Market Frictions: A Unified Model of Search and Switching Costs

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Abstract. Despite the existence of two vast literatures, very little is known about the potential differences or interactions between search and switching costs. This paper demonstrates the benefits of examining the two frictions in unison. First, the paper shows how subtle distinctions between the two costs can provide important differences in their effects upon consumer behaviour and market prices. In many cases, policymakers may prefer to reduce search costs rather than switching costs. Second, the paper illustrates a simple methodology for estimating the magnitude of both costs while demonstrating the potential bias that can arise from a single-cost approach.

1. Introduction

In many markets, from bank accounts or mortgages to washing powder or computer software, consumers' choices are constrained by different forms of market friction. Consumers' ability to change suppliers is often restricted by *both* the costs of collecting information about alternative options and the costs of organising or adjusting to an actual changeover. Despite this, the two vast literatures on search costs and switching

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costs have remained largely independent of each other[1]. Indeed, very little is known about the potential differences or interactions between the costs and worse, the two frictions are often referred to synonymously. This paper demonstrates the benefits of examining the effects of search and switching costs in unison. It hopes to enable better policy in two ways. First, the paper shows how subtle distinctions between the two costs can provide important differences in their effects upon consumer behaviour and market prices. Second, the paper illustrates a simple methodology for estimating the magnitude of both costs while demonstrating the potential bias that can arise from a single-cost approach.

Farrell and Klemperer (2007) suggest 'a consumer faces a switching cost between sellers when an investment specific to his current seller must be duplicated for a new seller'. Examples include the costs from lost compatibility, lost loyalty benefits and the cost of rearranging transactions. While search costs may appear consistent with the last of these examples, it seems natural to make four major distinctions. Unlike switching costs, search costs cannot be incurred by a fully informed consumer (Distinction 1), search costs may be incurred more than once by searching across multiple firms (Distinction 2), search costs may be incurred without then necessarily choosing to switch suppliers (Distinction 3), and in a dynamic context, search costs may be incurred before any initial market purchase (Distinction 4). For the purposes of this paper therefore, the two costs are defined as follows. While these distinctions and definitions could be viewed as arbitrary, care is later taken to demonstrate the importance of each distinction on the market equilibrium.

Definition 1. *Search costs are the costs incurred by a consumer in identifying a firm's or set of firms' product and price offerings, regardless of whether the consumer then buys the product from the searched firm(s) or not.*

[1] For reviews, see Baye, Morgan and Scholten (2006) on search costs, Klemperer (1995) or Farrell and Klemperer (2007) on switching costs, and Waterson (2003) for a general overview.

Definition 2. *Switching costs are the costs incurred by a consumer in changing suppliers that do not act to improve the consumer's information.*

Section 3 introduces the model. Within a standard differentiated products framework (Perloff and Salop 1985), it presents a static game of a mature market where all consumers must incur both search and switching costs in order to move away from their existing supplier[2]. Section 4 starts the analysis by offering an original characterisation of a consumer's optimal 'search to switch' strategy, describing how extensively a consumer should search a market and to which firm, if any, the consumer should switch. By then endogenising the equilibrium price, Section 5 provides a unified model of search and switching costs where the results of standard 'single-cost' models, such as Wolinsky (1986) and Anderson and Renault (1999), can be illustrated as special cases.

Section 6 examines the comparative statics. The mechanisms by which the two costs affect competition have some important differences. Search costs weaken the incentive for firms to cut prices by reducing the willingness of consumers to start searching and by decreasing the extensiveness of any search activity across multiple firms. Switching costs also deter initial search activity but they do not affect the extensiveness of any search because they cannot be incurred across multiple firms (Distinction 2). However, because switching costs can still be incurred by fully informed consumers, (Distinction 1), they also enhance the loyalty of consumers that have searched the entire market. As a result, parameters can be selected such that either cost can have the larger marginal impact on equilibrium prices. Nevertheless, the paper shows that in many cases, search costs have the more powerful effect on market power. This follows from Distinction 3 which ensures that search costs have the stronger effect in deterring initial search activity. When evaluating whether to start searching, consumers place less weight on switching costs because, unlike search

[2] The analysis focuses on Distinctions 1-3. As later discussed, any further dynamic mechanisms from Distinction 4 are only likely to strengthen the results.

costs, they are incurred only with the probability of finding an attractive alternative. Within the context of the model therefore, an authority may often wish to reduce search costs by improving consumers' access to information, rather than taking actions to regulate or ease consumers' costs of switching[3].

In line with reality, the model offers an attractive description of market interaction where some consumers choose not to search, some consumers choose to engage in costly search and costly switching, and some consumers select to partake in costly search but then refrain from switching[4]. Section 7 uses this feature with data from a consumer survey to recover some quick 'back of the envelope' measures for both search and switching costs within eight different UK markets. In contrast to the approach taken by previous empirical studies that provide rich and general structural estimates of the actual value of either search costs or switching costs[5], the paper emphasises the potential importance of accounting for both frictions. Indeed, it is shown that by incorrectly attributing all imperfections to only one cost, a 'single-cost' methodology can exhibit an upward bias.

2. Previous Literature

This section reviews the limited number of previous papers that have analysed both frictions in unison. Compared to previous theoretical work, our model is substantially

[3] The paper also notes a result that appears to have been overlooked within the switching cost literature. Independent of the existence of search costs, the equilibrium price can be independent of the level of switching costs for certain parameters in the case of duopoly. While recent dynamic models have stressed the possibility that the incentives generated by switching costs can be pro-competitive (Doganoglu 2005, Dubé, Hitsch and Rossi 2008 and Cabral 2008), this result stems from a wholly static analysis.

[4] For example, 27% (23%) of consumers recently searched (switched) in the UK mobile phone market (Chang and Waddams 2008).

[5] Search costs have been estimated from price data alone (Hong and Shum 2006, Moraga-González and Wildenbeest 2008), from price and quantity data (Hortaçsu and Syverson 2004) and from data on prices and consumer search behaviour (De los Santos 2008). Shy (2002) estimates switching costs with a quick method using price and market share data.

less restrictive. In a model of switching costs, Padilla (1992) allows new consumers to either have zero or infinite search costs, while Sturluson's (2002a) duopoly model assumes that consumers have either search costs or switching costs, but not both. Schlesinger and von der Schulenburg (1991) allow consumers to face both costs in a circular city, but offer a counter intuitive equilibrium where, if anything, search costs have an equivalent effect. Closest to this paper, is the theoretical section of Knittel (1997). As a foundation for an empirical analysis into the US telephone industry, he provides an initial description of consumers' optimal behaviour for any level of search and switching costs. We build on this to offer a full characterisation of consumer behaviour, while endogenising firms' pricing decisions. In other empirical work, Sturluson (2002b) and Giulietti, Waddams Price and Waterson (2005) use survey data with proxies for each cost to suggest search costs have a smaller or larger effect on consumer switching behaviour, respectively. Giulietti, Otero and Waterson (2007) use observations of tariff dispersion within the UK electricity market to make some separate inferences about the trends of search and switching costs over time. Closest to the empirical part of our paper is Moshkin and Shachar (2002). Using a different mechanism to the current paper and not allowing for the possibility that consumers may face both costs, they develop a model capable of identifying whether consumers' television viewing behaviour is more consistent with the existence of search or switching costs. They show the former is true for 71% of consumers, and estimate the average search cost to be relatively larger than the average switching cost[6].

