Hopping on the Methadone Bus

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Abstract

This paper investigates the impact of a 'free drug program' on the market equilibrium of drugs. We introduce a screening model of the hard drug market in which dealers use payment and punishment options to screen between high and low risk users. We show that, if a free drug program selects sufficiently many high risk drug users, the pure-strategy separating market equilibrium ceases to exist and a symmetric mixed-strategy equilibrium results, in which drug users derive a higher expected utility. This encourages new drug users to enter the market. The novelty of the paper is the transmission mechanism for this effect, which is via the influence on market price.

JEL codes: D11, D82, I18

Keywords: Drugs, Drug Policy, Drug Dealing, Free Drug Programs, Screening

1 Introduction

Free drug programs supply severely affected addicts with drugs or their substitutes, such as methadone, under the supervision of professional medical staff. Advocates of such programs argue that the initiatives reduce drug-related criminality and help addicts take control of their problem.[1]

Empirical evaluations of methadone-maintenance treatment programs to date have typically restricted their attention to existing users of heavy drugs. They focus on determining

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[1] See Cussen and Block (2000), Frey (1997), or Prinz (1994).

whether users successfully reduce their drug use or eventually refrain from returning to regular drug use altogether,[2] whether there are positive effects of these programs on vocational rehabilitation of users,[3] or whether users are successful in leaving the drug-crime spiral.[4]

In this paper, we argue that free drug programs not only have an impact on individuals who are already in the drugs market, but – through a less costly distribution – also on potential users. We argue that by targeting severely addicted users, who typically need to resort to crime to finance their habit, free drug programs reduce the average cost of distributing drugs. This would be reflected in the attractiveness of the deals available and therefore affect the decisions of whether or not to enter the drug market as a user.

Non drug users might begin their consumption because of price: lower the price and more non-users will try a drug, and consequently become addicted. The price is determined by supply and demand, and thus the cost conditions on the supply side determine the cost and the equilibrium price: lower the supply curve, and price is lower and consumption higher. The point of our paper is that distributing methadone to drug addicts lowers the supply cost of competitive drug dealers, and so lowers price and increases demand.[5]

To this end, we introduce a screening model which analyzes the drug dealer's adverse selection problem and identifies the optimal purchase behavior of different types of drug users and derive the following result: Supplying severely affected addicts with free drugs changes the composition of the drug buyer population in a way that may lead to a situation where a pure strategy market equilibrium does not exist. We show that in this case a symmetric mixed strategy equilibrium exists in which all drug users derive a higher expected utility. This will encourage more drug users to enter the market. To the best of our knowledge, this negative side effect has not been identified in the literature yet.

We feel we contribute to the literature as we show how a health programme of intervention on a group of individuals will have an effect on the behaviour of individuals outside the group, and these effects need to be taken into account when evaluating the costs and benefits of this programme. This is not new, although it is rarely understood by politicians and the

[2]There is a large amount of literature on that topic. For recent contributions, see for example Dekimpe et al. (1998), Cox (2002), Dickinson et al. (2006), Harris et al. (2006), or Black et al. (2007).

[3]For example, Magura et al. (2004), Lidz et al. (2004), or Staines et al. (2005) focus on these effects.

[4]Brewster (2001) or Holloway et al. (2006) for example study whether drug users that were part of such a methadone maintenance program are less likely to engage in criminal activity.

[5]We would like to thank an anonymous referee for spelling out the mechanics of our model in these words.

media. The novelty of our paper is the transmission mechanism for this effect, which is via the influence on market price, rather than some kind of deterrence effects,[6] which would be well understood.[7] As such, this mechanism is more general than our application to free drugs programs suggests. It applies to any program or any opening of a new market that selects participants from a related market such that the costs of firms in that related market are affected. If these firms are – at least imperfectly – competitive, these cost changes will be passed through to buyers and affect their behavior.[8]

In the literature on rational addiction, initiated by Becker and Murphy (1988), it is argued that, by making it easier for drug users to exit drugs, more novice drug users might be attracted.[9] We do not argue along these lines. We also abstract from issues warranting a merit goods argument, such as users underestimating their probability of getting addicted or users buying low-quality drugs, which have been argued as a concern in the context of markets for illegal substances. Our approach is complementary to these. Instead of concentrating on the demand side, we rather focus on the cost of supplying the market for illegal drugs. In our model, entry of new addicts into the drugs market does not rely on the addicts' calculated (or miscalculated) decision to become addicted, but is driven by a reduction in the dealers' distribution costs, which will be passed through to users as a reduction of the price for drugs.

The paper is structured as follows. First we outline our model. Second we characterize the equilibria of the model and show conditions for their existence. We then discuss the implications of our model on free drug programs and give policy implications and conclude the paper.

[6]See for instance Becker and Murphy (1988) and the following literature on rational addiction.

[7]We would like to thank an anonymous referee for pointing out that insight in these words.

[8]An example from outside the health industry would be the impact that the emergence of a sizable venture capital industry had on conventional financing of innovative projects. Venture capital firms have been shown to have an advantage in selecting good R&D projects over industry investors and banks. If the population of innovative projects seeking financing is constant over time, the VC industry's emergence must have affected the average quality of projects financed by other investors negatively, adding to their costs, and reducing the number of projects financed through these investors by more than just the projects that are now financed through venture capital.

[9]See, e.g., Stevenson (1994) or, for a recent survey, MacDonald (2004).

2 The Model

Free drug programs target severe addicts that engage in criminal activities to finance their drug habits. The European Monitoring Centre for Drugs and Drug Addiction (2007, p.2) describes this category of users as follows: All offenders in this category support their addiction by some form of illegal income. This illegal income can come from both consensual crimes such as drug selling or prostitution (where criminalized), and acquisitive crimes such as shoplifting, robbery and burglary. The level of criminal activity is determined by the type and pattern of substance use, socioeconomic situation and extent of deviant lifestyle. Some users therefore will always rely on criminal activities to raise funds while others may try to regulate their drug consumption according to their financial resources and drug prices, or attempt to increase their legitimate income (e.g. social benefits, employment or pawning goods).

Based on these observations we build our model as follows. Consider the segment of the market for drugs which is populated by drug addicts. Assume that a typical addicted user i in this market has a valuation of V_i for one unit of drugs; where V_i is independently and identically distributed on $\left[\underline{V}, \overline{V}\right]$[10]. As long as this valuation exceeds the expected cost of buying the unit of drugs, the user will buy it, otherwise he will not buy. There are two types of drug users, characterized by the uncertainty of their income stream: High risk users (H) and low-risk users (L). Neither type of addicts is able to pay their drugs on the spot if they do not engage in criminal activity. However, users of type H have to engage in criminal activity at any point in time to finance their drug habit whereas users of type L would only need to do so if they had to pay their drugs on the spot and not if they were allowed deferred payment. The initial probability that a user is of type H is given by $\lambda \in]0, 1[$. A user's type is her private information and assumed to be independent of her valuation.[11]

Depending on the realization of their future income, drug users may need to default on an agreed deferred payment. If a user defaults, we assume the drug dealer may inflict a punishment to the user, e.g., by requiring her to deal or smuggle drugs for the dealer, in

[10]We choose the valuations to vary over users in order to allow for entry of users into the market if the cost of acquiring drugs falls.

