Value-at-Risk versus Non-Value-at-Risk Traders

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VALUE-AT-RISK VERSUS NON VALUE-AT-RISK TRADERS

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ABSTRACT

In the paper, I simulated the game with a joint presence of 95% VaR-rule and return-rule groups of agents in the game. Simulations highlighted the level of omniscience, next being the rule, which agents follow at the decision-making, and the third the presence of liquidity agents in the game. Omniscient agents make different decisions than non-omniscient agents with non-omniscient return-rule agents performed a little better than the omniscient return-rule agents did, and omniscient VaR-rule agents performed slightly better than non-omniscient VaR-rule agents did. VaR-rule agents clearly outperform return-rule agents, with omniscient return-rule agents performing the worst. The role of liquidity agents has proved to be very significant with none of the two observed performed worst in the neither case.

Keywords: social networks, portfolio decision-making, stochastic finance, Value-at-Risk.
JEL Classification: G11, G32, C73, Z13.

1 Introduction

In the world of incomplete knowledge and strategic behavior, the course of action depends of the knowledge of selfish individuals, the network to which such individuals belong, and the preferences they have, with the course of action influencing backwards the beliefs and knowledge of individuals. For every individual this is a continuous and intertwined process and a stochastic process as well, as the beliefs of individuals are affected by the course of his action and the action of the group. Namely, the ability to learn is the most important ability individuals possess, by which they improve their future decisions. When going from an individual to the group of individuals, whether they form a small community or an economy, the sum of such processes of individuals also result in the stochastic process.

Let us define a stochastic process in short. A stochastic process is defined on the probability space $(\Omega, \Im, P)$, with Ω representing the outcome space of all possible outcomes of events $\omega \in \Omega$, $\Im$ is a $\sigma -$algebra, the collection ob subsets of a sample space Ω that occur with the probability P, with $P(\Im) \geq 0$ and $P(\Omega) = 1$. We say that an event $\Im$ is certain if the assigned probability of the event equals one; that is $P(\Im) = 1$.

In the world of incomplete knowledge, risk-taking is indispensable. When looking the developments on financial markets, we see that it clearly is a stochastic process with stock prices moving both inter-day and intraday, and neither develops in a predictable way nor could be foreseen. In fact, unpredictability and the stochastic nature of stock process are implied in the process of the trade itself. If the prices were certain and that be the common knowledge among the traders, the trade is not possible as none would be willing to sell below the common knowledge price and none would be willing to buy above that price. Certain events are possible only theoretically and would prevent the trade if present. In reality, the only common knowledge is that of Hayek (1945) that “knowledge of the circumstances of which we must make never exist in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all separate individuals possess.” Portfolio theory provides a broad context of dealing with such stochastic processes, in finance defined by Markowitz (1952).

I deal with the question of a portfolio selection under uncertainty and different circumstances with individuals communicating through the social networks in a number of papers (Steinbacher 2008a, b; 2009a, b, c). In my previous work, I was interested in answering the question how individuals with different preferences and the level of omniscience manage their portfolios under different circumstances when being part of larger groups of individuals. In all the models, I use a small world network by which individuals communicate with each other (Watts and Strogatz 1998). The use of a small world network in such games is very intuitive, as it assumes that individuals have both local and some global connections. Although individuals have many more friends, the use of small world network is in line with the observation that groups of people tend to talk much more about information signals that they already share than individual-specific signals (Stasser, Taylor and Hanna 1989). However, the use of small world leads to herding and synchronous decisions, with the rate synchronization depending on the level of omniscience.

In all the models, the level of omniscience of individuals turned out to be very significant element of the decision-making, in a sense that more omniscient individuals get to the more synchronous decisions, despite the volatilities in the alternatives. On the other hand, non-omniscient individuals did not reach such synchronous behavior. Second feature of the papers gives note to the expected return of the alternatives where even short periods of time do matter. In all the cases, the presence of liquidity individuals proved to be of a decisive importance to the overall performance. Their role was especially significant with the omniscient individuals who very quickly react to the favorable conditions in the one short period of time, despite the chosen alternative be less favorable in the longer term. A consequence of such fast reaction is that they lose the information of other alternatives, which were less favorable in one short time period. Finally, the introduction of the Value-at-risk (VaR) as the reference for the decision-making demonstrated that individuals that take loss-aversion as a reference make quite different decisions from those individuals who are interested only in the returns of alternatives. To consider for VaR is to consider for an expected loss of a given portfolio due to changes in market prices of assets in the portfolio over a specified holding period with the given confidence level (Jorion 2006). It has been widely documented that in finance individuals consider risks more comprehensively (Kahneman and Tversky 1979; Thaler 1980; Tversky and Kahneman 1991; Hirshleifer 2001), and using VaR as a measure of risk is a step in that direction.

