https://mpra.ub.uni-muenchen.de/12505/>

MPRA Paper No. 12505, posted 06 Jan 2009 06:00 UTC

# Partnership Dissolution and Proprietary Information\*

Jianpei Li<sup>‡</sup>

January 2009

## Abstract

This paper studies different rules in dissolving a common value partnership where one partner holds proprietary information. In winner's bid auction (WBA) and loser's bid auction (LBA), there exists a unique mixed strategy equilibrium. "Pay-off equivalence" is established in the sense that partners' expected payoffs are the same under the two auction formats. The informed partner benefits from an information rent while the uninformed suffers from an "ownership's curse". When cake-cutting mechanism (CCM) is applied, whether pure strategy equilibrium exists or not depends on the identity of the proposer. If the uninformed partner is the proposer, the informed partner receives an information rent in a pure strategy equilibrium. If the informed partner is the proposer, the asset value is shared equally between the two parties in a mixed strategy equilibrium. The paper then compares the information rents and prices offered for proprietary information by the three rules.

**JEL classifications:** C72, D44, D86, L33.

**Keywords:** partnership dissolution, proprietary information, winner's bid auction, loser's bid auction, cake-cutting mechanism

---

\*I am grateful to Elmar Wolfstetter, Christian Kirchner, Xinhua Jiang, Jiong Gong, and seminar participants at Humboldt University of Berlin, the 4th annual conference of AsLEA, and the 5th biennial HKEA conference for helpful comments. <sup>‡</sup>School of International Trade and Economics, University of International Business and Economics, East Huixin Street 10, Beijing 100029, China, LiJianpei2007@gmail.com.

# 1 Introduction

By partnership, we refer to some indivisible asset jointly owned by more than one partner. In many situations, the partners may no longer want to own the asset together, hence triggering a dissolution of the partnership. As the rules governing the division of the asset are typically either legislated or negotiated using lawyers and the courts, who are poorly informed about the value of the asset, it is important to study simple price-generating mechanisms that make use of partners' private information. Commonly used mechanisms are winner's bid auction (WBA), loser's bid auction (LBA), and cake-cutting mechanisms (CCM).<sup>1</sup>

In WBA and LBA, partners submit sealed bids and the one with higher bid gets the asset. In WBA, the higher bid makes the final transaction price, while in LBA the lower bid makes the final transaction price. In CCM, one partner proposes a price for the asset, and the other partner has the option of buying out or selling out at the proposed price.

This paper considers a common-value partnership which is initially jointly owned by two partners but is to be dissolved for exogenous reasons. One partner receives a private signal about the value of the asset while the other partner holds only public information.<sup>2</sup> We analyse bidders' equilibrium strategies and compute the partners' expected payoffs. We then compare the information rents and the prices for the proprietary information generated by the three dissolution rules.

The main findings of the paper are as follows. First, unique equilibrium of WBA and LBA is in mixed strategies. Uninformed partner randomizes his bids as if he has received a private signal drawn from the same distribution as that of the informed partner. Second, "payoff equivalence" between WBA and LBA is established in the sense that partners obtain the same expected payoffs under the two auction formats. The informed partner expects an information rent while the uninformed suffers from a loss, which does not exist if that partner does not share ownership of the asset, and hence is referred to as an "ownership's curse". Third, in CCM with uninformed partner as the proposer (referred to

---

<sup>1</sup>WBA and LBA are special cases of the  $k + 1$ -auctions first analyzed by [Cramton, Gibbons, and Klemperer \(1987\)](#). CCM is also referred to as "I-cut-you-choose", "divide and choose", and "buy-sell provision" in the literature.

<sup>2</sup>Such proprietary information structure is a natural assumption in many real business operations. One prominent example is a partnership with an active and a sleeping partner where the active partner runs daily operation while the sleeping partner only shares profits. One has good reason to believe that the active partner knows more about the true value of the partnership than the sleeping partner.

as  $CCM_B$  hereafter), the informed partner receives an information rent in a pure strategy equilibrium. In  $CCM$  with the informed partner as the proposer (referred to as  $CCM_A$  hereafter), the two partners share the asset value equally in a mixed strategy equilibrium. Finally, comparison of the dissolution rules show that  $CCM_B$  may generate higher or lower information rents than  $WBA$  and  $LBA$ , depending on the distribution of asset value, while  $CCM_A$  generates zero information rent. Using the ranking of information rents, we are able to compare the prices different rules offer for the proprietary information.  $CCM_A$ , though allocates the asset value equally between the two partners, pays the lowest price for proprietary information and hence provides the lowest incentive for information acquisition.

The rest of the paper is organized as follows. In the remaining part of this section, we outline related literature. In Section 2, we present the setup of the model. In Sections 3, 4, and 5, we analyze the equilibrium and compute the partners' expected payoffs if the dissolution rules are respectively  $WBA$ ,  $LBA$  and  $CCM$ . In Section 6, we compare the three rules on the basis of the information rents they generate to the informed partner and examine the prices for proprietary information. Section 7 concludes. All proofs not in the text are relegated to the appendix.

## Related literature

There is a sizable literature on the three dissolution rules, focusing on the efficiency question whether a particular rule allocates the ownership of the partnership to a partner that values it the most.<sup>3</sup> Cramton, Gibbons, and Klemperer (1987) first analyzed the performance of a more general form of  $WBA$  and  $LBA$ ,  $k$ -double auctions, and show that auctions implement efficiency in a symmetric private value framework if partners hold equal shares of the partnership. Lengwiler and Wolfstetter (2005) establish revenue equivalence between  $WBA$  and  $LBA$  in such a framework, assuming the existence of an auctioneer who collects a fixed fraction of the auction revenue. de Frutos (2000) provides results on the existence and uniqueness of pure strategy equilibria in  $WBA$  and  $LBA$  for the case that partners' valuations are asymmetrically distributed. Kittsteiner (2003) solves for the equilibria of  $k$ -double auctions when partners' valuations are interdependent

---

<sup>3</sup>The more broad partnership dissolution literature focuses on the issue whether efficiency can be achieved, including Cramton, Gibbons, and Klemperer (1987), Engelbrecht-Wiggans (1994), Fieseler, Kittsteiner, and Moldovanu (2003), Jehiel and Paudner (2006), Ornelas and Turner (2007) and Chien (2007). Moldovanu (2001) surveyed most of these contributions.

and shows that they generally fail to implement efficiency in that case.

McAfee (1992) compares the performance of WBA and LBA with CCM for risk-averse agents in a symmetrically independent private value framework. Morgan (2004) solves for the equilibria of the three rules if a partnership exhibits common value and both partners have some noisy signals about that value. He then compares fairness of the three rules and concludes that none of them are ex post fair.

de Frutos and Kittsteiner (2008) study a two-stage game where dissolution is preceded by a negotiation stage where partners determine the identity of the proposer, and show that CCM allocates the asset efficiently in this game. Li and Wolfstetter (2007) endogenize partners' dissolution decision and analyze the properties of CCM and its variants in an environment where partners' dissolution incentives interact with investment incentives. Brooks and Spier (2004) analyze CCM in a framework similar to the current paper, compare the performance of CCM with simple buy/sell offer, and show that CCM exploits all possible gains from trade.

This paper contributes to the literature in two-folds: 1) it analyzes WBA and LBA in a framework where the partnership exhibits common value but one of the partners holds proprietary information about that value, which has not been done before.<sup>4</sup> The analysis shows that when proprietary information exists, the properties of equilibria are very different from the case where no such information exists, and pure strategy equilibria generally do not exist, in contrast to the existing contributions that focus on the existence of Nash equilibrium in pure strategies. 2) Using the asymmetric information setup, the paper is able to compare the information rents generated by the three dissolution rules and hence the prices offered by the three rules for proprietary information.

The paper is also related to contributions on competitive bidding with proprietary information, including Wilson (1967), Milgrom and Weber (1982), Engelbrecht-Wiggans, Milgrom, and Weber (1983), Hausch (1987), Hendricks and Porter (1988), Hendricks, Porter, and Wilson (1994), and Dubra (2006). This paper extends that analysis to a partnership dissolution environment where both the winner and loser of the auctions receive some share of the auction revenue. Similar to a first price auction, in both WBA and LBA, the uninformed partner bids as if he has drawn a private signal according to the distribution of the informed partner's signal. Different from that, the loser, instead of assuring himself a zero payoff, always suffers from a loss.

