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Steinbacher, Matej and Steinbacher, Matjaz and Steinbacher,  
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# TO WORK OR NOT? SIMULATING INSPECTION GAME WITH LABOR UNIONS

Matej Steinbacher, Matjaz Steinbacher, Mitja Steinbacher

E-mail: [matej.steinbacher@gmail.com](mailto:matej.steinbacher@gmail.com), [matjaz.steinbacher@gmail.com](mailto:matjaz.steinbacher@gmail.com),  
[mitja.steinbacher@gmail.com](mailto:mitja.steinbacher@gmail.com)

## ABSTRACT

The model of social network is used to analyze the impact of the power of labor unions in the labor relations. We find that labor union capable to affect a pecuniary compensation of shirking employees lessens the motivation of employees to work and improve to the unionization rate. As a result, the performance of the firm is significantly deteriorated and its existence endangered. On the other hand, the inspection proved to be a successful method for “motivating” employees to work. By using non-omniscient agents, we also estimated the cost of that non-omniscience, which proved to be significant in all cases.

## **1. Introduction**

Incentives can be defined as any action that provides motives for a particular behavior of individuals and represent a significant element in the decision-making processes of individuals. On the other hand, disincentives can be defined as any action that lessens motives for a particular behavior of individuals thus having negative motivational effects to individuals' behavior. When deciding which action to pursue, individuals compare the gains they believe an action will bring them to the costs of an action and consider some risks on both sides, which they perceive relevant. Self-motivated individuals also tackle their preferences and other individual-specific characteristics to the feasibility constraints. Incentives and disincentives alter the feasibility constraint in the above cost-benefit analysis, which makes them very significant in the decision-making. Because incentives directly enter the cost-benefit analysis, they induce reallocation of activities towards those that are subject of an incentive and away from those that are not, or are subject of a disincentive (Holmstrom and Milgrom 1991; Baker 1992; Prendergast 1999; Jensen and Murphy 1990). The answer how incentives and disincentives induce moral hazard is not universal, as individuals differ in their preferences, knowledge, attitudes towards risk, ties in the social network and other individual-specific characteristics, whereas in the environment of Hayekian knowledge (Hayek 1945), individuals learn and pursue their rational self-interest. Schelling (1978) observes that such self-interests on an individual basis might have huge global effects, whereas Montgomery (1991) stresses the importance of social ties for the salary of employees.

A significant element in the process of incentives and disincentives in industrial relations is represented by labor unions, which might have several different outcomes, and that question

we tackle in the paper. We extend the inspection game of Dresher (1962) by adding a labor union. Labor unions have yet not been introduced into the inspection game, although their influence in the labor relations could not be ignored. In the game, a principal (employer or a firm) employs an agent (or agents or employees or workers), and assigns a task to him for which the latter receives a payment, if the task successfully accomplished. The arrangement results in the well-known principal-agent problem, because the two participants have opposite interests (Grossman and Hart 1983; Mookherjee 1984). In particular, while the employer wants the task to be accomplished, the employee tries to receive his payment with as little effort as possible. The dilemma is tackled by a costly inspection going at the expense of the employer, and is intended to reveal the true effort of the employee. Employee that is caught shirking does not get paid, whereas he does not get fired. Therefore, in order to motivate agents to work, principal might increase the level of the inspection. Because of the communication processes among employees, principal does not need to inspect every single agent in order to “motivate” him to work. Therefore, we search for the optimal inspection rate of the principal.

A simple labor union is introduced into the game as a potential escape hatch for shirking workers by warranting a partial pecuniary compensation for those unionized. The level of this compensation is exogenously determined in the model, and it directly determines benefits, which employees can get by joining the union. We could say that the level of this pecuniary compensation determines the power of the union. Such concept defines the union power in terms of the share of any “rent” captured by the union. Therefore, the decision to join the union or not interacts with the decision to shirk, and when deciding whether to join the union or not, employees weigh expected benefits with the established union-membership fee, and keep these expectations in mind (Hammermesh 1977).

The power a labor union has in the game can affect working incentives of selfish employees, which further affects the level of inspection done by a principal and the profitability of a firm. Clark (1984) and Freeman and Kleiner (1999) estimate that unionized firms, which operate in similar environment than non-unionized firms, are associated with 10 to 20 percent lower profits. As Freeman and Medoff (1984) state, the profitability effect is not present *per se*, but “*what matter are the market conditions and routes by which unionism alters profits.*” We find that under certain conditions labor union has quite a negligible impact on the work incentives of employees and on the performance of a firm. However, increasing its power has extremely negative effect on the working habits of employees, both union and non-union members, and the performance of the firm as well (DiNardo and Lee 2004).

The principal-agent inspection game in the paper is simulated bottom-up, using a social network of many local and global interactions among individuals (Axelrod 1984; Milgram 1967; Watts and Strogatz 1998; Wasserman and Faust 1994). With such social network, we consider the fact that workers in firms work together in small groups and communicate with each other by using different cognitive methods, that they become friends, while they also have acquaintances from other such groups. Workers in firms also daily meet at different occasions, e.g. at the workplace, at lunch, when coming to work and going home, etc. By connecting workers from such groups with their acquaintances and acquaintances of acquaintances and so on, we assume that workers from the entire firm are connected with such local acquaintances.

The paper is organized as follows. Section 2 gives a short note on the social network that is used in the paper. In the Section 3, we develop the game in three different forms. In particular,

we analyze a baseline inspection game without labor union and two versions with a labor union; i.e. exogenously and endogenously determined. In Section 4, we apply the small world network to these three models, and analyze the results in the Section 5. In the Section 6, we give some conclusions.

## 2. Small world network

The network  $g = (A, E)$  is a set of vertices  $A = \{A_1, A_2, \dots, A_n\}$ , representing agents-employees, and edges  $E = \{e_1, e_2, \dots, e_m\}$ , representing their unique pairwise connections. If two agents are connected, we denote  $ij \in g$ , while  $ij \notin g$  represents two unconnected agents. Using adjacency matrix,  $ij = 1$  if  $ij \in g$  and  $ij = 0$  if  $ij \notin g$ . We use undirected graph with no loops, where edges are unordered pairs of vertices and are either connected or not, thus if  $ij = 1 \Leftrightarrow ji = 1$  and no  $ii = 1$  is possible for all  $i \in N$ . In a small world network, people have many local and some global connections with others, which we get by randomly rewiring some of the connections from a regular network at some small probability (Watts and Strogatz 1998). The probability that a connection from an agent  $A_i$  is rewired to the randomly chosen agent  $A_j$  in the network equals  $p = 0.01$ . The number of agents to which an agent  $A_i$  is directly connected represents a degree of an agent  $A_i$  and is denoted  $k(A_i)$ . The average degree of agents in the network equals  $\bar{k}_i(g) = 6$ . The network remains unchanged once agents are populated and connections are rewired. There are no isolated agents in the network.

## 3. The game

### 3.1 General framework

We begin with a simple multi-agent inspection game. Owner of the firm, or a principal, denoted  $P$ , employs a nonempty, finite set of employees, or agents, denoted  $A_i$ , where  $i = \{1, 2, \dots, 1000\}$ . In every time period each agent has to choose between two discrete choices, to work, denoted with  $W$ , or shirk, denoted with  $S$ , which makes his set of pure strategies in every time period to be given as  $S_i = \{S, W\}$ .

Each agent  $A_i$  contributes  $v_i(s_i \in S_i) \in \{0, 1\}$  to the total output with  $v_i = 1$  if working and  $v_i = 0$ , if not. For the sake of simplicity, we assume that workers are equally productive and do not allow for any kinds of stochastic production shocks. Then the total output of the firm equals to the sum of outputs of all individual agents, thus  $v = \left\{ \sum_i v_i(s_i) \mid s_i = W \right\}$ . Although principal could also observe the output value of the firm to monitor the efficiency of employees, we assume that costly inspection, denoted  $h$ , that is borne out by the principal, is the only way how a principal can reveal the true state of each agent. Principal thus have two alternatives from which to choose, to inspect  $\{I\}$  or not  $\{N\}$ , making his set of pure strategies  $S_p = \{I, N\}$ .

