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The Long-run Effects of Household Liquidity Constraints and Taxation on Fertility, Education, Saving, and Growth

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Abstract

This paper investigates economic growth under liquidity constraints by taking into account the choices of fertility, human capital and saving. In a model of four overlapping generations, parents are altruistic towards their offspring and finance their education investment. The government provides education subsidies to young adult parents and levies taxes on income of the adult generation. Sensitivity analysis on borrowing limits and tax parameters highlights effects with opposite sign on the main endogenous variables at steady state. A lift in liquidity constraints decreases savings and capital accumulation and this effect is responsible for the ambiguous sign of comparative statics on the rate of fertility and on human capital investment. From model simulation, we derive an inverted U-shaped curve relating the borrowing limit with fertility, education and growth, meaning that financial reforms in the less developed countries have positive effects on the economy in the long-run, even if they raise fertility and reduce savings. Greater government subsidies to human capital investments and lower income taxes have positive effects on savings and fertility. The same parameters present ambiguous effects on education investments and growth. Numerical simulations show that a) human capital investment has an inverted U-shaped relation with income taxes and education subsidies; b) economic growth decreases with greater income taxes and increases with higher education subsidies.

Jel codes: O40, O16, J13, D91.

1 Introduction

The family has a central role in the modern theory of economic growth and development that considers human capital accumulation as the engine of economic dynamics in the long term (e.g., Becker, Murphy and Tamura, 1990; Galor and Weil, 1996). In this strand of the literature, recent research has investigated the consequences of limited access of households to the credit market on aggregate economic outcomes. In this paper we study the effects of borrowing constraints on economic growth by taking into account all the major decisions of households: fertility, child education and savings. Although the importance of the connections among these sides of household behavior is clear in the microeconomics of the family(e.g., Becker, 1991; Cigno, 1991; Schultz, 1997), the existing literature on economic growth still lacks a comprehensive study of the issue in economic environments characterized by credit market imperfections. In this analysis we also consider an even more neglected issue in growth models: family taxation with endogenous fertility, which is an important feature of modern economic systems since several forms of market failure have pervasive effects on household behavior.

The existing literature analyzes particular aspects of household choices under borrowing constraints. The seminal article by Jappelli and Pagano (1994) highlights the consequences of liquidity constraints on saving and growth. If household expenditure on consumption is limited, then savings are greater and capital accumulation is stronger, and this effect can cause higher economic growth. De Gregorio (1996) shifts the focus on family behavior from savings to investment in human capital. Young individuals who attend school face an opportunity cost given by forgone earnings. If they cannot fully finance this cost by borrowing on the credit market, then they will reach a lower level of human capital, which can be detrimental to economic growth. De Gregorio and Kim (2000) and Azariadis and de la Croix (2006) extend this approach to analyze the evolution of income distribution and growth in economies with imperfect capital markets. All these papers assume exogenously given liquidity constraints, while some recent interesting research (Lochner and Monge-Naranjo, 2002; Andolfatto and Gervais, 2006; Papagni, 2006; de la Croix and Michel, 2007) follows the approach of Kehoe and Levine (1993)[1] to endogenize borrowing limits in life-cycle models of human capital investment. It must be noted that only Lochner and Monge-Naranjo, (2002), and Azariadis and de la Croix (2006) study the joint dynamics of physical and human capital relying on numerical simulations, while only in Papagni (2006) is fertility choice endogenous in a model of a small-open economy[2].

[1] See also Azariadis and Lambertini, 2003.

[2] A different but important strand of the literature analyzes the effects of social security

In this paper, our aim is to study how liquidity constraints influence steady-state economic growth through the interactive effects they have on the most important decisions of parents: number of children, their level of education, consumption and savings. In order to preserve a comprehensive approach to family decisions, we retain from the literature the assumption of exogenous borrowing limits.

Intergenerational linkages and borrowing/lending household behavior are clearly specified in a model of four overlapping generations. Agents acquire human capital when young, then work and have children in the first age of adulthood. In this period, they can choose to rely on the credit market to finance consumption and expenditure on child education. However, parents face a limit to borrowing which could ration their current expenditures. In the next age, children leave the family and adult workers save to preserve their future well-being under retirement. Imperfections in the credit market justify government intervention which consists in subsidies to child education expenditure financed by flat-rate taxes on labor income of the second generation of adults who are free from child support.

The model owns a unique stationary equilibrium. Numerical simulations show that the steady state has the stability properties of a local saddle point. Sensitivity analysis on borrowing limits and tax parameters highlight effects with opposite sign on the main endogenous variables at steady state. A lift in liquidity constraints decreases savings and capital accumulation as in Jappelli and Pagano (1994),[3]and this effect is responsible for the ambiguous sign of comparative statics on the rate of fertility and on human capital investment. From model simulation, we derive an inverted U-shaped curve relating the borrowing limit with fertility, education and growth, meaning that financial reforms in the less developed countries have positive effects on the economy in the long-run, even if they raise fertility and reduce savings. The same reform in countries with significant financial development could be ineffective or detrimental to economic growth. These results seem confirmed by econometric analyses. Indeed, at the micro level, Pitt et al. (1999) find that female and male participation to microcredit programs in Bangladesh increases fertility, which is consistent with the shape of the curve relating the credit limit with fertility at low levels of financial development. A similar non-monotonicity of the function of the growth rate seems confirmed by

on fertility and growth (e.g., Boldrin, et al., 2005; Ehrlich and Kim, 2007). The first paper also considers the degree of financial market development. Indeed, it shows that when households can access to better financial instruments for saving fertility is lower because they can rely less on their children's aid in the old age.

[3]This effect is confirmed by econometric analyses of Jappelli and Pagano (1994) and Loayza et al. (2000).

studies based on aggregate data. Indeed, Jappelli and Pagano (1994) and De Gregorio (1996) find some evidence of a positive relation between liquidity constraints and growth in estimates on data from OECD countries, while De Gregorio's (1996) estimates show a negative relation when the sample refers to developing countries.

According to comparative statics of steady-state equilibrium, greater government subsidies to human capital investments and lower income taxes have positive effects on savings and fertility. The same parameters present ambiguous effects on education investments and growth. In fact, in this model there are several channels through which parameters influence the endogenous variables often in opposite directions. Numerical simulations resolve this sign indeterminacy since they show that a) human capital investment has an inverted U-shaped relation with income taxes and education subsidies; b) economic growth decreases with greater income taxes and increases with higher education subsidies. Most of these comparative-statics effects are new for the literature on economic growth with endogenous fertility, where the only similar papers are Zhang and Casagrande (1998) who find that the fertility rate does not depend on fiscal policy parameters[4], and Papagni (2006) where in a model of multiple equilibria, the effects of fiscal policy depend on the level of steady-state fertility rate.

The whole set of results of this paper represents a contribution to the analysis of the ways credit market imperfections and public policy affect household behavior and economic growth in the long run. They provide some insights into the channels through which these phenomena interact, which emerge from a comprehensive account of intergenerational relations among members of the family and those expressed by the state.

The rest of the paper is organized as follows. Section 2 presents an OLG model of economic growth. Section 3 derives general equilibrium and characterizes the dynamic properties of the steady state. Section 4 presents the results of sensitivity analysis of steady state variables with respect to parameters proxy of credit availability and fiscal policy. Section 5 concludes the paper.

2 The Model

We put forward an overlapping-generations model of economic growth with endogenous fertility. The economy is populated by identical individuals

[4] An interesting reference is Jappelli and Pagano (1999) which deals with fiscal policy in a model of liquidity constraints and growth. Boldrin et al. (2005) and Ehrich and Kim (2007) investigate the effects of social security policy on fertility and economic growth.

whose life is summarized in four periods, such that they are young in the first, young adult in the second, adult in the third and old in the fourth. Agents attend school when young, work and take care of children during the first period of adulthood, work and save when adult, and retire and consume saving returns when they are old. The credit market is affected by imperfections which bring about constraints to the household borrowing ability. Young agents do not work, nor have access to the financial market and education costs are borne by their altruistic parents. The economy produces one homogeneous good and is closed to international markets. A crucial assumption is that individuals are endowed with perfect foresight.

2.1 Technology

Time is discrete and is denoted by $t = 0, 1, 2, ...\infty$. The labor force is made by population of the two adult generations. We denote with $N_t \equiv N_t^{t-1}$ the number of young adults born in time $t-1$ and living in period t, such that $n_t = \frac{N_{t+1}^t}{N_t^{t-1}}$ represents the number of children born of a young adult at time t. Every adult is endowed with human capital e_t that she acquired during childhood by attending school. This is a productive process which requires resources in terms of goods and services that must be drawn from other uses. Teachers, books and other inputs (television, journals, travels, etc.) can be considered within a general definition of learning technology. Here, we specify a simple human capital production function:

$$e_t = \Lambda b_t \ , \tag{1}$$

where Λ is the level of learning technology, and b_t stands for the amount of resources employed in the learning process. Human capital does not depreciate and can be used in production during two generations of agent life.

