Stochastic Processes in Finance and Behavioral Finance

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ABSTRACT
In the paper, we put the foundations for studying asset pricing and finance as a stochastic and behavioral process. In such process, preferences and psychology of agents represent the most important factor in the decision-making of people. Individuals have their own ways of acquiring the information they need, how to deal with them and how to make predictions and decisions. People usually also do not behave consistent in time, but learn. Therefore, in order to understand the behavior on the markets, a new paradigm is needed.

Keywords: asset pricing and finance, stochastic processes, behavior of agents.
JEL Classification: G12, G14, C91.

STOCHASTIC PROCESSES IN FINANCE AND BEHAVIORAL FINANCE

1 Introduction

Daily movements in asset prices are the main characteristic in financial markets, by now very hard to be forecasted with certainty. The reason for that is that it is impossible to predict the behavior of agents in the markets, as people use their own reasoning in the decision-making. Therefore, the only common knowledge in the society could be summed in the words of Hayek (1945, pp. 519) that "knowledge of the circumstances of which we must make never exist in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess."

This explains why market participants make their own predictions of the stock prices according to their own knowledge and their own "subjective formation of expectations,"[1] which makes such agent's behavior and price formation impossible to forecast with certainty. On the other hand, such daily price movements give evidence that agents on the market agree that they disagree about the prices. Therefore, the fundamental question people face in their asset management is what is the best strategy for playing the stock market and to a what extent can the past movements in stock prices be used to make predictions of the future prices? Is it better to be focused on "fundamentals," whatever they are and anyhow they can be measured, or to follow the "psychology of the market," whatever this is? These questions also lie in the heart of the Markowitz's portfolio selection (Markowitz 1952) and provided "answers" within two concepts: finance as a predictable deterministic process and as a non-predictable stochastic process.

To give some insights into the financial market, we present finance as a stochastic process, where psychology of people is the most important element. To define the stochastic process first, we would say that a stochastic process is a variable that evolves over time in a way that is at least partly random. Weather is a stochastic process. Namely, variation in temperatures is partly deterministic and predictable, we can expect to have warm summers and cold winters and that temperatures rise during the day and fall at night, and partly random and unpredictable.[2] However, if we can predict what the temperature of the January 1 will be over the next two years or even what temperature we will have tomorrow, it is impossible to predict the price of a stock of Google over the next two years. Why is that? This is because weather is a (weakly) stationary process, which means that statistical properties of the variable are constant over time, while asset prices are non-stationary, meaning that the expected value can grow, or fall, without bounds. Precisely this stochastic nature of asset prices is what makes opportunities to individuals for increasing their wealth, but also threats of losing it. Therefore, individuals are taking risks.

[1] See Arthur (1995).
[2] See Dixit and Pindyck (1994).

The paper proceeds as follows. Section 2 describes finance as a stochastic process, which is followed with an agent-based representation of finance in Section 3. Section 4 is about how knowledge of people affects market behavior, giving credits to the prospect theory. Section 5 gives some insights into the utility maximization principles under risk, while Section 6 is about the herd behavior. The paper ends with short conclusions.

2 Finance as a stochastic process

Markov processes

The fundamental variable in asset pricing is the price. We generally say that a price is the information that is determined through the two processes of supply and demand. We can also consider the formation of prices through the bargaining processes within the matching framework and say that bargaining power of both agents after including their preferences defines the price. Of course, neither supply nor demand develops according to some predefined rule, but through knowledge and expectations of people, which change in time very irregularly. Therefore, prices also change in time in a non-predictive way (see Tab. 1). Note that matching is about the expectations of people and that returns just reflect changes in prices.

TABLE 1: *Stock market returns 1962-1994*

Security	Mean	SD	Skewness	Kurtosis	Min	Max
Daily Returns						
IBM	0,039	1,42	-0,18	15,48	-22,96	11,72
General Signal	0,054	1,66	0,01	6,35	-13,46	9,43
Wrigley	0,072	1,45	-0,00	14,03	-18,67	11,89
Interlake	0,043	2,16	0,72	15,35	-17,24	23,08
Raytech	0,050	3,39	2,25	62,40	-57,90	75,00
Ampco-Pittsburgh	0,053	2,41	0,66	8,02	-19,05	19,18
Energen	0,054	1,41	0,27	8,91	-12,82	11,11
General Host	0,070	2,79	0,74	9,18	-23,53	22,92
Garan	0,079	2,35	0,72	10,13	-16,67	19,07
Cont. Materials	0,143	5,24	0,93	9,49	-26,92	50,00
Monthly Returns						
IBM	0,81	6,18	-0,14	3,83	-26,19	18,95
General Signal	1,17	8,19	-0,02	4,87	-36,77	29,78
Wrigley	1,51	6,68	0,30	4,31	-20,26	29,72
Interlake	0,86	9,38	0,67	7,09	-30,28	54,84
Raytech	0,83	14,88	2,73	25,70	-45,65	142,11
Ampco-Pittsburgh	1,06	10,64	0,77	5,04	-36,08	46,94
Energen	1,10	5,75	1,47	15,47	-24,61	48,36
General Host	1,33	11,67	0,35	4,11	-38,05	42,86
Garan	1,64	11,30	0,76	5,30	-35,48	51,60
Cont. Materials	1,64	17,67	1,13	6,33	-58,09	84,78

Source: Campbell, Lo, and MacKinlay (1997).

Economists and other financial experts have since ever asked them a question, whether and in what extent could history be used in analyzing future price movements? Contrary to chartists who assume that history repeats itself, proponents of stochastic finance assume, that future price movements are no easier to be forecasted than the path of a series of cumulated random numbers, because price changes have no memory but are independent in time.[3] History fills people with experience, but it does not *per se* provide any knowledge, which could directly be used for estimating the future. Very frequent representation of such process is to describe it as a Markov chain process.

