



Munich Personal RePEc Archive

## **Productivity growth in Australian manufacturing: a vintage capital model**

Bloch, Harry and Madden, Gary G

University of Tasmania, Department of Economics, Hobart, Australia, Curtin University of Technology, School of Economics and Finance, Perth WA 6845, Australia

1995

Online at <https://mpra.ub.uni-muenchen.de/13003/>  
MPRA Paper No. 13003, posted 09 Apr 2009 06:41 UTC

# Productivity growth in Australian manufacturing: a vintage capital model

Harry Bloch

*Department of Economics, University of Tasmania, Hobart,  
Australia and*

Gary Madden

*Department of Economics, Curtin University, Perth, Australia*

## Introduction

Recent contributions by Hulten (1992) and Gort *et al.* (1993) indicate a renewed interest in using capital-embodied technology models to understand the sources of productivity growth. An advantage of models with capital-embodied technology is that current productivity is related to the prior time path of investment. This provides a potential dynamic link between past market conditions and current productivity performance. In particular, models with capital-embodied technology provide a possible explanation for the positive relationship between productivity growth and the rate of investment, particularly investment in capital equipment, found in cross-country studies (see, for example, Wolff (1991) and De Long and Summers (1992)).

Capital-embodied models of technology achieved some notoriety in the 1960s, especially with the vintage capital models of Jøhansen (1959), Solow (1960) and Salter (1966). We adapt Salter's model, but cast our analysis in continuous time rather than in discrete time. This adapted analysis is used to derive relationships determining average labour productivity growth under alternative assumptions about the age structure of industry capital equipment.

Regressions in the form of the relationships derived from the analysis are estimated using data for a cross-section of Australian manufacturing industries. Variables suggested by the analysis of the vintage capital model contribute significantly to the explanation of differences in average labour productivity growth across the sample industries. However, specific restrictions on coefficient values derived from the analysis are rejected by the regression results. The implications of this mixed support for the application of the vintage capital model to explaining labour productivity growth in Australian manufacturing are discussed.

---

The able research assistance of Matthew Stubbs is gratefully acknowledged as is financial support from the Institute for Research into International Competitiveness at Curtin University and the School of Business and Law Strategic Research fund at the University of Tasmania. The views expressed in the article are, however, solely those of the authors.

**Competitive equilibrium with capital-embodied technical change**

Salter (1966) provides a model of competitive equilibrium under conditions in which technology is embodied in capital equipment. At each point in time there is a particular technology available in newly purchased equipment. This technology is defined by the level of the fixed input requirements for both capital and labour. All technologies operate with constant returns to scale, so that under competitive conditions the cost per unit of output from any technology is constant for all output levels. Technical progress is introduced by having both the amount of labour required to produce a unit of output and the unit cost of production decrease with each successive vintage of equipment.

Salter uses his model to show that the range of vintages employed in competitive equilibrium depends on both the relative productivity of the vintages and on the prices of both capital and labour inputs. Furthermore, the distribution of output over the vintages within this range depends on changes in demand. Thus, average productivity for an industry depends on demand growth and relative factor prices as well as the technical progress embodied in capital equipment.

We adapt Salter's model for the purpose of examining productivity growth in Australian manufacturing. Salter's characterization of capital-embodied technical change is formalized as a continuous-time model. This contrasts with Salter's use of discrete-time analysis.

The essence of any model of capital-embodied technical change is that the level of output depends on the distribution of the capital stock over vintages of equipment. A general discrete-time production function for this type of model can be written as

$$q_t = f(L_t, I_t, I_{t-1}, I_{t-2}, \dots, I_{t-m}), \quad (1)$$

where  $q_t$  is the current level of output,  $L_t$  the level of current labour input,  $I_{t-i}$  the level of gross investment in capital equipment of vintage  $t-i$  and  $m$  is the number of vintages of capital in use in the current period.

In Salter's version of the general model [1], there are assumed to be fixed labour and capital input coefficients that apply to each vintage of equipment. He further assumes that the labour input coefficients fall with each successive vintage of equipment, while the capital coefficients are constant over vintages. This implies that the level of labour input required for a given level of output is lowest when the newest capital equipment is used in production. Therefore, if the amount of capital stock available for each vintage is given, minimizing the level of labour input for efficient production requires allocation of production to new equipment before old equipment.

