Conflict and Conflict Managment with Asymmetric Stakes (The Bad-Cop and the Good Cop part II)

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A Model of Conflict and Conflict Management
with Asymmetric Stakes

(The Good-Cop and the Bad-Cop Game, Part II.)

by

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A model of Conflict and Conflict Management with Asymmetric Stakes

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Abstract

This paper considers a partial equilibrium model of conflict where two asymmetric, rational and risk-neutral opponents evaluate differently the contested stake. Differently from common contest models, agents have the option of choosing a second instrument to affect the outcome of the conflict. The second instrument is assumed to capture positive investments in 'conflict management' - labelled as 'talks'. It will be demonstrated that, under some conditions, an asymmetry in the evaluation of the stake can lead to a concession from one agent to the other. In particular, the agent with the higher evaluation of the stake would make a concession, proportional to the optimal choice of 'talks'. The existence of a concession paves the way for establishing a feasible settlement region (FSR) given that both parties can be better off while expending resources in 'talks'. Eventually the basic model is extended in order to consider the existence of asymmetries in technological capabilities. Whenever the agents exhibit sufficiently asymmetric productive characteristics a FSR can be established even if a concession is not ensured. However a concession can enlarge the FSR. Finally, throughout the paper, the concept of entropy is applied as a tool for the measurement and evaluation of conflict and conflict management.

KEYWORDS: Conflict, Contest, Conflict management, Asymmetry in evaluation, Entropy, Returns to Scale, Concession, Guns, Talks.

JEL CODE: D7, D72, D74, D74, D82.

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A MODEL OF CONFLICT AND CONFLICT MANAGEMENT WITH ASYMMETRIC STAKES

(THE GOOD-COP AND THE BAD-COP GAME, PART II)

1. Basic Ideas and Cornerstones

Most conflicts involve remarkable bargaining efforts between the antagonists. Beyond violence, as applied when sending actual or potential threats, agents use to apply other instruments to successfully end any contest. During a war, for example, the exploitation of actual violence is often interlinked with diplomatic efforts. Diplomatic negotiations are often conducted while troops are deployed on the battlefield. In international interactions, the exploitation of potential or actual violence cannot be disentangled from partial openings and cooperative behaviours.

A story which immediately recalls this simple intuition is the story of *The Good-Cop and Bad-Cop Game*. Consider two cops arresting a suspect. Imagine also that they lack sufficient evidence to convict him. Then, they have to spend efforts in order to induce prisoner to confess. Next, as usually happens in American movies, in the questioning room cops have to play the good-cop and bad-cop game. The bad cop has to appear more aggressive, rude and less conciliatory. He would send exactly what students of strategy would define a 'credible threat'. On the other hand, the good cop has to appear less rude and more conciliatory expounding the advantages of confessing. The Cops' dilemma will be how much efforts in both behaviours should be spent. Of course, the outcome of questioning will depend upon the interdependent impact of complement instruments, rudeness and persuasion. On the other hand, the suspect has to choose whether to confess (cooperating through partial openings) or to stick to his presumption of innocence.

This story can be simply considered as a conflict. A conflict interaction involves interdependent decisions in the presence of coercion and anarchy. By coercion, I intend that kind of behaviour that is shaped and influenced by the existence of a threat. The importance of threat has been brilliantly expounded first by Thomas Schelling (1960/1966) and Kenneth Boulding (1963). The existence of a threat recalls the idea of deterrence and sheds light on a characteristic feature of conflict – namely, that while involved in a conflict the choices of an agent are choices made under coercion. Even though agents have options to make a choice, this is not purely voluntary.[1]

[1]Of course, if one had to develop this idea, it would be useful to single out exactly what is 'voluntary' and what is 'coercive', but this goes far beyond the aim of this paper. Basu (2006) deals with the distinction between Coercion and Voluntariness. To expound the concept, he provides a brilliant example drawn from his own life: [...] *In 1971 when I was a student in Delhi I was mugged at knife point one winter evening on the Delhi University campus. Three men in*

In recent economic literature, Jack Hirshleifer pioneered the work on modelling conflict, whose foundations are in Hirshleifer (1987a, 1988, 1989). The economic theory of conflict[2] rests to a large extent upon the assumption that agents involved in conflict interactions have to choose an optimal level of efforts or resources devoted to the unproductive activity of conflict. A significant element in economic theory of conflict is that investing resources in conflict is necessarily detrimental for welfare. This is central to the theory of conflict as well as to the theory of rent-seeking and contests. Given the partial-equilibrium framework adopted in this work, the analysis produced can be generalized to all these theoretical categories. In the spirit of the definition provided by Bhagwati (1982), who proposes a general taxonomy for a broader range of economic activities representing ways of making profit in spite of being directly unproductive, conflicts, contests and rent-seeking can be considered directly unproductive activities (DUP). According to this view, such activities yield pecuniary returns but do not produce goods and services which enter a utility function, either directly or indirectly through increased production or availability to the economy of goods that enter a utility function. This is the rationale behind the label of *directly unproductive profit-seeking activities* (DUP).

What is mainly outlined in recent literature is that while conflict models are usually general equilibrium models, rent-seeking, and contest models are partial equilibrium models. This means that conflict models should involve a trade-off between productive and unproductive activities and the prize (or the rent) of the contest is endogenous. The stake of the conflict is interpreted as a joint production which depends on the productive efforts of the agents. At the same time, the cost function is represented by the foregone production. In such a construction the greater the number of the agents, the greater the 'pie' to be split. In rent-seeking and contest models, the prize (or the rent) is given exogenously. In such a case, even if the number of contestants becomes larger, the rent does not change. Moreover, rent-seeking and contest models can involve

shawls came up to me on an ill-lit road, and one of them whipped out a knife and asked me for my watch. It took me a few seconds to decide what I should do. I took off my watch and handed it over to the man with the knife (somehow I seem to recall I thanked him) and walked back to my dorm. The question is: Did I part with my watch voluntarily or under coercion? Clearly, everybody will agree that this is coercion. If this is not coercion, then pretty much nothing is. But notice that this was not a situation of no choice. When the man pointed the knife at me and asked for my watch, he was giving me a choice: I could give him my watch or my life. I chose to keep my life. In fact, it was a bargain since mine was a cheap, unreliable watch. So having a choice cannot be equated with noncoercion.[...]

[2] In more recent years several studies extended Hirshleifer's basic model. See among others: Grossman (1991/1998), Skaperdas (1992), Garfinkel (1990/1994), Grossman and Kim (1995), Skaperdas and Syropoulos (1996), Neary (1997a), Anderton et al. (1999), Anderton (1999/2000), Garfinkel and Skaperdas (2000), Alesina and Spolaore (2003/2005), Dixit (2004), Spolaore (2004), Caruso (2006b). This growing literature has been recently surveyed in Garfinkel and Skaperdas (2006).

unconstrained optimization, whereas conflict models necessarily imply constrained optimization. Neary (1997b) and Hausken (2005) propose a comparison of conflict and contest models along these lines.

This paper presents a partial equilibrium model of conflict featuring two asymmetric, rational and risk-neutral opponents. It is intended to develop the literature on conflict by tackling three main points:

(i) the existence of a second type of effort (instrument) to win the conflict;
(ii) an asymmetry in the evaluation of the stake of the conflict;
(iii) the existence of a concession to favour an agreement between agents.

In particular note that the definition of conflict interaction given above has notable formal implications for this work.

First, as noted above, the existence of coercion shapes and influences agents' behaviour. This, clearly, marks a difference from rent-seeking and contests. Needless to say, in rent-seeking activities, an interest group can voluntarily choose whether or not to participate into the competition for public funds. In a sport contest – e.g. a race – an athlete can decide not to start. By contrast, a conflict interaction is not a voluntary choice. Agents have to participate into it and cannot give up. Of course, this assumption does exclude the possibility of escape. In formal terms, what is needed is an appropriate mathematical function which does not allow for zero efforts in conflict. This implies that equilibria where agents do not spend efforts and resources in conflict interaction will not be allowed. This assumption does constitute a precise choice and partly contrasts with the existing literature. The economic theory of conflict takes as a cornerstone the Contest Success Function (hereafter CSF for brevity). [3]The CSF is a mathematical relation that links the outcome of a contest and the efforts of the players. It summarises the relevant aspects of what Hirshleifer defines the *technology* of conflict. In particular, the outcome for one agent is decreasing in the efforts of other agents. There are two functional forms of CSF adopted in literature: the *ratio form* and the *logistic form*. Hirshleifer (1989), Baik (1998) and Anderton (2000) analyse the different impact of these two different functional forms for CSF. In the first case, the outcome depends upon the ratio of the efforts applied, whilst in the second case it depends upon the difference between the efforts committed. The main difference between the two functional forms of CSF becomes clear when one agent, say agent 1, puts zero in conflict effort.

In the simplest two-agents case, let $p_i\left(z_i, z_j\right)$ denote the probability of winning the contest (or alternatively the fraction of the stake) for agent

[3] Selective seminal contributions are by Tullock (1980), O'Keeffe et al. (1984) and Rosen (1986). Dixit (1987) develops a general framework for contests using the general properties of logit functions. See then Skaperdas (1996) and Clark and Riis (1998) for a basic axiomatization. See also Amegashie (2006).

i with $z_i, z_j, i = 1, 2, i \neq j$ indicating the efforts. The probability of winning of agent i is increasing in agent i's efforts and decreasing in the efforts of the other agent. The ratio form of the CSF implies that if one of the two contestants does not exert any positive efforts, the other grabs all the contested stake, namely $p_i(z_i, 0) = 1, \forall z_i \in (0, \infty)$. By contrast, using the logistic form, an agent committing zero effort can achieve some degree of success. If peace and cooperation have to be defined as the absence of violent efforts (with $z_i = z_j = 0$), peace can never occur as an equilibrium under the ratio form of CSF, because either agent would be likely to defect and invest any small positive magnitude in order to raise its fraction of the stake to 100%. Then, the choice of ratio form of the CSF is consistent with the assumption of coercion as a characteristic feature of conflict. Under coercion and credible threat one agent can choose the optimal level of conflict efforts but cannot give up its own irreversible investment. The existence of threat would not allow for the logistic form of CSF.

The second characteristic feature of conflict is anarchy. By 'anarchy' I simply mean the absence of rules, norms and institutions governing agents' behaviour. This implies that conflict can be managed and solved only in the presence of endogenous 'rules-of-the-game' governing the interaction. It will be shown that endogenous concessions can constitute an integrative mechanism leading to an equilibrium. This does not mean that a mediator, a court or an existing and legitimate institution cannot play any role in managing and solving conflicts. It means that in presence of anarchy only endogenous commitment are assumed to be credible and self-enforcing. Then, it also could favour a settlement. Broadly speaking, an unilateral concession will be considered as a self-enforcing constraint.

As presented above, the aim of this paper is that of studying a conflict between two risk-neutral agents that evaluate differently the stake and that can use different instruments in order to pursue their own maximum utility. Therefore, the outcome of the contest will arise from the interaction of such different instruments. In this view, the standard one-instrument models commonly adopted in literature can be considered as a special case of multi-instruments models.

Therefore, the limiting assumption of this paper is that once involved in conflict interactions, agents face the option of choosing also a second instrument in order to improve the outcome of the conflict. Thus, in the continuation of this work I will interpret the second instrument in a broad view. It is assumed to capture the vast majority of potential *conflict management efforts*. In reality, It can take different shapes. It can involve, among others, elements of communication, negotiation and signalling. Under the assumption of complete information, the second instrument must be perfectly observable. Thus to summarise,

(i) the use of a second instrument needs not to be "payoff-irrelevant": it must have a direct impact on both agents' payoffs;

(ii) the second instrument must also be costly. There is no room for *cheap talk*. In fact, what is needed is a "credibility-cost". Under the assumption of complete information, an observable costly effort is also assumed to be credible;

(iii) investment in conflict management must be irrevocable;

(iv) the two instruments must be complements.

In the theory of contest the use of a second instrument is not a novelty, although such approach has not been developed extensively.[4] In particular, this paper is close to a model proposed by Epstein and Hefeker (2003), who model a contest where, the use of a second indtrument creates an advantage for the player with the higher stake.

It must be underlined that the two instruments are intended to be complements. In fact, it is reasonable to assume that the outcome of a conflict depends upon the mixed effect of violence *and* negotiation. This means that opponents do no give up their willingness to pursue the maximum possible payoff. This is in line with the approach presented by Genicot and Skaperdas (2002), who present a general equilibrium model of conflict with investment in conflict management.

Henceforth, for expository convenience, in the continuation of the work I shall refer to the second instrument as "*talks*" whereas the first instrument will be indicated through the traditional '"*guns*".[5]

Eventually, another goal of this paper will be represented by the identification of a *Feasible Settlement Region* (henceforth FSR for sake of brevity) as the set of possible peaceful agreements. The limiting hypothesis is that a settlement region is feasible if and only if both agents choose to expend positive efforts in the second instrument, namely in "talks". At the same time, a FSR must be an incentive-compatible structure. Both agents have to be better off under a FSR. To summarise formally, henceforth let

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[4] Baik and Shogran (1995) study a contest between players with unknown relative ability. Under the assumption of decreasing aversion to uncertain ability, agents are allowed to expend resources in order to reduce such uncertainty through spying. Konrad (2003) enriches a model of rent-seeking considering the interaction between two types of efforts: (i) the standard rent-seeking efforts to improve their own performance in the view of winning a prize; (ii) a sabotaging effort in order to reduce the effectiveness of other agents' efforts. In this model, sabotage is targeted towards a particular rival group and reduces this group's performance. The point of interest is that through sabotage a group can increase its own probability of winning the prize as well as the other contestants'. Thus, the model predicts that sabotage disappears whenever the number of contestants becomes large. Caruso (2005b) presents two different models of contest with two instruments. The analysis is applied to sport contests in order to consider the phenomena of match-fixing and doping. Arbatskaya and Mialon (2005) analyse in depth the equilibrium properties of a two-instruments contest model and compare the results to those attainable in standard one-instrument models.

[5] Of course, being in a partial equilibrium framework the classical tradeoff between 'butter' and "guns" is not considered.

$h_i, i = 1,2$ denote the investments in 'talks' and let $\pi_i, i = 1,2$ denote the payoffs accruing to the agents. Then, a FSR can be defined as:

$$FSR \equiv \left\{ \left(\gamma_i, \gamma_j\right) \in \square ; \left(h_i, h_j\right) \in \square : \gamma_i > 0, \gamma_j > 0, h_i > 0, h_j > 0, i = 1,2, i \neq j \right\}$$

where $\gamma_i = \pi_i^a - \pi_i^b$ captures the positive difference between payoffs attainable in scenario A and in scenario B respectively. Of course, whenever $\pi_i^a > \pi_i^b$ agent i would prefer scenario A to scenario B. Broadly interpreted, a FSR can be considered as a bargaining space.

The remainder of the paper is organised as follows: in the first part, a basic partial equilibrium model of conflict is presented. In the second part, a basic model allowing for the second instrument is presented. The third part is focused on the issue of measurement. In particular, entropy is presented as an alternative tool to measure conflict and conflict management. In the following sections, a first extension will deal with (a) the existence of an unilateral concession leading to the establishing of a FSR; (b) the existence of an asymmetry in productive characteristics and its impact on the establishing a FSR. Finally, the last section summarises the results and provides suggestions for future research.

