Endogenous Timing with Government's Preference and Privatization

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Abstract

By introducing the government's preference for tax revenues into an extended game with observable delay, this study provides new insight into the trade-off between the government and the public firm's payoff in a government's optimal policy of privatization. The results show that: (i) regardless of the government's preference for tax revenues, the government does not have an incentive to privatize in an endogenous timing context even though there are conflicts of interest between the public firm and the government and (ii) under a mixed duopoly, each sequential-move equilibrium varies with the level of the government's preference for tax revenues.
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1 Introduction

From perspectives on public choices, White (1996), Poyago-Theotoky (2001), and Myles (2002) showed that when governmental interventions, such as production subsidies, are incorporated into a mixed oligopoly, all firms' profits and social welfare are identical before and after the privatization of the public firm in a mixed oligopoly. This is irrespective of whether the public firm moves simultaneously with the private firms or the public firm acts as a Stackelberg leader or all firms behave as profit-maximizers. On the other hand, Fjell and Heywood (2004) demonstrated that when the public leader is privatized and becomes the private leader, the optimal subsidy, output, and social welfare are reduced. Moreover, by introducing taxes (ad valorem or specific) in a mixed oligopoly, Mujumdar and Pal (1998) showed that privatization can increase both social welfare and tax revenues, where an increase in tax does not change the total output but increases the output of the public firm and the tax revenues.

However, most papers on mixed oligopolies make a standard assumption on firms' objectives when governmental intervention is incorporated into the mixed oligopoly: private firms are profit-maximizers while the public firm, as well as the government, is a social-welfare maximizer. It has been understood that the public firm, as well as the government, traditionally maximizes the sum of the tax revenues or subsidies and the consumers' and producers' surpluses. In particular, endogenous timing in a mixed oligopoly, with the acceptance of some conflicts of interest between the public firm and the government, has not been addressed.

Recently, some papers have investigated endogenous timing in a mixed oligopoly since an alternate order of moves often produces significantly different results and thus, leads to a different level of welfare. Most studies that formulate endogenous timing in a mixed oligopoly adopt the extended game with observable delay of Hamilton and Slutsky (1990). For example, Pal (1998) analyzed an endogenous order of moves in quantity choice in a mixed oligopoly that consisted of a single public firm and n domestic private firms[1]. Matsumura (2003) considered the endogenous roles of firms in a mixed-duopoly market where a state-owned public firm and a foreign private firm compete. Lu (2006) investigated endogenous timing in a mixed oligopoly with a public firm, domestic private firms, and foreign private firms. All these papers found that in equilibrium, the (single) public firm never chooses quantities simultaneously with private firms. On the other

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[1]Jacques (2004) and Lu (2007) developed slight corrections of Proposition 4.1 of Pal (1998).

hand, the study of Barcena-Ruiz (2007) is a notable exception; it shows that in a mixed duopoly, all firms (public and private) simultaneously choose prices. However, in the literature on mixed oligopolies, most studies assume that the government and the public firm act as a coherent entity.

In this study, we introduce differential objective functions for the public firm and the government in an extended game with observable delay. For the government, it has been argued in the literature that there is another way to limit the discretionary power of governments when a Leviathan government exists (see Brennan and Buchanan, 1980). For example, Oates (1985) and Zax (1989) found empirical support for a Leviathan government, while Forbes and Zampelli (1989) rejected the assumption of a Leviathan government[2]. Therefore, the literature on mixed oligopolies contains a number of puzzles for fiscal centralization and the size of the public sector. (Oates, 1989). These two contrasting views clearly reflect different perceptions of policy-making. Firstly, government is a benevolent maximizer of social welfare. Secondly, it intrinsically is a tax-revenue maximizer. In a departure from the framework of traditional models that involves a monolithic entity that seeks to maximize social welfare across the public firm and the government, we assume that the public firm gives full weight to the social welfare[3], which is defined as the sum of the consumers and producers' surpluses, while the government attaches weight to both the social welfare and its preference for tax revenues.