We assume that gross investment occurs continuously over time, so that the amount of output produced at time  $t$  is given by

$$q_t = \int_{t-m}^t \kappa^{-1} I_j dj \quad (2)$$

where  $\kappa$  is the capital input requirement for a unit of output produced, assumed to be constant over time, and  $t-m$  is the oldest vintage of capital equipment utilized. The amount of labour input required to operate this equipment is given by

$$L_t = \int_{t-m}^t \lambda_j \kappa^{-1} I_j dj \quad (3)$$

where  $\lambda_j$  is the labour input requirement for a unit of output produced with capital of vintage  $j$ . Finally, the amount of capital equipment actually utilized at time  $t$  is the total of gross investment still in use and is given by

$$K_t = \int_{t-m}^t I_j dj. \quad (4)$$

The average productivity of labour at any point in time is given by dividing the level of output in (2) by the level of labour input in (3) to yield

$$z_t = q_t / L_t = (\int_{t-m}^t \kappa^{-1} I_j dj) / (\int_{t-m}^t \lambda_j \kappa^{-1} I_j dj). \quad (5)$$

Average productivity in (5) depends on technology as specified in the capital and labour coefficients,  $\lambda_j$  and  $\kappa$ , the distribution of investment over time as specified in a density function for  $I_j$  and the range of vintages in use as specified by the limits of integration. Salter's assumption that there is a continual decrease in labour input coefficients is incorporated into the model by specifying that

$$\lambda_t = \lambda_0 e^{-\theta t}, \theta > 0. \quad (6)$$

In a simple case corresponding to that analysed by Salter, constancy over time is assumed for the gross investment variable and the time span between the newest vintage and the oldest vintage of equipment utilized. These assumptions together with a constant capital-to-output coefficient imply that the level of output produced is constant over time. In this case, average labour productivity from (5) is given by

$$z_t = \lambda_t^{-1} [-\theta m / (1 - e^{-\theta m})]. \quad (7)$$

The term in brackets in (7) is less than one when both  $\theta$  and  $m$  are positive and decreases with either  $\theta$  or  $m$ . Thus, average labour productivity is less than the labour productivity for capital equipment of the newest vintage, as given by  $\lambda_t^{-1}$ . Furthermore, the ratio of average productivity to productivity of the newest vintage equipment falls either with the rate of labour-saving technical change, given by  $\theta$ , or with  $m$ , the variable that measures the time span between the newest and oldest vintages of capital equipment in use.

The difference between average labour productivity and the productivity of new equipment does not affect the rate of growth of labour productivity in the simplest case. When  $\theta$  and  $m$  are fixed, the rate of growth of average labour productivity is equal to the rate of labour-saving technical change. This can be seen by taking the time rate of change of (7) after substituting for  $\lambda_t$  from (6) to yield

$$\dot{z}_t = \theta. \quad (8)$$

Salter determines both the level of gross investment at any point in time and the range of vintages in use by imposing conditions of competitive market equilibrium. The labour coefficient for each vintage of capital equipment is fixed, so that both the marginal and average cost of production for any vintage are constant with respect to output under the competitive condition that inputs are available to the individual producer at a fixed price. Full competitive market equilibrium requires that no producer be able to expand output at a marginal cost below market price using existing equipment and that market price equals the marginal and average cost of using new equipment.

Imposing the condition that price equals the marginal and average cost of using new equipment implies that

$$p_t = c_t = w_t \lambda_t + r_t \mathcal{K} \quad (9)$$

where  $p_t$  is the price of a unit of output,  $c_t$  is the average and marginal cost of output from new equipment at time  $t$ ,  $w_t$  is the corresponding wage rate and  $r_t$  is the corresponding rental price of a unit of capital equipment. The marginal cost of using existing equipment varies continuously with the vintage of equipment under the assumption of continuous labour-saving technical change. This means that the requirement that producers not be able to expand at a price exceeding marginal cost implies that price equals the marginal cost for the oldest vintage of capital equipment in use, so that

$$p_t = w_t \lambda_{t-m}. \quad (10)$$

Salter determines the range of labour productivity for capital equipment that is in use by combining (9) and (10) to yield