A single homogeneous good, Y, is produced in the economy according to a production function with constant returns to scale with respect to capital, K_t, and labor, L_t, inputs:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \ , \quad \alpha \in (0, 1) \, ,$$

where A denotes the exogenous level of technology. Labor input L is made by raw labor and efficiency units supplied each time by two generations of adults: $L_t = N_t e_t + N_{t-1} e_{t-1}$. As usual, we express the production function per unit of effective labor terms as:

$$y_t = Ak_t^\alpha \ , \tag{2}$$

where $y_t = \frac{Y_t}{L_t}$, and $k_t = \frac{K_t}{L_t}$.

The sector of good production is competitive as are the markets for factors of production. Accordingly, profit maximization and market equilibrium imply that wage per efficiency unit of labor, w_t, equals marginal productivity:

$$w_t = A(1-\alpha)\,k_t^{\alpha} \;, \tag{3}$$

and the rental rate of capital, R_t, equals capital marginal productivity:

$$R_t = A\alpha k_t^{\alpha-1} \;. \tag{4}$$

2.2 Household preferences and budget constraints

The family is composed by a single parent and by children that she has in the second period of her life. The first generation of individuals is concerned mainly with schooling. Children do not work and their consumption and expenditure on education - both included in b_t - derive from the income of their young parents. Indeed, we distinguish adulthood into two periods according to the kind of relationship between children and parents. During the first period adults have children and take care of them until they reach the age which allows them to work and be self-sufficient. After this stage of adult life, children leave their parents' house and intergenerational linkages in the family disappear[5].

To simplify the analysis we assume that childhood is a dummy generation meaning that consumption is an input of education and does not provide any utility to children. Furthermore, children do not have resources and cannot borrow to finance human capital investment, hence this decision is made by their altruistic parents[6]. Adults decide consumption over their life-cycle, number and human capital of children, saving and debt with the aim of intertemporal utility maximization. Taking the point of view of a young adult, we denote with $C_{i,t+i} \equiv C_{i,t+i}^{t-1}$, $i = 0, 1, 2$, consumption of an agent born at time $t-1$, where i is the age of her life varying from young adulthood to old age.

Parents appreciate the presence of children in the family and spend part of their earnings to rear them. Altruism in the family also motivates the preference of the parents for well-educated children. The intertemporal utility

[5] We do not deal with bequest, because we concentrate on the effects of liquidity constraints on *inter vivos* transfers which according to the empirical literaure (e.g., Cox and Jappelli, 1990) dominate bequest.

[6] These assumptions over child and parent behavior have been made in several papers, e.g., de la Croix and Doepke, (2004)

at period t of an adult born at time $t-1$ is represented by the function

$$V\left(C_{0,t}, C_{1,t+1}, C_{2,t+2}, e_{t+1}, n_t\right) =$$
$$\log(C_{0,t}) + \beta \log(C_{1,t+1}) + \beta^2 \log(C_{2,t+2}) + \varphi \log\left(e_{t+1}\right) + U\left(n_t\right),$$

where $\beta \in (0,1)$ is a discount factor, $\varphi > 0$ measures the importance of children's education in the preferences of a young adult, e_{t+1} denotes human capital of children born at time t which will be adult in the next period, and $U\left(n_t\right)$ is a continuous function of fertility satisfying the assumptions:

$$U_n\left(n_t\right) > 0,\ U_{nn}\left(n_t\right) < 0,\ \lim_{n\to 0} U_n\left(n_t\right) = \infty,\ \lim_{n\to\infty} U_n\left(n_t\right) = 0.$$

In this economic environment adults make all decisions. During the first period of adulthood households enjoy consumption and take care of children. Raising children is an expensive activity and we assume parents spend a share $\tau \in (0,1)$ of their labor income on child rearing with a cost of $\tau n_t w_t e_t$. Young adults can increase their resources by borrowing on imperfect credit markets, or could save to increase consumption during the next ages. In the case of a loan lower than the maximum amount allowed by the credit system (i.e., unconstrained borrowing), young adults face the following budget constraint:

$$C_{0,t} + \tau n_t w_t e_t + (1-v)\, n_t b_{t+1} = w_t e_t + D_t, \tag{5}$$

where D_t denotes borrowed resources, and $v \in (0,1)$ is the share of children's education costs that is subsidized by the government. Note that D_t can also be negative in the case of households choosing to save for the future. After the first period of adulthood, children leave the family and parents become worried about their welfare in the years in which they will have retired. Hence, adults repay debts D_t previously incurred (or enjoy saving returns if $D_t < 0$), consume and save part of their remaining income for consumption in old age. Their choices are consistent with the following budget constraint:

$$C_{1,t+1} + R_{t+1} D_t + S_{t+1} = (1-t_w)\, w_{t+1} e_t, \tag{6}$$

where S_{t+1} denotes saving of an adult born in period $t-1$, and $t_w \in (0,1)$ denotes a tax rate on adult income[7]. In old age individuals retire and cannot borrow since nobody will repay their loan. Hence they finance consumption with the returns from saving in the previous age:

$$C_{2,t+2} = R_{t+2} S_{t+1}. \tag{7}$$

[7] Here we are assuming that wages of adults are subject to taxes while those of young adults are not. This is a simplifying assumption that can be easily analytically justified by considering that young adults are the recipients of education subsidies.

The life-cycle present value budget constraint derives from single-period budget constraints eqs. (5)-(7):

$$C_{0,t}+\frac{C_{1,t+1}}{R_{t+1}}+\frac{C_{2,t+2}}{R_{t+2}}+\tau n_t w_t e_t+(1-v)\,n_t b_{t+1}=w_t e_t+\frac{(1-t_w)\,w_{t+1}e_t}{R_{t+1}}. \quad (8)$$

More realistically, households are not allowed to borrow any amount of resources consistent with their life-cycle income because the credit market is affected by important imperfections. A recent literature has endogenized liquidity constraints in life-cycle models (e.g., Azariadis and Lambertini, 2003) following the framework of Kehoe and Levine (1993) where agents who default on a loan contract cannot borrow in the future, and credit is allowed only to those who have the incentive to repay their debt. Several applications of this approach concern the finance of human capital investment in models of dynamic general equilibrium under some simplifying hypotheses (e.g., Lochner and Monge-Naranjo, 2002; Andolfatto and Gervais, 2006; Papagni, 2006; de la Croix and Michel, 2007). All these paper but Papagni (2006) consider fertility exogenous, while Andolfatto and Gervais (2006) and Papagni (2006) make the assumption of a small open economy which implies exogenous capital accumulation. While, endogenizing liquidity constraint makes an important contribution to the analysis of household behavior in the life-cycle general equilibrium model, assumptions which neglect the number of children in household decision-making deny the possibility that parents who face liquidity constraints modify their choices with respect to the number of children. Indeed, constrained households could either reduce the quantity and quality of their children or could have fewer children but invest more in their human capital. The aggregate consequences of these alternatives can differ significantly.

To take account in the model of all the major dimensions of household decisions in the life cycle, we assume exogenous credit constraints as do Jappelli and Pagano (1994), Buiter and Kletzer (1995) and De Gregorio (1996) amongst others. In particular, we assume that the borrowing limit $\overline{D_t}$ is defined implicitly by the following rule:

$$w_t e_t+\overline{D_t}=\Psi\left[w_t e_t+\frac{(1-t_w)\,w_{t+1}e_t}{R_{t+1}}\right], \quad (9)$$

Equation (9) establishes that for any value of the given parameter Ψ there is a specific amount of credit limit $\overline{D_t}$. Ψ can be thought of as the share of life cycle income available to young adults in each period. It is a positive constant that assumes its minimum value, $\underline{\Psi}$, at:

$$w_t e_t = \underline{\Psi} \left[w_t e_t + \frac{(1 - t_w) w_{t+1} e_t}{R_{t+1}} \right],$$

in which case households cannot borrow: $\overline{D_t} = 0$. The opposite case is that of unconstrained access to the credit market which occurs when Ψ assumes its maximum value: $\overline{\Psi} = 1$, and $\overline{D_t} = \frac{(1-t_w)w_{t+1}e_t}{R_{t+1}}$. Accordingly, when the parameter Ψ assumes values lower than one, households are constrained in their borrowing ability if the ratio of current expenditure on consumption and children to the present value life-cycle income is greater than Ψ, which means $D_t > \overline{D_t}$ and young adults cannot finance their desired expenditure:

$$C_{0,t} + \tau n_t w_t e_t + (1 - v) n_t b_{t+1} \leq \Psi \left[w_t e_t + \frac{(1 - t_w) w_{t+1} e_t}{R_{t+1}} \right]. \tag{10}$$