[11]One might assume that valuations and probability to repay are negatively correlated. In this case, low-valuation – low-risk individuals will enter the market as a result of the introduction of free drug programs and our results will be reinforced.

order to repay (part of) the value owed. A deal $\gamma = (P, R, T)$ between the dealer and the user is characterized by punishment P in case the user cannot meet the payment obligation, a monetary payment R, and point in time of the payment $T \in \{I, D\}$, where I stands for immediate payment and D for deferred payment. The probability of repayment of the (H) $-$type and the (L) $-$type in case of a deferred payment is θ_H and θ_L, respectively, with $\theta_L > \theta_H > 0$.[12] [13] We assume that punishment P of the user has value δP to the dealer, with $\delta \in (0, 1)$, i.e., punishment is less valuable to the drug dealers than it is costly to the drug users. The marginal cost of drugs is c and there are n dealers in the market competing in deals specifying P, R, and T. Users accept the best possible deal. If a user is indifferent between two deals we assume, as a tie-breaking rule, that she chooses the one with the lower P.

We assume that users who engage in criminal activity receive a disutility $0 < \zeta \ll \underline{V}$ from engaging in criminal activity. Therefore, if a user can avoid getting involved in criminal activity, keeping everything else constant, she would strictly prefer to do so. As $H-$type users cannot avoid to become criminal, ceteris paribus, they are indifferent between accepting a deal with a deferred payment of R and an immediate payment of $\theta_H R$. In this case, we assume they choose the deal with immediate, but lower repayment.

As her type is a user's private information, dealers are unable to distinguish the users of different risk directly. They can do so only indirectly, designing different deals (γ_H, γ_L) such that users sort themselves. Summarizing, the expected utility of drug user of type $t \in \{L, H\}$ with valuation V_i from a contract with deferred payment is given by $U_{iH} = V_i - \theta_H R_H - (1 - \theta_H) P_H - \zeta$ and $U_{iL} = V_i - \theta_L R_L - (1 - \theta_L) P_L$ for the two types, respectively; and that for immediate payment by $U_{it} = V_i - R_t - \zeta$. The corresponding expected profit of the dealer is given by $\Pi_t = \theta_t R_t + (1 - \theta_t) \delta P_t - c$ for a contract with deferred payment and by $\Pi_t = R_t - c$ for a contract with immediate payment.

Finally, we assume dealers and users to honor their deals. A rationale for this might come from underlying reputation mechanisms: Each side might loose their ability to deal with anyone in the future if they renege on their deals.

[12] This distinction captures the fact that some users have to take greater risks to finance their drug habit, e.g. to engage in criminal activity with uncertain return.

[13] For a description of the illegal drug market see Donohue III and Levitt (1998) and Levitt and Venkatesh (2000).

3 Analysis

In this section, we will derive symmetric equilibria in pure and mixed strategies for the game just laid out. Any equilibrium in this game has to fulfill three conditions, which entail (1) dealers to make zero profits for each type; (2) incentive compatibility (or self-selection); and (3) that no dealer can offer another deal on which he obtains a strictly positive profit.

Pure strategy equilibria There are two possible types of pure strategy equilibria, separating and pooling equilibria. In a separating equilibrium, dealers offer deals γ_H^* for $H-$type users and γ_L^* for $L-$type users with $\gamma_H^* \neq \gamma_L^*$. Incentive compatibility, thus, implies that $\forall s \neq t$, with $s, t \in \{H, L\}$, $U_t(\gamma_t^*) \geq U_t(\gamma_s^*)$. We derive the two separating equilibrium candidates,

$$\begin{aligned}
\gamma_H^S &= \left(P_H^S, R_H^S, T_H^S\right) = (0, c, I), \\
\gamma_L^S &= \left(P_L^S, R_L^S, T_L^S\right) = \left(\frac{(\theta_L - \theta_H)\, c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, \frac{c\,(1-\delta) - c\,(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, D\right)
\end{aligned}$$

and

$$\begin{aligned}
\gamma_H^S &= \left(P_H^S, R_H^S, T_H^S\right) = \left(0, \frac{c}{\theta_H}, D\right), \\
\gamma_L^S &= \left(P_L^S, R_L^S, T_L^S\right) = \left(\frac{(\theta_L - \theta_H)\, c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, \frac{c\,(1-\delta) - c\,(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, D\right)
\end{aligned}$$

in the appendix. The corresponding expected utilities in both these separating equilibria are $U_H\left(\gamma_H^S\right) = V_i - c - \zeta$ and $U_L\left(\gamma_L^S\right) = V_i - \theta_L \frac{c(1-\delta)-c(\theta_H-\delta\theta_L)}{\theta_L-\delta\theta_H-\theta_H\theta_L+\delta\theta_H\theta_L} - (1-\theta_L)\frac{(\theta_L-\theta_H)c}{\theta_L-\delta\theta_H-\theta_H\theta_L+\delta\theta_H\theta_L}$. Note that there are two separating equilibrium candidates, both of which entail the same deal for $L-$types. Even though they are different for the high-risk users, one of them entails immediate payment and the other one deferred payment, both the dealers and the $H-$users are indifferent between the two deals.

In a pooling equilibrium dealers offer deals γ_H^* for $H-$type users and γ_L^* for $L-$type users with $\gamma_H^* = \gamma_L^* = \gamma$, which implies that any candidate fulfills self-selection trivially. We derive the only pooling equilibrium candidate,

$$\overline{\gamma} = \left(\overline{P}, \overline{R}, \overline{T}\right) = \left(0, \frac{c}{\lambda\theta_H + (1-\lambda)\,\theta_L}, D\right),$$

in the appendix. The user's expected utility in the pooling equilibrium candidate is $U_{iL}\left(\overline{\gamma}\right) = V_i - \theta_L \frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}$ and $U_{iH}\left(\overline{\gamma}\right) = V_i - \theta_H \frac{c}{\lambda\theta_H+(1-\lambda)\theta_L} - \zeta$, respectively.

We show in the appendix that a pooling equilibrium does not exist. We also show that a profitable deviation by the dealers from the separating equilibrium candidate to the pooling equilibrium candidate is possible if and only if $\lambda < \frac{\theta_L-\delta\theta_L-\theta_L^2+\delta\theta_L^2}{2\theta_L-\theta_H-\delta\theta_L-\theta_L^2+\delta\theta_L^2}$. In this case, no pure strategy equilibrium exists.

