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THE PURCHASING POWER PARITY IN THE MAGHREB COUNTRIES : A NONLINEAR PERSPECTIVE

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Abstract

The main objective of this paper is to test the validity of the purchasing power parity in the Maghreb countries (namely, Algeria, Morocco and Tunisia). We apply the threshold autoregressive non-linear model (TAR) proposed by Caner and Hansen (2001). First, a review of literature on PPP is presented, analysing its empirical validity and the econometric techniques that have been applied. After that, and investigating for the joint hypothesis of nonlinearity and non-stationarity in the exchange rate behaviour, the TAR model is presented and used for the PPP in the Maghreb countries. The results indicate that the RER shows nonlinear behaviour. Moreover, The Moroccan Tunisian (DH/DT) bilateral exchange rate is found to be highly persistent and follows a random walk, whereas the two others(Algerian Moroccan and Algerian Tunisian bilateral real exchange rates) are characterised by partial unit roots. This implies that PPP holds in one threshold regime but not in the other.

Key Words: *Purchasing Power Parity (PPP) - Real Exchange Rate (RER) Threshold Autoregressive Model (TAR) - Non-linearity- Maghreb countries.*

1 Introduction

The Purchasing Power Parity (PPP) concept is one of the oldest and most controversial relationships in the theory of exchange rates. Although the term "purchasing power parity" was coined by Cassel (1918), it has a very much longer history in economics (See, Frenkel, 1978). It is also one of the most widely tested economic hypotheses. PPP is based on the law of one price (LOOP) and implies that exchange rates should equalize the national price levels of different countries in terms of a common currency. Although Long run PPP is a very simple proposition about exchange rate behaviour, it has attracted the attention of researchers for a long time. Indeed, it has been viewed as basis for international comparison of income and expenditures, an efficient arbitrage condition in goods and assets, an equilibrium condition, and a theory of exchange rate determination (Officier ,1976; Frenkel, 1978; Dornbush, 1987; Isard, 1987; Breuer, 1994; Froot and Rogoff, 1995; Taylor, 1995; Rogoff, 1996; Sarno and Taylor, 2001).

Many studies in international finance have examined the validity of PPP over the long run either by testing whether nominal rates and relative prices move together in the long run or by testing whether the real exchange rate has a tendency to revert to a stable equilibrium level over time. These studies have yielded different result outcomes depending of the testing procedures employed.

Moreover, the empirical literature on Purchasing Power Parity seems to have arrived at the consensus that real exchange rates tend toward PPP in the very long run. However, as Rogoff (1996) points out, the slow rate of convergence to PPP, with deviations having a half life about three to five years, remains a puzzle.

New developments in PPP extend the traditional approaches in two important ways. First, they recognize the nonlinearities created by information, transaction and transportation costs, and other trade impediments. Second, they recognize the importance of time for commodity arbitrage. This new approach refers to the modern theories of LOP and PPP (Peppinger, 2004). The nonlinearity helps resolve a number of puzzles concerning the persistence and volatility of real exchange rates.

A number of empirical studies support such a non-linear adjustment of real exchange rates toward long-run equilibrium. However, those studies generally assume smooth transition between different threshold regimes and focus on developed countries. A discrete transition is likely to be more appropriate for developing countries with a history of macroeconomic instability. In this paper, we empirically explore the possibility of non-linear mean reversion, or different threshold regimes in terms of stationarity, in the case of the Maghreb real exchange rates. We will be using the Caner and Hansen methodology (2001) that allows us to simultaneously investigate non-stationarity and non-linearity under a discrete transition between regimes. Stationary real exchange rates would provide support for the empirical validity of PPP in the Maghreb whereas non-stationary exchange rates would not. The practical implications of deviations from PPP are especially meaningful for the exchange rate behaviour in the

Maghreb in the time of the creation of the euro-Mediterranean free trade area.

The rest of the article is structured as follows. In the next section we provide a short review on empirical validity of the PPP. Section 3 discusses the caner and Hansen methodology. Section 4 describes the data. The results are reported in section 5. A final section briefly summaries and concludes.

2 Empirical evidence on PPP

2.1 Theoretical basis on PPP

The Purchasing Power Parity (PPP) concept is one of the oldest and most controversial relationship in the theory of exchange rates. Among the most popular versions of PPP, there exist the "absolute" version which states that the exchange rate between two currencies of any pair of countries should equal the ratio of the aggregate price levels in the two currencies, and the "strict" version which relates changes in exchange rates to in inflation differential rates.