2.3 Optimization

In the first period of life individuals accomplish plans made by their parents. In the next age, households become adult and make programs over the main aspects of their life. Hence, at period t a young adult chooses consumption of the remaining three periods of life, how many children she will have and their level of education according to the following utility maximization problem:

$$\max_{C_{0,t}, C_{1,t+1}, C_{2,t+2}, e_{t+1}, n_t} V (C_{0,t}, C_{1,t+1}, C_{2,t+2}, e_{t+1}, n_t)$$

subject to

$$C_{0,t} + \tfrac{C_{1,t+1}}{R_{t+1}} + \tfrac{C_{2,t+2}}{R_{t+2}} + \tau n_t w_t e_t + (1 - v) n_t b_{t+1} = w_t e_t + \tfrac{(1-t_w)w_{t+1}e_t}{R_{t+1}};$$

$$C_{0,t} + \tau n_t w_t e_t + (1 - v) n_t b_{t+1} \leq \Psi \left[w_t e_t + \tfrac{(1-t_w)w_{t+1}e_t}{R_{t+1}} \right].$$

To solve this problem we specify a Lagrangian function in which λ_t denotes the multiplier associated with the intertemporal budget constraint eq. (8) and μ_t is the multiplier of the borrowing constraint, eq. (10). The following assumptions on the utility function

- A1: $-U_{nn} (n_t) n_t^2 > \varphi$, $\forall n_t \in R_+$;
- A2: $U_{nn} (n_t) n_t + U_n (n_t) > 0$, $\forall n_t \in R_+$;

ensure: a) the utility maximization problem is convex[8]; b) the young adult chooses a non-negative number of children; c) a trade-off between quality and quantity of children in parents' decisions. Hence, the first order conditions:

$$\begin{aligned}
&\tfrac{1}{C_{0,t}} - (\lambda_t + \mu_t) = 0; \\
&\tfrac{\beta}{C_{1,t+1}} - \tfrac{\lambda_t}{R_{t+1}} = 0; \\
&\tfrac{\beta^2}{C_{2,t+2}} - \tfrac{\lambda_t}{R_{t+1}R_{t+2}} = 0; \\
&\tfrac{\varphi}{b_{t+1}} - (\lambda_t + \mu_t)(1-v)\, n_t = 0; \\
&U_n(n_t) - (\lambda_t + \mu_t)\left[(1-v)\, b_{t+1} + \tau w_t e_t\right] = 0.
\end{aligned} \tag{11}$$

are sufficient for a maximum. In the case of non-binding liquidity constraints, the young adult household would choose consumption and children expenditure as:

$$C_{0,t} + \tau n_t w_t e_t + (1-v)\, n_t b_{t+1} = \Omega(n_t)\left[w_t e_t + \frac{(1-t_w)\, w_{t+1} e_t}{R_{t+1}}\right],$$

where

$$\Omega(n_t) \equiv \frac{U_n(n_t)\, n_t + 1}{1 + \beta + \beta^2 + U_n(n_t)\, n_t}.$$

Accordingly, liquidity constraints are binding if $\Psi < \Omega$, in which case the household cannot borrow the amount that would maximize her utility. In the following, we will maintain this hypothesis for any period t.

Concerning conditions (), the first three refer to consumption choice and have straight interpretation. The saving rate of an adult of the generation $t-1$ can be derived from the f. o. c.

$$S_{t+1} = \frac{\beta^2 (1-\Psi)\, R_{t+1}}{\beta + \beta^2}\left[w_t e_t + \frac{(1-t_w)\, w_{t+1} e_t}{R_{t+1}}\right]. \tag{12}$$

Hence, adult saving is a function of the life cycle income and increases with the tightness of liquidity constraint (see Jappelli and Pagano, 1994).

[8] Indeed, the quantity-quality of children trade-off is known to introduce non-convexity into household optimization problems. If we rewrite the utility maximization problem defining the new variable $q_{t+1} = n_t b_{t+1}$ and replacing b_{t+1} with $\frac{q_{t+1}}{n_t}$ then we get a new problem that under A1-A2 is convex (see also Willis, 1973).

The fourth condition refers to the decision of the parent on investment in children's education, and equates marginal utility of higher education to the marginal cost which is clearly dependent on the number of children to be educated. After manipulation of equations (11) we get the following rule for child investment:

$$b_{t+1} = \frac{\varphi\Psi}{(1-v)\,n_t\,(U_n\,(n_t)\,n_t+1)}\left[w_t e_t + \frac{(1-t_w)\,w_{t+1}e_t}{R_{t+1}}\right], \tag{13}$$

which states that parents spend on their children's education a fraction of their life-time income which depends positively on credit availability, Ψ, and negatively on fertility.

The last of the first order conditions derives from the parent's choice of fertility, and equates the marginal increase in utility of one more child to its marginal cost which is made by two components: child rearing and education. Even in the case of fertility the solution of the household's decision problem provides us with the following rule that implicitly describes the factors behind choosing the number of children:

$$\frac{n_t\,(U_n\,(n_t)\,n_t+1)}{(U_n\,(n_t)\,n_t-\varphi)} = \frac{\Psi}{\tau}\left[1+\frac{(1-t_w)\,w_{t+1}}{w_t R_{t+1}}\right]. \tag{14}$$

It can be easy to verify that under assumptions A1-A2 the left side of equation (14) increases with n_t, and this effect implies that parents have more children if their resources are higher, if child cost is lower, and if they appreciate less child education (lower φ).

2.4 Intergenerational fiscal policy

Household liquidity constraints are market imperfections that justify state intervention. Here, we do not deal with all kinds of intergenerational fiscal policies, nor with their optimal design. Instead, we focus on intergenerational transfers which can, at least in part, release families from limited access to the credit market. According to this policy, the government supports families with children by subsidizing their expenditure on child education in a proportion given by v. This public expenditure is financed with flat-rate taxes on wages of the next generation of adults who are free from chid support. The government balances its budget in each period, which implies:

$$v n_t b_{t+1} N_t = t_w w_t e_{t-1} N_{t-1},$$

where the left side represents public expenditure on subsidies received at time t by the young adult generation which has n_t children per capita, while the right side is the amount of resources collected by the state at time t as taxes on the wages of adults born at time $t-2$. The government budget constraint can also be written as:

$$vn_t b_{t+1} = \frac{t_w w_t e_{t-1}}{n_{t-1}}, \tag{15}$$

and in this form it highlights the dynamic interactions between fertility and education implied by public policy: lower past fertility and higher adult education allow greater subsidization of education investment of the present generation.

3 Equilibrium

The previous section dealt with optimal decisions of households and firms. The goods market clears when production equals the demand for consumption and investment in human and physical capital; factor markets also clear when each factor price equals marginal productivity. In the market for loans households face a limit to borrowing which implies market rationing. Firms need resources to finance their investment and the supply of capital is provided by savings of households. We assume that capital depreciates fully in one period, hence the capital market-clearing condition is

$$K_{t+1} = -D_t N_t + S_t N_{t-1},$$

according to which the supply of savings comes from adult savings net of young adult debts. This condition can be written more conveniently as

$$k_{t+1}\left(n_t e_{t+1} + e_t\right) = -D_t + \frac{S_t}{n_{t-1}}. \tag{16}$$

Substitution of optimal individual policies for debt and saving in this equation gives the equilibrium rule for capital accumulation:

$$\begin{aligned} k_{t+1}\left(n_t e_{t+1} + e_t\right) &= w_t e_t - \Psi\left[w_t e_t + \frac{(1-t_w)\, w_{t+1} e_t}{R_{t+1}}\right] + \\ &+ \frac{\beta^2 (1-\Psi)\, R_t}{\left(\beta + \beta^2\right) n_{t-1}}\left[w_{t-1} e_{t-1} + \frac{(1-t_w)\, w_t e_{t-1}}{R_t}\right]. \end{aligned}$$

The last equation completes the description of the components of the model economy. The equilibrium is characterized by the clearing of product markets, factor markets, and financial markets in which credit to households is rationed. In equilibrium human capital accumulation follows from the optimal decisions of young parents subject to a credit constraint which is in part relaxed by the intervention of the government whose budget constraint is balanced in every period. Starting from historical values of state variables $(N_0,\ e_0,\ n_0,\ k_0,\ k_1)$, the evolution of the economy derives from optimal fertility behavior, human and physical capital accumulations, equations (1), (13), (14), and (16).

In order to simplify the analysis, we define two new variables. As in de la Croix and Michel (2002), we set:

$$x_t \equiv \frac{w_{t+1}}{w_t R_{t+1}}, \tag{17}$$

which has the meaning of a growth factor of the discounted life-cycle wage. In this way, the life-cycle income becomes:

$$\left[w_t e_t + \frac{(1-t_w)\, w_{t+1} e_t}{R_{t+1}}\right] = w_t e_t \left[1 + (1-t_w)\, x_t\right].$$

The second new variable is:

$$j_t \equiv \frac{\varphi \Psi \left[1 + (1-t_w)\, x_t\right]}{(1-v)\left(U_n\left(n_t\right) n_t + 1\right)}, \tag{18}$$

which can be substituted in equation (13) to give:

$$n_t b_{t+1} = j_t w_t e_t.$$

Hence, j_t can be thought of as the proportionality factor which explains the educational expenditure of each young parent in terms of her present income. j_t increases with the wage growth factor, x_t, and decreases with the fertility rate.

