Growth, External Debt Constraints and Budgetary Policies

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April 1992

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Working paper of BETA N° 9208

April, 1992

Abstract This paper extends MURPHY's (1991) analysis of alternative lending arrangements. We incorporate the capital accumulation into the two sector model and departure from traditional model of this kind by making two assumptions. One of them is to postulate rigid wage and unemployment, and the another is to assume sector-specific capital goods. We analyze then the long-run behavior and the short-run adjustment path for macroeconomic variables such as stock of external debt, real exchange rate, capital stocks, investment and private consumption of a small developing country in response to different budgetary policy changes under three alternative lending arrangements. The results suggest that, first, the behavior of the economic system under total debt and debt-ratio arrangements are the same in the long-run and not very different in the short- and intermediate-run; secondly, the budgetary policies can have an influence over the external debt in the short- and intermediate-run under the total debt ratio arrangement.

Keywords: lending arrangement, Creditworthiness, external debt, real exchange rate and capital accumulation.

J.E.L. classification number: F41, F34

[1] I would like to thank A. VAROUDAKIS and H. STAHN for a number of helpful comments. All remaining errors are, of course, my own.

1 Introduction

The small open developing countries often require external capital. Typically, in a favorable environment, they need more resources than they can generate themselves to realize their anticipated potential of growth. Over-borrowing, resulting from inadequate perception of domestic growth potential, has occurred occasionally. In the 1980's, we have observed serious external debt problem in many developing countries. Most discussions concerning the international debt problem in these countries have focused on how to restructure lending to developing countries in the future. It has been suggested that there should be an explicit link established between a limited set of quantifiable economic criteria and the terms on which a country can borrow from abroad. This view emphasizes the creditworthiness of the borrowing country and focuses on the loan arrangement as a mechanism for imposing discipline on the borrower's actions. MURPHY (1991) has given a formal analysis of proposals for tying the terms of loan arrangement to measures of creditworthiness. Three possible lending arrangements are considered: linking the interest rate on borrowing to the country's total external debt, to its trade-balance deficit and to the ratio of its debt to traded-output.

In a setting of two-sector small open economy, MURPHY has demonstrated that the arrangement linking interest rates on lending to the outstanding stock of external debt is characterized by a stable macroeconomic adjustment process, whereas arrangement that ties the interest rate to performance on the trade account is likely to be unstable. For certain types of lending arrangements, a policy of reducing government spending to lower the budget deficit can have very different effects on the long-run level of debt, depending on the manner in which spending is reduced. In particular, a reduction in government spending on non-traded goods may lead to a long-run increase in the stock of debt. This is the case when the interest rate is tied to the debt and tradable output ratio.

In a context of growth, BHANDARI, HAQUE and TURNOVSKY (1990) have studied the case where the developing economy faces an upward-sloping supply schedule for debt, which embodies the risk premium associated with lending to a sovereign borrower. That corresponds to the first case of MURPHY, where the interest rate on the economy's borrowing is related to its total net external debt. In extending the basic neoclassical growth model to an open economy context, OTANI and VILLANUEVA (1989) study the growth of a debtor developing country facing an interest rate related to its debt-export ratio. But their emphasis is on the role of the government in promoting the technical progress and the formation of human capital.

External debt constraints can take another form. That is, the lenders, in some cases, impose credit ceilings on borrowers, as assumed by BORENSZTEIN and GHOSH (1989) in an intertemporal optimizing two-sector model. There, the debtor country can choose to repudiate the debt but has to be subjected to some sanctions, such as a permanent exclusion from foreign borrowing, and some trade-related measures that reduce the advantages of international trade for the debtor country. [2]

The external constraints are imposed in the models of MURPHY (1991) and BHANDARI, HAQUE and TURNOVSKY (1990), because the representative consumer with constant time preference rate is adopted to represent all the consumers of the small country. In contrast, the BLANCHARD's (1985) small open economy model with overlapping perpetual youth generations does not need these externally imposing constraints for the small economy to choose a stable level of external debt, although the fixed time preference rate and the exogenous interest rate are adopted. BORENSZTEIN (1989) builds a model in which two period life overlapping generations framework is adopted for the consumption side, the external debt position can then be determined as in BLANCHARD (1985). Staying in the infinite horizon intertemporal optimizing model, ENGEL and KLETZER (1989) study a two sector small economy with representative consumer doted of an endogenous rate of time preference, which is parameterized as did UZAWA (1968). In this model, they show that the stages in the balance of payments, and hence the stages of the external debt position, can be traced out in an optimizing framework. This kind of stages of debt can not, of course, be generated in MURPHY (1991), BHANDARI, HAQUE and TURNOVSKY (1990) and others, due to their adoption of infinite lifetimes and constant rate of time preference.

If the assumption that the aggregate behavior of all the consumers of the small economy are well represented by that of a typical constant time preference rate consumer, then the approach of BHANDARI, HAQUE and TURNOVSKY (1990) and MURPHY (1991) is relevant. This is the beginning point of our work.

The model of BHANDARI, HAQUE and TURNOVSKY has a too simple struc-

[2]In all these models cited above, a developing country is often taken as a debtor country. But the reality is more complicated. For the latin american countries: at one hand, the government is seriously indebted, at the other hand there is much flighted capital of private sectors. If we add the government foreign debt and the private external credit position, these latin american countries will be net creditors. This phenomena is studied by GERTLER and ROGOFF (1990) in a model of asymmetric information.

ture. In contrast, the two-sector model of MURPHY does not incorporate the capital accumulation. The simple structure is responsible for their clear-cut results, of which one is that when the interest rate prime is tied to total debt, budgetary policy not only has no effects on the external position in the long run, but also in the short- and intermediate- run.

Two usual assumptions in a neoclassical two sector model with capital accumulation, i.e. the flexible wage and full employment assumption and the short-run perfect mobility of labor and capital across sectors, appear quite unsatisfactory to me. As to the flexible wage and full employment assumption, we know that one important problem in a under-developed country is that there exists a permanent unemployment which cannot even be eliminated in a century, not to say that it can disappear in a overnight time. The wage rate in these countries are generally pushed to increase, at least not to decrease automatically, due to the exposed sector which interacts with developed world market. This permanent upward pressure on wage can then induce the country into a permanent unemployment or under-employment situation, a situation which resembles to that of involuntary unemployment in the sense of Keynes. Finally, it is not strange that there is often unemployment in a developing country because the industrialization process is simply unachieved. If unemployment is general and important, then the labor supply is not a constraint for the expansion of the economy. The assumption that there is perfect mobility of capital between sectors is quite counter-factual, especially in a developing country. The tradable and non-tradable sector can be very different in a developing country due to the fact that the tradable sector is aspired by the modern industrial capitalism, or that the traditional sector has inherited from the country's tradition and cannot absorb quickly the new technology of modern production.

Our analysis of budgetary policy in a indebted developing country is based on these observations. It will be interesting to integrate these two approaches in introducing capital accumulation into a two sector model. That is what we try to do here. We choose to introduce capital accumulation process into the two-sector model of MURPHY as do BHANDARI, HAQUE and TURNOVSKY in their one-sector model.

The usual assumption concerning a small open economy can be stated clearly as follows. The small debtor developing country produces and consumes traded and non-traded goods. The price of traded goods is determined in world market and is taken as given by the home country. We assume that the country's traded goods sector faces perfectly elastic world demand and supply. The price of non traded-goods is determined in the home country by the market clearing condition.

Domestic residents and government can borrow and lend to at world financial markets at the variable interest rate determined by the lending arrangements. For domestic residents, this interest rate is taken as given, because each of them has a relative small market power in relation to the national and world financial markets and thus has negligible influence on financial market conditions. [3] To install the new equipments, the country has to engage some resources to adjust the existent stock of capital. This is modeled in using the idea of adjustment cost function. Our principal interest is to examine the influence of capital accumulation over the interaction between the debt and the real exchange rate and private consumption dynamics.

