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Abstract

We study the effect of tariffs in a one-sector small open economy that imports oil. We find that (1) the model may exhibit local indeterminacy and sunspots when tariff rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures (or lump-sum tansfers); and (2) indeterminacy disappears if the government finances endogenous public spending and transfers with fixed tariff rates. Under the first type of balanced budget formulation, we provide numerical (calibration) examples to illustate that the government shouldn't distort the oil price paid by firms with tariffs in order to avoid aggregate instability. Under the second type of balanced budget formulation, we prove that the economy exhibits equilibrium uniqueness, regardless of the existence of lump-sum transfers.

Key Words : Indeterminacy, Endogenous Tariff Rate, Small Open Economy, Balanced-budget Rule

JEL Classification Number: Q43, F41

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1. Introduction

It is well understood by now that a standard neoclassical growth model with factor income taxes, as in Schmitt-Grohe and Uribe (1997, henceforth SGU), may possess an indeterminate steady state and thus a continuum of perfect foresight equilibria.[1] Unlike one-sector models with sufficiently strong productive externalities, as in Benhabib and Farmer (1994) and Farmer and Guo (1994), the upward-sloping (equilibrium) labor demand curve needed for indeterminacy is due to the presence of fiscal increasing returns caused by endogenous labor income tax rates. Specifically, Guo and Harrison (2004, henceforth GH) show that SGU's indeterminacy result depends on a balanced-budget requirement whereby the tax rate decreases with the output and indeterminacy disappears once the goverment finances endogenous public spending and transfers with fixed income tax rates. One common feature in those models is that self-fulfilling beliefs of agents can be an independent shock to endogenous business cycles.[2] In open economies, tariff as a tax on trade acts like a factor income tax and we ask whether it generates indeterminacy in a similar way to factor income tax.

To address this question, we first briefly summarize the literature on trade taxes. It has been known that the government can raise revenue by using the tariff instruments.[3] Many authors, such as Atolia (2006) and Leung (1999), point out that in an open economy model, public investment or spending can be financed by a tariff (and income tax). Ramsey's (1927) analysis suggests that tariff rates can be endogenized. Two examples are Loewy (2004) and Mourmouras (1991) who use endogenous tariffs in a two-country open economy endogenous growth model and a small open economy OLG model respectively.

[1] In a one-sector neoclassical growth model with factor income taxes, SGU show that the model can exhibit indeterminacy if taxes rates are endogenously determined by a balanced budget rule with a pre-set level of government expenditures (or lump-sum transfers).

[2] See Benhabib and Farmer (1999) for an excellent survey of the literature. In this paper, I use the terms "indeterminacy", "sunspots" and "self-fulfilling beliefs" interchangeably.

[3] The revenue motive behind the imposition of trade taxes is well documented. See, Kindleberger and Lindert (1978, p. 143), and Riezman and Slemrod (1987).

In this paper, we introduce government tariff policy into a one-sector small open economy that imports energy (say, oil) and extend Schmitt-Grohe and Uribe's analysis by considering different balanced-budget rules whereby government expenditures and/or transfers can be financed by the energy tariff revenue.[4] In particular, we consider a countercyclical (flat) tariff policy whereby constant (endogenous) government expenditures (or lump-sum transfers) are financed by endogenous (exogenous) tariff rates.[5] It turns out that a countercyclical tariff policy is needed to generate indeterminacy in this model while a flat tariff policy can make the economy immune to indeterminacy regardless of the existence of lump-sum transfers. All of these results suggest that tariffs generate indeterminacy in a similar way to factor income taxes.[6] Moreover, under the balanced-budget rule with a countercyclical tariff policy, for empirically plausible values of steady state tariff rates (or energy taxes), we provide calibration examples to illustate that the government shouldn't distort the energy (oil) price paid by firms with tariffs in order to avoid aggregate instability.

The intuition for how regressive tariffs generate indeterminacy in the one-sector small open economy is tantamount to understanding how fiscal increasing returns arise. Suppose the proverbial representative agent expects future tariff rates to increase. This implies, for any given stock of capital, future imports of foreign inputs and the marginal product of capital will be lower. It will lower the current demand for foreign inputs, thus leading to a fall in total output. If the tariff rate is regressive with respect to the output (under the balanced budget rule with a pre-set level of govern-

[4] For simplicity, we assume that the government doesn't impose consumption taxes on the tradable goods or factor income taxes on the production factors. The tariff revenue in this model can also be interpreted as oil tax revenue. Hence the implication of this model is not limited to open economies with trade taxes, which means that the main result in this paper also applies to domestic energy taxes. The foreign input can also be interpreted as non-reproducible natural resources extracted domestically.

[5] Under the balanced budget rule with a constant level of government expenditures (or lump-sum transfers), tariff rates are endogenously determined since the government is forced to lower the tariff rates as total output (or tax base) rises. This means that the tariff rates are countercyclical.

[6] In the two sector model without "fiscal increasing returns" induced by factor income taxes, Bond, Wang and Yip (1996) and Meng and Velasco (2003) prove that distortionary factor taxation nonetheless causes indeterminacy in a closed-economy, endogenous growth model and a small open RBC model respectively. Does the "channel equivalence" between factor income taxes and tariffs to generate indeterminacy still hold in a small open economy two sector model? This is one issue which deserves further research.

ment expenditures (or lump-sum transfers)), the tariff rate today will increase, thus validating the agent's initial expectations. By contrast, the above mechanism for indeterminacy is eliminated under the balanced budget rule that consists of fixed tariff rates and endogenous public spending. More precisely, constant tariff rates together with diminishing marginal products of productive inputs will reduce the higher anticipated returns from belief-driven labor and investment spurts, thus making sunspot fluctuations less likely to occur.

It turns out that in our benchmark model, under the assumption that government expenditures are constant and labor is indivisible, we can explicitly derive the necessary and sufficient condition for the balanced budget rule to generate indeterminacy. It requires that the steady state tariff rate be greater than the share ratio of capital and imported factors in production and less than the share ratio of labor and imported factors in production. This result suggests that, in general, we should either impose some restrictions on the government ability to adjust tariff rates or reduce the level of tariff rates levied on the imported factors in order to avoid aggregate instability. We consider the current high tariff rate which is prevailing in European countries (especially in year 2002). Some countries like Denmark and Netherlands, whose economies depend on the imported exhaustible natural resources, can be easily pushed into destabilization.[7] We use the Aguiar-Conraria and Wen's (2005, 2007, and 2008, henceforth ACW) estimation of the imported energy share in the two countries and find that the high tariff rate on oil in the EU leads the two countries into destabilization. Similarly, the energy taxes which the EU countries have tried to impose recently also bring the potential dangers of destabilization into those countries whose economies depend on non-reproducible resources. Those countries like Denmark and Netherlands should pay close attention to the control of energy taxes in order to stabilize the economy. As an optimal import tariff, energy taxes seem to be very high in these two countries (see Newbery (2005)).

[7] Although throughout the paper, we analyze the model for the developed countries, the result also holds for the less-developed countries which productions are dependent on the imported factors.

