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Abstract

The aim of this work is to show how automated traders can operate a futures market. First, we established some hypothesises on the properties of the 'correct' price pattern which translates accurately the underlying moves in the supply/demand balance and the nominal price, then mathematical measures were derived allowing to estimate the efficiency of a given trading strategy. As a starting step, we applied our approach to a simplified market setup where only two automated traders, a producer and a consumer, can trade. They receive a stream of forecasts on supply and demand levels and they should react instantaneously by adjusting these forecasts, then issuing sale and buy orders. Later, we suggested a parameterized trading strategy for the two automatons. Finally, we obtained by simulation the optimal parameters of this strategy in some particular cases.

1   Introuction

The futures market is a major part of nowadays commercial Exchanges, it is the place where *futures contracts* are traded. A futures contract is a binding agreement between a seller and a buyer, it is related to a specific commodity[1], like crude oil, gold, metals, grains, oilseeds, etc. A specific feature of a futures transaction is that the price of the commodity is fixed at the present time, whereas the effective delivery of the merchandize, from the seller to the buyer, will occur in a future date, which could be several months or years later [4]. The majority of raw commodities producers, processors, consumers, and merchants buy and/or sell futures contracts in order to hedge their price risk, i.e. protect themselves against unforseen sharp price variations. Speculators are also investing heavily in these markets for profit

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[1]There could also be futures contracts on financial instruments like stocks, indexes, foreign currencies, etc.

taking, at the same time they create the necessary liquidity that allows to these markets to operate efficiently [8, 18]. A detailed mathematical description of the futures market's platform was provided in [14], the market mechanism was explained as well as the evolution of traders' positions. However, the current study focuses on a futures market with only two players: a producer and a consumer. The purpose is to show how automated traders can operate this market.

So far, automated trading was limited to computerizing Exchanges' platforms which once were operated by human pit brokers receiving orders, by telephone or other means, from external human traders, then proceed to their execution in an open outcry market. This automatization process has met a great success with the advent of electronic platforms, consequently pit brokerage is progressively disappearing [2, 19]. However, human traders, representing the interest of commercial companies (producers, farmers, refiners, consumers, etc.) are still operating. They constantly asses the market supply and demand balance alongside with their specific needs, then translate their judgments into sale or buy orders [4]. The current investigation is an attempt to expand the automatization process into a new border by eliminating human traders in the decision making process. Instead of humans, we designed automated traders who analyze the market fundamentals (supply and demand) and send buy and sale orders to the Exchange.

A large part of studies found in the literature of financial markets deal with the issue of price formation from the *downstream* perspective, analyzing time series of the observed phenomena, like price, volume, open interest, etc., to establish market properties and make predictions [5, 7]. However, the most successful approaches in the community of traders are *technical analysis* [11] and *fundamental analysis* [6]. Economists have struggled to suggest authentic methods for trading like Shelton [17] who, based on game theory tools, designed a trading strategy where a player (a trader) is facing Nature (the market) and attempts to take the right decisions depending on the trader's appetite for risk and the mood of the market. Studies inspired from physics principles, like statistical mechanics phase transitions, were also conducted to examine the links between investors behavior and price formation [10].

On the other hand, *upstream* approaches are becoming frequent, their essence is to build the price time series from the interaction of independent artificial agents equipped with specific features; these agents play the role of traders in real markets. The Santa Fe Stock Market Simulator [9] is a typical computer model of the stock market allowing to carry out simulations and tests the effects of different scenarios on the behavior of the price [12]. A genetic approach developed in [1] helped to clarify the link between fundamental trading and technical trading and showed how bubbles occur. Preist [13] has suggested an agent-based technic for trading commodities via the Internet; a set of agents, representing the participants, enter into negotiation in a series of double auctions in order to determine the market price. The purpose of this class of approaches is to examine the effect of individual trader's features on the price behavior and showing how phenomena like bubbles and crashes occur.

Our investigation can be categorized as an upstream approach, but our goal differs from the one pursued by this class of approaches. Indeed, our purpose is

more practical than theoretical, it consists to the creation of automated traders able to negotiate the price of futures contracts in a futures market. For this reason, a discussion over the price's role in the market is necessary. We started by establishing some hypothesises on the properties of the 'correct', or benchmark, price pattern. Then we formulated the corresponding mathematical criteria allowing to measure the relative distance between the benchmark pattern and a given price pattern provided by any trading strategy, this measures also the performance of this strategy. Later, we suggested a framework of an artificial futures market composed only by an automated seller and an automated buyer representing the interests of a producer and a consumer respectively. The automatons are fed with a regular stream of forecasts on the supply and demand levels over a trading horizon of m periods. They react by adjusting the forecasts then issuing sale and buy orders from which the price is formed. To run the model, we suggested a parameterized trading strategy based on the gap supply-demand function and price bands built around the nominal price. Finally, we used simulation to search for the optimal parameters of the trading strategy by maximizing an aggregate performance function.

Some economic circles may feel septic, or even show their opposition to an automatic pricing approach, because it could appear to them that automation is thwarting the free market concept and liberal economy in general. They might also argue that producers will likely become unmotivated in case of a regulated market where everything is done automatically, leading to an absence of opportunities. Hence, what could be the benefits of an automatic pricing approach? A clear advantage of this approach is bringing a new contribution to the era of computerization and automation by freeing humans from the tedious task of pricing, and also providing a laboratory tool to scientists and economists, allowing them to carry on simulations and analyze the effects of various trading hypothesises.

However, the utmost benefit of this approach could be the additional rationality it brings in the price fixation process. Indeed, major financial crashes and speculative bubbles [16] were caused by irrational behavior of human traders: mimic, panic, greed and fears, etc. These are the main challenges facing human traders. It is likely that by replacing human traders by automated ones, their would be less financial crisis since automated traders will be pursuing rational strategies taking into account the real status of the market. The resulting rationality could be an economic factor for stability provided that these automatons will be able to generate a price pattern playing its role as a *market regulator*, i.e. continuously balancing supply and demand levels.