With these new variables we restate the equations describing economic equilibrium. First, let us consider the optimal expenditure in education, eq. (13). From substitution of equations (14), (18), and $e_t = \Lambda b_t$ in equation (13) we get the gross rate of growth of human capital γ_t:

$$\gamma_t \equiv \frac{e_{t+1}}{e_t} = \frac{\Lambda \tau w_t j_t}{\Psi \left[1 + (1-t_w)\, x_t\right]}. \tag{19}$$

Then, substitution of (17) in (14) provides the relation between fertility rate n_t and x_t:

$$\frac{n_t \left(U_n\left(n_t\right) n_t + 1\right)}{\left(U_n\left(n_t\right) n_t - \varphi\right)} = \frac{\Psi}{\tau}\left[1 + (1-t_w)\, x_t\right]. \tag{20}$$

Since individual optimal decisions must be consistent with the government budget constraint, we plug equations (13), (18) into equation (15) and obtain the first-order difference equation:

$$j_t = \frac{t_w}{v \Lambda w_{t-1} j_{t-1}} \equiv \phi\left(w_{t-1}, j_{t-1}\right). \tag{21}$$

The Cobb-Douglas technology allows the derivation of the following dynamic relation between w and x:

$$w_t = A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} w_{t-1} x_{t-1} \equiv \omega\left(w_{t-1}, x_{t-1}\right). \tag{22}$$

The dynamics of physical capital can be reformulated in terms of the variable x_t as follows:

$$\begin{aligned} k_{t+1}\left(n_t \frac{e_{t+1}}{e_t} + 1\right) &= w_t - \Psi w_t \left[1 + \left(1 - t_w\right) x_t\right] + \\ &+ \frac{\beta^2 \left(1-\Psi\right) R_t w_{t-1} e_{t-1}}{\left(\beta + \beta^2\right) n_{t-1} e_t} \left[1 + \left(1 - t_w\right) x_{t-1}\right]. \end{aligned} \tag{23}$$

After substitution in equation (23) of equations (19), (21), (22) and the following relations

$$k_{t+1} = \frac{\alpha}{1-\alpha} x_t w_t,$$

$$R_t w_{t-1} = x_{t-1} w_t,$$

- derived from equations (2) and (3) and from the definition of x_t - the difference equation of k_t becomes a first-order difference equation in x_t :

$$\begin{aligned} x_t &= \frac{\left(1-\Psi\right)\left(1-\alpha\right) w_{t-1} x_{t-1} j_{t-1} + \frac{\beta^2 (1-\Psi)(1-\alpha) A}{\left(\beta+\beta^2\right)\Lambda} \left[1 + \left(1-t_w\right) x_t\right]}{\left[\alpha + \Psi\left(1-\alpha\right)\left(1-t_w\right)\right] w_{t-1} x_{t-1} j_{t-1} + \frac{A t_w \alpha^{1+\alpha} (1-\alpha)^{1-\alpha}}{v} w_{t-1}^{\alpha} x_{t-1}^{1+\alpha}} \\ &\equiv \chi\left(w_{t-1}, x_{t-1}, j_{t-1}\right). \end{aligned} \tag{24}$$

3.1 The dynamic system

The intertemporal equilibrium is characterized by capital accumulation, human capital investment and reproductive behavior of the population. It can be analyzed in terms of the derived variables x_t, w_t, j_t.

Definition 1 *A dynamic equilibrium of the economy is a sequence* $\{x_t, w_t, j_t\}_{t=0}^{\infty}$ *that satisfies the dynamical system*

$$\begin{cases} x_t = \chi\left(x_{t-1}, w_{t-1}, j_{t-1}\right). \\ \\ w_t = \omega\left(x_{t-1}, w_{t-1}\right) \\ \\ j_t = \phi\left(w_{t-1}, j_{t-1}\right), \end{cases} \tag{25}$$

where $(N_0,\ e_0,\ n_0,\ k_0,\ k_1)$ *are exogenously given.*

Definition 2 *A steady-state equilibrium of the economy is a triple* $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ *such that:*

$$\begin{cases} \widehat{x} = \chi\left(\widehat{x}, \widehat{w}, \widehat{j}\right). \\ \\ \widehat{w} = \omega\left(\widehat{x}, \widehat{w}\right) \\ \\ \widehat{j} = \phi\left(\widehat{w}, \widehat{j}\right). \end{cases} \tag{26}$$

Accordingly, the model implies that in a steady-state equilibrium the capital-labor ratio $k_t = \widehat{k}$ remains constant as well as the rate of growth of population $n_t = \widehat{n}$, and the rate of growth of human capital $\gamma_t = \widehat{\gamma}$. Hence, when the economy reaches a stationary equilibrium the variables in level, Y_t, K_t, L_t, grow at the constant rate $\widehat{g} = \widehat{n}\widehat{\gamma} - 1$. The existence and uniqueness of a steady-state equilibrium for the overlapping generations economy is the argument of the following proposition:

Proposition 1 *A steady-state equilibrium* $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ *of the dynamical system (25) exists and is unique.*
Proof. *In appendix.* ■

To study the local stability of the steady state $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ we consider the linear approximation of the dynamical system (25) in the neighborhood of the steady state. The relative Jacobian matrix evaluated in the point $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ provides information on the local stability of the steady state. Given the complexity of the analytical expressions for the eigenvalues of the Jacobian, we rely on a numerical simulation. As in Azariadis, Bullard and Ohanian, (2001), we calculate the steady state $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ and the relative eigenvalues by drawing randomly the values of the parameters $\{\beta,\ \varphi,\ \alpha,\ v,\ \tau,\ t_w,\ \Psi\}$ from uniform distributions. More precisely, we assume that each parameter takes values in a given range: $\beta \in (0.5,\ 2)$; $\varphi \in (0.5,\ 2)$; $\alpha \in (0.25,\ 0.5)$; $v \in (0.05,\ 0.3)$; $\tau \in (0.05,\ 0.2)$; $t_w \in (0.05,\ 0.4)$; $\Psi \in (0.2,\ 1)$. We also set the parameters A and Λ, which define the scale of good production and that of human capital accumulation, to a constant value: $A = 1$, $\Lambda = 1$.

From the random selection of 100 numerical configurations of the parameters we obtain values of the steady state $\left\{\widehat{x}, \widehat{w}, \widehat{j}\right\}$ and the eigenvalues $\{\lambda_x, \lambda_w,\ \lambda_j\}$for each parameter configuration. The results are summarized in Figure 1. Two features characterize these simulations: first, two eigenvalues have modulus smaller than one, while the other eigenvalue is greater than one in all but four cases; second, the stable eigenvalues are equal. Note that initial conditions $(N_0,\ e_0,\ n_0,\ k_0,\ k_1)$ imply the following historical values: $x_0 = k_1/A\alpha k_0^{\alpha}$, $w_0 = A\,(1-\alpha)\,k_0^{\alpha}$, and $j_0 = \frac{\varphi\Psi[1+(1-t_w)x_0]}{(1-v)[U_n(n_0)n_0+1]}$. Accordingly, all but four of the economies defined in these numerical simulations present a steady state which has the local stability properties of a saddle point, while the remaining four show asymptotically stable dynamics in the neighborhood of steady states. Repeated eigenvalues have modulus smaller than one and value in many cases positive but in some cases negative, hence convergence of the system to the steady state is not monotone (Galor, 2007).