However, in my previous papers I did not allow for different types of individuals being present in the games at once, in a sense that one group would follow a returns-rule as a reference for the decision-making and the other the VaR-rule, but simulated games with either individuals pursuing returns-rule or the individuals pursuing VaR-rule. In this paper, I simulate the games with the presence of both groups at once. Individuals in the games still have four alternatives available and might opt either for pure strategies of having the portfolio only of stocks of CreditSuisse or Citigroup, or make a portfolio of the two.

The remainder of the paper is organized as follows. Section 2 defines the model, Section 3 the data, and Section 4 simulation results. A final section provides some conclusions.

2 The model

Assume there are $n = 1000$ agents, populated on the ring lattice to form a small world network (Watts and Strogatz 1998). On average, each individual is connected to six others.

The network, also called a graph, $g = (V, E)$ is a set of vertices $V = \{v_1, v_2, \ldots, v_{1000}\}$, representing agents, and edges $E = \{e_1, e_2, \ldots, e_{3000}\}$, representing their pairwise relations. If two agents in the network are connected, we denote $ij \in g$, whereas $ij \notin g$ represents two unconnected vertices. Using adjacency matrix, we can also put $ij = 1$ if $ij \in g$ representing two connected agents, and $ij = 0$ if $ij \notin g$ representing two unconnected agents in the network. Undirected network is used in the games, with edges being unordered pairs of vertices, thus if $ij = 1 \Leftrightarrow ji = 1$. In a small world network, people have many local and some global connections with others. We get such global connections by rewiring some of the local connections. In the model, agents are rewired with the probability $p = 0.01$.

Agents in the network are split upon the two different and independent factors; by their initial preferences over the type of a stock, and by the way, in which they make decisions.

As regards the first factor of initial preferences, agents are split into two groups with those in the first group preferring stocks of CreditSuisse. Their share is $0 \le u \le 1$, whereas the portion of $(1-u)$ agents in the second group prefer stocks of Citigroup. As in the previous papers, agents in both groups are allowed either to opt for a pure strategy, making a portfolio of only the stocks that one prefer, or make a portfolio out of the two stocks available. Agents who prefer stocks of CreditSuisse and make a portfolio only of the CreditSuisse stocks are denoted CS, and CSp if they opt for a portfolio of the two. Contrary, with C are denoted agents that prefer stocks of Citigroup and opt for a pure Citigroup portfolio, whereas they are denoted Cp if they opt for the portfolio of the two. In either case, portfolio is selected from the part of stocks one prefers, $0 \le pi \le 1$, and the remainder, $(1-pi)$, that represents the part of stocks in a portfolio of a non-preferred company.

As regards the second factor of the agents' heterogeneity, agents are again split into two groups, with those in the first group making decision upon the VaR and those in the second upon the return of the alternatives. In either case, agents have the knowledge of the value of VaR of their portfolio as well as the knowledge of the return of their portfolio. Therefore, agents that make decisions upon the VaR rule and are connected with the agents who decide upon the return of the alternatives are able to report them the return of their alternative if asked, and vice versa. I assume that agents numbered from $1 \le n \le 500$ pursue VaR rule and the remainder the return-rule. In either case, agents accumulate their wealth in time.

As in Steinbacher (2009b, c), so are also here liquidity agents present in the games. The characteristic of such agents is that they never change their initial strategies. In the games, I assume that $l = 0.1$ of the entire population are liquidity agents and distribute them into two islands in the network. In the first, liquidity agents are numbered $451 \le n \le 500$ and the second $951 \le n \le 1000$. In the games, liquidity agents are halved to the groups of VaR-rule and return-rule agents and pursue all the alternatives. To each of the liquidity agent, the strategy is set at the beginning of the game and they pursue it for the whole period.

Agents accumulate wealth in time according to the strategy they pursue, whereas they might choose an alternative from the following four available.

$$
\begin{aligned}
&W_{t+1}\left(A_C\right)=W_t\left(A_C\right)\cdot\left[1+Cr\right]\\
&W_{t+1}\left(A_{Cp}\right)=W_t\left(A_{Cp}\right)\cdot\left[1+Cr\cdot pi+CSr\cdot\left(1-pi\right)\right]\\
&W_{t+1}\left(A_{CSp}\right)=W_t\left(A_{CSp}\right)\cdot\left[1+CSr\cdot pi+Cr\cdot\left(1-pi\right)\right]\\
&W_{t+1}\left(A_{CS}\right)=W_t\left(A_{CS}\right)\cdot\left[1+CSr\right]
\end{aligned}
\tag{1}
$$

$W_{t+1}(\bullet)$ and $W_t(\bullet)$ represent wealth of an individual in time t and $t+1$, whereas $(\bullet)$ denotes the strategy played by an individual in time. Returns of stocks, denoted Cr and CSr, are exogenous to the individuals and they cannot foresee them, neither do they know the system of price changes in time. This is the application of an efficient market hypothesis as defined by the theory of stochastic finance (see Fama 1965).