The next section is a discussion over the price's role in the market and its important link to the supply and demand balance. This will lead to formulating some hypothesises on the 'correct' price pattern, then deriving analytical criteria measuring the performance of a given trading strategy. In the third section, we describe the setup of the futures market for the producer-consumer case. The forth section presents an example of a trading strategy for the automated traders. Illustrative tabular and graphical results are outlined in the last section.

2 Measuring performance of a trading strategy

The price plays at least two important roles in any market. First, it serves as a reporting tool of the prevailing market's state, e.g. a rising price could be a sign of a deficit (demand exceeding supply), an vice versa. Secondly, the price allows to adjust the supply and demand balance. The price could be seen as a barometer, measuring the pressure of the market, then taking the right decision to equilibrate the market by regulating supply and demand levels. In fact, if supply exceeds demand then the market is in a surplus status, implying a price decline, which in turn will be perceived as a buying opportunity by consumers, consequently consumption is encouraged to grow. In parallel, this price decline should be a signal towards the producers to reduce their output, or momentarily halt it at all, in order to erase the surplus status; consequently, this price decline has allowed to solve the prevailing surplus problem. Similarly, in a deficit market, the contrary of what was described heretofore should occur: the price should increase in order to reduce consumption and encourage production. If production capacity is insufficient then investors will be attempted to invest more to raise their production capacity.

2.1 Some hypothesis on the correct price pattern

What should be the properties of the 'correct' price pattern? This is a debatable subject. However, the strategies of the automated traders will be derived directly from this debate. Nevertheless, it is commonly agreed that supply and demand balance is the main driver of the price. Herein are some hypothesises on the properties that should have an ideal price pattern.

Definition 2.1 (Nominal price). *To produce one unit of a given product, say C, we need to use c_k units of the input C_k which costs $p_k(t_{j-1})$ at instant t_{j-1}, with $k = 1, \dots, K$ and $j = 1, \dots, m$; thus, including a profit margin ratio r, the nominal (or rational) price of product C at instant t_j should be*

$$p^*(t_j) = (1 + r) \sum_{k=1}^{K} c_k \, p_k(t_{j-1}). \tag{1}$$

Assumption 2.1 (Nominal price). *We assume that, at any instant t_j belonging to the time horizon, the nominal (or rational) price, $p^*(t_j)$, of the product of interest is known. This nominal price could vary from time to time due to price variations emanating from other markets, impacting on its production cost.*

Let $\mathbf{S} = \{S(t_j)\}_{j=0,m}$ be a set of supply forecasts, and $\mathbf{D} = \{D(t_j)\}_{j=0,m}$ a set of demand forecasts related to the periods $t_0, t_1, \dots, t_m$. We set γ as the trading strategy used by the two automatons. The price series[2] $\mathbf{p} = \{p(t_j)\}_{j=1,m}$ is obtained by $\mathbf{p} = f_1(\gamma, \mathbf{S}, \mathbf{D})$ and the traded quantities series $\mathbf{q} = \{q(t_j)\}_{j=1,m}$ is obtained by $\mathbf{q} = f_2(\gamma, \mathbf{S}, \mathbf{D})$, where the functions f_1 and f_2 summarize the market functioning mechanism (see sections 3 and 4 for full details).

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[2]Note that transactions start at instant t_1, instead of t_0 as it was the case with S and D.

Hypothesis 2.1 (S&D determines the trend of the actual price). *The price pattern should inversely follow the balance of supply and demand (S&D) pattern, i.e. if the S&D balance declines then the price should increase, and vice-versa. In a mathematical form, this can be described by*

$$S\&D(t_j) < S\&D(t_{j-1}) \Rightarrow p(t_j) > p(t_{j-1}),$$
$$S\&D(t_j) > S\&D(t_{j-1}) \Rightarrow p(t_j) < p(t_{j-1}).$$

If no change happens on the S&D levels between instants t_{j-1} and t_j, then their shouldn't be any change in the price as well, that is

$$S\&D(t_j) = S\&D(t_{j-1}) \Rightarrow p(t_j) \simeq p(t_{j-1}).$$

Hypothesis 2.2 (The price is driving S&D's next move). *A significant increase in the price would encourage investment, therefore increasing the supply level, at the same time consumption will be reduced (due to the higher price), and vice-versa. This could be formulated as follows*

$$
\begin{aligned}
p(t_j) > p(t_{j-1}) &\Rightarrow S\&D(t_{j+1}) > S\&D(t_j) \\
p(t_j) < p(t_{j-1}) &\Rightarrow S\&D(t_{j+1}) < S\&D(t_j) \\
p(t_j) = p(t_{j-1}) &\Rightarrow S\&D(t_{j+1}) \simeq S\&D(t_j)
\end{aligned}
$$

Hypothesis 2.3 (Nominal price determines the level of the actual price). *In case of a surplus, the market price should fall below the nominal price in order to discourage production and encourage consumption, that is*

$$S(t_j) > D(t_j) \Rightarrow p(t_j) < p^*(t_j).$$

Inversely, in case of a deficit, the market price should be above the nominal price in order to encourage production and ration consumption, that is

$$S(t_j) < D(t_j) \Rightarrow p(t_j) > p^*(t_j).$$

If at any time t_j, the supply level is equal to the demand level, then the market price, $p(t_j)$, should be equal to the nominal price, $p^*(t_j)$, that is

$$S(t_j) \simeq D(t_j) \Rightarrow p(t_j) \simeq p^*(t_j).$$

Hypothesis 2.4 (S&D volatility transferred to the price). *The volatility of S&D should induce an equivalent volatility in the price pattern, that is in any subset of time $\{t_k, \ldots, t_{k+h}\} \subset \{t_1, \ldots, t_m\}$, the following is satisfied*

$$\sigma_{S\&D}(t_k, t_{k+h}) \simeq \sigma_p(t_k, t_{k+h}),$$

where $\sigma_{S\&D}(t_k, t_{k+h})$ and $\sigma_p(t_k, t_{k+h})$ are the standard deviations of the S&D and price respectively over the period (t_k, t_{k+h}), with $h \in \mathbb{N}$ and $h < m$.