4 Effects of liquidity constraints and taxation

The local stability properties of the dynamical system allow analysis of the stationary equilibrium displacement after changes in credit availability and fiscal policy. Since our interest focuses on fundamental variables of the model, namely fertility rate, human capital accumulation, saving and growth rate, we rewrite the steady-state system (26) in terms of the variables $\widehat{n}$ and $\widehat{k}$ and obtain (see appendix) the following equations:

$$
\begin{aligned}
U_n(\widehat{n})\,\widehat{n} &= \frac{\Psi\varphi}{1-v}\sqrt{\frac{(1-\alpha)\,vA}{t_w\Lambda}}\left[1+\frac{(1-t_w)}{A\alpha}\widehat{k}^{1-\alpha}\right]\widehat{k}^{\frac{\alpha}{2}}-1, && (27)\\
\widehat{\gamma} &= \left(\frac{\Psi\varphi\Lambda}{1-v}\right)\frac{A(1-\alpha)\,\widehat{k}^{\alpha}+(1-t_w)\,\frac{1-\alpha}{\alpha}\widehat{k}}{\left[U_n(\widehat{n})\,\widehat{n}+1\right]\widehat{n}}, && (28)\\
\widehat{g}+1 &= \left(\frac{\Psi\varphi\Lambda}{1-v}\right)\frac{A(1-\alpha)\,\widehat{k}^{\alpha}+(1-t_w)\,\frac{1-\alpha}{\alpha}\widehat{k}}{\left[U_n(\widehat{n})\,\widehat{n}+1\right]}, && (29)\\
\Theta\left(\widehat{k}\right) &\equiv B_1\widehat{k}^{1-\alpha}+B_2\widehat{k}^{1-\frac{\alpha}{2}}-B_3\left[\widehat{k}^{\frac{\alpha}{2}-1}+(1-t_w)\,\widehat{k}^{-\frac{\alpha}{2}}\right]+\\
-(1-\Psi)(1-\alpha) &= 0, && (30)
\end{aligned}
$$

where B_1, B_2, B_3 are the following functions of parameters:

$$
\begin{aligned}
B_1 &\equiv \frac{\alpha+\Psi(1-\alpha)(1-t_w)}{A\alpha};\\
B_2 &\equiv (1-\alpha)^{\frac{1}{2}}\,\Lambda^{\frac{3}{2}}\sqrt{\frac{t_w}{Av}};\\
B_3 &\equiv \frac{\beta^2(1-\Psi)\,\alpha\sqrt{v(1-\alpha)\,A}}{\left(\beta+\beta^2\right)\Lambda^{\frac{3}{2}}\sqrt{t_w}}.
\end{aligned}
$$

The first equation explains the rate of fertility and shows how the positive effect of $\widehat{k}$ on $\widehat{n}$ passes through life-cycle income. The second refers to the rate of human capital accumulation which shows a positive dependence of $\widehat{\gamma}$ on the capital to labor ratio and a negative relation with the rate of fertility. A similar shape has the third equation of the gross rate of growth of aggregate income. The last implicit equation derives from the equilibrium in the capital market and takes account of all the influences of the other endogenous variables on $\widehat{k}$.

The four equations (27)-(30) fully describe the model economy at the steady-state equilibrium and can be used to study the effects of both greater credit availability, and changes of education subsidies and income taxes on fertility, rate of human capital accumulation, saving and economic growth.

Relaxing borrowing constraints

Liquidity constraints have a pervasive influence on the endogenous variables of the model: lifting borrowing constraints (increasing Ψ) provides young adults with greater resources for consumption and investment in children, with direct partial effects on households' choices. However, the full effect depends on how $\widehat{n}$ and $\widehat{k}$ change. As far as physical capital is concerned we prove the following:

Proposition 2 *The steady-state capital to labor ratio $\widehat{k}$ increases with a tightening of liquidity constraints (lower Ψ).*

Proof. Applying the implicit function theorem to (30) we get:

$$\frac{d\widehat{k}}{d\Psi} = \frac{-\Theta_{\Psi}\left(\widehat{k},\Psi\right)}{\Theta_{\widehat{k}}\left(\widehat{k},\Psi\right)}.$$

Since B_1, B_2, B_3 are positive constants, $\Theta_{\widehat{k}}\left(\widehat{k},\Psi\right) > 0$ easily follows. A brief inspection of (30) is enough to verify that $\Theta_{\Psi}\left(\widehat{k},\Psi\right) > 0$, which completes the proof of the proposition. ■

Proposition (2) confirms a well-known comparative statics result obtained by Jappelli and Pagano (1994). Actually, in our model the number of children and their education enter the utility function of the parents as their consumption does. Hence, young adults, facing tighter access to credit, reduce their debts and increase resources for capital accumulation. De Gregorio (1996) derives an ambiguous effect of borrowing constraints on savings in a different model in which there is no altruism in the family and young agents choose to allocate their time endowment to work or to human capital investment.

The level effect of borrowing constraints on capital intensity interacts with fertility and education investment decisions to determine the growth effect (De Gregorio, 1996; Azariadis and de la Croix, 2005). The rate of fertility shows clearly two effects of greater Ψ: a partial positive effect which is countervailed by the negative effect of Ψ on capital intensity. Hence, at a steady state, greater credit allows parents to spend a higher share of their life-cycle income on child rearing, but reduces the present value of their current and future income. Hence, the net effect of greater Ψ is ambiguous. This is also the case of comparative statics of Ψ on $\widehat{\gamma}$ which depends on fertility and capital intensity. We rely on numerical simulations of the model to characterize the relations between Ψ and $\widehat{n}$ or $\widehat{\gamma}$. In fact, we specify the following CRRA utility function of fertility:

$$U(n) = \left(1 - \frac{1}{\sigma}\right)^{-1} n^{1-\frac{1}{\sigma}},$$

which under $\sigma > 1$ satisfies assumption A2. We specify the parameters by referring to values common in the existing literature. Hence, we assume that one period in the evolution of the model economy is fifteen years, and the yearly discount factor is $1/1.06$ which implies that $\beta = 0.417$ (see Lochner and Monge-Naranjo, 2002). Relying on evidence that can be found in the literature on the cost of raising children (e.g., de la Croix and Doepke, 2004), we set: $\tau = 0.15$. The capital share parameter α assumes the value 0.33 which is the usual choice in the literature. Subsidies to education are found world-wide and cover a significant share of the costs. However, human capital investment also includes several informal learning activities which are not usually subsidized. Hence, we think that a value of 0.3 for the subsidization rate v represents a good estimate of the real average value. The government funds such subsidies to human capital by levying taxes on wages. Since our model does not consider other forms of public expenditures, and the ratio of taxes on GDP varies across many countries in a range between 0.3 and 0.5, we set the rate of income taxes t_w equal to 0.2. The rest of the parameters were chosen to obtain reliable results from simulations. Accordingly, the weight of child human capital in parents' preferences is set to $\varphi = 0.5$, while utility of the number of children is fixed by the parameter $\sigma = 1.5$[9]. We also calibrated the model by choosing the scale of production technology: $A = 5$, while the scale parameter of investment in education assumes the value $\Lambda = 10$.

The equations (27)-(30) were simulated under the above specified set of parameter values in order to numerically draw the functions of $\widehat{k}$, $\widehat{n}$, $\widehat{\gamma}$, and $\widehat{g}$ with respect to Ψ, which varies in the range $(0.2 - 0.9)$. Figures (2)-(4) represent such relations[10]. Figure (2) shows how the number of children responds positively to less tight credit constraints, although the curve decreases at high values of Ψ. This pattern implies that the direct positive effect of Ψ on $\widehat{n}$ overcomes the negative effect due to decreasing labor income which derives from the negative influence of greater household credit on capital intensity. The last effect prevails over the direct when Ψ is close to 1. Hence, simulations support a view of the effects of financial reforms in which fertility and saving take opposite directions. Such a result has a crucial role in shaping the relation between credit availability and investment in education. Indeed, equation (29) shows that $\widehat{\gamma}$ decreases with $\widehat{n}$, and increases with $\widehat{k}$. Figure (3) presents a simulated curve with an inverted U shape on the plane $(\Psi, \widehat{\gamma})$, which tells of a positive influence of better access to the credit market on child education when the market is underdeveloped. In this type of economy, if young parents are allowed to borrow greater resources they choose to have more children and to make a greater investment in each

[9]When $\varphi = 0.5$ and $\sigma = 1.5$, assumption A1 is satisfied if the gross fertility rate, $\widehat{n}$, takes values greater than 0.45, which is the case in our simulations.

[10]We performed simulations under different parameter configurations and the shape of the simulated relations did not change.

child's education (the direct effect of Ψ). This effect on child quality exceeds the negative effect which derives from greater fertility and lower labor income. However, the positive effects of greater household credit vanish when Ψ reaches a significant value and the financial sector becomes well developed. The rate of growth of aggregate income (Figure 4), which equals the rate of accumulation of aggregate human capital, L, follows a trend similar to that of $\widehat{\gamma}$ with greater growth as a consequence of better credit access of households in the first stages of financial development and a limit to the benefits that may be gained by such a policy.

Our results would appear to reconcile the existing econometric evidence on the relation between household borrowing constraints and economic growth with economic theory. Indeed, Jappelli and Pagano (1994) find an increasing relation between the two phenomena in estimates of a model on a sample of developed countries (OECD and others), and this evidence is not clearly denied by the results of De Gregorio (1996) from estimates on a similar sample of countries. On the other hand, De Gregorio (1996) also finds significant evidence of a positive effect on growth of the ratio of credit from the banking system to the nonfinancial private sector and GDP, from estimates on data of 63 developing countries. According to the results of our model, financial development can be beneficial for economic growth of poor countries since it boosts investment in human capital, while in developed countries greater credit availability might bring about lower economic growth through increased fertility and decreased saving.

