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Reexamination of the Purchasing Power Parity (PPP) under Cournot Competition

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Abstract This paper reexamines the Purchasing Power Parity (PPP) in an open economy macroeconomic model with Cournot competition in the international trade of a unique good. Foreign and domestic firms have a Cournot perception of foreign and domestic markets and make separate quantity decisions for each market taking the quantity decisions of the other firm as given. In introducing the money, the balance of payments relation and the nominal exchange rate, the PPP implied by the law of one price can be reexamined. We show that the imperfect competition and symetric trade barriers are not enough to break the absolute and relative PPP. But under symmetric trade barriers, asymmetric policy and macroeconomic conditions are sufficient for the absolute PPP not to hold, while asymmetric trade barriers are enough to break it. These results are developed without calling on the role of non-tradable goods, shifts in the consumers' tastes or technological changes, which are commonly considered as the principal factors to break the absolute and relative PPP. Conditions under which the relative PPP is not verified are discussed.

JEL Classification N°: F31
Keywords: Purchasing Power Parity (PPP), Cournot perception firms

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I. Introduction

The Purchasing Power Parity (PPP) is well known in international macro-economics and finance. It is often seen as the fundamental relation when we examine the issue of the nominal exchange rate.

According to Dornbusch,[1] there are two reasons which justify the role of the PPP as the essential element of open economy macroeconomics. First, it is a benchmark by which to judge if the level of a nominal exchange is over or undervalued. Second, the PPP is to serve as a prediction model for nominal exchange rates under different policy or exogenous shocks. He talks, of course, of the role of the relative PPP, as the absolute PPP is very badly verified in the short-run as in the long-run. He admits, meanwhile, that the relative PPP is not well verified in the experience of the post-Bretton-Woods floating exchange rate regime. Some explanations are given to the prediction failure of both absolute and relative PPP. But they rely often on the role of non-tradable goods, shifts in the consumers' tastes or technological changes, etc.

The role of the trade barriers and transport costs are generally discussed as if they imply possibly international distortion of prices and violation of the law of one price which is the foundation, if not the exclusive one, of both the absolute and the relative PPP. But a thorough understanding of the mechanism underlying the law of one price and the PPP is lacking in the absence of non-tradable goods, shifts in the consumers' tastes and technological changes.[2]

To understand the PPP fully, we have then to sort out the effect of the real exchange rate on the PPP. A model with international trade in a unique good will be necessary. It means that we need imperfect competition models, the advantage of which is that the international trade flux is well defined even if there is only one good. We could not use the perfect competition concept, for which the trade volume is not well determined in this case.

[1] Who gives a masterful survey on the PPP issues in the New Palgrave, 1987a.

It is already suggested that the law of one price may be weakened in an imperfectly competitive context as indicated by Dornbusch (1987a) and Krugman and Obstfeld (1988). In particular, the imperfect competition reinforces the distortion effects of the trade barriers and transport costs. One question is that: can the existence of these factors implies necessarily the violation of the law of one price. As indicated by Dornbusch, there is not yet a large literature on the relation between imperfect competition and nominal exchange rate. It is not clear whether imperfect competition, trade barriers and transport costs are sufficient to break the PPP. In this literature, the role of other policy and macroeconomic factors in breaking the relative or absolute PPP is neglected. A natural question is whether the PPP can be verified even under these conditions.

The Cournot competition model[3]is appropriate for our purpose of finding the factors which break the law of one price and so the PPP, or the conditions under which they are verified. It suggests that the existence of rivalry between oligopolistic firms leads to two-way trade in identical products.[4]

In this model, the consumers do not perceive the national and foreign products as differentiated goods, otherwise the change of the nominal exchange rate may be seen as due to that of the relative price of different goods produced in two countries. With this kind of model, it then becomes easy and more meaningful to talk of the relation between the nominal exchange rate and the PPP. As there is only one good, the violation of PPP is not due to changes in terms of exchange.

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[2] All of them influence the real exchange rate and hence the nominal exchange rate.

[3] A partial Cournot equilibrium model is used by Brander and Krugman (1983). In this reciprocal dumping model, there is the possibility of breaking the law of one price when there are transport costs. As Brander and Krugman are focusing on international trade issues, neither the nominal exchange rate nor the balance of payments (or trade balance) are given appropriate treatments. Dornbusch (1987b) has used a multi-firm Cournot competition model to discuss the relation between nominal exchange rate and price of goods.

[4] The crucial element which determines the trade in similar good is the 'segmented markets' perception as referred to by Helpman (1982): each firm perceives each country as a separate market and makes distinct quantity decisions for each of them.