Similar issues have been explored by Miguel and Manzano (2006) in a small open economy that imports oil. Unlike this paper, they assume that the government finances an exogenous flow of public spending by using consumption and oil taxes (or tariffs) and by issuing debt. Since the tariff rates are exogenous in their model, indeterminacy can not arise. The main finding in their paper is that the government should not distort the oil price paid by firms with taxes, even when consumption of oil is considered and the government distinguishes between the taxes paid by the households and the firms. That is due to the fact that the optimal tax on intermediate goods (such as oil) should be zero in order to maintain aggregate production efficiency. This paper confirms their point of view from another perspective. That is, in order to avoid aggregate instability caused by endogenous tariffs, the government should not distort the energy price.

To the best of our knowledge, the papers that study indeterminacy in the open economy are Lahiri (2001), Weder (2001), Meng and Velasco (2004) and ACW (2005, 2007 and 2008). Unlike this paper, Lahiri, Weder, and Meng and Velasco are concerned with multiple equilibria in the economy that uses capital and labor as inputs. Only ACW extend the Benhabib-Farmer model to an open economy by introducing imported foreign factors as a third production input. But their model relies on decreasing marginal costs (or increasing returns), as in Lahiri (2001) and Weder (2001), to generate indeterminacy. Although Meng and Velasco analyze the effects of distortionary factor taxation in generating indeterminacy, none of them explores the mechanism for indeterminacy due to the presence of fiscal increasing returns.

The remainder of this paper is organized as follows. In section 2, we consider a standard neoclassical growth model that incorporates foreign energy as a third production factor. The energy price is assumed to be set in the world markets and taken as given. In this setup, we prove our key results: (1) the model may exhibit local indeterminacy and sunspots when tariff rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures (or lump-sum transfers); and (2) indeterminacy disappears if the government finances endogenous public spending and transfers with fixed tariff rates. In section 3, we compare this model with Benhabib and Farmer, SGU and ACW models and find that (1) the indeterminacy condition in our model has a close correspondence with the one obtained in the increasing returns model of Benhabib and Farmer (1994); (2) if the imported factor is mainly a labor substitute, indeterminacy may not easily arise; and (3) the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. In section 4, we discuss the robustness of our indeterminacy result and in section 5, we conclude the paper.

2. An Economy with Tariffs

This paper incorporates two different formulations of the government budget constraints into a standard neoclassical growth model that incorporates foreign energy as a third production factor. We assume that labor is indivisible (as in Hansen (1985)) and the only source of government revenue is a tariff. In particular, the balanced-budget rule consists of exogenous (and/or endogenous) government purchases (and/or transfers), and endogenous (and/or exogenous) tariff rates levied on the imported input.

2.1. Firms

We introduce government tariff policy into the continuous time framework of ACW without productive externalities.[8] There is a continuum of identical competitive firms with the total number normalized to one. The single good is produced by the representative firm with constant returns to scale Cobb-Douglas production function

[8]Without productive externalities, the model of ACW is identical to the standard neoclassical growth model that imports oil as a third productive input.

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0}, \tag{1}$$

where y_t is total output, k_t is the aggregate stock of capital, n_t is the aggregate labor supply, $a_k + a_n + a_0 = 1$ and the third factor in the production, say oil (o_t), is imported.[9] Perfect competition in factor and product markets implies that factor demands are given by

$$w_t = a_n \frac{y_t}{n_t}, \tag{2}$$

$$r_t + \delta = a_k \frac{y_t}{k_t}, \tag{3}$$

and

$$p^o(1 + \tau_t) = a_0 \frac{y_t}{o_t}, \tag{4}$$

where $(r_t + \delta)$ denotes the user cost of renting capital, w_t denotes the real wage, p^o denotes the real price of oil (the imported goods) and τ_t is the tariff rate levied on the imported oil and uniform to all firms.[10] Here we should emphasize that (1) p^o is the relative price of the foreign input in terms of the single good, which is the numeraire and tradable; and (2) the variable τ_t represents the endogenous (or exogenous) tariff rate levied on the foreign input and we require that $\tau_t \geq 0$ to rule out the existence of import subsidies.[11]

Since we assume that the economy is open to importing energy (oil), the agent can use the tradable good to buy the foreign input. The energy price is assumed to be exogenous and the foreign input is assumed to be perfectly elastically supplied.[12] These imply that the energy price, p^o, is

[9] The third inputs can be non-reproducible natural resources.

[10] $\delta \in (0, 1)$ denotes the depreciation rate of capital and r_t the rental rate of capital.

[11] If tariff rates are exogenous, $\tau_t = \tau$ holds for all t.

[12] The model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models have been used widely to study the business-cycle effects of oil price shocks. This literature includes Finn (2000), Rotemberg and Woodford (1996), Wei (2003), and ACW (2005, 2007, and 2008).

independent of the factor demand for o_t. Hence by substituting out o_t in the production function using $o_t = a_0 \frac{y_t}{p^o(1+\tau_t)}$, we can obtain the following reduced-form production function

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{5}$$

Here the term $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ acts as the "technology coefficient" in a neoclassical growth model, which is inversely related to the foreign factor price and τ_t. In this reduced-form production function, the "effective returns to scale" is measured by

$$\frac{a_k + a_n}{1 - a_0} = 1. \tag{6}$$

2.2. Households

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time and maximizes the intertemporal utility function

$$\int_0^\infty e^{-\rho t}(\log c_t - bn_t)dt, \; b > 0, \tag{7}$$

where c_t and n_t are the individual household's consumption and hours worked, and $\rho \in (0, 1)$ is the subjective discount rate. We assume that there are no intrinsic uncertainties present in the model.

The budget constraint of the representative agent is given by

$$\dot{k}_t = r_t k_t + w_t n_t - c_t + T_t, \; k_0 > 0 \text{ given}, \tag{8}$$

where $\dot{k}_t$ denotes net investment and $T_t \geq 0$ is the lump-sum transfers.

The first order conditions for the household's problem are

$$\frac{1}{c_t} = \Lambda_t, \tag{9}$$

$$b = \Lambda_t w_t, \tag{10}$$

$$\dot{\Lambda}_t = (\rho - r_t)\Lambda_t, \tag{11}$$

where Λ_t denotes the marginal utility of income.

2.3. Government

The government chooses the tariff/transfer policy $\{\tau_t, T_t\}$, and balances its budget in each period. At each point in time, the budget constraint of the government can be stated as follows

$$p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1 + \tau_t)} = G_t + T_t, \tag{12}$$

where $G_t \geq 0$ represents government expenditures. Finally, market clearing requires that aggregate demand equal aggregate supply

$$c_t + G_t + \dot{k}_t + \delta k_t + o_t p^o = y_t. \tag{13}$$

Note that the international trade balance is always zero. Foreigners are paid in goods. This is clear in the above equation, according to which domestic production is divided between consumption, investment, imports and government expenditures ($c_t + i_t + p^o o_t + G_t = y_t$, $i_t = \dot{k}_t + \delta k_t$). So part of what is produced domestically is used to pay for the imports. This is the interpretation of Finn (2000), Wei (2003) and ACW (2005 and 2008).

2.4. Analysis of the model dynamics

As in GH (2004), we assume that tariff revenues can be either consumed by the government (i.e. $G_t \geq 0$ for all t) or returned to households as transfers (i.e., $T_t \geq 0$, for all t). It is easy to verify that the economy in which the government finances endogenous public spending and/or transfers with fixed tariff rates is immune to indeterminacy. That is due to the following proposition.