THE PURE CONFLICT MODEL

Consider two risk-neutral agents, indexed by $i = 1,2$, that are identical in abilities, but at the same time they have different evaluations of the stake in the conflict. Then, Let $x_i \in (0,\infty), i = 1,2$ denote the stake of the conflict. Given the asymmetry in evaluation, it would be possible to write that $x_1 \neq x_2$ where the subscripts indicate the evaluation of agent 1 and agent 2 respectively. In particular, hereafter assume that agent 1 has a higher evaluation than agent 2, namely $x_1 > x_2$. Let $\delta \in (0,1)$ denote the degree of asymmetry between the stakes of the two agents, namely $\exists \delta \in (0,1)\, s.t. x_2 = \delta x_1$. For sake of notational simplicity, throughout the paper I shall use agent 1's evaluation as a kind of numeraire and it will be simply denoted by x. There is common knowledge about such hypotheses.

Under the assumption of risk-neutrality, agents interpret the outcome of the non-cooperative interaction as deterministic. That is, given the assumption of risk-neutrality, agents are indifferent between conflict and sharing a stake in accordance with the winning probabilities. Let $z_i \in (0,\infty), i = 1,2$ denote the positive amount of 'guns' and $h_i \in [0,\infty), i = 1,2$ denote 'talks'.

As noted above, a partial equilibrium model of conflict with an exogenous prize is not technically distinguishable from the standard rent-seeking model. The outcome of the conflict is determined through a CSF. The *ratio* form of the CSF used here is:

$$p_i = \frac{z_i}{z_i + z_j} \qquad \text{for } i = 1,2 \text{ and } j \neq i \tag{1}$$

Equation (1) is differentiable and follows the conditions below:

$$\begin{cases} p_1 + p_2 = 1 \\ \partial p_i / \partial z_i > 0 \quad \partial p_i / \partial z_j > 0 \\ \partial^2 p_i / \partial^2 z_i < 0 \quad \partial^2 p_i / \partial^2 z_j > 0 \\ \partial^3 p_i / \partial^3 z_i > 0 \quad \partial^3 p_i / \partial^3 z_j < 0 \end{cases} \tag{1.1}$$

The functional form adopted in equation (1) implies that the conflict is not decisive, namely there is no preponderance of an agent over the other. This is of course a limiting assumption, even if many conflicts fall in this category. Under the assumption of risk-neutrality the outcome of the CSF denotes the proportion of appropriation going to agent i for $i = 1,2$. Eventually, the payoff function is given by:

$$\pi_i^{pc}\left(z_i, z_j\right) = \frac{z_i}{z_i + z_j} x_i - z_i, i = 1,2, i \neq j \quad (2)$$

Applying a maximization[6] process, using $x_2 = \delta x_1$ and suppressing subscripts for notational simplicity, the equilibrium choices of 'guns' (denoted by stars superscripted) are given by:

$$z_1^{pc*} = \frac{\delta}{\left(\delta+1\right)^2} x; z_2^{pc*} = \frac{\delta^2}{\left(\delta+1\right)^2} x \quad (3)$$

It is clear that $z_1^{pc*} > z_2^{pc*}$ and also that $\partial z_i^{pc*} / \partial x > 0, \partial z_i^{pc*} / \partial \delta > 0, i = 1,2$. Note also that $\partial^2 z_1^{pc*} / \partial^2 \delta < 0,$ and $\partial^2 z_2^{pc*} / \partial^2 \delta > 0 \Leftrightarrow \delta < 1/2$.
Eventually the payoffs are given by:

$$\pi_1^{pc*} = \frac{1}{\left(\delta+1\right)^2} x; \pi_2^{pc*} = \frac{\delta^3}{\left(\delta+1\right)^2} x. \quad (4)$$

Simple to verify that $\pi_1^{pc*} > \pi_2^{pc*}, \partial \pi_i^{pc*} / \partial x > 0, i = 1,2$. Note also that $\partial \pi_1^{pc*} / \partial \delta < 0, \partial^2 \pi_1^{pc*} / \partial^2 \delta > 0$ and $\partial \pi_2^{pc*} / \partial \delta > 0; \partial^2 \pi_2^{pc*} / \partial^2 \delta > 0$.

To sum up, both agents expend positive investments in 'guns' which are increasing in the evaluation of the stake. They both get positive payoffs and agent 1, namely the agent with a higher evaluation of the stake, is capable of getting a higher payoff by means of a higher level of 'guns'.

Figure 1 shows the relationship between the payoffs of both agents and the degree of asymmetry in the evaluation of the stake (with an arbitrary value $x = 100$ attached to the agent 1's evaluation of the stake). As the asymmetry in evaluation decreases, the difference between the attainable payoffs decreases as well.

[6] The First Order Conditions and the Second Order Conditions are given by

$$\frac{\partial \pi_i^{pc}}{\partial z_i} = \frac{z_j}{\left(z_i + z_j\right)^2} x_i - 1 = 0, \frac{\partial^2 \pi_i^{pc}}{\partial^2 z_i} = -\frac{2 z_j x_i}{\left(z_i + z_j\right)^3} < 0, i = 1,2, i \neq j$$

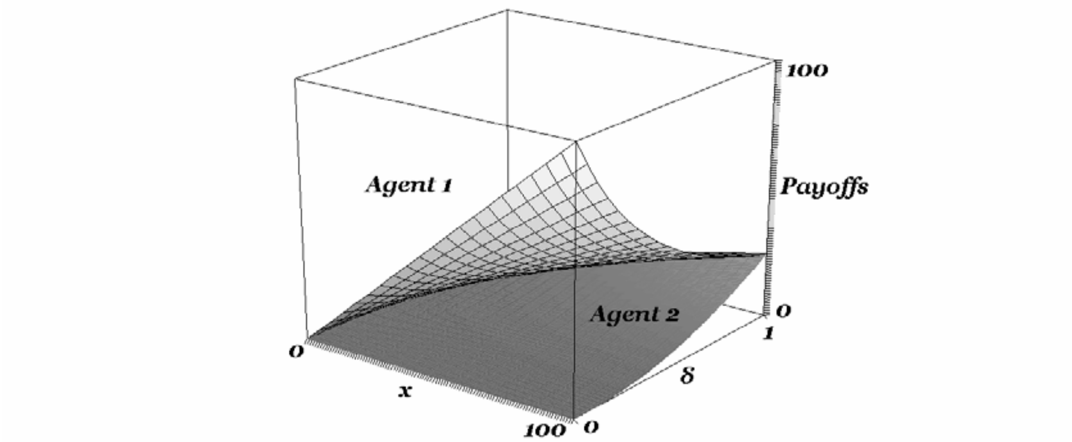

FIGURE 1. – PAYOFFS IN PURE CONFLICT

INVESTING IN CONFLICT MANAGEMENT

Consider now the option of a second instrument. Agents commit themselves to the use of a second instrument in order to affect the outcome of the contest. As mentioned above, the basic model presented hereafter follows and partly modifies the one proposed in Epstein and Hefeker (2003). The ordinary CSF is modified in order to allow for a second instrument. The two instruments are assumed to be complementary to each other. Then, the use of the second instrument would strengthen the effect of the first instrument. Eventually the CSF becomes:

$$p_i^{cm} = \frac{z_i\left(h_i+1\right)}{z_i\left(h_i+1\right)+z_j\left(h_j+1\right)}, i=1,2 i \neq j. \tag{5}$$

and follows the conditions below:

$$\begin{cases} \partial p_i^{cm}/\partial z_i > 0 & \partial^2 p_i^{cm}/\partial z_i < 0 & \partial^3 p_i^{cm}/\partial z_i > 0 \\ \partial p_i^{cm}/\partial z_j < 0 & \partial^2 p_i^{cm}/\partial z_j > 0 & \partial^3 p_i^{cm}/\partial z_j < 0 \\ \partial p_i^{cm}/\partial h_i > 0 & \partial^2 p_i^{cm}/\partial h_i < 0 & \partial^3 p_i^{cm}/\partial h_i > 0 \\ \partial p_i^{cm}/\partial h_j < 0 & \partial^2 p_i^{cm}/\partial h_j > 0 & \partial^3 p_i^{cm}/\partial h_j < 0 \end{cases} \tag{5.1}$$

and the other cross-partial derivatives with respect to x_i are given by:

$$\begin{cases} \dfrac{\partial^2 p_i^{cm}}{\partial z_i \partial h_i} > 0 & \Leftrightarrow \quad h_i z_i - h_j z_j + z_i - z_j < 0 \\ \dfrac{\partial^2 p_i^{cm}}{\partial z_i \partial z_j} > 0 & \Leftrightarrow \quad h_i z_i - h_j z_j + z_i - z_j > 0 \\ \dfrac{\partial^2 p_i^{cm}}{\partial z_i \partial h_j} > 0 & \Leftrightarrow \quad h_i z_i - h_j z_j + z_i - z_j > 0 \end{cases} \tag{5.2}$$

Eventually, assuming linear cost functions for both instruments the payoff function for each agent become:

$$\pi_i^{cm} = \frac{z_i\left(h_i+1\right)}{z_1\left(h_1+1\right)+z_2\left(h_2+1\right)} x_i - z_i - h_i, i = 1,2. \tag{6}$$

Also in this case, a Nash-Cournot behaviour for both agents is assumed. Therefore, each agent maximizes[7] its own payoff. The optimal choices of 'guns' and 'talks' are given by:

$$\begin{cases} z_1^{cm*} = \dfrac{\delta^2}{\left(\delta^2+1\right)^2} x & h_1^* = \dfrac{\delta^2}{\left(\delta^2+1\right)^2} x - 1 \\ \\ z_2^{cm*} = \dfrac{\delta^3}{\left(\delta^2+1\right)^2} x & h_2^* = \dfrac{\delta^3}{\left(\delta^2+1\right)^2} x - 1 \end{cases} \tag{7}$$

First, note that the agent with a higher evaluation of the stake arms more than the opponent ($z_1^{cm*} > z_2^{cm*}$). Moreover, it is also clear that $z_i^{cm*} > 0, \partial z_i^{cm*} / \partial x > 0, \partial z_1^{cm*} / \partial \delta > 0; i = 1,2$. That is, irreversible investments in 'guns' of both agents are increasing in the value of the stake. At the same time, 'guns' are both decreasing in the asymmetry of evaluation, namely as the evaluations of the stake converge agents arm more. Whenever the degree of asymmetry is close to zero, the level of 'guns' is almost equal.

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[7] The first order conditions for maximization are:

$$\frac{\partial \pi_i^{cm}}{\partial z_i} = \frac{z_j\left(h_i+1\right)\left(h_j+1\right)}{\left(h_i z_i + h_j z_j + z_i + z_j\right)^2} x - 1 = 0$$

$$\frac{\partial \pi_i^{cm}}{\partial h_i} = \frac{z_i z_j\left(h_j+1\right)}{\left(h_i z_i + h_j z_j + z_i + z_j\right)} x - 1 = 0$$

$i = 1,2, i \neq j$

What about the optimal level of 'talks'? First note that:

$$h_1^* > 0 \Leftrightarrow x > \frac{\left(\delta^2+1\right)^2}{\delta^2}; h_2^* > 0 \Leftrightarrow x > \frac{\left(\delta^2+1\right)^2}{\delta^3} \qquad (8)$$

That is, in order to have a positive investment in 'talks' the value of the stake must be relatively large. Secondly, it is clear that agent 2 has a narrower range allowing for positive values of h. For example, for $x = 100$ it is simple to verify that $h_1^* > 0 \Leftrightarrow \delta > .1$ whereas $h_2^* > 0 \Leftrightarrow \delta > .22$. Moreover, it is clear that $\partial h_i^* / \partial x > 0; \partial h_i^* / \partial \delta > 0, i = 1,2$. Note in particular, that 'talks' are increasing in the degree of asymmetry. Another point of interest is that the difference of both instruments exactly equals. That is, in formal terms, $z_1^{cm*} - z_2^{cm*} = h_1^* - h_2^*$.

Eventually, in order to verify whether the critical points $\left(z_1^{cm*}, z_2^{cm*}, h_1^*, h_2^*\right)$ represent a global maximum, it is possible to consider the Hessian matrices for both agents. In the appendix are reported the results. The analysis shows that $\left(z_1^{cm*}, z_2^{cm*}, h_1^*, h_2^*\right)$ does constitute only a local max.

The payoffs in equilibrium are given by:

$$\pi_1^{cm*} = \frac{x\left(1-\delta^2\right)}{\left(\delta^2+1\right)^2} + 1 \qquad (9.1)$$

$$\pi_2^{cm*} = \frac{x\delta^3\left(\delta^2-1\right)}{\left(\delta^2+1\right)^2} + 1 \qquad (9.2)$$

Given $\delta \in (0,1)$, it would be simple to verify that $\pi_1^{cm*} > \pi_2^{cm*}$. Note also that $\partial \pi_1^{cm*} / \partial \delta < 0$ whilst $\partial \pi_2^{cm*} / \partial \delta > 0 \Leftrightarrow \delta > 0.68$. Moreover, consider that agent 1's payoff is unambiguously larger than zero, ($\pi_1^{cm*} > 0$), whereas $\pi_2^{cm*} > 0 \Leftrightarrow \delta^5 x + \delta^4 - \delta^3 x + 2\delta^2 + 1 > 0$. Figure 2 clearly shows that there is a large range where agent 2's payoffs turn negative.

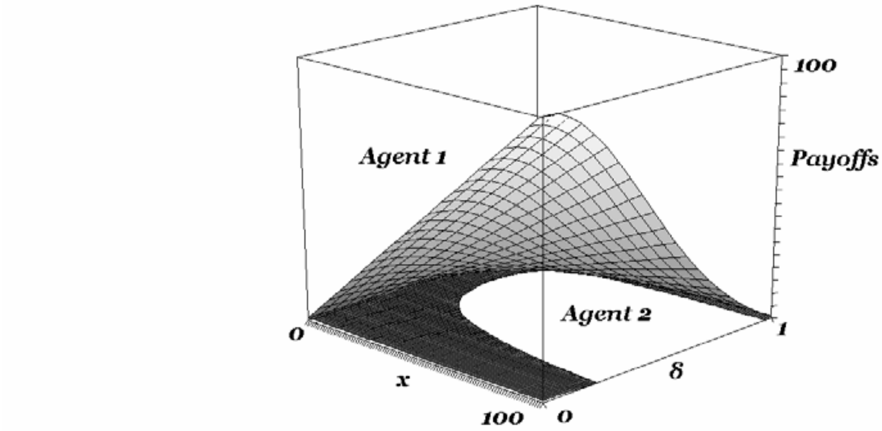


FIGURE 2. – PAYOFFS IN CONFLICT MANAGEMENT

In order to illustrate the impact of the second instrument on agents' behaviour I apply a numerical example with an arbitrary value attached to the stake of the conflict, say $x = 100$. Moreover, it is possible now to analyse by means of a standard comparative statics what would be the scenario chosen by agents comparing the attainable payoffs under pure conflict and under conflict management. The results, as presented in Table 1, show that there is no room for a FSR. Agents will prefer pure conflict to conflict management. However, analysing the figures, some differences emerge that are worth mentioning:

First, there is a range that I would define *Conflict Trap*. In such a situation no agent is willing to invest resources in 'talks', namely $h_1^* < 0; h_2^* < 0$. This emerges when the asymmetry in the evaluation of the stake is extremely large. Consider briefly that for $x = 100$ no agent is going to invest in 'talks' when $0 < \delta < 0.1$. As previously indicated by inequalities (8) in order to have positive investments in 'talks' the value of the stake must be relatively large. The intuition behind is clear. Given the assumption of common knowledge, agent 1 is aware that agent 2 is not going to invest a massive amount of resources neither in 'guns' nor in 'talks'.