Some assumptions specific to our model permit us to obtain a number of interesting results different from the Murphy's ones and from these of BHANDARI, HAQUE and TURNOVSKY. First, we give up the flexibility of real wage and full employment assumption. An exogenous given wage rate, in terms of traded goods, is introduced. [4] We assume, for simplicity, they are the same for the two sectors. This exogenously given wage is too high to ensure the full employment of total labor offer. As we noted before, in fact, the indebted developing countries are often characterized by a high unemployment rate. Secondly, it is assumed that the tradable goods are used as investment and installation goods in the tradable sector of economy and the non-tradable goods plays the same kind of role in the non-tradable sector. It is not difficult to find out in a developing country an abrupt separation between traditional and modern industrial sector. There exists little inter-sector exchange. Although this is the case, the capital can be reallocated in the intermediate- and long-run.

The results suggest that the three alternative lending arrangements considered in MURPHY degenerate into two; that is, the one where the interest rate related to total net external debt and the one where the interest rate is linked to trade-balance deficit. The arrangement which links the interest to the ratio of debt to traded output coincides now with the total net external debt arrangement, as they present no difference in the long-run behavior and no significant difference in the short- and intermediate-run dynamics. As remarked by MURPHY, the stability properties of the remaining two dynamic systems corresponding to two very dif-

[3] BHANDARI, HAQUE and TURNOVSKY (1990, p. 393) note that an individual agent can take account of the influence of his decision on the aggregate debt of the economy and therefore on the prevailing domestic interest rate even if the number of such agents is large.

[4] In VAROUDAKIS (1990b), this rigid wage rate is justified as efficient wage in a cadre of an intertemporally optimizing firm.

ferent lending arrangements (i.e. total debt and trade-balance deficit as proxies for creditworthiness) are also changed and become more complicated as the consequences of incorporation of capital accumulation. It is in this dynamic setting we proceed to study some debt-reduction budgetary policy propositions.

In section 2, we construct the model with capital accumulation and unemployment, and we study the short-run equilibrium; in section 3, we study the stability of the dynamic systems under the different external debt constraints, in section 4, we lay out long-run effects of spending reduction budgetary policies, in section 5, the macroeconomic dynamic adjustment profile of different variables to non-anticipated permanent budgetary policies will be analyzed in doing a simulation exercise ; we conclude finally in section 6.

2 The model

2.1 The firms

The domestic firms produce traded and non-traded goods using capital and labor as inputs through a production function

$$Y = F(L, K), \tag{1}$$

which is assumed to possess standard neoclassical properties

$$F_L, F_K > 0, \quad F_{LL}, F_{KK} < 0, \quad F_{LL}F_{KK} - F_{LK}^K > 0;$$

we assume also that this production function satisfies a normality condition which takes the following form: [5]

$$F_{LL}^T F_K^T - F_L^T F_{LK}^T < 0.$$

Net profit of the representative firm of the non-traded sector at each point of time is therefore given by

$$\pi^N = F_N(L^N, K^N) - \frac{w}{p}L^N - \Psi^N(I^N), \tag{2}$$

where the superscript "N" indicates that all these variables concern the non-traded sector. K^N, L^N and w/p are respectively the existent stock of capital , the level

[5]The normality condition of a production function implies that when the total cost increases, the demand of every production input augments. This concept is derived by analogy to the normality of consumption goods in utility.

of employment and the real wage rate in the non-traded sector. The function $\Psi(.)$ represents the total investment and installation costs associated with the purchase and installation of I^N units of new capital goods. It is assumed to be a convex function of I^N, that is, $\Psi_I^N > 0$ and $\Psi_{II}^N > 0$. This formulation follows the original specification of installation costs introduced by LUCAS (1967) and GOULD (1968). ABEL(1979) and HAYASHI(1982) postulate explicitly an installation cost function that depends upon the existent stock of capital and the new investment. This alternative formulation is also adopted by SEN and TURNOVSKY(1989), and BORENSTZEIN(1989).

Assuming perfect competition on the non-traded goods market, the representative firm maximizes the present value of its profits (cash-flows). We also assume that the firm finances its investments purely by retained earnings and therefore does not need to borrow. The same assumption is maintained when study the traded sector representative firm.

The non-tradable sector representative firm's problem can be written as following:

$$\max \int_0^{\infty} [F^N(K^N, L^N) - \frac{w}{p} L^N - \Psi^N(I^N)] e^{-rt} dt, \tag{3}$$

under the dynamic capital stock constraint:

$$\dot{K}^N = I^N - \delta K^N. \tag{4}$$

The hamiltonian of this problem is

$$H = [F^N(K^N, L^N) - \frac{w}{p} L^N - \Psi^N(I^N)] + q^N(I^N - \delta K^N), \tag{5}$$

where q^N is the multiplier associated to the constraint (4).
The first-order conditions are:

$$F_L^N = \frac{w}{p}, \tag{6}$$

$$q^N = \Psi_I^N, \tag{7}$$

$$\dot{q}^N = (r + \delta) q^N - F_K^N, \tag{8}$$

$$\dot{K}^N = I^N - \delta K^N, \tag{9}$$

$$\lim_{s \to \infty} I^N e^{-rs} = 0. \tag{10}$$

The variable q^N is the non-tradable sector capital stock's ratio of evaluation, it is measured in terms of non-tradable.

The tradable sector representative firm's problem can be written as following:

$$\max \int_0^\infty [F^T(K^T, L^T) - wL^T - \Psi^T(I^T)]e^{-rt}dt, \tag{11}$$

under the dynamic capital stock constraint:

$$\dot{K}^T = I^T - \delta K^T. \tag{12}$$

The first-order conditions are:

$$F_L^T = w, \tag{13}$$

$$q^T = \Psi_I^T, \tag{14}$$

$$\dot{q}^T = (r+\delta)q^T - F_K^T, \tag{15}$$

$$\dot{K}^T = I^T - \delta K^T, \tag{16}$$

$$\lim_{s\to\infty} I^T e^{-rs} = 0. \tag{17}$$

where q^T is the multiplier associated to the constraint (12) and the capital stock, K^T, is mesured in terms of tradable.

The labor market is generally characterized by a non-binding constraint, that is $L^N + L^T < \bar{L}$, the total employment in the two sector is inferior to the level of full employment, $\bar{L}$. At one hand, the first-order conditions give out dynamic efficient evolution of capital stocks and its marginal values in the short- and intermediate-run. At other hand, giving the sector specific capital stocks [6] and non-binding labor market constraint [7], we can derive, from the first-order conditions of the firms problem, an instantaneous offer function for every sector. From the equation (6), we can derive

$$\frac{\partial L^N}{\partial p} = -\frac{F_L^{N2}}{pF_{LL}^N}, \ \frac{\partial L^N}{\partial K^N} = -\frac{F_{LK}^N}{F_{LL}^N}.$$

Then in using these partial derivatives and normality condition of production factors in the non-tradable sector, we can find out that:

$$y^N(K^N, p), \quad y_K^N > 0, y_p^N > 0.$$

[6]Which implies that the capital is not reallocated across the sector in the short-run, but through the new investment according to the long-run equalizing return rule.

[7]This means that increasing employment in one sector does not induce a corresponding reduction in the employment in the other sector as in the standard flexible wage and full employment models. This excludes out the possible movement of tradable production due to reallocation of labor between sectors.