A discrete time stochastic process $\{x_t\}$ is said to be a Markov process, if for all $k \geq 1$ and all t, $\Pr\left(x_{t+1} \left| x_t, x_{t-1}, x \ldots, x_{t-k}\right.\right) = \Pr\left(x_{t+1} \left| x_t\right.\right)$. This means that in time t_0 an individual in state x_0 makes a decision a_t which determines his outcome in t_1 that equals x_1. Once we know x_t, all the previous values become unimportant. The present state is the product of the past, but only a current state influences to the future decisions.

Markov property is in accordance with the weak form of the efficiency market hypothesis, that the current price of an asset involves all the past information about such asset. However, an important implication of a Markov process is that past returns do not warrant future returns. The intuition is close to that of von Mises (1996, pp. 31) that "every experience is something that passed away, [while] there is no experience of future happenings." Albert Einstein (1923, pp. 3) once raised an interesting question: "How can mathematics, a product of human reason that does not depend on any experience, so exquisitely fit the objects of reality? Is human reason able to discover, unaided by experience through pure reasoning the features of real things?" and answered: "As far as the theorems of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."

Returns as martingales

The concept of martingales in probability theory was introduced by Paul Pierre Lévy. The intuition of martingales tries to answer the logical question: why would someone sell the stock in time period t if he knew that he would be able to get higher return in the period $t+1$? Some answers can be found in the prospect theory of Kahneman and Tversky (1979), as people do have different knowledge and capabilities of logical reasoning, from which they make different expectations of the same things.

A stochastic process is a martingale if $E_t\left(x_{t+1} \left| \Omega_t\right.\right) = x_t$, where Ω_t denotes information matrix in time t, and E_t denotes expectations in time t. The equation says that in time t an expected price of an asset in the next period t_{t+1} as regards the set of information (or knowledge) in time t equals the price in time t. Individuals that would not pursue to such rule would refuse to some expected profit. There would be some room for the arbitrage in a case $x_{t+1} \neq x_t$.

[3] Theoretically, both theories hold true. Because people make prices by their bid and ask actions, so long as all of them believe in Chartism, stock prices behave in that way. So long that some do not believe in the power of history, prices do not behave according to their historical paths. Because not everyone believes in the historical behavior of prices, they behave in a stochastic way. An important argument in favor of that is the changing economic conditions and prospects of firms.

Efficient stock markets are usually construed to mean that the price of a stock follows a martingale, while this implies an implicit assumption of the knowledge of people to be a common knowledge. When using martingales in asset pricing, no stock bubbles are possible.

After including probabilities into the martingale equation, we get a sub-martingale equation $E_t\left(x_{t+1}\left|\Omega_t\right.\right)\geq x_t$, which allows stock bubbles, and a super-martingale equation, respectively $E_t\left(x_{t+1}\left|\Omega_t\right.\right)\leq x_t$, which allows stock crashes. We can also transform the sub-martingale equation and get, $E_t\left(x_{t+1}\left|\Omega_t\right.\right)=x_t+\varepsilon_t$, where ε_t represents a martingale error, and super-martingale into $E_t\left(x_{t+1}\left|\Omega_t\right.\right)=x_t-\varepsilon_t$, respectively.

However, when using the theory of martingales it is impossible to predict future returns using the information matrix Ω_t. Namely, the probability for assets to either rise or fall is the same. Barberis et al. (1998) give some arguments for that, as they argue that when some information becomes public, individuals respond either very insignificant or excessive, which corresponds to their previous expectations, and this all fills the martingale error term in any of the two directions.

The aim of introducing the martingale measure is twofold: firstly, it simplifies the explicit evaluation of arbitrage prices of derivative securities, and secondly, it describes the arbitrage-free property of a given pricing model for primary securities in terms of the behavior of relative prices. However, their use is only intuitive.[4]

Random walk in finance

Random walk describes a sequence that develops in time according to $x_{t+1}=x_t+\varepsilon_{t+1}$, where ε_{t+1} represents a white noise. A white noise is a sequence not correlated in time, with the zero expected value and variance σ^2; [$E_t\left(\varepsilon_{t+1}\right)=0$, $E_t\left(\varepsilon_{t+1}^2\right)=\sigma^2$ and $E_t\left(\varepsilon_t,\varepsilon_\tau\right)=0;\ t\neq\tau$].

As argued by Fama (1965), random walk theory involves two separate hypotheses, one is that price changes are independent, and the other that price changes conform to some probability distribution. We say that the sequence is independent if $\Pr\left(x_t\left|x_{t-1},x_{t-2},\ldots\right.\right)=\Pr\left(x_t=x\right)$.

In random walk models, in time t the expected future value of the variable x in time $t+1$ equals its value in time t: $x_t=E_t\left(x_{t+1}\right)$, which can be reformulated into $E_t\left(\frac{x_{t+1}}{x_t}\right)=1$. From this expression we see, that the future returns of assets cannot be predicted, as the sequence has no historical memory that could be used to predict future returns (Fama 1965). When

[4] A detailed survey on martingales is available in Duffie (1996).

treating a random walk model as a special case of an AR(1) process, then the coefficient x_t is unity, which implies a random walk to be a unit-root non-stationary time series.[5]

Some modifications of the random walk hypotheses add a drift to the model. Such sequence of the random-walk changes into $x_{t+1} = \mu + x_t + \varepsilon_{t+1}$, where μ represents the drift, and $\mu = E_t\left(x_{t+1} - x_t\right)$. If we assume a stationary sequence, the expected drift is constant in time and represents a time trend. As is argued by Tsay (2002), it is important to understand the meaning of a constant term in a time series model, where for a random walk with a drift, the constant term becomes the time slope.