In a two-country macroeconomic model where the consumer demand behaviors, the money, the nominal exchange rate and the balance of payment are explicitly introduced, we reexamine the PPP under the floating exchange rate regime in considering different trade, policy and macroeconomic conditions. The existence of differences in value-added tax rates in the two countries, import taxes or export subsidies (even with the same rates), transport costs, constraint of full employment, export or import quotas, self-limiting export quota can all, if not always, imply a violation of the law of one price and the absolute PPP. Particular attention is paid to the role of the ratio between nominal wage rate and aggregate demand. When there are transport costs, the role of the ratio of the nominal unitary transport costs to the aggregate demand is considered. Here it is assumed that the nominal wages are rigid, but this is not a sufficient condition to invalidate the two PPP. In contrast, both PPP can be well verified under nominal wage rigidities. The restrictive conditions under which both absolute and relative PPP are verified are discussed. From the results of this paper, question can be raised about whether the PPP in its actual form can effectively play, in a non Walrasian world, the role mentioned by Dornbusch.

In Section II, we lay down the basic model, i.e. the simplest case where the PPP is verified. In Section III, we discuss the different cases where the PPP can be verified or not. We conclude in Section IV.

II. The Model

Let us assume there are two qualitatively identical countries, one ‘‘domestic’’ and one ‘‘foreign’’ and each country has one representative household and one firm producing the same good as does its foreign rival. The government of each country spends its revenues (which may come from different kinds of taxes or emission of money) only on the national

market. The money is only held by national residents in each country, so there is no problem of competition between good and bad currencies.

A. The Problem of Household

It is assumed that the domestic representative household has the following preference function:[5]

$$C^{c}\left[\frac{M}{P}\right]^{1-c}, \qquad (1)$$

where C, M, P, c represent respectively the real consumption, the nominal quantity of money to be held by the household for future transactions, the price level of the unique good on the domestic market and a constant parameter.

The household supplies a fixed quantity of labor, noted $\overline{N}$, which is the only factor of production. The household's budget constraint is:

$$PC + M \leq WN + \Pi + M_{-1} - T, \qquad (2)$$

where W, N, Π, M_{-1}, T are respectively the rigid nominal wage[6], the level of employment for $N \leq \overline{N}$, the nominal profits, the initial nominal money balances and the lump-sum tax levied by the government.

In assuming that the household optimizes its utility function given out in (1) under the constraint (2), nominal money and good demands represent respectively a proportion *(1-c)* and *c* of the total disposable revenue of the household corresponding to the right hand of (2). As the supply of money is perfectly controlled by the central bank under the floating

[5] More general utility function can be considered. For analytical tractability, it is the simple Cobb-Douglas function which is adopted here.

[6] We do not give explicitly the reason of rigidity of the nominal wage. One can refer to Stiglitz (1986) for a discussion. In their two-country model, Fender and Yip (1994) based the nominal rigidity of wage on the nominal unemployment benefit rigidity.

exchange rate regime, by the equilibrium condition on the domestic money market, the nominal disposable revenue of the household during the current period is given by:

$$WN + \Pi + M_{-1} - T = \frac{M}{1-c}, \qquad (3)$$

which is considered, by the domestic and foreign firms, as given.

Using the asterisks to denote the foreign corresponding variables[7] and assuming that the foreign household has identical preference function and behavior, the equilibrium condition on the foreign money market implies that the foreign nominal revenue is given by

$$W^* N^* + \Pi^* + M^*_{-1} - T^* = \frac{M^*}{1-c^*}, \qquad (4)$$

which is considered similarly to be given for the firms.

B. The Firm's Behavior

The basic idea is that each firm regards the other country as a separate market. Consequently, it chooses the profit-maximizing quantity for each country separately. In considering that the other firm will hold fixed the quantity of its output sold in both countries, each firm has then a Cournot perception.

The domestic firm will sell a quantity x at price P on the domestic market and a quantity x^* at price P^* on the foreign market. The foreign firm will sell a quantity y at price P on the domestic market and a quantity y^* at price P^* on the foreign market. They have respectively an increasing return technology of production of the type:[8]

$$x + x^* = N - \underline{N},$$

$$y + y^* = N^* - \underline{N}^*,$$

[7] In the following, the asterisks generally denote corresponding variables associated with the foreign country.
[8] This assumption is introduced to justify the Cournot competition, see Dixon (1994).

with respectively a minimum of labor $\underline{N}$, $\underline{N}^*$necessary in beginning the period production in each country. We assume in this paper, that there is no free entry and exit[9]. The fact that the domestic firm is totally owned by the domestic residents, and the absence of non-monetary financial assets imply that there is no problem of international investment in pursuing higher profits.

We assume that the domestic and foreign governments levy respectively a tax on the import good (similarly for the case of export subsidies) at the rates τ, τ^*.The value-added tax rate is respectively θ, θ^*on the domestic and foreign markets.[10] Selling to the other country implies a fixed monetary cost per unit of good sold, which is respectively φ, φ^* for domestic and foreign firms.[11].