The real rate of return of the capital in the non-tradable sector $R^N = F_K^N$ can be shown to have the following implicit form:

$$R^N(K^N, p), \quad R_K^N > 0, R_p^N >, < 0, \text{ if} F_{KL}^N >, < 0.$$

In using (13), and the input normality condition, we can obtain that the production function y^T has a derivative relative to capital stock as

$$y_K^T = F_K^T + F_L^T \frac{dL^T}{dK^T} = F_K^T - F_L^T \frac{F_{LK}^T}{F_{LL}^T} > 0.$$

The offer function of traded goods can be written then as

$$y^T(K^T), \quad y_K^T > 0.$$

The real return of the tradable sector capital stock $R^T = F_{KL}^T$ is a decreasing function of K^T only:

$$R^T(K^T), \quad R_K^T < 0.$$

2.2 The consumer

The infinitely-lived representative consumer maximizes the discounted sum of instantaneous utility:

$$\max \int_0^\infty u(c^T, c^N) e^{-\rho t} dt, \tag{18}$$

under the intertemporal budgetary constraint

$$\dot{a} = ra + c^T + pc^N + \Psi^T(I^T) + p\Psi^N(I^N) + \tau - y^T - py^N. \tag{19}$$

Here, instantaneous utility is a function of consumption of non-traded goods C^N and traded goods C^T, and ρ is the constant instantaneous rate of time preference. We assume that the instantaneous utility function is concave in its two variables and that both consumption goods are normal. [8] The debt accumulation equation for the representative consumer is then given as (19), which is also his intertemporal budget, with a as the stock of debt, measured in terms of traded goods, held by the representative consumer; and with τ as the lump-sum tax paid to the government.

[8] The normality of the goods means that in the more than two goods utility function, we have $u_{ii}u_j - u_i u_{ji}$. This permits us to eliminate some ambiguities in the following discussion.

The hamiltonian of this problem can be written as:

$$H = u(c^T, c^N) - \lambda(ra + c^T + pc^N + \Psi^T(I^T) + p\Psi^N(I^N) + \tau - y^T - py^N), \quad (20)$$

where λ is the multiplier associated to the constraint (19).
The first-order conditions are

$$u_T = \lambda, \quad (21)$$

$$u_N = p\lambda, \quad (22)$$

$$\dot{\lambda} = (\rho - r)\lambda, \quad (23)$$

$$\lim_{s\to\infty} ae^{-rs} = 0. \quad (24)$$

These conditions are usually derived in the literature. We ignore here the distinction between capitalists and workers, that between the employed and the unemployed and that between those employed in different sectors. Consumers are assumed to own all factors of production and to receive rental payments that equal the value of production. They are permitted to borrow and lend on the internal and the world financial markets at an interest rate specific to their country, subject to an intertemporal budget constraint.

The instantaneous marginal utility of traded consumption evolves according to (23). We note that λ can be interpreted as the marginal value of wealth in terms of tradable goods. It is also equal, at every moment, to the instantaneous marginal utility of traded consumption, since $\partial u_N/\partial u_T = \lambda$; and numerical subscripts denote the partial derivative of functions. The relation (23) means that, if the subjective rate of discount exceeds (falls short of) the interest rate, then the consumption of the two goods must be adjusted so as to raise (lower) the marginal utility of traded consumption.

2.3 The government

The government is assumed to operate in accordance with its budget constraint:

$$\dot{b} = rb + pg^N + g^T - \tau, \quad (25)$$

where b is the stock of government debt, g^N and g^T are respectively government spending on non-traded goods and traded goods. These spendings are assumed to have no direct effect on consumer's utility and productive technology. A deficit is produced if the government spendings plus the interest obligations on outstanding debt is more important than its current revenues. It must be financed either by imposing additional lump-sum taxes or by issuing additional debt, this has no importance since there is a infinitely-lived representative agent. The government's intertemporal budget constraint requires that the present value of the stream of future surplus exactly offsets the value of its current debt.

2.4 The short-run equilibrium

2.4.1 The short-run macroeconomic relations

The short-run macroeconomic relationships can be represented as follows:

$$y(K^N, p) = c^N + \Psi^N(I^N) + g^N, \tag{26}$$

$$u_T = \lambda, \tag{27}$$

$$u_N = p\lambda, \tag{28}$$

$$q^N = \Psi_I^N, \tag{29}$$

$$q^T = \Psi_I^T, \tag{30}$$

$$\dot{q}^N = (r+\delta)q^N - R^N(K^N, p) \tag{31}$$

$$\dot{K}^N = I^N - \delta K^N, \tag{32}$$

$$\dot{q}^T = (r+\delta)q^T - R^T(K^T), \tag{33}$$

$$\dot{K}^T = I^T - \delta K^T, \tag{34}$$

$$\dot{\lambda} = (\rho - r)\lambda, \tag{35}$$

$$\dot{B} = rB + c^T + pc^N + \Psi^T(I^T) + p\Psi^N(I^N) + g^T + pg^N - y^T - py^N. \tag{36}$$

The equations (26), (27), (28) can be used to determine c^T, c^N and p in terms of λ, q^N, K^N and g^N. The equations (29), (30) can be used to determine I^N and I^T respectively in terms of q^N and q^T. The equation of external debt, (36), is a simple summation of the private and governmental intertemporal budget constraints ((19) and (25), with $B = a + b$ and $\dot{B} = \dot{a} + \dot{b}$.

By assumption, the non-traded goods markets clear at each instant of time. This condition (equation (26) determines the equilibrium path of the relative price of non-traded goods. This relation can also be used to derive a dynamic relation of real exchange rate, p in terms of other dynamic variables. But then, we have first to determine c^N in terms of λ and p, to derive finally a dynamic equation of p as in Murphy (1991, equation (2.11)), but the resulting dynamic relation will include also q^N and K^N as determinants. This approach is simply abandoned here due to its induced complication.

2.5 The short-run dynamic equilibrium analysis

It is necessary, before doing the stability analysis of the system under alternative lending arrangements, to study the the short-run reaction function of c^N, c^T, p, I^N, I^T in terms of the dynamic variables. It is easy to see that:

$$I^N = I^N(q^N),\ I_q^N > 0;$$

$$I^T = I^T(q^T),\ I_q^T > 0;$$

For the relationship between c^T, c^N, p and the dynamic variables λ, q^N, K^N and political variable g^N, we have to proceed to a total differentiation of the three equations (26), (27), (28). [9] The variation of q^N, k^N and g^N have some different effects over c^T according to the fact that the two goods are complementary or substitutable in the utility function in the sense of Edgeworth. In the case of Edgeworth complementarity, we have:

$$c^T = c^T(\underset{-}{\lambda}, \underset{-}{q^N}, \underset{+}{K^N}, \underset{-}{g^N});$$

in the case of Edgeworth substitutability, we will have:

$$c^T = c^T(\underset{-}{\lambda}, \underset{+}{q^N}, \underset{-}{K^N}, \underset{+}{g^N})$$

The short-run reaction functions c^N and p do not depend on the relationship between the two goods in the instantaneous utility function. They can be expressed as:

$$\begin{aligned} c^N &= c^N(\underset{-}{\lambda}, \underset{-}{q^N}, \underset{+}{K^N}, \underset{-}{g^N}) \\ p &= p(\underset{-}{\lambda}, \underset{+}{q^N}, \underset{-}{K^N}, \underset{+}{g^N}) \end{aligned}$$

3 Stability analysis under alternative lending arrangements

3.1 Stability analysis under total debt arrangement

The scheme of lending arrangement using total debt as a proxy for creditworthiness is to link the interest rate paid on debt to the total outstanding debt held both by

[9] See Appendix 7 for formal derivation.

the private sector and the government. Under this arrangement, the interest rate is positively related to total net debt, which we view as a proxy for the level of creditworthiness:

$$r = r^* + v(B) = r(B), \quad v' > 0,\ r' > 0; \tag{37}$$

where r^* is the interest rate prevailing internationally and $v(B)$ the country-specific risk premium.