Although some theoretical studies have already succeeded in explaining mixed oligopolies, Matusumura (1998), Saha and Sensarma (2008), and Kato (2008) explicitly investigated differential objective functions for the public firm and the government in a mixed duopoly. In comparison with prior research results, the outcome of differential objective functions for the government and the public firm in a mixed duopoly is a new finding since, thus far, the literature that assumes the same objective function for both the government and the public firm in a mixed oligopoly has yielded various robust results. In this sense, Matusumura (1998), Saha and Sensarma (2008), and Kato (2008) made a contribution to the literature. More specifically, Kato (2008) showed that without an extended game with observable delay, the government's privatization of the public firm depends on its preference for tax revenues. This is because the government is assumed to give more weight to tax revenues than to social welfare, whereas the public firm is only concerned with maximizing social welfare.

In this paper, to study the effects that arise when the objective functions of the government and the (sole) public firm are different, we extend Kato's (2008) model, which focuses on the efficiency of privatization by allowing the extended game with observable delay of Hamilton and Slutsky (1990) in the context of a quantity-setting mixed duopoly where the firms choose the timing at which they decide their quantities. First, in contrast to Kato (2008), we find that regardless of the government's preference for tax revenues, the government does not have an incentive to privatize in the endogenous-timing game even though there are conflicts of interest between the public firm and the government. Second, a unique sequential-move equilibrium is always found in the case of endogenous timing in a mixed duopoly if the government's preference for tax revenues is sufficiently small, while we have two sequential-move equilibria since the public firm and the private firm care about the sequential moves if the government's preference for tax revenues is sufficiently large.

The organization of the paper is as follows. In Section 2, we describe the model, and present

[2]In theoretical studies of a Leviathan government, Edwards and Keen (1996) and Rauscher (2000) used formalized tax-competition models to address the issue and showed that the results of tax competition are ambiguous. For a more detailed treatment of the Leviathan government, recent theoretical as well as empirical studies include Keen and Kotsogiannis (2002) and Brülharty and Jamettiz (2007, 2006).

[3]Following De Fraja (1991), we assume that the public firm only cares about the sum of the consumer and producer surpluses.

fixed-timing games; we also determine the endogenous timing under a mixed and privatized duopolistic market. Section 3 determines the public firm's endogenous choice of privatization motives. Section 4 closes the paper.

2 The Model

Consider a mixed-duopoly situation for a homogeneous good that is supplied by a public firm and a private firm. Firm 1 is a profit-maximizing private firm and firm 0 is a public firm that maximizes the social welfare. Assume that the inverse demand is characterized by $p = 1 - x_0 - x_1$ where x_0 is the output level of the public firm and x_1 is the output level of the private firm.

Given that both firms share the same production technology that is represented by a quadratic cost function $\frac{x_i^2}{2}$, $i = 0, 1$, each firm's profit is as shown in

$$\pi_i = px_i - tx_i - \frac{x_i^2}{2}, \quad i = 0, 1, \tag{1}$$

where t is the specific-tax rate. In what follows, we assume that a specific tax rate is imposed on the public and private firm. This is because calculation is greatly simplified if a specific tax rate is imposed on both firms, rather than ad valorem, without losing any of implication of our model.

On the other hand, the public firm's objective is to maximize the welfare, W, is to maximize welfare, which is defined as the sum of the consumer surplus and the profits of the individual firms. Thus, the public firm aims to maximize its objective function as follows;

$$W = \frac{X^2}{2} + \pi_1 + \pi_0 = \frac{X^2}{2} + (1 - X)X - \frac{(x_0^2 + x_1^2)}{2} - T, \tag{2}$$

where $X = x_1 + x_0$ is the total output and $T = t(x_0 + x_i)$ is the tax revenue.

In the manner of Kato (2008), we also assume that the government's payoff is given by

$$G = W + (1 + \alpha)T,$$

where α is the parameter that represents the weight of the government's preference for tax revenues. Here, $\alpha \geq 0$, i.e., the government values the tax revenues, T, more than the social welfare, W.