A second remarkable point refers to an asymmetry amongst the choices of agents. In particular agent 1 is willing to invest resources ($h_1^* > 0$) in the second instrument to manage the conflict. Instead agent 2 is not going to invest in talks, namely $h_2^* < 0$. This result also appears to be driven mainly by the evaluation of the stake. Given the asymmetry in the evaluation, agent 2 has clearly less incentives in expending positive investmens in 'talks'.

A third point highlights positive investments of both agents in 'talks' but they are not able to attain higher payoffs than the pure conflict scenario. This could be defined as an *Unfeasible Settlement Region (USR).* To build an intuition upon this result, consider that investing in both instruments is costly. Then, even if agents have an incentive given by the value of the stake, the benefits of 'talks' do not outweigh the costs.

TABLE 1. EFFORTS AND PAYOFFS

x	δ	z_1^{cm*}	z_2^{cm*}	h_1^*	h_2^*	π_1^{pc*}	π_2^{pc*}	π_1^{cm*}	π_2^{cm*}	$\pi_1^{cm*}-\pi_1^{pc*}$	$\pi_2^{cm*}-\pi_2^{pc*}$
100	0.05	0.25	0.01	-0.75	-0.99	90.70	0.01	100.25	0.99	9.55	0.98
100	0.15	2.15	0.32	1.15	-0.68	75.61	0.26	94.50	0.68	18.88	0.43
100	0.25	5.54	1.38	4.54	0.38	64.00	1.00	84.04	-0.30	20.04	-1.30
100	0.35	9.72	3.40	8.72	2.40	54.87	2.35	70.64	-1.99	15.77	-4.34
100	0.45	14.00	6.30	13.00	5.30	47.56	4.33	56.15	-4.03	8.59	-8.36
100	0.55	17.83	9.81	16.83	8.81	41.62	6.93	42.11	-5.84	0.49	-12.77
100	0.65	20.88	13.57	19.88	12.57	36.73	10.09	29.54	-6.84	-7.19	-16.92
100	0.75	23.04	17.28	22.04	16.28	32.65	13.78	18.92	-6.56	-13.73	-20.34
100	0.85	24.35	20.70	23.35	19.70	29.22	17.94	10.35	-4.74	-18.87	-22.69
100	0.95	24.93	23.69	23.93	22.69	26.30	22.55	3.69	-1.31	-22.60	-23.86

To sum up it is possible to write:

PROPOSITION 1: *When agents are asymmetric in their evaluation of the stake and identical in fighting abilities: (i) they prefer a pure conflict scenario; (ii) when the asymmetry in the evaluation of the stake is extremely large a conflict trap emerges.*

MEASURING CONFLICT

Conflict is susceptible to measurement. In the standard partial equilibrium contest theory the resources expended do constitute the social cost of contest. In rent-seeking literature it is defined as the Rent Dissipation. Then, recall the optimal choices of violent efforts. It would be possible to write that the total cost under pure conflict is given by:

$$TC^{pc} = z_1^{pc*} + z_2^{pc*} = \frac{\delta}{(\delta+1)} x \tag{10}$$

Recalling (7) the total cost of contest when both agents expend efforts in a second instrument is given by:

$$TC^{cm} = z_1^{cm*} + h_1^* + z_2^{cm*} + h_2^* = 2\left(x\frac{\delta^2(\delta+1)}{(\delta^2+1)^2} - 1 \right) \tag{11}$$

Define $M = \left(\delta^2 \left(\delta+1\right)\right) / \left(\delta^2+1\right)^2$ for compactness. Then, it is possible to write that $TC^{cm} > TC^{pc}$ for $x > \left(2/M\right)$.

However, as noted by Epstein and Hefeker (2003) since agents evaluate the stake differently it is necessary to look at the relative rent dissipation (RRD). It is defined as follows:

$$RRD = \frac{TC}{p_1^* x + p_2^* \delta x} \tag{12}$$

Then in case of pure conflict the RRD is given by:

$$RRD^{pc} = \frac{\delta}{\delta^2+1} \tag{13}$$

while in the case of CM would be given by:

$$RRD^{cm} = 2\left[\frac{\delta^2\left(\delta+1\right)}{\left(\delta^2+1\right)\left(\delta^3+1\right)} - \frac{\left(\delta^2+1\right)}{x\left(\delta^3+1\right)}\right] \tag{14}$$

Then it is possible to compare (13) and (14). The plot below scales the value of the stake against the level of asymmetry in evaluation.

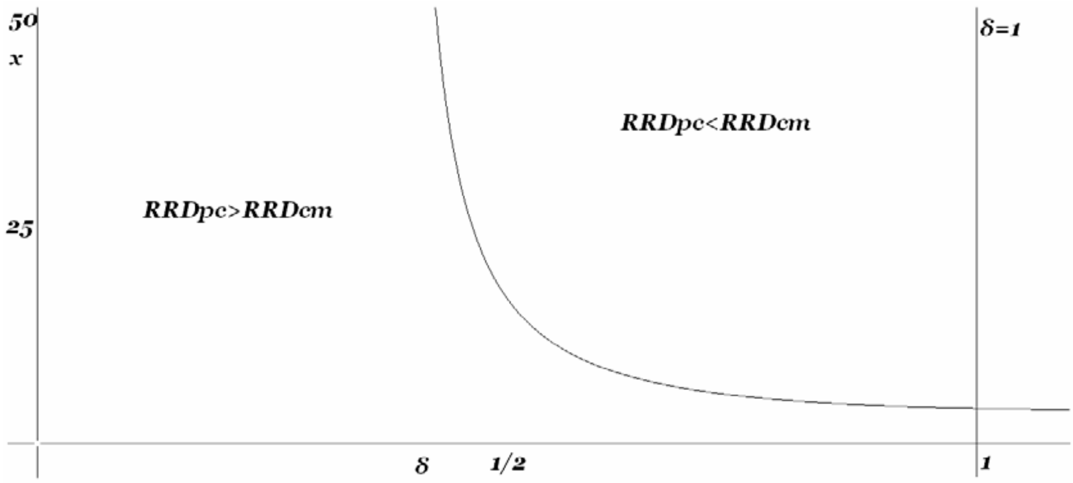


FIGURE 3. – RELATIVE RENT DISSIPATION IN PURE CONFLICT AND CONFLICT MANAGEMENT

Therefore, analysing both RD and RRD it appears clear that a conflict with two instruments would be more detrimental for welfare and less efficient from an economic point of view than a conflict with only one instrument. Hence, establishing a FSR would be welfare-immiserizing. In such a narrow sense, however, a pure conflict scenario would be paradoxically preferable, given that establishing a FSR would be less efficient than pure conflict. Of course, this kind of conclusion would be sensitive to the modelling adopted but such a 'positive' impact would not be theoretically excluded from the start.

However, it is clear that such a measurement could be unsatisfactory to analyse the realm of conflicts. If efficiency were a criterium for policy decision no conflict would emerge. As a result, further analysis is necessary. It would also be reasonable to identify a complementary measure for conflict and conflict management. An appealing idea for a more useful evaluation can be related to those of disorder and randomness. In fact, since conflict is a destructive interaction between two or more parties, it seems reasonable to consider also the degree of uncertainty it spreads. In actual violent appropriative conflicts, uncertainty about the final outcome does clearly constitute a characteristic element that should be considered in developing devices to solve the conflict itself.

The measure of uncertainty as the degree of disorder can be captured through the concept of $entropy$. In communication theory and physical sciences, entropy is commonly adopted as a measure of the degree of disorder, uncertainty or randomness in a system.[8] The famous reference is the work of Shannon and Weaver (1949), which posed the quantitative foundations of information theory. In such a framework, entropy is defined as:

$$E(p_1,\ldots,p_n) = -k\sum_{i=1}^{n} p_i \ln p_i, \tag{15}$$

where k is an arbitrary constant which can be set to unity without loss of generality.[9] Note that, following the prevailing literature, p_i can be interpreted in two different ways. First, it can represent a probability. Secondly, it can represent a share of some total quantity. Then, this flexible interpretation does fit well with the assumption of risk-neutrality and the following properties of the CSF.

The greatest disorder would occur when all outcomes have the same probability, i.e. $p_i = 1/n$ for $i = 1,\ldots n$. The degree of disorder is given by: $E(1/n,\ldots,1/n) = k\ln n$. For instance, in the limiting case of $n = 2$ and $k = 1$ the degree of disorder will be given by $E = \ln(2)$. Then, consider the pure-conflict case when agents use only one instrument. Thus, in such a case it would be simple to demonstrate that entropy is given by:

[8] Consider, among others, some applications of entropy to social sciences: the Nobel graduate in physic Dennis Gabor applied entropy to the measurement of social and economic freedom in Gabor and Gabor (1958). Entropy has also been proposed as a measure of competitiveness and diversification in market structure: see Attaran and Zwick (1989) and Horowitz and Horowitz (1968).

[9] The form adopted here is the one presented in Campiglio (1999), ch.4.

$$E^{pc}(p_1^*, p_2^*) = \frac{(\delta+1)\ln(\delta+1) - \delta\ln(\delta)}{(\delta+1)} \tag{16}$$

Consider now the case of conflict management. In such a case the entropy is given by:

$$E^{cm}\left(p_1^*, p_2^*\right) = \frac{\left(\delta^2+1\right)\ln\left(\delta^2+1\right) - 2\delta^2\ln(\delta)}{\left(\delta^2+1\right)} \tag{17}$$

It would not be difficult to show that $E^{pc} > E^{cm}$ for $\delta \in (0,1)$. This means that in presence of efforts devoted to conflict management, the degree of disorder is lower. In particular, the point of interest is that as the asymmetry in evaluation decreases, the degree of disorder and turbulence increases. This point sheds further light upon the results of the foregoing sections. It has been showed that as the asymmetry decreases, agents have no longer incentives to invest in 'talks'. Then, as the incentives to conflict increase, the degree of disorder increases. In particular, as the degree of asymmetry approaches the unity, the difference in the degree of disorder decreases.

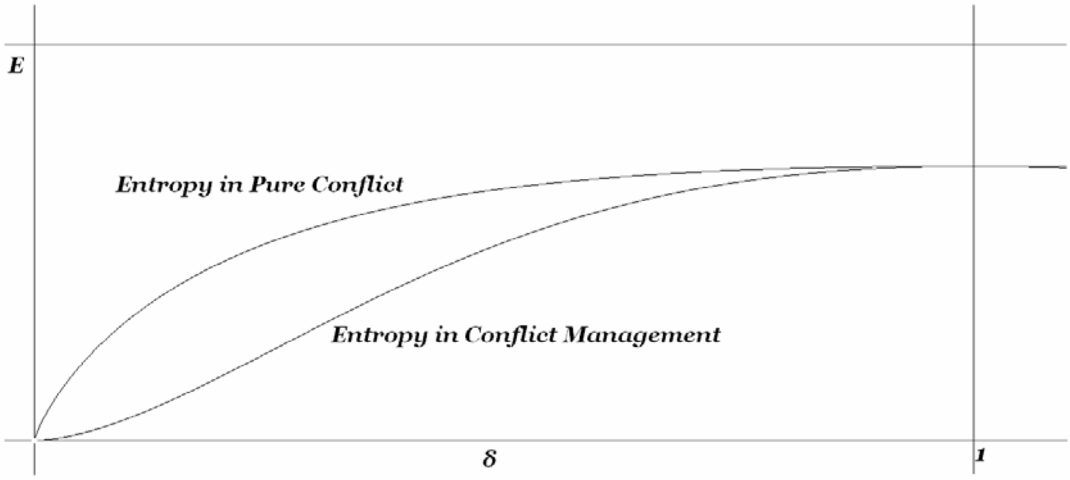


FIGURE 4. – ENTROPY IN PURE CONFLICT AND CONFLICT MANAGEMENT

In order to refine the use of entropy for measurement of conflicts, it would also be useful to introduce the concept of *relative entropy*. Relative entropy is defined as the ratio of the actual to the maximum entropy in a system. Relative entropy does not give any information about the degree of disorder That is, it would be useful to recognize the extent to which the degree of disorder approaches the maximum level attainable. In formal terms it is possible to write the relative entropy as: $RE = E / Ln(n)$. Then, relative entropy for pure conflict and conflict management respectively will be:

$$RE^{pc}(p_1^*, p_2^*) = \frac{(\delta+1)\ln(\delta+1) - \delta\ln(\delta)}{(\delta+1)\ln(2)} \tag{18}$$

$$RE^{cm}\left(p_1^*, p_2^*\right) = \frac{\left(\delta^2+1\right)\ln\left(\delta^2+1\right) - 2\delta^2\ln(\delta)}{\left(\delta^2+1\right)\ln(2)} \tag{19}$$

The relative entropy ratio would range from a value of zero for no entropy to a value of one when the maximum degree of entropy is attained.

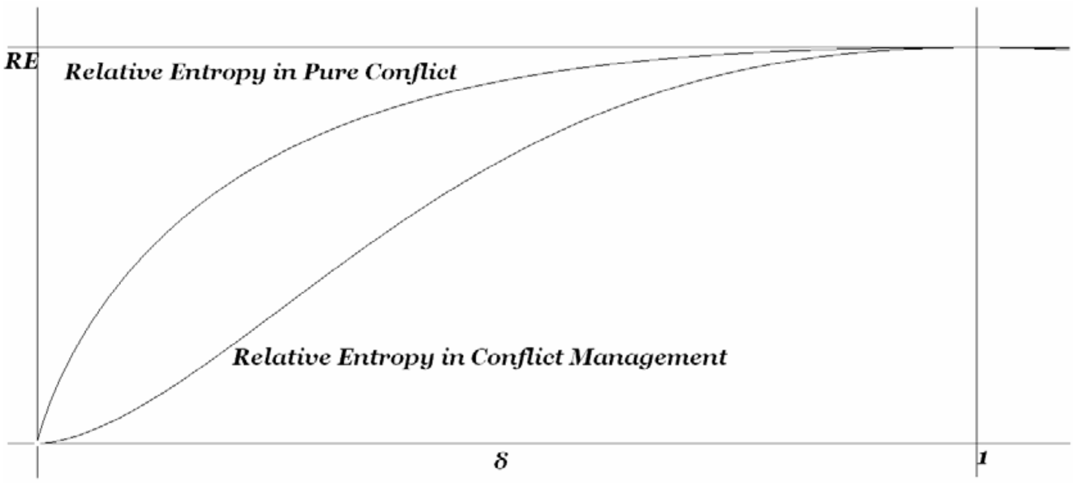


FIGURE 5. – RELATIVE ENTROPY IN PURE CONFLICT AND CONFLICT MANAGEMENT

Table 2 presents the calculations for entropy and relative entropy respectively.