Brownian motion in finance

Brownian motion is the implementation of the random walk in a continuous time. In discrete time, we denoted the exogenous shock as a white noise sequence, but in the continuous time, a stochastic process is implemented through the set of stochastic differential equations as a Brownian motion.

Fundamentals of the Brownian motion were set up in 1827, when a botanist Robert Brown observed a disordered motion of particles of pollens of plants in the water. A couple of years later, in 1900, Louis Bachelier used the Brownian motion to study stochastic movements in finance, while Albert Einstein set up the physical theory of Brownian motion in 1905. Einstein suggests that this motion is random and has the following properties. It has independent increments; the increments are Gaussian random variables; and the motion is continuous. Due to Norbert Wiener, Brownian motion is also known as the Wiener process, for which it is usually denoted as w_t and the change of the Wiener process is given as $\Delta w = \varepsilon_t \sqrt{\Delta t}$, with $\varepsilon_t \sim N\left(0, 1\right)$.

To develop Brownian motion, let us return to the discrete-time sequence of the random walk; $x_{t+1} = x_t + \varepsilon_{t+1}$, where ε_{t+1} is the stochastic white noise sequence. If the process starts at $x_0 = 0$ then the equivalent representation of the process equals to $x_{t+1} = \varepsilon_0 + \varepsilon_1 + \ldots + \varepsilon_t + \varepsilon_{t+1}$, where $x_{t+1} \sim N\left(0, t+1\right)$. Without a loss of generality, we can also write $x_t = \varepsilon_0 + \varepsilon_1 + \ldots + \varepsilon_t$, where $x_t \sim N\left(0, t\right)$.

Let us now give the change of the variable x in two different time dates, t and s, which can be represented as $x_s - x_t = \varepsilon_{t+1} + \varepsilon_{t+2} + \ldots + \varepsilon_s$, where $N\left(0, \left(s-t\right)\right)$. By partitioning the change of the variable between the two periods into N sub-periods, we get

[5] AR(1) process is an autoregressive process of order one and is defined as $x_{t+1} = \rho x_t + \varepsilon_{t+1}$, where ε_{t+1} is a white noise with zero mean and variance σ^2. For $\rho = 1$, AR(1) process becomes a random walk process.

$x_t - x_{t-1} = \varepsilon_{1t} + \varepsilon_{2t} + \ldots + \varepsilon_{Nt}$, where $\varepsilon_{it} \sim N(0, 1/N)$ and $x_t - x_{t-1} \sim N(0, 1)$. The limit as $N \to \infty$ is a continuous-time stochastic process, known as Brownian motion.[6]

Generalized Brownian motion is of the form $dx_t = \mu dt + \sigma dw_t$, where μ is the drift, w_t denotes a Brownian motion and σ the variance.

Carr and Wu (2004) strictly oppose the use of a Brownian motion in asset pricing for several reasons. First are stochastic shocks, where neither direction nor their size could be foreseen. The second argument against is the stochastic volatility in time, while variance is the principal variable in the Brownian motion. The last is due to the correlation between returns and volatility.

3 Agent-based finance

3.1 Agent-based finance in review

Agent based finance give some completely new insights into the asset pricing, which are based on a rejection of the Arrow-Debreu equilibrium and in much smaller use of mathematics. Hayek and Arthur give some arguments in favor of out-of-equilibrium approach to economic activities. In these respect, Arthur (2006, pp. 2) argues, “Economic agents – banks, consumers, firms, investors – continually adjust their market moves, buying decisions, prices, and forecasts to the situation these moves or decisions or prices or forecasts together create. … individual behaviors collectively create an aggregate outcome; and they react to this outcome. … Behavior creates pattern; and pattern in turn influences behavior.” Almost seventy years before Hayek (1937, pp. 37) wrote, “The situation is, however, different with the plans determined upon simultaneously but independently by a number of persons. … in order that all these plans can be carried out, it is necessary for them to be based on the expectation of the same set of external events, since, if different people were to base their plans on conflicting expectations, no set of external events could make the execution of all these plans possible.” Hayek (1937, pp. 38) then adds, “It is necessary for the compatibility of the different plans that the plans of the one contain exactly those actions which form the data for the plans of the other.”

The agent-based economics thus follows these Hayekian ideas as also the intellectual work of Herbert Simon (1955, 1957), Thomas Schelling (1978) and Robert Axelrod (1984). Some of the first attempts of agent-based modeling in finance were done by Garman (1976). Arifovic (1996) introduced genetic algorithm models into modeling exchange rates, which involves extensive learning of agents. On the other hand, Brock and Hommes (1998) and Brock and LeBaron (1996) endogenized the decision-making of people, made it the process of the interaction among them. Artificial stock exchange market was done within the Santa Fe institute (Palmer et al. 1994, LeBaron et al. 1999). Tay and Linn (2001) introduced fuzzy logic in agent’s learning processes. Some other models involve herd behavior Gode and

[6] For detailed representation of a Brownian motion, see Hamilton (1994), Tsay (2002), Duffie (1996).

Sunder (1993), Lux (1997), and Levy et al. (1994). Odean (1999) argues of the overconfidence of agents. Ellsberg (1961) and Elster (1998) argue that emotions represent an important aspect of people's decision-making. Barsky and DeLong (1993) link the different expectations of people with variation in economic growth rates. On the other hand, Smith et al. (1988) show the occurrence of bubbles by using asset pricing laboratory experiments. For the rest of the literature on agent-based asset pricing and psychology, David Hirshleifer (2001) is a good reference.[7]

3.2 Agents in agent-based finance

Preferences of agents

Agents are central to the agent-based modeling. Namely, how we model agents in the game influences the ways in which the simulations will proceed. Therefore, omitting some important variables might reduce the predictive power of the simulation and results very much.

