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Abstract: Canonical analysis of the classical general equilibrium model demonstrates the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. This paper shows that the analogous result is not true in urban economies. An open subset of economies where none of the rational expectations equilibria fully reveal private information is found. There are two important pieces. First, there can be information about a location known by a consumer who does not live in that location in equilibrium, and thus the equilibrium rent does not reflect this information. Second, if a consumer's utility depends only on information about their (endogenous) location of residence, perturbations of utility naturally do not incorporate information about other locations conditional on their location of residence. Existence of a rational expectations equilibrium is proved. Space can prevent housing prices from transmitting information from informed to uninformed households, resulting in an inefficient outcome. (*JEL Classifications*: D51; D82; R13)

Keywords: Urban Economics; General Equilibrium; Private Information; Rational Expectations.

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1 Introduction

1.1 Motivation

People can never fully comprehend the quality and the circumstances of a city until they experience a significant part of their life living in that city. Information on physical amenities of a city (i.e., weather, parks, museums, crime, traffic jams) is easily acquired by both consumers and researchers, so there is institutional and academic work on the quality of life in cities.[1] However, people cannot completely ensure that they choose the right city or location within the city for their family before they start experiencing life there. For example, there could be uncertainty about the quality of schools, congestion of commuting routes contingent on resident and business location, or even major highway closures. Current occupants of the city, or people with friends living in the city, might have information that others don't have. Moreover, even though the current environment of the city can be understood, it is not surprising that the future developments of cities are not known with certainty, but might be known better by current occupants.[2]

On the one hand, information about life in a city is reflected in the demand for and thus the price of housing in the city.[3] Since people are rational in understanding and using the relationship associating a specific state of nature with a specific equilibrium price, depending on what model people have in mind for how equilibrium prices are determined, the price of housing can be a signal for people in choosing a city best suited to their life style. Recall that the concept of rational expectations equilibrium requires agents to use models that are not obviously controverted by their observations of the market. Therefore, the question of whether the price of housing can play

[1] For example, Rosen (1979), Roback (1982), and Blomquist, Berger, and Hoehn (1988) develop a quality of life index for urban areas (QOLI), that measures or implicitly prices the value of local amenities in urban areas.

[2] For example, Cronon (1991) discusses the success of Chicago in surpassing other competitive cities, such as St. Louis, in the early development of the Midwest.

[3] It can also be reflected in wages, but for simplicity we focus on rent.

a significant role in transmitting information from informed people to uninformed people not only addresses the question of the efficiency of housing markets, but is also related to the issue of the existence of rational expectations equilibrium in urban economics.

Available information is utilized by agents in a rational expectations equilibrium, especially the information conveyed by equilibrium prices. Radner (1979) shows that in a particular asset trading model, if the number of states of initial information is finite then, generically, rational expectations equilibria exist where all traders' private initial information is revealed. In contrast to Radner's model, that fixes state-dependent preferences and then focuses on the information concerning traders' conditional probabilities of various events, Allen (1981) considers a space of economies that is defined by state-dependent preferences and confirms Radner's conclusion in that context. When state space is infinite, Allen (1981) shows that the generic existence of fully-revealing rational expectations equilibria depends on the condition that the price space must have at least as high a dimension as the state space. Jordan (1980) considers a model where information revealed by endogenous variables can be affected by expectations, and then characterizes the data that allow the generic existence of rational expectations equilibria. Jordan concludes that unless the public prediction is based on a very narrow class of data, a statistically correct expectation may fail to exist even for otherwise well-behaved economies.

The existence of rational expectations equilibria where prices do not fully reveal the state of nature motivates the development of this paper. As shown in standard general equilibrium models in the literature, fully revealing rational expectations equilibrium demonstrates the efficiency of market prices in information transmission. The cases where the rational expectations equilibrium is not fully revealing are more interesting, for they admit a positive value of private information (that cannot be learned by observing prices) and space for discussing purchases of and strategic behavior using private information. In contrast with standard models, this paper focuses on the existence of non-fully revealing rational expectations equilibrium. In contrast with Allen (1981), who proves the existence of an open and dense subset of economies that possess fully-revealing rational expectations equilibria in the standard general equilibrium model with a finite number of states, this paper shows that the analogous result does not hold in urban economies. An open subset of economies is found, where all the rational expectations equilibria of these economies do not fully reveal private information.

Though in different settings, the common intuition behind these economies is consistent. First of all, households' bid rents reflect their ex ante valuations for housing, and the expected valuations reflect households' information (and their prior distributions) about the states. However, the equilibrium bid rent reveals only the winner's valuation, instead of being determined by all households' valuations. Therefore, in urban economics, *the equilibrium price of land reflects only the ex ante valuation and the information of the household with the highest willingness-to-pay for a location.* In contrast, the standard general equilibrium model has aggregate excess demand that is dependent on every household's demand. This generates complete information revelation in equilibrium generically, if there are enough prices. The difference between the models is due to the standard assumption in urban economics that each person can be in only one place at one time. In this circumstance, the equilibrium price might not fully reveal households' private information, even if there are many prices and few states. *For example, if in equilibrium a household living in one location has information about another location, this information might not be revealed in equilibrium rents.*

The other important component, that yields an open set of economies with not all information revealed in equilibrium, concerns perturbations of utility functions. The set of states affecting utility of households living in one location is assumed to be different from the set of such states in another location; in other words, we use a product structure for the state space.

This is what we mean when we say spatially local perturbations of utility. Thus, when we consider perturbations of utility functions, we do not allow the utility of households living in one location to depend even a little on states belonging to other locations. This is what we mean when we say perturbations are spatially local.

The model that we present covers both within-city locations and the comparison of different cities, though the latter case is the focus of this paper. This paper is organized as follows: Two explicit examples give the intuition behind the non-existence of fully-revealing rational expectations equilibrium in Section 2. For generic results, in Section 3, we find an open subset of economies with no fully-revealing rational expectations equilibrium, provided that perturbations are spatially local. In Section 4, the existence of rational expectations equilibrium is demonstrated. When some household is insensitive (to be defined precisely in this section), there exists a unique non-fully revealing rational expectations equilibrium. When all households are not insensitive, there exists a fully-revealing rational expectations equilibrium. When spatially non-local perturbations are considered, the results are the same as the ones in standard general equilibrium models, namely generic existence of fully revealing rational expectations equilibrium. In this case, generically households are not insensitive. In Section 5, it is shown that the introduction of financial markets into our model can restore the existence of a fully-revealing rational expectations equilibrium, also restoring efficiency of equilibrium allocations. Whether the introduction of financial markets is reasonable is also examined. Section 6 concludes.

2 The Examples

Before stating formally and proving the results, let us examine a few examples. In the first example, one of the households is fully informed, whereas the other has no information. In the second example, both households have partial information about the states of nature in different locations. In both examples, the equilibrium prices are the same in different states, and hence illustrate an economy where the rational expectations equilibria do not fully reveal the private information of households. Examples similar to these appear in the literature on rational expectations in the standard general equilibrium model, though in that literature they belong to the complement of a generic set, and have a very different flavor.

2.1 The Framework

Suppose there are n households indexed by $j \in N \equiv \{1, ..., n\}$ and n cities, $k \in K \equiv \{1, ..., n\}$, each endowed with a fixed land supply of $\bar{x}_k$. We consider the case where consumers obtain different utilities from living in different locations. These could represent either areas within a city or in different cities. Beside locations, each household j has to choose the lot size of his/her house and the consumption of composite good in city k, denoted by s_{jk}, z_{jk}, respectively. Since it is impossible to consume a house at the same instant in two locations, $s_{jk} > 0$ implies $s_{jk'} = 0$, $\forall k' \neq k$. To placate urban economists, we shall introduce a commuting cost, but all of our arguments hold when commuting cost is set to zero and there is only a utility difference between locations. Consider city 1 as a core-city and others as periphery-cities. Following Fujita, Krugman, and Venables (2001), there is only commuting from city k, $k > 1$ to city 1. Denote T_k to be the commuting cost from city k to the core, it is assumed $0 = T_1 < T_2 < ... < T_n < \min (Y_j)_{j \in N}$ to ensure that there is no vacant city.

There are more than two states in each city, $\omega_k \in \Omega_k$, $k \in K$, representing preference differences in our model, each realized with a probability that is common knowledge. To focus on an exchange economy, standard in both rational expectations general equilibrium and urban economics models, suppose that household j earns a fixed income Y_j of composite good. Let Ψ_k denote the rent per unit of housing in city k, $k \in K$, and normalize the price of freely mobile composite consumption good to be 1. Households can augment their private information by and only by using the information conveyed by prices.[4] The rents are collected and consumed by an absentee landlord who owns all the housing and whose utility is $u_L((s_{Lk})_{k\in K}, z_L) = z_L$ in all states. The landlord is endowed with an inelastic supply of housing in all cities.

Each household can consume housing in only one city. Denote household j's consumption in city k in state ω as $\varphi_{jk} \equiv (s_{jk}, z_{jk})$ and let $\varphi_j \equiv (\varphi_{jk})_{k\in K}$ denote j's consumption in all cities. In state ω, given φ_j, the ex post utility function of household j is

$$u_j^{\omega}(\varphi_j) = \max\{(u_{jk}^{\omega}(\varphi_{jk}))_{k\in K}\},$$

$\omega \in \Omega$. Let Ψ_k be the rent of housing in city k, let $\psi_k \equiv (\Psi_k, 1)$ be the prices of housing and composite good in city k where composite good is numeraire, and let $\Psi \equiv (\psi_k)_{k\in K}$ denote the prices in all cities. The general optimization problem for household j with n cities, given his/her information structure $\mathcal{F}_j$, is:[5]

$$\begin{aligned}
&\max_{\varphi_j^{\omega}} \; Eu_j(\varphi_j^{\omega}|\mathcal{F}_j)\\
&\text{s.t.} \;\; \sum_{k\in K}\psi_k\varphi_{jk}^{\omega} + \sum_{k\in K}\ulcorner\frac{s_{jk}^{\omega}}{\sum_{k'\in K}s_{jk'}^{\omega}}\urcorner\, T_k \leq Y_j,\\
&\qquad \varphi_{jk}^{\omega} \neq 0 \text{ implies that } \varphi_{jk'}^{\omega} = 0, \; \forall k, k' \in K, k' \neq k\\
&\qquad \varphi_j^{\omega} \in \mathbb{R}_+^{2n} \text{ is } \mathcal{F}_j\text{-measurable.}
\end{aligned} \tag{1}$$

Given a price Ψ, the information that it conveys to all agents is denoted by $\sigma(\Psi)$, the sub-σ-field of $\mathcal{F}$ generated by the vector-valued random variable Ψ. Let μ denote a (countably) additive probability measure defined on $(\Omega, \mathcal{F})$.