Table 2. Entropy and Relative Entropy

	Pure Conflict		Conflict Management	
Asymmetry in Evaluation	Entropy	Relative Entropy	Entropy	Relative Entropy
0.05	0.19	0.28	0.02	0.03
0.15	0.39	0.56	0.11	0.15
0.25	0.50	0.72	0.22	0.32
0.35	0.57	0.83	0.34	0.50
0.45	0.62	0.89	0.45	0.65
0.55	0.65	0.94	0.54	0.78
0.65	0.67	0.97	0.61	0.88
0.75	0.68	0.99	0.65	0.94
0.85	0.69	1	0.68	0.98

0.95	0.69	1	0.69	1

The figures clearly show that entropy is lower in the presence of 'talks'. At the same time, it is worth noting that whenever agents expend resources in conflict management, the system fails to achieve its maximum possible degree of entropy at a relatively lower rate. In sum, it would be possible to write:

PROPOSITION 2: *When agents are asymmetrical in their evaluation of the stake and identical in fighting abilities, the conflict management scenario appears to be less turbulent than the pure conflict scenario.*

Although entropy appears to be an appealing concept to evaluate conflicts and contests, some points should be highlighted. First, a remarkable point of interest which would deserve further attention is related exactly to the functional form of CSF adopted. In particular, if entropy is used as a measure of the degree of disorder, it would be clear that it will depend directly on (i) the technology of conflict; (ii) the number of contestants; (iii) the abilities of contestants; (iv) the existence of institutional constraints or noises. The result of this section also raises questions on the trade-off between efficiency losses and the degree of disorder. There could be equilibria where a lower degree of disorder could be attainable with a higher waste of resources. However, the social waste of resources is higher than in a pure conflict scenario. This simple consideration would represent a crucial point for a future research agenda. A trade-off between the loss of resources and the degree of turbulence could clearly emerge.

A CONCESSION TO ESTABLISH AN INTEGRATIVE RELATIONSHIP

This section extends the basic model of conflict management in order to evaluate a concession made by agent 1, namely the agent with the higher evaluation of the stake. Making a concession falls in a broad sense in the category of integrative grants. The concept of integrative grant I apply does partly fit the idea of integrative relationship as developed and expounded by Boulding (1973). A grant is an unilateral transfer from an individual, a group or a social unit to another. In the simplest two-agents case, a grant provided by the first agent must enter the utility function of the second party. The recipient's payoff function must be increasing in the grant. The peculiar feature of a grant is its unilateral commitment. As noted in the introduction, a concession chosen endogenously has to be considered self-enforcing. In fact, in presence of an exogenously chosen concession an ordinary problem of cooperation and collective action emerges.[10]

[10] in Caruso (2006b) I explored the 'peaceful' impact of an exogenous institutional constraint

A possible criticism is that only a unilateral concession is considered. In the study of conflict many would maintain that only reciprocal concessions are effective. The choice of considering only an unilateral concession is based upon the results of the foregoing sections which showed how, under some conditions, only one agent is willing to expend positive efforts in 'talks'. The driving force in foregoing sections was the asymmetry in evaluation of the stake. Then I shall assume that agent 1, namely with a higher evaluation of the stake, is going to concede unilaterally to the opponent. This appear to be as a reasonable assumption given that in the presence of a relatively large asymmetry in the evaluation of the stake an asymmetrical willingness to manage the conflict emerges ($h_1 > 0, h_2 < 0$). The concession has to be considered as an irrevocable self commitment which is unambiguously and perfectly observable and measurable. The rationale behind this self-commitment is to establish a FSR.

Of course, it is questioned whether making the concession can modify the incentives to conflict management for both agents. Thus, through a positive transfer, agent 1 commits itself to influence the opponent's behaviour. This influence is provided not only by means of coercion but through an integrative approach. Essential to understand the impact of an integrative grant is the awareness that agents do not give up their rational and maximizing behaviour. They are still utility-maximizers and behave simultaneously à la Cournot.

Another remarkable and potentially critical point is the assumption of simultaneity. In such a framework this assumption can be accepted mainly because of complete information. In fact, given the assumption of complete information, each agent's payoff function is common knowledge among agents. Therefore, agent 2 knows what would be the maximum payoff going to agent 1 and then what would be its dominant strategy. Moreover, the level of asymmetry is also common knowledge. It is reasonable to think that if the complete information assumption is relaxed, a sequential protocol would fit much better the mechanism of making a concession.

Then, suppose that such an integrative grant is worth a fraction of the optimal level of resources expended for conflict management. A simple example can be drawn from International Relations. States invest resources in military expenditures and diplomacy. This does clearly fit with the idea of 'guns' and 'talks'. Then take foreign aid. Foreign aid flowing from one state to another commonly falls within the budget of diplomacy. Through foreign aid, the donor state attempts to influence the behaviour of the recipient. In fact, although foreign aid is supposed to be a unilateral transfer provided to address issues of poverty and development, it is also designed to pursue foreign policy objectives of donor countries.

Many other examples can be drawn form the real world, and some of them need not to indicate a benign or a desirable conduct. They can take the shape of extortion. Take a shopkeeper and a racketeer agent of a criminal organization. A shopkeeper opens a shop in an area where the criminal organization extracts rents from shopkeepers. The racketeer usually makes money by means of credible threats of violence. The stake of the conflict is represented by the value of the shop managed by the shopkeeper. The value of the shop corresponds to the discounted value of future earnings. Taking into account also personal beliefs and perceptions, the shopkeeper attaches a higher evaluation to the stake of the conflict since he knows that the shop is worth his life's work. The racketeer has a lower evaluation of the stake since it constitutes only one source of income among the shops available for racketeering. The shopkeeper anticipates the threat of the racketeer and will acquiesce to the extortion. He will concede a positive fraction of his future income. Then, in such a case the concession takes the shape of extortion.

To sum up, a concession – interpreted as an unilateral transfer from agent 1 to agent 2 – will enter additively the payoff function of agent 2. Then let $\sigma \in (0,1)$ the fraction of 'talks' devoted to making the concession. While choosing the optimal level of 'talks' agent 1 will choose also the level of the concession given the value of such a fraction. Of course, whenever $\sigma = 1$ agent 1 commit itself to concede the entire amount of resources expended in 'talks'. In its general form, the payoff function for agent i become $\pi_i^c = \pi_i^c(z_1, z_2, h_1, h_2, x, \sigma)$.Then it is possible to write:

$$\pi_1^c = \frac{z_1(h_1+1)}{z_1(h_1+1)+z_2(h_2+1)}x - z_1 - h_1 - \sigma h_1 \tag{20.1}$$

$$\pi_2^c = \frac{z_2(h_2+1)}{z_1(h_1+1)+z_2(h_2+1)}\delta x - z_2 - h_2 + \sigma h_1 \tag{20.2}$$

after the ordinary process of maximization the optimal choices of both agents are:

$$\begin{cases} z_1^{c*} = \dfrac{(1+\sigma)\delta^2}{(\sigma\delta^2+\delta^2+1)^2}x & h_1^{c*} = \dfrac{\delta^2 x - \delta^4(1+\sigma)^2 - 2\delta^2(1+\sigma) - 1}{(\sigma\delta^2+\delta^2+1)^2} \\ \\ z_2^{c*} = \dfrac{(1+\sigma)\delta^3}{(\sigma\delta^2+\delta^2+1)^2}x & h_2^{c*} = \dfrac{\delta^2 x(1+\sigma) - \delta^4(1+\sigma)^2 - 2\delta^2(1+\sigma) - 1}{(\sigma\delta^2+\delta^2+1)^2} \end{cases} \tag{21}$$

Note that $z_i^{c*} > 0, i = 1,2$ but also that:

$$\begin{cases} h_1^{c*} > 0 \quad \Leftrightarrow \quad \delta^2 x > \delta^4(1+\sigma)^2 + 2\delta^2(1+\sigma) + 1 \\ \\ h_2^{c*} > 0 \quad \Leftrightarrow \quad \delta^3 x(1+\sigma) > \delta^4(1+\sigma) + 2\delta^2(1+\sigma) + 1 \end{cases} \tag{22}$$

First, it is clear that $z_1^{c*} > z_2^{c*}$. Instead $h_1^{c*} > h_2^{c*} \Leftrightarrow \delta < 1/(1+\sigma)$. Plotting this inequality in the parameter space (δ,σ) makes clear that as the concession becomes larger and larger $(\sigma \to 1)$ and the asymmetry in the evaluation decreases ($\delta \to 1$) agent 2 will expend more resources in 'talks' than its opponent.

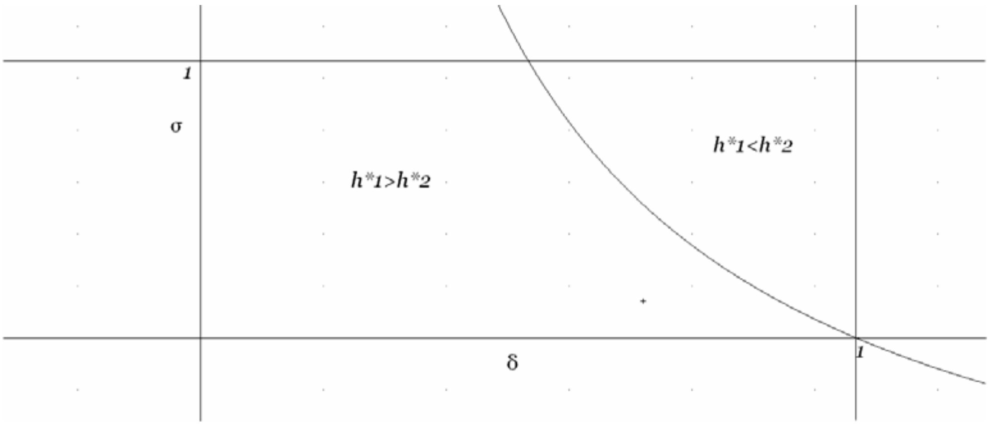


FIGURE 6 . – COMPARISON OF INVESTMENTS IN TALKS IN PRESENCE OF A CONCESSION

Another interesting comparison that can be made is that with optimal choices of 'guns' in the 'pure conflict' scenario. Recalling (3) it would be possible to verify that:

$$z_i^{c*} < z_i^{pc*} \Leftrightarrow \frac{(1-\delta)(\delta+1)^2}{\delta^3} < \sigma < \frac{(1-\delta)}{\delta}, i=1,2 \tag{23}$$

that is, optimal choices of 'guns' in the presence of a unilateral concession are not always lower than optimal choices under 'pure conflict'. Eventually, the attainable payoffs are given by:

$$\pi_1^{c*} = (1+\sigma) - \frac{x}{\left(\sigma\delta^2+\delta^2+1\right)^2}; \tag{24.1}$$

$$\pi_2^{c*} = (1-\sigma) - \delta^2 x \frac{\left(\delta^3(1+\sigma)^2 - \delta(1+\sigma)+\sigma\right)}{\left(\sigma\delta^2+\delta^2+1\right)^2} \tag{24.2}$$

It is possible to show that $\partial\pi_1^{c*}/\partial\sigma<0$ [11], $\partial\pi_1^{c*}/\partial\delta<0$ and that $\partial\pi_1^{c*}/\partial x>0 \Leftrightarrow \delta^2<1/(1+\sigma)$. It is more interesting to note the behaviour of agent 2's payoffs. First, agent's payoffs are increasing in the value of the stake only for specific combination of the parameters. That is, $\partial\pi_2^{c*}/\partial x>0 \Leftrightarrow \delta^3(1+\sigma)^2-\delta(1+\sigma)+\sigma>0$. Moreover, how do agent 2' payoffs change as δ is varied? Agent 2's payoff are increasing in δ if and only if $\partial\pi_2^{c*}/\partial\delta>0 \Leftrightarrow \delta^5(1+\sigma)^3+6\delta^3(1+\sigma)^2-2\delta^2\sigma(1+\sigma)-3\delta(1+\sigma)+2\sigma>0$.

Table 3 - Payoffs and FSR in presence of Concession ($x=100;\sigma=0.5$)

x	σ	δ	π_1^{pc*}	π_2^{pc*}	z_1^{cm*}	z_2^{cm*}	h_1^{cm*}	h_2^{cm*}	π_1^{pc*}	π_2^{cm*}	$\pi_1^{cm*}-\pi_1^{pc*}$	$\pi_2^{cm*}-\pi_2^{pc*}$	*FSR*
100	0.5	0.05	90.70	0.01	0.37	0.02	-0.75	-0.98	100.38	0.61	9.68	0.59	○
100	0.5	0.15	75.61	0.26	3.16	0.47	1.11	-0.53	91.92	1.09	16.30	0.84	○
100	0.5	0.2	69.44	0.56	5.34	1.07	2.56	0.07	85.16	1.28	15.72	0.72	●
100	0.5	0.21	68.30	0.63	5.82	1.22	2.88	0.22	83.66	1.30	15.35	0.67	●
100	0.5	0.22	67.19	0.72	6.31	1.39	3.21	0.39	82.11	1.32	14.92	0.60	●
100	0.5	0.23	66.10	0.80	6.81	1.57	3.54	0.57	80.53	1.33	14.43	0.52	●
100	0.5	0.24	65.04	0.90	7.32	1.76	3.88	0.76	78.91	1.34	13.87	0.44	●
100	0.5	0.25	64.00	1.00	7.84	1.96	4.22	0.96	77.26	1.34	13.26	0.34	●
100	0.5	0.26	62.99	1.11	8.36	2.17	4.57	1.17	75.58	1.33	12.59	0.23	●
100	0.5	0.27	62.00	1.22	8.89	2.40	4.92	1.40	73.87	1.33	11.87	0.10	●
100	0.5	0.35	54.87	2.35	13.11	4.59	7.74	3.59	59.75	1.12	4.88	-1.23	○
100	0.5	0.45	47.56	4.33	17.87	8.04	10.91	7.04	42.46	0.86	-5.10	-3.48	○
100	0.5	0.55	41.62	6.93	21.47	11.81	13.31	10.81	27.35	1.21	-14.28	-5.72	○
100	0.5	0.65	36.73	10.09	23.74	15.43	14.83	14.43	15.22	2.76	-21.51	-7.33	○
100	0.5	0.75	32.65	13.78	24.82	18.62	15.55	17.62	6.10	5.86	-26.56	-7.91	○
100	0.5	0.85	29.22	17.94	24.96	21.22	15.64	20.22	-0.43	10.60	-29.65	-7.35	○
100	0.5	0.95	26.30	22.55	24.44	23.21	15.29	22.21	-4.89	16.86	-31.18	-5.69	○

To give some indication of the significance of the results, table 3 offers a numerical example. As before, I set the value of the stake equal to one hundred ($x=100$) and I also use $\sigma=0.5$ to indicate the fraction of positive investments in 'talks' devoted to make a concession. That is, imagine that a government sets the amount of foreign aid as the half of total budget for diplomacy. The results show that a FSR is achievable in presence of a large asymmetry in the evaluation of the stake. In particular, under the arbitrary values chosen, a FSR is achievable if and only if $0.2\le\delta\le0.27$. This seems to imply that in order to reach an agreement two agents – risk-neutral and identical in abilities – have to adequately differ in evaluation. At the same time, they must not differ too much. In fact, when the asymmetry is extremely large (namely as $\delta\to0$), they have no incentive in investing in 'talks'. Even in the presence of a concession they are willing to be involved

[11] It would be possible to show that $\partial\pi_1^{c*}/\partial\sigma>0$ when δ is slightly larger than zero. However, as the value of x increases agent 1's payoffs are no longer increasing in σ.

in pure conflict. I labeled this scenario as *Conflict Trap*. As the degree of asymmetry decreases ($\delta \to 1$) there is no room for a FSR. Agents fall again in the *Unfeasible Settlement Region USR* and they prefer a **Pure Conflict** scenario. Then, in order to have a FSR the actual degree of asymmetry δ must fall within a range $\left[\underline{\delta},\overline{\delta}\right]$.