An individual is an agent and chooses among different alternatives, by which an agent suits his goals. In solving this problem, an agent chooses according to what is feasible to him, what is desirable to him, and what is the best alternative according the desirability of an individual given the feasibility constraints. However, in their decision-making agents prefer both optimal and simple strategies, which might be in an opposition under some circumstances and which forces agents into the strategic decision-making. In doing that, as Kahneman and Tversky (1979) argue, agents have strong desire to avoid the feeling of regret. However, they like to use efficient strategies. This means that agents are not prone of changing their strategies, which bring them satisfactory outcomes, despite the strategies they use do not bring them the highest possible outcome. When people are involved in a social interaction with others, Levine and Pesendorfer (2007) argue that they are prone to modify their preferences in such way that they select the most efficient strategies from the set of strategies of their friends. However, authors do not exclude the possibility that agents behave in a herd, which means that there is a possibility that agents do not change their behavior in order to take the most efficient strategies, but imitate the strategies of others no matter how efficient they are (see also Banerjee 1992, Lux 1995). In this respect, Aumann (1997), Rubinstein (1998), Selten (1975) argue for the *trembling hand perfection* principle.

When modeling agents, it is very important, as Tesfatsion (2006) argues, that privately motivated agents in an agent-based framework include economic, social, biological and physical entities, and that agents are able to communicate with each other by using different techniques. It is also important to allow that artificial agents have learning capabilities and are able to develop in time.

[7] In one of my previous work (Steinbacher et al. 2008), I simulate agents' decision-making of their portfolios using a small-world network.

People differ in the level of knowledge they have, capacity to think and reason, skills and experiences, emotions, social networks they are involved in, attitude towards risk, time, and different types of assets, wealth, luck and many other characteristics, which all are important elements in building one's preferences, which are so important for asset markets.[8] Gary Becker (1996) argues that individuals possess two kinds of capital: personal capital and social capital. Personal capital of an individual involves all of the previous experiences that influence or determine the current knowledge and behavior of an individual, as also his future knowledge and behavior. On the other hand, Becker defines a social capital as the sum of all external effects of other people that contribute to the current and future knowledge of people and contribute to the decision-making of people. Of course, personal and social capitals represent only a part of the entire human capital of people.

Akerlof and Kranton (2000) incorporate the identity of people, a person's sense of self, into their decision-making processes and consider how this influences their behavior and decision-making. We can expect that identity does affect the economic outcomes in the areas of consumption and savings behavior, attitude towards risk, labor relations and many other areas, which all affect the ways in which financial markets function. Becker (1996) sets out tastes of people. Besides tastes, other factors also influence people's decision-making.

Some others point out emotions of people (Ellsberg 1961 and Elster 1998). We can define emotions as a process or a state of a human mind, which arises from the relation of a human and an environment. What makes emotions so special and important in a decision-making of people is that they disarrange valuations, for which they should not be ignored in modeling behavior of agents. "Inflamed with passion, man sees the goal as more desirable and the price he has to pay for it as less burdensome than he would in cool deliberation."[9] However, what distinguishes people from animals is that people are able to rationalize their behavior.

Many argue that there are also habits and (dis)incentives because of the political decisions. In his paper, *Why do Americans Work so Much more than Europeans*, Prescott (2004) argues that marginal labor tax rates produce a great part of the difference, as taxes have an impact on labor/leisure decisions of people and investment decisions of businessmen. On the other hand, Alesina and Fuchs-Schündeln (2007) demonstrated that different economic regimes have a huge impact on the preferences of people. One of their findings is that people living under socialist regimes have much greater preferences towards redistribution, government intervention and higher taxes than people living in capitalist economies do have. This effect, as they argue, is especially strong for older cohort. This means that they are much more risk-averse and do lack for self-initiative and individual responsibilities.

[8] On the other hand, neither are assets fully substitutes for one another. Therefore, we have heterogeneous agents and different types of assets.

[9] See von Mises (1996, pp. 16).

Autonomy of agents

The most fundamental characteristic of people is action. People act and action is a purposeful behavior and a will put into operation, by which people satisfy their goals by employing the means if we take the definition of von Mises.

To say that people are autonomous beings is to say that there is no upside down control over their decision-making, but that people make decisions by themselves. By definition, people always make decisions by themselves, despite the fact that others might influence to the set of alternatives they have available. People are target-oriented beings, where their characteristics and foremost capacity to think and reason represent the sources of their decision-making. Therefore, when they face a problem, agents make decisions in order to solve it.

Even though agents behave in herds, they remain autonomous. This means that they acquire information, which contributes to the knowledge they have and from which they make expectations and make decisions according to their preferences. But their final decisions are always autonomous.

Social interaction

What makes an agent-based approach different from other computational and econometric techniques is that it allows for interaction between people, by which they improve their knowledge.[10] These not only accords to the fact that people are not alike but also that they are not isolated entities, have relatives, friends, colleagues and other acquaintances with whom they communicate and by which they acquire new information and improve their knowledge, which is then used for their future decisions.[11] Interaction among agents is required because the knowledge does not exist in a concentrated form but is dispersed, as argued by Hayek (1937, 1945).

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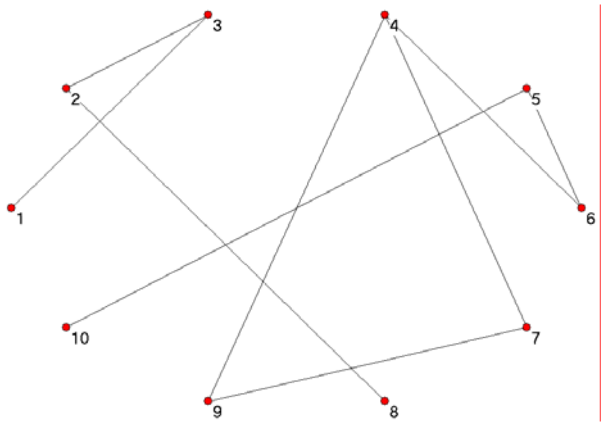
[10] Social interaction contributes to the changes of economic variables through following channels (Granovetter 2005). A social interaction influences the ways, in which individuals receive new information. The second is through reward and punishment. The third is through trust.