Each firm maximizes its nominal profits with respect to its own quantity decisions in assuming the quantity decisions of the other as given. Their profit functions are respectively:

$$\Pi = P(1-\theta)x + EP^*(1-\theta^*)(1-\tau^*)x^* - W(x + x^* + \underline{N}) - \varphi x^*, \qquad (5)$$

$$\Pi^* = \frac{P(1-\theta)(1-\tau)y}{E} + P^*(1-\theta^*)y^* - W^*(y + y^* + \underline{N}^*) - \varphi^* y \qquad (6)$$

where E is the nominal exchange rate of foreign currency in terms of domestic currency. Given the domestic government nominal spending G, the domestic central bank supply of money, and the quantity of good sold by the foreign firm, y, the domestic firm will perceive the following objective demand curve on the domestic market:

$$P = \frac{G + c(WN + \Pi + M_{-1} - T)}{x + y}, \qquad (7)$$

taking into account the equation (3), it can be written as:

[9] We can introduce the free entry and exit as Huw D. Dixon does (1994).
[10] The distribution costs can be similarly modeled as the Value-added-taxes. It is well know, these costs are much higher in Japan than in United-States.
[11] An alternative assumption will be to introduce a fixed unitary transport cost in real terms. And it is equivalent to consider the transport costs to be pure wastes or to be wage payments to national residents.

$$P = \frac{\Omega}{x+y}, \qquad P_y^{'} = P_x^{'} = \frac{-\Omega}{(x+y)^2}, \qquad \text{with } \Omega = G + \frac{cM}{1-c}. \tag{8}$$

Similarly, the objective demand for its product on the foreign market can be written as:

$$P^* = \frac{\Omega^*}{x^* + y^*}, \qquad P_{y^*}^{*'} = P_{x^*}^{*'} = \frac{-\Omega^*}{(x^* + y^*)^2}, \text{ with } \Omega^* = G^* + \frac{c^* M^*}{1-c^*}. \tag{9}$$

The first-order conditions of the firms' optimizing problems are:

$$\Pi_x^{'} = P_x^{'}(1-\theta)x + P(1-\theta) - W = 0, \tag{10}$$

$$\Pi_{x^*}^{'} = EP_{x^*}^{*'}(1-\theta^*)(1-\tau^*)x^* + EP^*(1-\theta^*)(1-\tau^*) - W - \varphi = 0, \tag{11}$$

$$\Pi_y^{*'} = \frac{(1-\theta)(1-\tau)P_y^{'}y}{E} + \frac{(1-\theta)(1-\tau)P}{E} - W^* - \varphi^* = 0, \tag{12}$$

$$\Pi_{y^*}^{*'} = P_{y^*}^{*'}(1-\theta^*)y^* + P^*(1-\theta^*) - W^* = 0. \tag{13}$$

It is easy to verify that the second-order conditions of optimality are satisfied for the firms' problems. This will garantee the existence of a unique equilibrium with trade in the unique goods as noticed by Brander and Krugman (1983).

III. Implication of Fiscal Policies and Macroeconomic Asymetry

We can complete the model with the balance of payment equation under the floating exchange rate regime. Under the assumption of no capital and labor movement, the balance of payments, identical to the trade balance, must be in equilibrium:

$$EP^*(1-\theta^*)(1-\tau^*)x^* = P(1-\theta)(1-\tau)y. \tag{14}$$

With the partial derivatives of the objective demand functions (8), (9), and the trade balance equilibrium relation, form the first-order conditions, the reaction functions of the firms can be deduced as:

$$y - \frac{W}{(1-\theta)\Omega}(x+y)^2 = 0, \tag{15}$$

$$yy^* - \frac{W+\varphi}{(1-\theta)(1-\tau)\Omega}(x+y)(x^*+y^*)x^* = 0, \tag{16}$$

$$xx^* - \frac{W^*+\varphi^*}{(1-\theta^*)(1-\tau^*)\Omega^*}(x+y)(x^*+y^*)y = 0, \tag{17}$$

$$x^* - \frac{W^*}{(1-\theta^*)\Omega^*}(x^*+y^*)^2 = 0, \tag{18}$$

The complete model constituted of (8)-(9) and (14-18) has a unique equilibrium with positive international trade flux.[12] This equilibrium is compatible with the firms' reaction functions and the trade balance equilibrium. It is easy to see from the equations (15) and (18), at this equilibrium, we must have $0 < x^* < \frac{(1-\theta^*)\Omega^*}{W^*}$, $0 < y < \frac{(1-\theta)\Omega}{W}$, for $x + y \le \overline{N}$, $x^* + y^* \le \overline{N}^*$. With this model, we will be able to examine the PPP under several commonly observed conditions. Some can be considered as barriers to trade, such as import taxes or export subsidies, transport costs, import and export quotas, voluntary restriction of exportation. These conditions are indicated in the literature as factors which can break the PPP, but they have not yet been examined in an explicit manner.[13] Others are referred to domestic fiscal policy such as value-added taxes and to particular macroeconomic situation such as full employment. Under some of these conditions, we can discuss the role of the asymmetry in the ratios of nominal wage rate to nominal aggregate demand.