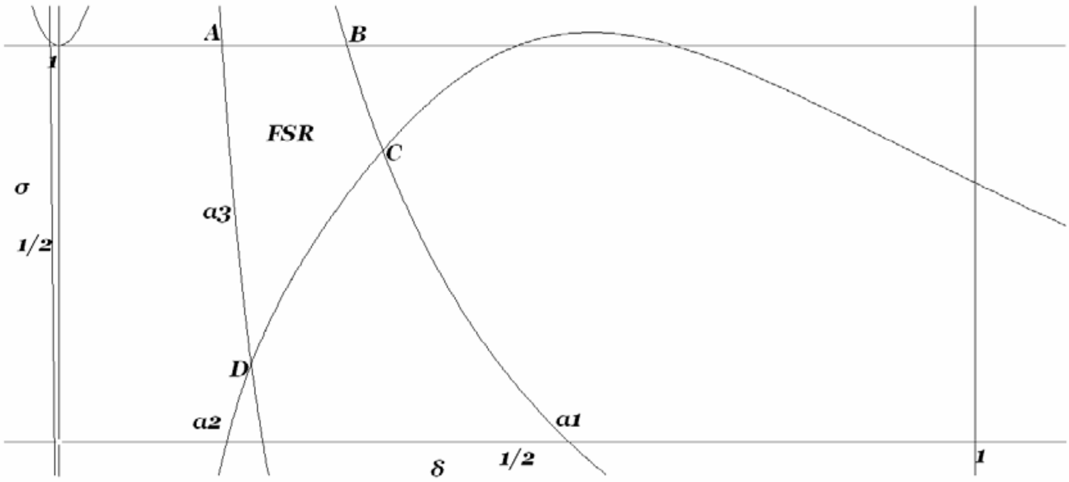


FIGURE 7 – FSR IN THE PRESENCE OF CONCESSION

Given the analytical complexity I skip all the formulas I present the FSR in the figure 7 The FSR is presented in the parameter space (δ,σ) for $x = 100$. The area ABCD is the FSR. In fact, consider the curves $a1, a2, a3$. All the points on the left of $a1$ curve respect the inequality $\pi_1^{c*} > \pi_1^{pc*}$ whereas all the points on the left of $a2$ follows the inequality $\pi_2^{c*} > \pi_2^{pc*}$.

In order to better assess the results of the analysis I let the σ vary and I also set a higher value for the value of the stake ($x = 1000$). The table below shows that as the arbitrary value attached to the stake becomes larger the FSR enlarges. However, it is interesting to note that it enlarges shifting to a lower degree of δ. Consider in particular the case for $\sigma = 0.5$. Setting $x = 1000$ the upper bound of a FSR shifts to 0.23 and the lower bound goes down to 0.09, ($0.09 \le \delta \le 0.23$). To sum up, as the value of the stake increases it is necessary to have a larger asymmetry to achieve a FSR. Note also that in both cases considered as the fraction approaches the unity ($\sigma \to 1$) the FSR shrinks.

Table 4 – FSR and Concession

σ	FSR ($x = 100$)	FSR ($x = 1000$)
0.1	No	No
0.15	No	$0.1 \le \delta \le 0.11$
0.2	$\delta = 0.21$	$0.1 \le \delta \le 0.12$
0.25	$\delta = 0.21$	$0.1 \le \delta \le 0.14$

0.3	$0.21 \leq \delta \leq 0.22$	$0.1 \leq \delta \leq 0.16$
0.35	$0.21 \leq \delta \leq 0.24$	$0.1 \leq \delta \leq 0.17$
0.4	$0.2 \leq \delta \leq 0.25$	$0.1 \leq \delta \leq 0.19$
0.45	$0.2 \leq \delta \leq 0.26$	$0.09 \leq \delta \leq 0.21$
0.5	$\mathbf{0.2} \leq \delta \leq \mathbf{0.27}$	$\mathbf{0.09} \leq \delta \leq \mathbf{0.23}$
0.55	$0.2 \leq \delta \leq 0.29$	$0.09 \leq \delta \leq 0.25$
0.6	$0.2 \leq \delta \leq 0.3$	$0.09 \leq \delta \leq 0.27$
0.65	$0.19 \leq \delta \leq 0.32$	$0.09 \leq \delta \leq 0.29$
0.7	$0.19 \leq \delta \leq 0.34$	$0.09 \leq \delta \leq 0.32$
0.75	$0.19 \leq \delta \leq 0.35$	$0.09 \leq \delta \leq 0.33$
0.8	$0.19 \leq \delta\, 0.34$	$0.09 \leq \delta \leq 0.32$
0.85	$0.19 \leq \delta \leq 0.33$	$0.09 \leq \delta \leq 0.32$
0.9	$0.19 \leq \delta \leq 0.32$	$0.09 \leq \delta \leq 0.31$
0.95	$0.18 \leq \delta \leq 0.32$	$0.09 \leq \delta \leq 0.3$

PROPOSITION 3: *When agents are identical in fighting abilities and asymmetrical in the evaluation of the stake a FSR can be established if and only if: (i) the agent with the higher evaluation of the stake provides the opponent with a positive concession which is assumed to be proportional to the level of investment in talks; (ii) the agents are sufficiently asymmetric in the evaluation of the stake. Namely, the actual degree of asymmetry must fall within a range* $\left[\underline{\delta},\overline{\delta}\right]$ *that is increasing in value of the stake.*

As measurement of this scenario it is possible now to consider the relative entropy in the presence of a concession. It is given by:

$$RE^c = \frac{\left(\delta^2\left(1+\sigma\right)+1\right)\ln\left(\delta^2\left(1+\sigma\right)+1\right)-\delta^2\left(1+\sigma\right)\ln\left(\delta^2\left(1+\sigma\right)\right)}{\left(\delta^2\left(1+\sigma\right)+1\right)\ln(2)} \qquad (25)$$

The Figure 8 shows how the proportional concession can have an impact on the level of relative entropy. It is interesting to note that, as the proportional concession increases, the level of entropy is higher. This happens whenever a relative large asymmetry exists. Consider that it has been shown that as $\delta \to 1$ a FSR cannot be established. In such a case, making a concession would be useless and the scenario would become more turbulent. At the same time, making a larger concession appears to be more turbulent. This is also clear, given that agent 1 is a rational utility-maximizer involved in a non-cooperative environment it would prefer a small concession to a large concession.

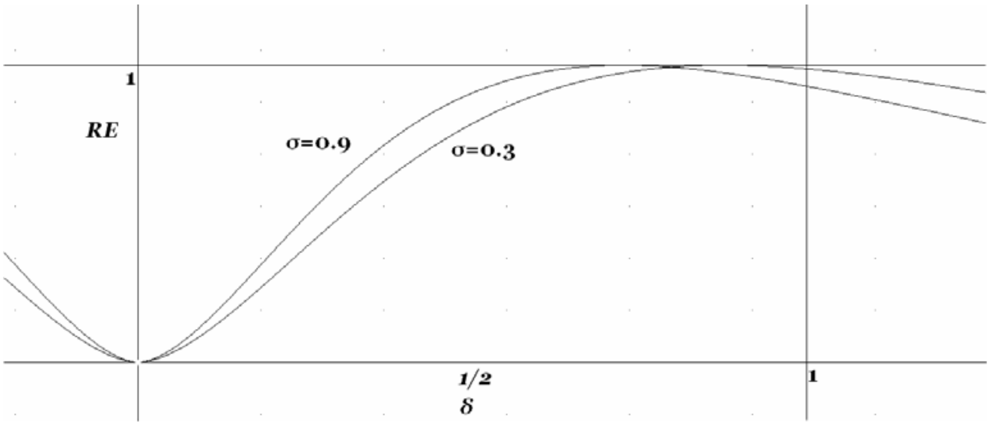


FIGURE 8. - RELATIVE ENTROPY IN THE PRESENCE OF AN UNILATERAL CONCESSION

It is possible to compare the level of relative entropy in the presence of concession ($\sigma = 0.5$) with the level of relative entropy in pure conflict. Figure 9 unambiguously shows that relative entropy is lower in the presence of concession.

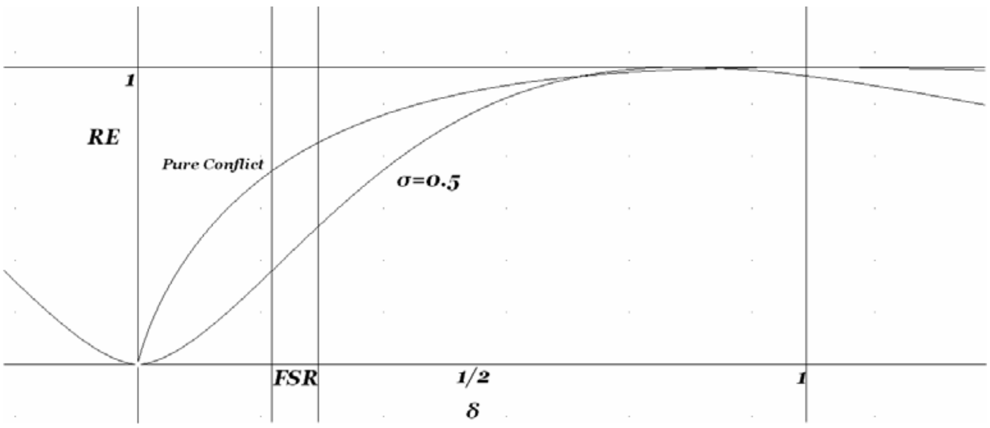


FIGURE 9. – RELATIVE ENTROPY IN THE PRESENCE OF AN UNILATERAL CONCESSION AND FSR

Proposition 4: *When agents are asymmetrical in their evaluation of the stake and identical in fighting abilities: (i) the conflict management scenario in the presence of a concession appears to be less turbulent than the pure conflict scenario; (ii) the greater is the concession the more turbulent appears to be the conflict management scenario.*

CONFLICT MANAGEMENT AND RETURNS TO SCALE

In the foregoing sections, I have assumed that agents retain different evaluations of the contested stake. In particular, it has been showed that the asymmetry in evaluation is a powerful force driving choices of agents.

However, I gave no explanation about the source of such asymmetry. In this section I shall try to provide a very simple tentative explanation for this. The basic assumption here is that agents are heterogeneous in production functions. Furthermore, agents compete for the appropriation a factor of production. Then, the stake of the conflict is the represented by an input as it enters a production function.

Historical and anecdotal narratives of territorial disputes provide plenty of examples in this respect. In ancient times, competition between farmers and gatherers can be interpreted along this view. Furthermore, take a potential conflict between an oil company and farmers over the exploitation of soil. Oil production exhibits increasing returns to scale while traditional agricultural productions exhibit decreasing returns to scale. Consider also the famous current dispute over the shrimps aquaculture in the subtropical coastal lowlands in America and Asia. Apart from environmental and social considerations it can be examined according to this view. In fact, shrimp farming can exhibit economies of scale whilst the traditional displaced productions as salt-flats, coastal fishing and agriculture exhibit decreasing returns to scale.

Then, productive characteristics can play a role. Dacey (1992/1996) pointed out how the existence of specific productive characteristics deserves a deep attention because they can have on impact on the level of hostility.

In this respect, this section is intended to take into account the productive characteristics of a factor of production. In particular, I try to analyse the relationship between the incentives to conflict and to negotiate and the existence of returns to scale for the stake of the conflict. The role of returns to scale is highly relevant, but hard to pin down. The economic literature on conflict commonly disregards the importance of returns to scale. The prevailing literature focuses mainly on the technology of conflict leading to different non-cooperative results. [12] What I want to maintain here is that agents look at the stake of the conflict taking into account the technical properties of their production processes. Agents attach to the value of the stake the technical properties of the production function. This also implies that, under the assumption of common knowledge, while appropriating a positive fraction of a contestable input each agent does take into account its own production function as well as that of its opponent.

Then, only for expository needs, hereafter imagine agents exhibiting heterogeneous production functions as: $y_1 = x^{\alpha}; \alpha \in (0,\infty)$, $y_2 = x^{\beta}; \beta \in (0,\infty)$ where y_1, y_2 denote the level of production of agent 1 and 2 respectively. The exponents α, β denote the degrees of returns to scale for agent 1 and 2 respectively. Both functions are standard single-stage production functions where the final output is a function of the flow of a single input denoted by x which does constitute the stake of the conflict.

[12] To my knowledge, a first attempt is Hirshleifer (1995), that deals with the existence of returns to scale through attaching a productivity parameter to a contestable income. However, it does not affect the result of the conflict.

Therefore, for $\alpha \neq \beta$ it is clear that $x^{\alpha} \neq x^{\beta}$. For sake of simplicity assume that $\alpha > \beta$. Of course, within this simple setting, it is clear that agents retain a different evaluations of the contested stake. In its general form, the payoffs function for agent i become $\pi_i^{rs} = \pi_i^{rs}\left(z_1, z_2, x, \alpha, \beta\right); i = 1,2$. Then it is possible to write:

$$\pi_1^{rs} = \frac{z_1}{z_1 + z_2} x^{\alpha} - z_1; \pi_2^{rs} = \frac{z_2}{z_1 + z_2} x^{\beta} - z_2 \tag{26}$$

Following an ordinary process of maximization[13] the optimal choices of 'guns' are given by:

$$z_1^{rs*} = \frac{x^{2\alpha+\beta}}{\left(x^{\alpha} + x^{\beta}\right)^2}; z_2^{rs*} = \frac{x^{\alpha+2\beta}}{\left(x^{\alpha} + x^{\beta}\right)^2}. \tag{27}$$

Under the assumption of $\alpha > \beta$, it would be possible to say that $z_1^{rs*} > z_2^{rs*} \Leftrightarrow x > 1$. Moreover, $\partial z_1^{rs*} / \partial \alpha > 0 \Leftrightarrow x > 1$.