[6] Specifically, they identify the costs by showing that a consumer who is constrained by switching costs will be equally likely to switch following a reduction in the quality of the current product choice relative to an equivalent increase in the quality of an alternative product, whereas a reduction in the quality of the current choice will produce an asymmetrically larger effect in a consumer constrained by search costs.

3. Model

The model builds on Perloff and Salop's (1985) differentiated products framework where each consumer places an idiosyncratic 'match value' on the product of each firm and where consumers seek to purchase from the firm that best suits her tastes. Although qualitatively similar results can be demonstrated within a 'price-search' model, this framework is chosen because it can better capture the full effects of Distinction 1.

Formally, let there be $n \geq 2$ firms that each sell a single good with zero production costs. A unit mass of consumers have a zero outside option and each possess a unit demand for the market good. Let consumer m gain a utility, $u_{mi} = \varepsilon_{mi} - p_i$, from choosing to buy from firm i at a price p_i, where the 'match value', ε_{mi}, is an independent draw from a distribution $G(\varepsilon)$ with positive density $g(\varepsilon)$ on $[\underline{\varepsilon}, \overline{\varepsilon}]$, where $\overline{\varepsilon} > \underline{\varepsilon}$.

Consumers are symmetrically located such that $(1/n)$ consumers are 'local' to each firm. Due to either physical proximity or some unmodelled previous relationship, consumers are assumed to face costs of search and switching with regard to all firms other than their local firm. Specifically, if consumer m is local to firm i, she must incur $c \geq 0$ to learn the match value and price, $\{\varepsilon_{mj}, p_j\}$, of any non-local firm $j \neq i$ and then further incur $s \geq 0$ if she wishes to trade with firm j. However, consumer m is free to learn the value of $\{\varepsilon_{mi}, p_i\}$ and trade with firm i at zero cost[7]. Consumers are free to search any number of non-local firms (sequentially with costless recall) before deciding to which searched firm, if any, to trade with. A one-shot simultaneous move game is considered where firms each select a single price, p_i, and where consumers select their optimal 'search to switch' strategy. Such a strategy must prescribe the extent to which the market should be searched, if at all, and to which firm, if any,

[7] Introducing costly local search does not change the main analysis. The later assumption that some positive fraction of consumers make a non-local search, $\widehat{x} > \max\{\underline{\varepsilon}, p^*\}$, ensures all consumers would find it optimal to make a local search, $\widehat{x} > p^*$.

the consumer should then switch.

4. Searching to Switch

To begin the analysis, this section derives the optimal search to switch strategy for a given consumer, under the assumption that the price of the consumer's local firm is p_i and that the price of all non-local firms is p^*. It suggests that the optimal strategy can be simplified to the use of two reservation utilities. A consumer should begin search only if her local match value and price compare unfavourably with the local reservation utility, $\widehat{x} - s$, and then stop search and switch only if a discovered non-local match value exceeds a second, standard reservation utility, $\widehat{x}$. Intuitively, this second reservation utility is equivalent to that found in standard search problems without switching costs because the decision to further search between non-local firms is independent of the level of switching costs. If search continues and no such match value is found at any of the firms, the now fully informed consumer can choose between the entire set of market offers (net of switching costs). Lemma 1 follows.

Lemma 1. *Given a search cost, c, and switching cost, s, the optimal search to switch strategy consists of the following.*

Step 1: If $\max\{0, \varepsilon_i - p_i\} + p^* \geq \widehat{x} - s$, *buy from firm i without search. Otherwise search any unsearched firm.*

Step 2: Keep searching until some firm k is found such that $\varepsilon_k \geq \widehat{x}$. Stop searching and switch to firm k.

Step 3: If no such firm is found amongst all the alternatives, trade with the firm offering the best offer, b, iff $b > 0$, with $b = \{\varepsilon_i - p_i, \varepsilon_j - p^* - s\}$ $\forall j \neq i$.

Where $\widehat{x}$ is the unique value of x that solves $c = \int_x^{\overline{\varepsilon}} (\varepsilon - x) g(\varepsilon) d\varepsilon$, if $\widehat{x} - s \in [\underline{\varepsilon}, \overline{\varepsilon}]$, and $\widehat{x}$ equals $\underline{\varepsilon}$ otherwise.

To derive the optimal strategy, one could formally cast the problem as a dynamic programming decision with finite options, where any single initial or latter search decision should be evaluated by its payoffs conditional on behaving optimally thereafter. However, as standard search results demonstrate, such problems can be solved equivalently by considering the myopic benefits of searching a single firm, disregarding its effects on future decisions (e.g. Kohn and Shavell 1974).

Step 1 of the optimal strategy provides a rule for deciding whether to initiate search beyond the local firm. Using the above logic, this reduces to a comparison of the benefits from not searching, $\max\{0, \varepsilon_i - p_i\}$, with the expected benefits of making a single search where, for a cost of c, a new surplus offer of $(\varepsilon_j - p^* - s)$ can be discovered. Such an offer will only improve upon the local offer iff $\varepsilon_j > \max\{0, \varepsilon_i - p_i\} + p^* + s$. For convenience denote $x_1 \equiv \max\{0, \varepsilon_i - p_i\} + p^* + s$. The consumer will be indifferent if $\max\{0, \varepsilon_i - p_i\} = -c + \int_{x_1}^{\overline{\varepsilon}} (\varepsilon_j - p^* - s) g(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{x_1} \max\{0, \varepsilon_i - p\} g(\varepsilon) d\varepsilon$. Search will then be optimal whenever $x_1 \equiv \max\{0, \varepsilon_i - p_i\} + p^* + s < \widehat{x}_1$, where $\widehat{x}_1$ is the unique value of x_1 that solves $c = \int_{x_1}^{\overline{\varepsilon}} (\varepsilon - x_1) g(\varepsilon) d\varepsilon$. If no solution exists such that $\widehat{x}_1 - s \in [\underline{\varepsilon}, \overline{\varepsilon}]$ then $\widehat{x}_1$ can be set equal to $\underline{\varepsilon} + s$ without loss as then search will never be optimal.