[11] See Ellison and Fudenberg (1995) for a short discussion on cognitive methods in acquiring information.

FIGURE 1: *Interaction among individuals*



Communication process is thus put into the center of studying economics as a process of many heterogeneous agents. Fig. 1 represents one such illustration among ten individuals. As we can see, it is important how many connections individuals have in such network and how much information and knowledge agents have.

An important characteristic of any social network is the “prestige” of individuals in the network. Two parameters define this: centrality and betweenness (Freeman 1977). More connections one has with others who also have many connections, usually such individual is more prestigious. However, it is important that people know people who have the right information.

For the efficient use of a social network, it is also important for individuals how capable they are in receiving information from others, how good the information they get from others is, how efficient they are to incorporate information into their knowledge and how good they use this in their decision-making. Another important aspect that affects the stock of individuals’ knowledge is how prone agents are in sharing the information and knowledge they have with others. People might overcome this difficulty by purchasing information on the market for information.

Responsiveness and reactive learning

In such social interaction, actions done by agents influence the others, while agents not only learn how to play such games, but also learn with whom to play. Agents that learn respond to the changes in their neighborhood, adapt their preferences, goals, means for achieving goals and strategies. These are dynamic agents. Contrary to them, static agents do not change their behavior in time.

In choosing their connections, agents face a difficulty, as individuals are not able to know the characteristics of all other agents and thus cannot know, with whom to go into the relation. In this context, Bavelas (1950) defines a concept of centrality, which is defined through the distance between particular agent on the society and others. When such distance from one agent to others is small, such agent is able to communicate with others very fast. Using this concept, Stephenson and Zelen (1989) argue that when choosing with whom to be connected, agents prefer individuals who have more connections with other agents. If the two agents want to be connected, such link must be agreed by both of them, while disagreement from only one agent suffices for breaking it.[12] However, the size of the network and the costs of an individual to be part of it determine the utility of an individual he receives from the network. Being involved within the network with very influential individuals could bring much higher utility its members, while membership in the network with non-influential individuals is expected to bring its members smaller utility.

An important element in reactive learning is the memory. Rubinstein (1998) defines it as a special kind of a knowledge, which defines what an agent knows at the certain time period about what has he knew at some previous date. And this is not perfect. People have imperfect recall, which limits their abilities of learning from their past behavior and actions. Imperfect recall is also the reason that actions of people might be inconsistent in time.

Rubinstein also brings forward some of the causes for the existence of constraints on the information held by an agent. The acquisition of information is costly; the information acquired often has to be stored in the memory before its use, and the memory is not unbounded; while information are received through the process of interaction that also has some of its own limits. If other agents are not prone of sharing their knowledge to others, such interaction does not spread the knowledge around the network.

Arthur (1991, 1992, 1994) argues that deductive reasoning does not work in making one's expectations, because of the limited capabilities of people's reasoning and logical thinking. Therefore, it is likely that agents use inductive reasoning instead.

Pro-activeness of agents

Individuals are target-oriented beings and they adjust their actions according to their preferences and expectations. "Action is will put into operation and transformed into an agency, is aiming at ends and goals, is the ego's meaningful response to stimuli and to the conditions of its environment, is a person's conscious adjustment to the state of the universe that determines his life."[13]

Among the most meaningful questions in agent-based economy is how agents adapt their strategies on the market? This could be done on several different ways. Bergh et al. (2002) distinguish among several different types of adaptation. Weak adaptation means that agents determine their action from their percepts in a static and fixed mapping. At the level of semi-weak adaptation mapping from the percepts can be modified, while in semi-strong adaptation agents can change also their goals, i.e. agents might change their attitude towards risk. In

[12] Such intuition is close to the concept of Nash equilibrium (Nash 1951).
[13] See von Mises (1996, pp. 11).

strong adaptation, agents can modify their intentions and manage the strategies for achieving their design goal. In managing their portfolios, investors' decisions about particular investments appear to be considered in isolation from the remainder of the investor's portfolio (Barberis et al. 2003).

However, in observing others, as also when receiving new information, agents must first know how to identify messages of others or those on the market. Other agents are usually not very much inclined to deliver their knowledge to others, especially not about important issues. In such situations, agents might try to decode the signals and messages that other agents had left on the market by their actions. An obstacle in following others is that agents can observe the situation after the action has been done, for which they are a step behind the leaders, unless they find the logic of their actions.

4 Market behavior and prospects of people

In the most part of the history of modeling in economics, the question of strategic thinking, i.e. what people know and how this affects human behavior and the decision-making, were considered unimportant. Such belief was through the years embodied in the Arrow-Debreu general equilibrium paradigm, which is based on the premise of homogeneous agents who have perfect knowledge and perfect foresight.

Contrary to the general equilibrium paradigm, several experimental studies provided some clear evidence that indifference curves are not independent of the current entitlements but have a kink around the current endowment level, which clearly contradicts with the neoclassical doctrine (List 2004). Such belief is also much closer to the reality and logical reasoning.

Experiences and knowledge of people represent a significant element in determining value function of individuals through expectations of people. Kahneman and Tversky (1991) argue that value functions of agents are convex for losses and concave for gains. They also argue that value functions are initially steeper for losses than gains, which contradicts the concave utility function advocated by neoclassical economic theory. In addition, Mehra and Prescott (1985) have proved that expected utility curve cannot simultaneously explain both the small-scale and large-scale risk attitudes.