We can now put the payoff function of each agent  $A_i$ , Equation (1), and of the principal  $P$ , Equation (2), as

$$u_i(s_i | s_p): S_i \rightarrow \mathfrak{R}, \quad (1)$$

$$u_p(s_p | s_i): S_p \rightarrow \mathfrak{R}. \quad (2)$$

Agent who is not found shirking gets payment  $w$ , no matter whether he was working or not. The level of payment is given in the relative terms as a share of produced output and is put exogenous into the model. This also means that the principal is not able to condition the level of wages according to the output.

Let us imagine that working requires efforts, which is costly, for which agents are better off when not working. We neglect the fact that some agents might consider shirking less satisfactory than working, as von Mises (1949) argue. Therefore, if an agent  $A_i$  decides to work, his decision is a subject of work related costs whatever they are, which we denote with  $g$ . For the sake of simplicity, we assume homogeneous agents in this respect that all have equal opportunity costs, therefore  $g_i = g > 0$  for all  $i \in N$ .

On the other side, principal not only decides whether to inspect or not, but also how much to inspect if he decides to inspect. Because inspection is subject to costs to the principal, we assume that a principal inspects each agent with the given probability  $r \in [0,1]$ , whereas the value of the probability is unknown to agents. Every level of probability that is lower than unity represents the incentive for agents to shirk on the hope not to be inspected. Recall that agent who is found for shirking gets  $w = 0$ . As long as the probability for inspection is unity, agents have dominant strategies to work.

The time in the model is discrete defined on  $t = 1, 2, 3, \dots, T$ . During a single iteration of the game, each agent  $A_i$  communicates with other agents to which he is connected and plays the game with the principal  $P$ , where both choose their strategies simultaneously at the beginning of every time period. Depending on the choices they make, the two receive payoffs described succinctly by the payoffs matrix (Fudenberg and Tirole 1994):



$A_i/P$	$I$	$N$
$S$	$0/-h$	$w/-w$
$W$	$w-g/v-w-h$	$w-g/v-w$

### 3.2 A one shot game

In a one-shot game we assume that there is only one time period with  $(w > g > 0) \wedge (w > h > 0)$ , meaning that none of the strategies is weakly or strictly dominated.

Under such circumstances, the game does not have Nash equilibrium in pure strategies.

To prove the first part, let assume that  $g > w > 0$ . Then the payoff matrix applies that  $u_i(S, s_p) \gg u_i(W, s_p) \forall s_p \in S_p$ , which indicates working to be strictly dominated strategy for an agent  $A_i$ . Second part can be proved using the similar reasoning. Assume now that  $h > w > 0$ , then the payoff matrix applies to  $u_p(s_i, N) \gg u_p(s_i, I) \forall s_i \in S_i$ , making the inspection strictly dominated strategy.

Let  $\sigma_i \in \Sigma$  be a mixed strategy of an agent  $A_i$ . Then the set of mixed strategies can be written as  $\Sigma_i = \left\{ \sigma_i \mid S_i \rightarrow [0,1], \sum_{s_i \in S_i} \sigma_i(s_i) = 1 \right\}$ . Expected payoff of each agent  $A_i$  playing such mixed strategy  $\sigma_i \in \Sigma_i$  then equals

$$u_i(\sigma \mid s_p) = \sum_{s \in S_i} \prod_{j \in N} \sigma_j(s_j) u_i(s_i). \quad (3)$$

Similar conditions hold true also for the principal  $P$ . We can put his set of mixed strategies as

$\Sigma_P = \left\{ \sigma_P \mid S_P \rightarrow [0,1], \sum_{s_P \in S_P} \sigma_P(s_P) = 1 \right\}$ , where the expected payoff from playing a mixed

strategy  $\sigma_P \in \Sigma_P$  is given by

$$u_P(\sigma \mid s_i) = \sum_{s_P \in S_P} \prod_{j \in N} \sigma_j(s_j) u_P(s_P). \quad (4)$$

We now turn to the best responses of each agent against the principal. Assume that  $r$  is the probability that an agent  $A_i$  is inspected and that  $\alpha_i$  is the probability that this agent is shirking. Then the expected payoff of an agent  $A_i$ , when he plays  $\{\alpha_i, 1 - \alpha_i\}$ , while a principal plays  $\{r, 1 - r\}$ , is

$$E[A_i \mid \alpha_i, r] = (w - g) + \alpha_i (g - rw). \quad (5)$$

From the Equation (5), we see that the expected payoff of an agent  $A_i$  is increasing in  $\alpha_i$  for  $(g - rw) > 0$  and decreasing elsewhere. Therefore, the correspondence of his best responses  $B_c(r)$  is defined as

$$B_c(r) = \begin{cases} \{S\} & r \in [0, g/w) \\ \{S, W\} & r = g/w \\ \{W\} & r \in (g/w, 1] \end{cases}. \quad (6)$$

### 3.3 A one shot game with a labor union

We now introduce a labor union into the game and are particularly interested in the effects a labor union induce on working habits of employees, wages, and the profitability of the firm.

We still have a principal  $P$  in the model who employs a nonempty and finite set of agents  $A_i$ . However, we now assume that part of agents are unionized, we denote them  $u$ , while the rest of them  $(1-u)$  remain non-union members.<sup>1</sup> We denote  $u_i = 1$  if an agent  $A_i$  is a union member and  $u_i = 0$  if he is not. To facilitate the notation, let  $U_i = \{A_i(u_i) | u_i = 1\}$  indicate a union member and  $L_i = \{A_i(u_i) | u_i = 0\}$  a non-union member.

Unionization is not free of charge, but is subject of paying a membership fee, which is denoted  $f$ . Because membership fee goes at the expense of each union member  $U_i$ , paying it per se lowers the utility of a member. Therefore, union members expect to get some benefits in return (Hammermesh 1977). We introduce the pecuniary compensation into the model, denoted  $c \in [0,1]$ , which a shirking union member receives if caught shirking. Pecuniary compensation represents the fraction of the wage  $w$  that union members get although being caught by the principal  $P$  while shirking. The level of such compensation represents a benefit of being unionized, while it also represents an incentive for union members not to work and for non-union members to become unionized. The level of this compensation could also reflect the power of a union, with higher the value more influential the labor union.

We first consider the case where  $u_i = 1$  for all  $i \in N$ . Therefore, the strategy space of an agent  $A_i$  remains unaltered as each agent may still work or shirk. To facilitate the notation, let

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<sup>1</sup> Note that  $u$  denotes utility as well. But it should be clear from the text whether in particular cases  $u$  relates to utility or the unionization.