After deciding to initiate search and on finding a new offer, $(\varepsilon_j - p^* - s)$, the consumer must decide when to stop searching in Step 2. If $(\varepsilon_j - p^* - s) \leq \max\{0, \varepsilon_i - p_i\}$ the consumer should clearly continue searching. However, on finding a potentially attractive offer, the benefits of terminating search immediately must be compared to making an additional search, where, for a further cost of c, a further offer of $(\varepsilon_k - p^* - s)$ will improve upon the current offer only if $\varepsilon_k > \varepsilon_j$. By denoting $x_2 \equiv \varepsilon_j$, indifference requires $(\varepsilon_j - p^* - s) = -c + \int_{x_2}^{\overline{\varepsilon}} (\varepsilon_k - p^* - s) g(\varepsilon) d\varepsilon_k + \int_{\underline{\varepsilon}}^{x_2} (\varepsilon_j - p^* - s) g(\varepsilon) d\varepsilon_k$. Further search will be optimal only when $x_2 \equiv \varepsilon_j < \widehat{x}_2$, where $\widehat{x}_2$ is the unique value of x_2 that solves $c = \int_{x_2}^{\overline{\varepsilon}} (\varepsilon - x_2) g(\varepsilon) d\varepsilon$. If no solution exists within $[\underline{\varepsilon}, \overline{\varepsilon}]$, search will not be optimal and $\widehat{x}_2$ can be defined equal to $\underline{\varepsilon}$ without loss[8].

Finally, to complete the derivation, Step 3 follows trivially for the scenario in which the consumer has searched and initially rejected all the alternatives. Here, the

[8] As the definitions for $\widehat{x}_1$ and $\widehat{x}_2$ coincide, Lemma 1 simply re-labels them both as $\widehat{x}$.

consumer is then free to trade with the firm offering the best market deal (net of switching costs), provided such a deal is preferred to the outside option of zero.

5. EQUILIBRIUM

The paper now considers the firms' pricing decisions. For tractability, attention is focussed on the uniform distribution; $G(\varepsilon) = (\varepsilon - \underline{\varepsilon})/(\overline{\varepsilon} - \underline{\varepsilon})$ and $g(\varepsilon) = 1/(\overline{\varepsilon} - \underline{\varepsilon})$. From Lemma 1, $\widehat{x}$ then reduces to (1) if $\widehat{x} - s \in [\underline{\varepsilon}, \overline{\varepsilon}]$, and $\underline{\varepsilon}$ otherwise.

$$\widehat{x} = \overline{\varepsilon} - \sqrt{2c(\overline{\varepsilon} - \underline{\varepsilon})} \tag{1}$$

First, it is clear that the firms are able to sustain the monopoly price[9], $p^m = \max\{\overline{\varepsilon}/2, \underline{\varepsilon}\}$, if no consumers wish to search in equilibrium, $\max\{\underline{\varepsilon}, p^m\} \geq \widehat{x} - s$. Consequently, the symmetric equilibrium price, p^*, is now found under the assumptions that i) some positive fraction of consumers do search in equilibrium, $\widehat{x} - s > \max\{\underline{\varepsilon}, p^*\}$, and ii) $s, c \geq 0$ such that $\widehat{x} \leq \overline{\varepsilon}$. Given that the price of all other firms is p^*, and using the terminology of Armstrong, Vickers and Zhou (2007), firm i's residual demand, $D_i(p_i, p^*)$, can be composed into the sum of fresh and return demand, $D_i(p_i, p^*) = F_i(p_i, p^*) + R_i(p_i, p^*)$, where

$$F_i(.,.) = \frac{1}{n}[1 - G(\widehat{x} - s + p_i - p^*) + G(\widehat{x} - s)(1 - G(\widehat{x} + p_i - p^*)) \sum_{k=0}^{n-2} G(\widehat{x})^k] \tag{2}$$

$$R_i(.,.) = \frac{1}{n}[\int_{\max\{\underline{\varepsilon}, p_i\} + p^* - p_i + s}^{\widehat{x}} G(\varepsilon)^{n-1} g(\varepsilon) d\varepsilon + (n-1) \int_{\max\{\underline{\varepsilon}, p^*\} + s}^{\widehat{x}} G(\varepsilon)^{n-2} G(\varepsilon - s) g(\varepsilon) d\varepsilon] \tag{3}$$

Firm i's fresh demand, (2), originates from consumers who are following Step 1 or Step 2 of the optimal strategy. From Step 1, firm i's $(1/n)$ local consumers choose

[9] The monopoly price follows from the fact that if no consumers search, each firm has a demand of $(1/n)$ if $p_i < \underline{\varepsilon}$, $1 - G(p_i)$ if $p \in [\underline{\varepsilon}, \overline{\varepsilon}]$, and 0 if $p > \overline{\varepsilon}$.

to buy without search if $\max\{0, \varepsilon_i - p_i\} \geq \widehat{x} - s - p^*$ and $\varepsilon_i - p_i > 0$. From the assumption, $\widehat{x} - s > p^*$, the first condition always ensures the second, and so such consumers buy with $\Pr(\varepsilon_i > \widehat{x} - s + p_i - p^*) = 1 - G(\widehat{x} - s + p_i - p^*)$. From Step 2, firm i receives a demand from non-local consumers who choose to visit during their search process and find it optimal to stop and buy. The number of visits can be expressed by $(1/n)[G(\widehat{x} - s) + G(\widehat{x} - s)G(\widehat{x}) + \ldots G(\widehat{x} - s)G(\widehat{x})^{n-2}] = (G(\widehat{x} - s)/n) \sum_{k=0}^{n-2} G(\widehat{x})^k$ and the probability of stopping at firm i, conditional on visiting, equals $\Pr(\varepsilon_i > \widehat{x} + p_i - p^*)$. Firm i's share of return demand, (3), stems from consumers within Step 3 of the optimal strategy, who have searched the entire market without finding a match worth stopping for, but then realise that firm i offers them the best deal. Its derivation is more complicated and is contained within the appendix.