Before this, we give out the basic character of the model in assuming that VAT rates, transportation costs, import taxes and export subsidies are zero. In this simple benchmark case, the only positive solution compatible with the existence of international trade and

[12] The proof can be obtained from the author upon request.
[13] See Dornbusch (1987a).

positive production is[14], $x^* = y = \frac{1}{(\sqrt{\alpha}+\sqrt{\beta})^2}$, $x = \frac{\sqrt{\beta}}{\sqrt{\alpha}(\sqrt{\alpha}+\sqrt{\beta})^2}$, $y^* = \frac{\sqrt{\alpha}}{\sqrt{\beta}(\sqrt{\alpha}+\sqrt{\beta})^2}$, with $\alpha = \frac{W}{\Omega}$, $\beta = \frac{W^*}{\Omega^*}$.

It is very easy to check that the law of one price and the absolute PPP is valid, as $\frac{EP^*}{P} = \frac{y}{x^*} = 1$. The verification of the relative PPP follows from that of the absolute PPP. The imperfect competition is not sufficient to break the PPP as we can imagine easily. As there is no constraint limiting the arbitrage of a firm, each firm will try to exploit the higher price in one country until the law of one price is applied and no more profits can be earned at lower price. The question is whether or not the combinaison of imperfect competition with other factors will imply the violation of the PPP.

A. Value-Added-Tax Rates

In empirical studies of PPP, the influence of the VAT reforms is not documented. The VAT rates are not the same for different countries and are subject to frequent changements. These reforms changent the price behaviors of firms and so the nominal exchange rate between two currencies. For clarifying the relation between nominal excahnge rate and VAT rates, we assume that the import taxe and export subsidies, and tranport costs are zero. After resolving the simplified model, we have the following proposition.

Proposition 1. *For the unique solution of the model with value-added taxes a modified PPP is verified, i.e.* $\frac{EP^*(1-\theta^*)}{P(1-\theta)} = \frac{y}{x^*} = 1$. *Or equivalently, we have* $\frac{EP^*}{P} = \frac{1-\theta}{1-\theta^*}$. *If the value-added tax rates are the same in the two countries, the absolute PPP is verified.*

[14] See Appendix A.

The proof is very easy as we can solve explicitly the model.[15] The intuition is that, interesting in (taxes deduced) net prices, the firms will make the same export quantity decisions which are the best for them. The rule of one net price is prevailing as there is no other constraint limiting arbitrage. Otherwise, the net price (measured in same currency) of the same good will be different in two countries as we impose the trade balance equilibrium. If the value-added tax rates are unchanged, the relative PPP is expected to be true. If the VAT reform is taken in one country, it will influence not only the price level but also the nominal exchange rate between the two currencies in question.

For exemple, if the VAT in the domestic country is increased, the domestic price will be higher and foreign price lower and the national currency must appreciate to garantee the external equilibrium. This is due to the fact that the net margin of profit in the domestic country diminue if the domestic price level and the nominal exchange rate stay unchanged. Domestic firm and foreign firm will try to sell more in the foreign country. As import decreases and export increases, the domestic market supply will be less than before, that contributes to increase the domestic price level. At the same time, more export and less import imply the domestic currency must be appreciated to equilibrate the trade balance. The absolute and relative variation of the nominal exchange rate will not be explained totally by the absolute and relative variations of prices in the two countries. If the VAT reform is combined with a price stabilization policy, its impact on the absolute and relative variation of the nominal exchange rate can be doubled.

B. Import Taxes or Export Subsidies

We assume that there are not transportation costs and the domestic and foreign governments do not levy VAT but a tax on the import good (similarly for the case of export subsidies) respectively at the rates τ, τ^*.The taxes that each firm pay to the government of the other country on its sale in this country reduce its incentive to export. In contrast, the export subsidies will stimulate the exports of the firm beneficing them.

Proposition 2. *i. A unique import tax rate ($\tau = \tau^*$) applied by the two countries is sufficient to break up the absolute PPP if the two economies are not globally symmetric, i.e. $\alpha \neq \beta$; Otherwise the absolute PPP can be true.*

ii.If $\tau \neq \tau^*$, the absolute PPP can also be verified if the tax asymmetry compensates the macroeconomic asymmetry, i.e. $\frac{(1-\tau^*)\tau}{(1-\tau)\tau^*} = \frac{\sqrt{\beta}}{\sqrt{\alpha}}$.