Figure 1 illustrates the equilibrium candidates of the model. For now, restrict attention to the pure-strategy equilibrium candidates $\left(\gamma_H^S, \gamma_L^S\right)$ and $\overline{\gamma}$. They are located on the corresponding zero-profit lines for deals offered to $H-$users, to $L-$users, and to both types of users in an eventual pooling deal, Π_H^0, Π_L^0, and $\overline{\Pi}^0$, respectively. Arrows indicate direction of increasing profits. The $H-$users' and $L-$users' indifference curves at utility levels achieved in γ_H^S and γ_L^S are given by I_H^S and I_L^S, respectively. Once more, the arrows indicate the direction of increasing utilities. In the separating equilibrium candidate, self selection is fulfilled as each type of user is better off by accepting the deal designed for them than by accepting the deal designed for the other type. The pooling equilibrium candidate, $\overline{\gamma}$, fulfills the zero-profit condition, as it is on $\overline{\Pi}^0$. Self-selection would be fulfilled trivially as dealers would only offer one deal to both type of users.

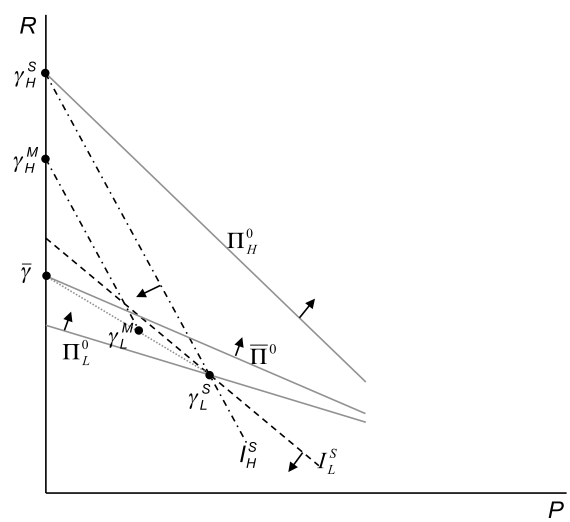

Figure 1: Equilibrium candidates of the model. $\left(\gamma_H^S, \gamma_L^S\right)$ is the pure-strategy separating equilibrium candidate, $\overline{\gamma}$ is the pure-strategy pooling equilibrium candidate, and $\left(\gamma_L^M, \gamma_H^M\right)$ is a typical pair of deals played in the mixed strategy.

To determine the existence of a pure-strategy separating equilibrium, the location of $\overline{\gamma}$ with respect to the intersection of the I_L^S line and the R axis is crucial. The separating equilibrium candidate will be broken by a dealer offering $\overline{\gamma}$ if both types of users receive higher a utility in $\overline{\gamma}$ than in their respective γ_H^S and γ_L^S. This is the case if $\overline{\gamma}$ involves a payment $\overline{R}$ that is smaller than the R in the intersection of the I_L^S line and the R axis, as in Figure 1. Given that $\frac{d\overline{R}}{d\lambda} > 0$, a free drug program that eliminates selectively sufficiently many high-risk users from the population dealers deal with, will eventually reduce $\overline{R}$ to this point.

Reducing the proportion of high risk users increases the expected profits of a dealer from a given deal that pools types. Competition drives dealers to pass on these gains to users, increasing their expected utility. Eventually the proportion of bad types in the population becomes so low that also the low risk types are better off accepting this pooling deal.

Given the slopes of the users' indifference curves, the pooling equilibrium candidate, however, is not an equilibrium either: It is always possible to offer a deal that is between the zero-profit lines for $H-$users and the pooling zero-profit line, which only $H-$users accept if the pooling deal is still offered. As we will derive next, in this case, a mixed-strategy equilibrium exists, which leaves all users with a higher expected utility than the pure-strategy separating equilibrium.

Mixed strategy equilibria We have shown that a pure strategy market equilibrium fails to exist if and only if $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. Several authors have shown that in markets with asymmetric information and no pure strategy equilibrium, a mixed strategy equilibrium may exist (see e.g. Dasgupta and Maskin, 1986; Rosenthal and Weiss, 1984). We follow Rosenthal and Weiss (1984) in our derivation of the symmetric mixed strategy equilibrium in our model. The conditions for the mixed strategy equilibrium are the same as for the pure strategy equilibria.

The strategies played in the mixture are pairs of deals, which look like "linear combinations" of the candidates for pure strategy equilibria (separating and pooling). A typical deal for the $H-$type user, $\gamma_H^M = \left(0, R_H^M, D\right)$, lies on the line between γ_H^S and $\overline{\gamma}$. The corresponding deal for the $L-$type user, $\gamma_L^M = \left(P_L^M, R_L^M, D\right)$, lies on the the line between γ_L^S and $\overline{\gamma}$ such that the $H-$type is indifferent between γ_H^M and γ_L^M as shown in Figure 1.

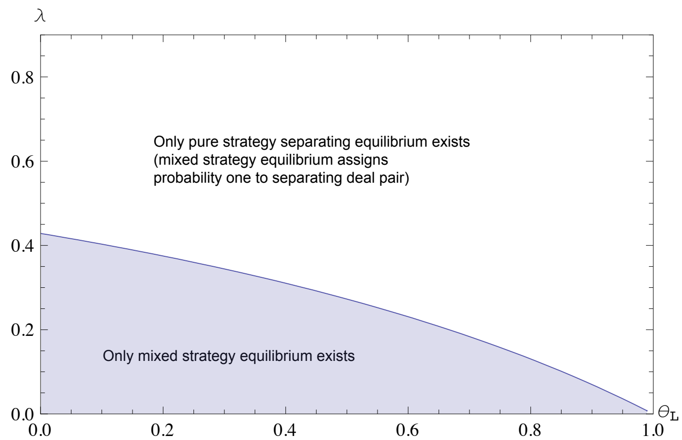

Figure 2: Equilibria for $\frac{\theta_H}{\theta_L} = 0.6$ and $\delta = 0.7$

The symmetric mixed strategy can now be identified with a cumulative distribution function (cdf) F defined on the variable R_H^M in the interval $[\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}, \frac{c}{\theta_H}]$. Define $r_L := \frac{1-\theta_L}{\theta_L}$ and $r_H := \frac{1-\theta_H}{\theta_H}$. Then a symmetric mixed-strategy equilibrium results if all n dealers play independently $\left(\gamma_L^M, \gamma_H^M\right) = \left(\left(0, R_H^M, D\right), \left(P_L^M, R_L^M, D\right)\right)$ according to the cdf

$$F\left(R_H^M\right) = \begin{cases} 0 & \text{for } R_H^M < \frac{c}{\lambda\theta_H+(1-\lambda)\theta_L} \\ 1 - \psi\left(c - \theta_H R_H^M\right)^{\frac{-\phi}{(n-1)\lambda\left(r_L - r_H\right)}} & \text{for } \frac{c}{\lambda\theta_H+(1-\lambda)\theta_L} \leq R_H^M < \frac{c}{\theta_H} \\ 1 & \text{for } \frac{c}{\theta_H} \leq R_H^M \end{cases}$$

where

$$\phi = \frac{1}{\theta_H}\left((1-\lambda)\left(\theta_L - \frac{\delta\left(1-\theta_L\right)^2}{\theta_L}\right) + \lambda\left(\frac{\theta_H\left(1-\theta_L\right)}{\theta_L} - \left(1-\theta_H\right)\right)\right),$$

$$\psi = \left(c - \theta_H \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L}\right)^{\frac{\phi}{(n-1)\lambda\left(r_L - r_H\right)}}.$$

It exists if and only if $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. See Appendix for the derivation of the mixed strategy equilibrium and the proof of this existence condition.