With this definition of external borrowing constraint, we can proceed now to the stability analysis of the dynamic system which describes the small country. The differentiation of the dynamic system of equations (31)–(36) around the steady-state equilibrium gives:

$$\begin{bmatrix} \dot{q}^N \\ \dot{K}^N \\ \dot{q}^T \\ \dot{K}^T \\ \dot{\lambda} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & 0 & 0 & -R_p^N p_\lambda & r'q^N \\ I_q^N & -\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & r+\delta & -R_K^T & 0 & r'q^T \\ 0 & 0 & I_q^T & -\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -r'\lambda \\ c_q^T & c_K^T & \Psi_I^T I_q^T & y_K^T & c_\lambda^T & r'B + r \end{bmatrix} \begin{bmatrix} q^N - \bar{q}^N \\ K^N - \bar{K}^N \\ q^T - \bar{q}^T \\ K^T - \bar{K}^T \\ \lambda - \bar{\lambda} \\ B - \bar{B} \end{bmatrix} \tag{38}$$

with $\varphi_1 = r + \delta - R_p^N p_q, \varphi_2 = -R_K^N - R_p^N p_K$. The determinant of stability matrix of system (38) can be written in the following form:

$$|J| = r'\lambda \left\{ c_\lambda^T \begin{bmatrix} \varphi_1 & \varphi_2 \\ I_q^N & -\delta \end{bmatrix} \begin{bmatrix} r+\delta & -R_K^T \\ I_q^T & -\delta \end{bmatrix} - R_p^N p_\lambda \begin{bmatrix} I_q^N & -\delta \\ c_q^T & c_K^T \end{bmatrix} \begin{bmatrix} r+\delta & -R_K^T \\ I_q^T & -\delta \end{bmatrix} \right\}$$

since we know that $r' > 0$, $c_\lambda^T < 0$, $p_q > 0$, $R_K^N < 0$, $p_K < 0$, $I_q^N > 0$, $R_K^T < 0$, $I_q^T > 0$, $\Psi_I^T > 0$, $y_K^T > 0$, $c_\lambda^T < 0$, $B > 0$, then a sufficient condition for $|J|$ to be negative is that the two goods are substitutable in the utility function and the capital and labor are substitutable in the non-tradable sector. If this is the case, we have $R_p^N = -\frac{F_L^N F_{KL}^N}{pF_{LL}^N} < 0$, and $c_q^T > 0$, $c_k^T < 0$, we can demonstrate, using the solution of short-run, that: [10]

$$I_q^N c_K^T + c_q^T \delta = I_q^N \frac{\lambda u_{TN} y_K^N}{\Delta} - \delta \frac{\lambda u_{TN} \Psi_I^N I_q^N}{\Delta} = \frac{\lambda I_q^N u_{TN}}{\Delta}(y_K^N - \delta \Psi_I^N).$$

$u_{NT} < 0$ by assumption, and $y_K^N - \delta\Psi_I^N = c_K^N > 0$ by the non-tradable goods market equilibrium condition, then the last expression is negative. We can also

[10]For the expression of partial derivations of reaction function, see 7.

demonstrate [11] that $-R_K^N - R_p^N p_K > -F_{KK}^N + F_{KL}^N \frac{F_K^N}{F_L^N} > 0$ under the input normality condition, $F_L^N F_{KK}^N + F_K^N F_{KL}^N < 0$.

From there, we deduce that the determinant $|J|$ is negative. It is possible when there is some reasonable complementarity between the two goods in the utility function and between the capital and labor in the non-tradable sector. We can also have $|J| < 0$. This signifies that the system can have one, three or five stable engenvalues which have a negative real part.

While one cannot rule out *a priori* the possibility of a completely unstable system with more than three unstable engenvalues, it is easy to show, in following BUITER (1989), that there exist parameter values for which there are three unstable roots (corresponding to the non-predetermined state variables q^N, q^T and the costate variable λ) and three stable roots (corresponding to the predetermined state variables K^N, K^T and B). If, e.g. $r' = 0$, the six characteristic roots are:

$$\lambda_1 = 0, \quad \lambda_2 = r,$$

$$\lambda_{3,4} = \frac{1}{2}\{r - R_K^N p_q \pm \sqrt{(r - R_K^N p_q)^2 - 4[I_q^N(R_K^N + R_p^N p_K) - \delta(r + \delta - R_K^N p_q)]}\},$$

$$\lambda_{5,6} = \frac{1}{2}\{r \pm \sqrt{r^2 - 4[I_q^T R_K^T - \delta(r + \delta)]}\};$$

where, given $I_q^N, I_q^T, p_q > 0$, $R_K^N + R_p^N p_K, R_K^N, R_K^T < 0$, λ_4 et λ_6 sont négatives.

Engenvalues [12] are continuous functions of values of the coefficients of matrix. A "small" increase of r' from zero will make the determinant of the state matrix in (38) negative, since $\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ are far from zero, these characteristic roots won't change sign. The zero root will become negative.

With three positive and three negative roots (all assumed to be real) the unique continuously convergent path to the new equilibrium after, say, a reduction in public spending, will be non-oscillatory.

[11] See Appendix 8 for formal derivation.

[12] They have to be understood as complex numbers.

3.2 Stability analysis under total debt per tradable output ratio arrangement

For the scheme linking the interest rate on international debt to the ratio of debt to traded output, we have:

$$r = r^* + z[\frac{B}{y^T(K^T)}], \quad z' > 0, \tag{39}$$

$$= r[\frac{B}{y^T(K^T)}], \; r' > 0; \tag{40}$$

because of the unemployment and the sector specific capital assumptions, the production of traded goods does not depend on the real exchange rate p. This is not the case in MURPHY, where the production of tradable is assumed to depend negatively on the real exchange rate.

Using the definition (40), the total differentiation of the dynamic system of equations (31) – (36) around the steady-state equilibrium gives:

$$\begin{bmatrix} \dot{q}^N \\ \dot{K}^N \\ \dot{q}^T \\ \dot{K}^T \\ \dot{\lambda} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & 0 & \phi q^N & -R_p^N p_\lambda & r' q^N / y^T \\ I_q^N & -\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & r+\delta & -R_K^T + \phi q^T & 0 & r' q^T / y^T \\ 0 & 0 & I_q^T & -\delta & 0 & 0 \\ 0 & 0 & 0 & \phi\lambda & 0 & -r'\lambda / y^T \\ c_q^T & c_K^T & \Psi_I^T I_q^T & y_K^T + \phi B & c_\lambda^T & r' B / y^T + r \end{bmatrix} \begin{bmatrix} q^N - \bar{q}^N \\ K^N - \bar{K}^N \\ q^T - \bar{q}^T \\ K^T - \bar{K}^T \\ \lambda - \bar{\lambda} \\ B - \bar{B} \end{bmatrix} \tag{41}$$

where $\phi = -r' a y_K^T / (y^T)^2 < 0, \varphi_1 = r + \delta - R_p^N p_q, \varphi_2 = -R_K^N - R_p^N p_K$.

The determinant of stability matrix of system (41) can be written in the following form:

$$|J| = \frac{r'\lambda}{y^T} \left\{ c_\lambda^T \begin{bmatrix} \varphi_1 & \varphi_2 \\ I_q^N & -\delta \end{bmatrix} \begin{bmatrix} r+\delta & -R_K^T \\ I_q^T & -\delta \end{bmatrix} - R_p^N p_\lambda \begin{bmatrix} I_q^N & -\delta \\ c_q^T & c_K^T \end{bmatrix} \begin{bmatrix} r+\delta & -R_K^T \\ I_q^T & -\delta \end{bmatrix} \right\}.$$

We find that the dynamic stability properties of the system (41) is not much different from the system (38). For r' reasonably small, we can affirm that the system has three stable characteristic roots under the substitutability assumption adopted before.

3.3 Trade balance arrangement

This scheme of lending arrangement is the one in which the interest rate on international lending is positively related to the trade deficit. In other words, it links debt payments to creditworthiness measured by the non-interest component of the current account. We have then:

$$\begin{aligned} r &= r^* + u(M), u' > 0, && (42)\\ &= r(M),\ r' > 0; && (43)\end{aligned}$$

where the trade balance deficit, M, is defined as the difference between the domestic total absorption and production of tradable, which can be represented as:

$$M = c^T + \Psi^T(I^T) + g^T - y^T(K^T) = M(\lambda, q^N, K^N, g^N, K^T, g^T),$$

if the creditworthiness of the small economy is a function of the trade balance, it is not very difficult to find that this system is not stable.