Despite all the agents in the network accumulate their wealth in time as defined by (1), a portion of them consider risk more directly and decide upon the VaR. VaR is calculated dynamically in time as the time goes by. I assume that VaR-oriented agents opt for the alternative with the highest value of VaR. By doing that, they take the alternative with the smallest risk and the lowest expected loss of a portfolio.

The computation of VaR is very straightforward. In the model, I take the variance-covariance approach. Some other approaches of VaR computation are available in Jorion (2006). Upon Engle and Bollerslev (Engle 1982, Bollerslev 1986, Bollerslev et al. 1994), the new sets of ARCH and GARCH type models and their alternatives are used to estimate the variance in the financial time series to correct for the autoregressive heteroscedasticity in time.

The variance is computed on realized returns up to time t in time with $t=1,2,\ldots,2457$ and is updated as the time goes by. This means that as daily prices of the stocks change, so do also the mean return and the variance of both stocks and portfolio change daily. The calculation of portfolio standard deviations using a variance approach is defined as

$$
\sigma_t=\left[\frac{1}{t-1}\sum_{t=2}^{T}\left(x_t-\mu_t\right)^2\right]^{\frac{1}{2}}
\tag{2}
$$

where σ_t denotes the estimated standard deviation of a portfolio in time t, x_t indicates realizations of market changes of a portfolio in time and μ_t represents the mean value of outcomes of portfolio realized to the point in time.

With a given standard deviation, VaR is then computed as a corresponding percentile of the computed standard deviation. This is done by multiplying the estimated standard deviation of realized returns of both stocks with the number of standard deviations at the corresponding confidence level, i.e. -1.65 if estimating 95% VaR, and by -2.33 if estimating the value of a 99% VaR. In simulations below, I use 95% VaR on a daily basis.

The level of omniscience of agents is introduced into the model through the probability function and the coefficient κ as (Szabó and Tőke 1998)

$$\wp_{VaR} = \left[1+\exp\left[\left(VaR(A_i)-VaR(A_j)\right)/\kappa\right]\right]^{-1} \tag{3a}$$

In every time period t a VaR-rule agent A_i chooses one of the individuals to which he is directly connected, A_j, and compares his own value of VaR, $VaR(A_i)$, to the value of VaR of a selected individual, $VaR(A_j)$, as defined in (3a). This means that higher the value of κ, more non-omniscient the agents. In simulations, I use $\kappa = 1.0$ to define non-omniscient agents and $\kappa = 0.001$ for omniscient agents. If the chosen agent j is a return-rule individual, an agent i still compares the value of agent's j VaR to his value of VaR.

$$\wp_{W} = \left[1+\exp\left[\left(W(A_i)-W(A_j)\right)/\kappa\right]\right]^{-1} \tag{3b}$$

Return-rule agents compare their realized wealth $W(A_i)$ in time t with the realized wealth of a chosen agent j in time, thus $W(A_j)$ as defined in (3b). If the chosen agent j is a VaR-rule individual, an agent i still compares the value of agent's j realized wealth to his wealth. This is possible because irrespective of the mode one uses agents possess knowledge of their both, wealth and VaR.

3 Data

I take data from finance.yahoo.com portal and refer to adjusted closed prices of both stocks from 21.1.1999 until 19.11.2008. An adjusted closed price is a price adjusted for splits and dividends. In order to use the same time-period for both stocks, I omit adjusted closed prices for the stock for a time units if the other stock was inactive on that day. In the model, daily returns are used, calculated from the raw data of stock prices. I do no use logarithmical approximation of returns, but insert calculated returns into the code.

4 Simulation results

4.1 Game 1

In the first game, I simulate a game with omniscient endogenous agents with $\kappa = 0.001$ who are allowed to change their preferences over the stocks as the game proceeds. However, agents are not allowed to change their rule of making decisions. This means that VaR-rule agents follow this rule throughout the game, and so do return-rule agents pursue return-rule to the end of the game, as well. In the start of the game, $pi = 0.3$ and $u = 0.5$, meaning that

$C[0] = Cp[0] = CSp[0] = CS[0] = 0.25$. Agents numbered $1 \le n \le 500$ pursue VaR rule and the remainder the return-rule. Initial strategies are randomly set to the agents of both groups. pi is constant. VaR-rule agents pursue 95% VaR. Figure 1a and Figure 1b represent fractions of agents of both groups of agents that pursue each alternative in time.

