Endogenous Wealth-Depending Time Preference and Fiscal Policy in Open Economy

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Abstract : In this paper, we study the effectiveness of fiscal policies in a framework of a small open economy where the behaviour of representative consumer is characterised by endogenous time preference which depends on wealth and consumption.

Key words : time preference, fiscal policy, open economy

JEL classification : H50, F41, F34,

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1. Introduction

The open economy extension of Ramsey's model is characterised by external position indeterminacy. This is due to the fact that the representative agent has fixed time preference. Authors like Obstfeld (1980, 1981, 1982), Svensson and Razin (1983), Pitchford (1989, 1991), Engel and Ketzler (1989), Devereux and Shi (1991) have introduced variable time preference[1] in open economy study. One can address two critics to these developments. First, an increase in fiscal pressure will induce a sharp reduction in consumption and a surplus of balance of payments in order to keep the same level of long term consumption. This is contrary to traditional view that the consumer will to smooth his life time consumption. Second, as time preference rate depends only on consumption, for some level of fixed world real interest rate, the small open economy will accumulate a rather significant wealth, likely to make its actions significant on international financial market, so invalidating the hypothesis of small open economy without market power. One solution to these two problems is to do like ZEE (1997 who introduce a discount function depending on consumption and output ratio. But, as national output represents only a source of revenue. It is not a necessarily a determinant of time preference. The time preference is a result of arbitrage between the current and futures consumption which is better represented by accumulated wealth. With this idea in mind, we will introduce in this paper the wealth effect on the determination of endogenous time preference.

2. The Model

Two alternatives formulations in our line can be formulated :

1) The accumulated wealth enters the subjective discount rate function :

[1] The theory of endogenous time preference is first developped by Boehm-Bawerck (1912) and Fisher (1930). Koopmans (1960), Koopmans, Diamond and Willamson (1964), Uzawa (1968), Epstein and Hynes (1983), Espstein (1987a, b) and Obstfeld (1990) have further contributed to it. Ryder and Heal (1973), Becker and Murphy (1988) and others have applied this « habit formation » to divers problems.

$$U[C(0)] = \int_0^{\infty} u[c(t)] e^{-\int_0^t \theta[c(s), w(s)] ds} dt, \tag{1}$$

where, U[.] represents the life time utility of the representative consumer, $C(0)$ his life time consumption, $u[.]$ the temporary utility function with $c(t)$ as consumption at time t, $\theta(.,.)$ the subjective discount function which depends on current consumption and accumulated wealth (w).

2) The ratio of accumulated wealth to output (y, a fixed dotation) enters the subjective discount rate function :

$$U[C(0)] = \int_0^{\infty} u[c(t)] e^{-\int_0^t \theta[c(s), \frac{w(s)}{y(s)}] ds} dt, \tag{2}$$

where the ratio w/y can be considered as consumer's sacrifice ratio during its past life. It can be positive or negative.

These two formulations have some similar implications for fiscal policy in open economy. So, we resolve only the consumer's program the first formulation. The concavity conditions of the utility function are verified (i.e., $u(c) > 0$, $u'(c) > 0$, $u''(c) < 0$). We admit also, $\theta(.,.) > 0$, $\theta_2^{'} > 0$, $\theta_{22} > 0$, $\theta_1^{'} > 0$, $\theta_{11}^{'} > 0$ $\theta_{12} = \theta_{21} > 0$.

Defined $\Theta = \int_0^t \{\theta[c(s), w(s)] - r^*\} ds$ with r^* as the world real interest rate and $\Theta(0) = 0$. Its time derivative is given out as:

$$\dot{\Theta} = \theta[c(t), w(t)] - r^*. \tag{3}$$

The consumer's problem becomes then:

$$U[C(0)] = \int_0^{\infty} u[c(t)] e^{-\Theta(t)} e^{-r^* t} dt, \tag{4}$$

under the dynamic constraint (3) and the following budgetary constraint :

$$\dot{w} = r^* w + y(t) - c(t) - \tau(t), \tag{5}$$

where $\tau(t)$ represents taxes or redistribution revenues ; w can be seen as the external position of the small economy ; r^* is the interest rate on the international financial market. When w is positive, the small economy lends to the rest of the world. Inversely, when w is negative, it borrows from the rest of the world at the same condition.