The effects of education subsidy and tax changes

The aggregate dynamics of the model economy depend not only on the degree of financial development, but also on state intervention toward the family. The intergenerational distribution policy specified in the present model is made by subsidies to young parents' expenditure on children's education and a proportional tax on labor income of the adult generation who do not have to care for their grown-up children. This simplified tax scheme could be augmented with other forms of taxes and family benefits without changing the main predictions of the model. The comparative statics of the steady-state equilibrium relies on equations (27)-(30), and provides the following

Proposition 3 *1. The steady-state capital to labor ratio $\widehat{k}$ increases if education subsidies, v, increase and decreases if the rate of income tax, t_w, increases.*

2. The steady-state rate of fertility $\widehat{n}$ increases with education subsidies, v, and decreases with the rate of income tax, t_w.

Proof. *In appendix.* ■

Intuition behind the results of Proposition3 can be gained by considering that greater subsidies to human capital make education less expensive and consequently young adults have fewer children and invest more in their human capital, which in turn increases both lifetime income and savings. Taxes on adult wages reduce their disposable income and savings. Changes in the rates v and t_w affect the number of children per young adult through income and substitution effects and the endogenous changes in $\widehat{k}$.

Greater subsidies to child quality increase the disposable income but make the number of children more expensive, with opposite effects on $\widehat{n}$. The positive one is reinforced by that of subsidies on capital intensity which raises wages per efficiency unit. According to Proposition 3, the net effect of v on $\widehat{n}$ is positive.

Higher taxes on adult wages decrease the parents' discounted lifetime income which implies that they face a tighter credit constraint. Hence, young parents have less resources for consumption and investment in children. Furthermore, higher taxes reduce adult disposable income and this has a negative effect on savings. Hence, the negative partial impact of t_w on $\widehat{n}$ is reinforced by that on the level of human capital and on capital intensity which further decreases the discounted life-time earnings of the parents. Our results are at odds with those of Zhang and Casagrande (1998) who develop comparative statics analysis of a growth model with endogenous fertility and education, and find no effect of subsidies and income taxes on the equilibrium rate of population growth.

Notwithstanding the unambiguous sign of comparative statics effects of v and t_w on $\widehat{n}$ and $\widehat{k}$, those on the rate of human capital accumulation and the growth rate cannot be determined. Such difficulties arise because of the negative influence of $\widehat{n}$ on both $\widehat{\gamma}$ and $\widehat{g}$. Indeed, greater subsidies make education less expensive and raise capital intensity, but they also raise fertility, which countervails the former positive effects on the rate of human capital accumulation. The same situation with opposite sign effects applies to the total derivative of $\widehat{\gamma}$ with respect to the rate of income taxes. Here, again, we simulate the model to get insights into the relation between v, t_w and $\widehat{\gamma}$ and $\widehat{g}$[11]. Figures 5-8 present the simulation results. Figure 5 shows how greater income taxes increase investment in human capital at low values of t_w, and then they decrease it. This non-monotonic relation is confirmed in the case of education subsidies by Figure 7, where $\widehat{\gamma}$ increases with v till it reaches a maximum and then decreases for high values of the subsidy rate. However,

[11] In this case, we set $\Psi = 0.4$.

Figures 6 and 8 present two monotone simulated curves of the growth rate $\widehat{g}$ as a function of respectively t_w and v. Indeed, economic growth decreases with greater income taxes and increases with higher education subsidies. Hence, intergenerational fiscal policy to foster economic growth maintains the usual effects even in this model with endogenous fertility.

5 Conclusions

This paper presented a dynamic general equilibrium investigation of household behavior under borrowing constraints in which the number of children is endogenous. The analysis of the model shows how fertility, education and savings interact under liquidity constraints, and comparative statics highlights non-monotonic effects of financial reforms on endogenous variables and growth at the steady state. In order to derive analytical results, it is assumed that the limit to borrowing is exogenously given. Hence, the results of the paper can be considered a useful reference for further analysis with endogenous credit constraints. Furthermore, intergenerational public policy provides some new hinsights into the effects of subsidies to education and income taxes on economic growth with endogenous fertility. The study of optimal fiscal policy in a dynamic general equilibrium with endogenous fertility choice setup remains a task for future research. Empirical investigation of the issues of this paper still remain to be done since the existing literature examines single sides of the behavior of households under liquidity constraints, but fails to provide a full account of it.

APPENDIX

Proof of Proposition 1. The difference equations (21) and (22) at a steady state become:

$$\widehat{j}\widehat{w} = \frac{t_w}{v\Lambda\widehat{j}}, \tag{A1}$$

$$\widehat{w} = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\,\widehat{x}^{\alpha}\widehat{w}^{1-\alpha}; \tag{A2}$$

which jointly provide the following equation:

$$\widehat{x} = \widehat{j}^{\frac{-2(1-\alpha)}{\alpha}}\frac{1}{A^{\alpha}\alpha}\left[\frac{t_w}{\Lambda v(1-\alpha)}\right]^{\frac{1-\alpha}{\alpha}}. \tag{A3}$$

Furthermore, at steady state equation (24) can be written as:

$$\widehat{x}\left[\alpha+\Psi\left(1-\alpha\right)\left(1-t_{w}\right)\right]+\Lambda\alpha\widehat{x}\widehat{w}\widehat{j}=$$

$$\left(1-\Psi\right)\left(1-\alpha\right)+\frac{\beta^{2}\left(1-\Psi\right)\left(1-\alpha\right)A}{\left(\beta+\beta^{2}\right)\Lambda}\frac{\left[1+\left(1-t_{w}\right)\widehat{x}\right]}{\widehat{x}\widehat{w}\widehat{j}}. \tag{A4}$$

Substitution of (A2) and (A3) in (A4) gives:

$$\widehat{j}^{\theta}\left[\alpha+\Psi\left(1-\alpha\right)\left(1-t_{w}\right)\right]B+\widehat{j}^{\theta-1}\frac{\alpha t_{w}}{v}B=$$

$$\widehat{j}^{1-\theta}\frac{\beta^{2}\left(1-\Psi\right)\left(1-\alpha\right)Av}{\left(\beta+\beta^{2}\right)t_{w}}B+\widehat{j}\frac{\beta^{2}\left(1-\Psi\right)\left(1-\alpha\right)\left(1-t_{w}\right)Av}{\left(\beta+\beta^{2}\right)t_{w}}+\left(1-\Psi\right)\left(1-\alpha\right);$$

where

$$\theta\equiv\frac{-2\left(1-\alpha\right)}{\alpha}<0,\ \text{and } B\equiv\frac{1}{A^{\alpha}\alpha}\left[\frac{t_{w}}{\Lambda v\left(1-\alpha\right)}\right]^{\frac{1-\alpha}{\alpha}}>0.$$

It can be easily seen that this equation in $\widehat{j}$ has on the left side, $l(\widehat{j})$, a decreasing convex function with:

$$\lim_{\widehat{j}\to 0}l(\widehat{j})=\infty,\ \text{and }\lim_{\widehat{j}\to\infty}l(\widehat{j})=0.$$

The right side, $r(\widehat{j})$, is an increasing concave function with:

$$\lim_{\widehat{j}\to 0}r(\widehat{j})=\left(1-\Psi\right)\left(1-\alpha\right),\ \text{and }\lim_{\widehat{j}\to\infty}r(\widehat{j})=\infty,$$

hence the left side crosses the right side only at one positive value of $\widehat{j}$.■

Derivation of equations (27)-(30) Let us consider (21) at the steady state in which we substitute the definition of j_t, eq. (18), $\widehat{w}=A\left(1-\alpha\right)\widehat{k}^{\alpha}$, and $\widehat{x}=\left(A\alpha\right)^{-1}\widehat{k}^{1-\alpha}$, then the following implicit equation for $\widehat{n}$ derives:

$$U_{n}\left(\widehat{n}\right)\widehat{n}=\frac{\Psi\varphi}{1-v}\sqrt{\frac{\left(1-\alpha\right)vA}{t_{w}\Lambda}}\left[1+\frac{\left(1-t_{w}\right)}{A\alpha}\widehat{k}^{1-\alpha}\right]\widehat{k}^{\frac{\alpha}{2}}-1.$$

Applying the same substitutions to equation (19) we get the rate of human capital accumulation:

$$\widehat{\gamma} = \left(\frac{\Psi\varphi\Lambda}{1-v}\right) \frac{A(1-\alpha)\widehat{k}^{\alpha} + (1-t_w)\frac{1-\alpha}{\alpha}\widehat{k}}{\left[U_n(\widehat{n})\widehat{n}+1\right]\widehat{n}},$$

which immediately gives the gross rate of growth of aggregate income:

$$\widehat{g} + 1 = \left(\frac{\Psi\varphi\Lambda}{1-v}\right) \frac{A(1-\alpha)\widehat{k}^{\alpha} + (1-t_w)\frac{1-\alpha}{\alpha}\widehat{k}}{\left[U_n(\widehat{n})\widehat{n}+1\right]}.$$