The payoffs accruing to the agents are:

$$\pi_1^{rs*} = \frac{x^{3\alpha}}{\left(x^{\alpha} + x^{\beta}\right)^2}; \pi_2^{rs*} = \frac{x^{3\beta}}{\left(x^{\alpha} + x^{\beta}\right)^2}. \tag{28}$$

Given $\alpha > \beta$ it is clear that $\pi_1^{rs*} > \pi_2^{rs*}$. To better assess the results consider a very simple numerical example. For $x = 100, \alpha = 1.2, \beta = 1$ the payoffs accruing to the agents are $\pi_1^{rs*} = 128.5, \pi_2^{rs*} = 8.1$. The 3D plot depicts both agents' payoffs.

[13] The First Order Conditions for the maximization problem are:

$$\frac{\partial \pi_1^{rs}}{\partial z_1} = \frac{z_2}{\left(z_1 + z_2\right)^2} x^{\alpha} - z_1 = 0; \frac{\partial \pi_2^{rs}}{\partial z_2} = \frac{z_1}{\left(z_1 + z_2\right)^2} x^{\beta} - z_2 = 0$$

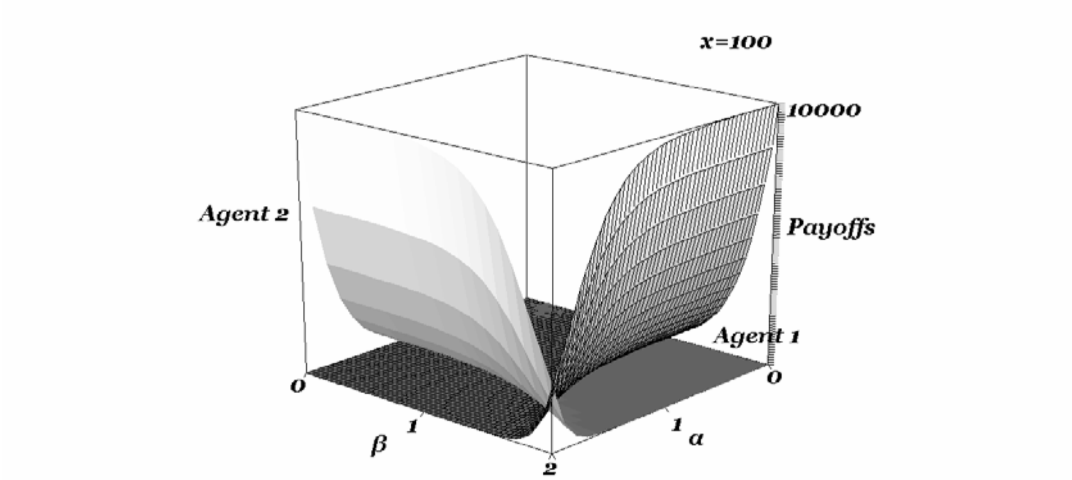


FIGURE 10. – PAYOFFS IN THE PRESENCE OF RETURNS TO SCALE ($x = 100$).

Consider now the option for the second instrument in the presence of a concession. Of course for $\sigma = 0$ there is no concession. The payoff functions become:

$$\pi_1^{rsc} = \frac{z_1(h_1+1)}{z_1(h_1+1)+z_2(h_2+1)} x^{\alpha} - z_1 - h_1 - \sigma h_1 \qquad (29.1)$$

$$\pi_2^{rsc} = \frac{z_2(h_2+1)}{z_1(h_1+1)+z_2(h_2+1)} x^{\beta} - z_2 - h_2 + \sigma h_1 \qquad (29.2)$$

Following an ordinary maximization process the optimal choices of both agents are given by:

$$\begin{cases} z_1^{rsc*} = \dfrac{x^{3\alpha+2\beta}(1+\sigma)}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)^2} & h_1^{rs*} = \dfrac{x^{3\alpha+2\beta}}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)^2} - 1 \\ \\ z_2^{rsc*} = \dfrac{x^{2\alpha+3\beta}(1+\sigma)}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)^2} & h_2^{rs*} = \dfrac{x^{2\alpha+3\beta}(1+\sigma)}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)^2} - 1 \end{cases} \qquad (30)$$

The payoffs are given by:

$$\pi_1^{rsc*} = \frac{x^{5\alpha} - x^{3\alpha+2\beta}(1+\sigma)}{\left(x^{2\alpha} + x^{2\beta}(1+\sigma)\right)^2} + (1+\sigma);$$

$$\pi_2^{rsc*} = \frac{x^{4\beta}\left((1+\sigma)^2\left(x^{\beta}+1-\sigma\right)\right) - x^{4\alpha}(\sigma-1) + \sigma x^{3\alpha+2\beta} - x^{2(\alpha+\beta)}(1+\sigma)\left(x^{\beta}+2(\sigma-1)\right)}{\left(x^{2\alpha} + x^{2\beta}(1+\sigma)\right)^2}.$$

. (31)

Also in this case, consider a simple numerical example. For $x = 100, \sigma = 0.5, \alpha = 1.2, \beta = 1$ the payoffs accruing to the agents are $\pi_1^{rsc*} = 126.48, \pi_2^{rsc*} = 1.66$. The 3D plot depicts both agents' payoffs (plot with panels indicate agent 1's payoffs) with $x = 100, \sigma = 0.5$. Analyzing the plot, it appears clear that there are combinations of α and β that push agent 1's payoffs down to the negative range.

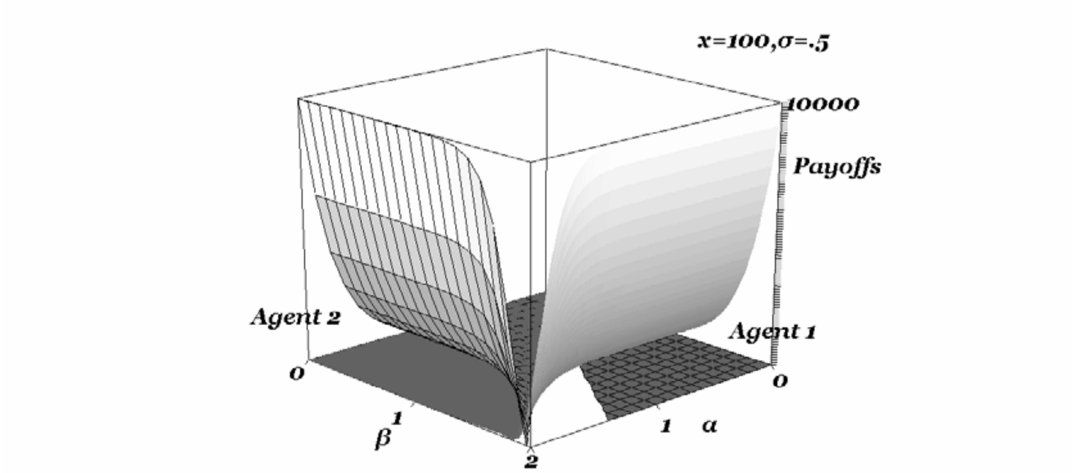


FIGURE 11 – PAYOFFS IN THE PRESENCE OF RETURNS TO SCALE AND A CONCESSION, $x = 100$ **,** $\sigma = 0.5$

In order to verify whether a FSR is attainable in the presence of a concession, I present first two plots and then a numerical example whose results are reported in the tables below.

What about the establishing of a FSR? Consider first the case of no concession $\sigma = 0$ with $x = 100$. For sake of simplicity I set $\chi = \alpha - \beta$ to denote the difference in the aggregate degrees of returns to scale. Under the assumption $\alpha > \beta$ the difference χ is unambiguously larger than zero. The FSR is the area delimited by ABC. First when $\chi \cong 0$ it is clear that no FSR can be attainable. At the same time, Figure 12 shows how a FSR can be reached when agents' aggregate degrees of returns to scale are sufficiently asymmetric.

Consider for example first the case of agent 1 exhibiting constant returns to scale $\alpha = 1$. In such a case to have a FSR it is necessary to have the difference between α and β falling within a range whose bounds are given by $\underline{\chi} \approx .369, \overline{\chi} \approx .494$. It would be the same to say that β must fall within a range whose bounds are given by: $\underline{\beta} \approx .506; \overline{\beta} \approx .631$.

Take agent 1 exhibiting decreasing returns to scale, say $\alpha = 0.85$, In such a case to have a FSR it is necessary to have the difference between α and β falling within a range whose bounds are given by $\underline{\chi} \approx .313, \overline{\chi} \approx .420$. The aggregate degree of returns to scale of agent 2 β must fall within a range whose bounds are given by: $\underline{\beta} \approx .43; \overline{\beta} \approx .537$.

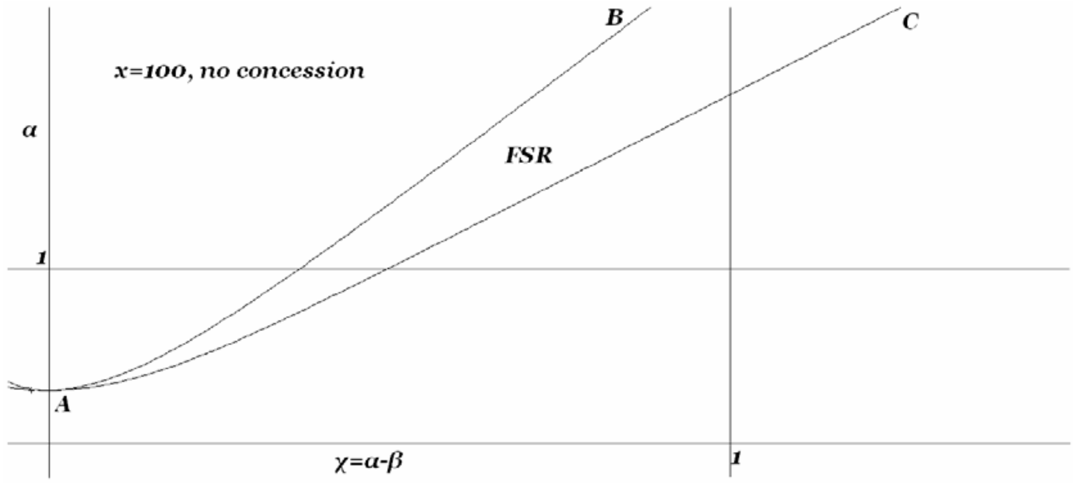


FIGURE 12 – FSR IN PRESENCE OF RETURNS TO SCALE ($x = 100$)

Then, differently from the foregoing section, while considering productive characteristics a FSR is attainable even if a concession is not ensured. Then, consider now the emergence of a concession. Figure 13 shows the FSR in the presence of returns to scale and concession with arbitrary values set at $x = 100$ and $\sigma = 0.5$.

Consider again the case of agent 1 exhibiting constant returns to scale. In such a case to have a FSR it is necessary to have the difference between α and β falling within a range whose bounds are given by $\underline{\chi}^c \approx .278, \overline{\chi^c} \approx .496$. It would be the same to say that β must fall within a range whose bounds are given by: $\underline{\beta}^c \approx .504; \overline{\beta^c} \approx .722$. (the superscript denotes 'concession'). Now consider again of agent 1 exhibiting decreasing returns to scale $\alpha = 0.85$. In such a case the bounds for χ are given by: $\underline{\chi}^c \approx .254, \overline{\chi^c} \approx .418$. Hence for β it is possible to write $\underline{\beta}^c \approx .432; \overline{\beta^c} \approx .596$.

To sum up briefly, it is clear that by means of a concession agent 1 is able to enlarge the FSR.

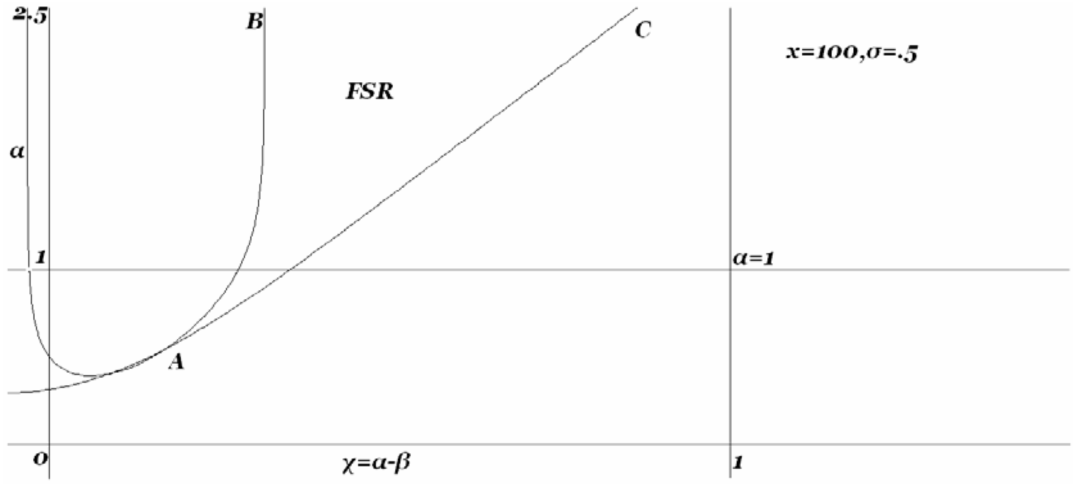


FIGURE 13 – FSR IN THE PRESENCE OF RETURNS TO SCALE AND CONCESSION ($x = 100, \sigma = 0.5$)

However, knowing that making a concession enlarges the FSR is not a shocking result. To assess and deepen better the results consider now the tables below. I set some arbitrary values for the β, I let vary α choosing values under the condition $\alpha > \beta$. Then, different values of a proportional concession are presented. The emergence of a FSR is denoted by bold symbols. Agents choose the 'conflict management ' scenario. Otherwise there is a persistence of a pure conflict scenario.

The first table presents the case of agent 2 exhibiting decreasing returns to scale ($\beta = .5$). The news here is that when both agents exhibit decreasing returns to scale the concession must be relatively large. That is, in the presence of small proportional concession the FSR is only slightly larger than one attainable without concession. Whenever agent 1 exhibits increasing returns to scale it is clear that even a small proportional concession appears to be effective in enlarging the FSR. In particular, when both agents exhibit increasing returns to scale a small concession appears to be more effective. Of course, it is still the asymmetry in the productive characteristics to lead towards the establishing of a FSR.