Figure 1a: Fractions of VaR-rule agents pursuing each alternative

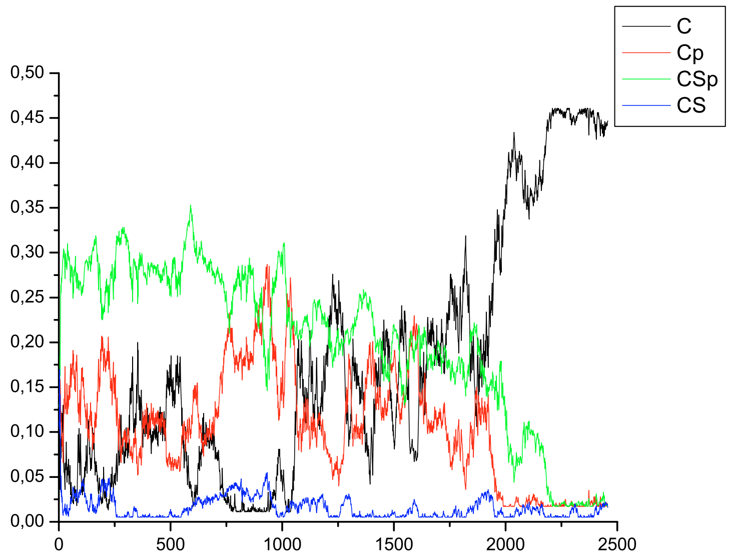


Figure 1b: Fractions of return-rule agents pursuing each alternative

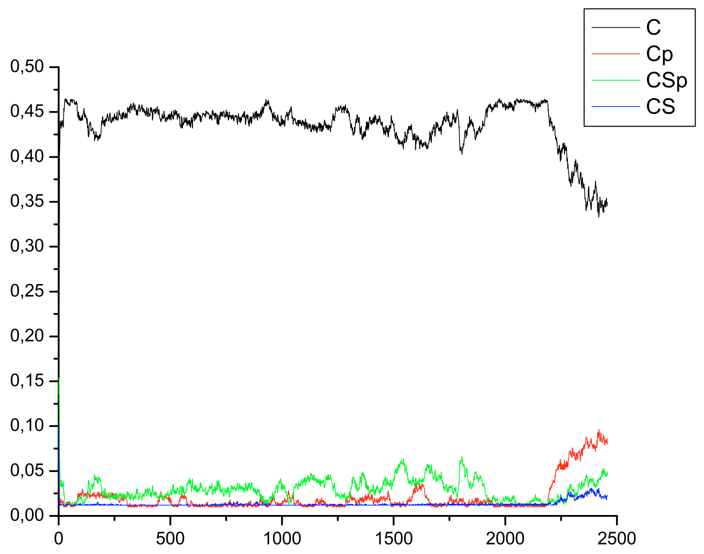
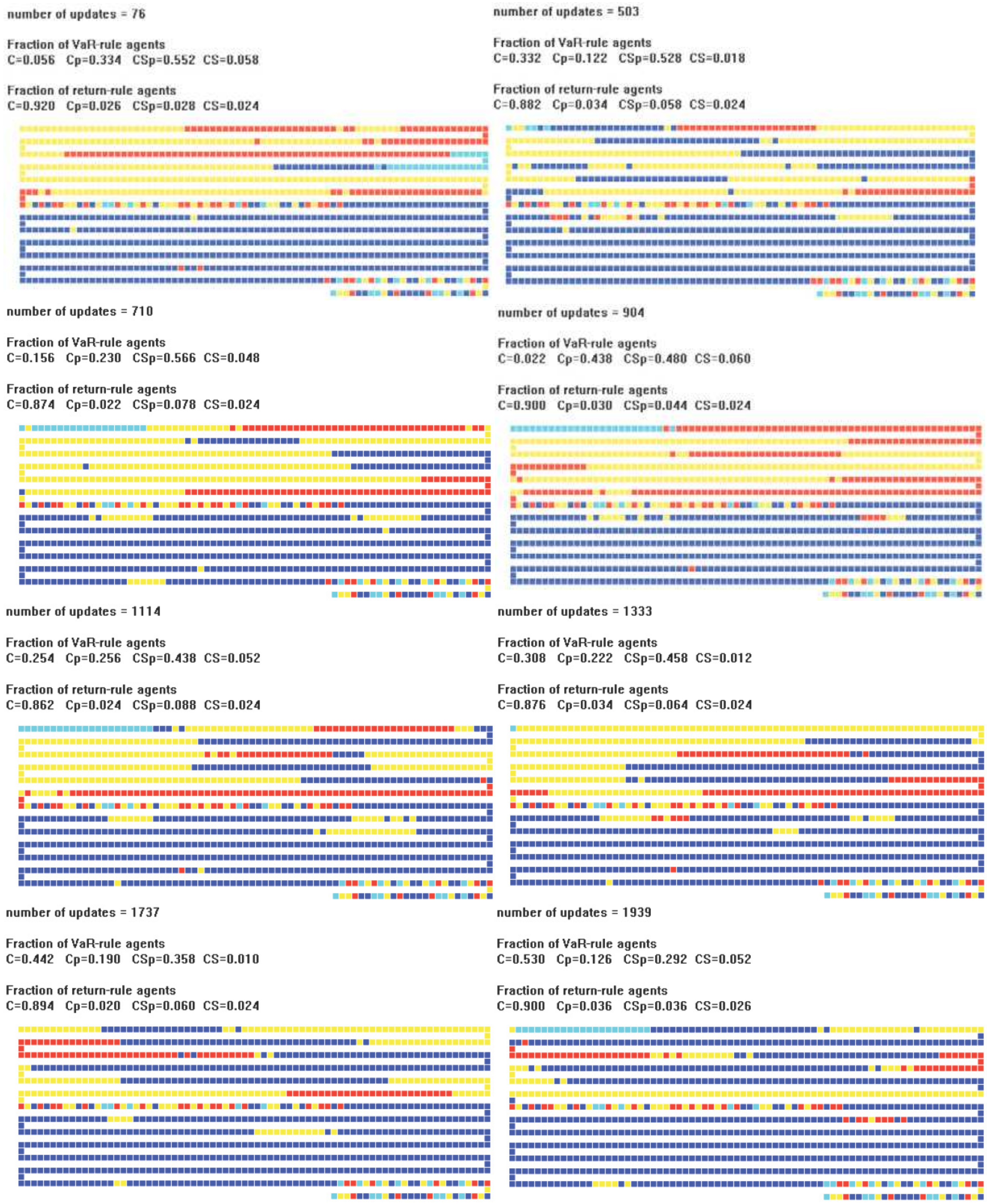
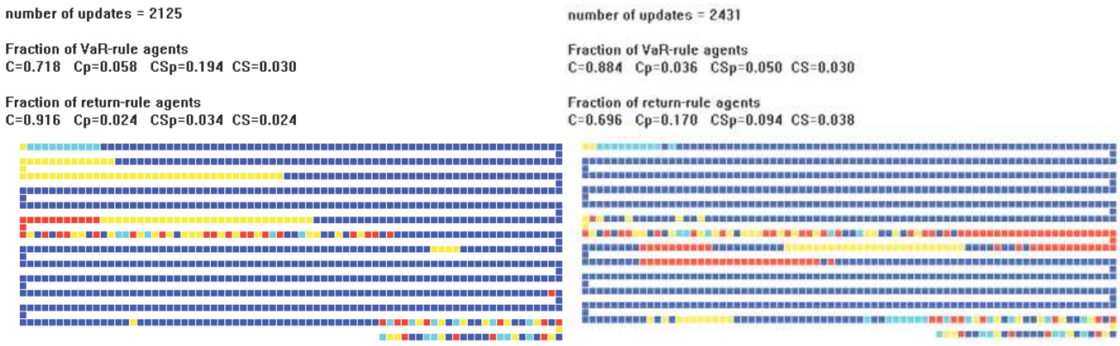