$S_i = \{SU, WU\}$ . The two alternatives of a principal are either to inspect or not, therefore  $S_p = \{I, N\}$ . The new payoffs matrix is now of the following form

$U_i/P$	$I$	$N$
$SU$	$cw - f / -cw - h$	$w - f / -w$
$WU$	$w - g - f / v - w - h$	$w - g - f / v - w$

Let us derive general condition for the optimal strategy of a union member  $U_i$ . When he plays  $\{\alpha_i, 1 - \alpha_i\}$  and the principal plays  $\{r, 1 - r\}$ , his expected payoff equals

$$E[U_i | \alpha_i, r, c] = (w - f) + \alpha_i(g - rw(1 - c)). \quad (7)$$

It is evident that the expected payoff of  $U_i$  is increasing in  $\alpha_i$  if  $(g - rw(1 - c)) > 0$  and decreasing elsewhere. Therefore, his best response correspondence  $B_c(r | c)$  is represented by

$$B_c(r | c) = \begin{cases} \{SU\} & r \in [0, g/w(1 - c)) \\ \{SU, WU\} & r = g/w(1 - c) \\ \{WU\} & r \in (g/w(1 - c), 1] \end{cases}. \quad (8)$$

We take some examples. We first assume a labor union with no power, therefore  $c = 0$ . In this case, the best response correspondence represented in Equation (8) reduces to Equation (6). However, notice that a union member  $U_i$  is worse off than non-union member  $A_i$ , because he must pay a membership fee  $f$ , for which he does not get any benefit in return. At

the other extreme case with a labor union of a maximum power, therefore  $c = 1$ , the best response strategy reduces to the  $\{SU\}$  for any  $r$  as  $\lim_{c \rightarrow 1} r = g[w(1-c)]^{-1} \rightarrow \infty$ .

### 3.4 Evolutionary inspection game

We now extend the framework to the multi-stage setting and allow for a mutual cooperation of agents with each other as represented by the small world network principle. Suppose now that when agents receive the wage, they spend a part of it, while saving the rest. Let  $s \in (0,1)$  be the savings rate, which is exogenously given to all agents. We may use such a simplification because we have homogeneous agents. Then the accumulated wealth of an agent  $A_i$ , denoted with  $e_i$ , for the  $t$ -th iteration is calculated according to

$$e_i(t) = s \sum_{h=0}^{t-1} q_i(h) + q_i(t). \quad (9)$$

In the Equation (9),  $q_i(h)$  and  $q_i(t)$  denote payoff of an agent  $A_i$  at iteration  $h$  and  $t$ , respectively.

After each agent in the network decides which strategy to pursue, he is rewarded with a payoff that contributes to his wealth. An important element in the game is the acquisition of knowledge, which agents adopt by communication processes with others in the network. We assume that agents not only learn from their own experiences, but also from experiences of others. Thus, we assume that when deciding which strategy to take, an agent  $A_i$  compares his accumulated wealth  $e_i$  and the strategy he played with the wealth and the strategy played by a

randomly chosen agent  $A_j$  to whom he is connected. The wealth of an agent  $A_j$  is denoted  $e_j$ . Such communication processes and the exchange of experiences among agents in the network affect the strategies agents take in the beginning of the next period, and so on. Because the network is fully connected with such interacting agents, information of individual agents circles around the whole network. Because of this feature, social networks usually imply herd behavior with strategies outperforming others usually being copied (Bikhchandani et al. 1998; Banerjee 1992). In the games, we assume that the quality of information does not weaken when circling through the network.

We now introduce the level of omniscience of agents into the game. This means that when agents compare the outcomes of different strategies with theirs, there is a probability that they do not take the most efficient strategy of other agents. The probability that an agent  $A_i$  adopts the strategy of one of his randomly chosen neighbor  $A_j$  is an increasing function of the wealth difference, defined as follows (Szabó and Töke, 1998)

$$\wp = \left(1 + \exp\left[\left(e_i - e_j\right)\kappa^{-1}\right]\right)^{-1}, \quad (10)$$

where  $0 < \kappa \ll 1$  is the parameter of non-omniscience, which is related to the strategy adoption. Closer to zero the value of parameter  $\kappa$ , more omniscient agents are. More omniscient agents are, more likely it is that they adopt the strategy with highest outcome. Equation (10) also implies that closer the payoffs of the two agents, less likely it is for an agent  $A_i$  to change his strategy. This is something Aumann (1997), Rubinstein (1998) and Selten (1975) define as “*trembling hand perfection*.” This means that the strategy of better performing agent is likely to be copied, while it is not completely impossible that agents adopt

the strategy of a worse performing agent. However, when the difference between payoffs of the two agents is very small, such probability is higher, and vice versa. This is in line with a psychological approach to the decision-making problem of agents (Kahneman and Tversky 1979; Kahneman 2003; Hirshleifer 2001).

### 3.5 Evolutionary inspection game with exogenous labor union

Consider now that  $U_i$  and  $L_i$  are nonempty finite sets of agents that are members of union and non-union members, whereas the level of unionization is exogenously given before the first iteration. Once unionization statuses are defined, such agents are not allowed to change it. The new payoffs matrix has the form

$A_i/P$	$I$	$N$
$S$	$0/-h$	$w/-w$
$W$	$w-g/v-w-h$	$w-g/v-w$
$SU$	$cw-f/-cw-h$	$w-f/-w$
$WU$	$w-g-f/v-w-h$	$w-g-f/v-w$

Here, the strategy space for each non-union-member  $L_i$  equals  $\{S, W\}$ , where the strategy space of each union-member  $U_i$  equals  $\{SU, WU\}$ . As before, the principal still has two choices available, whether to inspect or not, therefore  $\{I, N\}$ .

### 3.6 Evolutionary inspection game with endogenous labor union

We now make the labor union status endogenous. This means that along with the strategy adoption, each agent is also allowed to adopt unionization status of other agent. Under this circumstances, agents have four different alternatives available;  $S_i = \{S, W, SU, WU\}$ . Because of  $u_i(W, s_p) \gg u_i(WU, s_p) \forall s_p \in S_p \wedge \forall f > 0$ ,  $WU$  is strictly dominated strategy and is never played, for which the payoffs matrix can be reduced to

$A_i/P$	$I$	$N$
$S$	$0/-h$	$w/-w$
$W$	$w-g/v-w-h$	$w-g/v-w$
$SU$	$cw-f/-cw-h$	$w-f/-w$

We also assume that the opportunity cost of labor is strictly higher than the unionization fee, therefore  $g \gg f$ . For the opportunity cost of labor being lower than the unionization fee,  $SU$  is strictly dominated strategy for an agent  $A_i$ , as  $u_i(W, s_p) \gg u_i(SU, s_p)$  for all  $s_p \in S_p$  and for all  $c \in [0,1]$ .

#### 4. Simulation

Simulation process is given as follows. The model is populated with  $n = 1000$  agents, each of which initially has  $k_i(g) = 6$  connections. The probability that a connection of an agent  $A_i$  is rewired to the randomly chosen agent  $A_j$  in the network, equals  $p = 0.1$ . Once connections between agents are rewired and agents populated throughout the network, the network remains unchanged.



All inspection games are iterated forward in time, using a synchronous update scheme. With this, we let all agents to strike a deal with the principal according to the ascribed parameters and the probability  $r$  for principal to inspect. Throughout the game, agents simultaneously update their strategies, by considering the level of their non-omniscience according to the Equation (10).

In the simulations below, we set the output level of each agent  $v$  to unity, and adjust all other values accordingly in relative terms. In particular, each working agent earns  $w = 0.4$  and bears opportunity and other work related costs  $g = 0.125$ . We assume that workers save 10 percent of their wage, thus  $s = 0.04$ , while the union membership fee equals 5 percent of the wage, thus  $f = 0.02$ . Moreover, each inspection costs the principal  $h = 0.16$ , and the factor of non-omniscience in the strategy adoption equals  $\kappa = 0.1$ . Unionization rate, where applicable, equals  $u = 0.4$  and unionized agents are always randomly scattered in the network amongst non-union agents. In the version of exogenous labor unions, those agents that are union members retain that status through the whole game, while agents are free to change their status under endogenous setting. The time is set to  $T = 100.000$ .

To analyze the impact of small perturbations of  $r$  and  $c$  to the solution of the inspection game, we use the step of 0.02 units throughout the definition space  $[0,1]$  of both variables. Figures represent the averages of 20 independent realizations of games.

## 5. Results

### 5.1 Evolutionary inspection game without labor union

### *Firm's performance*

We first simulate the game without the labor union, which means that the first payoffs matrix applies. Figure 1 presents an equilibrium solution of the game. It shows the average firm performance per iteration  $\pi$ , subject to the probability rate of the inspection  $r$ .