A first intuition can be gained in studying the dynamic equation of the costate variable λ. The equation (35) means, if $r = r(M)$, the external debt B can not be determined. The remaining question is then, can the capital accumulation help to the determination of a stable level B? To have more justification of this intuition, we can produce the total differentiation of the dynamic system under the trade balance deficit arrangement as follows:

$$\dot{y} = Ay \tag{44}$$

with $\dot{y} = (\dot{q}^N, \dot{K}^N, \dot{q}^T, \dot{K}^T, \dot{\lambda}, \dot{B})'$, $y = (q^N - \bar{q}^N, K^N - \bar{K}^N, q^T - \bar{q}^T, K^T - \bar{K}^T, \lambda - \bar{\lambda}, B - \bar{B})'$, and

$$A = \begin{bmatrix} \varphi_1 + \kappa_1 q^N & \varphi_2 + \kappa_2 q^N & \kappa_3 q^N & \kappa_4 q^N & -R_p^N p_\lambda + \kappa_5 q^N & 0 \\ I_q^N & -\delta & 0 & 0 & 0 & 0 \\ \kappa_1 q^T & \kappa_2 q^T & r + \delta + \kappa_3 q^T & -R_K^T + \kappa_4 q^T & \kappa_5 q^T & 0 \\ 0 & 0 & I_q^T & -\delta & 0 & 0 \\ -\kappa_1 \lambda & -\kappa_2 \lambda & -\kappa_3 \lambda & -\kappa_4 \lambda & -\kappa_5 \lambda & 0 \\ c_q^T + \kappa_1 B & c_K^T + \kappa_2 B & \Psi_I^T I_q^T + \kappa_3 B & y_K^T + \kappa_4 B & c_\lambda^T + \kappa_5 B & r \end{bmatrix}$$

where $\varphi_1 = r + \delta - R_p^N p_q$, $\varphi_2 = -R_K^N - R_p^N p_K$, $\kappa_1 = r' M_{q^N}$, $\kappa_2 = r' M_{K^N}$, $\kappa_3 = r' M_{q^T}$, $\kappa_4 = r' M_{K^T}$, $\kappa_5 = r' M_\lambda$.

The state determinant can be simplified to:

$$|J| = -rr'M'_\lambda \lambda \begin{bmatrix} \varphi_1 & \varphi - 2 \\ I_q^N & -\delta \end{bmatrix} \begin{bmatrix} r + \delta & -R_K^T \\ I_q^T & -\delta \end{bmatrix}$$

$$-rR^N_p p_\lambda \begin{bmatrix} I^N_q & -\delta \\ -r'M'_{q^N}\lambda & -r'M'_{K^N}\lambda \end{bmatrix} \begin{bmatrix} r+\delta & -R^T_K \\ I^T_q & -\delta \end{bmatrix} > 0.$$

The positive determinant of the state matrix signifies that there are zero, two, four or six stable characteristic roots. It is easy to find that in a reasonable range value of $r'(>0)$, the system has two stable and four unstable engenvalues. Then the system cannot be stable. Since the state (and costate) matrix of (44) is decomposable into two distinct blocks, then each of them must be stable in a autonomous fashion for the total system to be stable. Then even, in the $r' < 0$ case, with three stable roots, [13] the system can not be stable. It is crucial to find out that the external position dynamic does not influence the other dynamic variables. The autonomy of this equation makes it non-convergent under whatever shock, as the expanding external position cannot engender an interactive force (feed-back) to stabilize it.

If the instability is the general case under trade balance arrangement, then the economy system can only be viable when there is an exogenously imposing feasible ceiling over the external debt, with occasional reduction of debt and of payment of interests when necessary. Its analysis, rather complicated, depends on the specification of the ceiling that the lending countries can impose. It can become an interesting case to study, but the inconvenient is that the dynamic system is broken down to the capital accumulation. The costate variable λ's dynamics cannot attain a stationary state if we assume that the time preference rate is different from the exogenously fixed interest rate. Since the lending countries fix the lending ceiling, there is no reason that they can impose a variable interest rate which is a function of the trade balance deficit. The trade balance deficit must always rest the same for the given external debt.

Without the possibility of borrowing further abroad, it is then not reasonable to assume that the domestic financial market adopt the same interest rate than the one on foreign debt over other domestic financial transaction, then it may exist a dual financial system: two interest rates coexist, one is imposing over the foreign borrowing, the other is endogenously determined by the domestic financial offers and needs. This case is then not to be analyzed here, since we are interested in the endogenous adjustment of foreign debt position of a small country facing to some budgetary policy shocks.

[13] In this case, the determinant can be negative in the counter-intuitive case, the system has three stable roots. The fact that the interest rate must diminish when trade balance deficit increases, $r' < 0$, is contrary to what is proposed, i.e. an increasing function of trade balance deficit as in Murphy (1991).

4 The budgetary policy effects at long-run equilibrium under total debt and debt-ratio arrangements

In this section, we will study the long-run effects of budgetary policies under two stable lending arrangements out of the three proposed schemes. The budgetary policies considered here are of permanent and non-anticipated nature. As the system in the long-run is deterministic, the temporary policies or anticipation cannot affect the long-run equilibrium. For simplicity, only balanced budgetary policies are considered. The saving that the government can realize from reducing the spending on tradable and non-tradable is entirely redistributed through reduction of per capita lump-sum tax.

4.1 The total debt arrangement

The long-run equilibrium of the small economy under the total debt arrangement can be written in imposing the stationary condition, i.e. $\dot{q}^N = \dot{K}^N = \dot{q}^T = \dot{K}^T = \dot{\lambda} = \dot{B} = 0$, and the external lending arrangement $r = r(B)$:

$$y^(K^N, p) = c^N + \Psi^N(I^N) + g^N, \tag{45}$$

$$u_T = \lambda, \tag{46}$$

$$u_N = p\lambda, \tag{47}$$

$$q^N = \Psi_I^N, \tag{48}$$

$$q^T = \Psi_I^T, \tag{49}$$

$$[r(B) + \delta]q^N - R^N(K^N, p) = 0, \tag{50}$$

$$I^N - \delta K^N = 0, \tag{51}$$

$$[r(B) + \delta]q^T - R^T(K^T) = 0, \tag{52}$$

$$I^T - \delta K^T = 0, \tag{53}$$

$$[\rho - r(B)]\lambda = 0, \tag{54}$$

$$r(B)B + c^T + \Psi^T + g^T - y^T = 0. \tag{55}$$

When the external debt position of the country is determined by equation (54), then equations (49), (52), (53) determine the long term values q^T, I^T, K^T; as we find out, all of them are independent of the budgetary policy (namely, spending over tradable or non-tradable goods). Of course, this does not mean that the budgetary policies have no influence over these variables. The contrary is true as it will

	c^T	c^N	λ	p	q^N	K^N	I^N	q^T	K^T	I^T	B
g^N	0	$-$	$+$	$+$	$-$	$-$	$-$	0	0	0	0
g^T	-1	$-$	$-$	$-$	$+$	$+$	$+$	0	0	0	0

Table 1: The long-run effects of national budgetary policies

be demonstrated in the following section when we will effectuate the transitional dynamic analysis.

As B, I^T, and K^T are unchanged by the budgetary policies, then by the last relation, (55), we find that c^T is only influenced by the public spending on tradable goods, g^T, at one-to-one basis. The spending on non-tradable goods, g^N, has only influence over c^N. The importance of the late policy over the private consumption has to be evaluated more carefully since it also change the relative price and the marginal value of the wealth and the offer conditions of the non-tradable sector. The public spending, g^T, has also an influence over c^N, p, q^N, I^N and K^N in the long-run which must also be asserted before we pass to a dynamic analysis.

The long-run effects of the budgetary policies can be represented in table 1.[14]

We can shortly comment the results as follows. The long-run effects of reduction of the spending on non-tradable goods does not modify the consumption of tradable goods, but reduces the consumption of non-tradable. If this policy reduces the pressure on the non-tradable market, it has to be stimulated up finally by a lower price (real exchange rate) level. From the external equilibrium condition of the economy, a lower price of non-tradable will discourage the tradable consumption under the substitutability assumption. For the equilibrium to be re-established, there must be a decrease in the marginal value of wealth. The diminution of price, inducing a higher real wage rate in the small country will incite the non-tradable sector to employ less labor which is substitutable to the capital, then the Tobin's q, the long-run capital stock and investment flux of the non-tradable sector will increase. Of course, in the case of complementarity between different kinds of goods and productive inputs, this policy's effect concerning the marginal value of wealth, λ, the Tobin's q, the optimal capital stock and investment flux will change drastically. More precisely, the signs of policy effects are reversed. Then the same policy will have positive effect on λ, negative effects on q^N, I^N and K^N. The rest of the effects is the same in the substitutability and complementarity case.