---

<sup>4</sup>Schweinzer (2008) analyzes a repeated bargaining game in a similar framework and show that information is gradually revealed by the informed partner as bargain goes on.

## 2 The Setup

Two risk neutral partners hold equal shares of an indivisible asset. If operated by a single partner, the asset generates a value  $X$  to its owner, which is a random variable drawn according to a continuous and differentiable distribution  $F : [0, \bar{x}] \mapsto [0, 1]$ , with associated density  $f$ . Partner A, the informed partner, receives a private signal  $x$  that reveals the realization of  $X$ . Partner B only holds public information about the distribution of  $X$ . For some exogenous reasons, if joint ownership continues, the value of the asset shrinks quickly to zero. Therefore, the partners wish to dissolve the partnership immediately by transforming joint ownership into single ownership. Dissolution rules under consideration are WBA, LBA, and CCM. No external buyers are available.<sup>5</sup>

## 3 The Winner's Bid Auction

In WBA, two partners simultaneously submit a bid for the asset. The partner with higher bid wins the auction, receives the asset and pays half of his bid to the losing partner. Ties are resolved by the flip of a coin.

**Lemma 1.** *There exists no equilibrium in which partner B plays pure strategy.*

Suppose there exists an equilibrium where partner B plays some pure strategy  $s$ . Partner A's best response is to slightly overbid if  $x > s$  and slightly undercut if  $x < s$ . Then for partner B bidding  $s$  is not a best response to partner A's strategy. If A quotes a price above  $s$ , partner B learns that the asset value must be above  $s$ , and therefore would like to overbid A to win the auction with a positive profit. Similarly,  $s$  is not optimal for B if A's bid is below  $s$ . Therefore, playing pure strategy is dominated for partner B. We denote partner B's randomization strategy by distribution  $G(\cdot)$  over his choice of bids on  $[0, \bar{x}]$ . In particular,  $G(s)$  is the probability that partner B's bid is below  $s$ .

Partner A's strategy is a mapping from his private signal  $X$  into real positive numbers, denoted by  $\beta(\cdot)$ .

---

<sup>5</sup>A real life example matching well with the setup is the dissolution of Hangzhou Huasan Telecommunication—a joint venture between Huawei (49%) and 3Com (51%)—in 2007. Before dissolution, major operations were under control of the management team sent by Huawei. The two partners agreed that no external buyers would be considered in case of dissolution. Future value of Huasan largely depended on the market structure and demand for telecommunication equipments. (Ming, Xu, and Chen (2007).)

**Lemma 2.** *In equilibrium, partner A's strategy  $\beta(x)$  is continuous and strictly increasing in  $x$ , with  $\beta(0) = 0$  and  $\beta(\bar{x}) := \bar{b} < \bar{x}$ . Partner B chooses his bid at random from the interval  $[0, \bar{b}]$  according to an atomless distribution  $G(s)$ .*

From Lemma 2, partner B uses the atomless strategy  $G(s)$  on  $[0, \bar{b}]$ . Hence, we can use  $g(s)$  to denote the associated density. Since partner A's bidding strategy is continuous and monotonic, the signal that leads to a bid  $b$  by partner A can be described by  $\beta^{-1}(b)$ , the inverse of  $\beta(\cdot)$ .

Given partner B's strategy  $G(s)$ , partner A with a signal  $x$  chooses his bid  $b$  to solve the following maximization problem:

$$\max_b U_A(x, b) = G(b)\left(x - \frac{1}{2}b\right) + \frac{1}{2} \int_b^{\bar{b}} yg(y)dy \quad (1)$$

By placing bid  $b$ , partner A wins the auction with probability  $G(b)$ . If winning, he receives the asset which values  $x$  to him and pays partner B half of his bid  $b$ . When  $b$  is below partner B's bid, a random variable with density  $g(\cdot)$ , partner A loses the auction, sells his share of the asset, and receives half of partner B's bid as compensation.

Taking the derivative of (1) with respect to  $b$  and setting it equal to zero yield the first order condition:

$$g(b)x - bg(b) - \frac{1}{2}G(b) = 0$$

Replacing  $b$  by  $\beta(x)$ , we get

$$g(\beta(x))x - \beta(x)g(\beta(x)) - \frac{1}{2}G(\beta(x)) = 0 \quad (2)$$

Given partner A's strategy  $\beta(X)$ , partner B chooses bid  $s$  to maximize his expected payoff:

$$U_B(s) = \int_0^{\beta^{-1}(s)} yf(y)dy - \frac{1}{2}sF(\beta^{-1}(s)) + \frac{1}{2} \int_{\beta^{-1}(s)}^{\bar{x}} \beta(y)f(y)dy \quad (3)$$

When bidding  $s$ , partner B expects to win the auction if and only if  $\beta(X) \leq s$ , equivalently  $X \leq \beta^{-1}(s)$ . Since partner A's private signal  $X$  is distributed according to  $F(\cdot)$ , partner B wins the auction with probability  $F(\beta^{-1}(s))$ . On winning, partner B receives the asset which generates an expected value  $E[X \mid X < \beta^{-1}(s)]$  to him, and pays half of his bid  $s$  to partner A. If losing, partner B sells his share to partner A and receives half of partner A's bid as compensation. As partner B plays mixed strategy, any action from his equilibrium strategy space shall bring him the same expected payoff. Therefore,  $U_B(s)$  shall be independent of  $s$  and its derivative with respect to  $s$  is 0:

$$\frac{d}{ds}U_B(s) = \beta^{-1}(s)f(\beta^{-1}(s))\frac{d\beta^{-1}(s)}{ds} - \frac{1}{2}F(\beta^{-1}(s)) - sf(\beta^{-1}(s))\frac{d\beta^{-1}(s)}{ds} = 0 \quad (4)$$

Since any  $s \in [0, \bar{b}]$  satisfies (4),  $s = \beta(x)$  satisfies (4) as well. Using  $s = \beta(x)$  and rewriting (4) give us:

$$\frac{1}{\beta'(x)}xf(x) - \frac{1}{2}F(x) - \beta(x)f(x)\frac{1}{\beta'(x)} = 0 \quad (5)$$

Equilibrium of the game is described by a pair  $(\beta(\cdot), G(\cdot))$  that solves simultaneously differential equations (2) and (5).

**Proposition 1.** *The winner's bid auction has a unique equilibrium  $(\beta(x), G(s))$  where:*

$$\beta(x) = x - \int_0^x \left( \frac{F(y)}{F(x)} \right)^2 dy \quad (6)$$

$$G(s) = F(\beta^{-1}(s)) = \Pr\{\beta(X) < s\} \quad (7)$$

Some features of Proposition 1 is noteworthy. The equilibrium bidding strategy of the informed partner,  $\beta(\cdot)$ , is the same as that in a WBA where both partners have private signals, symmetrically and independently distributed according to  $F$ .<sup>6</sup> Although partner B has no private information on the asset value, he adopts a randomization strategy in equilibrium such that the distribution of competing bid that partner A faces is the same as if he were confronting a bidder with private value that is independently distributed, also according to  $F$ . In the next proposition, we characterize the two partners' expected payoffs.

**Proposition 2.** *The two partners' expected payoffs are respectively:*

$$E[U_A] = \frac{1}{2}E[X] + \frac{1}{2} \left( \int_0^{\bar{x}} (F(x) - F(x)^2) dx \right) \quad (8)$$

$$E[U_B] = \frac{1}{2}E[X] - \frac{1}{2} \left( \int_0^{\bar{x}} (F(x) - F(x)^2) dx \right) \quad (9)$$

Since  $F(x) \geq F(x)^2$ ,  $E[U_A] > \frac{1}{2}E[X]$  and  $E[U_B] < \frac{1}{2}E[X]$ . The surprising part of the result is that the uninformed partner, partner B, suffers from an information loss: the uninformed partner expects to be worse off than his status quo, ownership of half of the expected asset value. Due to his ownership of the asset, the uninformed partner is entitled to half of the winning bid to exchange for his share of the asset. If the partnership is not dissolved, the value of his share diminishes quickly. This gives the informed partner an opportunity to hold him up if WBA is used to dissolve the partnership. The uninformed

---

<sup>6</sup>See [Lengwiler and Wolfstetter \(2005\)](#) for the derivation of equilibrium strategies under symmetric independent private values' framework.



partner thus experiences an “ownership’s curse”, which does not occur in standard first-price auctions where no share of the auction revenue goes to the loser and the uninformed partner can assure himself of no loss by placing a zero bid.