In the steady state, the equation (24) - which derives from equilibrium in the capital market - can be written in implicit form as:

$$\begin{aligned} &\widehat{x}\left[\alpha + \Psi(1-\alpha)(1-t_w)\right] + \Lambda\alpha\widehat{x}\widehat{w}\widehat{j} - (1-\Psi)(1-\alpha) + \\ &-\frac{\beta^2(1-\Psi)(1-\alpha)A}{\left(\beta+\beta^2\right)\Lambda}\frac{\left[1+(1-t_w)\widehat{x}\right]}{\widehat{x}\widehat{w}\widehat{j}} = 0. \end{aligned} \tag{A5}$$

Equation (21) at steady state can be written as

$$\widehat{j} = \widehat{k}^{-\frac{\alpha}{2}}\sqrt{\frac{t_w\Lambda}{Av(1-\alpha)}},$$

which, with $\widehat{x} = (A\alpha)^{-1}\widehat{k}^{1-\alpha}$ and $\widehat{x}\widehat{w} = \frac{1-\alpha}{\alpha}\widehat{k}$ - derived from the definition of $\widehat{x}$ - provide:

$$\widehat{x}\widehat{w}\widehat{j} = \frac{1-\alpha}{\alpha}\sqrt{\frac{t_w\Lambda}{Av(1-\alpha)}}\widehat{k}^{1-\frac{\alpha}{2}}. \tag{A6}$$

From substitution of (A6) in (A5) we get:

$$\Theta\left(\widehat{k}\right) \equiv B_1\widehat{k}^{1-\alpha} + B_2\widehat{k}^{1-\frac{\alpha}{2}} - B_3\left[\widehat{k}^{\frac{\alpha}{2}-1} + (1-t_w)\widehat{k}^{-\frac{\alpha}{2}}\right] - (1-\Psi)(1-\alpha) = 0$$

where B_1, B_2, B_3 are parameters:

$$\begin{aligned} B_1 &\equiv \frac{\alpha + \Psi(1-\alpha)(1-t_w)}{A\alpha};\ B_2 \equiv (1-\alpha)^{\frac{1}{2}}\Lambda^{\frac{3}{2}}\sqrt{\frac{t_w}{Av}}; \\ B_3 &\equiv \frac{\beta^2(1-\Psi)\alpha\sqrt{v(1-\alpha)A}}{\left(\beta+\beta^2\right)\Lambda^{\frac{3}{2}}\sqrt{t_w}}. \end{aligned}$$

Proof of Proposition 3.

1) Applying the implicit function theorem to equation (30) we get:

$$\frac{\partial \widehat{k}}{\partial a}=\frac{-\Theta_a\left(\widehat{k},v\right)}{\Theta_{\widehat{k}}\left(\widehat{k},v\right)},\ \ a=v,\ t_w.$$

From the proof of Proposition 2 we know $\Theta_{\widehat{k}}\left(\widehat{k},a\right)>0$. The derivative $\Theta_v\left(\widehat{k},v\right)<0$ is straight, and $d\widehat{k}/dv>0$ follows. Deriving equation (30) with respect to t_w we obtain:

$$\Theta_{t_w}\left(\widehat{k},t_w\right)=-\frac{\Psi\left(1-\alpha\right)}{A\alpha}\widehat{k}^{1-\alpha}+\frac{0.5}{t_w}B_2\widehat{k}^{1-\frac{\alpha}{2}}+\frac{0.5}{t_w}B_3\left[\widehat{k}^{\frac{\alpha}{2}-1}+\left(1-t_w\right)\widehat{k}^{-\frac{\alpha}{2}}\right]+ \\ +B_3\widehat{k}^{-\frac{\alpha}{2}}.$$

This derivative is positive if the technology parameter A is high enough, which can be assumed without maior consequences. This result completes the proof of the first part of the proposition.

2) Since A2 implies that $U_n\left(\widehat{n}\right)\widehat{n}$ is an increasing function of $\widehat{n}$, the sign of the effects of v and t_w on fertility derives from total differentiation:

$$\frac{d\left[U_n\left(\widehat{n}\right)\widehat{n}\right]}{da}=\frac{\partial\left[U_n\left(\widehat{n}\right)\widehat{n}\right]}{\partial a}+\frac{\partial\left[U_n\left(\widehat{n}\right)\widehat{n}\right]}{\partial\widehat{k}}\frac{d\widehat{k}}{da},\ \ a=v,\ t_w.$$

The partial derivative of $U_n\left(\widehat{n}\right)\widehat{n}$ with respect to v is clearly positive, as are the other two derivatives, which mean that $d\widehat{n}/dv>0$. Similarly, we have:

$$\frac{\partial\left[U_n\left(\widehat{n}\right)\widehat{n}\right]}{\partial t_w}<0;\ \ \frac{d\widehat{k}}{dt_w}<0,$$

from which we get $d\widehat{n}/dt_w<0$.■

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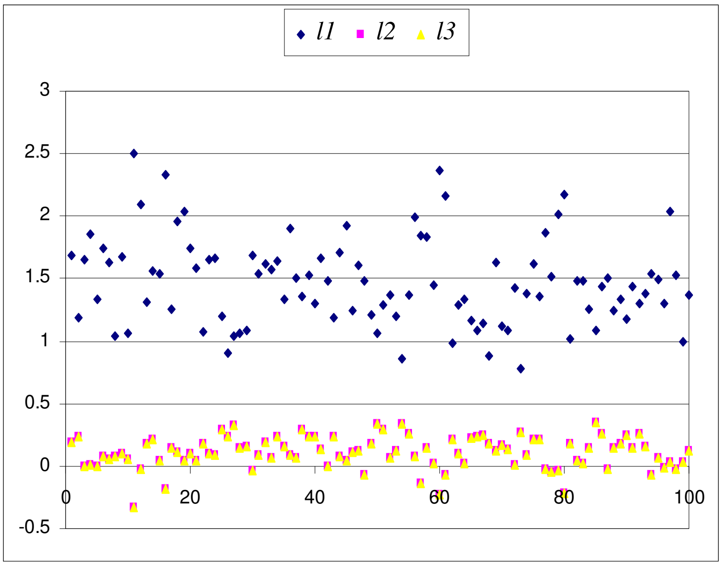

Figure 1: Eigenvalues of the dynamic system for 100 parameter configurations.

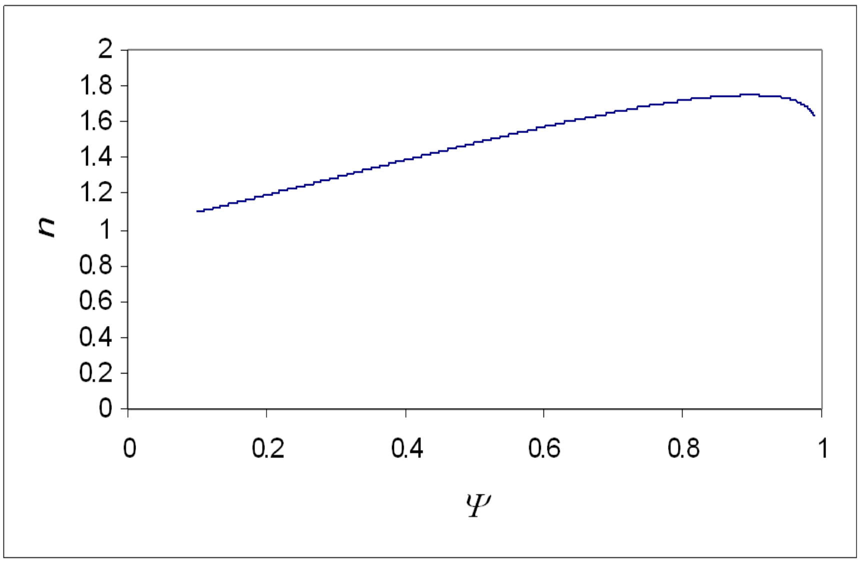

Figure 2: Fertility rate as a function of the relaxation of borrowing constraints.

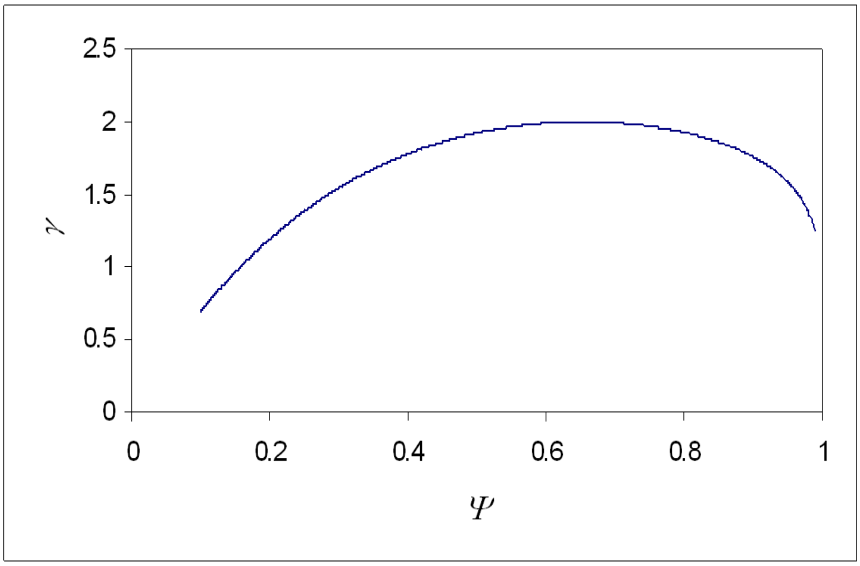

Figure 3: Growth rate of human capital as a function of the relaxation of borrowing constraints.