Table 5.1 - $x = 100; \beta = 0.5$

α	σ=0	σ=0.05	σ=0.15	σ=0.25	σ=0.40	σ=0.55	σ=0.7	σ=0.85	σ=1
0.55	○	○	○	○	○	○	○	○	○
0.6	○	○	○	○	○	○	○	○	○
0.65	○	○	○	○	○	○	○	○	○
0.7	○	○	○	○	○	○	○	○	○
0.75	○	○	○	○	●	●	●	●	●
0.8	●	●	●	●	●	●	●	●	●
0.85	●	●	●	●	●	●	●	●	●
0.9	●	●	●	●	●	●	●	●	●

0.95	●	●	●	●	●	●	●	●	●
1	○	○	○	○	○	○	○	○	○
1.05	○	○	○	○	○	○	○	○	○
1.1	○	○	○	○	○	○	○	○	○
1.15	○	○	○	○	○	○	○	○	○
1.2	○	○	○	○	○	○	○	○	○
1.25	○	○	○	○	○	○	○	○	○
1.3	○	○	○	○	○	○	○	○	○
1.35	○	○	○	○	○	○	○	○	○
1.4	○	○	○	○	○	○	○	○	○
1.45	○	○	○	○	○	○	○	○	○
1.5	○	○	○	○	○	○	○	○	○
1.55	○	○	○	○	○	○	○	○	○
1.6	○	○	○	○	○	○	○	○	○

Table 5.2 - x=100. b=0.75

α	**σ=0**	**σ=0.05**	**σ=0.15**	**σ=0.25**	**σ=0.40**	**σ=0.55**	**σ=0.7**	**σ=0.85**	**σ=1**
0.8	○	○	○	○	○	○	○	○	○
0.85	○	○	○	○	○	○	○	○	○
0.9	○	○	○	○	○	○	○	○	○
0.95	○	○	○	○	○	○	○	○	○
1	○	○	○	○	○	○	●	●	○
1.05	○	○	○	○	○	●	●	●	●
1.1	○	○	○	○	●	●	●	●	●
1.15	○	○	●	●	●	●	●	●	●
1.2	●	●	●	●	●	●	●	●	●
1.25	●	●	●	●	●	●	●	●	●
1.3	●	●	●	●	●	●	●	●	●
1.35	●	●	●	●	●	●	●	●	●
1.4	●	●	●	●	●	●	●	●	●
1.45	●	●	●	●	●	●	●	●	●
1.5	○	○	○	○	○	○	○	○	○
1.55	○	○	○	○	○	○	○	○	○
1.6	○	○	○	○	○	○	○	○	○
1.65	○	○	○	○	○	○	○	○	○
1.7	○	○	○	○	○	○	○	○	○
1.75	○	○	○	○	○	○	○	○	○
1.8	○	○	○	○	○	○	○	○	○
1.85	○	○	○	○	○	○	○	○	○

Table 5.3 - $x = 100; \beta = 1$

α	**σ=0**	**σ=0.05**	**σ=0.15**	**σ=0.25**	**σ=0.40**	**σ=0.55**	**σ=0.7**	**σ=0.85**	**σ=1**
1.05	○	○	○	○	○	○	○	○	○

1.1	○	○	○	○	○	○	○	○	○
1.15	○	○	○	○	○	○	○	○	○
1.2	○	○	○	○	○	○	○	○	○
1.25	○	○	○	○	○	○	●	●	○
1.3	○	○	○	○	○	●	●	●	●
1.35	○	○	○	○	●	●	●	●	●
1.4	○	○	○	○	●	●	●	●	●
1.45	○	○	○	●	●	●	●	●	●
1.5	○	○	●	●	●	●	●	●	●
1.55	○	●	●	●	●	●	●	●	●
1.6	●	●	●	●	●	●	●	●	●
1.65	●	●	●	●	●	●	●	●	●
1.7	●	●	●	●	●	●	●	●	●
1.75	●	●	●	●	●	●	●	●	●
1.8	●	●	●	●	●	●	●	●	●
1.85	●	●	●	●	●	●	●	●	●
1.9	●	●	●	●	●	●	●	●	●
1.95	●	●	●	●	●	●	●	●	●
2	○	○	○	○	○	○	○	○	○
2.05	○	○	○	○	○	○	○	○	○
2.1	○	○	○	○	○	○	○	○	○

Table 5.4 - $x = 100; \beta = 1.25$

α	σ=0	σ=0.05	σ=0.15	σ=0.25	σ=0.40	σ=0.55	σ=0.7	σ=0.85	σ=1
1.3	○	○	○	○	○	○	○	○	○
1.35	○	○	○	○	○	○	○	○	○
1.4	○	○	○	○	○	○	○	○	○
1.45	○	○	○	○	○	○	○	○	○
1.5	○	○	○	○	○	○	●	●	○
1.55	○	○	○	○	○	●	●	●	●
1.6	○	○	○	○	○	●	●	●	●
1.65	○	○	○	○	●	●	●	●	●
1.7	○	○	○	●	●	●	●	●	●
1.75	○	○	○	●	●	●	●	●	●
1.8	○	○	●	●	●	●	●	●	●
1.85	○	○	●	●	●	●	●	●	●
1.9	○	●	●	●	●	●	●	●	●
1.95	●	●	●	●	●	●	●	●	●
2	●	●	●	●	●	●	●	●	●
2.05	●	●	●	●	●	●	●	●	●
2.1	●	●	●	●	●	●	●	●	●
2.15	●	●	●	●	●	●	●	●	●
2.2	●	●	●	●	●	●	●	●	●
2.25	●	●	●	●	●	●	●	●	●
2.3	●	●	●	●	●	●	●	●	●
2.35	●	●	●	●	●	●	●	●	●
2.4	●	●	●	●	●	●	●	●	●
2.45	●	●	●	●	●	●	●	●	●

PROPOSITION 5 *–When agents are asymmetrical in productive characteristics, namely in the degree of return to scale, and identical in fighting abilities (i) there is room for establishing a FSR- agents would choose the 'conflict management scenario - if agents are also sufficiently asymmetric in their productive characteristics; (ii) an unilateral concession can enlarge the FSR; (iii) a proportional concession is more effective when both agents exhibit increasing returns to scale.*

It is possible now to compute the level of relative entropy in both scenarios. The relative entropy attainable in a pure conflict scenario is given by:

$$RE^{pc} = -\frac{x^{\alpha}\left(\ln\left(\frac{1}{x^{\alpha}+x^{\beta}}\right)+\alpha\ln(x)\right)+x^{\beta}\left(\ln\left(\frac{1}{x^{\alpha}+x^{\beta}}\right)+\beta\ln(x)\right)}{\left(x^{\alpha}+x^{\beta}\right)\ln(2)} \qquad (32)$$

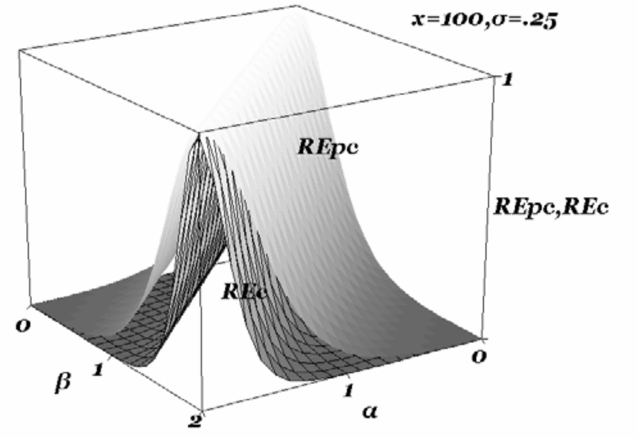
whilst the relative entropy in presence of CM and a proportional concession is given by:

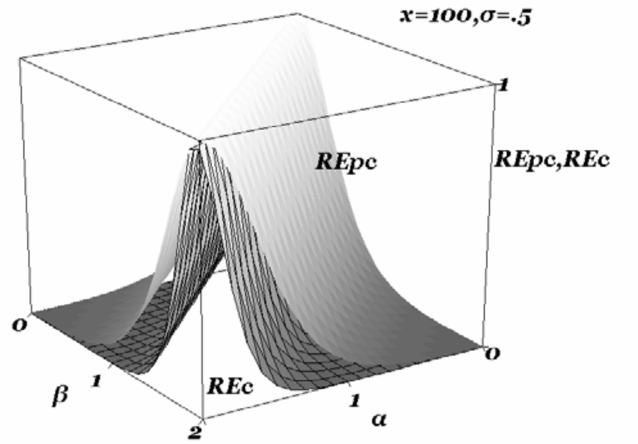
$$RE^{rsc} = -\frac{x^{2\alpha}\left(\ln\left(\frac{1}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)}\right)+2\alpha\ln(x)\right)+x^{2\beta}(1+\sigma)\left(\ln\left(\frac{(1+\sigma)}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)}\right)+2\beta\ln(x)\right)}{\left(x^{2\alpha}+x^{2\beta}(1+\sigma)\right)\ln(2)}$$

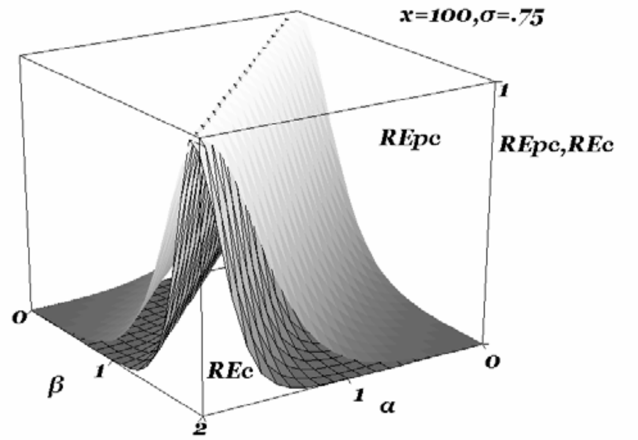
(33)

The results are illustrated in the plots below. I set different values for the proportional concession. The plots below offer a comparison of attainable levels of relative entropy for $x = 100$ and alternatively $\sigma = 0.25; \sigma = 0.5; \sigma = 0.75; \sigma = 1$.

The plots clearly show that relative entropy in conflict management is lower than in pure conflict. It is also interesting to note that when agents exhibit the same degree of returns to scale ($\alpha = \beta$) the entropy reach its maximum level. Interpreting entropy as a measure of degree of turbulence, a confirmation of previous results emerges. That is, when there is no asymmetry between agents the degree of turbulence is higher.


x=100,σ=.25
1
REpc
REpc,REc
REc
0
0
β
1
1
α
2


x=100,σ=.5
1
REpc
REpc,REc
0
0
β
1
REc
1
α
2


x=100,σ=.75
1
REpc
REpc,REc
0
0
β
1
REc
1
α
2

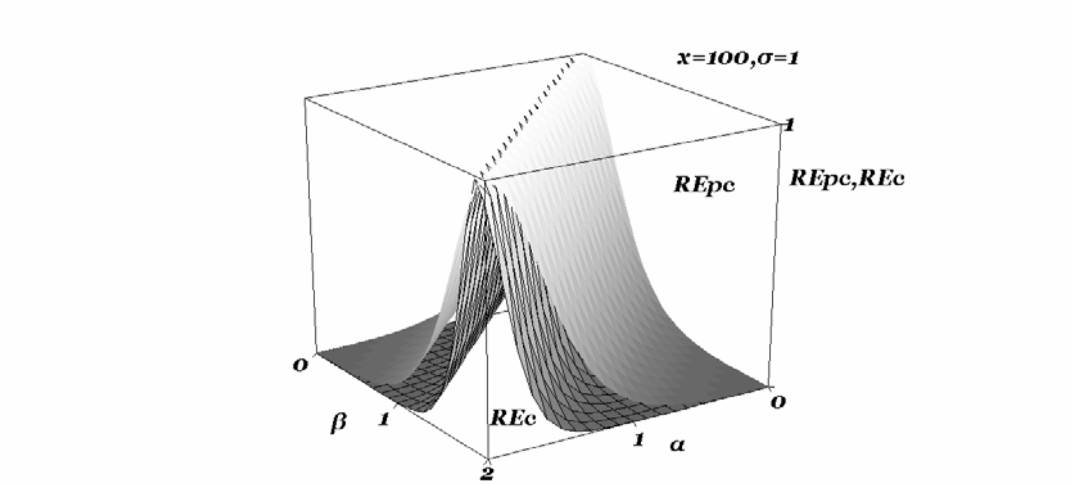


FIGURE 14 – RELATIVE ENTROPY IN THE PRESENCE OF RETURNS TO SCALE AND CONCESSION

CONCLUDING REMARKS

This paper analysed the incentives for risk-neutral agents to invest in conflict management in a conflict under different conditions. Through comparative statics, different scenarios have been studied. A Feasible Settlement Region (FSR) has been defined as the set of all possible settlement points. The key elements of the paper are: (i) an asymmetry in the evaluation of the stake; (ii) the existence of a transfer (concession) from one agent to the other. The main results are summarised below:

a) In the presence of agents with identical abilities and different evaluations of the stake, whenever the evaluations are sufficiently asymmetric, a FSR can be established if and only if the agent with the higher evaluation of the stake is willing to make a proportional concession to the opponent;

b) If the concession enters additively the payoff function of the recipient, both parties can be better off if the degree of asymmetry falls within a range $\left[\underline{\delta},\overline{\delta}\right]$. The existence of greater concessions expands the range of δ for which a FSR can be established;

c) As the evaluations of the stake converge, namely for $\delta \to 1$, a FSR is no longer attainable;

d) When the evaluations are extremely asymmetric, namely for $\delta \to 0$, a FSR cannot be attained even if a concession is ensured. Call this *Conflict Trap*.

These results show that an asymmetry in evaluation of the stake paves the way for ‘cooperation’ in a non-cooperative environment. Asymmetry in the evaluation can be translated in ‘asymmetry in incentives’. The other remarkable point is that whenever the asymmetry is extremely large, a conflict trap emerges. In other words – roughly speaking - agents have to be asymmetric but not too much. This result rests to a large extent upon the assumption of complete information. In fact, under complete information agent 1 knows perfectly that the investments in pure violence made by agent 2 will be negligible. The Pure conflict scenario will be the preferred option.

Taking into account productive characteristics of both agents sheds further light on the foregoing results. Of course, the contested stake has to be considered as a factor of production. It is differently evaluated because agents retain different productive characteristics captured through the idea of returns of scale. Recall that agent 1 is assumed to be more productive than agent 2. Putting bad news first, the main results are:

e) when agents are equal in productive characteristics there is no room for a FSR.

f) When agents are sufficiently asymmetric in the degree of returns to scale, a FSR is attainable;

g) Enlarging the FSR is possible through an unilateral concession provided by the most productive agent.

h) A concession appears to be more effective when both agents exhibit increasing returns to scale.

Broadly interpreted, this results suggest that an improvement in technology can favour an agreement between conflicting parties if the more productive agent provide an adequate compensation to the less productive agent. At the same time, being more productive make agents less prone to conflict.

The results of the paper can be also considered as a contribution to the study of self-enforcing arrangements. By means of a credible self-imposed concession, agent 1 is able to influence agent 2's behaviour. The FSR has been considered as a credible and incentive-compatible structure. Credibility has been assumed to be related to the cost of conflict management. Since efforts in conflict management – 'talks' - are costly and total outlays are irreversible, they are supposed to be credible. It is clear that the impact of credibility is strongly sensitive to the assumption of perfect information. This is the rationale behind the search for solutions involving only positive investments in conflict management, namely ($h_i > 0; i = 1, 2$).

However, the critical issue is the stability of such a solution. In reality, cheating does always constitute an option for participants. In this respect, it is significant that – formally speaking – the optimal choices in the 'conflict management' scenario do not constitute a global max. They do only constitute a local max. However, given the analytical complexity further investigation is needed on this point.

The integrative mechanism can also have another ambiguous impact. Under the hypothesis of common knowledge, agent 2, the recipient of the concession, can have an incentive to behave strategically: agent 2, albeit favouring a settlement, may be tempted to work against it expecting to get a monetary transfer. A classical problem of *moral hazard* can emerge.