**Corollary 1.** *Partner A’s ex ante expected payoff is larger than a half of the expected asset value. Partner B’s ex ante expected payoff is smaller than a half of the expected asset value. Partner A obtains an information rent*

$$\Delta_{WBA} = E[U_A] - E[U_B] = \int_0^{\bar{x}} (F(x) - F(x)^2) dx \quad (10)$$

**Example 1.** *Suppose  $X$  is uniformly distributed on  $[0, 1]$ . In WBA, the two partners’ equilibrium strategies are respectively  $\beta(x) = \frac{2}{3}x$  and  $G(s) = \frac{3}{2}s$ . The intervals of serious bids are identically given by  $[0, \frac{2}{3}]$ . Partner A’s equilibrium payoff is  $\frac{1}{3}$ . Partner 2’s expected payoff is  $\frac{1}{6}$ . Partner A gets an information rent equal to  $\frac{1}{6}$ .*

## 4 The Loser’s Bid Auction

In LBA, the partner with a higher bid wins the auction, receives the asset and pays half of the lower bid to the losing partner. Ties are again resolved by the flip of a coin.

In analogy to WBA, the only equilibrium that exists is in mixed strategies. Partner A plays a continuous monotonically increasing pure strategy  $\beta : [0, \bar{x}] \rightarrow [\underline{b}, \bar{x}]$  with  $\underline{b} > 0$ . Partner B plays mixed strategy by randomizing his bid according to an atomless distribution function  $G(\cdot)$  over the range  $[\underline{b}, \bar{x}]$ .

Given partner B’s equilibrium strategy  $G(\cdot)$ , partner A chooses bid  $b$  to maximize the following objective:

$$U_A(x, b) = xG(b) - \frac{1}{2} \int_0^b tg(t)dt + \frac{1}{2}b(1 - G(b))$$

By placing a bid  $b$ , partner A wins the auction with probability  $G(b)$ . When he wins, he receives the asset which values  $x$  to him and pays a half of partner B’s bid to partner B. Since partner B randomizes his bid, partner A conditions his expected payoff on the fact that partner B has placed a bid lower than his own bid  $b$ . If losing, partner A receives half of his own bid  $b$  and gives up his share of the asset to partner B. From the first order condition, we get:

$$2xg(b) - 2bg(b) + 1 - G(b) = 0$$

Replacing  $b$  by  $\beta(x)$ , we obtain

$$2xg(\beta(x)) - 2\beta(x)g(\beta(x)) + 1 - G(\beta(x)) = 0 \quad (11)$$

Given partner A's strategy  $\beta(X)$ , partner B chooses bid  $s$  to maximize:

$$U_B(s) = \int_0^{\beta^{-1}(s)} \left( t - \frac{1}{2}\beta(t) \right) f(t)dt + \frac{1}{2}s \left( 1 - F(\beta^{-1}(s)) \right) \quad (12)$$

When placing a bid  $s$ , partner B wins the auction if  $\beta(X) \leq s$ , or equivalently  $X \leq \beta^{-1}(s)$ . On winning, partner B updates his expectation of the asset value to  $E[X | X \leq \beta^{-1}(s)]$ , and pays partner A a half of  $E[\beta(X) | X \leq \beta^{-1}(s)]$ . Partner B loses with probability  $1 - F(\beta^{-1}(s))$ . If he loses, he gives up his own share of the asset and receives half of his own bid  $s$  from partner A as a compensation. As partner B plays mixed strategy in equilibrium, any choice of  $s$  must bring him the same expected payoff. Therefore, the derivative of  $U_B(s)$  with respect to  $s$  must be equal to 0.

$$\frac{d}{ds}U_B(s) = \left( \beta^{-1}(s) - s \right) f(\beta^{-1}(s)) \frac{d\beta^{-1}(s)}{ds} + \frac{1}{2} \left( 1 - F(\beta^{-1}(s)) \right) = 0 \quad (13)$$

Since the two partners' equilibrium strategy spaces are identical, for any  $s$  in partner B's strategy space, there must exist an  $x$  such that  $\beta^{-1}(s) = x$ . Equation (13) can be transformed into:

$$(x - \beta(x)) f(x) \frac{1}{\beta'(x)} + \frac{1}{2} (1 - F(x)) = 0$$

Rearranging terms, we get:

$$2xf(x) = 2\beta(x)f(x) - \beta'(x) (1 - F(x)) \quad (14)$$

Solutions to simultaneous differential equations (11) and (14) define the equilibrium strategies of the two partners.

**Proposition 3.** *The loser's bid auction has a unique equilibrium  $(\beta(x), G(s))$  where:*

$$\beta(x) = x + \int_x^{\bar{x}} \left( \frac{1 - F(t)}{1 - F(x)} \right)^2 dt \quad (15)$$

$$G(s) = F(\beta^{-1}(s)) = \Pr\{\beta(X) < s\} \quad (16)$$

From Proposition 3, we learn that similar to WBA, in LBA the uninformed partner randomizes his bid in such a way that the distribution of competing bid that partner A faces in equilibrium is the same as if he were confronting a competitor who has also drawn a private independent signal according to distribution  $F(\cdot)$ .

In the next proposition, we derive the *ex ante* expected payoffs of the two partners.

**Proposition 4.** *The two partners' expected payoffs are given by:*

$$E[U_A] = \frac{1}{2}E[X] + \frac{1}{2} \int_0^{\bar{x}} (F(x) - F(x)^2) dx \quad (17)$$

$$E[U_B] = \frac{1}{2}E[X] - \frac{1}{2} \int_0^{\bar{x}} (F(x) - F(x)^2) dx \quad (18)$$

When we compare the results of Propositions 2 and 4, we see that the partners' payoffs from WBA and LBA are identical. In Proposition 1 and Proposition 3 we established that the uninformed partner randomizes his bid in such a way that partner A faces a competitor who has drawn an independent signal from the same distribution  $F$ , both in WBA and LBA. Lengwiler and Wolfstetter (2005) proved the “payoff equivalence” of WBA and LBA in symmetric independent private value framework. It is therefore a straightforward outcome that the expected payoffs of the partners in the current asymmetric common value framework are also identical under the two auction formats. This is a special feature of the information structure and can be interpreted as an extension of “revenue equivalence” of standard first and second price auctions in symmetric independent private value framework.

**Corollary 2.** *WBA and LBA are “payoff” equivalent.*

We close this section with a numerical example on LBA, assuming that  $X$  is uniformly distributed.

**Example 2.** *Suppose  $X$  is uniformly distributed on  $[0, 1]$ . In LBA, the equilibrium strategies are respectively  $\beta(x) = \frac{2}{3}x + \frac{1}{3}$  and  $G(s) = \frac{3s}{2} - \frac{1}{2}$ . The intervals of serious bids are identically given by  $[\frac{1}{3}, 1]$ . Partner A's equilibrium payoff is  $\frac{1}{3}$ . Partner B's expected payoff is  $\frac{1}{6}$ . Partner A gets an information rent equal to  $\frac{1}{6}$ .*

## 5 The Cake-Cutting Mechanism

CCM stipulates that one partner proposes a price  $p$  for the asset, the other partner has the option either to buy out the proposer or sells his own share to the proposer at that proposed price. Depending on the identity of the proposer, the equilibrium outcomes are drastically different, as we show below.

### 5.1 Uninformed Partner as the Proposer ( $CCM_B$ )

If the uninformed partner is the proposer, given any price  $p$  he proposes, partner A buys partner B out if and only if A's payoff from buying is larger than that from selling. That is the case if  $x - \frac{1}{2}p \geq \frac{1}{2}p$ . Therefore, partner A buys B out if and only if  $x \geq p$ .