[4]When households condition their expectations on additional market variables, the equilibrium concept is defined as a generalized rational expectations equilibrium; see Allen (1998).

[5]The ceiling function, denoted by $\ulcorner\theta\urcorner$, is defined by the smallest integer greater than or equal to θ, i.e., $\ulcorner\theta\urcorner \equiv \min\{n \in \mathbb{Z}|\theta \leq n\}$. Notice that $\ulcorner\frac{s_{jk}}{\sum_{k'\in K}s_{jk'}^{\omega}}\urcorner$ can be either 1 or 0, depending on whether household j lives in city k or not.

Following Allen (1981), the concept of rational expectations equilibrium is formally defined as follows.

Definition 1 *A rational expectations equilibrium is defined as an equivalence class of $\mathcal{F}$-measurable house price functions $\Psi^* : \Omega \to \mathbb{R}_+^{2n}$, and for each $j \in N$, an equivalence class of $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable allocation functions $\varphi_j^* : \Omega \to \bigcup_{k \in K} \mathbb{R}_+^2$ such that*
(i) $\psi_k^{\omega *} \cdot \varphi_{jk}^{\omega *} \leq Y_j - T_k$ for μ-almost every $\omega \in \Omega$;
(ii) If $\varphi'_{jk} : \Omega \to \mathbb{R}_+^2$ satisfies the informational constraint that φ'_{jk} is $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable and the budget constraint that $\psi_k^{\omega *} \cdot \varphi_{jk}^{\omega \prime} \leq Y_j - T_k$ for μ-almost every $\omega \in \Omega$, then

$$\sum_{\omega \in \Omega} u_j^\omega(\varphi'_j)\ \mu(\omega | \mathcal{F}_j \vee \sigma(\Psi^*)) \leq \sum_{\omega \in \Omega} u_j^\omega(\varphi_j^*)\ \mu(\omega | \mathcal{F}_j \vee \sigma(\Psi^*)),\ \forall j = 1, 2;$$

(iii) $\sum_{j \in N} s_{jk}^{\omega *} = \bar{s}_k$, $\sum_{k \in K} \sum_{j \in N} z_{jk}^{\omega *} + z_L^{\omega *} + \sum_{k \in K} \sum_{j \in N} \ulcorner \frac{s_{jk}^\omega}{\sum_{k' \in K} s_{jk'}^\omega} \urcorner T_k = \sum_{j \in N} Y_j$, and for each j, $\varphi_{jk}^\omega \neq 0$ implies that $\varphi_{jk'}^\omega = 0$, $\forall k' \neq k$ for μ-almost every $\omega \in \Omega$.

It can be seen that $\sum_{\omega \in \Omega} u_j^\omega(\varphi_j)\ \mu(\omega | \mathcal{F}_j \vee \sigma(\Psi^*))$ is household j's expected utility of choosing φ_j, based on private information and the information given by Ψ^*.

This is the minimal perturbation of the standard general equilibrium model necessary to make it compatible with urban economics, i.e., it is the standard general equilibrium model that restricts each consumer to own housing in one and only one location. In what follows, we will solve for a bid rent equilibrium, that is equivalent to the solution of a standard market equilibrium. This device is common in urban economics, and is used "almost everywhere."

Definition 2 *Denoting $\Psi_{jk}^\omega \equiv \max_{\varphi_{jk}^\omega} \{ \frac{Y_j - T_k - z_{jk}^\omega}{s_{jk}^\omega} | E[u_j | \mathcal{F}_j \vee \sigma(\Psi^*)] = \bar{u}_j \}$, for μ-a.e. $\omega \in \Omega$, a bid rent equilibrium is defined by $(\Psi^{\omega *}, (\varphi_j^{\omega *})_{j \in N})$ such that* *for μ-almost every $\omega \in \Omega$, for each city $k \in K$:*

$$\Psi_k^{\omega *}(u) = \max_j \{\Psi_{jk}^{\omega}(u)\}; \tag{2}$$

$$\varphi_{jk}^{\omega *} = \begin{cases} \arg\max_{\varphi_{jk}^{\omega}} \{\frac{Y_j - T_k - z_{jk}^{\omega}}{s_{jk}^{\omega}} | E[u_j | \mathcal{F}_j \vee \sigma(\Psi^*)] = u\}, & \textit{if } \; j \in \arg\max_j\{\Psi_{jk}^{\omega}\}, \\ (0,0), \quad \textit{if } \; j \notin \arg\max_j\{\Psi_{jk}^{\omega}\}; \end{cases} \tag{3}$$

$$\sum_{j \in N} s_{jk}^{\omega *} = \bar{s}_k, \; \sum_{k \in K}\sum_{j \in N} z_{jk}^{\omega *} + z_L^{\omega *} + \sum_{k \in K}\sum_{j \in N} \ulcorner \frac{s_{jk}^{\omega}}{\sum_{k' \in K} s_{jk'}^{\omega}} \urcorner \, T_k = \sum_{j \in N} Y_j,$$

$$\textit{and } \varphi_{jk}^{\omega} \neq 0 \textit{ implies that } \varphi_{jk'}^{\omega} = 0, \; \forall k' \neq k, \forall j \in N. \tag{4}$$

Since each household can consume housing in at most one city, the consumption set is $\bigcup_{k \in K} \mathbb{R}_+^2$, and the ex post state-dependent preferences of living in city k, $k \in K$, can be specified by utilities $u_{jk} : \; \Omega_k \to \kappa_{jk}$, where κ_{jk} is a compact subset of $C^r(\mathbb{R}_+^2, \mathbb{R})$, $r \geq 2$, endowed with the weak C^r compact-open topology. Assume that for every ω_k, $u_{jk}^{\omega_k} \in \kappa_{jk}$ satisfies for each $\varphi_{jk} \in \mathbb{R}_+^2$:

(i) strict (differentiable) monotonicity: $D_{\varphi} u_{jk}^{\omega_k}(\varphi) \in \mathbb{R}_{++}$,

(ii) strict (differentiable) concavity: $D_{\varphi\varphi} u_{jk}^{\omega_k}(\varphi)$ is negative definite, and

(iii) smooth boundary condition: the closure in $\mathbb{R}^2$ of the upper contour set $\{\varphi' \in \mathbb{R}_{++}^2 | u_{jk}^{\omega_k}(\varphi') \geq u_{jk}^{\omega_k}(\varphi)\}$ is contained in $\mathbb{R}_{++}^2$.

These conditions ensure that every household's state-dependent preferences are smooth in the sense of Debreu (1972) for almost every state so that, conditional on any measurable $\omega \in \mathcal{F}$ and location, demands are well defined C^{r-1} functions. Our examples satisfy these assumptions.

Although it is well-known that bid-rent and competitive equilibria are closely connected (see for example Fujita, 1989), results in the literature cover only the context of no uncertainty. If the rational expectations equilibria were known to be fully revealing, this result could be applied state by state. We require an equivalence result in the context of uncertainty, especially when the rational expectations equilibrium might not be fully revealing. The proof uses classical duality.

Lemma 1 *Given that all households' preferences are representable by a utility function satisfying conditions (i), (ii), and (iii),* $(\Psi^{\omega*}, (\varphi_j^{\omega*})_{j\in N})$ *constitutes a bid rent equilibrium if and only if it constitutes a rational expectations equilibrium in a competitive economy.*

Proof. See Appendix A.

2.2 Example 1

Suppose that there are two households ($j \in \{1, 2\}$) with the same income ($Y_1 = Y_2 = Y$), and two cities ($k \in \{x, y\}$) with land endowments $\bar{x}$ and $\bar{y}$, respectively. Household 1's utility is state-dependent but the utility function of household 2 is independent of states. In each city k, there are two states $\omega_k \in \Omega_k \equiv \{L, H\}$, $k \in \{x, y\}$, which are equally likely to occur and the states in different cities are not correlated. What each agent can observe are events that are subsets of $\Omega \equiv \Omega_x \times \Omega_y$. Denote $\omega \equiv \omega_x \times \omega_y$ as an element of Ω. Furthermore, household 1 has no information, and household 2 knows what the state will be. That is, households' information are represented by $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y\}$, $\mathcal{F}_2 = \{\phi, \{H\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{L\}, \Omega_y\}$ which are sub-σ-fields of $\mathcal{F}$, where $\mathcal{F} \equiv \mathcal{F}_1 \vee \mathcal{F}_2$ is the smallest σ-field generated by the class $\mathcal{F}_1 \bigcup \mathcal{F}_2$ of subsets of $\Omega = \{HH, HL, LH, LL\}$.[6] Everything except the true state is common knowledge, so households are assumed to know the relationship between states and prices.

Given information structure $\mathcal{F}_1$, the superscripts on household 1's allocation can be ignored for simplicity until he/she learns something. The optimization problem for household 1 is to maximize expected utility subject to

[6]Following Aumann (1976), the join $\mathcal{F}_1 \vee \mathcal{F}_2$ denotes the coarsest common refinement of $\mathcal{F}_1$ and $\mathcal{F}_2$.

the budget constraint:

$$
\begin{aligned}
\max_{s_{1x}, s_{1y}, z_{1x}, z_{1y}} \quad & Eu_1(s_{1x}, s_{1y}, z_{1x}, z_{1y} | \mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x}) | \mathcal{F}_1],\ E[\beta_1^\omega \ln(s_{1y}) + \ln(z_{1y}) | \mathcal{F}_1]\} \\
\text{s.t.} \quad & \Psi_x s_{1x} + \Psi_y s_{1y} + z_{1x} + z_{1y} + \ulcorner \frac{s_{1y}}{s_{1x} + s_{1y}} \urcorner\, t \leq Y, \\
& s_{1k}\, s_{1l} = 0,\ s_{1k}\, z_{1l} = 0,\ z_{1k}\, z_{1l} = 0, \\
& s_{1k}, z_{1k} \geq 0, \quad \forall k, l = x, y,\ k \neq l;
\end{aligned}
$$

In contrast, since household 2's utility is state-independent, his/her optimization problem is for all $\omega \in \Omega$

$$
\begin{aligned}
\max_{s_{2x}^\omega, s_{2y}^\omega, z_{2x}^\omega, z_{2y}^\omega} \quad & u_2^\omega(s_{2x}^\omega, s_{2y}^\omega, z_{2x}^\omega, z_{2y}^\omega) \\
& = \max\{\alpha_2 \ln(s_{2x}^\omega) + \ln(z_{2x}^\omega),\ \beta_2 \ln(s_{2y}^\omega) + \ln(z_{2y}^\omega)\} \\
\text{s.t.} \quad & \Psi_x s_{2x}^\omega + \Psi_y s_{2y}^\omega + z_{2x}^\omega + z_{2y}^\omega + \ulcorner \frac{s_{2y}^\omega}{s_{2x}^\omega + s_{2y}^\omega} \urcorner\, t \leq Y, \\
& s_{2k}^\omega\, s_{2l}^\omega = 0,\ s_{2k}^\omega\, z_{2l}^\omega = 0,\ z_{2k}^\omega\, z_{2l}^\omega = 0, \\
& s_{2k}^\omega, z_{2k}^\omega \geq 0, \quad \forall k, l = x, y,\ k \neq l.
\end{aligned}
$$

Suppose that household 1 prefers city x more than household 2, and household 2 prefers y more than household 1, i.e., $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$.