The Hamiltonian associated with this problem is :

$$H = u[c(t)]e^{-\Theta(t)} + \dot{\lambda}(t)[r^* w + y(t) - c(t) - \tau(t)] - \dot{\phi}(t)\{\theta[c(t), w(t)] - r^*\}, \tag{6}$$

where $\dot{\phi}(t)$ and $\dot{\lambda}(t)$ are multipliers associated respectively with (3) and (5). The first-order necessary conditions are written as :

$$u'[c(t)]e^{-\Theta(t)} - \dot{\phi}(t)\theta_1'[c(t), w(t)] = \dot{\lambda}(t), \tag{7}$$

$$\dot{\dot{\lambda}} = \dot{\phi}(t)\theta_2'[c(t), w(t)],$$

(8)

$$\dot{\dot{\phi}} = r^* \dot{\phi} - u[c(t)]e^{-\Theta(t)}, \tag{9}$$

Noting $\lambda = \dot{\lambda}(t)e^{\Theta(t)}$ and $\phi = \dot{\phi}(t)e^{\Theta(t)}$, the previous conditions can be rewritten as:

$$u'[c(t)] - \phi(t)\theta_1'[c(t), w(t)] = \lambda(t), \tag{10}$$

$$\dot{\lambda} = \phi(t)\theta_2'[c(t), w(t)] + \lambda(t)\{\theta[c(t), w(t)] - r^*\}, \tag{11}$$

$$\dot{\phi} = \phi(t)\theta[c(t), w(t)] - u[c(t)], \tag{12}$$

The intertemporal time preference rate can be defined as follows:

$$\rho = \frac{d}{dt}\log\left\{[u'(c) - \theta_1'(c, w)\phi]e^{-\Theta t - r^* t}\right\}\Big|_{\dot{c}=0} = \theta(c, w) + \frac{\theta_{12}''(c, w)\dot{w}\phi + \theta_1'(c, w)\dot{\phi}}{u'(c) - \phi\theta_1'(c, w)}.$$

Its partial derivatives are given by: $\rho_c = -\dfrac{\theta_{12}''(c, w)}{u'(c) - \phi\theta_1'(c, w)}$, $\rho_w = \dfrac{\theta_2'(c, w)u'(c) + r^*\theta_{12}''(c, w)}{u'(c) - \phi\theta_1'(c, w)}$ and $\rho_\phi = \dfrac{\theta_1'(c, w)\theta(c, w)}{u'(c) - \phi\theta_1'(c, w)}$.

In differentiating (10) to time and in substituting the terms λ, $\dot{\lambda}$ and $\dot{\phi}$ by their expressions given by (10)-(12) in the resulting equation, one can obtain the short-run dynamic equation of consumption:

$$\dot{c} = \alpha(\rho + \frac{\phi\theta_2^{'}(c,w)}{u'(c) - \phi\theta_1^{'}(c,w)} - r^*), \qquad \text{with } \alpha = \frac{u'(c) - \phi\theta_1^{'}(c,w)}{u''(c) - \phi\theta_{11}^{''}(c,w)} , \tag{13}$$