Given the government's payoff, we consider the observable-delay game of Hamilton and Slutsky (1990) in the context of a quantity-setting mixed duopoly where the firms choose the timing for deciding their quantities. For simplicity of analysis, we assume that there are two periods for the choice of quantity and that each firm cannot produce over more than one period. A three-stage game is considered. In the first stage, the government sets the specific tax to maximize its own payoff between the privatized and the mixed duopoly. In stage two, both firms simultaneously announce in which period they will choose their quantities, given the specific-tax rate and the competition situations. In stage three, each firm chooses its quantity knowing each other's choice of the production period and the specific-tax rate. If both firms decide to choose their quantities at the same time, a Cournot-type game occurs whereas if both firms decide to choose their quantities in different periods, a Stackelberg-type game occurs wherein the leader is the firm that is the first to choose.

2.1 Fixed-Timing Games under a Mixed Duopoly

Before presenting the results of the equilibrium that is derived from the model with the observable-delay game, we discuss one Cournot- and two Stackelberg-type mixed-duopoly models with fixed timing.

[**Cournot-Type Game under a Mixed Duopoly**]: In the third stage, taking as t and solving the first-order conditions, (1) and (2), we obtain,

$$x_0 = \frac{1-t-x_1}{2}, \quad x_1 = \frac{1-t-x_0}{3}. \tag{3}$$

By solving the first-order conditions (3), we obtain the equilibrium output, x_i^*

$$x_0^* = \frac{2(1-t)}{5}, \quad x_1^* = \frac{1-t}{5}. \tag{4}$$

We now move to the second stage of the game. From (4), the government's payoff, G^*, in the mixed duopoly can be rewritten as follows:

$$\max_t \ G^* = \frac{(1-t)[8+t(7+15\alpha)]}{25}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^* = \frac{15\alpha - 1}{2(7+15\alpha)}. \tag{5}$$

Then, the substitution of (5) into (4) yields the equilibrium output, $x_1^* = \frac{3(1+\alpha)}{2(7+15\alpha)}$ and $x_0^* = \frac{6(1+\alpha)}{2(7+15\alpha)}$. The equilibrium objective functions, π_1^* and W^*, respectively, can be obtained as:

$$\pi_1^* = \frac{27(1+\alpha)^2}{8(7+15\alpha)^2}, \quad W^* = \frac{18(1+\alpha)^2}{(7+15\alpha)^2}. \tag{6}$$

[**Stackelberg-Type Game under a Mixed Duopoly**]: In this subsection, we discuss two Stackelberg-type models of fixed timing, in one of which the public firm is the leader while the private firm is the leader in the other.

First, consider the game where the public firm is the leader. Let $x_i^{lf}(i=0,1)$ denote the equilibrium quantity when the public firm acts as the leader. To solve for the quantities in this game through backward-induction, we use the private firm's reaction function $x_1 = (1-t-x_0)/3$, as in the simultaneous-move game. Thus, the public firm's best response function that maximizes W^{lf} is

$$\begin{aligned}\max_{x_0} W^{lf} =& \frac{9x_0^2 + (1-t-x_0)(5x_0+1-t)}{18} + \frac{(2-2x_0+t)(1+2x_0-t)}{9} \\ &- \frac{3x_0^2}{6} - \frac{(1-t-x_0)^2}{6} - \frac{t[3x_0+(1-t-x_0)]}{3}.\end{aligned} \tag{7}$$

The solution of the first-order condition, (7), yields

$$x_0^{lf} = \frac{5(1-t)}{14}, \quad x_1^{lf} = \frac{3(1-t)}{14}. \tag{8}$$

We now move to the first stage of the game. From (8), the government's payoff, G^{lf}, in the mixed oligopoly can be rewritten as follows:

$$\max_t \ G^{lf} = \frac{(126-252t+126t^2)+224(1+\alpha)t(1-t)}{392}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^{lf} = \frac{8\alpha - 1}{7 + 16\alpha}. \tag{9}$$