Given A's strategy, partner B chooses price  $p$  to maximize his expected payoff

$$U_B(p) = \int_0^p xf(x)dx - \frac{1}{2}pF(p) + \frac{1}{2}p(1 - F(p)) \quad (19)$$

Partner B becomes buyer with probability  $F(p)$ . When he buys, he pays  $\frac{1}{2}p$  for A's share and updates his expectation of the asset value with the fact that  $X < p$ . Partner B becomes seller with probability  $1 - F(p)$ . In that case, he gives up his share of the asset for  $\frac{1}{2}p$ .

**Proposition 5.** *In  $CCM_B$ , partner B proposes a price  $p$  equal to  $x^m$ , the median of  $X$ . Given proposed price  $p$ , partner A buys if  $x \geq p$  and sells if  $x < p$ .*

*Proof.* The second part of the proposition is clear from above discussions. Partner B's optimal pricing strategy is determined by maximizing  $U_B(p)$  with respect to  $p$ . The first order condition  $\frac{dU_B(p)}{dp} = 0$  gives  $F(p) = \frac{1}{2}$ . Therefore, it is optimal for partner B to propose a price  $p = x^m$ .  $\square$

The dissolution outcome of  $CCM_B$  is that partner A becomes single owner if  $x \geq x^m$  and partner B becomes the single owner otherwise.

**Proposition 6.** *In  $CCM_B$ , the two partners' ex ante expected payoffs are:*

$$E[U_A] = \int_{x^m}^{\bar{x}} xf(x)dx = E[X | X \geq x^m] \quad (20)$$

$$E[U_B] = \int_0^{x^m} xf(x)dx = E[X | X < x^m] \quad (21)$$

*Proof.* Partner B's payoff is obtained by substituting  $p = x^m$  into equation (19). Since partner A becomes the single owner if and only if  $x \geq x^m$ , his payoff is equal to  $x - \frac{1}{2}x^m$  if  $x \geq x^m$  and  $\frac{1}{2}x^m$  if  $x < x^m$ . Thus partner A's ex ante expected payoff is

$$\begin{aligned} E[U_A] &= \int_0^{x^m} \frac{1}{2}x^m f(x)dx + \int_{x^m}^{\bar{x}} \left(x - \frac{1}{2}x^m\right) f(x)dx \\ &= \int_{x^m}^{\bar{x}} xf(x)dx = E[X | X \geq x^m]. \end{aligned}$$

$\square$

Partner A's information rent is:

$$\Delta_{CCM_B} = E[U_A] - E[U_B] = \int_{x^m}^{\bar{x}} xf(x)dx - \int_0^{x^m} xf(x)dx \quad (22)$$

This is obviously nonnegative since  $E[U_A] \geq x^m \geq E[U_B]$ . Furthermore, the information rent depends on the distribution of  $X$  as  $\Delta_{CCM_B}$  decreases in  $x^m$ .

**Corollary 3.** *In  $CCM_B$ , Partner A always benefits from an information rent, which decreases in  $x^m$ .*

## 5.2 Informed Partner as the Proposer ( $CCM_A$ )

Now, suppose the informed partner, partner A, is the proposer. Partner B has to decide whether he would like to buy or sell given a price proposed by partner A. Firstly, as partner B has no private information on the value of the asset, playing pure strategy is dominated by mixing between buying and selling. Suppose he does play pure strategy by selling if  $p \geq \tilde{p}$  and buying otherwise. Then partner A's best strategy is to propose  $p = \tilde{p} + \epsilon$  if  $x > \tilde{p}$  and proposes  $p = \tilde{p} - \epsilon$  otherwise, where  $\epsilon$  is some arbitrarily small positive number. However, given A's strategy, partner B is better off if he buys instead when he observes  $p \geq \tilde{p}$  and sells when he observes  $p < \tilde{p}$ , which destroys the optimality of the assumed pure strategy. Moreover, it is optimal for partner B to randomize equally between buying and selling. Suppose partner B randomizes between "buying" and "selling" with probability  $(q, 1 - q)$ , with  $q > \frac{1}{2}$ . As partner B buys more frequently than sells, the optimal strategy for partner A is to propose a maximal price  $\bar{x}$ , independent of his signal  $x$ . Given price  $p = \bar{x}$ , partner B nevertheless would like to sell with probability 1 instead of mixing between buying and selling, since almost for sure the asset value is below  $\bar{x}$ . This destroys the optimality of  $q > \frac{1}{2}$ . A similar argument rules out  $q < \frac{1}{2}$  to be an equilibrium. Therefore, partner B's best strategy is to randomize equally between buying and selling.

**Proposition 7.** *In  $CCM_A$ , partner A proposes a price equal to his true signal, ie  $p = x$ , in equilibrium. The uninformed partner, partner B, randomizes equally between buying and selling.*

*Proof.* We have already established that partner B randomizes equally between buying and selling given any price  $p$ . Given B's strategy, partner A chooses price  $p$  to maximize his expected payoff

$$U_A(p) = \frac{1}{2}\left(x - \frac{p}{2}\right) + \frac{1}{2} \cdot \frac{p}{2} = \frac{1}{2}x$$

which is independent of the price he quotes. Therefore, proposing a price equal to his true signal is partner A's optimal strategy.  $\square$

Obviously, the outcome of  $CCM_A$  is that each partner becomes the single owner of the asset with equal probability, hence the asset value is shared equally between the two partners.

**Proposition 8.** *In  $CCM_A$ , the two partners share the asset value equally in equilibrium. Partner A does not get any information rent.*

*Proof.* Straightforward from Proposition 7. □

We close this section with a numerical example assuming that  $X$  is uniformly distributed.

**Example 3.** Suppose  $X$  is uniformly distributed on  $[0, 1]$ . In  $CCM_B$ , partner  $B$  proposes a price equal to  $\frac{1}{2}$ . Partner  $A$  sells if his signal is below  $\frac{1}{2}$  and buys otherwise. Partner  $A$ 's expected payoff is  $\frac{3}{8}$ . Partner  $B$ 's expected payoff is  $\frac{1}{8}$ . The information rent of partner  $A$  is  $\frac{1}{4}$ . In  $CCM_A$ , partner  $A$  and  $B$  buy the asset with equal probability. Each partner gets an expected payoff equal to  $\frac{1}{4}$ .

## 6 Prices for Proprietary Information

A comparison of information rents arising from the three institutions is given in the following proposition.

**Proposition 9.** *Information rents accruing to partner  $A$  under the three dissolution rules are ranked as follows*

$$\Delta_{CCM_B} \geq \Delta_{WBA} = \Delta_{LBA} > \Delta_{CCM_A} \quad \text{if } x^m \leq \tilde{x} \quad (23)$$

$$\Delta_{WBA} = \Delta_{LBA} > \Delta_{CCM_B} > \Delta_{CCM_A} \quad \text{if } x^m > \tilde{x} \quad (24)$$

where  $\tilde{x}$  is determined by

$$\int_0^{\tilde{x}} (F(x) - F(x)^2) dx = \int_{\tilde{x}}^{\bar{x}} xf(x)dx - \int_0^{\tilde{x}} xf(x)dx.$$

When information is exogenous,  $CCM_A$  is the ideal mechanism as it does not generate any information rent and allocates the asset fairly between the two partners.

When we consider instead endogenous information, for example an environment where neither partner holds any private information about the asset value  $X$  at the time dissolution is agreed upon but one partner has the opportunity of learning about  $x$  at fixed cost of  $c$ , the reward offered by different rules for proprietary information is given exactly by the information rents they generate to the informed partner.

**Corollary 4.** *The prices for proprietary information offered by different dissolution rules are ranked by (23) and (24).*

If and only if the price offered by a dissolution rule exceeds cost  $c$  will a partner invest in information acquisition. A direct observation is that  $CCM_A$ , though allocates the asset value fairly between the two partners, offers zero price for proprietary information and hence fails to provide any incentive for information acquisition. Therefore, if  $CCM_A$  is the dissolution rule in use, the asset value will not be revealed in transaction and dissolution will occur in complete ignorance of the true value of the partnership.