The earlier promises of the flexible exchange rates were that long-run trends in exchange markets would be denominated by relative rates of inflation, i.e. that exchange rates would follow the PPP (Friedman , 1953), and that temporary factors such as shifting interest rates might cause temporary deviations from PPP but such deviations are reduced because speculators force the market towards its long long-run equilibrium.

The two mentioned versions can be written as follows:

Absolute Version

$$lnS_t = a + bln\ (p/p^*)_t + U_t$$

Relative Version

$$lnS_t = bln(p/p^*) + V_t$$

where S_t= the exchange rate

(p/p^*) = the ratio of domestic to foreign price indices, the asterisk denotes the foreign country.

U_t, V_t = error terms

Δ = the first difference operator

a = the intercept term

b = the slope coefficient.

There is not, however, a unique view about which price index should be used in these versions. According to one extreme view, exchange rates should be held in line with general price indices, i.e. prices of both traded and non-traded goods. Advocates of this view emphasise the role of asset equilibrium in determining the exchange rate (Cassel, 1930). A second view focuses on commodity arbitrage as the international mechanism that correct purchasing power disparities and therefore argues that only prices of traded goods should be included in the calculation of the ratio of price indices. Supporters of this view are, for example, (Angell, 1922; Bunting, 1939; Hecksher, 1930; Pigou, 1930; Viner, 1937).

The third view goes further to account for non-traded goods only. According to Keynes, the use of prices of traded goods only, is no more than a tautology, because it simply means that the price of a commodity must be the same elsewhere when converted into a common currency. Hansen and Hodrick (1980) for example claimed for the use of production indices.

The choice of the price index is not the only deficiency to the PPP, other factors such as the choice of base period for relative PPP and the transportation costs may also bias the calculation of PPP. These deficiencies have weakened the theoretical basis of PPP.

The PPP doctrine is seen as an equilibrium relationship between an exchange rate and some designated ratio of price indices. This relationship implies that any divergence from the ratio will set in motion corrective forces acting to restore equilibrium. The question that can be asked here is which causes which? Is it the changes in prices that cause exchange rate movements or is it the opposite?

The majority of authors recognised that prices and exchanges rates are determined simultaneously. A minority, however, argued that there exists a causal relationship between prices and exchange rates. Cassel (1930), for example, claimed that the causality goes from prices to the exchange rate; Einzig (1937) claimed the opposite.

2.2 Violations of PPP

This section considers some empirical results concerning the validity of purchasing power parity. The body of empirical literature on PPP (Purchasing Power Parity) focused on developing countries is quite thin, both in absolute terms and when compared to that available for industrial economies (Breuer, 1994). This is probably a consequence of the developing countries' reluctance to adopt floating exchange rates following the breakdown of the Bretton Woods system. Indeed, the fact that the majority of these countries held on for a while to fixed exchange rate arrangements-as well as to all forms of restrictions on current and capital account transactions-made it both less pressing and less meaningful to use their data to test models that relied upon (or consisted of) PPP-based notions of the equilibrium exchange rate.

The situation started to change in the late 1980s. Since then, a growing number of studies have examined the time series properties of RER in various developing countries, in many cases testing explicitly for some version of PPP.

To classify the tests employed in the studies we followed the demarcation of the various stages of tests of PPP proposed by Breuer, 1994 and Froot and Rogoff, 1995, namely: simple tests of PPP as the null hypothesis, univariate tests of the time series properties of the RER series, and cointegrating tests of PPP, both bivariate and trivariate .

The results given by these studies capture some interesting features of empirical studies of RER and PPP in emerging economies. First, in terms of coverage, there is far more evidence available for Latin American economies than for developing countries in other parts of the world. Second, the periods covered by the studies are quite short. The majority of studies conducted tests on data series that covered less than 30 years and some of them did so on series that covered less than 15 years. Third, studies relied a bit more heavily on consumer price indices than on wholesale price indices to construct their measure of relative (domestic to foreign) prices. Fourth, the majority of studies relied on some type of univariate test to examine the main properties of the RER and the hypothesis. Only very few studies (McNown and Wallace, 1989, Liu, 1992, Gan, 1994 and Seabra, 1995) conducted bivariate cointegrating tests of PPP. And fifth, studies were generally unclear about the precise PPP hypothesis that was being tested.

An obvious consequence of the predominance of univariate tests of PPP is that the bulk of the findings obtained by the above studies revolve around the stationarity of various measures of the RER. By and large, the hypothesis that the RER is stationary in developing countries (and, thus, that some form of PPP condition holds in the long run) does not receive much support from these studies. In fact, Edwards, 1989 tested the random walk hypothesis for a combined total of 44 series, and rejected it in about 2/3 of the cases.