By using (9), we can compute the private firm's profit, π_1^{lf}, and the public firm's payoff, W^{lf} as follows;

$$\pi_1^{lf} = \frac{216(1+\alpha)^2}{49(7+16\alpha)^2}, \quad W^{lf} = \frac{144(1+\alpha)^2}{7(7+16\alpha)^2}. \tag{10}$$

Next, consider the game where the private firm is the leader. Let $x_i^{fl}(i = 0, 1)$ denote the equilibrium quantity when the private firm acts as the leader. Similarly, as in the case of public-firm leadership, we use the public firm's reaction function, $x_0 = \frac{1-t-x_1}{2}$. Thus, the private firm maximizes

$$\max_{x_1} \pi_1^{fl} = \frac{x_1(1 - x_1 + t)}{2} - \frac{x_1^2}{2} - tx_1$$

which yields $x_1^{fl} = x_0^{fl} = (1-t)/4$.

We now move to the second stage of the game. Similar to the case of public leadership, the government's payoff, G^{fl}, in the mixed duopoly can be rewritten as follows:

$$\max_t \; G^{fl} = \frac{5(1-t)^2 + 8t(1-t)(1+\alpha)}{16}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^{fl} = \frac{4\alpha - 1}{3 + 8\alpha}. \tag{11}$$

By using (11) and the optimal outputs, we obtain the equilibrium objective function levels as follows;

$$\pi_1^{fl} = \frac{3(1+\alpha)^2}{2(3+8\alpha)^2}, \quad W^{fl} = \frac{5(1+\alpha)^2}{(3+8\alpha)^2}. \tag{12}$$

2.2 Endogenous Timing under a Mixed Duopoly

Having derived the equilibrium for three fixed-timing games in the previous section and using the same notation for the timings as before, we can determine firm's endogenous timing that each takes the government's payoff as a given.

Let "F" and "S" represent first period and second period with regard to quantity choice respectively. When both firms have chosen "F" or "S", they will play a Cournot-type game in the second stage; when the public firm has chosen "F" while the private firm has chosen "S", a public-leader Stackelberg-type game arises in the third stage; when the private firm has chosen "S" while the public firm has chosen "F", a private-leader Stackelberg-type game arises in the second stage.

In this subsection, we will find the Nash equilibria in the second stage for any given payoff of the government under a mixed duopoly. The reduced endogenous-timing game can be represented by the following payoff table.

Table 1: The endogenous-timing game under a mixed duopoly

		Private firm	
		F	S
Public firm	F	W^*, π_1^*	W^{lf}, π_1^{lf}
	S	W^{fl}, π_1^{fl}	W^*, π_1^*

Straightforward computations show that

$$\pi_1^{fl} > \pi_1^* \Leftrightarrow 115 + 408\alpha + 324\alpha^2 > 0,$$
$$\pi_1^{lf} > \pi_1^* \Leftrightarrow 735 + 2646\alpha + 1856\alpha^2 > 0.$$

These inequalities tell us that the private firm prefers sequential production. We then find that

$$W^{lf} > W^* \Leftrightarrow 49 + 112\alpha + 8\alpha^2 > 0.$$

When we compare W^{fl} with W^*, by applying $83 + 186\alpha - 27\alpha^2 = 0$ to a discriminant and solving for the roots of this equation, we obtain $\alpha \risingdotseq 0.42$[4]. Hence,

$$W^{fl} > W^* \Leftrightarrow 83 + 186\alpha - 27\alpha^2 > 0, \quad \text{if} \quad 0 < \alpha < 0.42.$$
$$\text{Otherwise,} \quad W^{fl} < W^* \quad \text{if} \quad \alpha > 0.42.$$

These inequalities tell us that the public firm prefers sequential production if and only if $0 < \alpha < 0.42$. So, there exist subgame perfect Nash equilibria in the observable delay game. Thus, we have the following proposition:

Proposition 1: *: Suppose α denotes the government's preference for tax revenues. Then, there are three possible endogenous orders of moves. If $0 < \alpha < 0.42$, then the order is either private leader-public follower or public leader-private follower, and if $\alpha > 0.42$, the order is private leader-public follower.*