## 7 Conclusions

In dissolving a common-value partnership where one partner holds proprietary information, simple mechanisms (WBA, LBA, and CCM) have very different properties in comparison to situations where no such proprietary information exists. In WBA and LBA, the unique equilibrium is in mixed strategies. The informed partner receives an information rent and the uninformed partner suffers from an “ownership’s curse”. WBA and LBA are “payoff equivalent” as the partners receive the same expected payoffs under the two auction formats. When CCM is applied, whether pure strategy equilibrium exists or not depends on the identity of the proposer. If the informed partner is the proposer ( $CCM_A$ ), only mixed strategy equilibrium exists where the informed partner fully reveals his information and the two partners share the asset value equally. When the uninformed is the proposer ( $CCM_B$ ), the informed partner gets an information rent in a pure strategy equilibrium. Comparison of the dissolution rules show that  $CCM_B$  may generate higher or lower information rents than  $WBA$  and  $LBA$ , depending on the distribution of asset value, while  $CCM_A$  generates zero information rent. Using the ranking of information rents, we are able to compare the prices different rules offer for proprietary information.  $CCM_A$ , though allocates the asset value equally between the two partners, generates zero reward for proprietary information and hence does not provide any incentive for information acquisition.

## Appendix

**Proof of Lemma 1.** Suppose there exists an equilibrium where partner B employs a pure strategy and always bids a certain amount  $s$ . Given partner B’s strategy, partner A’s best response is to bid  $s + \epsilon$  if  $x > s$  and bid  $s - \epsilon$  if  $x < s$ , where  $\epsilon$  is an arbitrary

positive number. Therefore, partner B's expected payoff by bidding  $s$  is

$$\Pr\{X \geq s\} \frac{1}{2}(s + \epsilon) + \Pr\{X < s\} \left( E[X | X < s] - \frac{1}{2}s \right) \quad (25)$$

Now, given partner A's strategy, if partner B bids  $s + 2\epsilon$ , his expected payoff is

$$\Pr\{X \geq s\} \cdot \left( E[X | X \geq s] - \frac{1}{2}(s + 2\epsilon) \right) + \Pr\{X < s\} \cdot \frac{1}{2}(s - \epsilon) \quad (26)$$

For small  $\epsilon$ , we have

$$\begin{aligned} E[X | X \geq s] - \frac{1}{2}(s + 2\epsilon) &> \frac{1}{2}(s + \epsilon) \\ \text{and } \frac{1}{2}(s - \epsilon) &> E[X | X < s] - \frac{1}{2}s \end{aligned}$$

Therefore, (26) is strictly larger than (25). Partner B has an incentive to bid  $s + 2\epsilon$  rather than  $s$ , destroying the optimality of  $s$ . Hence it is confirmed that there is no equilibrium that partner B plays pure strategy.  $\square$

**Proof of Lemma 2.** Given partner B's bid  $S$ , we first show that  $\beta(x)$  is nondecreasing on  $[0, \bar{x}]$ . Let  $b' = \beta(x')$  and  $b'' = \beta(x'')$  with  $x' \geq x''$ . For  $b'$  and  $b''$  to be respectively the equilibrium bid when the signal is  $x'$  and  $x''$ , it is necessary to have  $U_A(x', b', S) \geq U_A(x', b'', S)$  and  $U_A(x'', b'', S) \geq U_A(x'', b', S)$ . That is:

$$\begin{aligned} \Pr\{S < b'\} \cdot (x' - \frac{1}{2}b') + \Pr\{S \geq b'\} \cdot \frac{1}{2}E[S | S \geq b'] \\ \geq \Pr\{S < b''\} \cdot (x' - \frac{1}{2}b'') + \Pr\{S \geq b''\} \cdot \frac{1}{2}E[S | S \geq b''] \end{aligned} \quad (27)$$

$$\begin{aligned} \Pr\{S < b''\} \cdot (x'' - \frac{1}{2}b'') + \Pr\{S \geq b''\} \cdot \frac{1}{2}E[S | S \geq b''] \\ \geq \Pr\{S < b'\} \cdot (x'' - \frac{1}{2}b') + \Pr\{S \geq b'\} \cdot \frac{1}{2}E[S | S \geq b'] \end{aligned} \quad (28)$$

Combining (27) and (28), we get:

$$(\Pr\{S < b'\} - \Pr\{S < b''\}) (x' - x'') \geq 0 \quad (29)$$

Since  $x' \geq x''$ , one must have  $\Pr\{S < b'\} \geq \Pr\{S < b''\}$ , which implies  $b' \geq b''$ .

Secondly,  $\beta(x)$  must be gapless. Suppose there is a gap  $[b', b]$ , then there must be a gap  $(b', b)$  in partner B's choice of bids  $S$ . For partner B, any bid from  $(b', b)$  is dominated by a bid  $s = b'$  because that lowers the price partner B pays in case of winning without affecting the probability of winning. Therefore, partner B will place zero probability on any bid in the interval  $(b', b)$ . But then given B's strategy, the type of partner A that



plans to bid  $b$  finds it better to lower his bid because that reduces the price he pays in case of winning without lowering his winning probability. Therefore,  $\beta(x)$  can not have any gap. This establishes the argument that  $\beta(x)$  is continuous and nondecreasing in  $x$ .

Thirdly, we establish that  $\beta(x)$  and  $G(\cdot)$  are atomless. Suppose for partner B there exists an  $s$  such that  $\Pr\{S = s\} > 0$ . Then there exists an  $\epsilon > 0$  such that partner A will assign zero probability to any bid in the interval  $(s - \epsilon, s)$ , creating a gap. Similar reasoning leads to the result that  $\beta(\cdot)$  can not have any atom either.

Since  $\beta(\cdot)$  is continuous, atomless and nondecreasing, partner A's strategy is strictly increasing. Continuity of  $\beta(\cdot)$  implies that it is differentiable almost everywhere on its domain.

We still need to determine the boundaries. We will first show that partner B's action space must be  $[\beta(0), \beta(\bar{x})]$ , and then show that  $\beta(0) = 0$  and  $\beta(\bar{x}) < \bar{x}$ . Given that partner A chooses his bid from  $[\beta(0), \beta(\bar{x})]$ , partner B does not have any incentive to choose an action outside that interval. Suppose  $\beta(\bar{x}) + \epsilon$  is also in his action set. It is then strictly dominated by  $\beta(\bar{x})$  at which he wins almost for sure but pays a lower price conditional on winning. Suppose there exists a small interval  $[\beta(0), \beta(0) + \epsilon]$  that is not a part of partner B's randomization strategy. Then partner A by placing a bid below  $\beta(0) + \epsilon$  loses for sure. Then there must exist a small interval below  $\beta(0) + \epsilon$  such that partner A would like to increase his bid to win with some positive probability, hence violating the assumption that  $\beta(\cdot)$  is the assumed equilibrium. Hence, the supports of the two partners' serious bids are identically given by  $[\beta(0), \beta(\bar{x})]$ .

When  $x = 0$ , suppose  $\beta(0) > 0$ . Then partner A wins the auction with a positive probability, in which case he gets the asset and pays half of his bid to partner B, resulting in a negative payoff. This is strictly dominated by bidding zero and ensuring himself a nonnegative payoff. Therefore,  $\beta(0) = 0$ .

When  $x = \bar{x}$ , partner A does not propose any price above  $\bar{x}$  because doing so is strictly dominated by proposing a price equal to  $\bar{x}$ . In both cases partner A wins the auction with probability 1 but the latter leads to a lower payment. By bidding  $\bar{x} - \epsilon$ , partner A still wins almost for sure but makes even a smaller payment when he wins. For small  $\epsilon$ , incremental payoff from a lower payment dominates the payoff decrease from a slightly lower winning probability. Thus,  $\beta(\bar{x}) = \bar{b} < \bar{x}$ .  $\square$

**Proof of Proposition 1.** The plan is first to show that  $\beta(\cdot)$  and  $G(\cdot)$  are mutually best responses, then establish the uniqueness of the equilibrium. Given partner B's random-

ization strategy  $G(\cdot)$ , if partner A with signal  $x$  bids  $\beta(x')$ , his payoff is:

$$U_A(x, x') = G(\beta(x'))(x - \frac{1}{2}\beta(x')) + \frac{1}{2} \int_{\beta(x')}^{\bar{b}} yg(y)dy \quad (30)$$

Its derivative with respect to  $x'$  is:

$$\frac{\partial U_A}{\partial x'}(x, x') = \left( g(\beta(x'))x - \frac{1}{2}G(\beta(x')) - \beta(x')g(\beta(x')) \right) \frac{d\beta(x')}{dx} \quad (31)$$