FIGURE 1: Optimal inspection rate and firm profit

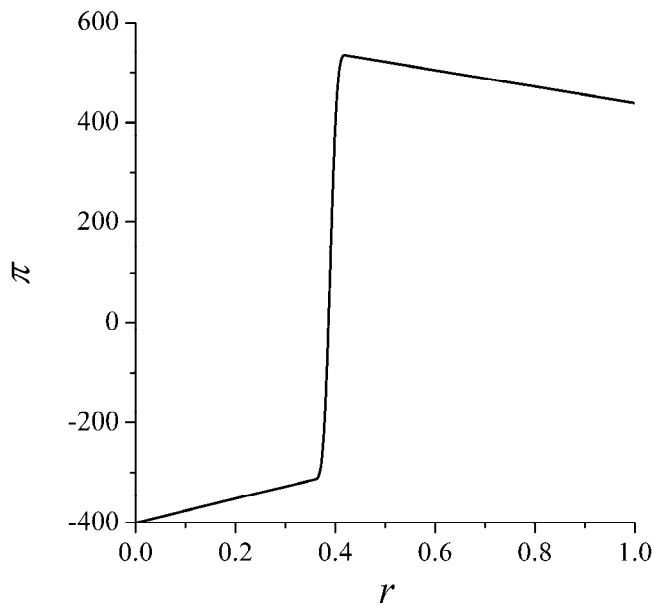


Figure 1 shows that it is optimal for the principal  $P$  to set the inspection rate at  $r = 0.4$  in order to maximize the work incentives of workers and his profit. In this case, as simulation

predicts, all agents work, since  $s_i(r) = \begin{cases} \{W\}; & r \geq 0.4 \\ \{S\}; & r < 0.4 \end{cases}$ . This means that principal does not need

to inspect every agent in order to make sure that everyone is working.

To calculate the average profit of the firm with respect to the probability of an inspection  $\pi(r)$ , we use a corresponding payoffs matrix. Profit equals total revenues less total costs borne by the principal, that is  $\pi(r) = n_1 v - n_2 w - n_3 h$ . Here,  $n_1$  denoted the number of workers working,  $n_2$  the number of workers receiving the wage  $w$ , and  $n_3$  the number of workers that are inspected. Let us make one example. For  $r = 0.34$ , optimal decision rule for an agent  $A_i$  is to shirk in the equilibrium, thus,  $n_1 = 0$ .  $r = 0.34$  means that principal  $P$  inspects  $n_3 = 340$  agents out of 1000 (34 percent of the agents in the network) and they all are caught shirking. This means that  $n_2 = 660$  agents are working. Putting all this together, we get the loss of the firm of  $\pi(r = 0.34) = -318.4$  units. All remaining analytical results and simulation results are given in Table 1.

TABLE 1: Firm's profit as a function of inspection rate

$r$	$\pi_s(r)$	$\pi_A(r)$	$ERR$	$r$	$\pi_s(r)$	$\pi_A(r)$	$ERR$	$r$	$\pi_s(r)$	$\pi_A(r)$	$ERR$
0	-400	-400	0	0.34	-318.414	-318.4	-4.3E-05	0.68	491.2015	491.2	-3E-06
0.02	-395.202	-395.2	-3.9E-06	0.36	-313.599	-313.6	4.15E-06	0.7	488.0066	488	-1.3E-05
0.04	-390.393	-390.4	1.73E-05	0.38	-308.802	-308.8	-5E-06	0.72	484.7876	484.8	2.57E-05
0.06	-385.606	-385.6	-1.5E-05	0.4	536.0058	536	-1.1E-05	0.74	481.5937	481.6	1.3E-05
0.08	-380.797	-380.8	9.11E-06	0.42	532.7909	532.8	1.7E-05	0.76	478.3979	478.4	4.47E-06
0.1	-375.995	-376	1.32E-05	0.44	529.6029	529.6	-5.4E-06	0.78	475.21	475.2	-2.1E-05
0.12	-371.21	-371.2	-2.6E-05	0.46	526.4099	526.4	-1.9E-05	0.8	472.0014	472	-3E-06
0.14	-366.389	-366.4	2.99E-05	0.48	523.2093	523.2	-1.8E-05	0.82	468.7939	468.8	1.3E-05
0.16	-361.599	-361.6	2.9E-06	0.5	519.9959	520	7.98E-06	0.84	465.607	465.6	-1.5E-05
0.18	-356.781	-356.8	5.4E-05	0.52	516.8062	516.8	-1.2E-05	0.86	462.3975	462.4	5.47E-06
0.2	-352.001	-352	-3.5E-06	0.54	513.5913	513.6	1.69E-05	0.88	459.2019	459.2	-4.2E-06
0.22	-347.206	-347.2	-1.7E-05	0.56	510.4008	510.4	-1.5E-06	0.9	455.9985	456	3.33E-06
0.24	-342.392	-342.4	2.25E-05	0.58	507.1889	507.2	2.2E-05	0.92	452.7938	452.8	1.37E-05
0.26	-337.599	-337.6	3.61E-06	0.6	503.9934	504	1.31E-05	0.94	449.6084	449.6	-1.9E-05
0.28	-332.805	-332.8	-1.6E-05	0.62	500.7986	500.8	2.76E-06	0.96	446.3995	446.4	1.12E-06
0.3	-328.004	-328	-1.4E-05	0.64	497.5991	497.6	1.83E-06	0.98	443.2006	443.2	-1.3E-06
0.32	-323.188	-323.2	3.84E-05	0.66	494.401	494.4	-1.9E-06	1	440	440	0

In the table  $r \in [0,1]$  denotes the inspection rate,  $\pi_s(r)$  denotes the simulation results of the firm profit  $\pi(r)$ ,  $\pi_A(r)$  denotes to the analytical solution, and  $ERR$  measures the relative error, which is the difference between the simulation result and the analytical result. Profit of the firm in the optimum equals  $\pi(r = 0.4) = 536$ . However, it is not Pareto efficient. If all agents had behaved omnisciently, then all would have worked already at the  $r = 0.3125$ .<sup>2</sup> Such behavior would have resulted in lower inspection costs and consequently in higher profit for the principal, as  $\pi^*(r = 0.3125) = 550$ . This is 2.6 percent higher than the best equilibrium solution. This difference reflects the effect of non-omniscient behavior of agents, which can be defines as the cost of non-omniscience.

#### *Agent's performance*

Simulation implies an optimal strategy for an agent, which is  $s_i(r) = \begin{cases} \{W\}; & r \geq 0.4 \\ \{S\}; & r < 0.4 \end{cases}$ . However,

Equation (6) predicts an optimal strategy for an agent to be  $s_i(r) = \{\{S\} | r \in [0, 0.3125)\}$  and  $s_i(r) = \{\{W\} | r \in (0.3125, 1]\}$  with the point of indifference at  $r = 0.3125$ . From this difference, we see that the non-omniscience of agents affects the payoff of agents even more than that of the firm.

We now calculate the average utility of an agent  $A_i$  by using corresponding payoffs matrix. We get it from  $u_i(r) = m_1 w - m_2 g$ , where  $m_1$  is the probability that a worker receives a wage  $w$ , while  $m_2$  measures the average time an agent  $A_i$  is working. For instance, for  $r = 0.34$  optimal decision rule for agent  $A_i$  predicts that he will shirk in the equilibrium, thus,  $m_2 = 0$ .

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<sup>2</sup> This results from the analytical solution of the Equation (6).

$r = 0.34$  denotes a 34 percent chance that principal  $P$  inspects agent  $A_i$ , who is caught while shirking. Therefore, we get  $m_i = 0.66$ . When putting all this together, we get  $u_i(r = 0.34) = 0.264$ . Results of the simulations and the rest of analytical results are shown in Table 2.