The fact that the public and private firms prefer sequential production if the government's preference for tax revenues is sufficiently small plays an important role in the derivation of the result. In our setting, besides two sequential-move equilibria that are always found in the case of endogenous timing in a mixed duopoly if $\alpha > 0.42$, we also obtain a unique sequential-move equilibrium if the government's preference for tax revenues (i.e., α) is sufficiently small. The intuition is as follows. Given that the private firm has the strictly dominant strategy, regardless of the government's preference for tax revenues, it always chooses the simultaneous-move game. As for the public firm, when the government's preference for tax revenues is sufficiently small, the public firm prefers sequential production to moving simultaneously. Thus, we get the unique subgame-perfect equilibrium stated in Proposition 1.

In addition, having derived the equilibria for the case of endogenous timing, we can state the following result.

Lemma 1: *Suppose that the government and the public firm have different objective functions. Then, the government's payoff in the first stage is given by*

$$G^{lf} = \frac{(1+\alpha)^2(256\alpha + 143)}{7(7+16\alpha)^2} \quad \text{or} \quad G^{fl} = \frac{4(1+\alpha)^2(1+2\alpha)}{(3+8\alpha)^2} \tag{13}$$

[4]A negative solution for α is excluded by the assumption that $\alpha > 0$.

if $0 < \alpha < 0.42$. *However, if* $\alpha > 0.42$*, then the government's payoff in the first stage is given by* G^{lf}.

In light of Lemma 1, a comparison of G^{fl} with G^{lf} yields[5]

$$G^{fl} > G^{lf} \quad \text{if} \quad 0 < \alpha < 0.17. \quad \text{Otherwise,} \quad G^{lf} > G^{fl}.$$

Thus, numerical analysis shows that when $\alpha \fallingdotseq 0.17$, the government's payoff under private-leadership is greater than under public-leadership. Hence the cutoff level is $\hat{\alpha} \fallingdotseq 0.17$ such that $G^{fl} > G^{lf}$ when $a \leq \hat{\alpha} \fallingdotseq 0.17$. Hence, from a comparison of the public firm's payoff and the government's payoff from Proposition 1, we can state the following results.

Proposition 2: *If* $\alpha > 0.17$*, the government prefers the public-leadership game, while the public firm prefers private-leadership game as long as* $0.17 < \alpha < 0.42$.

This proposition suggests that differences in the implementation of leadership depend on the structure of political power with regard to the public firm and the government. In other words, the government has an incentive to use the public-leadership game when the preference for tax revenue is sufficiently small while the public firm does not have such an incentive. The conflict between these two views of objective functions typically induces a conflict with regard to the endogenous timing.

2.3 Fixed-Timing Games under a Privatized Duopoly

The previous section examined the impact of a mixed duopoly in terms of the extent of the government's preference, viz., the parameter, α. This subsection compares the equilibrium of a mixed duopoly, which would be established in the case of a privatized duopoly. As discussed in the basic model, consider a privatized-duopoly situation for a homogeneous good that is supplied by firm $l = 1, 2$. Firm $l(l = 1, 2)$ is a private, profit-maximizing firm. Thus, the inverse demand is assumed by $p = 1 - x_1 - x_2$. Similar to the previous section, we discuss one Cournot- and two Stackelberg-type privatized-duopoly models of fixed timing, each of which takes the government's payoff as a given.

[**Cournot-Type Game under a Privatized Duopoly**]: In the third stage, taking as t and solving the first-order conditions (1), we obtain,

$$x_1^C = x_2^C = \frac{1-t}{4}. \tag{14}$$

We now move to the second stage of the game. From (14), the government's payoff, G^C, in the privatized duopoly can be rewritten as follows:

$$\max_t \; G^C = \frac{(1-t)[5 + t(3 + 8\alpha)]}{16}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^C = \frac{4\alpha - 1}{3 + 8\alpha}. \tag{15}$$

[5] See the Table A in the Appendix. It can be easily checked by using simple numerical examples.