$$\lambda_{t-m} - \lambda_t = r_t \mathcal{K} / w_t. \quad (11)$$

While this condition on the range of vintages utilized is determined by Salter using discrete-time analysis, it is equally applicable to our continuous-time analysis. Indeed, the use of continuous-time analysis avoids the potential for (10) being an inequality relation. Substituting from (6) for the values of  $\lambda$  in (11) gives the following exponential function:

$$\lambda_t (e^{\theta m} - 1) = (r_t / w_t) \mathcal{K}. \quad (12)$$

The age difference between the newest and oldest vintages of capital utilized in production is then determined by solving for the value of  $m$  from (12) as follows:

$$m = (1/\theta)[\ln(1 + (r_t \mathcal{K} / w_t \lambda))]. \quad (13)$$

Factors that influence  $m$  in (13) are relative input prices, the rate of labour-saving technical progress and the values of both the labour and capital coefficients. Substituting for  $m$  from (13) into the average labour productivity expression in (7) yields the following expression for average labour productivity:

$$z_t = \lambda_t^{-1} [(w_t \lambda_t / r_t \kappa) (\ln(1 + (r_t \kappa / w_t \lambda_t)))]. \quad (14)$$

The term in square brackets in (14) is always less than one and decreases with the share of capital cost (or rises with the share of labour cost) in competitive price, so that average labour productivity is less than the productivity of newest equipment by a proportion dependent on shares of labour and capital inputs in the costs associated with the newest vintage equipment.

Impacts of changes in technology and input prices on average labour productivity growth are found by taking the derivative of (14) with respect to time. After rearranging terms, this yields

$$\begin{aligned} \dot{z}_t &= \alpha \theta + \beta (\dot{w}_t - \dot{r}_t), \\ \text{where } \alpha &= [(1 + w_t \lambda_t / r_t \kappa) (\ln(1 + (r_t \kappa / w_t \lambda_t)))]^{-1} \\ \beta &= (1 - \alpha). \end{aligned} \quad (15)$$

The value of  $\alpha$  in (15) varies from zero to one as the share of labour in the cost of production with the newest vintage of equipment, given by  $w_t \lambda_t / r_t \kappa$ , varies from zero to infinity. Thus, the rate of growth of average labour productivity is positively related to both the rate of labour-saving technical change and the difference in the rate of increase in wages and the rental price of capital[1]. Furthermore, the relationship is linearly homogeneous.

The relationship in (15) is derived under the assumptions used to obtain (7), namely, the existence of an equilibrium with a constant level of gross investment,  $I_t$ , over the  $m$  periods since the installation of the oldest vintage equipment still in use. This limits application of the relationship to industries with constant capital stocks. If an industry's capital stock increases through a change in the level of current gross investment, the effect on average labour productivity is given by

$$\partial z_t / \partial K_t = (1 - \lambda_t z_t) \kappa^{-1} L_t, \quad \partial K_t > -I_{t-m}, \quad (16)$$

where the restriction on the values of  $\partial K_t$  is required to ensure that the maximum age of capital equipment in use is kept constant at  $m$  periods.

If equilibrium with a constant level of gross investment for at least  $m$  periods is followed by a change in the level of gross investment, the rate of change in average labour productivity is found by adding a term based on (16) to the relationship in (15). The resulting relationship, after substitution for  $\lambda_t z_t$  from (14), yields

$$\begin{aligned} \dot{z}_t &= \alpha \theta + \beta (\dot{w}_t - \dot{r}_t) + \gamma \dot{K}_t, \\ \text{where } \gamma &= (w_t \lambda_t / r_t \kappa) (\ln(1 + (r_t \kappa / w_t \lambda_t))). \end{aligned} \quad (17)$$

Alternatively, given that the capital-to-output ratio is assumed constant, the rate of capital growth is equal to the rate of output growth in competitive equilibrium, so that

$$\dot{z}_t = \alpha \theta + \beta (\dot{w}_t - \dot{r}_t) + \gamma \dot{q}_t. \quad (18)$$

**Determinants of productivity growth in Australian manufacturing**

Relationships for determining the rate of growth in average labour productivity given in (17) and (18) both allow for the effects of differential input price growth and growth in the capital stock. The relationships are equivalent if the capital-to-output ratio is constant over time. If appropriate measures of the capital stock were available, there would be no need for the use of the relationship in (18). However, the method used in calculating the available capital stock estimates is inconsistent with the vintage capital model[2]. As a result, the output growth variable may outperform the measured capital growth variable as a proxy for the rate of growth in the actual capital stock. Thus, estimates of the determinants of productivity growth based on (18) are provided below along with estimates based on the relationship in (17).