TABLE 2: Agent's utility as a function of inspection rate

$r$	$u_i^S(r)$	$u_i^A(r)$	$ERR$	$r$	$u_i^S(r)$	$u_i^A(r)$	$ERR$	$r$	$u_i^S(r)$	$u_i^A(r)$	$ERR$
0	0.400	0.400	0	0.34	0.2640138	0.264	5.21E-05	0.68	0.2749985	0.275	-5.3E-06
0.02	0.3920016	0.392	3.98E-06	0.36	0.2559987	0.256	-5.1E-06	0.7	0.2749934	0.275	-2.4E-05
0.04	0.3839932	0.384	-1.8E-05	0.38	0.2480015	0.248	6.17E-06	0.72	0.2750124	0.275	4.52E-05
0.06	0.3760059	0.376	1.56E-05	0.4	0.2749942	0.275	-2.1E-05	0.74	0.2750063	0.275	2.28E-05
0.08	0.3679965	0.368	-9.4E-06	0.42	0.2750091	0.275	3.29E-05	0.76	0.2750021	0.275	7.78E-06
0.1	0.359995	0.360	-1.4E-05	0.44	0.2749971	0.275	-1E-05	0.78	0.27499	0.275	-3.6E-05
0.12	0.352098	0.352	2.78E-05	0.46	0.2749901	0.275	-3.6E-05	0.8	0.2749986	0.275	-5.1E-06
0.14	0.343989	0.344	-3.2E-05	0.48	0.2749907	0.275	-3.4E-05	0.82	0.2750061	0.275	2.22E-05
0.16	0.335999	0.336	-3.1E-06	0.5	0.2750042	0.275	1.51E-05	0.84	0.274993	0.275	-2.5E-05
0.18	0.3279807	0.328	-5.9E-05	0.52	0.2749938	0.275	-2.2E-05	0.86	0.2750025	0.275	9.2E-06
0.2	0.3200012	0.320	3.81E-06	0.54	0.2750087	0.275	3.16E-05	0.88	0.2749981	0.275	-7E-06
0.22	0.3120061	0.312	1.95E-05	0.56	0.2749992	0.275	-2.8E-06	0.9	0.2750015	0.275	5.53E-06
0.24	0.3039923	0.304	-2.5E-05	0.58	0.2750112	0.275	4.05E-05	0.92	0.2750062	0.275	2.26E-05
0.26	0.2959988	0.296	-4.1E-06	0.6	0.2750066	0.275	2.4E-05	0.94	0.2749916	0.275	-3.1E-05
0.28	0.2880054	0.288	1.86E-05	0.62	0.2750014	0.275	5.02E-06	0.96	0.2750005	0.275	1.82E-06
0.3	0.2800044	0.280	1.59E-05	0.64	0.2750009	0.275	3.31E-06	0.98	0.2749994	0.275	-2.1E-06
0.32	0.2719876	0.272	-4.6E-05	0.66	0.2749991	0.275	-3.5E-06	1	0.275	0.275	0

Here  $r$  is the inspection rate probability,  $u_i^S(r)$  represents simulation results,  $u_i^A(r)$  represents the analytical result and  $ERR$  is a relative error again. We see that for  $r = 0.34$ , agents get inferior outcome, which is due to their level of non-omniscience. Because  $u_i(s_i = W) = 0.275$  for all  $r \in [0,1]$ , any strategy that would result in lower utility would not be optimal. Hence, for  $u_i(r) < 0.275$ , we have a strict preference relation  $W \succ S$ . Equilibrium solutions of agents in the range of  $r \in [0.32, 0.38]$  are thus suboptimal, as they

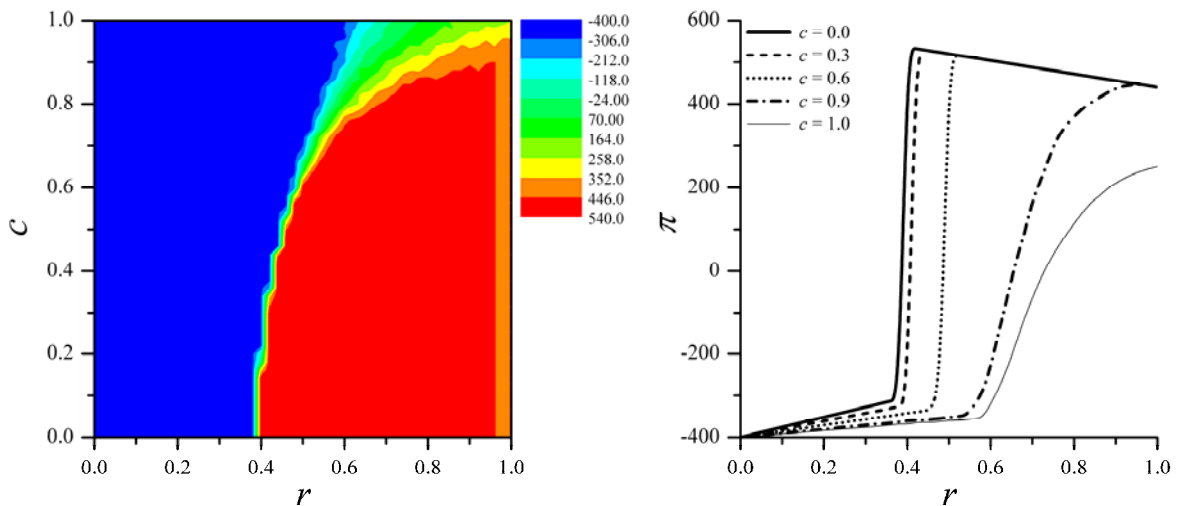
could attain higher utility if they had perfect knowledge, as the guaranteed utility value is  $u_i = 0.275$ . Therefore, the difference again denotes the cost of non-omniscience of agents.

## 5.2 Evolutionary inspection game with labor union

### Firm's performance

We now introduce exogenous labor union into the model. We first study the performance of the firm, with its profit (color coded in the figure) being a function of inspection rate and the power of labor union,  $\pi(r, c)$  and show the simulation results in Figure 2.

FIGURE 2: The average income of the firm when unionization is fixed



For  $c = 0$ , theoretical model simplifies to the baseline model. Results of the simulation are shown in Table 3 (columns  $\pi_s(r, 0)$ ) and, as predicted, nearly replicate those from the Table

1.  $c = 0$  indicates the labor union that has no power to influence the level of a pecuniary

compensation received by the shirking workers. Therefore, it is also not able to influence on the working habits of workers.

TABLE 3: Firm's profit as a function of inspection rate with  $c = 0$  and  $c = 1$

$r$	$\pi_s(r,0)$	$\pi_s(r,1)$	$r$	$\pi_s(r,0)$	$\pi_s(r,1)$	$r$	$\pi_s(r,0)$	$\pi_s(r,1)$
0	-400	-400	0.34	-318.42317	-372.569	0.68	491.19853	-113.686
0.02	-395.20083	-398.425	0.36	-313.60581	-371.11	0.7	488.00791	-66.5564
0.04	-390.39762	-396.812	0.38	-308.80256	-368.393	0.72	484.80347	-14.9423
0.06	-385.58618	-395.084	0.4	536.00323	-369.123	0.74	481.60481	-15.133
0.08	-380.8047	-393.701	0.42	532.78161	-364.719	0.76	478.39913	60.30182
0.1	-376.00264	-391.899	0.44	529.60345	-363.421	0.78	475.19648	65.66602
0.12	-371.20784	-390.337	0.46	526.39823	-365.034	0.8	472.00371	96.42518
0.14	-366.42701	-388.127	0.48	523.19696	-361.358	0.82	468.79724	107.7686
0.16	-361.59952	-387.325	0.5	520.00213	-359.373	0.84	465.60231	147.6994
0.18	-356.80424	-385.611	0.52	516.77261	-359.893	0.86	462.39441	177.4516
0.2	-351.99001	-383.383	0.54	513.59462	-357.653	0.88	459.19323	199.3029
0.22	-347.17982	-382.023	0.56	510.39575	-356.734	0.9	456.0032	191.5415
0.24	-342.40118	-380.497	0.58	507.19789	-353.559	0.92	452.79636	198.4991
0.26	-337.61035	-379.132	0.6	504.0069	-322.223	0.94	449.59776	224.7047
0.28	-332.79298	-377.815	0.62	500.80268	-228.407	0.96	446.40195	222.2397
0.3	-327.98289	-376.456	0.64	497.60059	-204.786	0.98	443.20104	221.3987
0.32	-323.20751	-375.038	0.66	494.38826	-166.661	1	440	255.6478