Thus, by using (14) and (15), we have each firm's equilibrium profit level as follows;

$$\pi_1^C = \pi_2^C = \frac{3(1+\alpha)^2}{2(3+8\alpha)^2}. \tag{16}$$

[**Stackelberg-Type Game under a Privatized Duopoly**]: Consider the game where the private firm 1 is the leader. To solve for the backwards-induction quantity of this game, we use the private firm 2's reaction function $x_2 = (1-t-x_1)/3$ as in the simultaneous-move games. To distinguish notations, the superscript LF is defined when the private firm 1 acts as the leader and FL is defined when the private firm 2 acts as the leader. The private firm 1's best response that maximizes

$$\max_{x_l} \pi_1^{LF} = \frac{x_1(2-2x_1+t)}{3} - \frac{x_1^2}{2} - tx_1. \tag{17}$$

By solving the first-order conditions (17), we obtain,

$$x_1^{LF} = \frac{6(1-t)}{21}, \quad x_2^{LF} = \frac{5(1-t)}{21}. \tag{18}$$

We now move to the first stage of the game. From (18), the government's payoff, G^{LF}, in the privatized duopoly can be rewritten as follows:

$$\max_t \ G^{LF} = \frac{(11-11t)[31-31t+42t(1+\alpha)] - (5-5t)^2 - (6-6t)^2}{882}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^{LF} = \frac{33\alpha - 7}{2(26+66\alpha)}. \tag{19}$$

By using (19) and symmetry among private firms, we have the equilibrium output levels as follows;

$$\pi_1^{LF} = \pi_2^{FL} = \frac{28(59+99\alpha)^2}{1176(26+66\alpha)^2}, \quad \pi_2^{LF} = \pi_1^{FL} = \frac{25(59+99\alpha)^2}{1176(26+66\alpha)^2}. \tag{20}$$

2.4 Endogenous Timing under a Privatized Duopoly

Similar to the previous subsection, using the same notation for timings, we can determine firms' endogenous timings when the government's payoff is taken as a given.

In this subsection, we will find the Nash equilibria in the second stage for any given payoff of the government. Hence, the reduced endogenous-timing game can be represented by the following payoff table.

Table 2: The endogenous-timing game under a privatized duopoly

Private Firm 2

Private Firm 1		F	S
	F	π_1^C, π_2^C	π_1^{LF}, π_2^{LF}
	S	π_1^{FL}, π_2^{FL}	π_1^C, π_2^C

To find the subgame-perfect Nash equilibrium, we need to compare profits. Straightforward computation shows that[6]

$$\pi_i^C > \pi_i^{LF} \quad \text{if} \quad 0 < \alpha < 0.27. \quad \text{Otherwise,} \quad \pi_i^C < \pi_i^{LF} \quad \text{if} \quad \alpha > 0.27.$$
$$\pi_i^C > \pi_i^{FL} \quad \text{if} \quad 0 < \alpha < 0.42. \quad \text{Otherwise,} \quad \pi_i^C < \pi_i^{FL} \quad \text{if} \quad \alpha > 0.42.$$

These inequalities tell us that both firms prefer simultaneous production if $0 < \alpha < 0.27$ and move sequentially if $\alpha > 0.42$; one firm prefers sequential production if $\alpha > 0.27$ and simultaneous production otherwise. So, there are subgame-perfect Nash equilibria, $\{F, F\}$ or $\{S, S\}$ (respectively, $\{F, S\}$ or $\{S, F\}$) in the observable delay game if $0 < \alpha < 0.27$ (respectively, $\alpha > 0.42$). However, if $0.27 < \alpha < 0.42$, we obtain a unique sequential-move equilibrium. Thus, we have the following proposition.