Figure 2 represents the course of the game at the selected time intervals of the game. Each figure represents the time, fraction of agents pursuing each strategy within the both groups and the colored blocks each representing individual agent in the game. As of the model, the first half is populated with the VaR-rule agents and the second half with the return-rule agents, with the liquidity agents scattered on two islands. Thus, the figures supplement the two figures of aggregate shares of agents pursuing each strategy (Figure 1a and Figure 1b).

Figure 2: Agents pursuing each alternative





Omniscient return-rule agents get to a unanimous decision quite soon in the game, contrary to the VaR rule agents who do not. In both cases, the role of liquidity agents is indispensable for the developments of the game as they keep the information of both pure strategies in the case of VaR-rule agents, whereas they prevent the alternative C to oust other alternatives of the game in the case of omniscient return-rule agents. Despite VaR-rule agents approach to both portfolios from the start of the game, they digress from them, with slowly approaching to the alternative C, the same as return-rule agents. In the last stage, the shares of VaR-rule agents pursuing each alternative within the entire VaR-rule group of agents is the following, $C = 0.894$, $Cp = 0.034$, $CSp = 0.04$ and $CS = 0.032$. In the last stage, the shares of return-rule agents pursuing each alternative within the entire return-rule group of agents is the following, $C = 0.698$, $Cp = 0.162$, $CSp = 0.093$ and $CS = 0.047$.

Figure 3 presents payoffs of selected agents in time as the game proceeds. To make comparison between the agents of different decision-making rules, I take two VaR-rule agents, $A[5]$ and $A[135]$, two return-rule agents, $A[678]$ and $A[850]$, and two liquidity agents, $A[475]$ and $A[978]$. It shall be noted that the selection of agents was arbitrary.