To illustrate the influence of the power of the labor union, we examine the case  $c = 1$ , meaning that shirking workers are fully compensated if being union members. This is presented in  $\pi_s(r,1)$  columns in the Table 3. It is evident that in this case the performance of the firm deteriorates significantly. Namely, workers, both unionized and non-unionized, become more prone to shirking, inducing the principal more costs of inspection. The reason for that is that in choosing the strategy to play, non-unionized workers compare also strategies of the unionized workers, who are more likely to shirk. Despite non-unionized workers are not allowed to adopt the unionized status from unionized workers, they are free to adopt the strategy of shirking from a unionized workers if they see it beneficial. Therefore, at the optimal level of inspection at  $r = 1$ , only 78.7 percent of all workers work in the equilibrium,

resulting in the profit of the firm to be  $\pi_s(1,1) = 255.65$ . This is a little more than 58 percent of the profit that firm gets in  $\pi_s(1,0) = 440$ , and a little less than 48 percent of the profit when comparing with the optimal solution  $\pi_s(0.4,0) = 536$ .

### *Agent's performance*

We now turn to the performance of agents and first assume  $c = 0$ . Table 4 depicts results of simulations.

TABLE 4: Agent's utility when  $c = 0$

$r$	$u_i^S(r)$	$u_i^U(r)$	$u_i^L(r)$	$r$	$u_i^S(r)$	$u_i^U(r)$	$u_i^L(r)$	$r$	$u_i^S(r)$	$u_i^U(r)$	$u_i^L(r)$
0	0.3921426	0.38	0.4	0.34	0.2560632	0.244023	0.264023	0.68	0.2667815	0.255001	0.275001
0.02	0.3840234	0.372001	0.392001	0.36	0.2481008	0.236006	0.256006	0.7	0.2670871	0.254992	0.274992
0.04	0.3761476	0.363998	0.383998	0.38	0.2401252	0.228003	0.248003	0.72	0.2669141	0.254997	0.274997
0.06	0.3678712	0.355986	0.375986	0.4	0.2668868	0.254997	0.274997	0.74	0.2668778	0.254995	0.274995
0.08	0.3603021	0.348005	0.368005	0.42	0.2671108	0.255018	0.275018	0.76	0.2670883	0.255001	0.275001
0.1	0.35204	0.340003	0.360003	0.44	0.2670192	0.254997	0.274997	0.78	0.2669809	0.255004	0.275004
0.12	0.3441328	0.332008	0.352008	0.46	0.2668792	0.255002	0.275002	0.8	0.2669437	0.254996	0.274996
0.14	0.336307	0.324027	0.344027	0.48	0.266938	0.255003	0.275003	0.82	0.2669804	0.255003	0.275003
0.16	0.3280069	0.316	0.336	0.5	0.2670505	0.254998	0.274998	0.84	0.2670377	0.254998	0.274998
0.18	0.3201192	0.308004	0.328004	0.52	0.26692	0.255027	0.275027	0.86	0.267023	0.255006	0.275006
0.2	0.3122924	0.29999	0.31999	0.54	0.2672654	0.255005	0.275005	0.88	0.2670042	0.255007	0.275007
0.22	0.3038824	0.29198	0.31198	0.56	0.2669917	0.255004	0.275004	0.9	0.2669268	0.254997	0.274997
0.24	0.2959962	0.284001	0.304001	0.58	0.2669371	0.255002	0.275002	0.92	0.2667586	0.255004	0.275004
0.26	0.2881478	0.27601	0.29601	0.6	0.2669705	0.254993	0.274993	0.94	0.2670096	0.255002	0.275002
0.28	0.280013	0.267993	0.287993	0.62	0.2671649	0.254997	0.274997	0.96	0.2672655	0.254998	0.274998
0.3	0.2720503	0.259983	0.279983	0.64	0.267132	0.254999	0.274999	0.98	0.266959	0.254999	0.274999
0.32	0.2641625	0.252008	0.272008	0.66	0.2673117	0.255012	0.275012	1	0.2671374	0.255	0.275

In the Table 4,  $r$  denotes the probability that the principal  $P$  inspects an agent  $A_i$ ,  $u_i^S(r)$  is the average utility of both union and non-union members,  $u_i^U(r)$  is the average utility of unionized worker  $U_i$ , and  $u_i^L(r)$  is the average utility of non-unionized worker,  $L_i$ . It is evident from the simulation that unionized workers are always worse off than non-unionized.



As unionized workers do not get any benefit if the union has no power, this happens for  $c = 0$ , this difference occurs due to paying a strictly positive unionization membership fee,  $f \gg 0$ .

On the other extreme, we have the case of a powerful union with the value  $c = 1$ . This means that unionized workers are fully compensated if caught for shirking. Simulation results are given in Table 5.