The symmetric equilibrium price can be found from the first order condition, $p^* = -D_i(p^*, p^*)/D_i'(p^*, p^*)$. Each firm's equilibrium demand, $D_i(p^*, p^*)$ can be expressed by (4), while $D_i'(p^*, p^*)$ can be derived by differentiating the sum of (2) and (3), and evaluating at $p_i = p^*$ in order to obtain (5), where $I(p^* \leq \underline{\varepsilon}) = 1$ iff $p^* \leq \underline{\varepsilon}$ and zero otherwise. An expression for the equilibrium price is then presented in (6)[10].

$$D_i(p^*, p^*) = (1/n)\left[1 - G(\max\{\underline{\varepsilon}, p^*\})G(\max\{\underline{\varepsilon}, p^*\} + s)^{n-1}\right] \tag{4}$$

[10]In the appendix, equilibrium existence is demonstrated for the case of large n. For smaller n, we implicitly assume that there no profitable deviations away from the price implied by the first order condition. Such an assumption is common in models of friction (e.g. Wolinksy 1986, Grossman and Shapiro 1984) yet Christou and Vettas (2008) show how large upward price deviations can be profitable for a small set of parameters in a related model of advertising. Here, one such candidate deviation price could be $p_i \geq \overline{\varepsilon} - \widehat{x} + s + p^*$ such that the expression for demand changes and firm i can only receive return demand. Nevertheless, Christou and Vettas show when a pure-strategy equilibrium exists, it is unique. Further, in a search model with a (standard) uniform distribution, similar to ours, Armstrong et al (2007) demonstrate existence for the entire parameter range. However, due to the addition of switching costs such a proof appears intractable here.

$$D_i'(p^*, p^*) = \frac{-1}{n(\bar{\varepsilon} - \underline{\varepsilon})}[1 + G(\widehat{x} - s)\sum_{k=0}^{n-2} G(\widehat{x})^k - I(p^* \leq \underline{\varepsilon})(s/(\bar{\varepsilon} - \underline{\varepsilon}))^{n-1}] \tag{5}$$

$$p^* = \frac{1 - G(\max\{\underline{\varepsilon}, p^*\})G(\max\{\underline{\varepsilon}, p^*\} + s)^{n-1}}{(\bar{\varepsilon} - \underline{\varepsilon})^{-1}[1 + G(\widehat{x} - s)\sum_{k=0}^{n-2} G(\widehat{x})^k - I(p^* \leq \underline{\varepsilon})(s/(\bar{\varepsilon} - \underline{\varepsilon}))^{n-1}]} \tag{6}$$

The expression for the equilibrium price breaks down into two possible cases, i) $p^* \leq \underline{\varepsilon}$ or ii) $p^* > \underline{\varepsilon}$. In the first case, labelled as market coverage, all consumers buy in equilibrium such that each firm's demand in equilibrium, (4), collapses to $(1/n)$ and the equilibrium price, p_C^*, reduces to (7). Note that as market frictions tend to zero, such that $\widehat{x}$ and $\widehat{x} - s$ tend to $\bar{\varepsilon}$, the equilibrium price converges to $(\bar{\varepsilon} - \underline{\varepsilon})/n$. This price corresponds to that found in Perloff and Salop (1985) and reflects the market power that derives purely from product differentiation. Consequently, a necessary condition for the existence of the market coverage case requires $(\bar{\varepsilon} - \underline{\varepsilon})/n \leq \underline{\varepsilon}$. As formalised in the next section, an increase in friction, decreases $\widehat{x} - s$ and increases the equilibrium price. As the price reaches $\underline{\varepsilon}$, either further increases in friction have no effect on the equilibrium price as $p^m \equiv \max\{\bar{\varepsilon}/2, \underline{\varepsilon}\} = \underline{\varepsilon}$ if $\underline{\varepsilon} \geq \bar{\varepsilon}/2$, or, if $\underline{\varepsilon} < \bar{\varepsilon}/2$, p^* increases beyond $\underline{\varepsilon}$ and the second case of non-market coverage becomes active.

$$p_C^* = \frac{1}{(\bar{\varepsilon} - \underline{\varepsilon})^{-1}[1 + G(\widehat{x} - s)\sum_{k=0}^{n-2} G(\widehat{x})^k - (s/(\bar{\varepsilon} - \underline{\varepsilon}))^{n-1}]} \tag{7}$$

In the case of non-market coverage, the equilibrium price excludes some consumers such that, in accordance with intuition, the expression for each firm's equilibrium demand, (4), reduces to $(1/n)(1 - (\Pr(\varepsilon < p^*)\Pr(\varepsilon < p^* + s)^{n-1}))$. An explicit expression for the equilibrium price, p_{NC}^*, is hard to obtain, but the original expression for the equilibrium price collapses to (8). Note that as $\widehat{x} - s \to p_{NC}^*$, such that all consumers refrain from searching, the equilibrium price converges to $p^m = \bar{\varepsilon}/2$.

$$p_{NC}^* = \frac{1 - G(p_{NC}^*)G(p_{NC}^* + s)^{n-1}}{(\bar{\varepsilon} - \underline{\varepsilon})^{-1}[1 + G(\widehat{x} - s)\sum_{k=0}^{n-2} G(\widehat{x})^k]} \tag{8}$$

Finally, before examining the comparative statics in more detail, it is worth considering some special cases. If one sets switching costs to zero, the price derived in (6) offers an original unification of the equilibrium prices found in the search models of Anderson and Renault (1999) and Wolinksy (1986) (which assume market coverage and non-market coverage, respectively). Second, by setting search costs to zero, such that $\widehat{x} = \overline{\varepsilon}$, the model collapses to a static analysis of switching costs which shares some similar features to the framework used in the infinite horizon (duopoly) models of Doganoglu (2005) and Cabral (2008)[11].

6. COMPARATIVE STATICS

In this section, some comparative statics are examined, while paying particular attention to the relative mechanisms by which changes in the level of search and switching costs affect competition. If the monopoly price can already be sustained, increases in either cost will have no effect. To avoid this case, the assumption that some consumers search in equilibrium is maintained throughout. Further, as much of the intuition can be understood in the more tractable and simpler case of market coverage, we focus on that case first, before discussing the additional effects within the non-market coverage case. All omitted proofs are contained in the appendix.

6.1. Market Coverage. To aid later understanding, Proposition 1 notes that increases in the number of competitors reduces the equilibrium price, (7). Intuitively, increases in n generate two effects on the price sensitivity of each firm's demand. Fresh demand becomes more price sensitive due to a decrease in each firm's share of local consumers ($1/n$) and an increase in the number of visits from the relatively more price sensitive non-local consumers. Return demand becomes more price sensitive as such consumers now have a larger number of options to choose from.