[14]For the formal derivation, see Appendix 9

For a permanent non-anticipated reduction in the public spending on the tradable goods, it is clear that this policy “crowds in” hundred percent the private consumption of tradable. This will increase the marginal value of wealth, which in turn renders the non-tradable market into excess demand by increasing the private consumption. The excess demand can only be removed by a corresponding increase of the real exchange rate, which will finally translate into negative variation of non-tradable sector capital stock and corresponding variation of marginal value of capital in this sector.

4.2 The long-run effects of budgetary policies under the debt-ratio arrangement

Under this arrangement, the variables as B, q^T, I^T and K^T are determined by equations (54), (49), (52) and (53), independently of other equations. The national budgetary policies have no influence over them in the long-run. As before, this does not mean that these variables are also shielded from the policy changes in the short-run. The effects of budgetary policies are totally similar to those in the case of the total debt arrangement. Since the long-run effects of budgetary policies are in the same way than those in the case of total debt arrangement, we can then refer to table 1 for them.

5 Transitional dynamics under policy shocks: a simulation exercise

The full dynamic interaction of the system under different lending arrangement is too complicated to be analyzed. For having an idea of the short- and intermediate-run movements of different variables, we give first a general formulation of resolution of the dynamics, then we proceed to a simulation exercise.

5.1 Total debt arrangement

With three stable roots (assumed to be real) and three predetermined variables, the system is expected to exhibit a saddle-point equilibrium configuration under perfect foresight assumption. Then, the adjustment path can take generally the following form: [15]

$$\begin{bmatrix} q^N(t)-\bar{q}^N \\ K^N(t)-\bar{K}^N \\ q^T(t)-\bar{q}^T \\ K^T(t)-\bar{K}^T \\ \lambda(t)-\bar{\lambda} \\ B(t)-\bar{B} \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \\ V_{41} & V_{42} & V_{43} \\ V_{51} & V_{52} & V_{53} \\ V_{61} & V_{62} & V_{63} \end{bmatrix} \begin{bmatrix} k_1 e^{-\lambda_1 t} \\ k_2 e^{-\lambda_2 t} \\ k_3 e^{-\lambda_3 t} \end{bmatrix} \tag{56}$$

where $(V_{1j}, ..., V_{6j})', j = 1, 2, 3$, are engenvectors associated to engenvalues $\lambda_1 < \lambda_2 < \lambda_3 < 0$. The constants $k_i, i = 1, 2, 3$, can be obtained from the following system

$$\begin{bmatrix} V_{21} & V_{22} & V_{23} \\ V_{41} & V_{42} & V_{43} \\ V_{61} & V_{62} & V_{63} \end{bmatrix} \begin{bmatrix} k_1 e^{-\lambda_1 t} \\ k_2 e^{-\lambda_2 t} \\ k_3 e^{-\lambda_3 t} \end{bmatrix} = \begin{bmatrix} K^N(t)-\bar{K}^N \\ K^T(t)-\bar{K}^T \\ B(t)-\bar{B} \end{bmatrix} \tag{57}$$

in letting $t = 0$, the coefficients k_1, k_2, k_3 are calculated in using the initial conditions $K^N(0), K^T(0), B(0)$. Since $K^N(0)-\bar{K}^N \neq 0, K^T(0)-\bar{K}^T = 0, B(0)-\bar{B} = 0$, they can be quite simplified.

A general remark that we can make about the solution in (56) is that, though the variables as B, q^T, K^T do not change in the long-run facing to whatever budgetary policies shocks, they are not stationary in the short- and intermediate-run. But the solution is too general to gain some intuitive comprehension of the dynamic adjustment profile of the system under disequilibrium, since there are too much factors which work in different ways. A simulation exercise is proposed here.

Given $r = 0.04$, $\delta = 0.05$, $R_p^N = -0.1$, $R_K^N = -0.21$, $p_q = 0.1$, $p_\lambda = -0.4$, $p_K = -0.1$, $q^N = 1.2$, $q^T = 1.3$, $R_K^T = -0.2$, $I_q^N = 2$, $I_q^T = 2.2$, $c_q^T = 0.08$, $c_K^T = -0.02$, $\Psi_I^N = 1.2$, $y_K^N = 0.5$, $c_\lambda^T = -0.6$, $B = 10$, λ, $r' = 0.003$, the characteristic roots of the state matrix of liberalized system (38) can be shown as, $\lambda_1 = -0.6489871086, \lambda_2 = -0.6122056431, \lambda_3 = -0.02206804787, \lambda_4 = 0.08009209496, \lambda_5 = 0.7012283941, \lambda_6 = 0.6619403129$. There are three stable and three unstable roots.

The dynamic adjustment corresponding to a reduction of public spending on non-tradable goods ($K^N(0) - \bar{K}^N > 0$) can be illustrated in the figure 1. It is shown by the simulation that, although the long-run value of q^T, K^T, a and I^T cannot be influenced by the policy, they can be modified in the short-run by

[15] For a detailed characterization of the stable manifold under rational anticipation condition, see VAROUDAKIS (1990a).

Figure 1: The short- and intermediate-run effect of a permanent non-anticipated reduction of spending on non-tradable goods.

Figure 2: The short- and intermediate-run effect of a permanent non-anticipated reduction of spending on tradable goods.

the dynamic response of the economy to this permanent shock which engenders initially a pressure on the non-tradable market. The transmission of the effects of the shock can be easily understood in this simple model. When the shock arrives, the depression on the non-tradable market is moved by reducing (more possibly) the real exchange rate more then necessary compared to its long-run level. The ambiguity persists due to the double influence of adjustment of consumption plan of consumers and the revaluation of the market value of non-tradable sector. The overshooting of real exchange rate cannot be established easily since there are two factors which interact to determine its pattern of initial adjustment. The variation of marginal value of wealth and the marginal value of non-tradable sector capital stock can be translated into initial overshooting, retarded overshooting or other form of adjustment of real exchange rate. In the substitutability case, the pattern of the variation of the tradable consumption is simple, it increases sharply to be reduced gradually later. This explain why there is an external debt (B) dynamics in the short-run, which in turn explains the transitory movement of q^T, K^T and I^T.

The reaction of the economy following a permanent non-anticipated reduction of spending on tradable goods can be illustrated in figure 2. The same kind of remarks as that regarding the 1 can be made.

5.2 Debt ratio arrangement

Given $r = 0.04$, $\delta = 0.05$, $R_p^N = -0.1$, $R_K^N = -0.21$, $p_q = 0.1$, $p_\lambda = -0.4$, $p_K = -0.1$, $q^N = 1.2$, $q^T = 1.3$, $R_K^T = -0.2$, $I_q^N = 2$, $I_q^T = 2.2$, $c_q^T = 0.08$, $c_K^T = -0.02$, $\Psi_I^N = 1.2$, $y_K^N = 0.5$, $c_\lambda^T = -0.6$, $B = 10$, λ, $r' = 0.03$, $y^T = 15$, the characteristic roots of the state (and costate) matrix of liberalized system (44) can be shown as, $\lambda_1 = -0.6469368852, \lambda_2 = -0.6121156831, \lambda_3 = -0.01707689893, \lambda_4 = 0.06925727297, \lambda_5 = 0.6619296437, \lambda_6 = 0.6949435444$. There are three stable and three unstable roots. The short-run behavior of the system under debt ratio arrangement is similar as under the total debt arrangement.