Results from the (few) studies that used cointegration tests were somewhat more supportive of the PPP hypotheses. The two studies that conducted trivariate tests of cointegration (Liu, 1992 and Seabra, 1995) found even stronger evidence of an equilibrium relationship between the exchange rate and domestic and foreign prices (18 of 20 cases). Notably, all the support for PPP obtained from these stage-three tests stemmed from data on Latin American countries; in fact, Gan, 1994 did not find evidence of cointegration between the exchange rate and prices in any of the five East Asian countries in his sample.

Seeing what the studies have to offer, one gets the distinct feeling that our knowledge of the basic time series properties of RER in developing countries and, in particular, of the relevance of PPP as a long-run benchmark for the equilibrium RER in these economies is fairly rudimentary. The most serious shortcoming is, without question, the low power of the tests (especially of cointegration tests) to distinguish among alternative hypotheses in the short periods covered by the studies a deficiency that cannot be fixed by the common practice of increasing the number of observations through the use of quarterly or monthly data (Froot and Rogoff, 1995, Oh, 1996). But this is hardly the only problem. The pervasive and severe data problems that one encounters in developing countries may well be at the root of these shortcomings, and it is quite possible that for many countries this constraint will not disappear for many years. But this does not alter the basic conclusion that the evidence on RER stationarity and long-run PPP contained in studies of individual developing countries does not enable us to discern which, if any, of the regularities of the long-run RER that have been found for industrial economies are also applicable to (or relevant for) the developing world.

2.3 Recent developments in PPP violations

Among the possible explanations for the violation of th law of one price and the purchasing power parity suggested by the empirical evidence, transportation costs, tariffs and non-tariff barriers are dominant. This has given rise to theoretical models of non-linear exchange rate arrangements (e.g. Williams and Wright, 1991; Dumas, 1992; Sercu, Uppal and Van Hulle, 1995, Sarno and Taylor, 2001). To test these models empirically Michael, Nobay, and Peel (1997) use the Lothian and Taylor (1996) long span of annual data on dollar-sterling and franc-sterling exchange rates as well as monthly data for three real exchange rates during the interwar period and show that statistically significant nonlinearity characterizes the adjustment toward equilibrium of the real exchange rate series examined, successfully modeled as exponential smooth-transition autoregressive processes (Granger and Terasvirta, 1993). Obsfeld and Taylor (1997) investigated for the nonlinear nature of the adjustment process in term of threshold autoregressive (TAR) model (Tong, 1990). Obsfeld and Taylor provide evidence that TAR models work well when applied to disaggregated data, and yield estimates in which the thresholds correspond to popular rough estimates of the order of magnitude of actual transport costs (Sarno and Taylor, 2002).

As far as developing countries are concerned, Taylor and Sarno (2001) examined the behaviour of the real exchange rates of nine transition economies (Bulgaria, the Czech Republic, Hungary, Latvia, Lithuania, Poland, Romania, the Slovak Republic, and Slovenia) during the1990s. They used a nonlinear multivariate generalization of the Beveridge-Nelson decomposition. They results were supportive to the nonlinear behaviour of real exchange rates. However, and to the best of our knowledge, no empirical work has been carried out to test the nonlinearity of exchange rates in the Maghreb countries.

3 Research Methodology

In this section we present the TAR model that we will be using in our empirical work. Appropriate hypotheses and test structures are discussed

3.1 TAR model

The threshold model used is as follows[1]

$$\Delta y_t = \left\{ \begin{array}{c} \theta_1' x_{t-1} + e_{1t} \ si \ z_{t-1} \prec \lambda \\ \theta_2' x_{t-1} + e_{2t} \ si \ z_{t-1} \geq \lambda \end{array} \right\} \tag{1}$$

with $t = 1$,...,T ; x_{t-1}= $(y_{t-1}\ r\prime_t\ \Delta y_t ... \Delta y_{t-k})$' ; $e_{1}t$ and $e_{2}t$ are two white noise i.i.d ; z_t is the switching variable that should be stationary, where $z_t = y_t$ - y_{t-m} for $m \geq$ 1(Caner and Hansen : 2001) ; $r_t$is a vector of deterministic components including an intercept with a possibility of a linear trend; λ is the threshold so as ; $\lambda \in [\lambda_1,\ \lambda_2]$ avec $p(z_t \leq \lambda_1) = 0.15$ and $p(z_t \leq \lambda_2) = 0.85$ (Andrews : 1993) ; $\theta_1 = \begin{bmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{bmatrix}, \theta_2 = \begin{bmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{bmatrix}$;

[1]See, BEC Frederick and others, 2002, for more details

ρ_1 et ρ_2 are scalar and are the slope coefficients on y_{t-1} in the two regimes ; β_1 and β_2 have the same dimension as rt and are the slopes on the deterministic components; $\alpha_1$and $\alpha_2$are the slope coefficients on $(\Delta y_t,....,\Delta y_{t-k})$ in the two regimes.