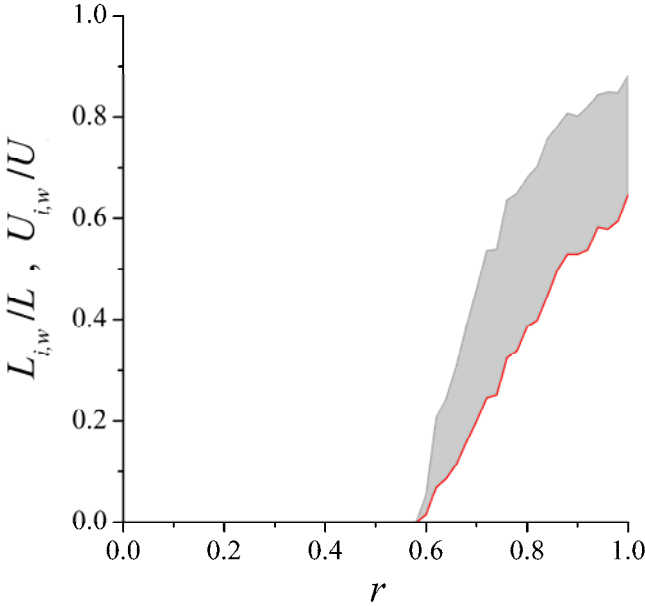
TABLE 5: Agent's utility when  $c = 1$

$r$	$u_i(r)$	$u_i^U(r)$	$u_i^L(r)$	$r$	$u_i(r)$	$u_i^U(r)$	$u_i^L(r)$	$r$	$u_i(r)$	$u_i^U(r)$	$u_i^L(r)$
0	0.39213	0.38	0.4	0.34	0.310202	0.38	0.263983	0.68	0.255352	0.360196	0.184945
0.02	0.387173	0.38	0.392007	0.36	0.305525	0.38	0.25603	0.7	0.256808	0.355097	0.191077
0.04	0.382394	0.38	0.383996	0.38	0.299753	0.38	0.248014	0.72	0.259131	0.349286	0.199278
0.06	0.377574	0.38	0.375987	0.4	0.296983	0.38	0.240004	0.74	0.257683	0.348594	0.195931
0.08	0.372836	0.38	0.367994	0.42	0.289719	0.38	0.231998	0.76	0.260372	0.339449	0.209815
0.1	0.367947	0.38	0.35999	0.44	0.285178	0.38	0.22401	0.78	0.260605	0.337656	0.209264
0.12	0.363167	0.38	0.352016	0.46	0.283234	0.38	0.21599	0.8	0.26025	0.331744	0.212736
0.14	0.357967	0.38	0.343999	0.48	0.276583	0.38	0.207996	0.82	0.260778	0.330184	0.214436
0.16	0.353683	0.38	0.335982	0.5	0.271435	0.38	0.199995	0.84	0.263429	0.324294	0.223399
0.18	0.348808	0.38	0.328002	0.52	0.268551	0.38	0.192017	0.86	0.262852	0.317812	0.227099
0.2	0.34354	0.38	0.320021	0.54	0.263173	0.38	0.183981	0.88	0.263462	0.313885	0.231242
0.22	0.33891	0.38	0.312012	0.56	0.258997	0.38	0.175988	0.9	0.262543	0.313879	0.228372
0.24	0.334162	0.38	0.304015	0.58	0.252762	0.38	0.167978	0.92	0.263802	0.312818	0.230921
0.26	0.329547	0.38	0.296016	0.6	0.251408	0.378077	0.166125	0.94	0.264365	0.307188	0.235951
0.28	0.324977	0.38	0.288007	0.62	0.254283	0.371299	0.177362	0.96	0.264577	0.307699	0.235918
0.3	0.320378	0.38	0.279985	0.64	0.253126	0.369173	0.176002	0.98	0.263973	0.305674	0.234527
0.32	0.315738	0.38	0.271996	0.66	0.254842	0.3656	0.17906	1	0.264932	0.299194	0.242126

We use the same notation as in Table 4. Results demonstrate that in the case of  $c = 1$ , union members almost surely outperform non-unionized workers. It is to be expected that if non-unionized workers had had an opportunity to become a union member, they would have eagerly do that and shirk for any  $r \in [0.06, 1]$ , where their cost-benefit analysis would show that union-related benefits outweigh the costs of the membership,  $f$ .

To present the influence of labor union on working habits of workers, we present the share of unionized workers who decide to work,  $U_{i,w}/U$ , and the share of those non-unionized workers who decide to work,  $L_{i,w}/L$ , respectively.  $U = \sum_{i=1}^n (A_i(u_i) | u_i = 1)$  is a set of unionized workers and  $L = \sum_{i=1}^n (A_i(u_i) | u_i = 0)$  is a set of non-unionized workers and in both cases a subscript  $W$  relates to those who are working. Figure 3 depicts the share of agents that work within both groups (unionized and non-unionized) when  $c = 1$ .

FIGURE 3:  $U_{i,w}/U$  and  $L_{i,w}/L$



Shaded area in the figure indicates the difference between the relative share of union-members who are working (bottom line) and the relative share of non-union-members who are working as well (upper line). The picture reveals that unionized workers are more prone to shirking than non-unionized workers are. Since unionized workers are fully compensated for

shirking, this is as expected. However, in the case of the full inspection, where  $r = 1$ , more than 60 percent of unionized workers are working in the equilibrium although they are fully compensated for shirking, which is surprising.

### *5.3 Evolutionary inspection game with endogenous labor union*

Finally, we relax the assumption of exogenously determined unionization status and make it endogenous. This means that when an agent  $A_i$  decides, which strategy of an agent  $A_j$  to adopt, he does not only consider the strategy he takes, but also the unionization status of an agent.

#### *Firm's performance*

Figure 4 depicts the performance of the firm (color coded) as a function of the inspection rate and compensation rate,  $\pi(r, c)$ . In the Table 6 we see that the maximum attained profit equals  $\pi = \{\pi(r, c) | r = 0.4 \wedge c \in [0, 0.26]\} = 536$  units. This is the same as in the no unionization case,  $\pi(r)$  in Table 1. However, for  $c$  approaching 1, the performance of the firm deteriorates significantly and in a case  $\pi(r = 1, c = 1) = -560$  it suffers a loss of 560 units. This is because no one works in the equilibrium, while principal is obliged to pay wages to the agents. Of course, such firm is incapable of surviving in the long run and is likely to go bankrupt.

FIGURE 4: The average income of the firm when unionization is not fixed

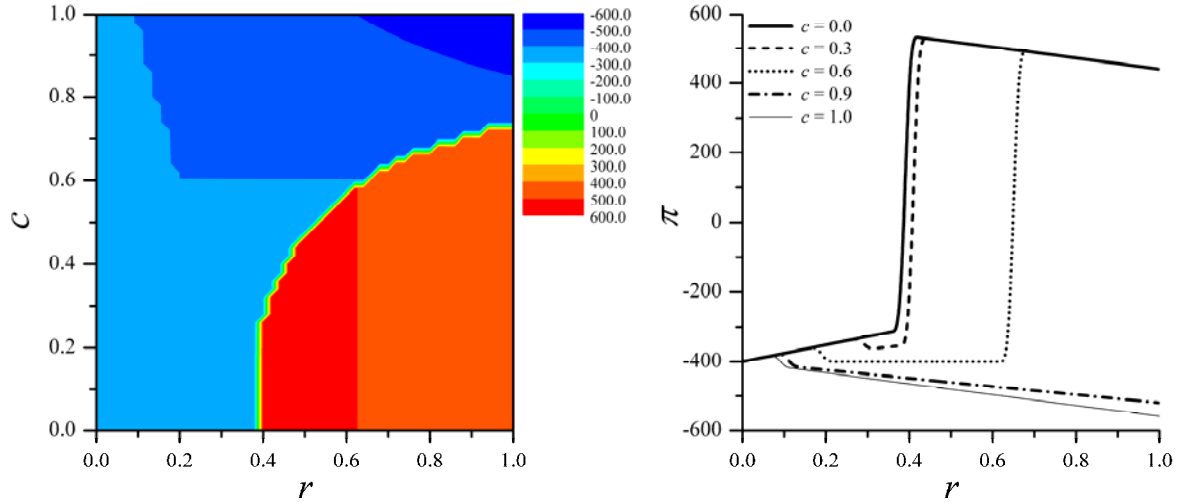


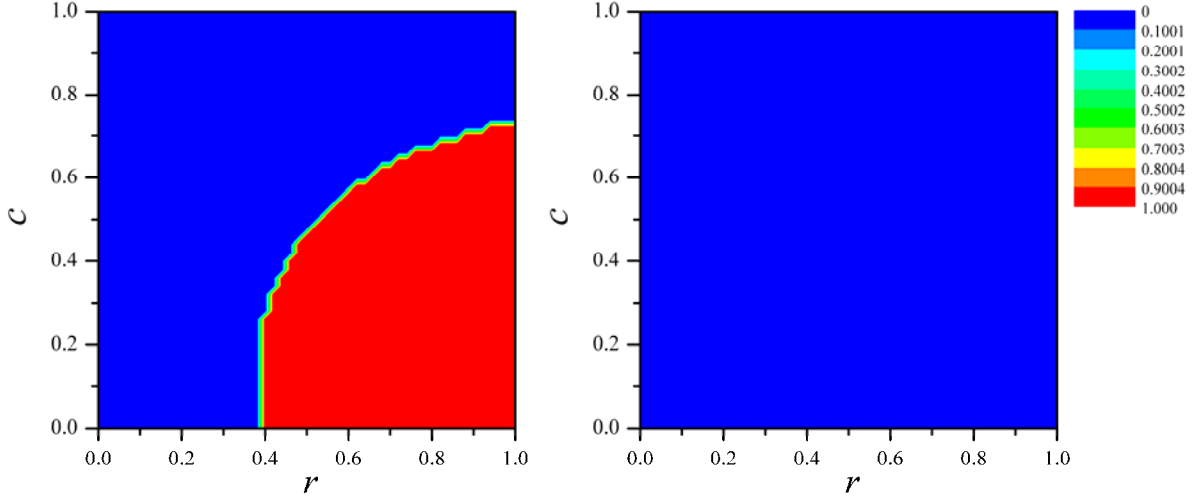
TABLE 6: Firm's profit as a function of inspection rate with  $c = 0$  and  $c = 1$