In urban economics, as studied by Alonso (1964), bid rent describes a particular household's willingness to pay for housing in terms of composite commodity, given a fixed utility level. Following Fujita (1989) and our Lemma 1, people live where their bid rents are maximal in equilibrium, and these bid rents are equilibrium rents. The bid rent functions of the two households for the housing in x and y are

$$\Psi_{1x}^\omega = \max_{s_{1x}} \frac{Y - e^{Eu_1}(s_{1x})^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{5}$$

$$\Psi_{1y}^\omega = \max_{s_{1y}} \frac{Y - t - e^{Eu_1}(s_{1y})^{-E[\beta_1^\omega]}}{s_{1y}}, \tag{6}$$

$$\Psi_{2x}^\omega = \max_{s_{2x}^\omega} \frac{Y - e^{u_2^\omega}(s_{2x}^\omega)^{-\alpha_2}}{s_{2x}^\omega}, \tag{7}$$

$$\Psi_{2y}^\omega = \max_{s_{2y}^\omega} \frac{Y - t - e^{u_2^\omega}(s_{2y}^\omega)^{-\beta_2}}{s_{2y}^\omega}, \tag{8}$$

where $\omega \in \Omega$. From first and second-order conditions, the optimal land lot sizes for households are

$$s_{1x}^{\omega *} = [\frac{e^{Eu_1}(1+E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{9}$$

$$s_{1y}^{\omega *} = [\frac{e^{Eu_1}(1+E[\beta_1^\omega])}{Y-t}]^{\frac{1}{E[\beta_1^\omega]}}, \tag{10}$$

$$s_{2x}^{\omega *} = [\frac{e^{u_2^\omega}(1+\alpha_2)}{Y}]^{\frac{1}{\alpha_2}}, \tag{11}$$

$$s_{2y}^{\omega *} = [\frac{e^{u_2^\omega}(1+\beta_2)}{Y-t}]^{\frac{1}{\beta_2}}. \tag{12}$$

From market clearing conditions $s_{jx}^{\omega *} = \bar{x}$ and $s_{jy}^{\omega *} = \bar{y}$, we have

$$Eu_1^* = \begin{cases} \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1+E[\alpha_1^\omega]], & \text{if household 1 lives at } x; \\ \ln[Y-t] + E[\beta_1^\omega]\ln[\bar{y}] - \ln[1+E[\beta_1^\omega]], & \text{if household 1 lives at } y, \end{cases} \tag{13}$$

$$u_2^{\omega *} = \begin{cases} \ln[Y] + \alpha_2\ln[\bar{x}] - \ln[1+\alpha_2], & \text{if household 2 lives at } x; \\ \ln[Y-t] + \beta_2\ln[\bar{y}] - \ln[1+\beta_2], & \text{if household 2 lives at } y, \end{cases} \tag{14}$$

for $\omega \in \Omega$. So the equilibrium bid rents of agents in the two cities in two states are

$$\Psi_{1x}^{\omega *} = \frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}, \tag{15}$$

$$\Psi_{1y}^{\omega *} = \frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]}\frac{Y-t}{\bar{y}}, \tag{16}$$

$$\Psi_{2x}^{\omega *} = \frac{\alpha_2}{1+\alpha_2}\frac{Y}{\bar{x}}, \tag{17}$$

$$\Psi_{2y}^{\omega *} = \frac{\beta_2}{1+\beta_2}\frac{Y-t}{\bar{y}}, \tag{18}$$

for $\omega \in \Omega$. The equilibrium bid rents are presented in Figure 1, where the horizontal axis represents the amount of transportation cost and the vertical axis represents the individuals' bid rents.

Since $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]} > \frac{\alpha_2}{1+\alpha_2}$ if and only if $E[\alpha_1^\omega] > \alpha_2$, given $E[\alpha_1^\omega] > \alpha_2$, the bid rent of household 1 for the housing in x is higher than that of household

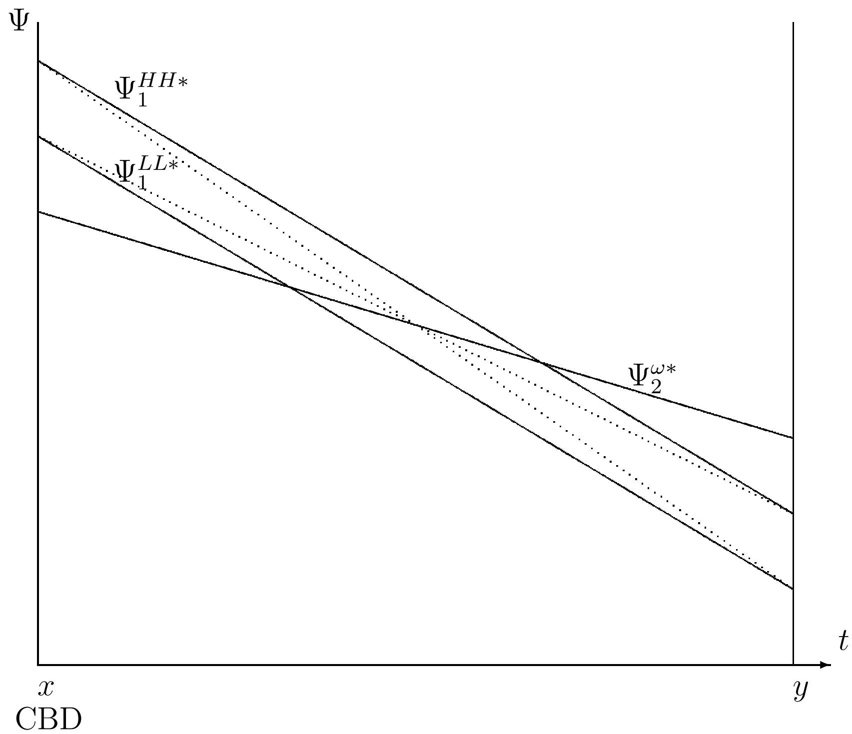

Figure 1: The bid rent functions in Example 1, where the dotted lines represent Ψ_1^{HL*} and Ψ_1^{LH*}, respectively.

2 for the housing in x in both states. Similarly, since $\frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]} < \frac{\beta_2}{1+\beta_2}$ if and only if $E[\beta_1^\omega] < \beta_2$, $E[\beta_1^\omega] < \beta_2$ implies that the bid rent of household 1 for the housing in y is lower than that of household 2 for the housing in y in all states. Therefore, the equilibrium location pattern where household 1 lives at x and household 2 lives in y is verified under the conditions we have assumed.

Notice that there is no equilibrium that fully reveals information. If in equilibrium $\Psi_x^{HH*} = \Psi_x^{HL*} \neq \Psi_x^{LH*} = \Psi_x^{LL*}$, the valuation of household 1 for the housing in city x differs in different states (in city x), which conflicts with the assumption that household 1 has no information about the state. Notice also that $\Psi_x^{\omega*}$ and $\Psi_y^{\omega*}$ depend only on the mean of α_1, β_2, and the values of Y, t, $\bar{x}$, and $\bar{y}$. Therefore, the equilibrium rents in the two cities are independent of the realized state, and there exists no fully-revealing rational expectations equilibrium.[7] Even though household 2 knows the state, since household 2 doesn't care about the state, equilibrium prices don't reveal it.

2.3 Example 2

Follow the same setting in the previous example, but suppose that household 1 knows the state in city y, but has no information about city x. On the other hand, household 2 knows only the state in city x, but not the state in y. Furthermore, the states in the two cities are not correlated. That is, let $\Omega \equiv \Omega_x \times \Omega_y$, where $\Omega_x = \Omega_y \equiv \{H, L\}$ represent the state spaces in cities x and y. $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y, \{H\}, \{L\}\}$, $\mathcal{F}_2 = \{\phi, \Omega_x, \{H\}, \{L\}\} \times \{\phi, \Omega_y\} \subseteq \mathcal{F}$ are sub-σ-fields representing private information. Again, the relationship between states and prices is common knowledge.

Each household chooses to live in one and only one city. Moreover, households make their decisions simultaneously. Given an event $\omega \in \Omega$, both households' utilities are state-dependent, so their optimization problems are

$$\begin{aligned}
\max_{s_{1x}, s_{1y}^\omega, z_{1x}, z_{1y}^\omega} \quad & Eu_1(s_{1x}, s_{1y}^\omega, z_{1x}, z_{1y}^\omega | \mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x}) | \mathcal{F}_1],\ \beta_1^\omega \ln(s_{1y}^\omega) + \ln(z_{1y}^\omega)\} \\
\text{s.t.} \quad & \Psi_x s_{1x} + \Psi_y s_{1y}^\omega + z_{1x} + z_{1y}^\omega + \ulcorner \frac{s_{1y}^\omega}{s_{1x}^\omega + s_{1y}^\omega} \urcorner t \le Y, \\
& s_{1x}\, s_{1y}^\omega = 0,\ s_{1x}\, z_{1y}^\omega = 0,\ z_{1x}\, s_{1y}^\omega = 0,\ z_{1x}\, z_{1y}^\omega = 0, \\
& s_{1x}, s_{1y}^\omega, z_{1x}, z_{1y}^\omega \ge 0;
\end{aligned}$$

$$\begin{aligned}
\max_{s_{2x}^\omega, s_{2y}, z_{2x}^\omega, z_{2y}} \quad & Eu_2(s_{2x}^\omega, s_{2y}, z_{2x}^\omega, z_{2y} | \mathcal{F}_2) \\
& = \max\{\alpha_2^\omega \ln(s_{2x}^\omega) + \ln(z_{2x}^\omega),\ E[\beta_2^\omega \ln(s_{2y}) + \ln(z_{2y}) | \mathcal{F}_2]\} \\
\text{s.t.} \quad & \Psi_x s_{2x}^\omega + \Psi_y s_{2y} + z_{2x}^\omega + z_{2y} + \ulcorner \frac{s_{2y}^\omega}{s_{2x}^\omega + s_{2y}^\omega} \urcorner t \le Y, \\
& s_{2x}^\omega\, s_{2y} = 0,\ s_{2x}^\omega\, z_{2y} = 0,\ z_{2x}^\omega\, s_{2y} = 0,\ z_{2x}^\omega\, z_{2y} = 0, \\
& s_{2x}^\omega, s_{2y}, z_{2x}^\omega, z_{2y} \ge 0;
\end{aligned}$$

[7] There is another way to prove the same result. Since household 1's bid rent in x and household 2's bid rent in y are independent of the state, it must be true that $\mathcal{F}_j \vee \sigma(\Psi^{\omega *}) = \{\phi, \Omega\}$, $\forall j = 1, 2$. If the equilibrium price is fully revealing, then $\sigma(\Psi^{\omega *})$ is finer than $\mathcal{F}_j \vee \sigma(\Psi^{\omega *})$, a contradiction.