3. Stability analysis

Equations (5), (12) and (13) constitute the dynamic system. Its linear form is given as follows:

$$\begin{bmatrix} \dot{c} \\ \dot{w} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \alpha\varphi_1 & \alpha\varphi_2 & \alpha\varphi_3 \\ -1 & r^* & 0 \\ \phi\theta_1^{'} - u' & \phi\theta_2^{'} & \theta \end{bmatrix} \begin{bmatrix} c - \bar{c} \\ w - \bar{w} \\ \phi - \bar{\phi} \end{bmatrix}, \tag{14}$$

with $\varphi_1 = -\dfrac{\phi\theta_{12}^{''}(c,w)[u''(c) - \phi\theta_{11}^{''}(c,w)]}{[u'(c) - \phi\theta_1^{'}(c,w)]^2}$,

$$\varphi_2 = \frac{[\theta_2^{'}(c,w)u'(c) + r^*\theta_{12}^{''}(c,w) + \phi\theta_{22}^{''}(c,w)][u'(c) - \phi\theta_1^{'}(c,w)] + \phi^2\theta_2^{'}(c,w)\theta_{12}^{''}(c,w)}{[u'(c) - \phi\theta_1^{'}(c,w)]^2},$$

$$\varphi_3 = \frac{\theta_1^{'}(c,w)\theta(c,w)[u'(c) - \phi\theta_1^{'}(c,w)] + \theta_2^{'}(c,w)u'(c)}{[u'(c) - \phi\theta_1^{'}(c,w)]^2}.$$

The determinant of the stability matrix:

$$|\Delta| = \alpha\theta(c,w)(r^*\varphi_1 + \varphi_2) + \alpha\varphi_3\{-\phi\theta_2^{'}(c,w) + r^*[u'(c) - \phi\theta_1^{'}(c,w)]\},$$

As $\theta_{22}^{''}(c,w) > 0$, $\theta_{12}^{''}(c,w) > 0$ [2], one has $\varphi_1, \varphi_2, \varphi_3 > 0$. The determinant is then negative if one can demonstrate that $\Omega = -\phi\theta_2^{'}(c,w) + r^*[u'(c) - \phi\theta_1^{'}(c,w)] > 0$. The last condition ensures also the positivity of the trace :

$$Tr. = \alpha\varphi_1 + r^* + \theta = \frac{-\phi\theta_2^{'}(c,w) + r^*[u'(c) - \phi\theta_1^{'}(c,w)] + \theta(c,w)[u'(c) - \phi\theta_1^{'}(c,w)]}{[u'(c) - \phi\theta_1^{'}(c,w)]^2} > 0.$$

The negativity of the determinant and positivity of the trace mean that the system possesses one stable engenvalue and two unstable engenvalues. As there are one predetermined variable (w) and two non predertmined variables (ϕ and c), the system has a saddle-point equilibrium.

[2] The condition $\theta_{22}^{''}(c,w) > 0$ is necessay for ensuring the concavity of objective function and the concavity of the Hamiltonian function in terms of w as $\dfrac{\partial^2 H}{\partial w^2} = -\phi\theta_{22}^{''} < 0$. The condition $\theta_{12}^{''}(c,w) > 0$ imposes a certain degree of complentarity between consumption and accumulated wealth in the time preference function.

To show that $\Omega = -\phi\theta_2^{'}(c,w) + r^*[u'(c) - \phi\theta_1^{'}(c,w)] > 0$, consider (10) and (11) at steady state. That gives :

$$\Omega = -\phi\theta_2^{'}(c,w) + r^*[u'(c) - \phi\theta_1^{'}(c,w)] = \theta(c,w)[u'(c) - \phi\theta_1^{'}(c,w)] > 0\,, \tag{15}$$

At the steady state, the diminution of the marginal value of wealth must be exactly equal to the excess of the real interest rate over the subjecctive discount rate. In models where the wealth does not enter the time preference function, one has simply: $r^* = \theta(\bar{c})$.