$r$	$\pi_s(r,0)$	$\pi_s(r,1)$	$r$	$\pi_s(r,0)$	$\pi_s(r,1)$	$r$	$\pi_s(r,0)$	$\pi_s(r,1)$
0	-400	-400	0.34	-318.39406	-454.40098	0.68	491.19724	-508.80282
0.02	-395.19664	-395.19641	0.36	-313.58888	-457.60955	0.7	487.98949	-511.99315
0.04	-390.3963	-390.39536	0.38	-308.82458	-460.80293	0.72	484.79725	-515.19845
0.06	-385.60074	-385.60082	0.4	535.99896	-463.98951	0.74	481.59888	-518.39487
0.08	-380.79271	-380.78985	0.42	532.80465	-467.20759	0.76	478.40308	-521.58528
0.1	-376.00013	-415.99823	0.44	529.59258	-470.39368	0.78	475.19713	-524.80106
0.12	-371.20487	-419.19994	0.46	526.39438	-473.60333	0.8	471.99057	-527.99664
0.14	-366.39383	-422.3977	0.48	523.19049	-476.7758	0.82	468.81289	-531.19181
0.16	-361.59437	-425.60253	0.5	519.98211	-479.98721	0.84	465.60043	-534.40178
0.18	-356.79382	-428.79709	0.52	516.81303	-483.19938	0.86	462.39849	-537.60126
0.2	-351.98737	-432.00354	0.54	513.59567	-486.3976	0.88	459.20009	-540.80515
0.22	-347.1803	-435.20227	0.56	510.41327	-489.60337	0.9	456.00296	-544.00109
0.24	-342.38659	-438.39881	0.58	507.18891	-492.79809	0.92	452.79736	-547.19967
0.26	-337.5918	-441.60554	0.6	504.01756	-496.01101	0.94	449.59835	-550.3998
0.28	-332.80126	-444.79377	0.62	500.81351	-499.19728	0.96	446.39617	-553.598
0.3	-328.00458	-447.99963	0.64	497.60529	-502.40141	0.98	443.19919	-556.79996
0.32	-323.20204	-451.21198	0.66	494.4136	-505.60043	1	440	-560

### Agent's performance

Figure 5 illustrates a fraction of non-unionized (left picture) and unionized workers (right picture) who work within non-unionized and unionized workers.

FIGURE 5: Fractions of agents that work in the firm with non-constant union membership



As predicted, unionized workers never work in the equilibrium. The reason for that is that  $WU$  is strictly dominated strategy for all  $r, c \in [0,1]$ . In the case  $c = 0$ , model simplifies to the one without labor union and the first payoffs matrix applies. In the equilibrium, no worker is unionized and  $U = \{ \}$  is an empty set for all  $r \in [0,1]$ . Such solution is a consequence of the fact that the unionization status is a subject of paying a strictly positive unionization fee  $f \gg 0$ , borne out by every unionization worker, and brings no benefits.

In the case  $c = 1$ , the strategy of each agent to work is strictly dominated for all  $r \in [0,1]$ . However, in such situation it is not always optimal for agents to become unionized. Table 7 depicts the utility of each agent in the game. Here  $r$  denotes to the probability that principal will inspect,  $u_i^s(r,1)$  denotes a utility of an agent  $A_i$ , and  $u_i \in \{0,1\}$  is a binary variable, which denotes a unionization status. If an agent is unionized  $u_i = 1$ , and  $u_i = 0$  if he is not.

TABLE 7: Agent's utility as a function of  $r \in [0,1]$  when  $c = 1$

$r$	$u_i^S(r,1,u)$	$u_i \in \{0,1\}$	$r$	$u_i^S(r,1,u)$	$u_i \in \{0,1\}$	$r$	$u_i^S(r,1,u)$	$u_i \in \{0,1\}$
0	0.400	0	0.34	0.380001	1	0.68	0.3800028	1
0.02	0.3919964	0	0.36	0.3800096	1	0.7	0.3799932	1
0.04	0.3839954	0	0.38	0.3800029	1	0.72	0.3799985	1
0.06	0.3760008	0	0.4	0.3799895	1	0.74	0.3799949	1
0.08	0.3679899	0	0.42	0.3800076	1	0.76	0.3799853	1
0.1	0.3799982	1	0.44	0.3799937	1	0.78	0.3800011	1
0.12	0.3799999	1	0.46	0.3800033	1	0.8	0.3799966	1
0.14	0.3799977	1	0.48	0.3799758	1	0.82	0.3799918	1
0.16	0.3800025	1	0.5	0.3799872	1	0.84	0.3800018	1
0.18	0.3799971	1	0.52	0.3799994	1	0.86	0.3800013	1
0.2	0.3800035	1	0.54	0.3799976	1	0.88	0.3800052	1
0.22	0.3800023	1	0.56	0.3800034	1	0.9	0.3800011	1
0.24	0.3799988	1	0.58	0.3799981	1	0.92	0.3799997	1
0.26	0.3800055	1	0.6	0.380011	1	0.94	0.3799998	1
0.28	0.3799938	1	0.62	0.3799973	1	0.96	0.3799998	1
0.3	0.3799996	1	0.64	0.3800014	1	0.98	0.380	1
0.32	0.380012	1	0.66	0.3800004	1	1	0.380	1

For  $r = 0.06$  and  $r = 0.08$ , a fully omniscient agent are better off than a non-omniscient agent from the simulation. Because the minimum level of utility that each fully omniscient agent can get is  $u_i(r,1,1) = 0.38$  for all  $r \in [0,1]$ . This result is strictly higher than  $u_i(0.06,1,1) = 0.376$  and  $u_i(0.08,1,1) = 0.368$ . Again, this is the cost of non-omniscience of agents.

## 6. Concluding remarks

In the paper, we applied small world network to the modified principal-agent inspection game, to which we included a labor union that appears as a potential escape hatch for shirking unionized workers. We were particularly interested how the power of a labor union influences working habits of employees, and thereby the performance of the firm. We get some inspiring results.

The presence of a labor union *per se* does not have an impact on the output of the firm. However, powerful labor unions that are capable to arrange for a fully compensation of shirking unionized workers by the firm, significantly enter the work/shirk tradeoff in favor of shirking and increased unionization rates, altering the efficiency of the firm significantly. In particular, if the benefits accruing from the power of a labor union exceed the costs of a membership, then adopting strategies that encourage unionization can increase the utility of agents, while they lower the utility of the firm. On the other hand, agents are not motivated to join the labor union if benefits of being unionized do not outweigh the costs. Our findings are in line with Black and Lynch (2001), who argue, that the effect of labor unions in the economy is not given but depends highly on the specific economic and labor relations environment in which unions operate. In the paper, this is represented through the level of pecuniary compensation to the shirking unionized workers.

Simulations proved that the inspection could be an efficient method, by which principal can make workers to work. Simulations also showed that it is not necessary to inspect every single worker in order to “motivate” him to work. However, as the power of the labor unions is increasing, this also forces principal to increase the level of inspection, which raise the costs of a firm and reduces the utility and the profits of a firm.

By using non-omniscient agents, we also estimated the costs of non-omniscience of workers, which can explain the “out-of-equilibrium” solutions. We find that the cost to be very significant for both, employees and the firm.

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