Proposition 1. *If the tariff rate is exogenous, production doesn't exhibit increasing returns to scale since A_t term is a constant for all t. (In this case, government expenditures are endogenous under the balanced budget rule.) Therefore, the economy exhibits saddle path stability, regardless of the existence of lump-sum transfers.*

GH prove that under perfect competition and constant returns-to-scale, if the government finances endogenous public spending and transfers with fixed income tax rates, a one-sector real business cycle model exhibits determinacy, regardless of the existence of lump-sum transfers. In this model, we have the same result. Once we fix the tariff rate (or oil tax rate) like Miguel and Manzano (2006), the model doesn't display increasing returns to scale, so indeterminacy cannot arise.

To remain comparable with SGU's analysis, we focus on the cases where the government either consumes all tariff revenues (i.e., $T_t = 0$) or transfers the revenue to the household in a lump-sum way (i.e., $G_t = 0$). In the following sections, we mainly discuss the case where $T_t = 0$ holds for all t. (In section 2.6, we extend the basic model to consider the case where $G_t = 0$ holds for all t.) Under this specific assumption $T_t = 0$, we replace the consumption with $\frac{1}{\Lambda_t}$ and transform wage rate and rental rate into functions of capital and labor, then the equilibrium conditions can be reduced to the following five equations

$$b = \Lambda_t a_n A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1}, \tag{14}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - a_k A_t k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}}, \tag{15}$$

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau_t})y_t - \delta k_t - \frac{1}{\Lambda_t} - G_t, \tag{16}$$

$$G_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}, \tag{17}$$

and

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{18}$$

First we claim that for a given tariff rate (i.e., $\tau_t = \tau$, for all t), the dynamical system possesses a unique interior steady state.

Lemma 1. *The dynamical system possesses a unique interior steady state when the government consumes all tariff revenues and the tariff rate is exogenous, i.e., $\tau_t = \tau$, for all t. (In this case, $A_t(\tau_t) = A(\tau)$ holds for all t.)*

Proof. To find such a steady state, set $\dot{\Lambda}_t$ in (15) equal to zero. We can solve the capital/labor ratio in the steady state, which is not independent of the tariff rate. $(\frac{k}{n})_{ss} = (\frac{\rho+\delta}{a_k A(\tau)})^{\frac{1-a_0}{-a_n}}$ is unique for the given tariff rate. Equation (14) can be solved for a unique and positive value of Λ in the steady state, i.e., $\Lambda_{ss} = \frac{b}{a_n A(\tau)}(\frac{\rho+\delta}{a_k A(\tau)})^{a_k/a_n}$. Using this value of Λ, the government budget constraint (17), and the fact that in the steady state $\dot{k}_t = 0$, we can write the market-clearing condition (16) as

$$(1-a_0)k_{ss}[A(\tau)(\frac{k}{n})_{ss}^{\frac{-a_n}{1-a_0}} - \delta] = \frac{a_n A(\tau)}{b}[\frac{\rho+\delta}{a_k A(\tau)}]^{-a_k/a_n}. \tag{(kss)}$$

Since $(\frac{k}{n})_{ss}$ is known given the tariff rate, we can find that k_{ss} (the steady state value of the capital stock) is unique and positive. Because both the capital stock and the capital/labor ratio are positive and unique in the steady state, n_{ss} (the steady state value of the labor supply) is also positive and unique. Finally, the steady state level of government purchases given by (17) is also unique and can be written as

$$G_{ss} = \frac{\tau}{1+\tau} a_0 A(\tau) k_{ss} (\frac{k}{n})_{ss}^{-\frac{a_n}{1-a_0}}, \tag{(g)}$$

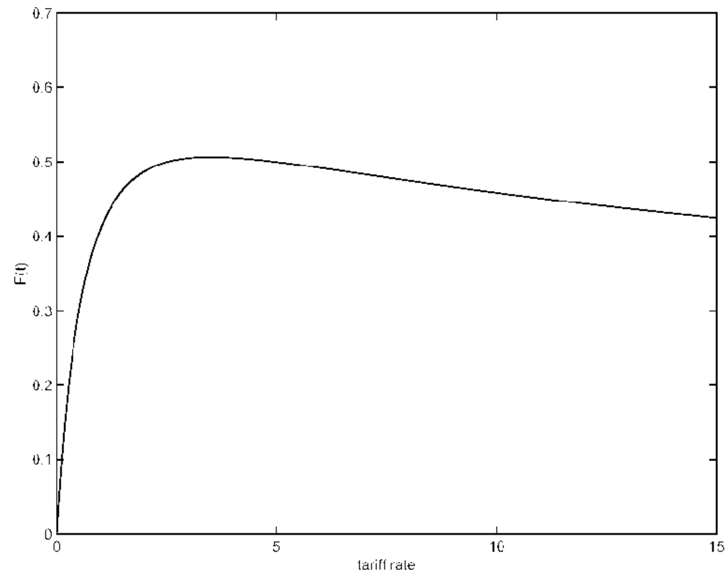
where k_{ss} is the solution to (kss) equation. It is clear that G_{ss} is continuous in τ.

It follows from (kss) and (g) that when τ is equal to zero, G_{ss} is also equal to zero because k_{ss} is in this case positive and finite. If the tariff rate is exogenous, we can prove that there exists a unique tariff rate that maximizes G_{ss}. It is $\tau_m = \frac{a_n}{a_0}$. ■

Secondly, for a given level of government expenditures, there is a (steady-state) Laffer curve-type relationship between the tariff rate and tariff revenue, which means that the number of steady state tariff rates that generate enough revenue to finance the pre-set level of government purchases will be general either 0 or 2. We prove it in the following lemma.

Lemma 2. *When tariff rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures, the steady state in the dynamical system which consists of (14)-(18) may exist and the number of steady state tariff rates (τ_{ss}) that generate enough revenue to finance the pre-set level of government purchases will be general either 0 or 2. If there are two steady states in the model, we only focus on the steady state associated with the low steady state tariff rate since the steady state associated with the high steady state tariff rate is always determinate.*

Proof. We derive the steady state values of these variables $\frac{k}{n} = (\frac{\rho+\delta}{a_k A(\tau_{ss})})^{\frac{1-a_0}{-a_n}}$, $\Lambda = \frac{b}{a_n A(\tau_{ss})}(\frac{\rho+\delta}{a_k A(\tau_{ss})})^{\frac{a_k}{a_n}}$ and $k = \frac{\frac{a_n A(\tau_{ss})}{b}(\frac{\rho+\delta}{a_k A(\tau_{ss})})^{-\frac{a_k}{a_n}}}{[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$, where $A(\tau_{ss})$ denotes the steady state value of A_t as τ_t is equal to its steady state value τ_{ss}. We also find that in the steady state, $G = \frac{\tau_{ss}}{(1+\tau_{ss})^{\frac{a_n+a_0}{a_n}}}$constant$= F(\tau_{ss})$



holds and the constant is $\frac{(\frac{a_0}{p^o})^{\frac{a_0}{a_n}} a_0(\rho+\delta)a_n(\frac{\rho+\delta}{a_k})^{-\frac{a_k}{a_n}}}{a_k b[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$. It is clear that $F(\tau_{ss})$ is non-monotone and the number of positive steady state tariff rates that generate enough revenue to finance a pre-set level of government purchases will be general either 0 or 2.