From B's strategy, we have:

$$G(\beta(x')) = F(\beta^{-1}(\beta(x'))) = F(x'), \quad g(\beta(x')) \frac{d\beta(x')}{dx} = f(x') \quad (32)$$

Applying (32) to (31), we get:

$$\begin{aligned} \frac{\partial U_A}{\partial x'}(x, x') &= f(x')x - \frac{1}{2}F(x') \frac{d\beta(x')}{dx} - \beta(x')f(x') \\ &= \frac{1}{2F(x')} \left( 2F(x')f(x')x - F(x')^2 \frac{d\beta(x')}{dx} - 2\beta(x')F(x')f(x') \right) \\ &= \frac{1}{2F(x')} \left( 2F(x')f(x')x - \frac{d(F(x')^2\beta(x'))}{dx} \right) \end{aligned}$$

Using (6), one gets:

$$\beta(x') = x' - \int_0^{x'} \left( \frac{F(y)}{F(x')} \right)^2 dy = \frac{1}{F(x')^2} \int_0^{x'} 2yf(y)F(y)dy$$

Therefore,

$$\begin{aligned} \frac{\partial U_A}{\partial x'}(x, x') &= \frac{1}{2F(x')} \left( 2F(x')f(x')x - \frac{d \int_0^{x'} 2yf(y)F(y)dy}{dx'} \right) \\ &= \frac{1}{2F(x')} (2F(x')f(x')x - 2F(x')f(x')x') \\ &= f(x')(x - x') \end{aligned}$$

Since  $\frac{\partial U_A}{\partial x'}(x, x')$  is positive if  $x' < x$  and negative if  $x' > x$ , the best choice for partner A is  $x' = x$ . This confirms that  $\beta(x)$  is indeed partner A's best response to the proposed strategy of partner B.

We shall now show that given  $\beta(X)$ , any bid of partner B from his action space leads to the same expected payoff. From Lemma 2, we learned that the supports of the two partners' equilibrium strategies are the same, ie  $[0, \bar{b}]$ . Therefore, for any of partner B's bid  $s \in [0, \bar{b}]$ , one can find a value  $x''$  such that  $s = \beta(x'')$ , or equivalently,  $\beta^{-1}(s) = x''$ . One can rewrite partner B's expected payoff (3) as a function of  $x''$  as follows:

$$U_B(x'') = \int_0^{x''} yf(y)dy - \frac{1}{2}\beta(x'')F(x'') + \frac{1}{2} \int_{x''}^{\bar{x}} \beta(y)f(y)dy$$

Partner B's maximization problem is equivalent to choosing an  $x''$  such that  $U_B(\cdot)$  is maximized. If any random bid  $s$  from  $[0, \bar{b}]$  leads to the same expected payoff for partner B, it must also be the case with the associated  $x''$  from interval  $[0, \bar{x}]$ . Taking the derivative of  $U_B(x'')$  with respect to  $x''$ , we get:

$$\begin{aligned}
\frac{dU_B(x'')}{dx''} &= x''f(x'') - \frac{1}{2}F(x'')\frac{d\beta(x'')}{dx} - f(x'')\beta(x'') \\
&= \frac{1}{2F(x'')} \left( 2x''F(x'')f(x'') - \frac{d\beta(x'')}{dx}F(x'')^2 - 2\beta(x'')F(x'')f(x'') \right) \\
&= \frac{1}{2F(x'')} \left( 2x''F(x'')f(x'') - \frac{d \int_0^{x''} 2yf(y)F(y)dy}{dx''} \right) \\
&= \frac{1}{2F(x'')} (2x''F(x'')f(x'') - 2x''F(x'')f(x'')) \\
&= 0
\end{aligned}$$

This confirms that the proposed strategies are indeed mutually best responses.

$\beta(\cdot)$  is uniquely determined by differential equation (5). Using the results from Lemma 2 that  $\beta(\bar{x}) = \bar{b}$  and the supports of the two partners' serious bids are identical in equilibrium,  $G(\bar{b}) = 1$ . Thus,  $G(\cdot)$  is uniquely determined by equation (2).  $\square$

**Proof of Proposition 2.** Denote partner A's equilibrium payoff when he has received signal  $x$  as  $U_A(x)$ . From (1), we get

$$U_A(x) = G(\beta(x))(x - \frac{1}{2}\beta(x)) + \frac{1}{2} \int_{\beta(x)}^{\bar{b}} yg(y)dy$$

Using

$$G(\beta(x)) = F(\beta^{-1}(\beta(x))) = F(x), \quad g(\beta(x)) \cdot \frac{d\beta(x)}{dx} = f(x),$$

partner A's equilibrium payoff can be transformed into:

$$\begin{aligned}
U_A(x) &= F(x) \left( x - \frac{1}{2}\beta(x) \right) + \frac{1}{2} \int_x^{\bar{x}} \beta(y)f(y)dy \\
&= F(x) \left( x - \frac{1}{2} \left( x - \int_0^x \left( \frac{F(y)}{F(x)} \right)^2 dy \right) \right) + \frac{1}{2} \int_x^{\bar{x}} \left( - \int_0^y \left( \frac{F(s)}{F(y)} \right)^2 ds \right) f(y)dy \\
&= \frac{1}{2}F(x)x + \frac{1}{2F(x)} \int_0^x F(y)^2 dy + \frac{1}{2} \int_x^{\bar{x}} yf(y)dy + \frac{1}{2} \int_x^{\bar{x}} \int_0^y F(s)dsd \left( \frac{1}{F(y)} \right)
\end{aligned}$$

Applying integration by parts to the third and fourth term, one obtains:

$$\begin{aligned}
U_A(x) &= \frac{1}{2}F(x)x + \frac{1}{2F(x)} \int_0^x F(y)^2 dy + \frac{1}{2} \left( \bar{x}F(\bar{x}) - xF(x) - \int_x^{\bar{x}} F(y)dy \right) \\
&\quad + \frac{1}{2} \left( \int_0^{\bar{x}} F(s)^2 ds - \frac{1}{F(x)} \int_0^x F(s)^2 ds - \int_x^{\bar{x}} F(s)ds \right) \\
&= \frac{1}{2}\bar{x}F(\bar{x}) - \int_x^{\bar{x}} F(s)ds + \frac{1}{2} \int_0^{\bar{x}} F(s)^2 ds
\end{aligned}$$

Partner A's *ex ante* expected payoff is:

$$\begin{aligned}
E[U_A] &= \int_0^{\bar{x}} U_A(x)f(x)dx \\
&= \frac{1}{2}\bar{x}F(\bar{x}) + \frac{1}{2}\int_0^{\bar{x}} F(s)^2 ds - \int_0^{\bar{x}} \int_x^{\bar{x}} F(s)dsf(x)dx \\
&= \frac{1}{2}\left(\bar{x}F(\bar{x}) - \int_0^{\bar{x}} F(x)^2 dx\right)
\end{aligned}$$

The unconditional expected asset value is:

$$E[X] = \int_0^{\bar{x}} xf(x)dx = \bar{x}F(\bar{x}) - \int_0^{\bar{x}} F(x)dx$$

Therefore, partner A's expected payoff can be written as

$$E[U_A] = \frac{1}{2}E[X] + \frac{1}{2}\left(\int_0^{\bar{x}} (F(x) - F(x)^2)dx\right)$$

Since the bid distributions of A and B have identical supports in equilibrium and  $\beta(\cdot)$  is a monotonic function, for any bid  $s$  of partner B, there must exist an  $x$  such that  $s = \beta(x)$ . Therefore, partner B's maximization problem can be equivalently rewritten as a function of  $x$ :

$$U_B(x) = \int_0^x yf(y)dy - \frac{1}{2}\beta(x)F(x) + \frac{1}{2}\int_x^{\bar{x}} \beta(y)f(y)dy$$

Substituting  $\beta(x)$  from (6), we get

$$\begin{aligned}
U_B(x) &= \int_0^x yf(y)dy - \frac{1}{2}F(x)\left(x - \int_0^x \left(\frac{F(y)}{F(x)}\right)^2 dy\right) + \frac{1}{2}\int_x^{\bar{x}} \left(y - \int_0^y \left(\frac{F(t)}{F(y)}\right)^2 dt\right) f(y)dy \\
&= \int_0^x ydF(y) - \frac{1}{2}F(x)x + \frac{1}{2F(x)}\int_0^x F(y)^2 dy + \frac{1}{2}\int_x^{\bar{x}} ydF(y) + \frac{1}{2}\int_x^{\bar{x}} \int_0^y F(t)^2 dt d\left(\frac{1}{F(y)}\right)
\end{aligned}$$