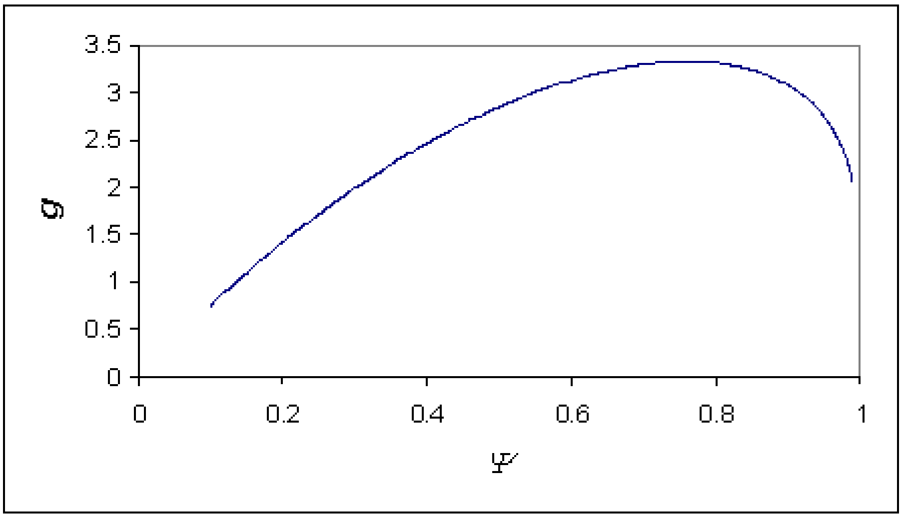

Figure 4: Economy growth rate as a function of the relaxation of borrowing constraints.

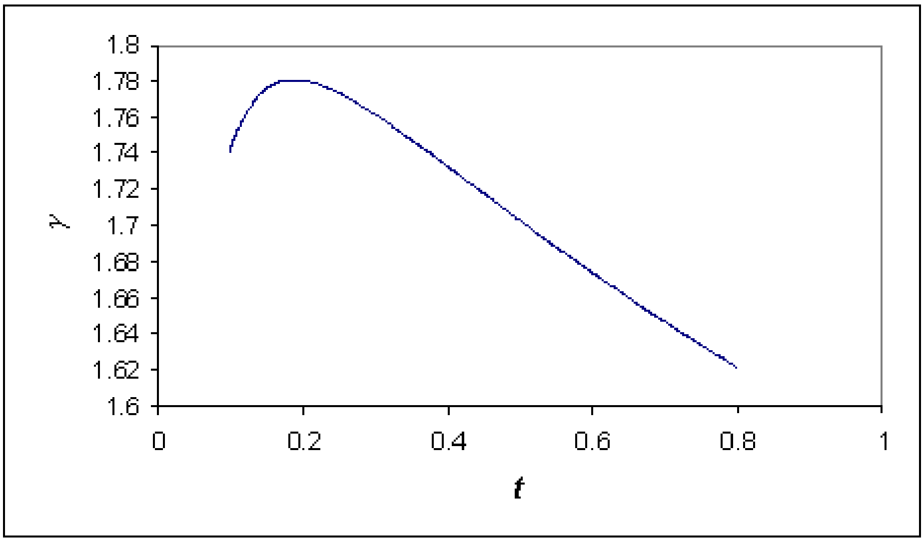

Figure 5: Growth rate of human capital as a function of income tax rate.

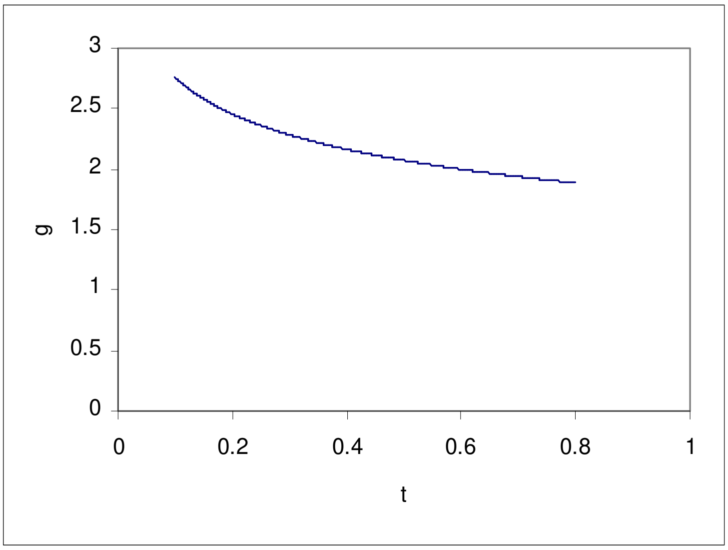

Figure 6: Economy growth rate as a function of income tax rate.

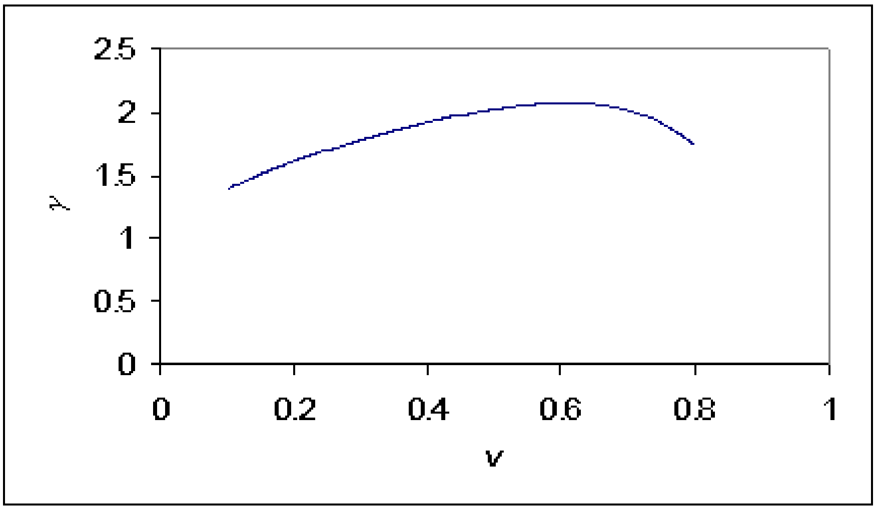

Figure 7: Growth rate of human capital as a function of the subsidy rate.

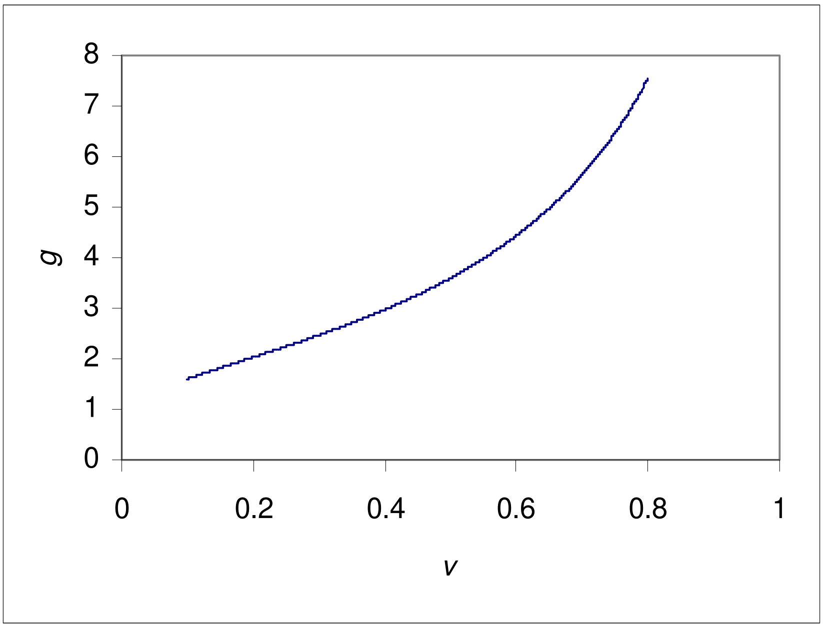

Figure 8: Economy growth rate as a function of education subsidy