The second interesting finding is that if τ is exogenous as in the above lemma, $\frac{\partial G_{ss}}{\partial \tau} = 0$ implies that there exists a unique exogenous tariff rate that maximizes G_{ss}. Its value is $\frac{a_n}{a_0}$. This is due to the fact that G_{ss} is equal to $\frac{\tau}{(1+\tau)^{\frac{a_n+a_0}{a_n}}}$constant. ■

In the third step, we show that when the government expenditures are exogenous, the tariff rate is countercyclical with respect to the tax base or the output under the balanced budget rule. The following proposition is the key to indeterminacy in this model.

Proposition 2. *If the government expenditures are exogenous, the tariff rate is regressive with respect to the tax base ($p^o o_t$), or the output under the balanced budget rule, i.e.* $\frac{\partial \tau_t}{\partial y_t} < 0$.[13] *The regressive (countercyclical) tariff rate ($\frac{\partial \tau_t}{\partial y_t} < 0$) can induce increasing returns to scale with respect*

[13] *This relation doesn't violate the evidence of a negative relationship between tariffs and growth, especially among the world's richest countries like those in EU, which is documented by Dejong and Ripoll (2005).*

to capital and labor.

Proof. $p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)} = G$ implies that $\frac{\partial \tau_t}{\partial y_t} < 0$. Considering the log-linearization of the following equations around the steady state $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$, $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, and $y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}$, it is easy to verify that $\hat{y_t} = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k_t} + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n_t}$.[14] This means that production exhibits increasing returns to scale with respect to capital and labor, i.e., $\frac{a_k+a_n}{1-a_0(1+\tau_{ss})} > 1$. Thus, an endogenous tariff rate could be a source of fiscal increasing returns. ■

GH illustrate that under perfect competition and constant returns-to-scale, Schmitt-Grohé and Uribe's indeterminacy result depends on a balanced-budget requirement whereby the tax rate decreases with the household's taxable income. In this model, we get a similar result that requires the countercyclical rate to generate indeterminacy.[15]

To facilitate the analysis of the model dynamics, we consider the log linear approximation of the equilibrium conditions around the steady state. Let λ_t, $\hat{k_t}$, $\hat{\tau_t}$ and $\hat{n_t}$ denote the log deviations of Λ_t, k_t, τ_t and n_t from their respective steady states. The log linearized equilibrium conditions then are

$$0 = \lambda_t - \frac{\tau_{ss}\hat{\tau_t}}{\frac{1-a_0}{a_0}(1+\tau_{ss})} + \frac{a_k}{1-a_0}(\hat{k_t} - \hat{n_t}), \tag{19}$$

$$\dot{\lambda_t} = (\rho+\delta)[\frac{a_n}{1-a_0}(\hat{k_t} - \hat{n_t}) + \frac{\tau_{ss}\hat{\tau_t}}{\frac{1-a_0}{a_0}(1+\tau_{ss})}], \tag{20}$$

[14] Log-linearizing equation $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ around the steady state, we have $(G - a_0 y_{ss})\hat{\tau_t} = a_0 y_{ss}\hat{y_t}$ and $G = \frac{\tau_{ss} a_0 y_{ss}}{(1+\tau_{ss})}$, where $\hat{\tau_t}$ and $\hat{y_t}$ denote the log deviations of τ_t and y_t from their respective steady states (i.e., τ_{ss} and y_{ss}). Combining these two equations yields $\hat{y_t} = -\frac{1}{1+\tau_{ss}}\hat{\tau_t}$. $A_t = (\frac{a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ implies that $\hat{A_t} = -\frac{\tau_{ss}\hat{\tau_t}}{\frac{1-a_0}{a_0}(1+\tau_{ss})}$. Log-linearizing the production function implies that $\hat{y_t} = \hat{A_t} + \frac{a_k}{1-a_0}\hat{k_t} + \frac{a_n}{1-a_0}\hat{n_t}$. It is straightforward to see that $\hat{y_t} = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k_t} + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n_t}$.

[15] We think that the progressive tariff rate may make the economy against the sunspots in ACW model.

$$\dot{\hat{k}}_t = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta]\hat{k}_t + \frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}\hat{n}_t + [-\delta + \frac{(1-a_0)}{a_k}(\rho+\delta)]\lambda_t, \qquad (21)$$

and

$$\hat{y}_t = -\frac{1}{1+\tau_{ss}}\hat{\tau}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t. \qquad (22)$$

Combining (19) and (22) yields

$$\hat{n}_t = \frac{\lambda_t}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}} + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\hat{k}_t.$$

Using this expression to eliminate the $\hat{n}_t$in (20) and (21) results in the following system of differential equations

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{\hat{k}}_t \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \begin{bmatrix} \lambda_t \\ \hat{k}_t \end{bmatrix} \text{ and } J = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}, \qquad (23)$$

where $J_{11} = -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}}$, $J_{22} = (\rho+\delta)\frac{1-a_0}{a_k-a_0\tau_{ss}} - \delta$, $J_{12} = (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k-a_0\tau_{ss}}$ and $J_{21} = (\rho+\delta)\frac{(1-a_0)}{a_k}\frac{1-(\tau_{ss}+1)a_0}{a_k-a_0\tau_{ss}} - \delta$. We can then compute the Jacobian matrix of the dynamical system (23) evaluated at the steady state. The trace and the determinant of the Jacobian are stated as follows

$$trace(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta, \qquad (24)$$

$$\det(J) = \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}\{\delta(a_n - a_0\tau_{ss}) - \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}[a_n(1-a_0) - a_0\tau_{ss}\frac{1-a_0}{a_k}(1-a_0(1+\tau_{ss}))]\}. \quad (25)$$

Proposition 3. *The necessary and sufficient condition for the indeterminacy of the equilibrium is* $J_{11} + J_{22} = trace(J) < 0 < J_{22}J_{11} - J_{12}J_{21} = \det(J)$, *or,* $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$.

Notice that since the dynamical system contains one predetermined variable, k_t, the equilibrium is indeterminate if and only if both eigenvalues of the Jacobian matrix have negative real parts. It is equivalent to requiring that the determinant be positive and the trace negative. It is easy to verify that, $trace(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta < 0$ if and only if $\tau_{ss} > \frac{a_k}{a_0}$. If the trace condition is satisfied, the term $\frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}$ on the right side of the determinant is negative. $det(J) > 0$ if and only if $G(\tau_{ss}) = [\frac{(\rho+\delta)a_0^2(1-a_0)}{a_k} - \delta a_0^2]\tau_{ss}^2 - \tau_{ss}[\frac{(\rho+\delta)a_0(1-a_0)^2}{a_k} - \delta a_0(1-a_0)] + [(\rho+\delta)a_n(1-a_0) - \delta a_n a_k] < 0$. It is easy to show $G(\frac{a_k}{a_0}) = 0$ and $G(0) > 0$. Then the necessary and sufficient condition for the equilibrium indeterminacy is equivalent to $G < 0$, or, $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$ where $\tau^* = \frac{[(\rho+\delta)a_n(1-a_0) - \delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0) - \delta a_0 a_k]} = \frac{a_n}{a_0} > \frac{a_k}{a_0}$.