Proof: For the same import tax rate, $\tau = \tau^*$, let us assume that the absolute PPP is verified, $\frac{EP^*}{P} = \frac{y}{x^*} = 1$. For it to be true, we must have $y = x^*$. In using $y = x^*$, under the assumption $\tau = \tau^*$, one can obtain, by the equations (15-16) and (18), $\sqrt{x^*} = \sqrt{y} = \frac{1-\tau}{(1-\tau)\sqrt{\beta} + \sqrt{\alpha}}$; and by the equations (15) and (17-18), one has $\sqrt{x^*} = \sqrt{y} = \frac{1-\tau^*}{(1-\tau^*)\sqrt{\alpha} + \sqrt{\beta}}$. If $\alpha \neq \beta$, we can check that there is a contradiction under the assumption $y = x^* > 0$. Then, part i of the proposition 2 is true. To check the part ii, we can equalize the two expressions of $\sqrt{x^*}, \sqrt{y}$.

Q.E.D.

[15] We can use the same method showed in the Appendix A.

Following the last proposition, the absolute PPP is generally not true when there are import taxes, i.e. $\frac{EP^*}{P} = \frac{y}{x^*} \neq 1$. Introducing import taxes can then block the perfect exchange rate arbitrage under the floating exchange rate regime. Even this is the general case, there exists the taxe rates which compensate the difference in nominal wage/nominal aggregate demand ratios.

For and only for $\tau = \tau^*$, the equality between the ratios of nominal wage rate per nominal aggregate demand in the two countries ($\alpha = \beta$) implies the absolute PPP.[16] But as $\alpha = \frac{W}{\Omega}, \beta = \frac{W^*}{\Omega^*}, \Omega = G + \frac{cM}{1-c}, \Omega^* = G^* + \frac{c^* M^*}{1-c^*}$, i.e. four parameters can influence α and β, their equality will be a particular situation. Then, if α, β do not evolve in the same proportion in the time, the relative PPP can not be verified. To understand this intuitively, consider an initial equilibrium where $\tau = \tau^*$ and $\alpha = \beta$, so the export and import quantities are equal for each country. In the case of a small rise in α (an increase in *W* or a diminution in *Ω*) for β unchanged, at the actual domestic price and foreign exchange rate, the initial quantity decision (domestic sale and export) of the domestic firm implies some loss for marginal sales according to equations (15) and (16). It will reduce its exoprts and sales on the domestic market. And there will be more import as the domestic price increases. All that means there will be a depreciation of national currency and an increase of national price level even though this is attenuated by more import. Foreign production will increase and the foreign price will be higher but increase less than the domestic price. In consequence, the one price law is broken, and the national currency seems as under-valued.

C. Transport Costs

The transport costs reduce each firm's incentive to export on the other market, as do the import taxes. We consider VAT rates, import taxe and export subsidy are zero in the two countries.

Proposition 3. *Under the floating exchange rate regime, the transport costs can be sufficient to break the absolute PPP if the two economies are not globally symmetric.*

This proposition can be formulated as the proposition 2 as the role of transport costs is similar to that of import taxes. The proof is similar also, so it is not given. Like import taxes, transport costs make the perfect exchange rate arbitrage impossible in general under the floating exchange rate regime, except in perhaps the case where the two countries have the same ratio of nominal unitary transport costs to nominal aggregate demand and the same ratio of nominal wage rate to nominal aggregate demand.

D. Voluntary and Involuntary Import or Export Quotas

The Cournot type competitive firms may press their governments to limit import from the other country to guarantee their profit margins. Sometimes the limitation to import from the other country takes the form of voluntarily limited exports as illustrated by the treaty between the Europeans and the Japanese on the trade of cars. But in this model, these nuances can not be expressed in mathematical terms. Consider then the simplified model where the VAT rates, import taxes and export subsidies and transport costs are zero.

[16] The demonstration is left to the reader.

As the export of each firm is fixed, i.e. $\bar{x}^*, \bar{y}$ are constant, the balance of payment equilibrium condition is written, under the effective quota condition which is given in the following proposition, as:

$$EP^* \bar{x}^* = P\bar{y}. \tag{3.26}$$

It is easy to see that for the simplified model, the quotas are effective if $\bar{x}^* < \frac{1}{(\sqrt{\alpha}+\sqrt{\beta})^2}$, $\bar{y} < \frac{1}{(\sqrt{\alpha}+\sqrt{\beta})^2}$ and $0 < \bar{x}^* < \frac{1}{\beta}$, $0 < \bar{y} < \frac{1}{\alpha}$.

Proposition 4. *For the effective quota-constrained solution of the model defined by the equations (3.24)-(3.26), the absolute PPP is generally not verified under the floating exchange rate regime.*

Using the equilibrium trade balance equation, we have $\frac{EP^*}{P} = \frac{\bar{y}}{\bar{x}^*} \neq 1$, if $\bar{x}^* \neq \bar{y}$. It is pure evidence that the PPP is verified only in the case where the import or export quotas are the same for the two countries.