Hypothesis 2.5 (Liquid market). *A good price pattern is the one where transactions take place almost at each period, that is the traded quantity, $q(t_j)$, at instant t_j should be positive most of the time, $j = 1, \ldots, m$.*

Hypothesis 2.6 (Homogenous volumes). *The volatility of the traded quantities, $q(t_j)$, should be kept to a minimum, that is in any subset of time $\{t_k, \ldots, t_{k+h}\} \subset \{t_1, \ldots, t_m\}$,*

$$\sigma_q(t_k, t_{k+h}) \simeq 0,$$

where $\sigma_q(t_k, t_{k+h})$ is the standard deviation of the variable q over the period (t_k, t_{k+h}), with $h \in \mathbb{N}$ and $h < m$.

2.2 Measuring strategy performances

We suggest herein a set of practical criteria allowing to quantify the efficiency of the trading strategy γ in respect to hypothesises 2.1-2.6. The principle of these criteria is to add up the number of times each hypothesis was satisfied over the periods $t_1, \ldots, t_m$, then build a ratio indicating the degree of satisfaction of each hypothesis.

First we define the gap function, G, which is simply the difference between supply and demand:

$$G(t_j) = S(t_j) - D(t_j), \tag{2}$$

with $j = 0, 1, \ldots, m$. The gap function G will be the analytical measure of the $S\&D$ balance evoked earlier.

In order to measure the efficiency of the trading strategy γ in respect to hypothesis 2.1, we use the following criterion

$$z_1(\gamma, \mathbf{S}, \mathbf{D}) = \frac{1}{m} \sum_{j=1}^{m} 1_{[\, sign(G(t_j) - G(t_{j-1})) \, = \, -sign(p(t_j) - p(t_{j-1}))\,]}, \tag{3}$$

where $1_{[\cdot]}$ is the condition function [15], z_1 is a ratio taking its values in the range $[0, 1]$ and measuring the efficiency of our strategy relatively to hypothesis 2.1 : if z_1 is close to 0 then the strategy has a very weak performance relatively to this hypothesis; inversely, if z_1 is close to 1 then this strategy fully respects this hypothesis. In the

same spirit, we suggest the following criteria for hypothesises 2.2-2.6,

$$z_2(\gamma, \mathbf{S}, \mathbf{D}) \;=\; \frac{1}{m-1} \sum_{j=1}^{m-1} \mathbf{1}_{[\,sign(G(t_{j+1})-G(t_j)) \,=\, sign(p(t_j)-p(t_{j-1}))\,]}, \tag{4}$$

$$z_3(\gamma, \mathbf{S}, \mathbf{D}) \;=\; \frac{1}{m} \sum_{j=1}^{m} \mathbf{1}_{[\,sign(G(t_j)) \,=\, -sign(p(t_j)-p^*(t_j))\,]}, \tag{5}$$

$$z_4(\gamma, \mathbf{S}, \mathbf{D}) \;=\; \frac{1}{m-h} \sum_{k=1}^{m-h} \mathbf{1}_{[\,|\sigma_G(t_k, t_{k+h})-\sigma_p(t_k, t_{k+h})| \le \epsilon\,]}, \tag{6}$$

$$z_5(\gamma, \mathbf{S}, \mathbf{D}) \;=\; \frac{1}{m} \mathbf{1}_{[q(t_j)>0]}, \tag{7}$$

$$z_6(\gamma, \mathbf{S}, \mathbf{D}) \;=\; \frac{1}{m-h} \sum_{k=1}^{m-h} \mathbf{1}_{[\sigma_q(t_k, t_{k+h}) \le \epsilon]}, \tag{8}$$

where $0 < \epsilon \ll 1$. We may favor one criterion over another, this is done by associating different weights $w_1, \dots, w_6$ to these criteria, with $0 \le w_k \le 1$, $k = 1, \dots, 6$, and $\sum_{k=1}^{6} w_k = 1$. The average performance of the trading strategy γ is

$$\bar{z}(\gamma, \mathbf{S}, \mathbf{D}) = \sum_{k=1}^{6} w_k \, z_k. \tag{9}$$

Now, assuming that two sets of representative samples of supplies and demands, $\mathbb{S} = \{\mathbf{S}^{(1)}, \dots, \mathbf{S}^{(K)}\}$ and $\mathbb{D} = \{\mathbf{D}^{(1)}, \dots, \mathbf{D}^{(K)}\}$, are available, and the strategy γ was parameterized by a parameter $\alpha \in \mathbb{A}$, then for a specific $\alpha_0 \in \mathbb{A}$, the average performance of strategy γ over the sets of samples $\mathbb{S}$ and $\mathbb{D}$, is

$$z(\gamma(\alpha_0), \mathbb{S}, \mathbb{D}) = \frac{1}{K} \sum_{k=1}^{K} \bar{z}(\gamma(\alpha_0), \mathbf{S}^{(k)}, \mathbf{D}^{(k)}). \tag{10}$$

Therefore, the optimal parameter for this strategy over $\mathbb{S}$ and $\mathbb{D}$, is

$$\alpha^* = \arg\max_{\alpha \in \mathbb{A}} z(\gamma(\alpha), \mathbb{S}, \mathbb{D}). \tag{11}$$

3   The producer-consumer market setup

We suggest herein a simple model for a futures market made up with one producer and one consumer only. An automated seller is designed to hedge the production of the producer, and an automated buyer is designed to hedge the needs of the consumer. The trading process starts at period t_1 and carry on until the final time t_m. An initial period, t_0, is added to the model to initialize some variables like S, D, etc., though no trade takes place at t_0. At each period t_j, $j = 1 \dots m$, forecasts of supply, $S(t_j)$, and demand, $D(t_j)$, are sent to the automated traders, as well as other information like the nominal price, $p^*(t_j)$, of this period. As such,

these automated traders are designed to react to the flow of forecasts, then generate automatically their actual decisions $\mathbf{u}_1(t_j)$ and $\mathbf{u}_2(t_j)$ respectively. Based on these decisions, a transaction may occur at a price $p(t_j)$ with a traded quantity $q(t_j)$ of futures contracts.