Proposition 1 *The pure-strategy separating equilibrium* $\left(\gamma_L^S, \gamma_L^H\right)$ *exists if and only if* $\lambda \geq \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$*. The mixed-strategy equilibrium characterized by* $\left(\gamma_L^M, \gamma_H^M\right)$ *and* $F\left(R_H^M\right)$ *exists if and only if* $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$*. There is no pooling equilibrium.*

Define the lowest λ, for which the pure-strategy separating equilibrium exists, as $\underline{\lambda} := \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. For arbitrarily chosen values of $\frac{\theta_H}{\theta_L} = 0.6$ and $\delta = 0.7$, Figure 2 plots $\underline{\lambda}$

as a function of θ_L. It visualizes that for $\lambda < \underline{\lambda}$, only the mixed strategy equilbrium exists, but neither of the pure strategy equilibria. If however $\lambda \geq \underline{\lambda}$, the pure-strategy separating equilibrium exists and the mixed strategy equilibrium we derived only exists in its degenerate form, assigning probability one on the pair of deals offered in the separating equilibrium.

Supply-side driven entry of new drug users due to free drug programs Free drug programs target severely affected addicts. Severely affected users are very likely to be those who need to finance their habit with drug-related crime at any point in time and are, thus, highly risky to deal with. In terms of our model, this implies that free drugs programs reduce the share of high-risk users, λ. In order to fully assess the impact of these programs, we thus need to take into account the effect that this change in the user population has on the utility derived by each user.

First note that the utility the users derive in the pure-strategy separating equilibrium does not depend on λ. That implies that, as long as this equilibrium exists, a change in the composition of the drug user population induced by a free-drug-program does not affect the utility of the users, and thus, does not lead to supply-side driven entry of new users into the market. Second, observe that in the mixed strategy equilibrium, users have a higher expected utility than in the pure-strategy separating equilibrium. As every pair $\left(\gamma_L^M, \gamma_H^M\right)$ played in the mixed strategy equilibrium is a linear combination of $\overline{\gamma}$ and γ_i^S and as $U_i\left(\overline{\gamma}\right) > U_i\left(\gamma_i^S\right)$ for the mixed strategy equilibrium to exist, the expected utility of every user in the mixed strategy equilibrium is higher than in the separating equilibrium.

Proposition 2 *The expected utility the users derive in the pure-strategy separating equilibrium does not depend on λ.*

Proposition 3 *The expected utility of every user in the mixed strategy equilibrium is higher than in the separating equilibrium.*

Taken together, propositions 1 - 3 imply the following for a free drug program.

Proposition 4 *If and only if a free drugs program leads to a reduction of the share of high-risk users to $\lambda < \underline{\lambda}$, it leads to supply-side driven entry of new users into the drugs market.*

Comparative statics It can be shown that $\frac{d\underline{\lambda}}{d\delta} = \frac{\left(\theta_L^2-\theta_L\right)(\theta_L-\theta_H)}{\left(2\theta_L-\theta_H-\delta\theta_L-\theta_L^2+\delta\theta_L^2\right)^2} < 0$. This means that the more valuable the mode of punishment of defaulting users is to the dealers, the smaller is the lowest λ, for which the pure-strategy separating equilibrium exists, and thus the bigger is the range of λ for which free drugs programs do not lead to supply-side driven entry of new users into the market. Similarly, it can be shown that $\frac{d\underline{\lambda}}{d\theta_H} = \frac{(1-\delta)\left(\theta_L-\theta_L^2\right)}{\left(2\theta_L-\theta_H-\delta\theta_L-\theta_L^2+\delta\theta_L^2\right)^2} > 0$ and $\frac{d\underline{\lambda}}{d\theta_L} = \frac{(1-\delta)\left(-\theta_H+2\theta_H\theta_L-\theta_L^2\right)}{\left(2\theta_L-\theta_H-\delta\theta_L-\theta_L^2+\delta\theta_L^2\right)^2} < 0$. In Figure 3, we plot $\underline{\lambda}$ as a function of θ_L for δ (bottom to top) and $\frac{\theta_H}{\theta_L}$ (left to right) values of 0.1, 0.5, and 0.9.