Note that in fact, the optimized utility of household 1 is state-dependent (state-independent) at y (x), denoted by $u_{1y}^{\omega *}$ (Eu_{1x}^{*}); $u_{2x}^{\omega *}$ and Eu_{2y}^{*} are similarly defined. To present an example of rational expectations equilibrium without revealing private information, suppose that $E[\alpha_1^{\omega}] > \alpha_2^{\omega}$ and $E[\beta_2^{\omega}] > \beta_1^{\omega}$, for all $\omega \in \Omega$.

Given these conditions, suppose that households 1 and 2 choose to live in cities x and y, respectively. Their bid rent functions are, $\forall \omega \in \Omega$,

$$\Psi_{1x}^{\omega} = \max_{s_{1x}} \frac{Y - e^{Eu_1} s_{1x}^{-E[\alpha_1^{\omega}]}}{s_{1x}}, \tag{19}$$

$$\Psi_{1y}^{\omega} = \max_{s_{1y}} \frac{Y - t - e^{u_1^{\omega}} s_{1y}^{-\beta_1^{\omega}}}{s_{1y}}, \tag{20}$$

$$\Psi_{2x}^{\omega} = \max_{s_{2x}} \frac{Y - e^{u_2^{\omega}} s_{2x}^{-\alpha_2^{\omega}}}{s_{2x}}, \tag{21}$$

$$\Psi_{2y}^{\omega} = \max_{s_{2y}} \frac{Y - t - e^{Eu_2} s_{2y}^{-E[\beta_2^{\omega}]}}{s_{2y}}. \tag{22}$$

Thus, the optimal land sizes for household 1 and 2 are, $\forall \omega \in \Omega$,

$$s_{1x}^{\omega *} = [\frac{e^{Eu_1}(1 + E[\alpha_1^{\omega}])}{Y}]^{\frac{1}{E[\alpha_1^{\omega}]}}, \tag{23}$$

$$s_{1y}^{\omega *} = [\frac{e^{u_1^{\omega}}(1 + \beta_1^{\omega})}{Y - t}]^{\frac{1}{\beta_1^{\omega}}}, \tag{24}$$

$$s_{2x}^{\omega *} = [\frac{e^{u_2^{\omega}}(1 + \alpha_2^{\omega})}{Y}]^{\frac{1}{\alpha_2^{\omega}}}, \tag{25}$$

$$s_{2y}^{\omega *} = [\frac{e^{Eu_2}(1 + E[\beta_2^{\omega}])}{Y - t}]^{\frac{1}{E[\beta_2^{\omega}]}}. \tag{26}$$

From $s_{jx}^{\omega *} = \bar{x}$ and $s_{jy}^{\omega *} = \bar{y}$, we have

$$Eu_1^{*}(\cdot|\mathcal{F}_1) = \begin{cases} \ln[Y] + E[\alpha_1^{\omega}] \ln[\bar{x}] - \ln[1 + E[\alpha_1^{\omega}]], & \text{if household 1 lives at } x; \\ \ln[Y - t] + \beta_1^{\omega} \ln[\bar{y}] - \ln[1 + \beta_1^{\omega}], & \text{if household 1 lives at } y, \end{cases} \tag{27}$$

$$Eu_2^{*}(\cdot|\mathcal{F}_2) = \begin{cases} \ln[Y] + \alpha_2^{\omega} \ln[\bar{x}] - \ln[1 + \alpha_2^{\omega}], & \text{if household 2 lives at } x; \\ \ln[Y - t] + E[\beta_2^{\omega}] \ln[\bar{y}] - \ln[1 + E[\beta_2^{\omega}]], & \text{if household 2 lives at } y. \end{cases} \tag{28}$$

Again, agents' equilibrium bid rents are

$$\Psi_{1x}^{\omega *} = \frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]} \frac{Y}{\bar{x}}, \tag{29}$$

$$\Psi_{1y}^{\omega *} = \frac{\beta_1^\omega}{1+\beta_1^\omega} \frac{Y-t}{\bar{y}}, \tag{30}$$

$$\Psi_{2x}^{\omega *} = \frac{\alpha_2^\omega}{1+\alpha_2^\omega} \frac{Y}{\bar{x}}, \tag{31}$$

$$\Psi_{2y}^{\omega *} = \frac{E[\beta_2^\omega]}{1+E[\beta_2^\omega]} \frac{Y-t}{\bar{y}}, \tag{32}$$

where $\omega \in \Omega$. The equilibrium bid rents are drawn in Figure 2, where the horizontal axis represents the transportation cost and the individual bid rents are represented by the vertical axis.

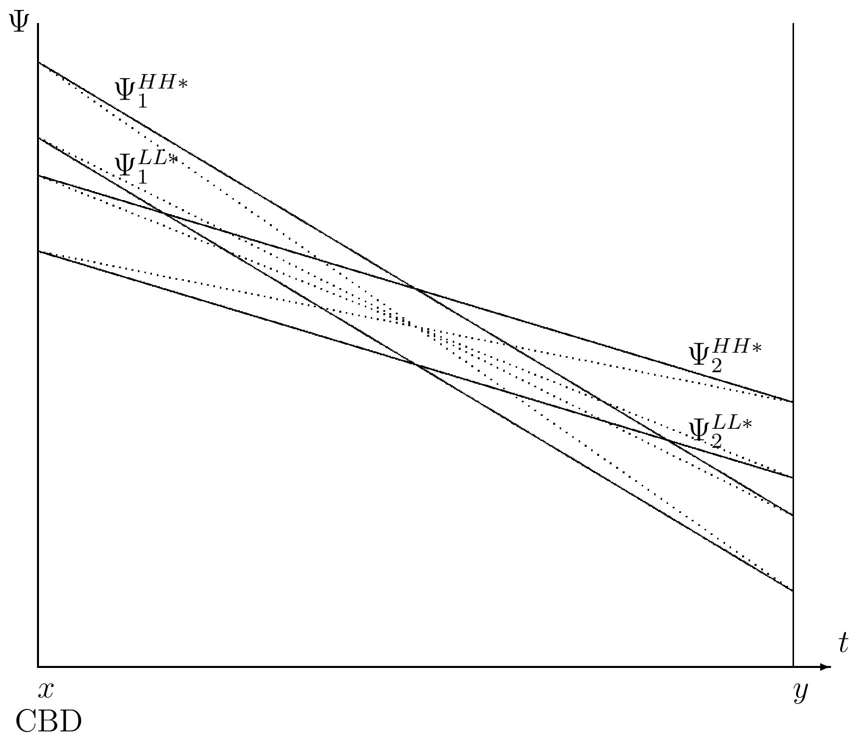

Figure 2: The bid rent functions in Example 2, where the dotted lines represent Ψ_1^{HL*}, Ψ_1^{LH*}, Ψ_2^{HL*}, and Ψ_2^{LH*}, respectively.

Inequalities $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, $\forall\omega$, imply that the bid rent of household 1 (household 2) for the housing in x (y) is always higher than that of household 2 (household 1). So the equilibrium location pattern where household 1 lives at x and household 2 lives at y is verified.[8]

Again, there is no fully revealing equilibrium in this example. Since Ψ_x^* and Ψ_y^* depend only on Y, t, the mean of the preference parameters and the endowments of land in cities, the equilibrium rents are the same in all the realized states. That is, the mapping from prices to preferences is not injective, which is the source of the non-existence of fully-revealing rational expectations equilibrium.

These examples illustrate different causes for the equilibrium not fully revealing private information: The first one comes from the fact that the informed household doesn't care about different states. The second one comes from the mismatch between informed households and their locations. In the next section, we show that these unfortunate circumstances can persist under arbitrarily small perturbations.

3 An Open Subset of Economies without Fully Revealing Equilibria

The examples represent two points in the space of utility functions with no fully revealing rational expectations equilibrium. In this section, we generalize the examples and show that, in economies under uncertainty where there is no market for contingent claims or financial contracts, fully revealing rational expectations equilibrium is not present for an open set of utility functions. But for all parameters satisfying a condition, there exists a rational expectations equilibrium that might not be fully revealing. This will be proved in the next section.

Suppose there are two households ($j \in N \equiv \{1, 2\}$), one landlord ($j = L$), and two cities ($k \in K \equiv \{x, y\}$). Let $\Omega \equiv \Omega_x \times \Omega_y = \{H, L\} \times \{H, L\}$

[8] Even when households can observe other households' consumption (of housing and composite good), given that the states in two cities are not correlated, the non-existence of fully-revealing generalized rational expectations equilibria (GREE) still holds in this example.

be the finite payoff-relevant state space of the economy; every element $\omega \equiv \omega_x \times \omega_y \in \Omega$ is called a state of the economy. Every household is endowed with the same initial state-independent endowment Y, a private information sub-σ-field $\mathcal{F}_j \subsetneqq \mathcal{F}$, and a state-dependent utility function $u_j \equiv \max\{u_{jx}, u_{jy}\}$ defined on $\Omega \times (\mathbb{R}^2_+ \cup \mathbb{R}^2_+)$, meaning that households must each choose a location. Households are assumed to maximize their conditional expected utilities, where the ex post state-dependent preferences of living in city k are specified by $u_{jk} : \Omega_k \longrightarrow \kappa_{jk}$, where κ_{jk} is a compact subset of $C^r(\mathbb{R}^2_+, \mathbb{R})$ functions, $r \geq 2$, which is endowed with the weak C^r compact-open topology. For each state ω, the economy $(Y, u^{\omega}_j(\cdot)_{j \in N})$ is a smooth economy as defined by Debreu (1972). It is important to notice that u_{jk} is payoff-relevant to only Ω_k, that is, we assume that people living in city k care only about the state in k. Later, we consider the perturbations that maintain this property.