Proposition 1. *The equilibrium price is decreasing in the number of firms, n, for*

[11] Doganoglu (2005) assumes uniformly distributed match values, while Cabral (2008) allows for more general distributions and assumes that firms price discriminate.

any $n \geq 2$*, provided there is some search in equilibrium,* $\widehat{x} - s > \underline{\varepsilon}$*.*

To now examine the comparative statics of either search or switching costs, notice that in the case of market coverage, one need only examine the effect on the denominator, $D_i'(p_C^*, p_C^*)$, as the size of the market in the numerator is left unchanged at unity by assumption. Consider first the effects of an increase in search costs, by writing the derivative of $(1/p_C^*)$ with respect to c as shown below. Proposition 2 then follows, using the fact that $\partial\widehat{x}/\partial c = -(\overline{\varepsilon} - \underline{\varepsilon})/(\overline{\varepsilon} - \widehat{x})$.

$$\frac{d(1/p_C^*)}{dc} = (\overline{\varepsilon} - \underline{\varepsilon})^{-1}[\frac{\partial G(\widehat{x} - s)}{\partial c} \sum_{k=0}^{n-2} G(\widehat{x})^k + G(\widehat{x} - s)\frac{\partial \sum_{k=0}^{n-2} G(\widehat{x})^k}{\partial c}] \tag{9}$$

Proposition 2. *The equilibrium price is increasing in the level of search costs, c, for any* $n \geq 2$*, provided there is some search in equilibrium,* $\widehat{x} - s > \underline{\varepsilon}$*.*

The equilibrium price is increasing in the level of search costs. Of more interest, are the mechanisms through which this occurs. Through inspection of (9), one can observe the existence of two mechanisms that both act to reduce the potential increase firm i's non-local demand that could be generated from a reduction in its price. First, holding the extensiveness of consumers' search activity beyond the first search constant (via $\widehat{x}$), a growth in search costs discourages non-local consumers from engaging in any initial search activity (via a reduction in $\widehat{x}-s$). Second, conditional on a consumer beginning search, an increase in search costs also reduces the extensiveness of any search via a reduction in $\widehat{x}$ Now consider an increase in the level of switching costs by inspection of (10). Proposition 3 then follows as $\partial(\widehat{x} - s)/\partial s = -1$.

$$\frac{d(1/p_C^*)}{ds} = (\overline{\varepsilon} - \underline{\varepsilon})^{-1}[\frac{\partial G(\widehat{x} - s)}{\partial s} \sum_{k=0}^{n-2} G(\widehat{x})^k - (n-1)(s^{n-2}/(\overline{\varepsilon} - \underline{\varepsilon})^{n-1})] \tag{10}$$

Proposition 3. *The equilibrium price is increasing in the level of switching costs, s, for any $n \geq 2$, provided there is some search in equilibrium, $\widehat{x} - s > \underline{\varepsilon}$.*

The equilibrium price is also increasing in the level of switching costs, but the mechanisms by which this occur have some differences to those observed following an increase in search costs. First, similar to the case of search costs, holding the extensiveness of consumers' search activity constant, a growth in switching costs discourages consumers from engaging in any initial search, via a reduction in $\widehat{x} - s$. Second, unlike the case of search costs, an increase in switching costs prompts return consumers that have already searched the entire market to show an increased loyalty to their local firm. Note however that, unlike the case of search costs, an increase in switching costs has no effect on the extensiveness of search as we know the decision to stop searching is independent of s.

The rest of this section examines the relative effects of the two costs and the role played by Distinctions 1-3 as listed in the introduction. To do so, it is useful to define $A = d(1/p_C^*)/dc - d(1/p_C^*)/ds$ such that search (switching) costs will then have the larger relative marginal effect if A is negative (positive). As shown below, A can be rearranged to consist of three expressions. The first expression is positive and concerns the impact of switching costs on inducing return consumers to remain loyal to their local firm. This effect relates to the assumption that switching costs are still active even when consumers are fully informed (Distinction 1)[12]. The second expression is the impact of search costs on reducing the extensiveness of search beyond the initial search. This effect is negative and results from the assumption that search costs can be incurred across multiple suppliers (Distinction 2). Switching costs produce no such effect. The final expression is the net impact of search costs relative to switching costs on deterring initial search activity. Both costs enhance inertia,

[12] Note that this effect would not be captured within a framework that does not permit the existence of return demand, as in many price-search models.

but importantly, note that this expression is negative such that search costs have the larger effect, with $\partial G(\widehat{x}-s)/\partial c - \partial G(\widehat{x}-s)/\partial s = -(\widehat{x}-\underline{\varepsilon})/(\overline{\varepsilon}-\widehat{x})(\overline{\varepsilon}-\underline{\varepsilon}) < 0$. This difference stems from the assumption that search costs may be incurred without necessarily choosing to switch (Distinction 3). In evaluating the expected benefits from beginning search, the consumer gives a greater weight to search costs as she expects to incur them with certainty, while only expecting to incur switching costs with the lesser probability of finding a worthwhile alternative. The inability of consumers to condition the payment of search costs on the exact value of any non-local offer makes them more powerful in deterring initial search.

$$A = (\overline{\varepsilon}-\underline{\varepsilon})^{-1}[(n-1)\left(\frac{s^{n-2}}{(\overline{\varepsilon}-\underline{\varepsilon})^{n-1}}\right)+G(\widehat{x}-s)\frac{\partial\sum_{k=0}^{n-2}G(\widehat{x})^k}{\partial c}+\sum_{k=0}^{n-2}G(\widehat{x})^k(\frac{\partial G(\widehat{x}-s)}{\partial c}-\frac{\partial G(\widehat{x}-s)}{\partial s})]$$

In aggregate, the comparison of the relative marginal effects therefore comprises of an evaluation of the effects of switching costs (via Distinction 1), versus the net effects of search costs (via Distinctions 2 and 3). While it is clear that search costs always have a larger marginal effect on equilibrium price when the level of switching costs is sufficiently small (as $A < 0$ for $s \to 0$), an assessment in the general case remains difficult. However, Proposition 4 can be stated.

Proposition 4. *When the number of firms is small, the marginal effects of the two costs on the equilibrium price, dp_C^*/dc and dp_C^*/ds, cannot be consistently ranked in order of magnitude. However, there exists n^* such that the marginal effect from an increase in search costs is always larger when $n > n^*$, for all s and c, provided there is some search in equilibrium, $\widehat{x}-s > \underline{\varepsilon}$, and if search costs are positive, $\widehat{x} < \overline{\varepsilon}$.*

Far from having equivalent effects, the mechanisms by which search and switching costs affect the market equilibrium are sufficiently different that the two costs can have significantly different marginal effects on the equilibrium price. When the number of firms is small, either cost can have the larger impact. However, when the number of firms is larger than some threshold, n^*, search costs have a consistently larger impact on market power. To understand why, note that from Proposition 1, increases in the number of competitors reduce the price sensitivity of fresh demand by increasing its composition towards visiting consumers from rival firms. Consequently, increases in either friction that deter such visits then have a larger impact on raising prices when the number of firms is larger. Further note from Proposition 1, that increases in the number of firms also reduce the price sensitivity of return demand, such that the loyalty-inducing effects of switching costs on such consumers also decline. Hence, for sufficiently large n, these changes mute the impact of Distinction 1 and allow the effects of search costs to dominate, through Distinctions 2 and 3. Proposition 4 can offer no general characterisation of the threshold number of firms, but some initial simulations suggest that n^* can often be as low as four[13].