E. Constraint of Full Employment

The labor supply of each country is assumed to be fixed, the two firms may be constrained by the supply of labor in their own country. To clarify the issue, we will deal with a simple case where only the domestic country is constrained by the labor supply. That means the domestic firm export in real terms is $\bar{N} - x$. If the fiscal and trade policy parameters are zero, the model (3.35) has a unique non trivial solution compatible with the reaction functions and the trade balance equilibrium under the conditions $\bar{N} - \underline{N} - \frac{1}{\beta} < x < \frac{\bar{N} - \underline{N}}{2}$, $\bar{N} - \underline{N} < \frac{3}{2\beta}$. (See Appendix B.)

Proposition 5. *For the unique non trivial solution of the model (3.35) with full employment constraint, the absolute PPP is not verified.*

Proof: From the equation (3.34), we have, $\frac{y}{x+y} = \frac{EP^*}{P}$. Under the restrictions $\overline{N} - \underline{N} - \frac{1}{\beta} < x < \frac{\overline{N} - \underline{N}}{2}$, $\overline{N} - \underline{N} < \frac{3}{2\beta}$, the quantity decisions variables x, y are strictly different from zero. Consequently, $\frac{EP^*}{P} < 1$. Q.E.D.

Measured in the same currency, the price on the foreign market will be lower than on the domestic market. On the domestic market, the demand tension is stronger as the full employment constraint is effective, the foreign firm can try to sell more in the domestic market at higher price. But the more it sells on the domestic market, the lesser the possibility for it to find national currency to bring back the value of its sale to its own country at attractive rate. The equilibrium real exchange rate, at which the foreign firm has no incentive to sell at higher price (as measured in the same currency) in the domestic country, will be between zero and one.

IV. Conclusion

A two-country macroeconomic model with cournot competition is built here in order to show under which conditions the PPP may be verified or not. First, it is clear from the results of this paper and other authors that the imperfect competition is not enough to break the law of one price. Second, the transport costs and trade barriers are equally insufficient to break the PPP. Third, but with imperfect competition, this law can be broken under some trade, policy

and macroeconomic conditions, i.e., when there are unequal quotas, unequal value-added taxes and import taxes, and when the macroeconomic conditions (ratio of nominal wage rate and nominal aggregate demand, unemployment rates and ratio of the nominal unitary transport costs to nominal aggregate demand) are not the same in the two countries.

But under symmetric quotas, symmetric employment situation, symmetric import and export taxes, with the same ratio of nominal wage rate to nominal aggregate demand, the same ratio of the nominal unitary transport costs to nominal aggregate demand and the same value-added tax rate, the PPP can continue to be verified. In the case where these factors are symmetric in the two countries, the underlying model can be used to calculate an explicit solution of the new parity, which may be useful in the prediction of the long-run movement of the nominal exchange rate.

In general, the efficiency of the exchange market does not rule out the possibility of one good being charged different prices (evaluated in the same currency in using the floating nominal exchange rate) in the two countries. It is due to the fact that the different constraints imposed on the international trade or national economic conditions do not permit the Cournot perception firms to exploit completely the apparent arbitrage opportunities on the good market with positive net profits. The resulting violation of the PPP is then compatible with the firms' profit maximization.

Several directions of extension can be considered. In particular, we can envisage the free entry and exit in the model. We may consider the case where the labor supply is elastic and the real and nominal wage rates are flexible. The intertemporal trade, capital and labor movement, international investment and currency competition, if modeled, may complicate the determination of the floating nominal exchange rate. Macroeconomic policy issues may be investigated in order to evaluate their effects on the welfare of each nation when the PPP is not verified.

Some words can be said about the relative PPP. In this model, if we assume that the nominal wage rate, the nominal public spending and the nominal transport costs evolve always in the same proportion as the nominal stock of money, and the consumers' average propensity to spend does stay unchanged, then the relative PPP can be verified. If these elements do not change in this way, the relative PPP can then deviate its usual course predicted by the monetarist economists. If these factors change in a non predictable manner in the time, the prediction of the movements of nominal exchange rate will be very arbitrary.