Before we prove the results, some characteristics of equilibrium must be defined. In a rational expectations equilibrium, the information can be fully revealing, which means that all households can learn the state of nature by observing the equilibrium price and using their private information. Alternatively, the information can be non-fully revealing in a rational expectations equilibrium, where at least one household cannot tell the state of nature from the equilibrium price and their private information. Their formal definitions are as follows.

Definition 3 *A fully-revealing rational expectations equilibrium is a rational expectations equilibrium such that*

$$\mathcal{F}_j \vee \sigma(\Psi^*) = \mathcal{F}, \quad \forall j \in N. \tag{33}$$

When there is at least one j such that the above equality does not hold, we say it is a non-fully-revealing rational expectations equilibrium.

In other words, conditioning on a fully revealing equilibrium price function is equivalent to knowing the pooled information of all households in the economy. Though Allen (1981) proves the existence of an open and dense subset of economies with fully-revealing rational expectations equilibrium in the classical framework, when perturbations location-by-location are considered, Theorem 1 will show that the same statement does not hold in urban economics. Utility functions defined location-by-location are termed formally as local utilities as follows.[9] We have been using them in this paper up to this point.

Definition 4 (Local Utilities)
The households' preferences are called local when their preferences satisfy $\forall j \in N, k \in K$*,* $u_{jk} : \Omega_k \to \kappa_{jk}$*. If for some* j, k*, there exists* k'*,* $k' \neq k$ *such that* $u_{jk} : \Omega_k \times \Omega_{k'} \to \kappa_{jk}$ *is not constant for some* $\omega_{k'}, \omega'_{k'} \in \Omega_{k'}$*, then it is called non-local.*

That is, saying that utilities are local requires that each household's utility in k is measurable with respect to only Ω_k when they live in city k. We shall require that when utility functions are perturbed, if they start local, they remain local. We call this a "spatially local perturbation." *Spatially local perturbation* means that if people living in a city care only about the state in the city where they live, then when their utility function is perturbed, it continues to have this property. Spatially local perturbations are more realistic than non-local perturbations in urban economics, since it is not persuasive to say that the perturbed preferences conditional on residence in city k depend on the state in another city. For example, when preference perturbations are considered, in most cases, the state of living in Chicago is irrelevant to the circumstances in New York. Therefore, in urban economics, it doesn't make sense to consider spatially non-local perturbations as used in standard models. Throughout this paper, to highlight the distinct essence of urban economics, we focus on spatially local perturbations.

It is possible to add other kinds of perturbations to the model, for example, national or regional uncertainty, but this would only complicate notation.

[9] Throughout this paper, only preference perturbations are considered since endowment perturbations give households more information, and perturbations of ex ante information are not smooth.

Theorem 1 *Given the discrete state space Ω, consider local perturbations of households' preferences. There exists an open subset of economies that possess no fully-revealing rational expectations equilibrium.*

Proof. Consider example 1 first. Notice that in equilibrium, household 1's marginal rate of substitution for housing in city x is $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}$. On the other hand, household 2's marginal rate of substitution for housing in x is $\frac{\alpha_2^\omega}{1+\alpha_2^\omega}\frac{Y}{\bar{x}}$. Let $\alpha_1^{HH}=\alpha_1^{HL}>\alpha_1^{LH}=\alpha_1^{LL}$ and $\beta_1^{HH}=\beta_1^{LH}>\beta_1^{HL}=\beta_1^{LL}$.

Since in the example $E[\alpha_1^\omega]>\alpha_2$ and $E[\beta_1^\omega]<\beta_2$, we can choose $\epsilon^\alpha=\frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}}>0$, $\epsilon^\beta=\frac{\beta_2-E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}}>0$, and $\epsilon=\min\{\epsilon^\alpha,\epsilon^\beta\}$. Recall that the equilibrium marginal utilities in example 1 are

$$v^*\equiv(D_{s_{1x}}Eu_1^*,D_{s_{1y}}Eu_1^*,D_{z_{1x}}Eu_1^*,D_{z_{1y}}Eu_1^*,D_{s_{2x}}u_2^{\omega*},D_{s_{2y}}u_2^{\omega*},D_{z_{2x}}u_2^{\omega*},D_{z_{2y}}u_2^{\omega*}).$$

Centered at v^*, consider all spatially local perturbations of utility functions within an open set in the weak C^r topology such that

$$D_{s_{1k}}Eu_1\in(D_{s_{1k}}Eu_1^*-\epsilon,D_{s_{1k}}Eu_1^*+\epsilon), \tag{34}$$

$$D_{z_{1k}}Eu_1\in(D_{z_{1k}}Eu_1^*-\epsilon,D_{z_{1k}}Eu_1^*+\epsilon), \tag{35}$$

$$D_{s_{2k}}u_2^\omega\in(D_{s_{2k}}u_2^{\omega*}-\epsilon,D_{s_{2k}}u_2^{\omega*}+\epsilon), \tag{36}$$

$$D_{z_{2k}}u_2^\omega\in(D_{z_{2k}}u_2^{\omega*}-\epsilon,D_{z_{2k}}u_2^{\omega*}+\epsilon),\qquad k=x,y. \tag{37}$$

These perturbations are evaluated at city k, $k=x,y$, individually, and are thus spatially local perturbations. Then it can be checked that all utilities within this neighborhood generate bid rents that are within ϵ of the equilibrium bid rents in example 1. Furthermore, household 1's realized marginal rate of substitution for housing in city x is always higher than the marginal rate of substitution of household 2; household 2's marginal rate of substitution for housing in city y is always higher than that of household 1.[10]

[10] In city x, for example, since the lowest MRS for household 1 is $\frac{E[\alpha_1^\omega]/\bar{x}-\epsilon}{(1+E[\alpha_1^\omega])/Y+\epsilon}$, and the highest MRS for household 2 is $\frac{\alpha_2/\bar{x}+\epsilon}{(1+\alpha_2)/Y-\epsilon}$, household 1's MRS is greater than household 2's MRS if and only if $\epsilon<\epsilon^\alpha=\frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}}$. Similarly, household 2's MRS in city y is greater than that of household 1 if and only if $\epsilon<\epsilon^\beta=\frac{\beta_2-E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}}$.

Now we can prove the non-existence of fully revealing rational expectations equilibrium for all economies in this neighborhood. Suppose for any set of preferences within these spatially local perturbations, there exists a fully revealing rational expectations equilibrium $(\varphi_1^*, \varphi_2^*, \Psi^*)$. Then the uninformed household (household 1) can infer the state of nature by observing Ψ^*. However, within the perturbations, the equilibrium bid rents are the same across states, contradicting that Ψ^* is a fully-revealing rational expectations equilibrium price.

Obviously, a similar argument works for the cases with more than 2 states and example 2. *Q.E.D.*

This paper shows that if one household has the information about a specific city, if he doesn't live there, the housing price in that city will not reveal his information. If a household lives in the city about which he is informed, there is an information gain (in that he can maximize ex post utility instead of expected utility), but also a information spillover to all other households in that they can learn private information about that city by observing the equilibrium housing price. When local utility and spatially local perturbations are considered, the information spillover plays no role for the households living in other cities. However, when spatially non-local perturbations are considered, a small perturbation makes the utility in city k relevant to the states of all cities. Then as shown in Allen (1981), generically there exists an open and dense set of economies possessing fully revealing rational expectations equilibrium.

Finally, we make a remark here: If there is no fully revealing rational expectations equilibrium, an equilibrium allocation can fail to be a Pareto optimum. Consider a variation of Example 1 shown in Figure 3. When the probability is quite evenly distributed over states in Ω_k, $k = 1, 2$, household 1's bid rent for city 1 is larger than that of household 2, and household 2's bid rent for city 2 is larger than that of household 1. So in equilibrium, household j lives in city j, $j = 1, 2$ in both states. However, in a Pareto optimum, household j lives in city $3 - j$, $j = 1, 2$ when $\omega = LH$. Therefore, we have an example with an equilibrium allocation that is ex ante but not ex post efficient.

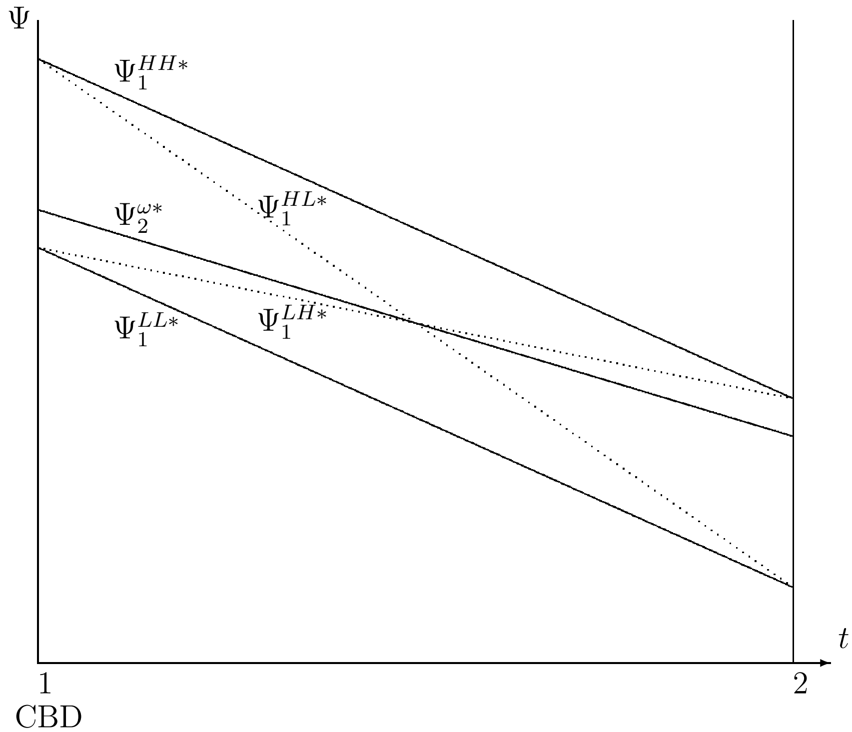

Figure 3: The non-fully revealing rational expectations equilibrium allocation can fail to be Pareto optimal.