The actual production of the producer will be ready at the final time t_m, and only at this time the exact amount of supply, $S(t_m)$, will be known; prior to this period, only forecasts were available, that is $S(t_j)$, $j = 0, \ldots, m-1$ were forecasts of the supply level that will occur actually at period t_m. The same applies on the demand side, i.e. $D(t_j)$, $j = 0, \ldots, m-1$ are just forecasts of the actual demand that will be known exactly at the final period t_m.
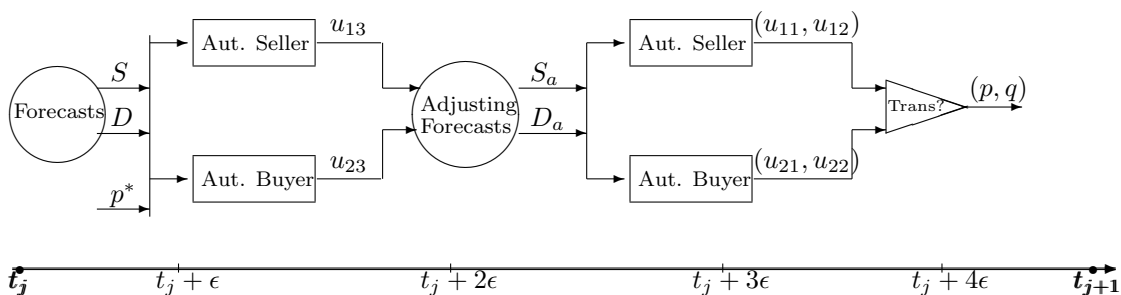

Figure 1: Unfolding of events within period t_j

To understand the unfolding of events in period t_j, one may assume that this period was split into tiny subintervals as shown in figure 1. First, the forecasted supply and demand $S(t_j)$ and $D(t_j)$ are received at instant $t_j + \epsilon$, then at instant $t_j + 2\epsilon$ both traders announce their adjustment factors, $u_{13}(t_j)$ and $u_{23}(t_j)$, allowing to compute the adjusted forecasts of supply and demand $S_a(t_j)$ and $D_a(t_j)$. At instant $t_j + 3\epsilon$, both traders send their sale and buy orders in the form of $(u_{11}(t_j), u_{12}(t_j))$ and $(u_{21}(t_j), u_{22}(t_j))$ respectively, then at instant $t_j + 4\epsilon$ a transaction, $(p(t_j), q(t_j))$, may occur. In the following, we will consider period t_j as a whole entity, but one should bear in mind that events unfold inside this period as described herein.

The decision of the automated seller in period t_j has the following form

$$\mathbf{u_1}(t_j) = (u_{11}(t_j), \ u_{12}(t_j), \ u_{13}(t_j)), \tag{12}$$

where $u_{11}(t_j)$ is the ask-price and $u_{12}(t_j)$ is the ask-quantity. In practice, we add a minus sign '-' to the ask-quantity in order to make a distinction between buy and sale orders; also, doing so, will put the current notations in line with those used in [14]. The third component, $u_{13}(t_j) \in [-1, +1]$, is an adjustment factor, that is a ratio by which the seller intends to increase, or decrease, his supply in regards to the last available price. Consequently, the *adjusted forecast of supply* will be

$$S_a(t_j) = S(t_j)(1 + u_{13}(t_j)). \tag{13}$$

For instance, if the price, $p(t_{j-1})$, of the prior period is under the nominal price, $p^*(t_{j-1})$, the producer may decide to cut supplies by 15%. Inversely, if the price is high, he may decide to hire an additional production capacity to boost his supplies, therefore he will announce an increase, say 10%, of forecasted supplies.

On the other hand, the order of the automated buyer has the following form

$$\mathbf{u_2}(t_j) = (u_{21}(t_j) ,\ u_{22}(t_j) ,\ u_{23}(t_j)) , \tag{14}$$

where $u_{21}(t_j)$ is the bid-price, $u_{22}(t_j)$ is the bid-quantity, $u_{23}(t_j) \in [-1, +1]$ is the adjustment factor on the demand side. For instance, if the price is so high, the consumer may decide to cut his consumption by 10%. Consequently, the *adjusted forecast of demand* will be

$$D_a(t_j) = D(t_j) \left(1 + u_{23}(t_j)\right) . \tag{15}$$

A transaction will occur if the two conditions:

i) : $u_{12}(t_j) \neq 0$ and $u_{22}(t_j) \neq 0$,

ii) : $u_{21}(t_j) \geq u_{11}(t_j)$,

are satisfied at the same time. In such event, the transactional price will be

$$p(t_j) = \frac{u_{11}(t_j) + u_{21}(t_j)}{2}, \tag{16}$$

and the transactional quantity will be

$$q(t_j) = \min\{|u_{12}(t_j)| ,\ u_{22}(t_j)\}. \tag{17}$$

If condition i), or ii), or both, is/are not satisfied then no transaction will occur in period t_j, i.e. t_j is a non-transactional time, and we set conventionally in this case

$$p(t_j) = p(t_{j-1}) \quad \text{and} \quad q(t_j) = 0. \tag{18}$$

For the moment, we assume that the automated seller will launch only sale orders, therefore he is not allowed to buy back something that he had sold before. Equally, the automated buyer is authorized to issue only buy orders.