6 Conclusion

This paper has extended MURPHY's (1991) analysis of alternative lending arrangements in incorporating capital accumulation process. Another departure from the cited analysis is that we do not assume the full employment. This unemployment assumption is responsible for some results in this paper, notably the quasi-equivalence of the total debt arrangement and debt-traded output ratio arrangement. We distinguish different kinds of policy changes according to their properties such as spending on traded and non-traded goods. MURPHY (1991) demonstrated that under total debt arrangement, the policy which consists to reduce spending on traded or non-traded goods has no effect on the country's net external debt position. A similar result is obtained by BHANDARI, HAQUE and TURNOVSKY (1990) in their one sector model, the public spending has not any effect neither on capital stock nor on external debt in the short- and long-run. While under debt ratio arrangement, MURPHY's results suggest that the reduction in spending on tradable goods reduces the steady state level of debt. A reduction in spending on non-traded goods will actually increase the steady-state level of debt.

The first interesting finding in our model is that, under the lending arrangement based on the ratio of debt to traded output as under the total debt arrangement, the permanent reduction in spending on tradable or non-tradable will not increase nor decrease external debt, in the long-run. The reason for this is that, under the rigid wage and unemployment assumption and the sector-specific investment and installation goods assumption, no reallocation of capital and labor between the two sectors is effectuated in the short-run, the instantaneous tradable offer function depends only on the existent tradable sector capital stock. In MURPHY, the trade balance arrangement is totally unstable. In our model, the global stability analysis suggests that, even if capital accumulation is introduced, this arrangement cannot change original instability of the economy, as if it is the case under perfect international financial market condition.
country's net

The second interesting finding is that, although the long-run effects of budgetary policy effects are absent in regard to the external debt, the capital stock, investment and marginal value of capital in the tradable sector, the different spending reduction policies can generate interesting short-run dynamic effect on external debt level, since then on the trade balance and the balance of payment. These transitory dynamic movements of external debt are accompanied by those of tradable sector capital stock, investment and marginal capital value. We remarked that, in this interaction context with more than one predetermined variable, the initial overshooting of real exchange rate — which is quite popular in rational anticipation open economy models — is more difficult to be re-established.

The deterministic steady state equilibrium debt level under the two stabilizing lending arrangements is quite contrasting to what can be determined in some constant time preference infinite horizon models which adopt perfect international financial market assumption. There, the steady state but can be influenced by the temporary policy changes and the anticipation of policy changes. [16]

The puzzling assumption that the representative consumer's subjective rate of discount is equal to the interest rate facing the country in question is quite crucial for ensuring steady state equilibrium in a small open developing country facing a perfect world financial market. It is somewhat contradictory to have a constant subjective rate of discount which has to be equal to the world interest rate facing the country. The different lending arrangements are one kind of departures from this standard troubling case, which permits us to do some interesting budgetary policy analysis in a developing country. One can also drop this assumption by endogenizing the rate of discount as did OBSTFELD (1981, 1990), ENGEL and KLETZER (1989). This will permit us to explain the developing country's initial debt and the stages of external debt position. Another possibility to drop this assumption is to use an overlapping generation framework (BLANCHARD, 1985; BUITER, 1988a, b).

The structure of this model has the property that the economy is either an exporter when there is a trade account surplus or an importer when there is a trade account deficit. In fact, any country will export and import at the same time; the trade deficit or surplus is only a small quantity compared to total commercial flux. A more realistic model will incorporate this commercial flux to avoid omitting some important economic phenomena. The policies considered here are quite simple. It may be interesting to consider the effects of more complex types of budgetary and fiscal policy mix. It may also be interesting to consider the effects of some struc-

[16] See MATSUYAMA (1988, 1991), SEN and TURNOVSKY (1989).

tural policies which consist to stimulate the growth in helping the development of new production sector in the developing country.

7 The short-run reaction function of consumption and real exchange rate

For the relationship between c^T, c^N, p and the dynamic variables λ, q^N, K^N and political variable g^N, we have to proceed to a total differentiation of the three equations (26), (27), (28) as follows:

$$\begin{bmatrix} u_{TT} & u_{TN} & 0 \\ u_{NT} & u_{NN} & -\lambda \\ 0 & -1 & y_p^N \end{bmatrix} \begin{bmatrix} dc^T \\ dc^N \\ dp \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & \Psi_I^N I_q^N & -y_K^N & 1 \end{bmatrix} \begin{bmatrix} d\lambda \\ dq^N \\ dk^N \\ dg^N \end{bmatrix} \tag{58}$$

The solution of this system can be obtained in using the Cramer's rule:

$$\begin{bmatrix} dc^T \\ dc^N \\ dp \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} y_p^N u_{NN} - \lambda & -u_{TN} y_p^N & -\lambda u_{TN} \\ -y_p^N u_{NT} & u_{TT} y_p^N & \lambda u_{TT} \\ -u_{NT} & u_{TT} & u_{TT}u_{NN} - u_{NT}^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & \Psi_I^N I_q^N & -y_K^N & 1 \end{bmatrix} \begin{bmatrix} d\lambda \\ dq^N \\ dK^N \\ dg^N \end{bmatrix} \tag{59}$$

$$= \frac{1}{\Delta} \begin{bmatrix} y_p^N (u_{NN} - p u_{TN}) - \lambda & -\lambda u_{TN} \Psi_I^N I_q^N & \lambda u_{TN} y_K^N & -\lambda u_{TN} \\ y_p^N (u_{TT} - p u_{NT}) & \lambda u_{TT} \Psi_I^N I_q^N & -\lambda u_{TT} y_K^N & \lambda u_{TT} \\ -u_{NT} + p u_{TT} & J \Psi_I^N I_q^N & J y_K^N & J \end{bmatrix} \begin{bmatrix} d\lambda \\ dq^N \\ dk^N \\ dg^N \end{bmatrix}$$

where $\Delta = (u_{TT}u_{NN} - u_{TN}^2) y_p^N - \lambda u_{TT} > 0$, and $J = (u_{TT}u_{NN} - u_{TN}^2) > 0$, $(u_{NN} - p u_{TN}) < 0, (u_{TT} - p u_{NT}) < 0, -u_{NT} + p u_{TT} < 0$, these last three relations are verified by normality condition.

8 The sign of an expression

$$\begin{aligned} -R_K^N - R_p^N p_K &= -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) - F_{KL}^N \frac{dL}{dp} p_K \\ &= -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) - F_{KL}^N \frac{dL}{dp} \frac{-(u_{TT}u_{NN} - u_{NT}^2) y_K^N}{(u_{TT}u_{NN} - u_{NT}^2) y_p^N - \lambda u_{TT}} \end{aligned}$$

$$
\begin{aligned}
&> -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) - F_{KL}^N \frac{dL}{dp} \frac{-(u_{TT}u_{NN} - u_{NT}^2)y_K^N}{(u_{TT}u_{NN} - u_{NT}^2)y_p^N} \quad \text{since } \lambda u_{TT} < 0, \\
&= -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) + F_{KL}^N \frac{dL}{dp} \frac{y_K^N}{y_p^N} \\
&= -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) + F_{KL}^N \frac{dL}{dp} \frac{(F_K^N + F_L^N \frac{dL}{dK})}{F_L^N \frac{dL}{dp}} \\
&= -(F_{KK}^N + F_{KL}^N \frac{dL}{dK}) + F_{KL}^N \frac{F_K^N}{F_L^N} + F_{KL}^N \frac{dL}{dK} \\
&= -F_{KK}^N + F_{KL}^N \frac{F_K^N}{F_L^N} > 0 \text{ if } F_L^N F_{KK}^N + F_K^N F_{KL}^N < 0.
\end{aligned}
$$