Estimates of relationships determining average labour productivity growth are obtained using ordinary least squares (OLS) regressions on data for a cross-section of 34 Australian manufacturing industries. The dependent variable in each regression is the average rate of growth of labour productivity. Explanatory variables are derived from the rate of growth of labour-saving technical change, the difference between the rates of change of wages and the rental price of capital and either the rate of growth of the capital stock measured in constant dollars or the rate of growth of value added also measured in constant dollars. Each variable is multiplied by the function of wage payments relative to capital payments that determines the value of the corresponding  $\alpha$ ,  $\beta$  or  $\gamma$  parameter, where these functions are as expressed beneath (15) and (17). All rates of change are measured as the average rate of change over the period 1954-55 to 1981-82 and the ratio of wage payments to capital payments is the average value over this period. Values of each variable are calculated from data presented in Australian Bureau of Industry Economics (BIE) publications[3].

The average value of the rate of growth of labour productivity in any industry can at best be expected to approximate the value that would obtain under the assumptions leading to the relationships derived in the previous section. Thus, there is reason to expect unexplained residuals in the variation of average labour productivity growth across industries even after accounting for the influence of all the variables identified in the derived relationships. These unexplained residuals may not have a zero mean across industries, so regressions are estimated both with and without a constant term, even though no constant term appears in (17) or (18).

Results from OLS regressions with and without a constant term for the coefficients from (17) and (18) are presented in Table I. The value in parentheses under each estimated coefficient is the corresponding  $t$  ratio calculated using standard errors from the heteroskedasticity-consistent covariance matrix generated by Shazam Version 7.0. Values of the corrected  $R^2$  are not reported for regressions without constant terms, as  $R^2$  values are calculated using the explained and unexplained variations from the mean value of the dependent variable. In regressions without a constant, the explanatory variables determine the variation of the dependent variable from zero rather than from

| Constant         | Estimated coefficients                               |   |  |                                      |  | Corrected<br>$R^2$ | Standard<br>error of<br>estimate |
|------------------|--|---|--|--------------------------------------|--|--------------------|----------------------------------|
|                  | Modified<br>labour-<br>saving<br>technical<br>change | Modified<br>input price<br>differential | Modified<br>capital<br>stock<br>growth | Modified<br>value<br>added<br>growth |  |                    |                                  |
| 0.0021<br>(0.33) | 0.7435 <sup>a</sup><br>(4.61)                        | 1.1167 <sup>a</sup><br>(5.41)           | 0.2316<br>(1.86)                       |                                      |  | 0.322              | 0.0106                           |
|                  | 0.7829 <sup>a</sup><br>(6.61)                        | 1.1797 <sup>a</sup><br>(7.97)           | 0.2450 <sup>b</sup><br>(2.37)          |                                      |  | na                 | 0.0104                           |
| 0.0022<br>(0.56) | 0.5736 <sup>a</sup><br>(5.01)                        | 0.8690 <sup>a</sup><br>(6.14)           |  | 0.4605 <sup>a</sup><br>(5.09)        |  | 0.605              | 0.0081                           |
|                  | 0.6181 <sup>a</sup><br>(7.05)                        | 0.9423 <sup>a</sup><br>(7.91)           |  | 0.4694 <sup>a</sup><br>(5.54)        |  | na                 | 0.0080                           |

*Notes:*

The figures in parentheses are *t* ratios calculated using the heteroskedasticity-consistent covariance matrix from Shazam Version 7.0

<sup>a</sup> Indicates the coefficient is statistically different from zero at the 1 per cent significance level using a two-tailed *t* test

<sup>b</sup> Indicates the coefficient is statistically different from zero at the 5 per cent significance level using a two-tailed *t* test

**Table I.**

Regressions explaining average rate of labour productivity growth

the mean. An alternative measure of goodness of fit that depends on only unexplained variation in the dependent variable from its mean value, the standard error of estimate, is listed for all regressions. A lower value of the standard error of estimate indicates an improvement in explanatory power.