Although it is much obvious that people have different knowledge, skills, attitudes towards risk and time, preferences, wealth, beliefs about the same things and even interpret differently the same signals, for which they adopt different strategies, economists have accepted this fact only recently.

These kinds of differences among the people all determine their behavior and, as far as stock market is concerned, make the volatility on the markets. In fact, Milgrom and Stokey (1982) argue that precisely such heterogeneous knowledge, beliefs and volatility make the trade possible. Their argument seems very self-explanatory, because if prices were the common knowledge to people, no one would be willing to pay more than the common-knowledge price, and no one would be willing to sell below that price. In such world, no speculation is possible and there is no trade.

Such omniscient world does not reflect the situation in the real world, which is populated with heterogeneous agents. In the real world, when agents decide to buy stocks speculative and more risk-dominant traders already see someone whom they will sell the stocks at higher prices, while long-run and more risk-averse traders expect to earn higher dividends on the stocks.[14] Harrison and Kreps (1979) argue that such beliefs force agents to buy stocks even tough they believe stocks are already above their "fundamental value," as they are convinced to find someone who would be willing to buy them at the higher price. Therefore, one buying an asset buys him an American option, which can be exercised at any time until its terminal date (Scheinkman and Xiong 2003). However, the time when people will exercise the option depends on their expectations of the future prices. In this game of incomplete knowledge, agents get the opportunity to earn some profit precisely from such price speculations and different expectations. We do not use the expression information (Stigler 1961, 1962, McCall 1970, Akerlof 1970), but stick to Hayekian knowledge (Hayek 1937, 1945), because people use knowledge to make decisions, while information represent only one of the ways of acquiring knowledge. On the other hand, people might have the access to the information, but if they do not build them into their knowledge, such information access is of no use. Never the less, people make different predictions from having the same information, i.e. Shiller (2002, pp. 71) wrote, "The history of speculative bubbles begins roughly with the advent of newspapers." However, Bloomfield et al. (2000) argue that investors and prices are more prone to overreact to unreliable than to reliable information.

Stock bubbles represent one of the outcomes of such dispersion of expectations, and are later usually followed by stock crashes. It is very common to speak of the bubble if the price of a stock is above its "fundamental value;" i.e. the value of the stream of dividends to which it is claimed. Standard neoclassical theory precludes the existence of stock bubbles, by backward induction in finite horizon models, and transversality conditions in the infinite horizon (Santos and Woodford 1997). Despite the definition, which is quite logical, it is hard to say that individuals care about the fundamental value of assets they buy. People in fact only care in making profits, which follows to the prospects they have on markets. Abreu and Brunnermeier (2003) point out that bubbles have often emerged in periods of productivity enhancing structural changes; i.e. railway boom, electricity boom, internet and telecommunications boom. In such situation, variability of returns is huge, while people do not know the limits of the productivity change, and this enhances expectations of people.[15]

Authors also argue that dispersion of knowledge permits bubble to grow, "despite the fact that the bubble bursts as soon as a sufficient mass of individuals sell out." This occurs when the probability of finding someone who would be willing to purchase the stock at higher price vanishes. Herd behavior only deepens that process and leads to a mass downward overshooting effect – stock crash. This indicates that many agents do not know fundamental values of assets they have and do not care for that.

[14] For intuition, see the theory of segmented markets on financial markets.
[15] We might also include recent investment boom in China.

One such event is when an asset has a finite maturity. Then its price tends to converge to the price that is quoted on an asset (Santos and Woodford 1997). The reason for that is quite simple: as the probability of finding someone who would be prepared to pay more for the stock approaches to zero as the maturity approaches to zero. In such case, the room for earning extra profit on incomplete knowledge is very limited and approaches zero, as the price and termination date become a common knowledge.

Therefore, an asset buyer acquires an American type option to sell the asset if other agents have more optimistic beliefs. When stock price increases the probability that a an asset holder will find someone who will be willing to pay for a stock more than he had paid approaches zero while the probability of a stock crash increases. In such situation, an expected profit of the transaction is likely to become negative, while the cost of an uncertainty rises. Consequently, the willingness of agents to buy stocks shrinks, and they all want to sell. Agents might also start minimizing their loss. At this point, the aversion of prospects of agents, especially of their eventual loss, enters into their decision-making.[16,17] Loss aversion stipulates that agents are much more sensitive to reductions in wealth than to increases. Psychology of agents in their decision-making lies in the center of the prospect theory (Kahneman and Tversky 1979).

Motivated with the idea of prospect theory and the fact that people overweigh outcomes that are considered certain (Allais 1953), Thaler (1980) identified an "*endowment effect*." It has been proved that goods that can be lost or given up are highly valued by individuals than when the same goods are evaluated as a potential gain (Kahneman et al. 1990, 1991, Tversky and Kahneman 1991).[18] This implies that agents behave much more careful when buying risky stocks, as they do not know whether stocks will bring them profit or induce a loss. As expected, Kahneman and Tversky showed that in the positive domain the certainty effect contributes to a risk-averse preference for a sure gain over a larger gain that is merely probable, while in the negative domain, the same effect leads to the risk-seeking preference for a loss that is merely probable over the a smaller loss that is certain. When buying shares agents face bigger uncertainty. Glosten and Milgrom (1985) argue that the problem of matching buyers and sellers is most accurate in trading with shares of small firms, which are usually characterized with high bid-ask spreads and very small trading volumes. Such high spreads decrease probabilities that options are exercised and thus increase the uncertainty of selling the stock further.