A sufficient condition for the set of tariff rates satisfying the necessary and sufficient condition to be nonempty is that the labor share is larger than the capital share (i.e., $a_n > a_k$). For steady-state tariff rates smaller than $\frac{a_k}{a_0}$ or greater than $\frac{a_n}{a_0}$, the determinant of J is negative and therefore the equilibrium is locally determinate. It should be emphasized that SGU show that the revenue maximizing tax rate is the least upper bound of the set of taxes rate for which the rational expectations equilibrium is indeterminate, this property also holds in our case.

The intuition behind the indeterminacy result is quite straightforward. Suppose that agents expect future tariff rates to increase. This implies that, for any given capital stock, future oil imports and the rate of return on capital will be lower (the latter is due to the fact that the marginal product of capital is increasing in the oil input). The decrease in the expected rate of return on capital, in turn, lowers the current oil demand, leading the current output decrease. Since the tariff rate is countercyclical $\frac{\partial \tau_t}{\partial y_t} < 0$, budget balance can cause the current tariff rate to increase, thus validating agents' initial expectations. (For certain choices of the parameter values, namely those satisfying $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$, the expectation of an above steady state tariff rate in the next period leads to an increase in tariff rates today that is larger than the one expected for next period.)

To help understand the intuition, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

$$\frac{c_{t+1}}{c_t} = \beta(1 - \delta + a_k \frac{y_{t+1}}{k_{t+1}}) = \beta[1 - \delta + (1 + \tau_{t+1})^{-\frac{a_0}{1-a_0}} r_{t+1}^{bt}], \tag{26}$$

where β denotes the discount factor, $r_{t+1}^{bt} = a_k (\frac{a_0}{p_0})^{\frac{a_0}{1-a_0}} k_{t+1}^{\frac{a_k}{1-a_0}-1} n_{t+1}^{\frac{a_n}{1-a_0}}$ the before-tariff return on capital and τ_{t+1} the tariff rate in period $(t+1)$. Households' optimistic expectations that lead to higher investment raise the left hand side of this equation, but result in a lower before-tariff return on capital r_{t+1}^{bt} due to the diminishing marginal products. The countercyclical tariff rate can increase the right hand side of the equation, thus validating the initial optimistic expectations. If the tariff rate is a constant under the balanced budget rule with endogenous public spending and/or transfers, the right–hand side of (26) falls. As a consequence, indeterminacy cannot arise in this specification, regardless of the details of the government's tariff policies.

Capital accumulation is crucial in generating indeterminacy in this economy. One can easily show that in the absence of capital accumulation, the equilibrium is determinate. The reason is that without capital, (14) becomes $b/\Lambda_t = a_n (\frac{a_0}{p^o(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, (16) becomes $1/\Lambda_t = (1 - \frac{a_0}{1+\tau_t}) y_t - G$, and (17) becomes $G/n_t = \tau_t (\frac{1}{p^o})^{\frac{a_0}{1-a_0}} (\frac{a_0}{1+\tau_t})^{\frac{1}{a_n}}$. These three equations yield locally unique solutions for τ_t, n_t, and Λ_t.

2.5. Calibrated Examples

In this section, following SGU (1997) and ACW (2005), we calibrate the model using structural parameters that are found in the real business cycle literature. The values used are 0.7 for the labor share (a_n), 0.21 for the enery share (a_o), 0.09 for the capital share (a_k), 0.1 for the annual depreciation rate (δ) and 0.04 for the annual real interest rate (ρ).[16] Assuming that the steady state tariff rate in the country for oil import is 0.6 (τ_{ss}), it is easy to find that the two roots of the Jacobian matrix are ($-0.2250 \pm 1.5714i$), which indicates that indeterminacy in this case arises. The high tariff rate (0.6) is estimated using the result of Newbery (2005) and consistent with the empirical data in those EU countries (especially in year 2002).[17]

Next, we consider the energy tax policy that is implemented in most of European countries. As the optimal tariff argument, the energy taxes are relatively high in some European countries.[18] For instance, oil is heavily taxed in Denmark, the effective tax rate on domestic fuels exceeds 0.8. We calibrate the model's structural parameters following SGU (1997) and ACW (2005). Namely, we set the time period in the model to be one year, as in SGU (1997, section III), the annual real interest rate $\rho = 0.04$, the capital share in Denmark $a_k = 0.1$, the labor share in Denmark $a_n = 0.7$, the annual depreciation rate $\delta = 0.1$. The steady state oil tax rate ($\tau_{ss} = 0.8$) in Denmark falls inside the range of values for which the equilibrium is indeterminate.[19]

In a short note, Chen and Zhang (2008) introduce intrinsic uncertainty in the form of exogenous productivity and government purchases shocks into the discrete time version of this model and

[16] In this numerical calibration example, following ACW (2005), we set a_0 to be 0.21 which is the cost share of foreign inputs in domestic production in Netherlands. a_n is 0.7 which is the cost share of labor input in domestic production in Netherlands. Other parameter values are taken from SGU (1997, section III).

[17] $\frac{\text{import tariff}}{\text{import price}} = \frac{15.6\$/bbl}{26\$/bbl} = 0.6$ is the optimal tariff rate of oil from Newbery (2005, section 3.1). One issue should be pointed out. As in SGU, if the only source of government revenues is a tax on consumption, the model will exhibit local determinacy when consumption tax rates are endogenously determined by a balanced-budget rule with a constant level of government revenues.

[18] The energy tax revenue is overwhelmingly oil tax revenue in some EU countries, see Newbery (2005).

[19] The parameter values of a_k and a_n are taken from ACW (2005). The values of δ and ρ are taken from SGU (1997). The steady state tax rate is from Newbery (2005). Indeterminacy arises since two roots are $0.1667 \pm 1.1213i$.

investigate the propagation mechanism of sunspot and fundamental shocks under various assumptions about their correlation. Following SGU (1997, section IV), we calibrate the model and compute the impulse responses assuming that the time unit is a quarter, the steady state tariff rate is 0.6, the serial correlation of two fundamental shocks is 0.9, the discount rate is 0.99, the depreciation rate is 0.025, and the remaining parameters take the same values as in the first numerical case. For this calibration of the model, we find that (1) under indeterminacy, the impulse responses of tariff rates, output, and hours to sunspot, technology and government purchases shocks are hump-shaped and highly persistent; and (2) neither the first-order serial correlations, the contemporaneous correlations with output, nor the standard deviation relative to output of tariffs, output, hours, and consumption is affected by the relative volatility of the sunspot shock or its correlation with the fundamental shock. Therefore it validates the equivalence between factor income taxes and tariffs, in the sense that they share similar propagation mechanisms of sunspot and fundamental shocks.

2.6. Extending the model

In this section, we extend the basic model to consider the case where $G_t = 0$ and $T_t =$constant hold for all t. We demonstrate that considering a constant level of lump-sum transfers does not alter my main result, and a similar condition to that obtained in proposition 3 on the endogenous tariff rates is all that is needed for indeterminacy.