On the other hand, each trader $i = 1, 2$, is characterized by two components: his position $y_i(t_j)$ and the average price, $x_i(t_j)$, of this position at instant t_j, $j = 0, \dots, m$ (see [14]). In this study, the average price, $x_i(t_j)$, is not relevant. The position $y_i(t_j)$, of trader $i = 1, 2$, is the number of contracts he had sold or bought since the beginning of the trade until instant t_j. At the starting time, we have

$$y_i(t_0) = 0, \quad i = 1, 2.$$

At instant t_j, $j = 1, \dots, m$, these positions should be updated:

$$y_1(t_j) = y_1(t_{j-1}) - q(t_j), \tag{19}$$
$$y_2(t_j) = y_2(t_{j-1}) + q(t_j). \tag{20}$$

4 Example of a trading strategy

Suggesting an automatic trading strategy, $\gamma = (\gamma_1, \gamma_2)$, consists to build mathematical formula allowing to compute the decisions $\mathbf{u}_1(t_j)$ and $\mathbf{u}_2(t_j)$ of the automatons at each period t_j, $j = 1, \ldots, m$, from the sets of available information [3], $\mathbb{I}_1(t_j)$ and $\mathbb{I}_2(t_j)$, for both automatons, i.e

$$\mathbf{u}_1(t_j) = \gamma_1(\mathbb{I}_1) \quad \text{and} \quad \mathbf{u}_2(t_j) = \gamma_1(\mathbb{I}_2). \tag{21}$$

We assume that our market is a *transparent* one, that is all automatons have access to all the information available since t_0 until t_j. In this event, $\mathbb{I}_1(t_j) = \mathbb{I}_2(t_j) = \mathbb{I}(t_j)$, where

$$\mathbb{I}(t_j) = \mathbb{I}(t_{j-1}) \cup \{S(t_j), D(t_j), p^*(t_j), \mathbf{u}_1(t_j), \mathbf{u}_2(t_j)\}, \tag{22}$$

for $j = 1, \ldots, m$, with $\mathbb{I}(t_0) = \{S(t_0), D(t_0), p^*(t_0)\}$.

In addition, we assume that strategy γ_1 of the seller is parameterized by parameters α_{11}, α_{12}, α_{13} and α_{14}. Similarly, strategy γ_2 of the buyer is assumed to be parameterized by parameters α_{21}, α_{22}, α_{23} and α_{24}. In section 5, optimal values of these parameters will be computed by simulation in some particular cases.

4.1 Computing the adjusting factors u_{13} and u_{23}

As a way to compute the adjusting factors $u_{13}(t_j)$ and $u_{23}(t_j)$ of the automated seller and the buyer respectively, we suggest an approach based on a system of *price bands and frequencies.*
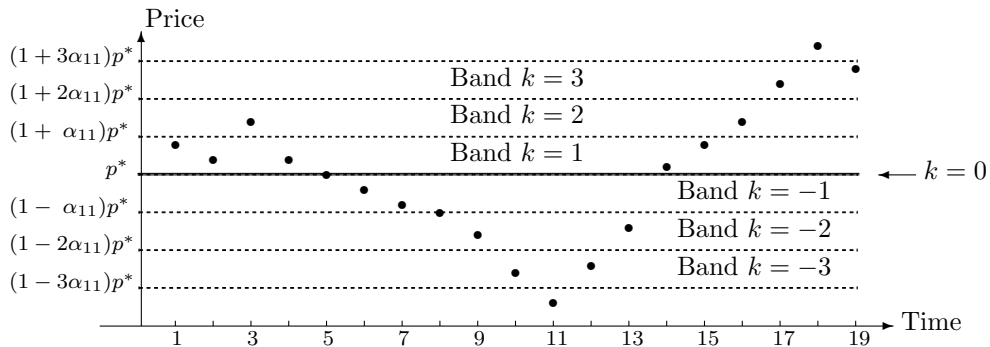

Figure 2: Seller's price bands around the nominal price

For the seller, we draw bands of width $\alpha_{11}p^*$ around the nominal price[3] p^*, with $0 < \alpha_{11} \leq 1$. If at any time t_j, the frequency of the price in any band has exceeded a specific value α_{12}, then an adjustment value, $u_{13}(t_j) \neq 0$, is generated, otherwise $u_{13}(t_j) = 0$.

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[3]On figure 2, the line of p^* is horizontal, i.e. it was assumed in this case that the nominal price was constant over the periods $t_1, \ldots, t_m$. However, in the general case, the line of the nominal price is a step-wise line, therefore the system of bands should be displaced accordingly each time p^* increases or decreases.

First, we initialize the number of times, $n_{1,k}(t_0)$, the price has been in the kth band of the seller:

$$n_{1,k}(t_0) = 0, \quad k = 0, \mp 1, \mp 2, \dots \tag{23}$$

The band $k = 0$ is a particular one as its width is not $\alpha_{11} p^*$, but it is formed only by the line of the nominal price as illustrated in figure 2. Then, each time the price access into the kth band, the counter $n_{1,k}$ should be incremented.

In the beginning of period t_j, i.e. at instant $t_j + \epsilon$ (see figure 1), we compute the relative change of the price $p(t_{j-1})$ compared to the nominal price $p^*(t_{j-1})$:

$$r(t_{j-1}) = \frac{p(t_{j-1}) - p^*(t_{j-1})}{p^*(t_{j-1})}. \tag{24}$$

After that, we determine the band, k_1, in which the price $p(t_{j-1})$ has been in the period t_{j-1} by solving the following

$$k_1(t_j) = sign(r(t_{j-1})) \times \arg_{k \in \mathbb{N}^+} \left\{ k - 1 < \frac{|r(t_{j-1})|}{\alpha_{11}} \leq k \right\}, \tag{25}$$

with $j = 1, \dots, m$. We readily note that if $r(t_{j-1}) = 0$ then $k_1(t_j) = 0$.