We can demonstrate such simple prospects of an agent A_i as $\pi(p) = (\pi_1, p; \pi_2, 1-p)$ where

[16] Due to lower trading volumes bid-ask spread increases and there is a problem of matching buyers and sellers.

[17] See Kahneman and Tversky (1979) for the intuition on prospect theory. Tversky and Kahneman (1992) also show that most people will reject a gamble with even chances to win and lose, unless the possible win is at least twice the size of the possible loss. However, agents on financial markets are usually more risk-dominant than average person is.

[18] In 1738, Bernoulli attempted to explain why people are generally more risk-averse and why risk-aversion decreases with increasing wealth (see Bernoulli 1954).

$$A_i = \begin{cases} \pi_1 = \dfrac{P_S - P_P - t}{P_P}; & P_S > P_P + t \\ \pi_2 = \dfrac{P_S - P_P - t}{P_P} & P_S \le P_P + t \end{cases}$$

In the response function of an agent π_1 defines the profit that an agent receives if he is able to sell the stock after he bought it and p is the probability of such outcome, while π_2 is the loss if he does not succeed in selling the stock. We can say that p denotes the probability whether P_S is higher than P_P, or not. P_S denotes the price at which one sells a stock and is stochastic, while P_P the price at which the stock was purchased. This price is known and stochastic, for which it causes uncertainty. Finally, t denotes transaction costs and other related costs (e.g. taxes, fees, etc.). The above general decision function of an agent is not stationary in time.

Thaler and Johnson (1990) argue that prior outcomes do affect the ways agents behave in the future. They conclude that after winning some gambles, individuals become less risk-averse and get more prone of taking more risk.

People also possess the ability to learn. The ability to learn is the most important ability of the people by which they improve their future decisions. In economics, "*learning-by-doing*" is a very popular expression, describing the process where people improve their skills when repeatedly perform the same work. Pretty much the same is within the human behavior, "A child who is being bullied learns either to fight better or to run away," as Skyrms and Pemantle (2000, pp. 9340) argue. Besides, in the process of acquiring their knowledge people also use social networks, which enable them, through social interaction with others, to learn from experiences of their colleagues and friends, thus improve their knowledge.

Despite the ability to learn, negative outcomes produce fear within people. Therefore, agent who suffers a loss is much more sensitive to additional setback, which increases his risk-aversion. After the loss, people tend to avoid from playing risky gambles in the future that they would otherwise take. Contrary to that, Thaler and Johnson introduced a "*house money effect*," meaning that agents have greater willingness to gamble with the money that was recently won, where the unpleasantness of an eventual loss of recently won money may be diluted by aggregating it with the earlier gain. Their finding rejects the assumption that people maximize their utility in every moment.

The other determinant that should be considered in the behavior of people is represented by how prone they are to changing their behavior and some habits and practices. People usually do not like to change their behavior very often, and once they form strong beliefs, they are often too inattentive to acquire such knowledge that would contradict their current knowledge and beliefs. Therefore, once people get convinced that one investment strategy is more lucrative than another is, they may not sufficiently attend to evidence suggesting the strategy is flawed, i.e. never dissuaded once convinced (Rabin 2003, Osborne and Rubinstein 1990).

When faced with the decision under uncertainty, people usually value simplicity (Rubinstein 1998). This means that when choosing among different alternatives, people like to have as efficient strategies as possible that would best serve their goals, while on the other hand, selected strategies should be as simple as possible. Or, not too complex. Among reasons, why agents prefer simplicity, Rubinstein argues that complex plans might easier break down, either are more difficult to be learnt, or may require some additional time to be implemented. Therefore, agents face a tradeoff, while the belief in one's own talents and abilities to adopt and execute one strategy is an important element. Every change represents a risk to people.

5 Decision-making under risk

It is in the nature of people to think and reason, and make decisions in order to be successful and happy. In doing this, people act by pursuing their rational self-interest. Everything in the nature has to act according to their rational self-interest or it dies. Taking a lion, a deer and a tree for the example, we see that a lion has to hunt or starve, while a deer has to run from the hunter or be eaten, and a tree has to find the way to the sunshine in order to survive.[19] In his *Wealth of Nation* Adam Smith made a comparison of the "*benevolent butcher*," who is interested only in selling the bread to people, while people are interested only in having bread to eat. Therefore, the utilitarian principle of the "invisible hand" is immanent to the people and the social structure is the outcome of countless actions of individuals pursuing their rational self-interest.

A significant fact in people's decision-making is the uncertain future. This is because actions of individuals do not follow to some deductive system. "If people knew the future, they would not have to choose and would not act, but would be like an automaton," said von Mises. For this, "the uncertainty of the future is already implied in the very notion of action."[20] Therefore, the most that can be attained with regard to reality is the use of probabilities.

To say that people implicitly consider the uncertainty when making decisions does not mean that people do not follow their rational self-interest and that they do not maximize their utility. It is not logical to say that people do something in order to be worse off. That would be a contradiction in terms. Namely, everything has its own reason. A psychotic behavior of people that do perceive the reality on a specific way aims to maximize the utility of such individuals as well, despite others might consider such behavior irrational. Therefore, it is oriented towards satisfying the utility of a psychotic individual.

Ramsey (1928) was among the first to model individuals' utility as the concave function of consumption, giving the rise of numerous of such concave functions. It is misleading to argue that people maximize only their consumption, as they also value their leisure and the uncertainty, at least.[21] Taking risk is not costless. Utility function is also not a static process,

[19] See Rand (1967) for the concept of rational self-interest.
[20] See von Mises (1996, pp. 105).
[21] For instance, in the hierarchy of people's needs, Maslow (1943) puts consumption at the bottom, while self-actualization and self-esteem on the top.

as is usually assumed by neoclassical economists, but is dynamic and changing, which reflects the fact that people learn and gain experience, which might alter the attitude of their preferences.