Assume that the government collects the revenue and makes a lump-sum transfer to the agent. Considering the (new) equilibrium conditions for the optimization problem, we note that (1) Eqs. (14)–(15) as well as (18) remain unchanged; and (2) Eqs. (16) and (17) have to be slightly modified:

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau_t})y_t - \delta k_t - \frac{1}{\Lambda_t}, \tag{16'}$$

$$T = \frac{\tau_t a_0 y_t}{(1+\tau_t)}. \tag{17'}$$

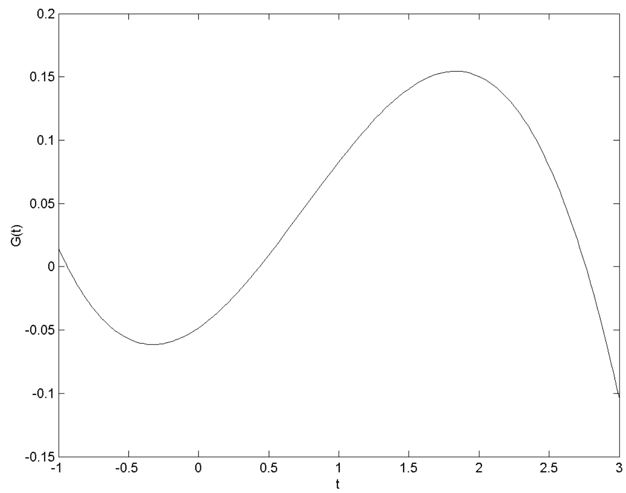
As in lemma 2 and proposition 2, we can easily prove that (1) the steady state in the dynamical system which consists of (14), (15), (16'), (17') and (18) may exist and the number of steady state tariff rates that generate enough revenue to finance a given level of lump-sum transfers will be general either 0 or 2; and (2) if the lump-sum transfers are exogenous, the tariff rate is regressive with respect to the tax base ($p^o o_t$), or the output under the balanced budget rule. Using the same method as given in the above sections, we can derive the Jacobian matrix J. In this case, four elements in J can be stated as follows $J_{11} = -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}}$, $J_{22} = (\rho+\delta)\frac{1-a_0}{a_k - a_0\tau_{ss}} - \delta$, $J_{12} = (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k - a_0\tau_{ss}}$ and $J_{21} = \frac{(\rho+\delta)}{a_k}\frac{(1-a_0)^2+\tau_{ss}[(1-a_0)^2 - a_n a_0] - \tau_{ss}^2 a_0}{(a_k - a_0\tau_{ss})(\tau_{ss}+1)} - \delta$. The necessary and sufficient condition for the model to exhibit indeterminacy is shown in the following proposition.

Proposition 4. *The equilibrium is indeterminate if and only if $\tau_2 < \tau_{ss} < \tau_3$ holds where $\frac{a_k}{a_0} < \tau_2 < \tau_3$. τ_2 and τ_3 are two positive roots of the polynomial $G(\tau_{ss}) = \Delta_1\tau_{ss}^3 + \Delta_2\tau_{ss}^2 + \Delta_3\tau_{ss} + \Delta_4$, where $\Delta_1 = a_0^2[\delta - \frac{(\rho+\delta)}{a_k}] < 0$, $\Delta_2 = -\delta a_0 a_n + a_0\{\frac{(\rho+\delta)[(1-a_0)^2 - a_n a_0]}{a_k} + \delta a_0 - \delta a_k\}$, $\Delta_3 = -\delta a_0 a_n + [-(\rho+\delta)a_n(1-a_0) + \delta a_n a_k] + a_0\frac{(\rho+\delta)}{a_k}(1-a_0)^2 - \delta a_k$, and $\Delta_4 = -(\rho+\delta)a_n(1-a_0) + \delta a_n a_k$.*[20]

In this case, we cannot explicitly derive the necessary and sufficient condition for the balanced budget rule to generate indeterminacy. That is because relaxing the assumptions of public spending will make the determinant of the Jacobian matrix become more complicated, up to a third order polynomial. But our indeterminacy result is robust to this extension. Here we provide a numerical example to validate it.

Example 1. *The purpose of this example is to illustrate the main result of the proposition–that indeterminacy in fact occurs with the empirical tariff rate–by one numerical experiment. We adopt*

[20] *See Zhang (2008b) for a detailed discussion of this proposition.*



the parameter values as in the first numerical case, i.e., $a_0 = 0.21$, $a_n = 0.7$, $a_k = 0.09$, $\delta = 0.1$, *and* $\rho = 0.04$. *We draw the graph of* $G(t)$ *for the numerical experiment and find that the negative root of* $G(\tau_{ss})$ *is* $\tau_1 = -0.9196$, $\frac{a_k}{a_0} = 0.4286 < \tau_2 = 0.4440$, *and* $\tau_3 = 2.7138$. *As* $\tau_2 < \tau_{ss} = 0.6 < \tau_3$, *indeterminacy arises.*

3. Comparison with Benhabib and Farmer, SGU and ACW Models

In this section, we first show that there exists a close correspondence between the indeterminacy condition of the model with endogenous tariff rates and constant government purchases presented in this paper and that of the increasing returns model of Benhabib and Farmer (1994). That is, the necessary condition for local indeterminacy is that the "equilibrium labor demand schedule" can be upward sloping and steeper than the labor supply schedule. Unlike the model of Benhabib and Farmer (1994), this model doesn't rely on increasing returns in production to make the "equilibrium labor demand schedule" upward sloping. In fact, the equilibrium labor demand schedule in our model is upward sloping because increases in the aggregate employment are accompanied by decreases in the tariff rate and increases in the after-tariff return on labor. The after-tariff labor demand function can be stated as follows (in log deviations from the steady state)

$$\hat{w}_t = \frac{a_k}{1-a_0}\hat{k}_t - \frac{a_k}{1-a_0}\hat{n}_t - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau}_t, \tag{27}$$

where $\hat{w}_t = \hat{w}_t^{bt} - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau}_t$ denotes the log deviation of the after-tariff wage rate from the steady state.[21] The firm's labor demand schedule is decreasing in $\hat{n}_t$. However, when we replace $\hat{\tau}_t$ with $\hat{k}_t$ and $\hat{n}_t$ using the balanced-budget equation (22), it is easy to obtain the equilibrium labor demand schedule:

$$\hat{w}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{-(a_k - a_0\tau_{ss})}{1-a_0(1+\tau_{ss})}\hat{n}_t. \tag{28}$$

As $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$, the equilibrium labor demand function is upward sloping since $\frac{-(a_k - a_0\tau_{ss})}{1-a_0(1+\tau_{ss})} > 0$. Because in our case $\hat{w}_t = \hat{c}_t$, the aggregate labor supply is infinitely elastic (for a given tariff rate and marginal utility of income), the labor demand schedule will be steeper than the labor supply schedule whenever $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$. It is worth emphasizing that our economy can be easily shown to be equivalent to SGU model since in both cases, the price-to-cost markup is countercyclical with respect to the output, which in turn gives rise to indeterminacy (see the appendix).

Secondly, we compare our model with SGU model. SGU prove that within a standard neoclassical growth model, a balanced budget rule can make expectations of higher tax rates self fulfilling if the fiscal authority relies on changes in labor income taxes to eliminate the short run fiscal imbalances. One may think if the import factor is a labor substitute, the endogenous tariff rate levied on the imported oil will make indeterminacy arise more easily. Although in the above sections, we follow ACW to assume that the imported factor is mainly a substitue for capital, we can not eliminate the possibility that imported factor is a substitute for labor.

[21] Here $\hat{w}_t^{bt}$ denotes the log deviation of the before-tariff wage rate from the steady state.