The results in Table I provide support for using the vintage capital to explain average labour productivity growth in Australian manufacturing. Each of the estimated coefficients of the rate of labour-saving technical change and the differential rate of increase in wages and the rental price of capital is positive and statistically different from zero at the 1 per cent significance level using a two-tailed *t* test. Further, the estimated coefficients of the value added growth variable in Table I are each positive and significant at the 1 per cent level, while the estimated coefficients of the capital growth variable are positive but only marginally significant.

A constant term is introduced into regressions in Table I to allow for a non-zero mean value of residuals unexplained by the relationships derived from the vintage capital model. Neither estimated constant term is statistically different from zero at the 10 per cent significance level using a two-tailed *t* test. Thus, there is no evidence that a constant term is necessary to a proper specification of the regression for explaining average productivity growth. A further test of the absence of misspecification is given by the application of Ramsey's reset test

using powers of the predicted explanatory variable. None of the reset statistics is significantly different from zero at the 5 per cent level using an  $F$ -ratio test.

The variables in the regressions correspond exactly to the terms,  $\alpha\theta$ ,  $\beta$  ( $w_t - \dot{r}_t$ ),  $\gamma\bar{K}_t$  and  $\gamma\dot{q}_t$ , from the relationships for average labour productivity growth given in (17) and (18). Thus, the vintage capital model implies a restriction that each explanatory variable in regressions without a constant term in Table I has a coefficient of unity. The joint restriction that each explanatory variable has a coefficient of unity is rejected at the 1 per cent significance level using an  $F$ -test for each of the regressions without a constant term. Thus, the vintage capital model cannot be taken to provide a completely satisfactory explanation of average labour productivity growth in the sample of Australian manufacturing industries.

Rejection of the restriction that each estimated coefficient has a value of unity in the regressions with a constant term in Table I may be related to defects in either the data used in the estimates or the specification of the regression equations. With regard to data problems it is interesting to note that the value added growth variable takes a coefficient closer to unity than the capital stock growth variable[4]. Capital stock growth appears directly in our model, whereas value added appears only when the capital-to-output ratio is assumed constant. As noted above, however, the data on capital stock used in the regressions are based on calculations inconsistent with the assumptions of the vintage capital model. Failure of the capital growth variable to outperform the value added growth variable in the regressions may reflect the inappropriate assumptions used in the calculation of the capital stock measures.

### Conclusions

Embodiment of technical change in capital equipment means that labour productivity reaches its full potential only when workers are equipped with the newest equipment. When the stock of equipment consists of a mixture of old and new vintages, average labour productivity falls short of the best practice level. Our analysis seeks to explain the course of average labour productivity growth under these circumstances. We find that productivity growth in these circumstances is related to both the rate of technical change and the age structure of the capital stock.

Regressions using data for cross-sections of Australian manufacturing industries suggest promise in using the vintage capital model to explain labour productivity growth. A positive and statistically significant relationship to average labour productivity growth is found for the rate of labour-saving technical change and for each of two variables that serve as proximate determinants of the age structure of the capital stock, namely measures of industry growth and the differential in growth rates between wages and the rental price of capital.

Much work remains to be done. The regression results leave unexplained a substantial portion of differences across industries in average labour productivity growth. Also, restrictions on the values of the estimated

coefficients implied by the vintage capital model are rejected using the regression results. Thus, the vintage capital model as developed in our analysis does not provide a fully satisfactory explanation of productivity growth in Australian manufacturing.

An obvious direction for future research applying vintage capital models to Australian manufacturing is to incorporate the details of the age structure of the stock of capital equipment. Some success has been achieved in overseas studies using summary measures of the age of capital in explaining average labour productivity growth, especially when allowance is made for cyclical variation in the utilization of capital of different vintages (McHugh and Lane, 1987). Also, it is possible to incorporate vintage effects into a more general model of technical change as indicated by Intriligator (1992). Finally, consideration can be given to market structure in terms of the influence of imperfect competition among domestic producers and the influence of exposure to foreign competition as discussed in Bloch and Madden (1994).