Appendix A An explicit solution of the model with no taxes and transport costs

The reduced model is as follows:

$$y - \alpha (x + y)^2 = 0, \tag{A.1}$$

$$yy^* - \alpha (x + y)(x^* + y^*)x^* = 0, \tag{A.2}$$

$$xx^* - \beta (x + y)(x^* + y^*)y = 0, \tag{A.3}$$

$$x^* - \beta (x^* + y^*)^2 = 0, \tag{A.4}$$

$$EP^* x^* = Py. \tag{A.5}$$

The manipulation off the equations (A.1) and (A.2) gives the following equations:

$$x = \frac{\sqrt{y}}{\sqrt{\alpha}} - y, \tag{A.6}$$

$$y^* = \frac{\sqrt{x^*}}{\sqrt{\beta}} - x^*. \tag{A.7}$$

In substituting (A.6) and (A.7) into the equations (A.2) and (A.3), we can obtain:

$$y(\frac{\sqrt{x^*}}{\sqrt{\beta}} - x^*) - \alpha \frac{\sqrt{y}}{\sqrt{\alpha}} \frac{\sqrt{x^*}}{\sqrt{\beta}} x^* = 0, \tag{A.8}$$

$$x^*(\frac{\sqrt{y}}{\sqrt{\alpha}} - y) - \beta \frac{\sqrt{y}}{\sqrt{\alpha}} \frac{\sqrt{x^*}}{\sqrt{\beta}} y = 0, \tag{A.9}$$

after simplification, we can get:

$$\sqrt{y}(\frac{1}{\sqrt{\beta}} - \sqrt{x^*}) - \frac{\sqrt{\alpha}}{\sqrt{\beta}} x^* = 0, \tag{A.10}$$

$$\sqrt{x^*}(\frac{1}{\sqrt{\alpha}} - \sqrt{y}) - \frac{\sqrt{\beta}}{\sqrt{\alpha}} y = 0. \tag{A.11}$$

In using the equation (A.10), it follows:

$$\sqrt{y} = \frac{\frac{\sqrt{\alpha}}{\sqrt{\beta}} x^*}{\frac{1}{\sqrt{\beta}} - \sqrt{x^*}}. \tag{A.12}$$

In substituting it into the equation (A.11), we deduce

$$\sqrt{x^*}\left(\frac{1}{\sqrt{\alpha}} - \frac{\frac{\sqrt{\alpha}}{\sqrt{\beta}} x^*}{\frac{1}{\sqrt{\beta}} - \sqrt{x^*}}\right) - \frac{\sqrt{\beta}}{\sqrt{\alpha}}\left(\frac{\frac{\sqrt{\alpha}}{\sqrt{\beta}} x^*}{\frac{1}{\sqrt{\beta}} - \sqrt{x^*}}\right)^2 = 0,$$

with the following transformations:

$$(1 - \sqrt{\beta}\sqrt{x^*})^2 - (1 - \sqrt{\beta}\sqrt{x^*})\alpha x^* - \alpha\sqrt{\beta}\sqrt{x^*}x^* = 0,$$

and,

$$1 - 2\sqrt{\beta}\sqrt{x^*} + \beta x^* - \alpha x^* + \alpha x^*\sqrt{\beta}\sqrt{x^*} - \alpha\sqrt{\beta}\sqrt{x^*}x^* = 0,$$

one gets:

$$(\beta - \alpha)x^* - 2\sqrt{\beta}\sqrt{x^*} + 1 = 0,$$

which yields:

$$\sqrt{x^*} = \frac{2\sqrt{\beta} \pm \sqrt{4\beta - 4\beta + 4\alpha}}{2(\beta - \alpha)} = \frac{\sqrt{\beta} \pm \sqrt{\alpha}}{\beta - \alpha}$$

i.e.

$$x_1^* = \frac{(\sqrt{\beta}+\sqrt{\alpha})^2}{(\sqrt{\beta}+\sqrt{\alpha})^2(\sqrt{\beta}-\sqrt{\alpha})^2} = \frac{1}{(\sqrt{\beta}-\sqrt{\alpha})^2},$$

$$x_2^* = \frac{(\sqrt{\beta}-\sqrt{\alpha})^2}{(\sqrt{\beta}+\sqrt{\alpha})^2(\sqrt{\beta}-\sqrt{\alpha})^2} = \frac{1}{(\sqrt{\beta}+\sqrt{\alpha})^2},$$

Using (A.6), (A.7) and (A.12), we can check easily the only good solution is $x^* = y = \frac{1}{(\sqrt{\alpha}+\sqrt{\beta})^2}$, $x = \frac{\sqrt{\beta}}{\sqrt{\alpha}(\sqrt{\alpha}+\sqrt{\beta})^2}$, $y^* = \frac{\sqrt{\alpha}}{\sqrt{\beta}(\sqrt{\alpha}+\sqrt{\beta})^2}$. The vector $(x, x^*, y, y^*) = (0,0,0,0)$ is also a solution of the model, but it is trivial compared to the reality of the world economy.

Appendix B The Full Employment Constraint in the Domestic Country

The firms' maximization problems are therefore as:

$$\Pi = Px + EP^*(\overline{N} - x) - W\overline{N}, \tag{B.1}$$

$$\Pi^* = \frac{Py}{E} + P^* y^* - W^*(y + y^* + \underline{N}^*), \tag{B.2}$$

where only the domestic firm's objective function is modified to reflect the full-employment constraint.