The next significant determinant of people's decision-making is rationality. Rationality can be defined as a statement about the choice of people, while people behave rational if they choose the best response according to their beliefs and preferences. It is obvious that rationality is by definition incorporated into people's behavior. It is logically that if a is preferred to b and b to c, than also a should be preferred to c. If it somehow happens that c is preferred to a, then such individual faces a problem of self-contradictory "*scale of value*," as a result of the deficiency of presence of mind, as the two actions are never synchronous and the value judgments not immutable.[22] Anyway, such person behaves rationally.

In the world of selfish individuals, a concept of Nash equilibrium (Nash 1951, Harsanyi 1967) is very intuitive in explaining behavior of individuals. It says that each individual's strategy choice is a best response to the strategies played by their rivals. Weakness of Nash equilibrium is that it assumes that individuals be right in their predictions, which means that it does not allow for incomplete knowledge, but despite that the concept us such cannot be denied.

In the world of incomplete knowledge, risk-taking is indispensable. Therefore, throughout this process of risk-taking the question of importance among selfish agents becomes not only who knows what, but also what people know about what others know, and what people know about what others know about what others know, etc (Samuelson 2004). This means that agents in the game appear in each other's equilibrium strategies (Nash 1951), but a little contrary to the concept of Nash, as they have incomplete knowledge. Precisely incomplete knowledge about the knowledge of others leads us to the point where there is no one else to buy a share. Therefore, when deciding whether to buy a share or not, agents reason whether they will find someone who would be willing to buy such share at the higher price. In such a game, agents do not think of the fundamental price of stocks neither whether the price is overvalued or not, but reason only what are the abilities to succeed in selling a share after it has been bought. Therefore, despite they all would know that the price is overvalued they do not know what strategies would others adopt, and therefore still see the opportunity to exhaust some profit in this period of finding the upper limit. Finding a peak is the most decisive moment of such game and those who are more successful in this, or even luckier, end with some profit, while others suffer a loss. For an infinite small agent, finding a peak is only a matter of luck and a gambling. At the peak, being in a short position is a desired strategy.

6 Herd behavior in people's decision-making

Specialization among people plays an important role in their life. It also influences the ways in which people make decisions. Namely, in their decision-making people are many times influenced by the decisions of others, to which specialization of knowledge leads. Adam

[22] See von Mises (1996).

Smith was among the first who emphasized the productivity of people as a byproduct of the division of labor and specialization of individuals. For our purpose, the two phenomena that arise from the specialization of knowledge are of a special interest: herd behavior and informational cascade.[23]

Keynes (1936) argues that people often do not trust their own knowledge and experiences, for which rather stick to some outside authority. Contrary to Keynes, Fischhoff et al. (1977) incorporate overconfidence into modeling human behavior, which leads to overestimation of the precision of their knowledge and abilities. Individuals become more overconfident after they have won some gambles. And in this context Odean (1999) adds that not only are people too willing to act when having too little information, but also they are also too willing to act when they are wrong.

Especially when faced with the lack of their own knowledge people behave as "*second-hand*" members of some social group, and are led by the decisions that were based on the reason and the knowledge of others whose knowledge they value higher then theirs own. Such "*second-hand*" agents have intent of copying others, while they act only after observing the behavior of these agents-leaders.[24] In pursuing such strategy of following some authority, agents face the problem of selecting the person to follow. Welch (2000) argues that the purchase recommendations for individual stocks by security analysts have a significant positive influence on the recommendations of other analysts, while the incentive to adopt behavior of one another agent increases in the number of previous adopters. Graham (1999) adds that analysts are more likely to herd when they are of a high reputation or have low ability, or if there is recognized public information that is inconsistent with the analyst's private information. Herding is also common when informative private signals are positively correlated across analysts.

An important factor in studying herd behavior is that it is not that leaders can overcome the uncertainty of the future, but second-hand individuals believe that due to their knowledge and experience, the variance of leaders should be far smaller than it is their own. For this reason, they choose to follow them. However, herd behavior leads to stock bubbles that are later followed by the stock crashes. "The reason is that capital gains lead to increasing actual returns. Once infection has reached the overwhelming majority of speculative traders, a change in basic sentiment occurs because the exhaustion of the pool of potential buyers causes price increases to diminish."[25]

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[23] While the terms informational cascade and herd behavior are used interchangeably in the literature, Smith and Sørensen (2002) emphasize that there is a significant difference between them. An informational cascade is said to occur when individuals ignore their private information in their decision-making, whereas herd behavior occurs when individuals make an identical decision, not necessarily ignoring their private information. Thus, informational cascade implies a herd, but herd does not imply an informational cascade. We do not distinguish between the two and will use only the expression herd behavior in the sequel.

[24] See Scharfstein and Stein (1990), Bikhchandani et al. (1992), Banerjee (1992, 1993).

[25] See Lux (1995, pp. 893).

7 Conclusion

In the paper, we put some arguments in favor of stochastic and behavioral finance. Namely, individuals make their predictions and decisions through a very complex system that is hard to be explained only through the set of differential equations. On the other hand, experiments have demonstrated that people usually also do not behave consistent in time, but use different ways of strategic thinking and reasoning, instead. This is especially true when dealing their wealth, which make the understanding of price movements much more complex. Although many attempts have been tried to define asset pricing through a kind of a stochastic process, i. e. Brownian motion or martingale, volatilities on the market indicate that new paradigm is needed.

Therefore, the question is what is to be done? Herbert Simon (1997, pp. 431) delivered the answer that "… economists must receive new kinds of research training, much of it borrowed from cognitive psychology and organization theory… [and] must learn how to obtain data about beliefs, attitudes, and expectations." The paper is a step in that direction.

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