6.2. Without Market Coverage. In addition to the previous effects, a further effect of switching costs exists if not all consumers purchase in equilibrium. Indeed, as switching costs rise, a consumer is less likely to find a deal worth more than his outside option of zero and each firm's demand in equilibrium shrinks, creating a downward pressure on the equilibrium price, $p^* = -D_i(p^*, p^*)/D_i'(p^*, p^*)$. Once this extra mechanism is taken into account, the effects of the two forms of frictions can be consistently ranked, as suggested in Proposition 5.

Proposition 5. *In the case of non-market coverage, the effect on price following an increase in search costs always exceeds the effect on price following an equivalent increase in switching costs, $dp^*_{NC}/dc > dp^*_{NC}/ds$, provided there is some search in equilibrium, $\widehat{x} - s > p^*_{NC}$, and search costs are positive, $\widehat{x} < \overline{\varepsilon}$.*

[13]For example, when $n = 4$, $A < 0$ for all $\widehat{x} \in (\underline{\varepsilon}, \overline{\varepsilon}]$, and for all levels of $\underline{\varepsilon}$ that permit market coverage, $\underline{\varepsilon} \in [(\underline{\varepsilon}/2), \overline{\varepsilon})$, for $\overline{\varepsilon} = 5, 10, 15, 20, 25, 50, 75$ or 100. Such results will be dependent upon the assumed uniform distribution.

Finally, an extra result is noted that appears to have been overlooked within the switching cost literature. As a special case when $n = 2$ and $\underline{\varepsilon} = 0$, the equilibrium price can be rewritten as $p^*/(\bar{\varepsilon}^2 - p^{*2}) = 1/(\bar{\varepsilon} + \widehat{x})$. Proposition 6 follows.

Proposition 6. *In the case of non-market coverage, the equilibrium price is independent of the level of switching costs when* $n = 2$ *and* $\underline{\varepsilon} = 0$, *provided there is some search in equilibrium,* $\widehat{x} - s > p^*_{NC}$.

Provided switching costs are low enough such that the monopoly price cannot be sustained, the equilibrium price is independent of the level of switching costs. This result is not dependent on the existence of search costs. Instead, it is generated by the fact that an increase in switching costs reduces the elasticity of demand and the size of the market in a way that leaves the price unchanged. While recent dynamic models have stressed the possibility that the existence of switching costs may enhance competition because the incentive to 'invest' in future market share may be larger than the incentive to 'harvest' locked-in consumers (Doganoglu 2005, Dubé et al 2008 and Cabral 2008), this result exists in a static context with no new consumers.

7. Data Application

As discussed in the introduction, previous empirical studies have focussed on providing rich and general structural estimates of the actual value of either switching costs or search costs. Instead, this section shows how some restrictions from the consumers' optimal search to switch strategy can be used with consumer survey data to quickly recover a set of 'back of the envelope' measures for both search and switching costs. The importance of considering both forms of friction simultaneously is demonstrated by showing how a 'single-cost' methodology can exhibit an upward bias.

To proceed, the results of Lemma 1 are used to select two restrictions on consumers' observable search and switching behaviour. These are then solved simultaneously to recover the two measures. Specifically, for any equilibrium price, for any number of firms greater or equal to two, and regardless of the market coverage assumption, the model suggests the following[14]. First, the proportion of consumers who choose not to search beyond their local firm, a, should equal $1 - G(x - s)$. From before, it follows that increases in either cost deter initial search activity, but search costs have the larger effect, $da/dc > da/ds > 0$. Second, the proportion of consumers who choose to search and then switch after only one search, b, should equal, $G(x - s)(1 - G(x))$. One can observe that increases in switching costs decrease b by reducing initial search activity, while higher search costs generate an ambiguous effect by deterring initial search and reducing the extensiveness of any search. Simultaneously solving these two restrictions and using the definition $\widehat{x} = \overline{\varepsilon} - \sqrt{2c(\overline{\varepsilon} - \underline{\varepsilon})}$, offers measures for both costs, scaled by the extent of product differentiation as shown below in (11).

$$\frac{\widehat{c}}{(\overline{\varepsilon} - \underline{\varepsilon})} = \frac{1}{2}(\frac{b}{1-a})^2 \text{ and } \frac{\widehat{s}}{(\overline{\varepsilon} - \underline{\varepsilon})} = a - (\frac{b}{1-a}) \tag{11}$$

Had the existence of switching costs been ignored under a 'single-cost' approach, an estimate of search costs could have been recovered from a restriction on the proportion of non-searchers alone. By setting $s = 0$, the model would suggest $a = 1 - G(x)$ yielding an estimate, (12). By attributing all the observed inertia to search costs alone, this method can generate estimates of search costs that exhibit an upward bias. Indeed, it is easy to show that the single-cost estimate is equal to, or larger than, that found under the two-cost methodology as $\widehat{c}_{\text{single}} \geq \widehat{c}$ if $b \leq a(1 - a)$, which is ensured by $\widehat{s} \geq 0$.

$$\frac{\widehat{c}_{\text{single}}}{(\overline{\varepsilon} - \underline{\varepsilon})} = \frac{1}{2}a^2 \tag{12}$$

[14] The costs can also be identified with a range of alternative restrictions, such as the total proportion of consumers choosing to switch. Though perhaps easier to measure empirically than b, such a restriction is more complex as it is dependent upon n and the market coverage assumption.

These measures are now calculated for eight different markets from the UK, using responses from the CCP survey as detailed in Chang and Waddams (2008). From a potential 2027 consumers, those who were household decision makers and aware of the possibility of choice in the relevant market were asked a series of questions about their search and switching behaviour. Data on a and b is obtained from two questions: whether or not the consumer had searched for alternative suppliers in the past three years and if they had switched in the past three years, how many suppliers the consumer had searched beforehand. The estimated results are displayed in Table 1.