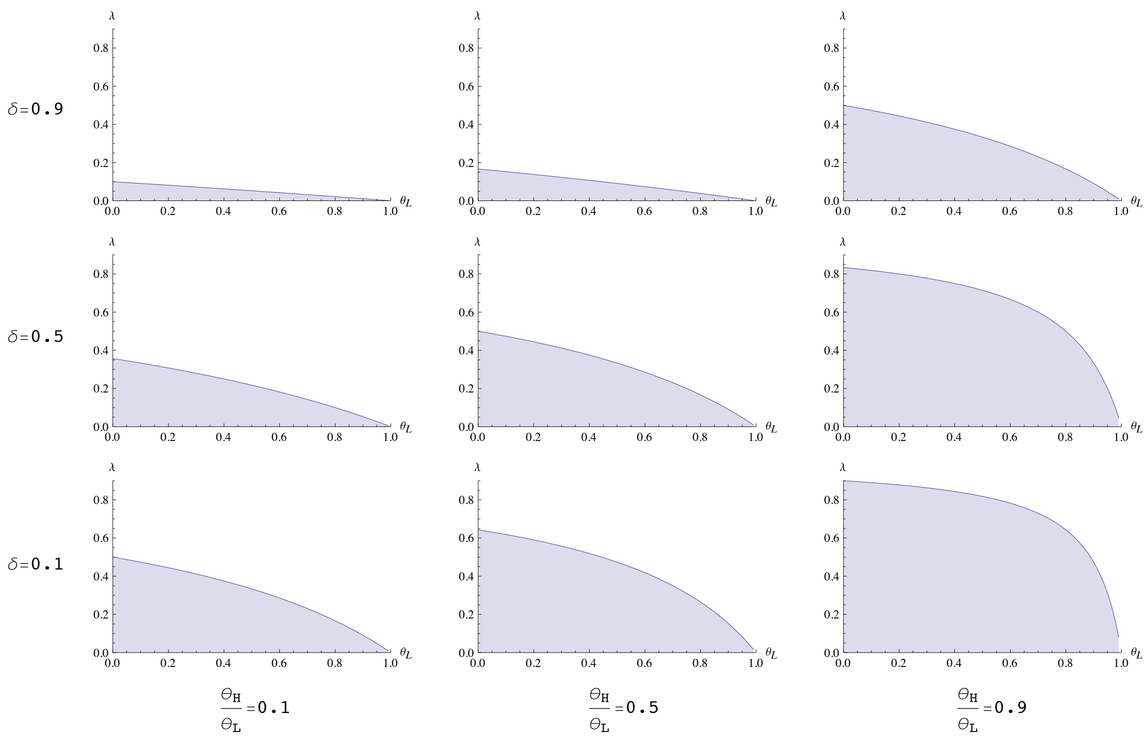

Figure 3: In the shaded areas, only the mixed-strategy equilibrium exists

These comparative statics imply that the higher θ_L, the lower θ_H, and the higher δ, the less likely does a free drugs program lead to supply-side driven entry of new users into the market.

Proposition 5 *$\frac{d\underline{\lambda}}{d\delta} < 0$, $\frac{d\underline{\lambda}}{d\theta_H} > 0$, and $\frac{d\underline{\lambda}}{d\theta_L} < 0$. Free-drug programs are, thus, more likely to lead to supply-side driven entry of new users into the market, the lower the value of punishment to dealers, δ, and the more different high-risk and low-risk users are in terms of* *their repayment abilities, θ_H and θ_L, respectively.*

The cheaper it is to distinguish types because (i) types are sufficiently different – θ_L high or θ_H low, ceteris paribus, – or because (ii) the punishment that is used creates high value to the dealers – δ high – the less frequently a free drugs program leads to a breakdown of the pure-strategy separating equilibrium and, as a consequence, the less frequently it leads to supply-side driven entry of new users.

Limit case If $\lambda \to 0$, there are only low-risk types in the population left. In this case, competition will (i) drive out costly punishment – it wastes resources – and (ii) imply a payment of $R = \frac{c}{\theta_L}$ with probability one. For continuity reasons, this implies that, in the approach of this limit case, the probability weight put on lower payments R_H^M, and thus on deals $\left(\gamma_L^M, \gamma_H^M\right)$ close to the pooling equilibrium deal $\overline{\gamma}$, is higher, the lower the λ. In addition, the lower λ, the lower the payment $\overline{R}$ in the pooling equilibrium deal. These two properties of the mixed-strategy equilibrium imply that the utilities from consuming drugs for both types of users increase in a falling λ. We can, thus, state the following.

Proposition 6 *The more effectively free-drug programs target high-risk users, the better off are all users, and thus, the more additional users will enter the drugs market.*

Robustness Three remarks on the robustness of our result are due. First, we have derived only one symmetric mixed-strategy equilibrium and shown that it is an equilibrium indeed. However, there may be more mixed-strategy equilibria, which we did not derive. Note that the pairs $\left(\gamma_L^M, \gamma_H^M\right)$ played in our mixed-strategy equilibrium have been derived using only the zero-profit and the self-selection conditions. These conditions must hold in any equilibrium; and, as consequence, our main result – that the expected utility of every user in the mixed strategy equilibrium is higher than in the separating equilibrium – must hold *in any mixed strategy equilibrium.* Second, in our model, we assume for the sake of simplicity perfectly competitive dealers. Our results, however, do not depend on this. As long as there is some (imperfect) competitive pressure, distribution cost savings will be passed on to users. Third, throughout this paper we have assumed that dealers and users honor their deals. A rationale might come from reputation mechanisms. Although it is interesting to study these mechanisms further, it is beyond the scope and purpose of this paper and we leave it for future research.

4 Discussion

Our study revealed a negative, supply-side driven effect of free drug programs on overall drug consumption. In an environment characterized by the outlined information asymmetries, free drug programs change the composition of drug users which, under specified conditions, lead to the non-existence of a pure strategy market equilibrium. If the proportion of low risk users gets sufficiently high, then neither a separating nor pooling equilibrium exists. In this instance, however, a symmetric mixed strategy equilibrium can be identified that leaves drug users with a higher utility level than in the pure strategy equilibrium. The higher level of utility is driven by a lowered effective price offered by drug dealers. The intuition of this result is as follows. By reducing the number of high risk users, dealing drugs becomes a less risky undertaking. This in turn will increase competition among dealers which will drive prices down. This supply-side driven effect is different to the already identified demand effect that causes prices to fall as a result of lower demand (see e.g. Clarke, 2003).

Empirical evidence suggests that heroin demands are very price elastic[14] so a reduction in effective price will encourage more people to consume drugs. Advocates of free drug programs need to be aware of this negative effect as ignoring it leads to an overestimation of the positive impact of these programs. The main message of our paper therefore is, that free drug initiatives should not exclusively focus on users that are in these programs (or that could be in them) but include the effects on potential users as well. The policy implications of this will be discussed in the final section.

5 Policy Implications

The key objectives of drug use prevention policies are how to (1) help users take control of their addiction; (2) reduce drug-related crime and (3) decrease overall drug consumption. Previous studies have shown that methadone programs meet objectives one and two.

[14] Demand for addicts is sometimes assumed to be inelastic. However, a highly elastic demand for "new" or "occasional" users implies an overall elasticity of demand for heroin (see Clarke, 2003; Saffer and Chaloupka, 1999).

Methadone satisfies a users desire for an opiate without producing a "rush" or "high" or the mental confusion associated with heroin. Although methadone is as addictive as heroin, the lack of a "high" makes it easier for users to overcome their addiction. Methadone programs also help to reduce drug-related crime. To finance drugs, high-risk users are more likely to commit crimes than new or low-risk users (Clarke, 2003). The provision of free heroin and methadone limits an addict's need to finance drugs.

Our analysis, however, shows that methadone programs may fail to deliver on the third objective by encouraging new users to enter the market or existing users to increase their level of consumption. This negative impact on overall drug consumption is amplified by policy makers that see methadone initiatives as substitutes for law enforcement measures. Reality is, that methadone programs rely on public funding which requires savings elsewhere – as a reduction of the number of high risk users lowers drug related crime, there is less need for police. We, however, suggest that methadone programs should be used as complements and not as substitutes to law enforcement measures. Free drug initiatives should lead to more police on the street not less.

Policy Implication 1 *Methadone programs should be used as complements and not as substitutes to law enforcement measures.*

Previous demand-side driven studies have already pointed towards the negative effect of methadone programs. Policy implications have been the introduction of legal penalties directed specifically at new users (Clarke, 2003). Our study, however, suggests under which conditions the utility increase of consumers will also be supply-side driven. When these conditions are met, in addition to targeting new or low-risk users, law enforcement measures should also be directed at drug dealers.