Applying integration by parts to the first, the fourth and the last term, one gets:

$$\begin{aligned}
U_B(x) &= \left(xF(x) - \int_0^x F(y)dy\right) - \frac{1}{2}F(x)x + \frac{1}{2F(x)}\int_0^x F(y)^2 dy + \frac{1}{2}\left(\bar{x}F(\bar{x}) - xF(x) - \int_x^{\bar{x}} F(y)dy\right) \\
&\quad + \frac{1}{2}\left(\int_0^{\bar{x}} F(y)^2 dy - \frac{1}{F(x)}\int_0^x F(y)^2 dy - \int_x^{\bar{x}} F(y)dy\right) \\
&= \frac{1}{2}\bar{x} - \int_0^{\bar{x}} F(x)dx + \frac{1}{2}\int_0^{\bar{x}} F(x)^2 dx \\
&= \frac{1}{2}E[X] - \int_0^{\bar{x}} (F(x) - F(x)^2) dx
\end{aligned}$$

which is indeed independent of  $x$ . □

**Proof of Proposition 3.** Similar to the proof of Proposition 1, the plan is to show that  $\beta(\cdot)$  and  $G(\cdot)$  are mutually best responses. Given partner B's randomization strategy  $G(\cdot)$ , if partner A with signal  $x$  bids an amount  $\beta(x')$ , his payoff is:

$$U_A(x, x') = xG(\beta(x')) - \frac{1}{2} \int_0^{\beta(x')} tg(t)dt + \frac{1}{2}\beta(x')(1 - G(\beta(x')))$$

Taking derivative with respect to  $x'$  gives

$$\frac{\partial U_A}{\partial x'}(x, x') = xg(\beta(x'))\frac{d\beta(x')}{dx'} - \beta(x')g(\beta(x'))\frac{d\beta(x')}{dx'} + \frac{1}{2}(1 - G(\beta(x')))\frac{d\beta(x')}{dx'}$$

Using partner B's strategy, we have

$$G(\beta(x')) = F(\beta^{-1}(\beta(x'))) = F(x'), \quad g(\beta(x'))\frac{d\beta(x')}{dx'} = f(x')$$

Therefore,

$$\begin{aligned} \frac{\partial U_A}{\partial x'}(x, x') &= xf(x') - \beta(x')f(x') + \frac{1}{2}(1 - F(x'))\frac{d\beta(x')}{dx'} \\ &= xf(x') + \frac{1}{2(1 - F(x'))} \left( -2\beta(x')f(x')(1 - F(x')) + (1 - F(x'))^2 \frac{d\beta(x')}{dx'} \right) \\ &= xf(x') + \frac{1}{2(1 - F(x'))} \frac{d(\beta(x')(1 - F(x'))^2)}{dx} \end{aligned}$$

Using

$$\beta(x') = -\frac{1}{(1 - F(x'))^2} \int_{x'}^{\bar{x}} td(1 - F(t))^2$$

we get

$$\begin{aligned} \frac{\partial U_A}{\partial x'}(x, x') &= xf(x') - \frac{1}{2(1 - F(x'))} \frac{d \int_{x'}^{\bar{x}} td(1 - F(t))^2}{dx} \\ &= xf(x') - \frac{1}{2(1 - F(x'))} (2x'(1 - F(x'))f(x')) \\ &= f(x')(x - x') \end{aligned}$$

Since  $\frac{\partial U_A}{\partial x'}(x, x')$  is positive if  $x > x'$ , negative if  $x < x'$ , and equal to zero if  $x = x'$ ,  $x = x'$  is the best choice for partner A. This confirms that  $\beta(\cdot)$  is indeed partner A's best response.

We now turn to show that given  $\beta(X)$ , if partner B chooses his bid according to  $G(s)$ , any  $s$  from  $[\underline{b}, \bar{x}]$  brings partner B the same expected payoff. For any  $s \in [\underline{b}, \bar{x}]$ , one can find a value  $x''$  such that  $s = \beta(x'')$ , or equivalently  $x'' = \beta^{-1}(s)$ . Rewrite partner B's expected payoff (12) as a function of  $x''$ , we get:

$$U_B(x'') = \int_0^{x''} \left( t - \frac{1}{2}\beta(t) \right) f(t)dt + \frac{1}{2}\beta(x'')(1 - F(x''))$$

Its derivative with respect to  $x''$  is

$$\begin{aligned}
\frac{dU_B(x'')}{dx} &= x''f(x'') - \beta(x'')f(x'') + \frac{1}{2}(1 - F(x''))\frac{d\beta(x'')}{dx} \\
&= x''f(x'') + \frac{1}{2(1 - F(x''))}\frac{d((1 - F(x''))^2\beta(x''))}{dx} \\
&= x''f(x'') + \frac{1}{2(1 - F(x''))}\frac{d\int_{x''}^{\bar{x}} td(1 - F(t))^2}{dx} \\
&= x''f(x'') - x''f(x'') \\
&= 0
\end{aligned}$$

This confirms that  $G(s)$  is indeed partner B's best response to partner A's strategy  $\beta(\cdot)$ .

Obviously, partner A's strategy  $\beta(\cdot)$  is uniquely determined by equation (14). It has been established that  $\beta(0) = \underline{b}$  and the two partners active bids have identical supports. Hence  $G(\underline{b}) = 0$ .  $G(\cdot)$  is therefore uniquely determined by equation (11).  $\square$

**Proof of Proposition 4.** Conditional on his signal  $x$ , partner A's payoff from LBA when both partners play the equilibrium strategies is:

$$U_A(x) = xG(\beta(x)) - \frac{1}{2}\int_0^{\beta(x)} tg(t)dt + \frac{1}{2}\beta(x)(1 - G(\beta(x)))$$

Using  $G(\beta(x)) = F(x)$  and  $g(\beta(x)) \cdot \frac{d\beta(x)}{dx} = f(x)$ , the above objective can be transformed into:

$$U_A(x) = xF(x) - \frac{1}{2}\int_0^x \beta(t)f(t)dt + \frac{1}{2}\beta(x)(1 - F(x))$$

Using (15), we obtain

$$\begin{aligned}
U_A(x) &= xF(x) - \frac{1}{2}\int_0^x \left( t + \int_t^{\bar{x}} \left( \frac{1 - F(y)}{1 - F(t)} \right)^2 dy \right) f(t)dt \\
&\quad + \frac{1}{2}\left( x + \int_x^{\bar{x}} \left( \frac{1 - F(t)}{1 - F(x)} \right)^2 dt \right) (1 - F(x))
\end{aligned}$$

Rearranging terms, we get

$$\begin{aligned}
U_A(x) &= xF(x) - \frac{1}{2}\int_0^x tf(t)dt - \frac{1}{2}\int_0^x \int_t^{\bar{x}} \left( \frac{1 - F(y)}{1 - F(t)} \right)^2 dy f(t)dt \\
&\quad + \frac{1}{2}x(1 - F(x)) + \frac{1}{2(1 - F(x))}\int_x^{\bar{x}} (1 - F(t))^2 dt \\
&= \frac{1}{2}x + \frac{1}{2}\int_0^x F(t)dt + \frac{1}{2(1 - F(x))}\int_x^{\bar{x}} (1 - F(t))^2 dt \\
&\quad - \frac{1}{2}\int_0^x \int_t^{\bar{x}} (1 - F(y))^2 dy \frac{f(t)}{(1 - F(t))^2} dt
\end{aligned}$$

Applying integration by parts to the last term, we get

$$\begin{aligned}
U_A(x) &= \frac{1}{2}x + \frac{1}{2} \int_0^x F(t)dt + \frac{1}{2(1-F(x))} \int_x^{\bar{x}} (1-F(t))^2 dt \\
&\quad - \frac{1}{2} \left( \frac{1}{1-F(x)} \int_x^{\bar{x}} (1-F(y))^2 dy - \int_0^{\bar{x}} (1-F(y))^2 dy + \int_0^x (1-F(t))dt \right) \\
&= \int_0^x F(t)dt + \frac{1}{2} \int_0^{\bar{x}} (1-F(y))^2 dy
\end{aligned}$$