Figure 3: Payoffs of selected agents in time

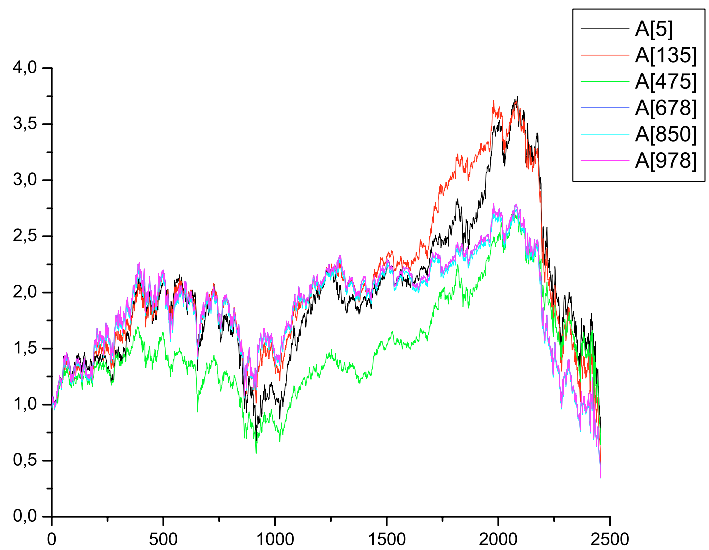


Figure 3 shows that VaR-rule agents not necessarily underperform return-rule agents. Even more, in all the cases of the agents from the figure, they outperform return-rule agents. Not only is this evident around the time interval $t = 2000$, but they outscore return-rule agents also in the final stage of the game, where agents attained the following wealth; $W(A[5]) = 0.7488$, $W(A[135]) = 0.4650$, $W(A[475]) = 0.6439$, $W(A[678]) = 0.3497$, $W(A[850]) = 0.3440$ and $W(A[978]) = 0.3497$. Results also demonstrate that none of the both liquidity agents performed worst.

4.2 Game 2

Now I change the level of omniscience and set $\kappa = 1.0$. All other characteristics of the game equal that of the Game one. Figure 4a and Figure 4b depict fractions of agents within both groups that pursue each alternative. Figure 5 depicts the course of the game on the basis of individual agents and at the selected time intervals of the game. Finally, Figure 6 presents payoffs of selected agents in time.