We get the following proposition:

Proposition 5. *If we assume that the imported factor is mainly a labor substitute instead of a capital substitute, which means that we fix a_k at a given level (say, $a_k = 0.3$) and let a_0 vary in the interval $(0, 1 - a_k - a_n)$,*[22] *indeterminacy may not easily arise under the labor substitute assumption.*

Proof. A formal proof can be stated as follows. Consider an economy with capital share (a) and labor share ($1 - a$). When we introduce into the model the foreign input with share b as a labor substitute, the indeterminacy region becomes $\frac{a}{b} < \tau_{ss} < \frac{1-a-b}{b}$. When we introduce into the model the foreign input with share b as a capital substitute, the indeterminacy region becomes $\frac{a-b}{b} < \tau_{ss} < \frac{1-a}{b}$. It is clear that the lower bound of the region under the labor substitute assumption is larger than the one obtained under the capital substitute assumption. ■

From this proposition, we find that although tariffs share with factor income taxes the similar mechanism for indeterminacy, they have different implications in generating indeterminacy. That is, the "equivalence" relationship between them only holds through fiscal increasing returns by endogenizing rates and making the government spending exogenous. ACW find that if the imported factor is a substitute for labor, a larger oil share (a_0) implies a smaller threshold value of the production externality although the reduction of externality is less dramatic. Here we find that for the same oil share (a_0), under the labor substitute assumption, the threshold value of the (steady state) tariff rate needed to generate indeterminacy (i.e., the lower bound of the indeterminacy region) can be larger than that obtained under the capital substitute assumption. In the numerical calibration example of Chen and Zhang (2008), we assume that the time unit is a quarter, the capital share is 0.09 (i.e., $a_k = 0.09$), the oil share is 0.21 (i.e., $a_o = 0.21$), the labor share is 0.7 (i.e., $a_n = 0.7$), the depreciation rate is 0.025 (i.e., $\delta = 0.025$), the discount rate in the discrete time version of this model

[22] *Under the capital substitute assumption, we fix a_n at a given level and let a_0 vary in the interval $(0, 1 - a_k - a_n)$.*

is 0.99 (i.e., $\beta = 0.99$) and the steady state oil tax rate is 0.6 (i.e., $\tau_{ss} = 0.6$).[23] Indeterminacy in this case arises. By contrast, if we assume that the imported oil is a labor substitute, indeterminacy will disappear. That is because indeterminacy would require that the steady state tariff rate be at least 10/7 under the labor substitute assumption (i.e., $a_k = 0.3$, $a_0 = 0.21$, and $a_n = 0.49$). But it is empirically unrealistic.

The economic intuition behind this result can be shown by considering the equilibrium condition in the labor market. Suppose that expectations of a future tariff increase shift the labor supply schedule up (since the firm will import more oil today to produce more output). Because the slope of the labor demand schedule is equal to $-\frac{a_k}{1-a_0}$, the absolute value of the slope under the capital substitute assumption ($|-\frac{a-b}{1-b}| = \frac{a-b}{1-b}$) is smaller than that obtained under the labor substitute assumption ($|-\frac{a}{1-b}| = \frac{a}{1-b}$). This implies that the decline in employment under the capital substitute assumption should be larger than that obtained under the labor substitute assumption. As a result, the increase in the tariff rate required to bring about budget balance under the capital substitute assumption should be larger than that obtained under the labor substitute assumption. Hence multiple equilibria become more likely to occur under the capital substitute assumption than under the labor substitute assumption.

Thirdly, we compare our model with ACW model. ACW show that heavy reliance on imported energy can have a significant effect on economic instability in the presence of increasing returns to scale: the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. We have the similar proposition below:

Proposition 6. *We fix a_n at a given level (say, $a_n = 0.7$), which implies that the imported input is a capital substitute (i.e., $a_k + a_0 = (1 - a_n)$ is fixed). The larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. Because the lower bound of the*

[23] The country in this case is Netherland and we assume that the foreign input is a capital substitute.

indeterminacy region $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$ decreases as a_0 increases, indeterminacy is easier to arise in the range of empirical tariff rates the larger a_0 is.

As a_0 decreases, the minimum tariff rate that generates indeterminacy increases (given that a_n is fixed). The intuition can also be shown by considering the equilibrium condition in the labor market. Suppose that expectations of a future tariff increase shift the labor supply schedule up. Because the slope of the labor demand schedule is equal to $-\frac{a_k}{1-a_0} = -\frac{1}{1+\frac{a_n}{a_k}}$, the smaller a_k is (given that a_n is fixed), the larger the decline in employment (since the slope of the labor demand schedule decreases in absolute value as a_k decreases). As a consequence, the increase in the tariff rate required to bring about budget balance is larger the smaller a_k is or the larger a_0 is, and hence multiple equilibria become more likely the larger a_0 is.

4. Robustness

In this section, we briefly discuss the robustness of our indeterminacy result in the economy with endogenously determined tariff rates. Specifically, we allow for income-elastic government spending and more general preferences.[24] First, we consider the tariff policy, where government expenditures are income elastic and tariff rates adjust in equal proportions to balance the budget in each period. Assuming that the key parameter values are taken from the first calibrated example, we calculate the smallest value of τ for which indeterminacy arises for a given value of the income elasticity of government expenditures, ε_{GY}.[25] As in SGU, we consider that government expenditures can be procyclical ($\varepsilon_{GY} = 0.5$), acyclical ($\varepsilon_{GY} = 0$) and countercyclical ($\varepsilon_{GY} = -0.5$). And we find that the smallest tariff rate that makes indeterminacy occur increases from 42.86 to 85.71 percent as ε_{GY} increases from zero to 0.5. Therefore we conclude that the more procyclical government expenditures

[24] As Jess Benhabib suggested to me, tariffs and factor income taxes are very close to each other in nature. Tariffs can inherit many characteristics of factor income taxes in the SGU model. So the method that we can use to analyze how **public debt** and **predetermined tax rates** affect indeterminacy will be essentially the same as in SGU (1997).

[25] We assume that $a_n = 0.7$, $a_0 = 0.21$, $a_k = 0.09$, $\rho = 0.04$ and $\delta = 0.1$.

are, the less likely it is that indeterminacy arises. That is because $\frac{G_t}{Y_t} = \frac{a_0}{1+\frac{1}{\tau_t}}$ (the budget balance) implies that the more procyclical government expenditures are, the smaller the required change in tariff rates necessary to balance the budget for a given change in output, the less likely it is that indeterminacy arises. If we consider the special case in which government expenditures are a fixed proportion of output ($\varepsilon_{GY} = 1$) and are financed by a tariff, the budget balance implies a constant tariff rate and hence the model is determinate. If we allow government expenditures to be countercyclical, the smallest tariff rate that makes indeterminacy occur decreases from 42.86 to 28.57 percent as ε_{GY} decreases from zero to -0.5. In other words, the more countercyclical government expenditures are, the more likely it is that indeterminacy arises.[26]

Secondly, we claim that the larger the share of public expenditures financed by tariffs, the more likely it is that indeterminacy arises. In section 2.6, we show that if $T_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ is exogenous (i.e., $G_t = 0$—the public expenditures financed by tariffs are zero), indeterminacy still arises but the lower bound of the indeterminacy region increases.[27]