To estimate equation (1) , we use the methodology developed by Hansen (1996) which propose to calculate the residuals variance for each possible threshold using ordinary least squares .We take the threshold value that minimise the residuals variance:

$$\Delta y_t = \left\{ \begin{array}{l} \widehat{\theta}_1'(\lambda)x_{t-1} + \widehat{e}_{1t}(\lambda) \ si \ z_{t-1} \prec \lambda \\ \widehat{\theta}_2'(\lambda)x_{t-1} + \widehat{e}_{2t}(\lambda) \ si \ z_{t-1} \geq \lambda \end{array} \right\} \tag{2}$$

$\widehat{e}_t(\lambda) = \widehat{e}_{1t}(\lambda)+\widehat{e}_{2t}(\lambda); \hat{\sigma}^2(\lambda) = T^{-1}\sum_{t=1}^{T}\hat{e}^2(\lambda); \hat{\lambda} = Min_{\lambda}\hat{\sigma}^2(\lambda); \hat{\theta}_1 = \hat{\theta}_1(\hat{\lambda}); \hat{\theta}_2 = \hat{\theta}_2(\hat{\lambda}); \widehat{e}_{1t} = \widehat{e}_{1t}(\hat{\lambda}); \widehat{e}_{2t} = \widehat{e}_{2t}(\hat{\lambda}).$

We can, then, rewrite the equation as follows

$$\Delta y_t = \left\{ \begin{array}{l} \widehat{\theta}_1'x_{t-1} + \widehat{e}_{1t} \ si \ z_{t-1} \prec \hat{\lambda} \\ \widehat{\theta}_2'x_{t-1} + \widehat{e}_{2t} \ si \ z_{t-1} \geq \hat{\lambda} \end{array} \right\}$$

Hansen (2000) propose a confidence interval for the threshold λ, based on the likelihood ratio : $\Gamma = \{\lambda : \ LR(\lambda) \leq c\}$ where $LR(\lambda) = T\frac{\hat{\sigma}^2(\lambda)-\hat{\sigma}^2(\hat{\lambda})}{\hat{\sigma}^2(\hat{\lambda})}$; C represent a confidence level (eg. $C = 95\%$) $c = c_{\xi}(C)$; and is the critical value (at level C) as tabulated by Hansen (2000) (we note that $LR(\lambda_0)$ is the likelihood ratio statistic for testing the hypothesis $H_0 : \lambda = \lambda_0$).

After estimating the model above, we, then go to test the threshold effect (linearity tests), followed by the stationarity tests.

3.2 Threshold effect tests

The question is to see if there is a threshold effect. This effect disappears under the null hypothesis:

$H_0 = \theta_1 = \theta_2$:

We use the standard Wald statistic (W_t). Since this statistic is a decreasing function of $\hat{\sigma}^2(\lambda)$, it can be written as follows :

$W_t = W_t(\hat{\lambda}) = \sup_{\lambda\in\Gamma} T\left(\overline{\frac{\sigma_0^2}{\hat{\sigma}^2(\lambda)}} - 1\right)$

where σ_0^2 is the residual variance of the estimated equation under the null hypothesis (The linear model), and $\hat{\sigma}^2(\lambda)$ is the residual variance of equation (2).

Hansen (1996) gives the asymptotic properties of the Wald statistic in the case where the threshold λ is not identified under the null hypothesis (in this case the test is non standard). Hansen proposed an approximation based on simulations.

Caner and Hansen (2001) argue that the presence of non stationarity in the data will affect the asymptotic distribution of the threshold test. According to them, the asymptotic distribution is nonpivotal and depends on the nuisance parameter function. The dependence is so complicated that the critical values cannot be tabulated. To account for this, Caner and Hansen (2001) propose two bootstrap approximations of the asymptotic distribution of Wald statistics. One is based on the unrestricted estimate and the other enforcing the restriction of a unit root.