Once k_1 of period t_j is determined, we increment the corresponding band's counter by setting $n_{1,k_1}(t_j) = n_{1,k_1}(t_{j-1}) + 1$; the other bands will not be incremented, that is $n_{1,k}(t_j) = n_{1,k}(t_{j-1})$ for $k \neq k_1$.

Now, we compute the frequency, f_{1,k_1}, of the price in the band k_1 at instant t_j by setting

$$f_{1,k_1}(t_j) = \frac{n_{1,k_1}(t_j)}{m}. \tag{26}$$

If this frequency has exceeded α_{12} then the automated seller should react by setting $u_{13}(t_j) = \alpha_{13} k_1(t_j) f_{1,k_1}(t_j)$, otherwise $u_{13}(t_j) = 0$. We can write this in a single relation by setting

$$u_{13}(t_j) = \alpha_{13} k_1(t_j) f_{1,k_1}(t_j) 1_{[f_{1,k_1}(t_j) \geq \alpha_{12}]}. \tag{27}$$

For instance, in figure 2, the number of trading periods is $m = 19$. The price has been 5 times in the band $k = 1$, hence its frequency at time $t_j = 19$ is $f_{1,1}(19) = 5/19$; whereas at instant $t_j = 7$, the frequency of the price in the band $k = 2$ was $f_{1,2}(7) = 1/19$.

In a similar way, the adjusting factor, u_{23}, of the buyer will be computed by

$$u_{23}(t_j) = \alpha_{23} k_2(t_j) f_{2,k_2}(t_j) 1_{[f_{2,k_2}(t_j) \geq \alpha_{22}]}, \tag{28}$$

where

$$n_{2,k}(t_0) \quad = \quad 0, \quad k = 0, \mp 1, \mp 2, \dots \tag{29}$$

$$k_2(t_j) \quad = \quad sign(r(t_{j-1})) \times \arg_{k \in \mathbb{N}^+} \left\{ k - 1 < \frac{|r(t_{j-1})|}{\alpha_{21}} \leq k \right\}, \tag{30}$$

$$n_{2,k_2}(t_j) \quad = \quad n_{2,k_2}(t_{j-1}) + 1, \quad n_{2,k}(t_j) = n_{2,k}(t_{j-1}) \text{ for } k \neq k_2, \tag{31}$$

$$f_{2,k_2}(t_j) \quad = \quad \frac{n_{2,k_2}(t_j)}{m}. \tag{32}$$

After u_{13} and u_{23} were calculated by (27) and (28), the adjusted forecasts of supply and demand, S_a and D_a, can be computed using (13) and (15) respectively.

4.2   Computing the adjusted forecast of the gap function

At this stage, the gap function (2) should be adapted to take into account the adjusted forecasts of supply and demand, S_a and D_a, hence the adjusted forecast of the gap will be computed by

$$G_a(t_j) = S_a(t_j) - D_a(t_j). \tag{33}$$

This function is susceptible to convey the new balance of the supply and demand. Furthermore, the relative change in the adjusted gap function

$$\delta G_a(t_j) = \frac{G_a(t_j) - G_a(t_{j-1})}{|G_a(t_{j-1})|}, \quad j = 1, \ldots, m, \tag{34}$$

measures the evolution of the balance of supply and demand between instants t_{j-1} and t_j. By intuition, we reckon that this change in the supply and demand would yield to an equivalent change in the price, this is described by the following relation

$$\tilde{\delta}p(t_j) = -\lambda\, \delta G_a(t_j), \tag{35}$$

where λ is a scale factor and $\tilde{\delta}\,p(t_j)$ should reflect the evolution of the price between t_{j-1} and t_j,

$$\tilde{\delta}p(t_j) = \frac{\tilde{p}(t_j) - p(t_{j-1})}{p(t_{j-1})}, \quad j = 1, \ldots, m, \tag{36}$$

where $\tilde{p}(t_j)$ is the projected price for period t_j. From the latter we obtain

$$\tilde{p}(t_j) = p(t_{j-1})\left[1 - \lambda\, \delta G_a(t_j)\right]. \tag{37}$$

In addition, most Exchanges fixe limits on price moves from one session to the next one in order to avoid large swings of the price in a short period of time which may disrupt the market functioning. For instance, a typical Exchange may impose that a price move from one session to the next one should not exceed 10% of the last price either side, or a ratio of 0.1. For this purpose, we add a second parameter, β, which is the ratio imposed by the Exchange to limit price moves, therefore the *actual* relative change in the gap, $\delta G_a(t_j)$, that will be used later is defined by

$$\delta_\beta G_a(t_j) = \begin{cases} \delta G_a(t_j), & \text{if } -\beta < \delta G_a(t_j) < \beta, \\ -\beta, & \text{if } \delta G_a(t_j) \leq -\beta, \\ +\beta, & \text{if } \delta G_a(t_j) \geq +\beta. \end{cases} \tag{38}$$

4.3 Computing the bid and ask prices and quantities u_{11}, u_{12}, u_{21}, u_{22}

This strategy relies on two simple ideas to generate respectively the price levels and the offered quantities. First, dealing with the price levels, this strategy suggests to react instantaneously to the variations of the supply and demand through the rate of change in the adjusted gap function $G_a(t_j)$. Practically, it suggests that the ask-price and bid-price of both automated traders are generated using relation (37) where parameter λ will be replaced by α_{14} for the seller and α_{24} for the buyer:

$$u_{11}(t_j) = p(t_{j-1}) \left[1 - \alpha_{14} \, \delta_\beta G_a(t_j)\right], \tag{39}$$

$$u_{21}(t_j) = p(t_{j-1}) \left[1 - \alpha_{24} \, \delta_\beta G_a(t_j)\right]. \tag{40}$$

Secondly, at each period t_j, the quantity to sell (resp. to purchase) is obtained by dividing the forecasted remaining quantity by the number of remaining periods. In a mathematical form, the ask-quantity is given by

$$u_{12}(t_j) = \frac{S_a(t_j) + y_1(t_{j-1})}{m - j + 1}, \tag{41}$$

and the bid-quantity is

$$u_{22}(t_j) = \frac{D_a(t_j) - y_2(t_{j-1})}{m - j + 1}. \tag{42}$$

Remark 4.1. The volatility of the forecasts from a period to period are likely to cause the values of u_{12} and/or u_{22}, proposed by this strategy, to be equal to zero, consequently causing an absence of a transaction in this period.