4 The Existence of Rational Expectations Equilibrium

After presenting an open subset of economies that possess non-fully-revealing rational expectations equilibrium, it is natural to ask: Can a rational expectations equilibrium fail to exist in urban economies? This can undermine the minimal requirement for further analysis in urban economics with uncertainty. In this section, the existence of (not necessarily fully-revealing) rational expectations equilibrium is examined, given the assumption of ordered relative steepness of bid-rents. First we describe how the existence of equilibrium depends on the number of locations relative to the number of households.[11]

When the number of locations is greater than the number of households, since each household can consume housing in at most one city, there must exist at least one city where no household lives. In these abandoned cities, by Walras' Law, the price of housing is zero. Therefore, unless the commuting cost is very high and these cities are far away from the core cities, households have an incentive to move into these cities to enjoy a higher utility. In this case, there is no equilibrium.

When the number of locations is the same as the number of households, the assumption of ordered relative steepness of bid rents ensures that every location is occupied by exactly one household in equilibrium. Therefore, we can settle households one-by-one from the core to periphery in the order of the slopes of their bid rents, constituting an equilibrium allocation.[12] Thus, we know ex ante what information will be revealed by equilibrium prices, so we can add this information to the consumer's optimization problem. The case when the number of households is larger than the number of locations is left to future work. This case is difficult because we don't know ex ante (due to an endogenous lot size) where consumers will reside in equilibrium, so we don't know what information will be revealed by equilibrium prices. This would be the case, for example, if there were a continuum of consumers.

Suppose there are n households and n locations. Before proving a theorem on the existence of equilibrium, we need to make following assumptions on households' bid rents. These assumptions are standard in urban economics;

[11]By contrast, in standard general equilibrium models, the focus is on the dimension of prices (which is the same as the number of cities in our examples) relative to the dimension of parameters.

[12]Without the assumption of ordered relative steepness of bid rents, we must find a fixed point in the information structure, which is hard.

see for example Fujita (1985, 1989).[13] Given a distance t from the core, a specific state ω, and a utility level u, denote $\Psi_j^\omega(t,u) \equiv \max_{\varphi_{jk}^\omega}\{\frac{Y_j - t - z_j^\omega}{s_j^\omega} | u_j = u\}$ to be household j's bid rent for the housing in the distance t, given ω and u.[14]

Assumption 1 (Ordered Relative Steepness of Bid Rent)
Households' bid rent functions are ordered by their relative steepnesses. That is, given $j < j' \leq n$, Ψ_j^ω is steeper than $\Psi_{j'}^\omega$: Whenever $\Psi_j^\omega(\bar{t}, u_j) = \Psi_{j'}^\omega(\bar{t}, u_{j'}) > 0$ for some $\bar{t}$, u_j and $u_{j'}$, then

$$\Psi_j^\omega(t, u_j) > \Psi_{j'}^\omega(t, u_{j'}) \;\; \forall\, 0 \leq t < \bar{t}, \tag{38}$$

$$\Psi_j^\omega(t, u_j) < \Psi_{j'}^\omega(t, u_{j'}) \;\; \forall\, t > \bar{t} \text{ where } \Psi_j^\omega(t, u_j) > 0. \tag{39}$$

When households have the same utility function but different incomes, and when housing is a normal good, ordered relative steepness of bid rents is naturally satisfied.[15] However, when households have different utilities but the same income, ordered relative steepness of bid rent is not implied. The assumption of ordered relative steepness of bid rents ensures that given arbitrary levels of utilities for two agents, for each state, their bid rents must single cross at one point as shown in Figure 4, where the bid rent curves shift down as the utility levels increase. For example, the Cobb-Douglas utilities in Example 1 and 2 satisfy the assumption of ordered relative steepness of bid rents, and so do quasi-linear utilities. In what follows, we prove the existence of rational expectations equilibrium given the assumption of ordered relative steepness of bid rents.

[13]In fact, in standard urban economics, the assumption of ordered relative steepness relates to only the uniqueness of equilibrium and makes the proof easier, but existence of equilibrium in urban economics can be proved without this assumption when there is no uncertainty; see Fujita and Smith (1987).

[14]Notice that though cities are discrete points on the line representing distance to the CBD, households' bid rents are in fact continuous functions of the distance from core.

[15]See Fujita (1989), page 28-29.

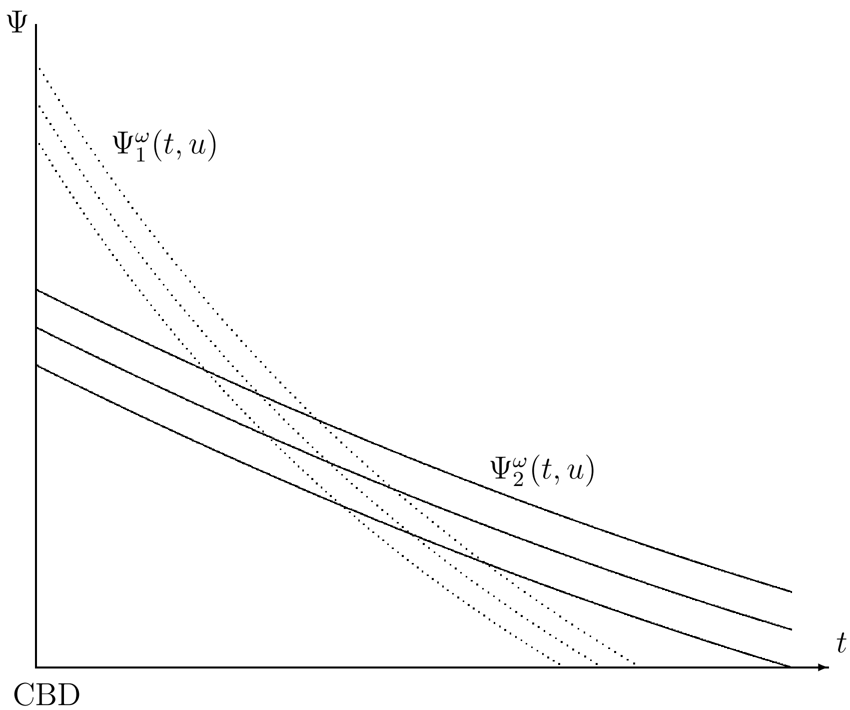

Figure 4: Example where households' bid rents satisfy ordered relative steepness of bid rents.

4.1 When households are insensitive

There are n households indexed by $j \in N \equiv \{1, ..., n\}$, and n cities, $k \in K \equiv \{1, ..., n\}$. Cities are indexed by the order of increasing distance from the core. That is, let T_k denote the commuting cost from city k to the core, $0 = T_1 < T_2 < ... < T_n < \min (Y_j)_{j \in N}$ is assumed to ensure that there is no vacant city. Let $\bar{s}_k$ be the land supply in city k. Also let $\tilde{\sigma}_k \equiv \sigma(\Omega_k) \times (\times_{k' \neq k} \{\phi, \Omega_{k'}\})$, which is the σ-algebra indicating that only the state in city k is known, whereas all states in other cities are completely unknown. To begin, given ordered steepness of bid rents and the same number of consumers and locations, use Assumption 1 to order consumers so that consumer 1 has the steepest bid rent, consumer 2 the next steepest, and so forth. Since the examples in Section 2 highlight the condition required for the existence of non-fully revealing rational expectations equilibrium, in what follows we focus on the case where households present insensitivity:

Definition 5 (Insensitivity)
There exist states $(\omega, \omega') \in \Omega \times \Omega$ *such that for each household* j *such that* ω *and* ω' *are in different partition elements of* $\mathcal{F}_j$,

$$\frac{D_{s_{jj}^{\omega}} u_{jj}^{\omega}}{D_{z_{jj}^{\omega}} u_{jj}^{\omega}} \Big|_{\varphi_{jj}^{\omega *}} = \frac{D_{s_{jj}^{\omega'}} u_{jj}^{\omega'}}{D_{z_{jj}^{\omega'}} u_{jj}^{\omega'}} \Big|_{\varphi_{jj}^{\omega' *}}, \tag{40}$$

but there exists j' *for whom* ω *and* ω' *are in the same element (with a positive probability) of* $\mathcal{F}_{j'}$, $u_{j'j'}^{\omega *} \neq u_{j'j'}^{\omega' *}$.

Let u^{**} solve $\Psi_k^{\omega *}(u) = \frac{D_{s_{kk}^{\omega}} E u_{kk}}{D_{z_{kk}^{\omega}} E u_{kk}} \Big|_{\varphi_{kk}^{\omega *}(u)}$; given housing is a normal good, Berliant and Fujita (1992) show that this solution is unique. So equilibrium always exists and is unique in our model, and the question then becomes whether it is fully revealing or not. Insensitivity is a necessary and sufficient condition for existence of a non-fully revealing rational expectations equilibrium.

The intuition for the condition of insensitivity is that for any household who has information, his/her marginal rate of substitution in city k is independent of two realized states. However, to ensure that the household's information is not trivial, we need the second part of the assumption which implies that his/her information about city k does matter for some other household. Insensitivity can result from one or more of several sources: utility could be quasi-linear, or information about conditions at another location can be irrelevant to a consumer, or some information is irrelevant to all consumers.

Now, consider a public partitional information function P that for every $\omega \in \Omega$, a nonempty subset $P(\omega)$ of Ω is assigned, where (1) for every $\omega \in \Omega$, $\omega \in P(\omega)$; (2) $\omega' \in P(\omega)$ implies $P(\omega') = P(\omega)$. Moreover, for every (ω, ω') satisfying insensitivity, $P(\omega') = P(\omega)$. This condition implies that when ω and ω' are insensitive, and ω' and ω'' are insensitive, then $P(\omega) = P(\omega') = P(\omega'')$. So it can be checked that

$$P(\omega) = \{\omega' \big| P(\omega') = P(\omega)\} \tag{41}$$

In other words, $P(\omega)$ is a partition element collecting states that are directly or transitively insensitive with ω. Intuitively, for all states in $P(\omega)$, either households have no information to distinguish them, or the informed households cannot reflect their information by differences in their marginal rate of substitution. The non-fully revealing rational expectations equilibrium is supported by the σ-algebra generated by the above public partitional information functions.