5. The long term effects of fiscal policies and the short-run dynamics

The steady state of the economy is described by the following equations

$$\bar{\phi}\theta(\bar{c},\bar{w}) - u(\bar{c}) = 0\,, \tag{16}$$

$$\bar{\rho} + \frac{\bar{\phi}\theta_2^{'}(\bar{c},\bar{w})}{u'(\bar{c}) - \phi\theta_1^{'}(\bar{c},\bar{w})} - r^* = 0\,, \text{ with } \bar{\rho} = \theta(\bar{c},\bar{w}) \tag{17}$$

$$r^*\bar{w} + y_0 - \bar{c}(t) - \tau_0 = 0\,. \tag{18}$$

. Consider a budget equilibrating fiscal policy $\Delta\tau = \Delta g$ (taxes = spending). Its effects on consumption, wealth and marginal value of wealth which can be deduced in differentiating (16)-(18) :

$$\begin{bmatrix} \alpha\varphi_1 & \alpha\varphi_2 & \alpha\varphi_3 \\ -1 & r^* & 0 \\ \phi\theta_1^{'} - u' & \phi\theta_2^{'} & \theta \end{bmatrix} \begin{bmatrix} d\bar{c} \\ d\bar{w} \\ d\bar{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} d\tau\,, \tag{19}$$

The determinant of the Jacobian is negative as demonstrated before:

$$|\Delta| = \alpha\theta(r^*\varphi_1 + \varphi_2) + \alpha\varphi_3[-\phi\theta_2^{'} + r^*(u' - \phi\theta_1^{'})] < 0$$

The solution of this system is:

$$\begin{bmatrix} d\bar{c} \\ d\bar{w} \\ d\bar{\phi} \end{bmatrix} = \frac{1}{|\Delta|}\begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \end{bmatrix} d\tau\,, \tag{20}$$

with,

$$A_{11} = \frac{-\alpha[\theta\theta_2^{'}(u'-\phi\theta_1^{'})^2 + \theta(r^*\theta_{12}^{''} + \phi\theta_{22}^{''})(u'-\phi\theta_1^{'}) + \theta_2^{'}\phi(\phi\theta_{12}^{''} - u'\theta_2^{'})]}{(u'-\phi\theta_1^{'})^2} > 0 \text{, if } \phi\theta_{12}^{''} - u'\theta_2^{'} > 0 .$$

$$A_{12} = \alpha\varphi_1 + \alpha\varphi_3(u'-\phi\theta_1^{'}) < 0 ,$$

$$A_{13} = -\alpha\varphi_1\phi\theta_2^{'} - \alpha\varphi_2(u'-\phi\theta_1^{'}) > 0 .$$

A fiscal policy will have a negative effect on long term consumption under the condition $\phi\theta_{12}^{''} - u'\theta_2^{'} > 0$, a positive effect on wealth and a negative effect on the marginal value of wealth.

As the system has one stable engenvalue, the solution under perfect foresight can be written as :

$$\begin{bmatrix} c(t) - \bar{c} \\ w(t) - \bar{w} \\ \phi(t) - \bar{\phi} \end{bmatrix} = \begin{bmatrix} V_{11} \\ V_{21} \\ V_{31} \end{bmatrix} k_1 e^{\lambda_1 t} , \tag{18}$$

where V_{11}, V_{21} and V_{31} sont les éléments of engenvector corresponding to λ_1, and k_1 can be found in resolving the following equation at $t = 0$:

$$w(t) - \bar{w} = V_{21}k_1e^{\lambda_1 t} ,$$

As w is a predetermined variable, its value at $t = 0$ is given by $w(0)$. Using this result, we have, $k_1 = [w(0) - \bar{w}]/V_{21}$. Then, we can rewrite the solution of dynamic system as follows :

$$c(t) - \bar{c} = \frac{V_{11}}{V_{21}}[w(0) - \bar{w}]e^{\lambda_1 t} ,$$

$$w(t) - \bar{w} = [w(0) - \bar{w}]e^{\lambda_1 t} ,$$

$$\phi(t) - \bar{\phi} = \frac{V_{31}}{V_{21}}[w(0) - \bar{w}]e^{\lambda_1 t} .$$