Policy Implication 2 *Methadone programs should be accompanied by law enforcement measures directed at drug dealers in order to avoid supply-side driven entry of new users into the drug market.*

Reducing the number of severely affected addicts lowers the risk of drug-dealing. Legal penalties and law enforcement measures need to compensate for this and make drug-dealing again a risky business.

6 Conclusion

In this paper, we analyze the effects and policy consequences of free drug programs within a screening model of the hard drug market in which dealers use payment and punishment options to screen between high and low risk users. We show that, if a free drug program selects suffciently many high risk drug users, the cost of distributing drugs is reduced. Due to competition of dealers, this will reduce the market price for drugs and attract new drug users. As a consequence, we recommend to treat free drug programs and law enforcement as complements. Targeting dealers with legal penalties and law enforcement must compensate for the reduction in the drug distribution costs due to free drug programs and ensure that drug-dealing stays a risky business.

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Appendix

A Pure strategies

A.1 Separating equilibrium candidate

Let us first concentrate on deals with deferred payments and introduce immediate payment deals later on. An equilibrium candidate with deferred payment $\left(\gamma_L^S, \gamma_H^S\right)$ needs to be on the zero profit lines for each deal, thus, $\Pi_t = \theta_t R_t^S + (1-\theta_t)\,\delta P_t^S - c = 0$. As punishment is wasteful, a dealer will offer punishment only to the $L-$type (the expected cost of punishment is lower for the $L-$type) and $P_H^S = 0$. It follows that $R_H^S = \frac{c}{\theta_H}$. Incentive compatibility requires to choose γ_L^S such that $U_H\left(\gamma_L^S\right) \leq U_H\left(\gamma_H^S\right)$, which is equivalent to $R_L^S \geq \frac{c}{\theta_H} -$

$\frac{1-\theta_H}{\theta_H} P_L^S$. Together with the zero profit requirement, this implies $P_L^S = \frac{(\theta_L - \theta_H)c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}$, and $R_L^S = \frac{c(1-\delta) - c(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}$.

Note now that both $H-$type users and dealers are indifferent between a deal with a deferred payment of R and an immediate payment of $\theta_H R$, whereas an $L-$type user is strictly better off with the deferred payment as she can hereby avoid to engage in criminal activity. Thus, we have derived the following result.[15]

Lemma 1 *Separating equilibrium candidates are* $\gamma_H^S = \left(P_H^S, R_H^S, T_H^S\right) = (0, c, I)$ *and* $\gamma_L^S = \left(P_L^S, R_L^S, T_L^S\right) = \left(\frac{(\theta_L - \theta_H)c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, \frac{c(1-\delta) - c(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, D\right)$*; and* $\gamma_H^S = \left(P_H^S, R_H^S, T_H^S\right) = \left(0, \frac{c}{\theta_H}, D\right)$ *and* $\gamma_L^S = \left(P_L^S, R_L^S, T_L^S\right) = \left(\frac{(\theta_L - \theta_H)c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, \frac{c(1-\delta) - c(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}, D\right)$*. The corresponding expected utilities in the separating equilibria are* $U_H\left(\gamma_H^S\right) = V_i - c - \zeta$ *and* $U_L\left(\gamma_L^S\right) = V_i - \theta_L \frac{c(1-\delta) - c(\theta_H - \delta\theta_L)}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L} - (1 - \theta_L) \frac{(\theta_L - \theta_H)c}{\theta_L - \delta\theta_H - \theta_H\theta_L + \delta\theta_H\theta_L}$.

Lemma 1 shows that there are two separating equilibrium candidates, both of which entail the same deal for $L-$types. Even though they are different for the high-risk users, one of them entails immediate payment and the other one deferred payment, both the dealers and the $H-$users are indifferent between the two deals.

A.2 Pooling equilibrium candidate

In a pooling equilibrium dealers offer deals γ_H^* for $H-$type users and γ_L^* for $L-$type users with $\gamma_H^* = \gamma_L^* = \gamma$. Also a pooling equilibrium candidate needs to satisfy the above outlined three conditions, namely self-selection (in pooling this is trivial), zero profits and impossibility to offer better deals. Let pooling equilibrium candidate values of our variables be indicated by upper bars. As punishment is inefficient, the only pooling equilibrium candidate is characterized by zero punishment and zero expected profit. Thus a pooling equilibrium candidate with deferred payment requires $\overline{\Pi} = \lambda\theta_H\overline{R} + (1-\lambda)\,\theta_L\overline{R} - c = 0$, which implies $\overline{R} = \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L}$ and $\overline{\gamma} = \left(\overline{P}, \overline{R}, \overline{T}\right) = \left(0, \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L}, D\right)$. Note that as $\theta_H < \theta_L$, the $H-$type users strictly prefer the pooled deal to a deal with immediate payment that lets the dealer break even. Therefore, a deal with immediate payment cannot be an equilibrium.

[15] This result relies on the tie-breaking rule specifying that if $H-$type users are presented to choose between accepting a deal with a deferred payment of R and an immediate payment of $\theta_H R$, everything else constant, they choose the deal with immediate, but lower repayment.

Note furthermore that $\overline{R}$ is increasing in λ, i.e., in the share of $H-$type users in the population of drug users. The user's expected utility in the pooling equilibrium candidate is $U_{iL}\left(\overline{\gamma}\right) = V_i - \theta_L \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L}$ and $U_{iH}\left(\overline{\gamma}\right) = V_i - \theta_H \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L} - \zeta$, respectively.

First note that a pooling equilibrium does not exist: As $\theta_H < \theta_L$, the indifference curve of the $H-$type user is steeper than that of the $L-$type user for any deal. Furthermore, the indifference curve of the $L-$type user is also steeper than the dealer's isoprofit curve as $\delta \in (0,1)$. Therefore, for any pooling deal, dealers can offer a profitable deal that only $L-$type users accept. Second, note that a deviation from the separating equilibrium candidate to the pooling candidate is possible if and only if $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. For this, both types have to be better off by accepting a deal that pools types. $H-$type users are always better off as the pooling candidate promises a lower repayment for zero punishment. $L-$type users are better off in the pooling candidate than in the pooling equilibrium candidate if $EU_L\left(\gamma_L^S\right) \leq EU_L\left(\overline{\gamma}\right)$, which simplifies to $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. Thus, if $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$, there is no pure-strategy equilibrium.[16]

B Mixed strategies

In the computation of the mixed strategies and the proof that there is not profitable deviation from the equilibrium in these mixed strategies, we borrow from Rosenthal and Weiss (1984).