Theorem 2 *Given Assumption 1 and that housing consumption is a normal good, under insensitivity, for $j = 1, ..., n$, there is an equivalence class of $\sigma(P(\omega))_{\omega\in\Omega}$-measurable bid rent functions $\Psi_j^* : \Omega \to \mathbb{R}_+$ and $\mathcal{F}_j \vee \sigma(\Psi_j^*)$-measurable consumption functions $\varphi_j^* : \Omega \to \mathbb{R}_+^2 \cup \mathbb{R}_+^2$ that constitute a unique non-fully revealing rational expectations equilibrium such that, for $k \in \{1, ..., n\}$,*

$$\Psi_k^{\omega*}(u) = \Psi_{kk}^{\omega}(u) = \max_{s_{kk}^{\omega}, z_{kk}^{\omega}} \{\frac{Y_k - T_k - z_{kk}^{\omega}}{s_{kk}^{\omega}} | E[u_{kk}|\mathcal{F}_k \vee \sigma(\Psi_k^{\omega*})] = u\}; \tag{42}$$

$$\varphi_{jk}^{\omega*}(u) = \begin{cases} (\bar{s}_k,\ Y_k - T_k - \Psi_k^{\omega*}(u)\, \bar{s}_k), & \text{if } j = k, \\ (0,0), & \text{if } j \neq k; \end{cases} \tag{43}$$

and the equilibrium utility level u^{**} can be solved by

$$\Psi_k^{\omega*}(u^{**}) = \frac{D_{s_{kk}^{\omega}} E[u_{kk}|\mathcal{F}_k \vee \sigma(\Psi_k^{\omega*})]}{D_{z_{kk}^{\omega}} E[u_{kk}|\mathcal{F}_k \vee \sigma(\Psi_k^{\omega*})]} \Big|_{\varphi_{kk}^{\omega*}(u^{**})}. \tag{44}$$

Proof. First, by Lemma 1, the rational expectations equilibrium corresponds to the bid rent equilibrium. Second, given Assumption 1, every city is occupied by exactly one household; otherwise, there exists an empty city with zero housing price (by Walras' Law) where all households will move. Since household 1 has the steepest bid rent, from equation (2) in Definition 2, he/she must occupy the housing in city 1 in equilibrium. After settling household 1, we can consider the problem as the one with $n-1$ households ($j \in \{2, ..., n\}$) and $n-1$ cities ($k \in \{2, ..., n\}$). Then, household 2 has a steeper bid rents than remaining households, so he/she wins the housing in city 2. Following the same logic, in equilibrium all households are arranged that household j

lives in city j, or say, city k is occupied by household k, and no one has an incentive to move. This is a standard argument in urban economics.

As shown in Figure 5, given that household k is located in city k, the intercept of budget line $Y_k - T_k$ and the housing supply $\bar{s}_k$ are determined by parameters. Now, given arbitrary u, the slope of budget line $\Psi_k^{\omega *}(u)$ and the corresponding $\varphi_{kk}^{\omega *}(u)$ are uniquely determined (by the budget line and $\bar{s}_k$). Furthermore, given consumption point $\varphi_{kk}^{\omega *}(u)$, since households' preferences are smooth, the slope of the indifference curve passing through $\varphi_{kk}^{\omega *}$ is uniquely determined. Finally, let $\Phi_{kk}^{\omega *}(u) \equiv \frac{D_{s_{kk}^{\omega}} E[u_{kk}|\mathcal{F}_k \vee \sigma(\Psi_k^{\omega *})]}{D_{z_{kk}^{\omega}} E[u_{kk}|\mathcal{F}_k \vee \sigma(\Psi_k^{\omega *})]} \mid_{\varphi_{kk}^{\omega *}(u)}$ the equilibrium utility level (and the equilibrium housing price in city k) is given by solving $\Psi_k^{\omega *}(u) = \Phi_{kk}^{\omega *}(u)$, as shown in Figure 5. Let $f(u) \equiv \Psi_k^{\omega *}(u) - \Phi_{kk}^{\omega *}(u)$, since $\Psi_k^{\omega *}$ and $\Phi_{kk}^{\omega *}$ are continuous in u, $f(u)$ is continuous in u. At $\bar{E}$, $f(u) < 0$ since $\Psi_k^{\omega *}(u) = 0$ at $\bar{E}$. Given $\bar{s}_k > 0$, by smooth boundary condition, $\Phi_{kk}^{\omega *}(u) \to 0$ as $z_{kk}^{\omega} \to 0$, which implies that $\exists \underline{u}$ such that $f(u) > 0, \forall u \leq \underline{u}$. Therefore, by the intermediate value theorem, there exists a u^{**} solving $f(u) = 0$ and thus there exists a rational expectations equilibrium. The uniqueness of equilibrium can be guaranteed by the condition that $\Phi_{kk}^{\omega *}(u)$ is increasing with u, which is true when the consumption of housing is a normal good as shown in Berliant and Fujita (1992).

Under insensitivity, we want to prove that the unique rational expectations equilibrium is non-fully revealing. Suppose the equilibrium is fully-revealing, then choose arbitrary k, we can have

$$\Psi_k^{\omega *} = \Psi_{kk}^{\omega} \neq \Psi_{kk}^{\omega'} = \Psi_k^{\omega' *}, \quad \forall \omega, \omega' \in \Omega. \tag{45}$$

First, for household k (living in city k in equilibrium), any pair of ω, ω' must be in different partition elements. That is, $\mathcal{F}_k \vee \sigma(\Psi_k^{\omega *}) = \mathcal{F}$. Second, from (42) and (44), $\Psi_k^{\omega} \neq \Psi_k^{\omega'}$ implies, $\forall \omega, \omega' \in \Omega$,

$$\frac{D_{s_{kk}^{\omega}} u_{kk}^{\omega}}{D_{z_{kk}^{\omega}} u_{kk}^{\omega}} \Big|_{\varphi_{kk}^{\omega *}} \neq \frac{D_{s_{kk}^{\omega'}} u_{kk}^{\omega'}}{D_{z_{kk}^{\omega'}} u_{kk}^{\omega'}} \Big|_{\varphi_{kk}^{\omega' *}}. \tag{46}$$

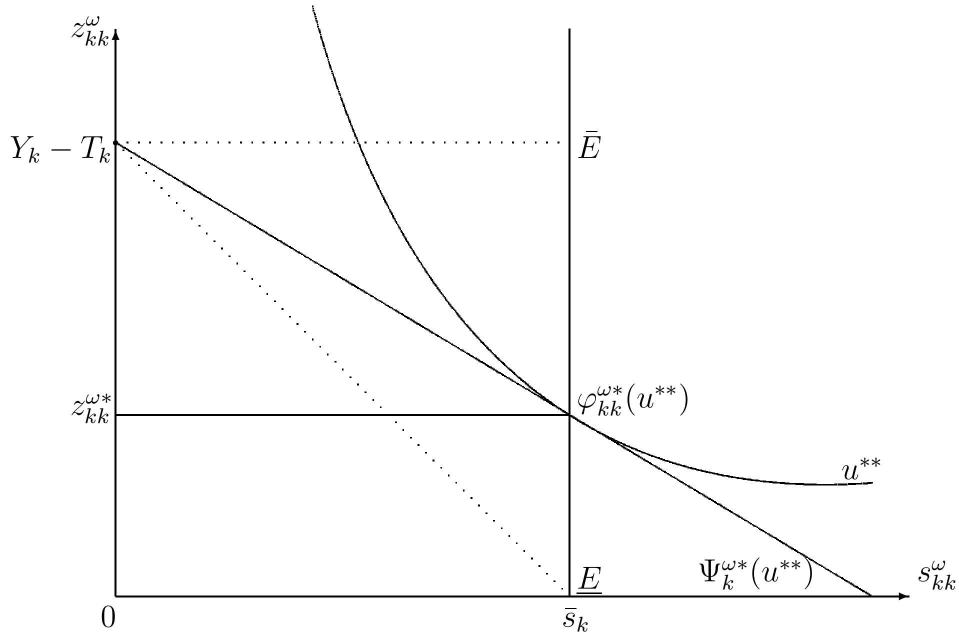

Figure 5: The determination of equilibrium housing price and equilibrium utility for household k in city k in state ω, $k \in \{1, ..., n\}$.

However, since every pair of ω, ω' must be in different partition elements, from insensitivity, there exist ω, $\omega' \in \Omega$ such that

$$\frac{D_{s^{\omega}_{kk}} u^{\omega}_{kk}}{D_{z^{\omega}_{kk}} u^{\omega}_{kk}} \Big|_{\varphi^{\omega *}_{kk}} = \frac{D_{s^{\omega'}_{kk}} u^{\omega'}_{kk}}{D_{z^{\omega'}_{kk}} u^{\omega'}_{kk}} \Big|_{\varphi^{\omega' *}_{kk}}, \tag{47}$$

a contradiction with (46). In fact, these non-fully revealing equilibrium prices reveal nothing beyond $\sigma(P(\omega))_{\omega \in \Omega}$ in equilibrium. *Q.E.D.*

Insensitivity is sufficient and necessary for the existence of a non-fully revealing rational expectations equilibrium. The first part of the definition of insensitivity ensures that any household who can distinguish ω and ω', his/her marginal rate of substitution reflects no difference in states. That is, there are states that the informed household does not care about. However, the second part of the definition implies that his/her information is useful for someone in maximizing utility, that is, there is some information needed which is not transmitted from informed to uninformed households. Therefore, the rational expectations equilibrium is non-fully revealing. Without insensitivity, the equilibrium can only be fully-revealing.

Theorem 3 *Given Assumption 1 and housing consumption is a normal good, under no insensitivity, there exists a unique rational expectations equilibrium that is fully revealing.*

Proof. From Assumption 1 and Lemma 1, it is shown that the rational expectations equilibrium exists and corresponds to the bid rent equilibrium. When insensitivity condition is violated, the realized marginal rates of substitution are different $\forall \omega_k \in \Omega_k$. Furthermore, for every $k \in K$, $\tilde{\sigma}_k \subseteq \mathcal{F}_k$, so the resulting bid rents are different in all states, implying that the rational expectations equilibrium is fully revealing. *Q.E.D.*

In the literature, an open and dense subset of standard economies with fully revealing rational expectations equilibrium is found. However, under the natural assumption of spatially local perturbations of utility functions, as shown in the previous section, an open subset of urban economies with only non-fully revealing equilibria is found. Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics. Non-fully revealing equilibrium is more interesting in highlighting the potential positive value and the strategic use of information. When non-local perturbations are considered, though they are not so reasonable in urban economics, the results are the same as the ones in standard general equilibrium models. That is, there is an open and dense subset of economies that possess a fully revealing rational expectations equilibrium.

As shown in the comparison in Table 1, the inefficiency in information transmission in a housing/land market rests on two key assumptions: spatially local utility perturbations and the standard setting in urban economics that every household can consume housing in only one place. When either of them is violated, the result in standard models is restored. That is, in economic circumstances where there is no location structure or no spatially local property of utility, generically, the efficiency of prices in information transmission is attained in a rational expectations equilibrium. We conclude that geographic structure, together with spatially local utility properties, can play a role in distorting the efficiency of the market in transmitting information from informed to uninformed households.