The first-order conditions of optimality for firms are:

$$x + x^* = \overline{N} - \underline{N}, \tag{B.3}$$

$$\Pi_x' = P_x' x + P - EP^* = 0, \tag{B.4}$$

$$\Pi_y^{*'} = \frac{P_y' y}{E} + \frac{P}{E} - W^* = 0, \tag{B.5}$$

$$\Pi_{y^*}^{*'} = P_{y^*}^{*'} y^* + P^* - W^* = 0. \tag{B.6}$$

The balance of payment equilibrium condition is modified as:

$$EP^* x^* = Py. \tag{B.7}$$

In using the domestic objective demand function, the equation (B.4) becomes:

$$\Pi'_x = P'_x x + P - EP^* = -\frac{Px}{x+y} + P - EP^* = 0; \qquad \text{(B.8)}$$

in the condition (B.7) and the equation (B.8), one has easily $x^* = \mathrm{x} + \mathrm{y}$.

We can then derive the reduced model as:

$$x + x^* = \overline{N} - \underline{N}, \qquad \text{(B.9)}$$

$$y + x = x^*, \qquad \text{(B.10)}$$

$$xx^* - \beta(x+y)(x^* + y^*)y = 0, \qquad \text{(B.11)}$$

$$x^* - \beta(x^* + y^*)^2 = 0. \qquad \text{(B.12)}$$

To guarantee the positivity of *y,* the equations (B.9) and (B.10) can be used to deduce that

$$x < \frac{\overline{N} - \underline{N}}{2}, \qquad \text{(B.13)}$$

To guarantee the positivity of y^*,one must have, by the equation (B.12):

$$0 < x^* < \frac{1}{\beta}, \qquad \text{(B.14)}$$

and from the equation (B.9), one must also have:

$$x > \overline{N} - \underline{N} - \frac{1}{\beta}. \qquad \text{(B.15)}$$

In using the equations (B.14), (B.9) and (B.10) into (B.11), one gets:

$$x^2 = \beta(\overline{N} - \underline{N} - x)(\overline{N} - \underline{N} - 2x)^2. \qquad \text{(B.16)}$$

The left hand of the equation (B.16) is a strictly convex increasing function in *x* and passes by the origin. Its right-hand (noted $\varphi(x)$) has a derivative $-\beta(\overline{N} - \underline{N} - 2x)\left[5(\overline{N} - \underline{N}) - 6x\right] < 0$ if $x < \frac{\overline{N} - \underline{N}}{2}$. That means that it is strictly decreasing for $0 < x < \frac{\overline{N} - \underline{N}}{2}$. Then, as $\varphi(\frac{\overline{N} - \underline{N}}{2}) = 0$, $\varphi(0) = \beta(\overline{N} - \underline{N})^3 > 0$, one unique solution of x exists for $0 < x < \frac{\overline{N} - \underline{N}}{2}$.

But it may be too great to generate a positive solution for y^*. For the case where $\overline{N} - \underline{N} - \frac{1}{\beta} < 0$, the unique solution is compatible with the positivity of all the quantity decisions variables. But for the case where $\overline{N} - \underline{N} - \frac{1}{\beta} > 0$, to have a valid solution, one must have, $\overline{N} - \underline{N} < \frac{3}{2\beta}$. So one has $x^2 < \beta(\overline{N} - \underline{N} - x)(\overline{N} - \underline{N} - 2x)^2$ for $\overline{N} - \underline{N} - \frac{1}{\beta} < x < \frac{\overline{N} - \underline{N}}{2}$. As shown by the following figure, there exists a unique solution satisfying the restrictions on x for it to be compatible with the positivity of the other quantity decisions variables.

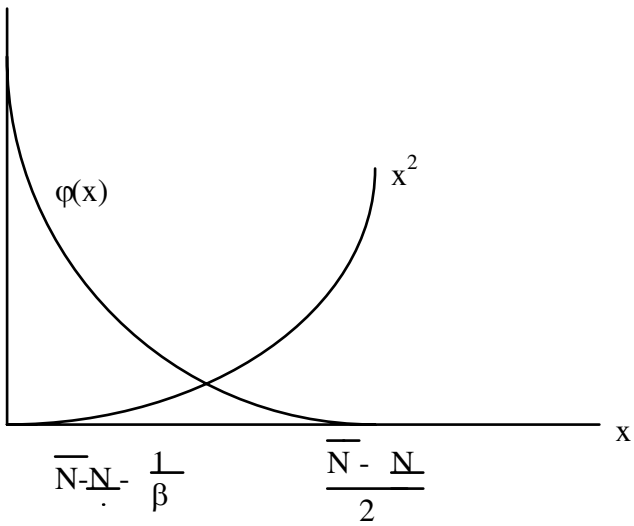

Figure 2. The existence of one unique equilibrium under the full-employment constraint.

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