Proposition 3: *Suppose that the government under a privatized duopoly has a preference for tax revenue, α. Then, there are four possible endogenous orders of moves depending on the value of α. If $0 < \alpha < 0.27$ the order is either $\{F, F\}$ or $\{S, S\}$; if $0.27 < \alpha < 0.42$, the order is $\{F, S\}$; if $\alpha > 0.42$, the order is either $\{S, F\}$ or $\{F, S\}$.*

Proposition 3 is in contrast to one of the findings in the mixed-oligopoly literature that an equilibrium outcome cannot sustain the simultaneous choice of quantities by both firms in a duopoly. In our setting, besides three sequential-move equilibria that are always found in the case of endogenous timing of a mixed duopoly, we have simultaneous-move equilibria in which each firm chooses either the first or second opportunity to produce output if the government's preference for tax revenues is sufficiently small (i.e., $0 < \alpha < 0.27$). The argument is in contrast to the one we used to explain why the public firm prefers simultaneous movement to being a follower when the private firm always prefers sequential production (see the paragraph below Proposition 1).

Finally, having derived equilibria for the case of endogenous timing, we can state the following result.

Lemma 2: *Suppose that the government has a preference, α, for tax revenues under a privatized duopoly. Then, if $\alpha > 0.27$, the government's payoff in the first stage is given by*

$$G^{FL} = G^{LF} = \frac{(59 + 99\alpha)(943 + 2838\alpha + 1089\alpha^2)}{252(26 + 66\alpha)^2}.$$

However, if $0 < \alpha < 0.27$, then the government's payoff in the first stage is given by

$$G^C = \frac{(1 + \alpha)^2}{3 + 8\alpha}.$$

Similar to the mixed duopoly, a comparison of G^C with $G^{LF} = G^{FL}$ yields[7]

$$G^C > G^{LF} = G^{FL} \quad \text{if} \quad \alpha > 0.84. \quad \text{Otherwise,} \quad G^{LF} = G^{FL} > G^C.$$

Hence, from a comparison of the private firm's payoff and the government's payoff, we can state the following result.

[6] The formal values of the firms' profits are obtained through tedious calculations and provided in Table B in the Appendix.

[7] See also the table A in the Appendix. It can be easily checked by using simple numerical examples.

Proposition 4: *If* $0.27 < \alpha < 0.84$*, the government and private firms prefer sequential production. However, if* $0 < \alpha < 0.27$ *(respectively,* $\alpha > 0.84$*), private firms (respectively, the government) prefer (respectively, prefers) sequential (respectively, simultaneous) production.*

Proposition 4 suggests that one significant result is derived in this paper: when comparing G^C with G^{LF}, the preference, α , for tax revenues plays an important role whereby the level of the government's payoff is determined by the critical value. More specifically, when we introduce the government's preference for tax revenues into the endogenous-timing model of a mixed duopoly, the negative trade-off relationship between the government's payoff and the private firm's profit is obtained when the government's preference for tax revenues falls in either of the ranges $0 < \alpha < 0.27$ and $\alpha > 0.84$.

In the endogenous-timing game, the government's objective function that attaches weight to both the social welfare and the tax revenues induces a trade-off relationship between the government's payoff and the private firm's profit if the government's preference for tax revenues is not in the middling range, i.e., it is either relatively large or relatively small.

3 Endogenous Motives for Government

Having found endogenous timing for government's payoff motives, we are now in a position to determine government's endogenous payoff motive. At the stage of announcement of the the specific tax, since the government's payoff under the mixed duopoly is larger than that under the privatized duopoly, the government prefers the mixed duopoly to the privatized duopoly[8]. Therefore, we can state the following results.

Proposition 5: *Suppose that the government and the public firm have different objective functions. Then, the government decides to choose the mixed duopoly rather than the privatized duopoly regardless of its preference for tax revenues. Thus, the government's payoff in the first stage is given by either* G^{lf} *or* G^{fl}*.*

From Proposition 5, the comparison between the government's payoff in the mixed duopoly and in the privatized duopoly can be interpreted as follows. Regardless of the government's preference for tax revenues, the government does not have an incentive to privatize in the endogenous-timing game even thought there are conflicts of interest between the public firm and the government, as in Proposition 2. The intuition is as follows. When comparing an endogenous motive for the privatized duopoly with that for a mixed duopoly, we are back to the mixed-duopoly case. We have a unique sequential-move equilibrium since the public firm and the private firm care about the sequential move if the government's preference for tax revenues is sufficiently small. On the other hand, if the government's preference for tax revenues is sufficiently large, the two firms that produce in different periods actually allow the public and private firms to produce more than in the simultaneous-move case.