B.1 Computation of $\left(P_L^M, R_L^M\right)$

A deal offered to a $H-$type user is of the form $\gamma_H^M = \left(0, R_H^M, D\right)$. For each $\gamma_H^M = \left(0, R_H^M, D\right)$, there is exactly one $\gamma_L^M = \left(P_L^M, R_L^M, D\right)$ for which $U_H\left(\gamma_H^M\right) = U_H\left(\gamma_L^M\right)$ and $\Pi = \lambda\Pi_H\left(\gamma_H^M\right) + (1-\lambda)\,\Pi_L\left(\gamma_L^M\right) = 0$. The slope of the line connecting γ^P and γ_L^S, b, is

$$b = -\frac{R^P - R_L^S}{P_L^S} = -\frac{\lambda\delta - \delta - \lambda + \lambda\theta_H + \delta\theta_L - \lambda\delta\theta_L}{\lambda\theta_L - \lambda\theta_H - \theta_L}.$$

Using this and $P_H^M = 0$, we can derive $\left(P_L^M, R_L^M\right)$ as a function of R_H^M, i.e.,

$$P_L^M = \frac{c\theta_H - R_H^M\theta_H\theta_L + \lambda R_H^M\theta_H\theta_L - \lambda R_H^M\theta_H^2}{\delta\theta_H - \theta_L + \lambda\theta_L - \lambda\delta\theta_H + \theta_H\theta_L - \lambda\theta_H\theta_L - \delta\theta_H\theta_L + \lambda\delta\theta_H\theta_L},$$

$$R_L^M = \frac{c\theta_H - c + \lambda R_H^M\theta_H + \delta R_H^M\theta_H - \lambda\delta R_H^M\theta_H - \delta R_H^M\theta_H\theta_L + \lambda\delta R_H^M\theta_H\theta_L - \lambda R_H^M\theta_H^2}{\delta\theta_H - \theta_L + \lambda\theta_L - \lambda\delta\theta_H + \theta_H\theta_L - \lambda\theta_H\theta_L - \delta\theta_H\theta_L + \lambda\delta\theta_H\theta_L}.$$

[16]See also Rothschild and Stiglitz (1976) for a proof.

B.2 Zero profit condition

To see that offering $\left(\gamma_H^M,\gamma_L^M\right)$ will always result in zero profits to a dealer when playing the mixed strategy consider the following: If $\left(\gamma_H^M,\gamma_L^M\right)$ offered by a specific dealer yields the highest utility to users, all drug users will buy from this dealer with probability one, generating a profit of

$$\Pi^M=\lambda\theta_H R_H^M+(1-\lambda)\left(\theta_L R_L^M+(1-\theta_L)\,\delta P_L^M\right)-c.$$

If a dealer is offering the pair $\left(\gamma_H^M,\gamma_L^M\right)$ that offer drug users the highest utility, he will get all drug users with probability one and have a profit of

$$\begin{aligned}
&\Pi^M\left(\gamma_L^M,\gamma_H^M\right)=\lambda\theta_H R_H^M\\
&+(1-\lambda)\,\theta_L\frac{c\theta_H-c+\lambda R_H^M\theta_H+\delta R_H^M\theta_H-\lambda\delta R_H^M\theta_H-\delta R_H^M\theta_H\theta_L+\lambda\delta R_H^M\theta_H\theta_L-\lambda R_H^M\theta_H^2}{\delta\theta_H-\theta_L+\lambda\theta_L-\lambda\delta\theta_H+\theta_H\theta_L-\lambda\theta_H\theta_L-\delta\theta_H\theta_L+\lambda\delta\theta_H\theta_L}\\
&+(1-\lambda)(1-\theta_L)\,\delta\frac{c\theta_H-R_H^M\theta_H\theta_L+\lambda R_H^M\theta_H\theta_L-\lambda R_H^M\theta_H^2}{\delta\theta_H-\theta_L+\lambda\theta_L-\lambda\delta\theta_H+\theta_H\theta_L-\lambda\theta_H\theta_L-\delta\theta_H\theta_L+\lambda\delta\theta_H\theta_L}-c=0.
\end{aligned}$$

If some other dealer offers deals that yield higher utility to users, then all users will buy from the other dealer and our dealer is left with zero profits.

B.3 Proof that F is a proper cdf

Define $r_L:=\frac{1-\theta_L}{\theta_L}$ and $r_H:=\frac{1-\theta_H}{\theta_H}$. Assume that the cdf that gives the probability of offering R_H^M below R, is

$$F\left(R_H^M\right)=\begin{cases}0 & \text{for } R_H^M<\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}\\ 1-\psi\left(c-\theta_H R_H^M\right)^{\frac{-\phi}{(n-1)\lambda(r_L-r_H)}} & \text{for } \frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}\le R_H^M<\frac{c}{\theta_H}\\ 1 & \text{for } \frac{c}{\theta_H}\le R_H^M\end{cases}$$

where

$$\phi=\frac{1}{\theta_H}\left((1-\lambda)\left(\theta_L-\frac{\delta(1-\theta_L)^2}{\theta_L}\right)+\lambda\left(\frac{\theta_H(1-\theta_L)}{\theta_L}-(1-\theta_H)\right)\right),$$

$$\psi=\left(c-\theta_H\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}\right)^{\frac{\phi}{(n-1)\lambda(r_L-r_H)}}.$$

Let us first check that the cdf is a proper cdf. Evaluating at $R_H^M=\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}$ gives

$$\begin{aligned}
F\left(\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}\right)&=1-\psi\left(c-\theta_H\frac{c}{\lambda\theta_H+(1-\lambda)\theta_L}\right)^{\frac{-\phi}{(n-1)\lambda(r_L-r_H)}}\\
&=1-1=0.
\end{aligned}$$

Evaluating at $R_H^M = \frac{c}{\theta_H}$ gives

$$F\left(\frac{c}{\theta_H}\right) = 1 - \psi\left(c - \theta_H \frac{c}{\theta_H}\right)^{\frac{-\phi}{(n-1)\lambda(r_L - r_H)}}$$
$$= 1 - 0 = 1.$$

Finally, $1 - F\left(R_H^M\right)$ gives us the probability of offering R_H^M above R. It is given by

$$1 - F\left(R_H^M\right) = \begin{cases} 1 & \text{for } R_H^M < \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L} \\ \psi\left(c - \theta_H R_H^M\right)^{\frac{-\phi}{(n-1)\lambda(r_L - r_H)}} & \text{for } \frac{c}{\lambda\theta_H + (1-\lambda)\theta_L} \leq R_H^M < \frac{c}{\theta_H} \\ 0 & \text{for } \frac{c}{\theta_H} \leq R_H^M \end{cases}.$$

The first derivative of $(1 - F(R))$ w.r.t. R_H^M is

$$\frac{d\left(1 - F\left(R_H^M\right)\right)}{dR_H^M} = \psi \frac{-\phi}{(n-1)\lambda(r_L - r_H)} \frac{\left(c - \theta_H R_H^M\right)^{\frac{\phi}{(n-1)\lambda(r_L - r_H)}}}{\left(c - \theta_H R_H^M\right)} (-\theta_H)$$
$$= \frac{-\phi}{(n-1)\lambda(r_L - r_H)} \frac{-\theta_H}{\left(c - \theta_H R_H^M\right)} \left(1 - F\left(R_H^M\right)\right).$$

As $\frac{d\left(1-F\left(R_H^M\right)\right)}{dR_H^M} < 0$, we have $\frac{dF\left(R_H^M\right)}{dR_H^M} > 0$ as long as $\lambda < \frac{\theta_L - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}{2\theta_L - \theta_H - \delta\theta_L - \theta_L^2 + \delta\theta_L^2}$. This is true as long as the pure strategy equilibrium does not exist. Therefore, F is a proper distribution function as long as the pure strategy equilibrium does not exist.