Table 1: Survey Responses and Estimated Measures of Search and Switching Costs

Market	a	b	$\widehat{c}/(\overline{\varepsilon}-\underline{\varepsilon})$	$\widehat{s}/(\overline{\varepsilon}-\underline{\varepsilon})$	$\widehat{c}_{\text{single}}/(\overline{\varepsilon}-\underline{\varepsilon})$
Electricity	0.69	0.02	0.001	0.641	0.241
Mobile Phone	0.66	0.01	0.000	0.627	0.216
Fixed Phone Line Rental	0.78	0.02	0.003	0.706	0.307
National + Overseas Calls	0.76	0.02	0.002	0.681	0.279
Broadband	0.51	0.02	0.001	0.476	0.129
Car Insurance	0.51	0.01	0.000	0.495	0.129
Mortgage	0.56	0.01	0.000	0.546	0.159
Current Bank Account	0.78	0.01	0.001	0.731	0.304

Several observations can be made. First, and most important, the expected upward bias of the single-cost methodology is confirmed. Had the role of switching costs been ignored, the estimates of search costs would have been overinflated and subsequent policy advice would have been misguided. In light of this, attempts to integrate the role of both forms of friction into future empirical studies would seem desirable. Second, within the data, the bias from neglecting switching costs is extremely large due to the fact that switching costs appear to be very important within the surveyed markets. Indeed, switching costs are estimated to be large relative to i) search costs and ii) the range of match values, $(\overline{\varepsilon} - \underline{\varepsilon})$. Comparison i) derives from the low proportion of switching consumers that were observed to make only one search. This indicates that search costs were low such that consumers chose to conduct more extensive searches before switching. Despite this, the observation that many consumers refrained from any search activity suggests switching costs were large relative to the potential benefits from improving their match value, which leads to comparison ii).

Finally, it must be stressed that the exact values of these estimates should be treated with caution due to the simplifying assumptions of the underlying model. It is hoped that further research can build upon this method of identification in order to provide a more general estimation procedure in the future. Some specific directions are suggested below.

8. Conclusions

To help policymakers better understand and measure market frictions, this paper has offered a unified analysis of search and switching costs. The paper has identified the mechanisms by which the two costs can generate different effects on competition. Largely due to the fact that search costs can be incurred without choosing to switch, it suggests that in many settings search costs may generate the larger anti-competitive effect. The paper has also presented a method for identifying the relative magnitude of the two costs. The method can be readily implemented using survey data and demonstrates the potential bias that can arise if one accounts only for a single cost.

To provide further help for policy, it would be useful for future research to extend the paper's findings to allow for asymmetric firms, more general product value distributions and heterogeneous consumers. It would also be of further interest to consider a dynamic model, although it is likely that this will only strengthen the findings. Standard results suggest that the introduction of dynamic competition often erodes the impact of switching costs by inducing firms to compete for the future profits of new consumers that are yet to be locked-in. However, Distinction 4 implies that no such effect may be present in the context of search costs because it is common for them to exist both pre- and post-purchase. Search costs are a pervasive, persistent and powerful impediment to competition. Their study remains a key area for ongoing research.

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9. APPENDIX:

Derivation of Return Demand, Equation (3).

The first term in (3) refers to firm i's $(1/n)$ own local consumers. They choose to search the entire market without choosing to stop but then buy from firm i with the probability that i) $\varepsilon_i \leq \widehat{x} - s + p_i - p^*$, ii) $\varepsilon_j \leq \widehat{x}\ \forall j \neq i$, iii) $\varepsilon_i - p_i \geq \varepsilon_j - p^* - s\ \forall j \neq i$ and iv) $\varepsilon_i \geq p_i$. As ii) is non-binding, this probability can be expressed by $\int_{\max\{\underline{\varepsilon},p_i\}}^{\widehat{x}-s+p_i-p^*} G(\varepsilon - p_i + p^* + s)^{n-1} g(\varepsilon)d\varepsilon$ or equivalently, $\int_{\max\{\underline{\varepsilon},p_i\}+p^*-p_i+s}^{\widehat{x}} G(\varepsilon)^{n-1}g(\varepsilon)d\varepsilon$. Now consider the second term in (3). A consumer from firm $j \neq i$ chooses to search the entire market without choosing to stop before switching to firm i if i) $\varepsilon_j \leq \widehat{x} - s$, ii) $\varepsilon_k \leq \widehat{x}\ \forall k \neq i, j$, iii) $\varepsilon_i \leq \widehat{x} + p_i - p^*$, iv) $\varepsilon_i - p_i - s \geq \varepsilon_j - p^*$, v) $\varepsilon_i - p_i \geq \varepsilon_k - p^*\ \forall k \neq i, j$ and vi) $\varepsilon_i \geq p_i + s$. The conditions i) and ii) are non-binding. Further, by rewriting iv) as $\varepsilon_j \leq \varepsilon_i - p_i + p^* - s$, observe that the probability that condition iv) is met is zero unless $\varepsilon_i \geq \underline{\varepsilon} + p_i - p^* + s$ and so with this further condition, the total probability can then be expressed by $\int_{\max\{p_i+s,\underline{\varepsilon}+p_i-p^*+s\}}^{\widehat{x}+p_i-p^*} G(\varepsilon - p_i + p^* - s)G(\varepsilon - p_i + p^*)^{n-2}g(\varepsilon)d\varepsilon$. Simplifying and multiplying by $(n-1)$ to sum over all non-local firms gives $((n-1)/n)\int_{\max\{\underline{\varepsilon},p^*\}+s}^{\widehat{x}} G(\varepsilon)^{n-2}G(\varepsilon - s)g(\varepsilon)d\varepsilon$.

Equilibrium Existence for Large n

For $n \to \infty$, we now show that there are no possible profitable deviations from the proposed equilibrium price, (6), which tends to $(1-G(\widehat{x}))/[(\overline{\varepsilon}-\underline{\varepsilon})(1-G(\widehat{x})+G(\widehat{x}-s))]$. As $n \to \infty$, return demand vanishes to zero, such that $D_i(p_i, p^*)/n$ equals 0 if $p_i \geq p^* + s + \overline{\varepsilon} - \widehat{x}$, $[1 - G(\widehat{x} - s + p_i - p^*) + G(\widehat{x} - s)(1 - G(\widehat{x} + p_i - p^*))/(1 - G(\widehat{x}))]$ if $p_i \in (p^* + s + \underline{\varepsilon} - \widehat{x},\ p^* + s + \overline{\varepsilon} - \widehat{x})$, $[1 + G(\widehat{x} - s)(1 - G(\widehat{x} + p_i - p^*))/(1 - G(\widehat{x}))]$ if $p_i \in (p^* + \underline{\varepsilon} - \widehat{x},\ p^* + s + \underline{\varepsilon} - \widehat{x})$, and $[1 + G(\widehat{x} - s)/(1 - G(\widehat{x}))]$ if $p_i \leq p^* + \underline{\varepsilon} - \widehat{x}$. Consequently, we need only show $\pi_i(p_i, p^*)$ is continuous, and strictly concave in p_i for $p_i \in (p^* + \underline{\varepsilon} - \widehat{x},\ p^* + s + \overline{\varepsilon} - \widehat{x})$ as demand is either zero or linear in p_i for all other $p_i \in \mathbb{R}^+$. This follows trivially, using $D_i'(p_i, p^*) < 0$ and $D_i''(p_i, p^*) = 0$ for all $\widehat{x} - s \in [\underline{\varepsilon}, \overline{\varepsilon}]$.