Therefore, partner A's ex ante expected payoff is:

$$\begin{aligned}
E[U_A] &= \int_0^{\bar{x}} \int_0^x U_A(x,x) f(x) dx \\
&= \int_0^{\bar{x}} \int_0^x F(t) dt f(x) dx + \frac{1}{2} \int_0^{\bar{x}} (1-F(y))^2 dy \\
&= \int_0^{\bar{x}} (F(t) - F(t)^2) dt + \frac{1}{2} \int_0^{\bar{x}} (1-F(y))^2 dy \\
&= \frac{1}{2} \left( \bar{x} - \int_0^{\bar{x}} F(y)^2 dy \right)
\end{aligned}$$

Making use of

$$E[X] = \int_0^{\bar{x}} x f(x) dx = \bar{x} - \int_0^{\bar{x}} F(x) dx$$

we can rewrite partner A's ex ante expected payoff as

$$E[U_A] = \frac{1}{2} E[X] + \frac{1}{2} \int_0^{\bar{x}} (F(x) - F(x)^2) dx$$

As partner B randomizes on the range  $[\underline{b}, \bar{x}]$ , his expected payoff can be derived easily when he chooses bid  $s = \bar{x}$  and wins with probability 1. His expected payoff is

$$U_B(\bar{x}) = \int_0^{\beta^{-1}(\bar{x})} \left( t - \frac{1}{2} \beta(t) \right) f(t) dt$$

Since  $\beta^{-1}(\bar{x}) = \bar{x}$ , we get

$$U_B(\bar{x}) = \int_0^{\bar{x}} t f(t) dt - \frac{1}{2} \int_0^{\bar{x}} \beta(t) f(t) dt$$

Substituting  $\beta(x)$  using (15), we get

$$\begin{aligned}
U_B(\bar{x}) &= \int_0^{\bar{x}} t f(t) dt - \frac{1}{2} \int_0^{\bar{x}} \left( t + \int_t^{\bar{x}} \left( \frac{1-F(y)}{1-F(t)} \right)^2 dy \right) f(t) dt \\
&= \int_0^{\bar{x}} t f(t) dt - \int_0^{\bar{x}} \int_x^{\bar{x}} t f(t) (1-F(t)) dt d(1-F(x))^{-1}
\end{aligned}$$

Applying integration by parts to the last term, we obtain

$$\begin{aligned}
U_B(\bar{x}) &= \int_0^{\bar{x}} t f(t) dt - \left( - \int_0^{\bar{x}} t f(t) (1-F(t)) dt + \int_0^{\bar{x}} x f(x) dx \right) \\
&= \int_0^{\bar{x}} t f(t) (1-F(t)) dt \\
&= \frac{1}{2} E[X] - \frac{1}{2} \int_0^{\bar{x}} (F(x) - F(x)^2) dx
\end{aligned}$$

□

**Proof of Proposition 9.** It is already established that  $CCM_A$  does not bring any information rent and WBA and  $LBA$  are “payoff equivalent”. We only need to compare the information rent from  $CCM_B$  and WBA. Recall that the information rent from WBA is given by:

$$\Delta_{WBA} = \int_0^{\bar{x}} (F(x) - F(x)^2) dx \quad (33)$$

which is a constant with respect to  $x^m$ . The information rent from  $CCM_B$  is given by:

$$\Delta_{CCM_B} = \int_{x^m}^{\bar{x}} xf(x)dx - \int_0^{x^m} xf(x)dx$$

which is a decreasing function of  $x^m$ . When  $x^m$  is close to zero,  $\Delta_{CCM_B}$  is close to  $E[X]$  and is larger than  $\Delta_{WBA}$ . When  $x^m$  is close to  $\bar{x}$ ,  $\Delta_{CCM_B}$  is close to 0 and thus smaller than  $\Delta_{WBA}$ . Recall the definition of  $\tilde{x}$ , then  $\Delta_{CCM_B} \geq \Delta_{WBA}$  if and only if  $x^m \leq \tilde{x}$ .  $\square$

## References

- BROOKS, R., AND K. SPIER (2004): “Trigger happy or gun shy? Dissolving Common-Value Partnerships with Texas Shootouts,” *Northwestern University, Kellogg Graduate School of Management, Working Paper*.
- CHIEN, H.-K. (2007): “Incentive Efficient Mechanisms for Partnership Dissolution,” *Mimeo*, University of Bonn.
- CRAMTON, P., R. GIBBONS, AND P. KLEMPERER (1987): “Dissolving a Partnership Efficiently,” *Econometrica*, 55, 615–32.
- DE FRUTOS, M. A. (2000): “Asymmetric Price-Benefits Auctions,” *Games and Economic Behavior*, 33, 48–71.
- DE FRUTOS, M. A., AND T. KITTSSTEINER (2008): “Efficient Partnership Dissolution under Buy/Sell Clauses,” *RAND Journal of Economics*, 39(1), 184–198.
- DUBRA, J. (2006): “A Correction to Uniqueness in “Competitive Bidding and Proprietary Information”,” *Journal of Mathematical Economics*, 42, 56–60.
- ENGELBRECHT-WIGGANS, R. (1994): “Auctions with Price-Proportional Benefits to All Bidders,” *Games and Economic Behavior*, 6(3), 339–46.
- ENGELBRECHT-WIGGANS, R., P. MILGROM, AND R. WEBER (1983): “Competitive Bidding and Proprietary Information,” *Journal of Mathematical Economics*, 11, 161–9.



- FIESELER, K., T. KITSTEINER, AND B. MOLDOVANU (2003): “Partnerships, Lemons and Efficient Trade,” *Journal of Economic Theory*, 113, 223–234.
- HAUSCH, D. (1987): “An Asymmetric Common-Value Auction Model,” *RAND Journal of Economics*, 18(4), 611–621.
- HENDRICKS, K., AND R. PORTER (1988): “An empirical study of an auction with asymmetric information,” *American Economic Review*, 78(5), 865–83.
- HENDRICKS, K., R. PORTER, AND C. WILSON (1994): “Auctions with oil and gas leases with an informed bidder and a random reservation price,” *Econometrica*, 63(1), 1–27.
- JEHIEL, P., AND A. PAUZNER (2006): “Partnership Dissolution with Interdependent Values,” *RAND Journal of Economics*, 37(1), 1–22.
- KITSTEINER, T. (2003): “Partnerships and the Double Auction with Interdependent Valuations,” *Games and Economic Behavior*, 44(1), 54–76.
- LENGWILER, Y., AND E. WOLFSTETTER (2005): “On some auction rules for amicable divorce in equal share partnerships,” in *Das Ethische in der Oekonomie, Festschrift f. Hans Nutzinger*, ed. by T. Beschorner, and T. Eger. Metropolis Verlag.
- LI, J., AND E. WOLFSTETTER (2007): “Partnership Dissolution, Complementarity, and Investment Incentives,” *Humboldt University of Berlin*, Working Paper.
- MCAFEE, R. P. (1992): “Amicable Divorce: Dissolving a Partnership with Simple Mechanisms,” *Journal of Economic Theory*, 56, 266–293.
- MILGROM, P., AND R. WEBER (1982): “The Value of Information in a Sealed-bid Auction,” *Journal of Mathematical Economics*, 10, 105–14.
- MING, S., K. XU, AND Z. CHEN (2007): “Purchasing 3Com,” *Caijing*, 21.
- MOLDOVANU, B. (2001): “How to Dissolve a Partnership,” *Journal of Institutional and Theoretical Economics*, 158, 66–80.
- MORGAN, J. (2004): “Dissolving a Partnership (Un)fairly,” *Economic Theory*, 23(4), 909–23.
- ORNELAS, E., AND J. TURNER (2007): “Efficient Dissolution of Partnerships and the Structure of Control,” *Games and Economic Behavior*, 60.

SCHWEINZER, P. (2008): “Sequential bargaining with common values,” *Bonn Econ Discussion Paper*, B-0000.

WILSON, R. B. (1967): “Competitive Bidding with Asymmetric Information,” *Management Science*, 13(11), 816–820.