As a novelty of this work, I would also quote the use of the concept of entropy as a tool for measurement of conflict. Following the common neoclassical approach, investing in conflict management would be welfare-immiserizing. In such a narrow sense, however, a pure conflict would be preferable to a scenario where agents invest resources in conflict management. Establishing a FSR would be less efficient than pure conflict. An appealing idea for a more useful evaluation can be related to those of disorder and randomness. In fact, since conflict is a destructive interaction between two or more parties, it seems reasonable to consider the degree of uncertainty it spreads. In actual violent appropriative conflicts, uncertainty about the final outcome does clearly constitute a characteristic element that should be considered while developing policies to solve the conflict itself. It has been shown that the level of entropy also depends on the level of the asymmetry in the evaluation of the stake. In particular, the point of interest is that as the asymmetry in evaluation decreases, the degree of disorder and turbulence increases. In particular, in presence of efforts devoted to conflict management, the degree of disorder is lower. Furthermore, an interesting point which emerged is that the greater is the concession the higher is the level of relative entropy. These results are confirmed when productive characteristics are assumed to be the source of the asymmetry.

The discussion related to the concept of entropy recalls the debate, famous among students of International Relations during the Cold War, about the stability of systems grounded on deterrence. In such a view, deterrence would be a stable system thanks to the existence of a credible threat. The results of this paper firmly contrasts this idea. A threat system (namely the ‘pure conflict’ scenario) is more turbulent than the ‘conflict management’ scenario. However, future research on this point could contribute to this enduring debate.

The analysis paves the way for several extensions. In particular, remarkable points deserving further extension are the impact of a larger time horizon and the setting of a learning process. The model expounded in this work is a timeless model. Nevertheless, consider a possible application to a multi-period interaction. Assume for example that a dynamic interaction involves a learning process. Then imagine that such a learning process can modify the asymmetry in evaluation. Consider for example that evaluations of the stake converge over time. Furthermore, you can also imagine that some peculiar features of agents modify (consider among others: production function, access to market, investment in new technologies etc). In such a case, in a future period (say $t+n$), the asymmetry in evaluation can decrease, namely $\delta_{t+n} > \delta_t$. In such a case, according to the results of the model, a settlement could be no longer possible. Parties could prefer a pure conflict. Broadly speaking, a superior information can have an ambiguous impact.

Moreover, for a future research agenda, consider that the CSF is used as a fundamental building block of several broader models. Applying the crucial modification of the CSF allowing for a second instrument can have an impact on the results emerging from these analyses.

Last but not least, what I would also claim as a point of interest is the relationship of 'conflict' with 'bargaining'. The results of the model show that conflict *can* evolve in a bargaining situation. It does if – and only if – some conditions are fulfilled. In other words, bargaining cannot be taken for granted. This, of course, has notable implications while designing policies and mechanisms to manage and solve conflicts.

APPENDIX

Throughout this appendix I shall check whether the critical points for a maximum computed constitute a global max, namely a NE. Thus, I have to check whether $\pi_1\left(z_1^*, z_2^*, h_1^*, h_2^*\right) \geq \pi_1\left(z_1, z_2^*, h_1, h_2^*\right), \forall\left(z_1, h_1\right) \in A$ and $\pi_2\left(z_1^*, z_2^*, h_1^*, h_2^*\right) \geq \pi_2\left(z_1^*, z_2, h_1^*, h_2\right), \forall\left(z_2, h_2\right) \in A$.

In order to check where the candidate critical points $\left(z_1^{cm*}, z_2^{cm*}, h_1^*, h_2^*\right)$ represent a maximum it is useful to compute the Hessian matrices for both agents. Let me denote $z_i = z_i^{cm}, i = 1,2.$ for notational simplicity. First, I compute the payoff function for agent 1 $\pi_1\left(z_1, h_1, z_2^*, h_2^*\right)$. The payoff function becomes:

$$\pi_1\left(z_1, h_1, z_2^*, h_2^*\right) = \frac{z_1\left(h_1+1\right)\left(\delta^4+2\delta^2+1\right)^2 x}{\delta^6 x^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)} - h_1 - z_1$$

And the Hessian matrix is given by:

$$H_1\left(z_1, h_1, z_2^*, h_2^*\right) = \begin{pmatrix} \dfrac{\partial \pi_1}{\partial z_1 z_1} & \dfrac{\partial \pi_1}{\partial h_1 z_1} \\ \dfrac{\partial \pi_1}{\partial z_1 h_1} & \dfrac{\partial \pi_1}{\partial h_1 h_1} \end{pmatrix} =$$

$$= \begin{pmatrix} -\dfrac{2\delta^6 x^3\left(h_1+1\right)^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^6 x^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} & \dfrac{\delta^6 x^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^6 x^2 - z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]}{\left[\delta^6 x^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} \\ \dfrac{\delta^6 x^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^6 X^2 - z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]}{\left[\delta^6 x^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} & -\dfrac{2\delta^6 x^3 z_1^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^6 x^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} \end{pmatrix}$$

Note that the Hessian matrix is symmetric. Let H_{1k} denote the k_{th} order leading principal submatrix of $H_1\left(z_1^{cm}, z_2^{cm}, h_1^*, h_2^*\right)$ for $k = 1,2$. The determinant of the kth order leading principal minor of $H_1\left(z_1^{cm}, z_2^{cm}, h_1^*, h_2^*\right)$ is denoted by $\left|H_{1k}\right|$. The leading principal minors alternate signs as follows:

$$\begin{aligned} &\left|H_{11}\right| < 0, \\ &\left|H_{12}\right| > 0 \Leftrightarrow \delta^6 x^2 - \left[3 z_1\left(\delta^8 + 4\delta^6 + 6\delta^4 + 4\delta^2 + 1\right)\left(h_1+1\right)\right] < 0. \end{aligned} \tag{9.3}$$

Then I compute the payoff function for agent 2 $\pi_2\left(z_1^*, h_1^*, z_2, h_2\right)$,

$$\pi_2\left(z_1^{cm*},h_1^{cm*},z_2,h_2\right)=\frac{z_2\left(h_2+1\right)\left(\delta^4+2\delta^2+1\right)^2\delta x}{\delta^4x^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)}-h_2-z_2 \qquad (9.4)$$

And the Hessian matrix is given by:

$$H_2\left(z_1^*,h_1^*,z_2,h_2\right)=\begin{pmatrix}\frac{\partial\pi_2}{\partial z_2 z_2} & \frac{\partial\pi_2}{\partial h_2 z_2}\\ \frac{\partial\pi_2}{\partial z_2 h_2} & \frac{\partial\pi_2}{\partial h_2 h_2}\end{pmatrix}=$$

$$=\begin{pmatrix} -\frac{2\delta^5x^3\left(h_2+1\right)^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^4x^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3} & \frac{\delta^5x^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^4X^2-z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]}{\left[\delta^4x^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3}\\ \frac{\delta^5x^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^4X^2-z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]}{\left[\delta^4x^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3} & -\frac{2\delta^5x^3z_2^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^4x^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3}\end{pmatrix}$$

(9.4)

Also in this case, let H_{2k} denote the k_{th} order leading principal submatrix of $H_1\left(z_1^{cm*},z_2^{cm},h_1^*,h_2\right)$ for $k=1,2$. The determinant of the kth order leading principal minor of $H_2\left(z_1^{cm*},z_2^{cm},h_1^*,h_2\right)$ is denoted by $|H_{2k}|$. The leading principal minors alternate in sign as follows:

$$\begin{aligned}&|H_{21}|<0,\\ &|H_{22}|>0\Leftrightarrow\delta^4x^2-\left[3z_2\left(\delta^2+1\right)^4\left(h_2+1\right)\right]<0.\end{aligned} \qquad (9.5)$$

since the Hessian matrices are not negative semidefinite it is necessary to deepen the analysis in order to show whether the critical points $\left(z_1^{cm*},z_2^{cm*},h_1^*,h_2^*\right)$ represent a global max. Then I compute the limits of both agents' payoffs. For the first agent we have:

$$\lim_{h_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = \frac{z_1 x\left(\delta^4 + 2\delta^2 + 1\right)^2}{\delta^6 x^2 + z_1\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_1$$

$$\lim_{h_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

$$\lim_{z_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -h_1 \tag{9.6}$$

$$\lim_{z_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

I do the same for agent 2.

$$\lim_{h_2 \to 0} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = \frac{z_2 \delta x\left(\delta^4 + 2\delta^2 + 1\right)^2}{\delta^4 x^2 + z_2\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_2$$

$$\lim_{h_2 \to \infty} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -\infty$$

$$\lim_{z_2 \to 0} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -h_1$$

$$\lim_{z_2 \to \infty} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -\infty \tag{9.7}$$

therefore for both agents it is still necessary to check for $h_i = 0, i = 1, 2$. Consider first the payoff function of agent 1:

$$\pi_1\left(z_1, z_2^*, 0, h_2^*\right) = \frac{z_1\left(\delta^4 + 2\delta^2 + 1\right)^2 x}{\delta^6 x^2 + z_1\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_1 \tag{9.8}$$

$$\frac{\partial \pi_1}{\partial z_1} = \frac{\delta^6 x^3 \left(\delta^4 + 2\delta^2 + 1\right)^2}{\left[\delta^6 x^2 + z_1 \left(\delta^4 + 2\delta^2 + 1\right)^2\right]^2} - 1 = 0 \tag{9.9}$$

$$z_1 = -\frac{\delta^3 x^{3/2} \left(\delta^3 x^{1/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \tag{9.10}$$

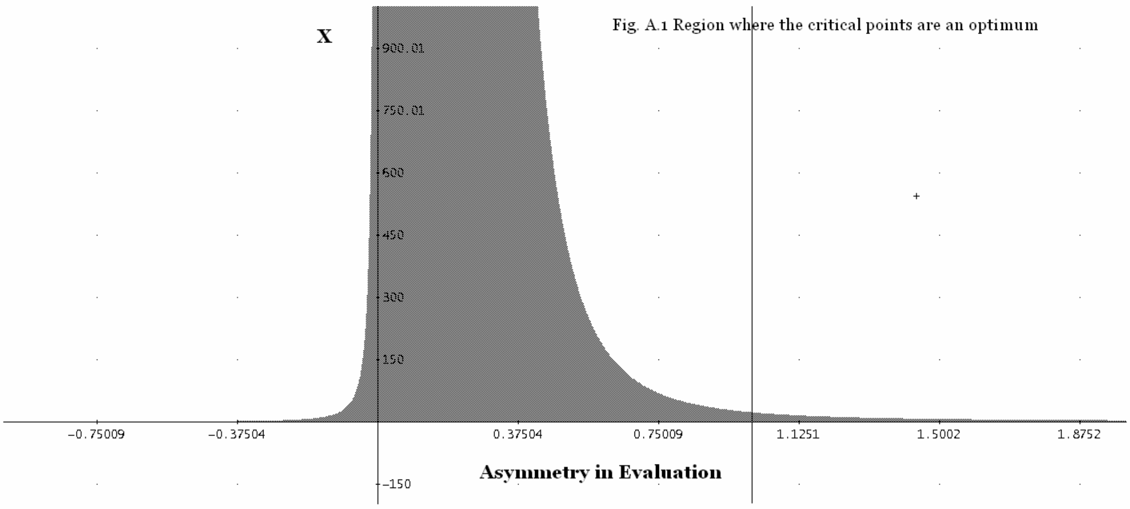
the payoff is:

$$\pi_1 = \frac{\left(\delta^3 x^{1/2} - \delta^4 - 2\delta^2 - 1\right) x}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \tag{9.11}$$

then, compare (9.1) and (9.11):

$$\frac{\left(\delta^2 + 1\right)^2 - x\left(\delta^2 - 1\right)}{\left(\delta^2 + 1\right)^2} > \frac{\left(\delta^3 x^{1/2} - \delta^4 - 2\delta^2 - 1\right) x}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \tag{9.12}$$

Due to the analytical complexity I present a plot in a (δ, x) space. The vertical axe corresponds to $\delta = 1$. The shaded area in the plot below shows the region where the critical points $\left(z_1^{cm*}, z_2^{cm*}, h_1^*, h_2^*\right)$ represent an optimum.



for agent 2 we have:

$$\pi_2\left(z_1^*, h_1^*, z_2, 0\right) = \frac{z_2 \left(\delta^4 + 2\delta^2 + 1\right)^2 \delta X}{\delta^4 X^2 + z_2 \left(\delta^4 + 2\delta^2 + 1\right)^2} - z_2 \tag{9.13}$$

$$\frac{\partial \pi_2}{\partial z_2} = \frac{\delta x z_2 \left(\delta^4 + 2\delta^2 + 1\right)^2}{\left[\delta^4 x^2 + z_2 \left(\delta^4 + 2\delta^2 + 1\right)^2\right]^2} - 1 = 0 \tag{9.14}$$

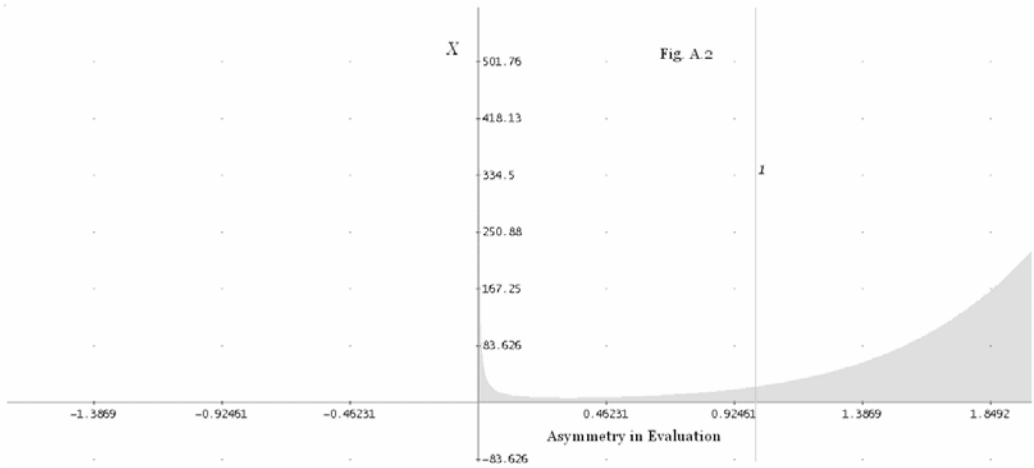
$$z_2 = -\frac{\delta^{5/2} x^{3/2} \left(X^{1/2} \delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \tag{9.15}$$

$$\pi_2 = \frac{\delta x \left(x^{1/2} \delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \tag{9.16}$$

compare (9.16) and (9.2)

$$\frac{\delta^3 x \left(\delta^2 - 1\right) + \left(\delta^2 + 1\right)^2}{\left(\delta^2 + 1\right)^2} > \frac{\delta x \left(x^{1/2} \delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2}$$

also in this case consider the plot below:



Then, it is clear that the critical points $\left(z_1^{cm*}, z_2^{cm*}, h_1^*, h_2^*\right)$ do not constitute a global maximum, namely a Nash Equilibrium (NE).

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