3.3 Stationarity tests

Enders and Granger (1998) give the critical values for testing the unit root hypothesis in he case of an asymmetric adjustment (as a TAR model). They demonstrated that the Dickey-Fuller test, and all its other extensions (eg, Philips and Perron), represent only one special case (the case of symmetric adjustment). According to them, the hypotheses to be tested are :

$H_o : \rho_1 = \rho_2 = 0$

$H_1 : \rho_1 \prec 0$ and $\rho_2 \prec 0$

Caner and Hansen (2001) propose a third hypothesis to be tested :

$$H_2 = \begin{bmatrix} \rho_1 \prec 0 \text{ et } \rho_2 = 0 \\ ou \\ \rho_1 = 0 \text{ et } \rho_2 \prec 0 \end{bmatrix}$$

In the case where H_0 is accepted, yt is integrated of order 1 ($I(1)$). However, when H_2 holds, the process y_t will behave as a unit root process in one regime and stationary in the other. Moreover, it is important to distinguish between the cases H_0, H_1, and H_2. Here, Caner and Hansen (2001) propose two Wald statistics (R_{1t} and R_{2t}) to test the hypothesis H_0 against the hypotheses H_1 and H_2 :

$R_{1t} = t_1^2 + t_2^2$ (test H_0 against the alternative $\rho_1 \neq 0$ ou $\rho_2 \neq 0$)

$R_{2t} = t_1^2 1_{(\hat{\sigma}_1 \prec 0)} + t_2^2 1_{(\hat{\sigma}_2 \prec 0)}$(test H_0 against the alternative $\rho_1 \prec 0$ ou $\rho_2 \prec 0$

Where, t_1 and t_2 are t-ratio for $\hat{\rho}_1$et $\hat{\rho}_2$ and from the OLS regression in equation (2).

Caner and Hansen (2001) put that although the two statistics (R_{1t} and R_{2t}) could justify rejection of unit root hypothesis, they cannot, however, distinguish between H_1 and H_2. They propose an examination of the individual statistics t_1 and t_2. If only one of these statistics is significant, then H_2 is accepted.

Caner and Hansen (2001) suggest that H_0 should be rejected for large values of R_t. In order to determine the significance (calculate $p-value$), they propose a bootstrap approximation of the sampling distribution of the test under H_0. Furthermore, Caner and Hansen (2001) put forward that the statistics distributions of R_t, t_1, t_2 depend on the presence of threshold effect. For this, they propose two bootstrap approximations to calculate the $p-value$: one bootstrap with a constraint of Threshold effect, and another without constraint. Because the rejection rate using the unidentified threshold model is seen to be less sensitive to the nuisance parameter, Caner and Hansen (2001) recommend the use the unconstraint bootstrap.

4 Data description

Monthly data on nominal[2] exchange rates as well as consumer price indices for the Maghreb countries (namely, Algeria, Morocco and Tunisia) are used in our analysis. The data are obtained from the International Monetary Fund's International Financial Statistics (IFS). Precisely, the sample period is 1974M1-2004M12 for Algeria; 1987M7-200412 for Tunisia; and 1974M1-2004M5 for Morocco.

In our empirical analysis we test the real exchange rate (RER) stationarity. The RER is calculated as follows :

$$q_t = s_t - p_t + p_t^*$$

q_t the real exchange rate ;
s_t the logarithm of nominal exchange rate ;
p_t the logarithm of consumer price index of the base country ;
p_t^* the logarithm of consumer price index of the foreign country

So we have three bilateral real exchange rates to test :

The first is between Algeria and Morocco for the period 1974M1-2004M5. The second between Algeria and Tunisia for the period 1987M7- 2004M12; and the third is between Morocco and Tunisia from for the period 1987M7-2004M6. However, we will not take the whole sample period in our empirical analysis.We will be using the period 1974M1- 2003M5 for the first ; 1987M7-2003M5 for the second; and 1987M7-2004M5 for the third one. The remaining of each period is used for out of sample forecasting.

5 Unit Root Test Results

We start our empirical study by conducting the ADF conventional tests on the three bilateral real exchange rates (Algerian Dinar - AD/Moroccan DIRHAM-DH, AD/Tunisian DINAR-DT, and DH/DT). The results are based upon an ADF regression with an intercept for the Tunisian Dinar/ Dirham(DT/DH= and the (AD/DT) real exchange rates, whereas for the DT/DH exchange rate the ADF regression equation is taken without an intercept.

We use the modified Akaike information criterion (MAIC) to choose the appropriate lag length, d. This test is proposed by Ng and Perron (2001)

The results in table 1, clearly show, with the exception of the DA/DH real exchange rate, that the unit root hypothesis cannot be rejected for two other bilateral exchange rates at the 1%, 5% and 10% level. For the DA/DH exchange

[2]These nominal exchange rates are bilateral rates (Algerian Dinar/Moroccan Dirham and hence (AD/DH) ; Algerian Dinar/Tunisian Dinar (AD/DT) ; And Tunisian Dinar/ Moroccan Dirham (DT/DH) calculated on the basis the nominal exchange rate of each country against the US Dollar.

rates, however, the unit root hypothesis is rejected at 5.49 % level. These results confirm that the purchasing power parity hypothesis is not accepted for the DA/DT and DH/DT exchange rates, whereas this hypothesis holds for the DA/DH exchange rates with a half-life of three months.