Thirdly, we can consider the general period utility function $U(c_t, n_t) = \log c_t - \frac{n_t^{1+\gamma}}{1+\gamma}$ ($\gamma \geq 0$), which implies less than perfectly elastic aggregate labor supply. In this case, the Frisch elasticity of labor supply with respect to wage is equal to $1/\gamma$. For a given value of the Frisch wage elasticity of labor supply (γ can be $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$), we calculate the smallest tariff rate that makes indeterminacy possible under the balanced budget rule with a pre-set level of government expenditures ($\tau_{ss}^{\text{lower bound}}$ will be 0.7989, 0.6246 and 0.5296 respectively).[28] And we find that the smallest tariff rate that generates

[26] It is easy to find that once we consider the income elasticity of government expenditures, four elements in the Jacobian matrix will become $J_{11} = -\frac{a_n(\rho+\delta)}{a_k + a_0(\varepsilon_{GY}-1)\tau_{ss}}$, $J_{22} = \frac{(1-a_0)(\rho+\delta)}{a_k + a_0(\varepsilon_{GY}-1)\tau_{ss}} - \delta$, $J_{12} = \frac{a_0\tau_{ss}(\varepsilon_{GY}-1)(\rho+\delta)}{a_k+a_0(\varepsilon_{GY}-1)\tau_{ss}}$ and $J_{21} = \frac{(1-a_0)(\rho+\delta)}{a_k}\frac{1-a_0+a_0(\varepsilon_{GY}-1)\tau_{ss}}{a_k+a_0(\varepsilon_{GY}-1)\tau_{ss}} - \delta$. The lower and upper bounds of the steady state tariff rate for indeterminacy are $\frac{a_k}{a_0(1-\varepsilon_{GY})}$ and $\frac{a_n}{a_0(1-\varepsilon_{GY})}$.

[27] In order to remain comparable with SGU's analysis, we only consider two cases: $T_t = 0$ and $G_t = 0$.

[28] We assume that $a_n = 0.7$, $a_0 = 0.21$, $a_k = 0.09$, $\rho = 0.04$ and $\delta = 0.1$. If we set γ to be 1, 1/2 and 1/4 as in SGU, we find that the model is indeterminate if the minimal values of the steady state tariff rate are 2.095, 1.540 and 1.095 respectively. It is easy to prove that four elements in the Jacobian matrix in this case are $J_{11} = -\frac{a_n(\rho+\delta)}{a_k - a_0\tau_{ss} + \gamma[1-a_0(1+\tau_{ss})]}$, $J_{22} = \frac{(1-a_0)(\rho+\delta)(1+\gamma)}{a_k - a_0\tau_{ss}+\gamma[1-a_0(1+\tau_{ss})]} - \delta$, $J_{12} = \frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})}\left[\frac{\gamma a_n[1-a_0(1+\tau_{ss})] - a_0 a_n \tau_{ss}}{a_k - a_0\tau_{ss}+\gamma[1-a_0(1+\tau_{ss})]} - a_0\tau_{ss}\right]$ and $J_{21} = \frac{(1-a_0)(\rho+\delta)}{a_k}\frac{(1+\gamma)[1-a_0(1+\tau_{ss})]}{a_k - a_0\tau_{ss}+\gamma[1-a_0(1+\tau_{ss})]} - \delta$. The lower and upper bounds of the indeterminacy region are $\frac{a_k+\gamma(1-a_0)}{a_0(1+\gamma)}$

indeterminacy is decreasing with respect to $1/\gamma$. We can analyze the relationship by considering the equilibrium condition in the labor market. The slope of the labor supply curve is γ, so employment falls by more the smaller γ is when expectations of a future tariff increase shift the labor supply curve up. As a result, the increase in the tariff rate needed to balance the budget is greater, the smaller γ is, and hence multiple equilibria become more likely the smaller γ is or the larger the wage elasticity of labor supply is.[29]

The numerical result shows that if the Frisch labor supply elasticity is less than or equal to 4, the smallest tariff rate for indeterminacy is empirically unrealistic. In order to make indeterminacy possible, the Frisch elasticity of labor supply should be sufficiently high. We conjecture that once we introduce a nontaxed sector (such as home production) into the model, the balanced-budget rule may induce indeterminacy under realistic tariff rates and labor supply elasticities.

5. Conclusion

We explore the "channel equivalence" between factor income taxes and tariffs to generate indeterminacy. The channel is through fiscal increasing returns by endogenizing rates and making the government spending (or lump-sum transfers) exogenous. We show that, in the presence of fiscal increasing returns caused by endogenous tariffs, it is easy for indeterminacy to occur in small open economies that import foreign energy and take as given the international energy price. The required steady state tariff rates can be empirically realistic. An implication of this paper is that economies largely dependent on non-reproducible natural resources may be vulnerable to sunspot fluctuations if the government finances public spending with endogenous energy taxes.

and a_n/a_0. The restriction on γ is $a_n > a_k(1+\gamma)$.

[29] In this case, the labor supply schedule is $\hat{w_t} = \hat{c_t} + \gamma \hat{n_t}$ and the equilibrium labor demand schedule is $\hat{w_t} = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k_t} + \frac{-(a_k - a_0\tau_{ss})}{1-a_0(1+\tau_{ss})}\hat{n}$. From (28), we can see that the slope of the equilibrium labor demand schedule is increasing in τ_{ss}. As $\frac{1}{\gamma}$ decreases, the slope of the labor supply curve increases; therefore, as $\frac{1}{\gamma}$ decreases, the steady state tariff rate has to increase in order for the labor demand curve to be steeper than the labor supply curve.

One future direction is to see under what circumstances, tariffs and capital income taxes are equivalent in generating indeterminacy since the essential element for indeterminacy in SGU model is the endogenous labor income tax rate.

6. Appendix:

We summarize the equilibrium conditions of the model with a balanced-budget rule, endogenous tariff rates, and constant government purchases presented in this paper. Consider the discrete time case of a tariff. The balanced-budget rule is given by

$$G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}.$$

The following equilibrium conditions hold for all t,

$$U_c(c_t, n_t) = \theta_t,$$

$$U_n(c_t, n_t) = w_t \theta_t,$$

$$\mathsf{Y}_t = c_t + k_{t+1} - (1-\delta)k_t,$$

and

$$1 = \beta \frac{\theta_{t+1}}{\theta_t}(1 - \delta + r_{t+1}),$$

where θ_t is the Lagrangian multiplier of the budget constaint of the agent. In this model, disposable income, Y_t, is given by

$$\mathsf{Y}_t = (1 - a_0) y_t = y_t - p^0 o_t - G,$$

G represents a fixed cost that ensures that firms do not make pure profits in the long run (given that the foreign firms take away their payments). The after-tariff wage rate w_t, and the after-tariff rental rate r_t are given by

$$r_t^{bt} = a_k (\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} = \mu_t r_t,$$

and

$$w_t^{bt} = a_n (\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} = \mu_t w_t,$$

where r_t^{bt} and w_t^{bt} denote the before-tariff rental rate and before-tariff wage rate respectively. In the balanced budget model, μ_t represents the wedge between marginal product and after tariff factor prices introduced by endogenous tariffs. It is easy to verify that the markup μ_t is countercyclical with respect to y_t since

$$\mu_t = (1 + \tau_t)^{\frac{a_0}{1-a_0}} = (1 - \frac{1 - a_0}{a_0} \frac{G}{\mathsf{Y}_t})^{-\frac{a_0}{1-a_0}} = \mu(\frac{G}{\mathsf{Y}_t}).$$

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