5 Numerical results

A Matlab code was written for the approach described in this study. This code can be used for different purposes. It generates samples of supply and demand forecasts following various probability distributions. Using the trading strategy, *bands and frequencies*, described earlier, the code determines the price pattern and traded quantities between the automated seller and buyer, then it measures the performances of this strategy in respect to the criteria suggested above. As inputs, the code is fed with profiles of the supply and demand forecasts phenomena, SP and DP, the nominal price p^*, and the spaces of parameters, $\mathbb{A}$ and $\mathbb{B}$, from which the optimum parameters should be picked up.

The supply profile SP (resp. demand profile DP) is a set of information characterizing the forecasts of supply (resp. demand) phenomenon. For instance, $SP = \{Normal; 50,000; 2\}$ stands for the supply forecasts follow a Normal distribution with a mean $\mu_S = 50,000$ and standard deviation $\sigma_S = 2$; and $DP = \{Uniform; 80,000; 90,000\}$ stands for the demand forecasts follow a Uniform distribution in the interval $[80,000\,;90,000]$. If the phenomenon does not belong to the
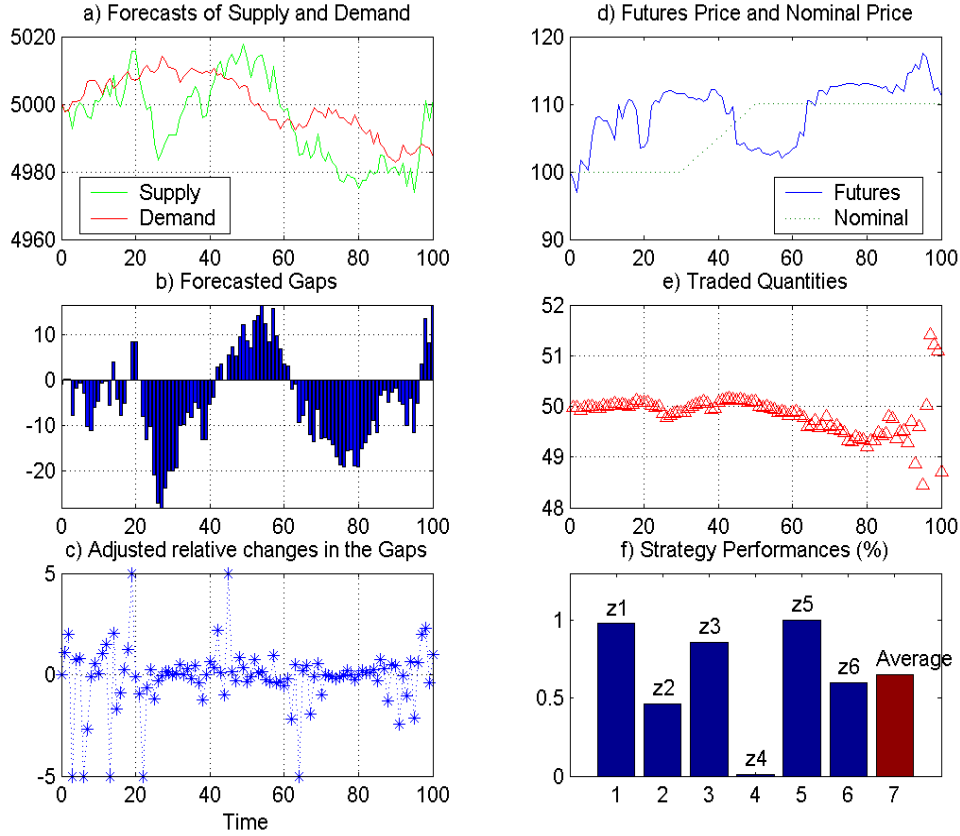
Figure 3: Graphical results provided by the Matlab code

set of known probability distributions, then the profile can be a tabular distribution $(x_1; \text{Probability}(x_1))$, $(x_2; \text{Probability}(x_2))$, ...

The set $\mathbb{A} = \{\alpha_{il} \in \mathbb{A}_{il}, \ i = 1, 2 \text{ and } l = 1, 2, 3, 4\}$ is the space of search of optimal parameters, with $\mathbb{A}_{il}$ is the set from which the optimal parameter α_{il}^* belongs to. The optimal parameter β^* belongs to $\mathbb{B}$.

Firstly, this code was used, with given inputs, to generate figure 3. As it can be seen on this figure, the futures price curve (figure 3c) follows closely the moves of supply and demand (figure 3a) and takes into account the underlying nominal price value. The performance of our trading strategy is displayed in figure 3f. The strategy has performed well in respect to all hypothesises of the correct price pattern, except hypothesis 2.4 which was weak with $z_4 = 0.0111$, that is the price was not as volatile as the S&D in this case; therefore, we need to find a new combination of α_{ij} and β which is maximizing z_4, without altering other criteria, or even increasing them.