Table[1] : ADF unit root tests

	Lag : MAIC	ADF	ρ_1	Half-life[3]
DA/DH	8	-2.831615 [0.0549]	-0.210624 (0.074383)	2.9306998/∞
DA/DT	5	-1.751252 [0.43035]	-0.104722 (0.059799)	∞
DH/DT	12	0.734806 [0.8725]	0.005187 (0.007059)	∞

Figures in parentheses and brackets are standard errors and p-values, respectively. .

Moreover, we used a simple Chow test for a structural break in the exchange rate ADF regressions. The null hypothesis of no structural break could not be rejected so that the estimated regressions are stable over the full period. These results are reported in table (2).

Table[2] : Chow breakpoint tests

	F-statistic	Log likelihood ratio
DA/DH	1.615703 [0.10093]	16.75673 [0.079923]
DA/DT	1.066243 [0.387012]	7.875185 [0.343729]
DH/DT	1.56022 [0.122777]	16.72158 [0.080756]

Figures in brackets are p-values.

Empirical studies have shown that ADF tests suffer from the low power problems, and require large sample data. Furthermore, many studies on the real exchange rate found that this latter follow a non linear behavior. Hence we will be conducting linearity tests on the ADF regressions of the real exchange rates, and then we take the non linearity aspect, if it exists, in unit root tests.

5.1 Linearity Tests

Two linearity tests are used to see possible aspects of nonlinearities in the ADF regression tests. The first test is the regression specification error test (RESET) developed by Ramsey (1969), and the second is the BDLS test by Brock and al. (1987) and developed in Brock, Dechert, Sheinkman, & LeBaron (1996)

[3] The half-life in months is calculated as: $ln(0.5)/ln(1+\rho_1)$.

Table[3] : The BDLS test for nonlinearity in residuals of the estimated ADF regressions

m	DA/DH	DA/DT	DH/DT
2	0.0422 [0.0000]	0.091215 [0.0000]	0.100655 [0.0000]
3	0.0766 [0.0000]	0.156817 [0.0000]	0.162653 [0.0000]
4	0.0970 [0.0000]	0.194222 [0.0000]	0.222476 [0.0000]
5	0.1033 [0.0000]	0.215996 [0.0000]	0.257832 [0.0000]
6	0.1077 [0.0000]	0.228059 [0.0000]	0.268719 [0.0000]

m is the embedded dimension. Bootstrap p-values are reported under the null hypothesis that the residuals are iid. The p-values are obtained using 10,000 bootstrap simulations.

The results of the BDLS test are reported in table 3, and show great evidence of nonlinearity in the residuals of the ADF regressions (All the $p-values$ close to zero). By contrast, the RESET test results reported in table 4, show that the ADF regressions are linear for the three real exchange rates. These tests do not impose any particular case of non linearity.

Table[4] : The RESET test for neglected nonlinearity in the auxiliary regression for ADF test

	Lag : MAIC	F-statistic	Log likelihood ratio
DA/DH	8	0.256195 [0.61309]	0.264789 [0.606849]
DA/DT	5	0.014538 [0.904166]	0.015194 [0.901897]
DH/DT	12	0.087326 [0.767983]	0.094847 [0.758103]

Figures in brackets are p-values

To overcome these conflicting results, we use the Wald test proposed by Caner and Hansen (2001) which will permit us to test a particular case of non linearity - Threshold Autoregressive models- (TAR).

5.2 Threshold test results

We run the Wald test to see the null hypothesis of non linearity against an alternative model which is TAR. The results are reported in table 5. the lag length d are determined so that we minimize the variance of the residuals in the TAR model. All the p-values show that the three real exchange rates are non linear but at different levels (3.81% for the DA/DH, 1.88% for the DA/DT and 5.53 for the DH/DT exchange rates). Thus, the three exchange rates follow TAR models, implying the misspecification of the ADF tests.

Table[5] : Threshold test with fixed delay parameters

	d	Wald	10% B.C.V	5%B.C.V	1%B.C.V
DA/DH	8	37.1 [0.0381]	32.4	35.9	44.2
DA/DT	3	48.7 [0.0188]	35.6	41	54.6
DH/DT	4	38.7 [0.0553]	34.9	39.5	49.2

Bootstrap critical values and p-values values are calculated from 10,000 replications. d is the delay parameter .