Figure 4a: Fractions of VaR-rule agents pursuing each alternative

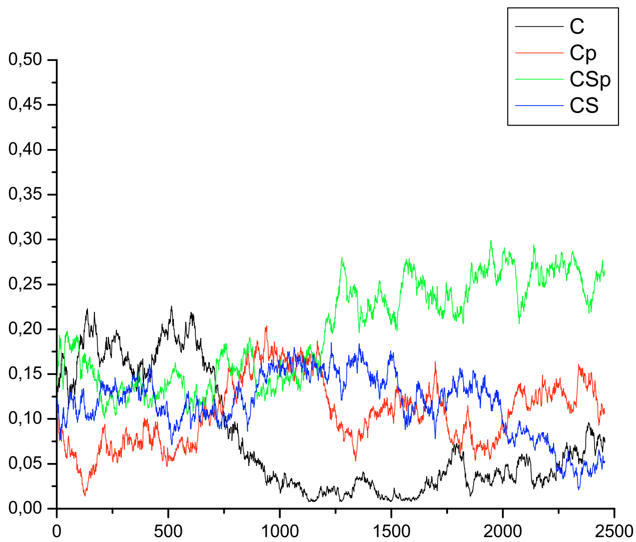


Figure 4b: Fractions of return-rule agents pursuing each alternative

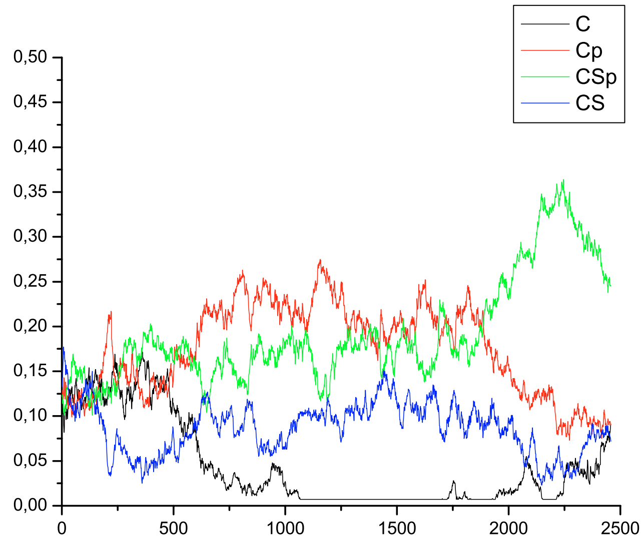
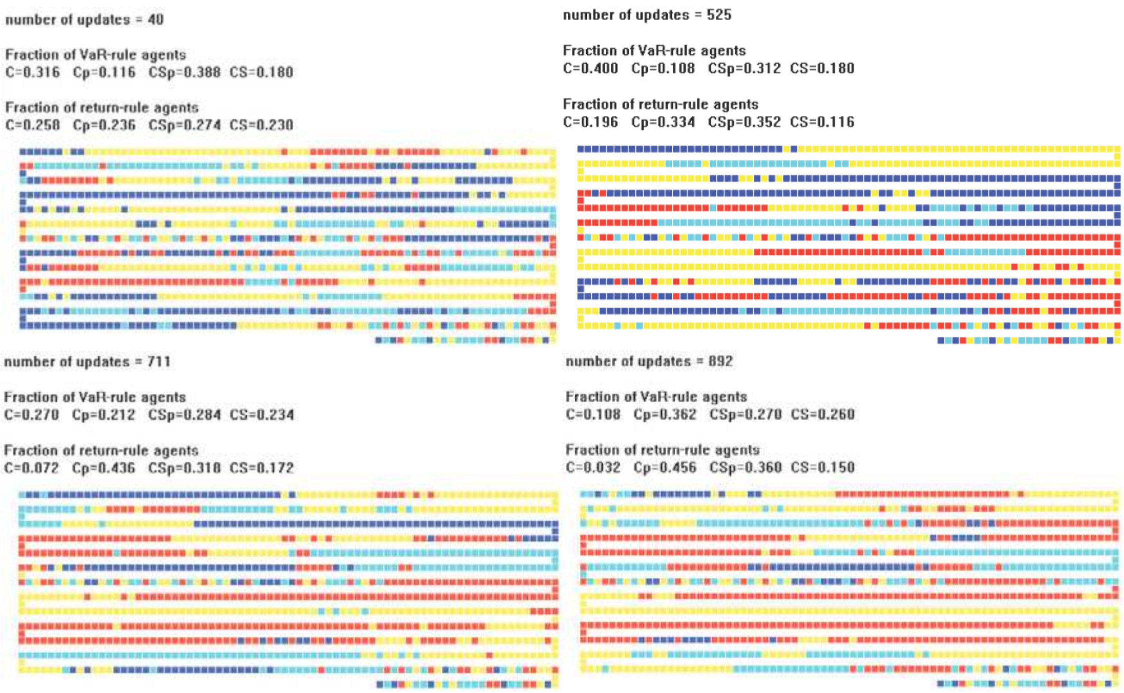
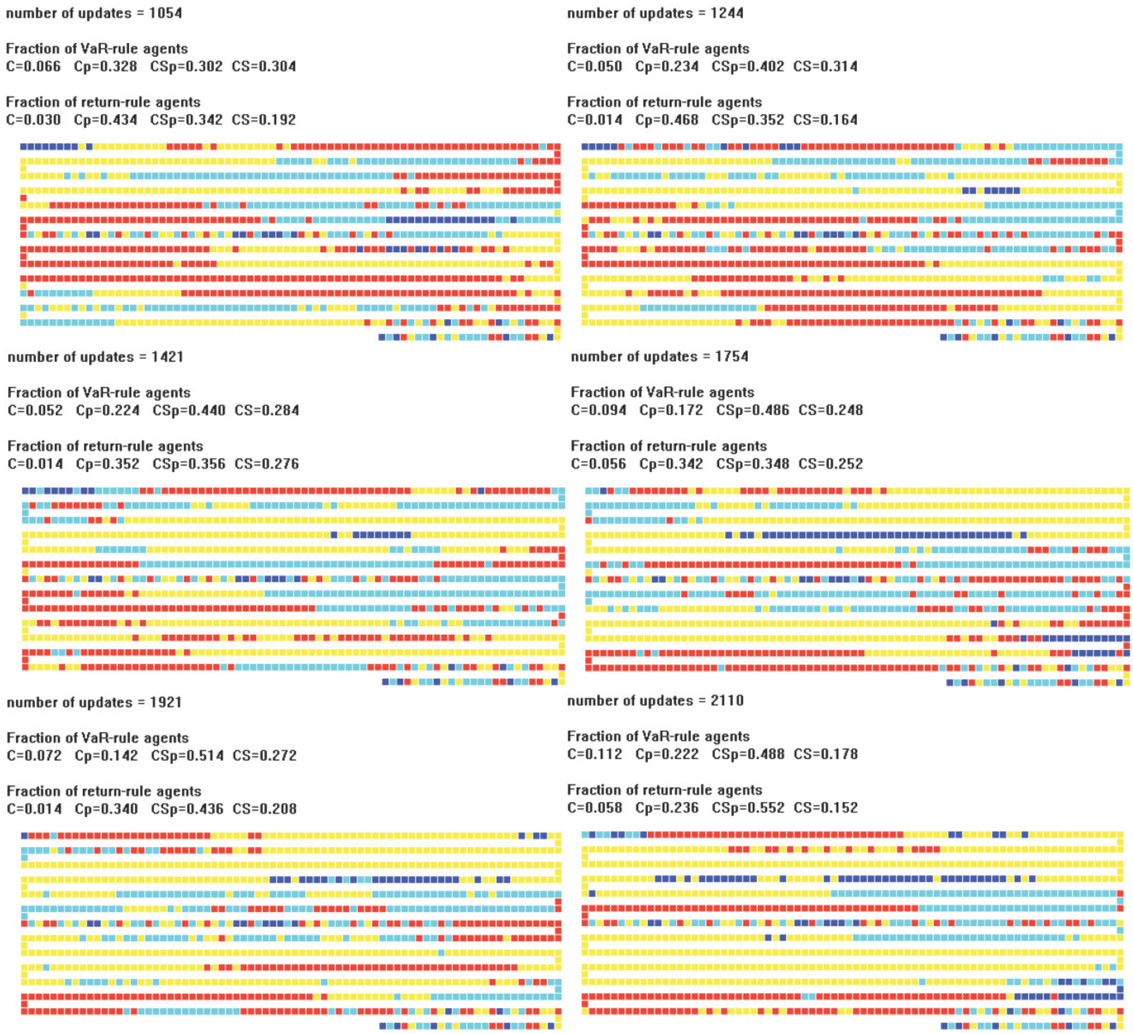


Figure 5 again represents the course of the game at the selected time intervals of the game. As of the model, the first half is populated with the VaR-rule agents and the second half with the return-rule agents, with the liquidity agents scattered on two islands.