	Households can consume housing in only one place	Ordinary consumption set
Spatially local utility perturbations	Open subsets of economies with fully revealing and non-fully revealing equilibria (Urban economics)	An open and dense subset of economies with fully revealing equilibria
Spatially non-local utility perturbations	An open and dense subset of economies with fully revealing equilibria	An open and dense subset of economies with fully revealing equilibria (Standard model)

Table 1: A comparison of the results in this paper with the results in the literature.

If households can be redistributed so that location is coincident with information, then we can create a fully-revealing rational expectations equilibrium. However, this idea seems impractical since in most cases, unless the households are very risk averse, households' subjective preferences for location do not necessarily depend on the information that they have. A classical way to induce households to reveal their private information, as shown in Debreu (1959) Chapter 7 and Arrow (1964), is to consider a contingent claim or financial market. This idea is discussed in the next section.

5 Adding Financial Markets

Some may wonder: When contingent claim or financial markets are included, do our examples with no fully-revealing rational expectations equilibrium survive? This interesting question is examined here.

Following the setting of Example 1, and in addition, following Magill and Quinzii (1996), consider that the two households can buy and sell state-contingent financial securities in financial markets before they consume composite good and housing. That is, consider a one-period, two-stage model as follows. At the beginning of the first stage, households are endowed with e_j^0 units of numeraire (composite consumer good), $j = 1, 2$. Household 2 has complete information about the states in two cities, and household 1 has no information. The financial markets are opened in stage 1, where the two households can buy and sell securities. Assume that the financial markets are complete in that the number of securities is the same as the number of states, so we can use the same index for securities and states. Specifically, the security ω, $\omega \in \Omega$, is a contract promising to deliver one unit of numeraire (income) in state ω, and 0 in other states, in the second stage. All securities are perfectly monitored and perfectly enforced. After closing the financial markets and the first stage, all security returns are paid at the beginning of the second stage. Then an absentee landlord trades with households in spot housing markets. The game is complete when the housing markets are closed. We want to know whether there is a fully-revealing rational expectations equilibrium under the new setting or not.

Denote e_j^ω to be household j's endowment in state ω in the second stage, and $\nu_j \equiv (\nu_j^\omega)_{\omega\in\Omega} \in \mathbb{R}^4$ be household j's portfolio. Let $q \equiv (q^\omega)_{\omega\in\Omega} \in \mathbb{R}^4$ and $V \equiv (V^\omega)_{\omega\in\Omega} \in \mathbb{R}^{16}$ where $q^\omega \in \mathbb{R}$ and $V^\omega \in \mathbb{R}^4$ represent the price of security ω and the payoffs of securities in state ω, respectively. That is, V^ω is a row vector of zeros except that the element representing state ω is 1, and $V^\omega \neq V^{\omega'}$, for all $\omega \neq \omega'$. The fully revealing rational expectations equilibrium under the new setting can be solved by backward induction as follows.

Suppose there exists a fully revealing rational expectations equilibrium. From Section 2.2, it is shown that given Y_j^ω, households' indirect utility functions with optimization in stage 2 are

$$U_1^\omega = \alpha_1^\omega \ \ln \bar{x} - \ln(1+\alpha_1^\omega) + \ln Y_1^\omega,$$

$$U_2^\omega = \beta_2^\omega \ \ln \bar{y} - \ln(1+\beta_2^\omega) + \ln Y_2^\omega.$$

Through monotonic transformations of these indirect utility functions, household j's optimization problems in stage 1 can be written as

$$\begin{aligned} \max_{\nu_j} \ & \tilde{U}_j^\omega \equiv \ln Y_j^\omega \\ \text{s.t. } & q \cdot \nu_j = e_j^0, \\ & Y_j^\omega - e_j^\omega = V^\omega \, \nu_j. \end{aligned}$$

Denoting the true state as $\hat{\omega}$, since households learn the true state by observing prices in a fully revealing rational expectations equilibrium, it is obvious that the equilibrium security prices must satisfy $q^{\hat{\omega}*} = 1$ and $q^{\omega*} = 0, \forall \omega \neq \hat{\omega}$. Since for arbitrary different $\hat{\omega}$, $\hat{\omega}'$, the corresponding equilibrium price vectors are not the same, each q^* reveals a unique $\hat{\omega}$. Therefore, it follows that q^* supports a fully-revealing rational expectations equilibrium.

Though we show that adding financial markets helps to reveal the informed household's private information, there are some issues with this idea. Grossman and Stiglitz (1980) argue that the informed household can use their private information to take advantage of uninformed households. Thus, if the financial markets and the corresponding fully-revealing equilibrium prices make private information publicly available to every household, the informed household could not earn an information rent (coming from asymmetric information) and has an incentive to hide his/her private information (by pretending to be uninformed). Therefore, though adding financial markets can restore the existence of a fully-revealing rational expectations equilibrium, there are reasons why these financial markets might not exist. Of course, if financial asset markets are incomplete for whatever reason, the problems we have discussed return.

6 Conclusions

Radner (1979), Jordan (1980), and Allen (1981) prove the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. Since in urban economies an open subset of economies without fully-revealing rational expectations equilibrium is found, this paper shows that Allen's theorem about the existence of a dense subset of economies possessing fully-revealing rational expectations equilibrium does not extend to urban economies when spatially local perturbations of utilities are considered. These perturbations retain the property that the utility of living at a location depends only on the consumption bundle at that location and the resolution of uncertainty about local variables only. Furthermore, since an open subset of economies with fully revealing rational expectations equilibria can easily be constructed, we cannot challenge the existence of an open subset of economies that possess fully-revealing rational expectations equilibria in the context of urban economies. Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics.

This paper highlights the important "local conditions" for the existence of rational expectations equilibria in urban economies. The existence of a unique rational expectations equilibrium is proved with the assumption of ordered relative steepness of bid rents. Whether the rational expectations equilibrium is fully revealing or non-fully revealing depends on the insensitivity condition: When insensitivity is satisfied, the unique rational expectations equilibrium is non-fully revealing; otherwise, the equilibrium is fully revealing. Though introducing financial markets can restore the existence of fully-revealing rational expectations equilibrium, many provisos also accompany it. In summary, geography can play a role in undermining the efficiency of market prices in transmitting information from informed to uninformed households.

One potential extension of this paper is to consider a continuum of households; however, the intuition that the mismatch of locally-informed households and their corresponding equilibrium locations is likely to yield an open subset of economies possessing only non-fully revealing rational expectations equilibria seems robust. Other topics for future research are to extend the intuition behind our results to other models. For example, in an overlapping generations model, time may play a role similar to the spatial structure in preventing information transmission. Moreover, when search/matching models are considered, stable equilibrium may also pick only the best of all potential matches. In either of these cases, we conjecture that there exists an open subset of economies with no fully-revealing rational expectations equilibrium, since agents with information about states in other lifetimes (in the overlapping generations framework) or in other equilibrium matches (in the search framework) might not have their information reflected in equilibrium prices.

Appendix A. Proof of Lemma 1

Comparing Definition 1 and Definition 2, since condition (iii) is the same as equations (4), $\forall\omega$ with $\mu(\omega) > 0$, we only need to prove that conditions (i) and (ii) are equivalent to equations (2) and (3).

First, to prove this, given that (2) and (3) are satisfied but either (i) or (ii) is not true, we want to show contradictions. If (i) is not true, there exists $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $\psi_k^*(\omega) \cdot \varphi_{jk}^*(\omega) > Y_k - T_k$, $\forall \omega \in \Omega_0$. Then for these $\omega \in \Omega_0$, we can have $\Psi_k^{\omega *} s_{jk}^{\omega *} + z_{jk}^{\omega *} > Y_k - T_k$, which implies

$$\Psi_k^{\omega *} > \frac{Y_k - T_k - z_{jk}^{\omega *}}{s_{jk}^{\omega *}}, \ \ \forall \omega \in \Omega_0,$$

a contradiction with (2), choosing the utility level the same as the optimalized level by Definition 1 ($u = u^*$).

On the other hand, if (ii) is not true, then $\exists j$ and $\varphi_j'(\omega)$ within the budget constraint such that

$$\int_\Omega u_j^\omega(\varphi_j')\, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)) > \int_\Omega u_j^\omega(\varphi_j^*)\, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)). \tag{48}$$

For this household j and for city k where he/she lives, we can choose $u = u^*$, and then by strict concavity and strict monotonicity, there exists $\epsilon > 0$ and $\varphi_j''(\omega) \equiv \frac{\varphi_j'(\omega) + \varphi_j^*(\omega)}{2} - \epsilon$ such that $E[u(\varphi_j''|\mathcal{F}_j)] = u^*$. Since $\psi_k^*(\omega) \cdot \varphi_{jk}''(\omega) < Y_k - T_k$ implies $\Psi_k^{\omega *} < \frac{Y_k - T_k - z_{jk}^{\omega''}}{s_{jk}^{\omega''}}$, let $\Psi_k^{\omega''} \equiv \frac{Y_k - T_k - z_{jk}^{\omega''}}{s_{jk}^{\omega''}}$, we find $\Psi_k^{\omega''} > \Psi_k^{\omega *}$ for a given $u = u^*$. Thus, $\varphi_{jk}^{\omega *}$ does not maximize $\Psi_{jk}^{\omega *}$, a contradiction with equation (3).

Secondly, given (i) and (ii) are true, but either (2) or (3) is not satisfied, we want to prove that there is a contradiction. If (2) does not hold, there exists k and j such that $\Psi_{jk}^\omega > \Psi_k^{\omega *}$ but j does not live in city k. Suppose j lives in city $k' \neq k$. Then for this household j, since he/she can pay less for the housing in k than the price that makes he/she indifferent between the housing in k and k', household j has an incentive to move into city k, a contradiction with condition (ii) that φ_j^* maximizes j's conditional expected utility.

If (3) does not hold, since the budget line with $\Psi_{jk}^{\omega *}$ is not tangent to the indifference curve for a given u, by strict concavity, there exists $\varphi_{jk}^{\omega'} \neq \varphi_{jk}^{\omega *}$ such that $E[u(\varphi_j'|\mathcal{F}_j)] = u^*$. By strict concavity, choosing $\varphi_{jk}^{\omega''} \equiv \frac{\varphi_{jk}^{\omega *} + \varphi_{jk}^{\omega'}}{2}$, then $\varphi_{jk}^{\omega''}$ is available for household j to achieve $E[u(\varphi_j''|\mathcal{F}_j)] > u^*$, a contradiction with (ii) that φ_j^* maximizes household j's expected utility.

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