B.4 Proof that there is no profitable deviation

As in Rosenthal and Weiss (1984), "with $(n-1)$ players playing independently according to F, we need to find the set of best pure-strategy responses by the $n-$th player to the cdf F^{n-1}". As in their paper, we can restrict attention to pair of deals both of which lie in $PSRT$ and one of which lies in $PSRQ$ in figure 4. The reason is that any deal above QR attracts no $L-$type and earns non-positive profits and any deal below PSR is dominated by one on PSR. As in their paper, inside $PSRQ$, profits are non-negative for $L-$types and non-positive for $H-$types. We need to show that any pair of deals consisting of the deal $(\widehat{x} + t, \widehat{y} - r_L t)$, with $(\widehat{x}, \widehat{y})$ on PR, and its most profitable companion deal for $H-$types $(0, \widehat{y} - r_L t + (\widehat{x} + t) r_H)$ cannot earn positive profits when played against F^{n-1}.

The expected payoff from deviating to that pair of deals is

$$H(t) = (1 - F(\widehat{y} - r_L t + (\widehat{x} + t) r_H))^{n-1} \lambda (\theta_H (\widehat{y} - r_L t + r_H (\widehat{x} + t)) - c)$$
$$+ (1 - F(\widehat{y} + r_H \widehat{x}))^{n-1} (1 - \lambda) (\theta_L (\widehat{x} + t) + (1 - \theta_L) \delta (\widehat{y} - r_L t) - c).$$

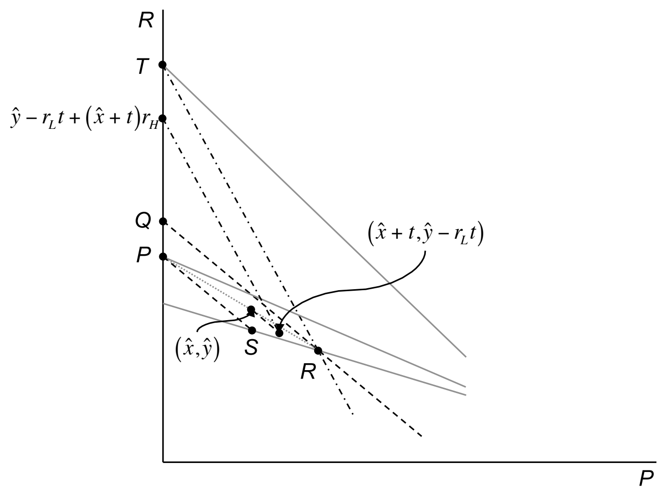

Figure 4: Best pure-strategy response to $(\widehat{x}, \widehat{y})$

Note that for $\frac{c}{\lambda\theta_H + (1-\lambda)\theta_L} \leq R_H^M < \frac{c}{\theta_H}$ the first derivative of $\left(1 - F\left(R_H^M\right)\right)^{n-1}$ w.r.t. R_H^M is

$$\begin{aligned}\frac{d\left(\left(1 - F\left(R_H^M\right)\right)^{n-1}\right)}{dR_H^M} &= (n-1)\frac{\left(1 - F\left(R_H^M\right)\right)^{n-1}}{1 - F\left(R_H^M\right)}\frac{d\left(1 - F\left(R_H^M\right)\right)}{dR_H^M} \\ &= \frac{-\phi}{\lambda\left(r_L - r_H\right)}\frac{-\theta_H}{\left(c - \theta_H R_H^M\right)}\left(1 - F\left(R_H^M\right)\right)^{n-1} > 0\end{aligned}$$

and

$$\frac{d^2\left(\left(1 - F\left(R_H^M\right)\right)^{n-1}\right)}{d\left(R_H^M\right)^2} = \frac{-\theta_H\left(\lambda\left(r_L - r_H\right) - \phi\right)}{\lambda\left(r_L - r_H\right)\left(c - \theta_H R_H^M\right)}\frac{d\left(\left(1 - F\left(R_H^M\right)\right)^{n-1}\right)}{dR_H^M}.$$

The marginal expected payoff is given by

$$\begin{aligned}\frac{dH(t)}{dt} = &-\lambda\theta_H\left(r_L - r_H\right)\left(1 - F\left(\widehat{y} - r_L t + \left(\widehat{x} + t\right) r_H\right)\right)^{n-1} \\ &- (1-\lambda)\left(\theta_L - \left(1 - \theta_L\right)\delta r_L\right)\left(1 - F\left(\widehat{y} + r_H\widehat{x}\right)\right)^{n-1} \\ &- \lambda\left(\theta_H\left(\widehat{y} - r_L t + r_H\left(\widehat{x} + t\right)\right) - c\right) \\ &\times \frac{d}{d\left(\widehat{y} - r_L t + \left(\widehat{x} + t\right) r_H\right)}\left(\left(1 - F\left(\widehat{y} - r_L t + \left(\widehat{x} + t\right) r_H\right)\right)^{n-1}\right)\left(r_L - r_H\right).\end{aligned}$$

Evaluating at $t = 0$, we can simplify this to

$$\frac{dH(0)}{dt} = 0.$$

Next, it can be shown that

$$\frac{d^2}{dt}H(t) = 2\lambda\theta_H (r_L - r_H)^2 \frac{d}{d(\widehat{y} - r_L t + (\widehat{x}+t) r_H)} (1 - F(\widehat{y} - r_L t + (\widehat{x}+t) r_H))^{n-1}$$
$$+ \lambda (\theta_H (\widehat{y} - r_L t + r_H (\widehat{x}+t)) - c) (r_L - r_H)^2$$
$$\times \frac{d^2}{d(\widehat{y} - r_L t + (\widehat{x}+t) r_H)^2} (1 - F(\widehat{y} - r_L t + (\widehat{x}+t) r_H))^{n-1} < 0.$$

Therefore, H is concave and has a maximum at $t = 0$. Thus, the best deviation from $(\widehat{x}, \widehat{y})$ on PR along an $L-$type indifference curve does not increase H and, thus, it does not pay to deviate from the mixed strategy equilibrium.