Threshold unit root test results are reported in table 6. The lag length, d, as determined by Wald test, is shown in the first column. In column (2) and (3), we report bootstrap $p-values$ of t_1 and t_2 that are used to test H_0 against H_2. These statistics are obtained using 10,000 bootstrap simulations.

Bootstrap p-value indicates that ρ_1 and ρ_2 are significantly equal to zero in the case of DH/DT exchange rates. Thus, we could no reject the unit root hypothesis (The two regimes are highly persistent or follow a random walk). As far the DA/DT exchange rate is concerned, the $p-value$ of t_1 and t_2 show that ρ_1 is significantly different from zero, whereas ρ_2 is significantly equal to zero which implies that the outer regime displays mean reversion (stationary), while the inner regime is highly persistent and follow a random walk.

For the DA/DH exchange rate, the same results are found but differently, i.e. the inner regime displays mean reversion (stationary), while the outer regime is highly persistent and follows a random walk.

In sum, the DH/DT exchange rate is found to be highly persistent and follows a random walk, whereas the two others are characterised by partial unit roots.

Another result can also be shown in table 6. R_{1t} and R_{2t} indicate that only the DA/DT exchange rate has a partial unit root. However, since R_{1t} cannot distinguish between H_1 and H_2, Our main concern will be on the statistics R_{2t}, t_1 and t_2.

We assume that the switching variable is stationary, which makes the process visit every regime infinitely often in the limit (Bec F. and others, 2002). Consequently, when the real exchange rate is mean-reverting in one regime at least, the whole process may be viewed as stationary. Thus, taking the statisticsR_{2t}, t_1 and t_2 we conclude that the DA/DH and DA/DT exchange rates could be regarded as stationary.

Furthermore, table 6 gives some other results such as: the threshold estimates, intercepts, ρ_1 and ρ_2 and the half-life of each exchange rate. For instance, the DA/DT exchange rate has a threshold estimate, λ, of -2.27 with 18.4% of observations in the outer regime (mean reverting with $\rho_1 = -0.45$ and a half-life of 1.16 months), and 81.6% of observations in the inner regime (random walk). It is clear that the estimated deviations from PPP, in this case, have a considerably shorter half-life than what has been found in the literature. Moreover, since the majority of observations lie inside the unit root regime, we can argue that the exchange rates are stationary but highly persistent. In this issue

Sekkioua (2005) argues that: "*This persistence coupled with threshold nonlinearity explains why standard ADF tests are biased towards the unit root null hypothesis*".

Table[6] : Threshold Unit Root Results

		DA/DH	DA/DT	DA/DH
	d	8	3	4
t_1	Outer	[0.867]	[0.0087]	[0.624]
t_2	Inner	[0.0779]	[0.752]	[0.204]
Unit root tests	R_{1t}	[0.202]	[0.0253]	[0.414]
Unit root tests	R_{2t}	[0.219]	[0.0281]	[0.450]
THRESHOLD	λ	0.0380	-2.27	0.0369
%	Outer	65.9	18.4	72.5
%	Inner	34.1	81.6	27.5
β_1	Outer	0.723(1.98)	8.30(4.25)	1.47(1.94)
β_2	Inner	8.11(3.21)	2.40(1.54)	3.75(3.47)
ρ_1	Outer	-0.0293(0.0926)	-0.450(0.203)	-0.0671(0.101)
ρ_2	Inner	-0.388(0.149)	-0.115(0.0711)	-0.361(0.181)
HALF-LIFE[4]	Outer	∞	1.1594250	∞
HALF-LIFE	Inner	1.4116389	∞	∞

Figures in parentheses and brackets are standard errors and p-values, respectively .

5.3 Linear versus TAR models

In this section, we give a comparison between linear models and TAR models, in order to see which perform best. To do this, both models were used to generate 1-, 3-, 6-, 9-, and 12-step ahead, using the periods left for forecasting (as noted earlier in the data description). Table 7 provides the results of RMSE tests of the two models (and to see how smaller is the RMSE). In fact, the results are in favour TAR models in the case of DA/DT and in all the steps (1, 3, 6, 9 and 12 months), whereas the linear model fit well the DA/DH exchange rate. In the case of DH/DT, the TAR model performs best at 60% (3 out of 5 steps).

It seems that TAR models perform better than linear models and are more accurate and appropriate. However, in the case of DA/DH exchange rate, one can explain the poor performance of TAR models to the fact that there may be some linearity in the forecast period (Sekkioua, 2005) and (Van Dijk et al., 2001)

[4]The half-life in months is calculated as: $ln(0.5)/ln(1 + \rho_1)$.