This proposition 5 differs from that of Kato (2008), which focused on comparing a mixed duopoly with a privatized duopoly when there is no endogenous-timing formulation under an observable-delay game. Furthermore, Kato (2008) demonstrated that if the government sufficiently prefers tax revenues, it does not privatize the public firm, while our paper shows that regardless of the government's preference for tax revenues, the government *never* has an incentive to privatize the public firm; this results in sequential-move equilibria.

[8]All calculations are given by the Table A in the Appendix.

4 Concluding Remarks

By introducing the government's preference for tax revenues into the theoretical framework of an extended game with observable delay, this study provides new insight into the trade-off between the government's payoff and the public firm's payoff in the government's optimal policy of privatization. Unlike extant literature on mixed oligopolies that is based on the assumption of a monolithic, Leviathan entity that involves the government and the public firm and that seeks to maximize the social welfare, we have found that the optimal privatization policies potentially differ from Kato (2008), which focused on the government's payoff for comparing mixed and privatized duopolies.

We have found that regardless of the government's preference for tax revenues, the government does not have an incentive to privatize in the endogenous-timing game even though there are conflicts of interest between the public firm and the government. This result may indicate that differences in the implementation of privatization depend on the political power structure between the public firm and the government. Moreover, a unique sequential-move equilibrium is always found in the case of endogenous timing in a mixed duopoly if the government's preference for tax revenues is sufficiently small, while we have two sequential-move equilibria since the public firm and the private firm care about the sequential move if the government's preference for tax revenues is sufficiently large.

Finally, we did not extend our results by considering a model where the public firm competes with n private firms or both domestic and foreign private firms, wherein the government seeks to simultaneously maximize tax revenues and social welfare. Also, in this paper, we have limited the policy analysis to privatization. However, richer policies, such as an ad valorem tax and subsidization policies towards both domestic and international mixed oligopolies, are worth considering in the framework of an extended game with observable delay. There could be important economic implications if the analysis is expanded in an extended game with observable delay. The extension of our model in these directions remains an agendum for future research.

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Appendix

In this case where we have been abbreviated, we present on separate page[9].

Table A: Government's Payoffs

α	G^{lf}	G^{fl}	$G^{LF} = G^{FL}$	G^{C}
⋮	⋮	⋮	⋮	⋮
0.15	0.387863772	0.389852608	0.316792803	0.314880952
0.16	0.386923898	0.38784872	0.31660885	0.314392523
0.17	0.386070449	0.385978874	0.316466023	0.31396789
0.18	0.38529882	0.384235046	0.316362186	0.313603604
⋮	⋮	⋮	⋮	⋮
0.82	0.41253904	0.382728594	0.346836276	0.346485356
0.83	0.413507518	0.383433223	0.347580467	0.347396266
0.84	0.414483619	0.384147403	0.348328506	0.348312757
0.85	0.415467157	0.384870887	0.349080299	0.349234694
0.86	0.41645795	0.385603436	0.349835757	0.350161943
⋮	⋮	⋮	⋮	

Table B: Firms' Profits

α	π_1^C	π_1^{fl}	π_1^{lf}
⋮	⋮	⋮	⋮
0.25	0.09375	0.082551403	0.092457571
0.26	0.092279435	0.08194958	0.09178353
0.27	0.090865558	0.08136799	0.091132149
0.28	0.089505274	0.080805639	0.090502316
⋮	⋮	⋮	⋮
0.40	0.07648283	0.075270351	0.084302793
0.41	0.07561539	0.074891005	0.083877926
0.42	0.074774534	0.074521904	0.083464532
0.43	0.073959097	0.074162641	0.083062158
0.44	0.073167978	0.073812831	0.08267037
⋮	⋮	⋮	⋮

[9]Table 1 is obtained using Microsoft Office Excel.