Proposition 1: The equilibrium price is decreasing in the number of firms, n, for any $n \geq 2$, provided there is some search in equilibrium, $\widehat{x} - s > \underline{\varepsilon}$.

Proof. Using, $\sum_{k=0}^{n-2} G(\widehat{x})^k = (1 - G(\widehat{x})^{n-1})/(1 - G(\widehat{x}))$, $d(1/p_C^*)/dn = -(\overline{\varepsilon} - \underline{\varepsilon})^{-1}[(G(x - s)G(x)^{n-1} \ln G(x))/(1 - G(x)) + (s/(\overline{\varepsilon} - \underline{\varepsilon}))^{n-1} \ln(s/(\overline{\varepsilon} - \underline{\varepsilon}))] > 0$, for $n \geq 2$ when $\widehat{x} \leq \overline{\varepsilon}$ and $s < \overline{\varepsilon} - \underline{\varepsilon}$ as ensured when $\widehat{x} - s > \underline{\varepsilon}$. ■

Proposition 2: The equilibrium price is increasing in the level of search costs, c, for any $n \geq 2$, provided there is some search in equilibrium, $\widehat{x} - s > \underline{\varepsilon}$.

Proof. Expanding (9) yields $(\overline{\varepsilon} - \underline{\varepsilon})^{-2}(\partial\widehat{x}/\partial c)[\sum_{k=0}^{n-2} G(\widehat{x})^k + G(\widehat{x} - s) \sum_{k=0}^{n-2} kG(\widehat{x})^{k-1}]$, which given $\partial\widehat{x}/\partial c = -(\overline{\varepsilon} - \underline{\varepsilon})/(\overline{\varepsilon} - x)$, is negative for all $n \geq 2$ when $\widehat{x} - s > \underline{\varepsilon}$. ■

Proposition 3: The equilibrium price is increasing in the level of switching costs, s, for any $n \geq 2$, provided there is some search in equilibrium, $\widehat{x} - s > \underline{\varepsilon}$.

Proof. Expanding (10) yields $(\overline{\varepsilon} - \underline{\varepsilon})^{-2}(\partial(\widehat{x} - s)/\partial s)[\sum_{k=0}^{n-2} G(\widehat{x})^k + (n-1)(s/(\overline{\varepsilon} - \underline{\varepsilon}))^{n-2}]$, which given $\partial(\widehat{x} - s)/\partial s = -1$, is negative for all $n \geq 2$ when $\widehat{x} - s > \underline{\varepsilon}$. ■

Proposition 4: When the number of firms is small, the marginal effects of the two costs on the equilibrium price, dp_C^*/dc and dp_C^*/ds, cannot be consistently ranked in order of magnitude. However, there exists n^* such that the marginal effect from an increase in search costs is always larger when $n > n^*$, for all s and c, provided there is some search in equilibrium, $\widehat{x} - s > \underline{\varepsilon}$, and if search costs are positive, $\widehat{x} < \overline{\varepsilon}$.

Proof. To prove the first claim, we need only show that there exists a range of parameters with low n where $A > 0$ such that $dp_C^*/dc < dp_C^*/ds$. As A is increasing in s, set s as large as possible within our assumptions, such that $A(s \simeq \widehat{x} - \underline{\varepsilon}) = (\overline{\varepsilon} - \underline{\varepsilon})^{-2}[(n-1)G(\widehat{x})^{n-2} - ((\widehat{x} - \underline{\varepsilon})/(\overline{\varepsilon} - \widehat{x})) \sum_{k=0}^{n-2} G(\widehat{x})^k]$. It then follows that $A(s \simeq \widehat{x} - \underline{\varepsilon}, n = 2) > 0$ when $\widehat{x}$ belongs to the non-empty interval, $[\underline{\varepsilon}, (\overline{\varepsilon} + \underline{\varepsilon})/2)$. To prove the second claim, we show that for all relevant parameters, i) $\partial A(s \simeq \widehat{x} - \underline{\varepsilon})/\partial n < 0$ and ii) $A(s \simeq \widehat{x} - \underline{\varepsilon}) < 0$ when $n \to \infty$. For i) note that $\partial A(s \simeq \widehat{x} - \underline{\varepsilon})/\partial n$ can be expressed as $(\overline{\varepsilon} - \underline{\varepsilon})^{-2}[G(\widehat{x})^{n-2}(1 + (n-1) \ln G(\widehat{x})) + ((\widehat{x} - \underline{\varepsilon})/(\overline{\varepsilon} - \widehat{x})) \cdot (G(\widehat{x})^{n-1})/(1 - G(\widehat{x})) \cdot \ln(G(\widehat{x})]$. This is increasing in $\widehat{x}$, yet negative for all $n \geq 2$, even when $\widehat{x}$ is set equal to the maximum value consistent with our assumption that $c > 0$, that is $\widehat{x} = \overline{\varepsilon}^{-}$. Finally, for ii) note that $A(s \simeq \widehat{x} - \underline{\varepsilon}) < 0$ when $n \to \infty$ for all $\widehat{x} \in (\underline{\varepsilon}, \overline{\varepsilon}]$. ■

Proposition 5: In the case of non-market coverage, the marginal effect on price following an increase in search costs always exceeds that from an equivalent increase in switching costs, $dp^*_{NC}/dc > dp^*_{NC}/ds$, provided there is some search in equilibrium, $\widehat{x} - s > p^*_{NC}$, and search costs are positive, $\widehat{x} < \overline{\varepsilon}$.

Proof. Using (8), define $H = p^*_{NC}(\overline{\varepsilon} - \underline{\varepsilon})^{-1}[1 + G(\widehat{x} - s)\sum_{k=0}^{n-2} G(\widehat{x})^k)] - 1 + G(p^*_{NC})G(p^*_{NC} + s) = 0$. From the implicit function theorem, it then follows that $dp^*_{NC}/dc > dp^*_{NC}/ds$ if i) $dH/dp^*_{NC} > 0$ and ii) $dH/dc < dH/ds$. Both are true, given $(\partial(\widehat{x} - s)/\partial c - \partial(\widehat{x} - s)/\partial s) = -(\widehat{x} - \underline{\varepsilon})/(\overline{\varepsilon} - \widehat{x}) < 0$ and our initial assumption, $\widehat{x} - s > p^*_{NC} > \underline{\varepsilon}$. ■