Table[7] : TAR versus Linear models (RMSE* tests)

			TAR RMSE		
Horizon	1	3	6	9	12
DA/DH	1.826502	1.549149	1.14719	1.31052	1.389479
DA/DT	1.013626	1.996119	1.46523	1.62947	2.456361
DH/DT	2.031927	3.072135	2.74736	2.46023	2.33836
			Linear RMSE		
Horizon	1	3	6	9	12
DA/DH	1.45059	1.03768	1.69882	1.38997	2.05830
DA/DT	1.37449	2.08433	1.55085	2.25149	2.92415
DH/DT	0.63362	2.12229	1.74013	1.52078	1.33915

*RMSE stands for Root mean square error. A lower value for RMSE indicates superior forecasting performance .

6 Conclusion

In this paper we have examined the behaviour of the real exchange rates in the Maghreb countries (namely, Algeria, Morocco and Tunisia).Our main concern is to investigate whether Purchasing Power Parity hold using bilateral exchange rates. If it holds, the real exchange rates should be stationary and mean reverting. Because the conventional ADF tests did not support the stationarity in the real exchange rates, We employed the empirical methodology developed by Caner and Hansen (2001), that allows us to simultaneously investigate non-stationarity and non-linearity under a discrete transition between regimes.

Using the ADF conventional test the results confirm that the purchasing power parity hypothesis is not accepted for the DA/DT and DH/DT exchange rates, whereas this hypothesis holds for the DA/DH exchange rates with a half-life of three months.

Then we have used two linearity tests to see any aspects of nonlinearities in the ADF regression tests. The first test is the regression specification error test (RESET), and the second is the BDLS. The results of the latter show great evidence of nonlinearity in the residuals of the ADF regressions. By contrast, the RESET test results show that the ADF regressions are linear for the three real exchange rates. To overcome these conflicting results, we used the Wald test proposed by Caner and Hansen (2001) which permit us to test a particular case of non linearity - *Threshold Autoregressive models*- (TAR). The results show that the three real exchange rates are non linear but at different levels (3.81% for the DA/DH, 1.88% for the DA/DT and 5.53 for the DH/DT exchange rates). Thus, the three exchange rates follow TAR models, implying the misspecification of the ADF tests.

Bootstrap $p-value$ indicates that ρ_1 and ρ_2 are significantly equal to zero in the case of DH/DT exchange rates. Thus, we could no reject the unit root hypothesis (The two regimes are highly persistent or follow a random walk). As far the DA/DT exchange rate is concerned, the $p - value$ of t_1 and t_2 show that ρ_1 is significantly different from zero, whereas ρ_2 is significantly equal to zero which implies that the outer regime displays mean reversion (stationary), while the inner regime is highly persistent and follow a random walk.

In sum, the DH/DT exchange rate is found to be highly persistent and follows a random walk, whereas the two others are characterised by partial unit roots.

Furthermore, the DA/DT exchange rate has a threshold estimate, of -2.27 with 18.4% of observations in the outer regime (mean reverting with $\rho_1 = -0.45$ and a half-life of 1.16 months), and 81.6% of observations in the inner regime (random walk). It is clear that the estimated deviations from PPP, in this case, have a considerably shorter half-life than what has been found in the literature. For instance , XU (2003) found that the estimated deviations from PPP have considerably shorter half life. It is about two years for CPI and WPI based real exchange rates, but only one year for the TPI based RER. Moreover, since the majority of observations lie inside the unit root regime, we can argue that the exchange rates are stationary but highly persistent.

Finally, we gave a comparison between linear models and TAR models, in order to see which perform best. In fact, the results are in favour TAR models in the case of DA/DT and in all the steps (1, 3, 6, 9 and 12 months), whereas the linear model fit well the DA/DH exchange rate. In the case of DH/DT, the TAR model performs best at 60% (3 out of 5 steps).

It seems that TAR models perform better than linear models and are more accurate and appropriate. However, in the case of DA/DH exchange rate, one can explain the poor performance of TAR models to the fact that there may be some linearity in the forecast period. One possible explanation of the above results is due to the "*border-effect*" since the frontiers are closed between Algeria and Morocco for more than ten years (since 1994), which has disturbed trade between Morocco and Algeria as well as between Morocco and Tunisia. For argument sake, Cuddington & Liang (2000), tried to explain why the PPP holds between the Franc Sterling but does not hold between the Dollar Sterling exchange rates. They argued that the geographic distance is greater between USA and Europe than between UK and France. However, one could say that effective distance has shrunk overtime due to the improvements in transportation and communication technology. This is why more research is needed to clarify this point.

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