Figure 5: Agents pursuing each alternative

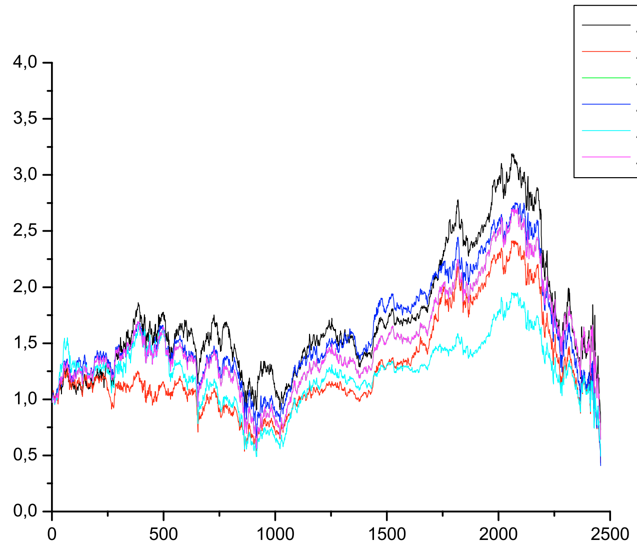




It can be seen from the figures that the behavior of non-omniscient agents is completely different from that of the omniscient. In the previous game, C was clearly a dominant strategy played by the omniscient return-rule agents, whereas both groups of non-omniscient keep it present only because of the liquidity agents. Non-omniscient agents do not reach the unanimous decision as fast as omniscient, or never reach it. In the last stage, the shares of VaR-rule agents pursuing each alternative within the entire VaR-rule group of agents is the following, $C = 0.148$, $Cp = 0.214$, $CSp = 0.532$ and $CS = 0.106$. In the last stage, the shares of return-rule agents pursuing each alternative within the entire return-rule group of agents is the following, $C = 0.182$, $Cp = 0.181$, $CSp = 0.493$ and $CS = 0.144$.

Finally, in the Figure 6, I demonstrate payoffs of selected agents in time. As in Figure 3, I take two VaR-rule agents, $A[5]$ and $A[135]$, two return-rule agents, $A[678]$ and $A[850]$, and two liquidity agents, $A[475]$ and $A[978]$. It shall be noted again that the selection of agents was arbitrary.

Figure 6: Payoffs of selected agents in time



As in the Figure 3, so does also non-omniscient VaR-rule agent A[5] outperforms others, finishing the game with the wealth of 0.7279. Other agents attained the following levels of wealth in the final stage, $W(A[135]) = 0.4343$, $W(A[475]) = 0.6439$, $W(A[678]) = 0.4092$, $W(A[850]) = 0.4918$ and $W(A[978]) = 0.6439$. Despite the highest level of wealth was attained by an omniscient agent, $A[5]$, non-omniscient agents used in the comparison also performed very well, beating all omniscient return-rule agents. When comparing the two games, non-omniscient return-rule agents performed a little better than the omniscient return-rule agents did. On the other hand, omniscient VaR-rule agents performed slightly better than non-omniscient VaR-rule agents did. Both liquidity agents once again performed pretty well.

5 Concluding remarks

In the paper, I simulated the game with a joint presence of 95% VaR-rule and return-rule groups of agents in the game. The paper has highlighted three features, one being the level of omniscience, next being the rule, which agents follow at the decision-making, and the third the presence of liquidity agents in the game.

It has been proved that omniscient and non-omniscient agents make quite different decisions, thus giving note to the cognitive analysis of the people's behavior. Game 1 demonstrated that the decision-making of omniscient VaR-rule agents varies to that of the return-rule agents, whereas non-omniscient agents behave much more alike.

Despite the highest level of wealth was reached by an omniscient agent, non-omniscient agents also performed very well, beating all omniscient return-rule agents. Surprisingly is that omniscient return-rule agents ended with the worst payoffs among the selected agents. When comparing the two games, non-omniscient return-rule agents performed a little better than the omniscient return-rule agents did. On the other hand, omniscient VaR-rule agents performed slightly better than non-omniscient VaR-rule agents did.

The presence of liquidity agents is very significant for the developments of the game. This is true for the VaR-rule agents as well as for the return-rule agents, and just the same for omniscient and for non-omniscient agents.

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