#### Notes

1. Average labour productivity varies with the prices of inputs in (15) even though there is no substitution between capital and labour in the production process for any vintage of capital equipment. The impact on average labour productivity is due solely to variation in the maximum age of equipment in use.
2. The capital stock measures used in estimating the determinants of productivity growth are taken from Bureau of Industry Economics (BIE) (1985). These estimates are calculated by assuming a fixed average life for plant and equipment and a constant rate of depreciation in equipment over this life (see BIE, 1985, Appendix 2). The vintage capital model allows for a variable life of capital equipment depending on the rate of labour-saving technical change and the differential in rates of change in wages and the rental price of capital. Furthermore, in the vintage capital model, equipment remains intact without depreciation until the equipment becomes obsolete.
3. Values of the average rate of growth of labour productivity, value added and capital stock are taken from tables in Appendix 6 of Bureau of Industry Economics (BIE) (1985). The average rate of growth of each variable is calculated as the compounded annual rate of growth required to explain the ratio of the 1981-82 value of the variable to the corresponding 1954-55 value. Values of the average rate of labour-saving technical change are the average annual rates of change in labour efficiency given in BIE (1986, Table 3.1). The difference between the rate of change in wage rate and rental price of capital is calculated from data given in the Data Appendix of the same publication, by subtracting the rate of growth of the rental price of capital for an industry from the corresponding rate of growth of wages and salaries per man-hour. Finally, the average value of the ratio of wage payments to capital payments is the ratio of the average wage share to the average capital share, using share values implicit in the data reported in BIE (1985, Table 5.2).
4. The standard error of estimate for the regressions with the value added growth variable and no constant term is also lower than for the corresponding regression with the capital stock growth variable. When both value added growth and capital stock growth are included in an encompassing regression, the estimated coefficient of the value added growth variable is statistically greater than zero at the 1 per cent significance level using a two-tailed *t* test. In the same regression, the estimated coefficient of the capital stock growth variable is negative and not statistically different from zero at the 5 per cent level

---

using the same test. These results support rejection of the model including the capital stock growth variable in favour of the model including the value added growth variable.

**References and further reading**

- Bloch, H. and Madden, G. (1994), "Tariffs and productivity when technology is embodied in capital equipment", Discussion Paper 94-06, Institute for Research into International Competitiveness at Curtin University, Perth.
- Bureau of Industry Economics (1985), *Productivity Growth in Australian Manufacturing Industry: 1954-55 to 1981-82*, Australian Government Printing Office, Canberra.
- Bureau of Industry Economics (1986), *Manufacturing Industry Productivity Growth: Causes, Effects and Implications*, Australian Government Printing Office, Canberra.
- De Long, B.J and Summers, L.H. (1992), "Equipment investment and economic growth: how strong is the nexus?", *Brookings Papers on Economic Activity*, Vol. 2, pp. 157-99.
- Gort, M., Bank, B.H. and Wall, R.A. (1993), "Decomposing technical change", *Southern Economic Journal*, Vol. 50, pp. 220-4.
- Hulten, C.R. (1992), "Growth accounting when technical change is embodied in capital", *American Economic Review*, Vol. 82, pp. 964-80.
- Intriligator, M.D. (1992), "Productivity and the embodiment of technical progress", *Scandinavian Journal of Economics*, Vol. 94, pp. s75-s87.
- Johansen, L. (1959), "Substitution versus fixed production coefficients in the theory of economic growth: a synthesis", *Econometrica*, Vol. 27, pp. 157-76.
- McHugh, R. and Lane, J (1987), "The age of capital, the age of utilized capital, and tests of the embodiment hypothesis", *Review of Economics and Statistics*, Vol. 69, pp. 362-7.
- Salter, W.E.G. (1966), *Productivity and Technical Change*, 2nd ed., Cambridge University Press, Cambridge.
- Solow, R.M. (1960), "Investment and technical progress", in Arrow, K., Karlin, S. and Suppes, P. (Eds), *Mathematical Methods in the Social Sciences, 1959*, Stanford University Press, Palo Alto, CA, pp. 89-104.
- Wolff, E.N. (1991), "Capital formation and productivity convergence over the long term", *American Economic Review*, Vol. 81, pp. 565-79.

---

International  
Journal of  
Manpower  
16,1

**32**

---