The second application of the Matlab code was to construct table 1 where the supply and demand profiles were assumed to be $SP = \{Normal; 5,000; 1\}$ and $SD = \{Normal; 5,001; 1\}$ respectively. Since the producer-consumer setup de-

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k	α_1	α_2	α_3	α_4	β	z_1	z_2	z_3	z_4	z_5	z_6	$\bar{z}$
1	0.10	0.05	0.2	0.02	0.15	0.639	0.506	0.618	0	1.000	0.901	0.610
2	0.10	0.05	0.2	0.02	1.00	0.724	0.504	0.652	0.075	1.000	0.901	0.643
3	0.10	0.05	0.2	0.20	0.15	0.637	0.507	0.623	0.096	1.000	0.901	0.627
4	0.10	0.05	0.2	0.20	1.00	0.815	0.507	0.592	0.015	1.000	0.901	0.638
5	0.10	0.05	4.0	0.02	0.15	0.837	0.525	0.606	0	1.000	0.901	0.645
6	0.10	0.05	4.0	0.02	1.00	0.933	0.505	0.608	0.014	1.000	0.901	0.660
7	0.10	0.05	4.0	0.20	0.15	0.880	0.512	0.582	0.042	1.000	0.901	0.653
8	0.10	0.05	4.0	0.20	1.00	0.797	0.515	0.591	0	1.000	0.901	0.634
9	0.10	0.20	0.2	0.02	0.15	0.908	0.515	0.740	0	1.000	0.901	0.677
10	0.10	0.20	0.2	0.02	1.00	0.958	0.519	0.759	0.031	1.000	0.901	0.695
11	0.10	0.20	0.2	0.20	0.15	0.850	0.507	0.747	0.051	1.000	0.901	0.676
12	0.10	0.20	0.2	0.20	1.00	0.966	0.515	0.832	0.001	1.000	0.901	**0.702**
13	0.10	0.20	4.0	0.02	0.15	0.947	0.517	0.707	0	1.000	0.901	0.678
⋮	⋮					⋮						⋮
31	0.50	0.20	4.0	0.20	0.15	0.903	0.515	0.711	0.010	1.000	0.901	0.673
32	0.50	0.20	4.0	0.20	1.00	0.878	0.513	0.689	0.001	1.000	0.901	0.664

Table 1: Average performances of specified parameters

scribed in this approach is a symmetric case (the two automatons have the same constraints and the same information), we have decided to perform our simulation on the particular case where both traders have the same parameters, i.e. $\alpha_{il} = \alpha_l$ with $l = 1, 2, 3, 4$. This is done so in order to reduce the number of parameters α_{il} to be optimized. Also, we assumed that the weight factors, w_l, were all the same, i.e. $w_l = 1/6$, for $l = 1, \ldots, 6$. The completed table 1 shows that over the space of search parameters used in this application, the highest average performance occurs in line 12, with $\bar{z} = 0.702$, i.e. this trading strategy has a 70.2% maximum performance; line 12 lists also the optimal combination of parameters α_l, β and the corresponding performances, $z_1, \ldots, z_6$, of each criterion.

σ_S	σ_D	α_1^*	α_2^*	α_3^*	α_4^*	β^*	$\bar{z}^*$
0.5	0.3	0.10	0.20	0.20	0.02	1.00	0.730
0.5	1.0	0.10	0.20	0.20	0.20	1.00	0.715
0.5	6.0	0.10	0.20	0.20	0.20	1.00	0.672
1.0	0.3	0.10	0.20	0.20	0.20	1.00	0.714
1.0	1.0	0.10	0.20	0.20	0.20	1.00	0.702
1.0	6.0	0.10	0.20	4.00	0.20	1.00	0.663
5.0	0.3	0.10	0.20	0.20	0.20	1.00	0.662
5.0	1.0	0.10	0.20	0.20	0.20	1.00	0.662
5.0	6.0	0.10	0.20	0.20	0.20	1.00	0.607

Table 2: Optimal parameters and performances of different S&D profiles

The third application of our code was computing optimal parameters and the corresponding maxima weighed performances for different supply and demand profiles; this is set out in table 2. In practice, such a table could help a designer of automatons to select the adequate parameters depending on the environment in which the automatons should intervene. For instance, if over a period of ten years of observation, it was shown that, for a particular commodity, supply forecasts follow a normal distribution with a standard deviation $\sigma_S = 0.5$, and the observed demand forecasts do the same with $\sigma_D = 0.3$, then to design automatons that should react properly to the forecasts in such an environment, and generate a correct price pattern, the designer should tune them with the following optimal combination of parameters $(\alpha_1^*; \alpha_2^*; \alpha_3^*; \alpha_4^*; \beta^*) = (0.10; 0.20; 0.20; 0.02; 1.00)$ leading to an optimal maximum weighed performance of 73.0%.

6   Conclusion and perspectives

We suggested herein a technical approach allowing to operate an artificial futures market with automated traders. The proposed producer-consumer setup retains the most essential features of real futures markets: it permits fixation of price ahead of the effective delivery of the merchandize, and more importantly this framework assumes uncertainty in the levels of supply and demand, and aggregates their updates easily. On the other hand, the obtained numerical results were appealing: the futures price follows closely the prevailing supply-demand balance and the underlying nominal price.

Automated trading remains an open research field necessitating new contributions in several directions. Looking forward, our approach needs to be generalized to the case of many producers and consumers, and even speculators should be involved to create more liquidity; the ultimate objective should be designing an efficient futures market structure operated entirely by automatons. In this optic, the main hypothesises of the correct price pattern need to be enriched by taking into account other market subtleties. In addition, the parametrization can be conducted in a different manner. For instance, it would be interesting to see what happens if the purchased quantities u_{12} and u_{22} were parameterized also. However, one should keep in mind that the number of parameters should be kept to a minimum in order to obtain an optimal solution in a reasonable time. Simulation can be bypassed by establishing the underlying mathematical model of the approach presented herein, then searching the optimal solution using numerical methods.

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