9 The derivation of long-run effect of budgetary policies

Using $I^N = K^N$ from equation (51), we can study the long-run effects of budgetary policies in totally differentiating equations (45) – (48), (50) and (55) as follows:

$$
\begin{bmatrix}
0 & -1 & 0 & y_p^N & 0 & y_K^N - \delta\Psi_I^N \\
u_{TT} & u_{TN} & -1 & 0 & 0 & 0 \\
u_{NT} & u_{NN} & -p & -\lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -\delta\Psi_{II}^N \\
0 & 0 & 0 & -R_p^N & r+\delta & -R_K^N \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
dc^T \\ dc^N \\ d\lambda \\ dp \\ dq^N \\ dK^N
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1
\end{bmatrix}
\begin{bmatrix}
dg^N \\ dg^T
\end{bmatrix}
\tag{60}
$$

The solution of this system gives

$$
\begin{bmatrix}
dc^T \\ dc^N \\ d\lambda \\ dp \\ dq^N \\ dK^N
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
A_{11} & -A_{16} \\
A_{21} & -A_{26} \\
A_{31} & -A_{36} \\
A_{41} & -A_{46} \\
A_{51} & -A_{56} \\
A_{61} & -A_{66}
\end{bmatrix}
\begin{bmatrix}
dg^N \\ dg^T
\end{bmatrix}
\tag{61}
$$

where

$$
\Delta = [\lambda - (u_{NN} - pu_{TN})y_p^N][-R_K^N + (r+\delta)\delta\Psi_{II}^N] - R_p^N(u_{NN} - pu_{TN})(y_K^N - \delta\Psi_I^N) > 0,
$$

since $R_p^N = F_{KL}^N \frac{dL}{dp} < 0$ under the assumption that $F_{KL}^N < 0$ (substitutability between labor and capital), we have to use an input normality condition which presents the following form:

$$
F_L^N F_{KK}^N - F_K^N F_{KL}^N < 0.
$$

It is then easy to demonstrate that the determinant is positive. For that it is sufficient to show that

$$-y_p^N R_K^N + R_p^N y_K^N > 0.$$

In fact we have $y_p^N = F_L^N \frac{dL}{dp}$, $R_p^N = F_{KL}^N \frac{dL}{dp}$, $y_K^N = F_K^N + F_L^N \frac{dL}{dK}$, $R_K^N = F_{KK}^N + F_{KL}^N \frac{dL}{dK}$, in substituting these expressions into the above inequality and in using the input normality condition, we find out that this inequality is verified.

The cofactors in (61) can be given as:

$$\begin{aligned}
A_{11} &= 0,\ A_{21} = \lambda[R_K^N - (r+\delta)\delta\Psi_{II}^N] < 0, \\
A_{31} &= -u_{TN}\lambda[-R_K^N + (r+\delta)\delta\Psi_{II}^N] > 0, \\
A_{41} &= (pu_{TN} - u_{NN})[-R_K^N + (r+\delta)\delta\Psi_{II}^N] > 0, \\
A_{51} &= (pu_{TN} - u_{NN})R_p^N \delta\Psi_{II}^N < 0,\ A_{61} = (pu_{TN} - u_{NN})R_p^N < 0, \\
A_{16} &= [\lambda - (u_{NN} - pu_{TN})y_p^N][-R_K^N + (r+\delta)\delta\Psi_{II}^N] - R_p^N(u_{NN} - pu_{TN})(y_K^N - \delta\Psi_I^N) > 0, \\
A_{26} &= (u_{NN} - pu_{TN})y_p^N[-R_K^N + (r+\delta)\delta\Psi_{II}^N] - R_p^N(u_{NN} - pu_{TN})(y_K^N - \delta\Psi_I^N) > 0, \\
A_{36} &= [-\lambda u_{TT} - (u_{NN} - pu_{TN})y_p^N][-R_K^N + (r+\delta)\delta\Psi_{II}^N] - R_p^N(u_{NT} - pu_{TT})(y_K^N - \delta\Psi_I^N) > 0, \\
A_{46} &= (u_{NT} - pu_{TT})[-R_K^N + (r+\delta)\delta\Psi_{II}^N] > 0, \\
A_{56} &= (u_{NN} - pu_{TN})R_p^N \delta\Psi_{II}^N < 0,\ A_{66} = (u_{NT} - pu_{TT})R_p^N < 0.
\end{aligned}$$

References

[1] ABEL Andrew Bruce, (1979), Investment and Value of Capital, Garland Publishing, Inc., New York and London

[2] BHANDARI Jagdeep S., HAQUE Ul Nadeem and TURNOVSKY Stephen J., (1990), Growth, External Debt, and Sovereign Risk in a Small Open Economy, IMF Staff Papers 37, No. 2, 388-417

[3] BLANCHARD Olivier J., (1985), Debt, Deficits, and Finite Horizons, Journal of Political Economy, vol. 93, No. 2, 223-247

[4] BLANCHARD Olivier J. and FISCHER Stanley, (1989), Lectures on Macroeconomics, MIT Press

[5] BORENSZTEIN Eduardo R., (1989), Fiscal Policy and Foreign Debt, Journal of international Economics 26, 53-75

[6] BROCK Philip L., (1988), Investment, the Current Account, and the Relative Price of Non-Traded Goods in a Small Open Economy, Journal of International Economics 24, 235-253

[7] BUITER, Willem H., (1988a), Death, Birth, Productivity Growth and Debt Neutrality, The Economic Journal, 98, 279-293

[8] BUITER, Willem H., (1988b), Structural and Stabilization Aspects of Fiscal and Financial Policy in the Dependent Economy, Oxford Economic Papers 40, 220-245

[9] BUITER, Willem H., (1989), Budgetary policy, International and Intertemporal Trade in the Global Economy, North-Holland

[10] ENGEL Charles and KLETZER Kenneth, (1989), Saving and Investment in an Open Economy with Non-traded Goods, International Economic Review, vol. 30, No. 4, November, 735-752

[11] GERTLER Mark and ROGOFF Kenneth, (1990), North-South lending and endogenous domestic capital market inefficiencies, Journal of Monetary Economics 26, 245-266

[12] GOULD, J. P., (1968), Adjustment Costs in the Theory of Investment of the Firm, Review of Economic Studies 35, 47-56

[13] HAYASHI Fumio, (1982), Tobin's Marginal q and Average q : a Neoclassical Interpretation, Econometrica, vol. 50, No. 1, 213-224

[14] LUCAS, R. E., (1967), Adjustment Costs and Theory of Supply, Journal of Political Economy, 75, 321-334

[15] MATSUYAMA Kiminori, (1987), Current account dynamics in a finite horizon model, Journal of International Economics 23, 299-313

[16] MATSUYAMA Kiminori, (1991), On exchange rate stabilization, Journal of Economic Dynamics and Control 15, 7-26

[17] MURPHY Robert G., (1991), Macroeconomic adjustment under alternative lending arrangements, Journal of Economic Dynamics and Control 15, 103-127

[18] OBSTFELD Maurice, (1990), Intertemporal dependence, impatience, and dynamics, Journal of Monetary Economics 26, 45-75

[19] OTANI Ichiro and VILLANUEVA Delano, (1989), Theoretical Aspects of Growth in Developing Countries, IMF Staff Papers, Vol. 36, No. 2, 307-342

[20] SEN Partha and TURNOVSKY Stephen J., (1989), Tariffs, Capital Accumulation, and the Current Account in a Small Open Economy, International Economic Review, Vol. 30, No. 4, November 811-831

[21] TREADWAY, A., (1969), On Rational Entrepreneurial Behavior and the Demand for Investment, Review of Economic Studies, 36, 227-240

[22] UZAWA Hirofumi, (1968), Time preference, the Consumption Function, and Optimum Asset Holdings, in J. N. Wolfe, ed., Value, Capital and Growth: Papers in Honor of Sir John Hicks (Chicago: Aline, 1968), also in "Selected Papers of UZAWA Hirofumi", Cambridge Press, Mass., 1988

[23] VAROUDAKIS Aristomène A., (1990a), Dynamique du taux de change et substitution des actifs financiers dans un modèle avec marché d'actions, Revue Economique, Vol.41, No.1, 95-118

[24] VAROUDAKIS Aristomène A., (1990b), Salaires d'Efficience, Compétitivité et Ajustements Macroéconomiques, document de travail, No. 90-02/E, Institut d'Etudes Politiques, Août