It is easy to show that : $\frac{V_{11}}{V_{21}} = r^* - \lambda_1 > 0$. In contrast, it is more difficult to determine the sign of

$\frac{V_{31}}{V_{21}} = \frac{|\Gamma|}{-\alpha\varphi_3}$, with $|\Gamma| = \begin{vmatrix} \alpha\varphi_1 - \lambda_1 & \alpha\varphi_2 \\ -1 & r^* - \lambda_1 \end{vmatrix}$. Using the fact :

$$\begin{bmatrix} \alpha\varphi_1 - \lambda_1 & \alpha\varphi_2 & \alpha\varphi_3 \\ -1 & r^* - \lambda_1 & 0 \\ \phi\theta_1^{'} - u' & \phi\theta_2^{'} & \theta - \lambda_1 \end{bmatrix} = 0$$

we have : $|\Gamma| = \frac{-\alpha\varphi_3[r^*(u'-\phi\theta_1^{'}) - \phi\theta_2^{'} - \lambda_1(u'-\phi\theta_1^{'})]}{\theta - \lambda_1}$. Using this result, it yields :

$$\frac{V_{31}}{V_{21}} = \frac{r^*(u'-\phi\theta_1^{'}) - \phi\theta_2^{'} - \lambda_1(u'-\phi\theta_1^{'})}{\theta - \lambda_1} > 0, \text{ as } \Omega = r^*(u'-\phi\theta_1^{'}) - \phi\theta_2^{'} > 0,\ u'-\phi\theta_1^{'} > 0.$$

Using the above result, one has

$$c(0) - \bar{c} = \frac{V_{11}}{V_{21}}[w(0) - \bar{w}] < 0$$

$$\phi(0) - \bar{\phi} = \frac{V_{31}}{V_{21}}[w(0) - \bar{w}] < 0.$$

As the consumption and the marginal value of wealth have an increasing time path, these results signify that they will experience an initial downward over-adjustment.

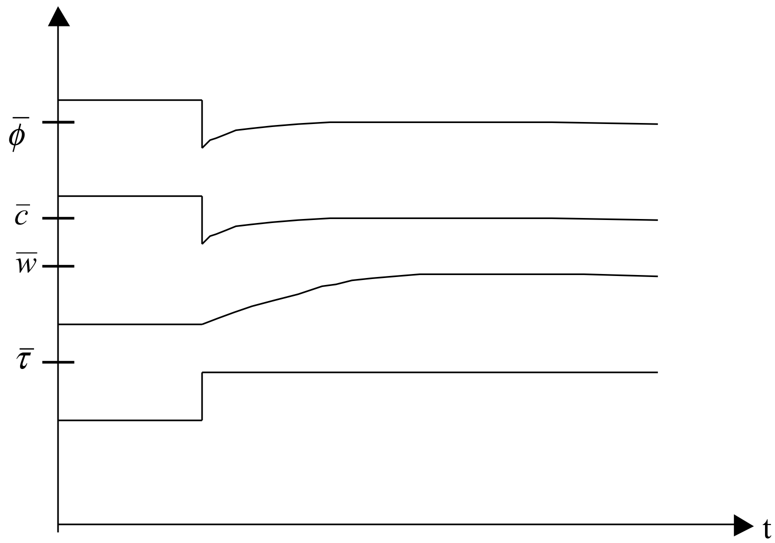

Figure 1. Time path of consumption, weath, marginal value of wealth and taxes.

6. Conclusion

The formulation of endogenous time preference is one fashion to escape from external position indeterminacy problem in representative agent open economy model. The first generation of the endogenous time preference has the non desired effect of accentuating the short-run movement of consumption and a strong accumulation of wealth when government increases its spending and taxes. In introducing wealth in the discount rate function, our formulation attenuate these effects and constitutes an interesting extension. The rationale for this formulation is that, the time preference is a relative variable which depends on relative importance between current consumption and wealth (future consumption).

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Effet de richesse sur le time preference???