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Abstract. The modern reformulation of Classical analysis of value and distribution built upon Sraffa's *Production of Commodities* provides quite a general and open framework to study distributive relationships, that deserves to be still explored. The present work aims to go through these relationships according to the perspective ensuing from the notion of "natural economic system", developed by Pasinetti.

The natural economic system is a sort of ideal configuration, which allows to accomplish the potentialities of the production system concerning growth, employment and the satisfaction of final wants. Actual (capitalist) economies do not normally fulfil the "natural" configuration. But this configuration can be considered a reference point in order to discriminate between the conditions that *have to be* verified for the system to reproduce itself and grow according to its potentialities, and the conditions that have to be regarded as pertaining to the institutional sphere.

On this perspective several indications can be drawn on various aspects. In this work we shall focus on the normative conclusions that can be drawn about income distribution. Firstly a description of the characteristics and the implications of the "natural" configuration of income distribution will be provided: we will focus on natural rates of profit (which are differentiated among sectors), on the natural level of wages and on the natural rate of interest. Secondly we will depict some instruments to orientate the actual set-up of capitalistic economies towards the natural configuration. In particular we will develop a notion of *uniform* natural profit rate and we will describe the consequent necessary ("natural") financial system which allows the natural accumulation of capital to be realized in a capitalistic context, where profit rates tend to be uniform.

Keywords: Average natural profit rate, Classical political economy, Labour theory of value, Natural system, Natural profit rates, Prices, Sraffa framework, Structural change, Surplus approach.

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1 Introduction

Theoretical analysis concerning the (functional) distribution of income has become an argument that has attracted less and less attention of theoretical economists during the last decades. The last renowned paper on this topic is probably that written by Nicholas Kaldor (1955-56), which individuates four 'alternative' theories of distribution: i) the Ricardian theory, ii) the Marxian theory, iii) the Neo-classical theories (which are distinguished between the marginal productivity theories and the 'degree of monopoly' theories) and iv) the (post-)Keynesian theory. Later on no new significant contribution has appeared in the literature. Obviously particular aspects of distributive relations have been dealt in several spheres of economic research. For example the wide literature on efficiency wages (see Solow (1979)) provides some insights to explain the determinants of wages and, as a residual, of profits. But in the generality of cases we have no systematic work on the topic, Nonetheless a coherent and widely accepted theoretical framework on income distribution is far from having been reached today.[1] The capital debates started in the Sixties have undermined deeply the Neoclassical theory of income distribution and, by consequence, the whole construct of general equilibrium analysis. The new general equilibrium framework established on the ashes of this debate appears quite "agnostic" on income distribution in its surface: it determines the rental price of each physical produced or non-produced input, without referring these prices to the actual magnitudes of wages, profits and rents. But, whenever it is necessary to develop considerations on these variables, the forces of demand and supply of land, labour and capital (this latter intended as a single magnitude) are however recalled, more or less consciously, at least at a starting point, even if they lack of logical grounds.

Sraffa on the other hand, in (1960) provided a coherent framework to *analyse* the *rela-*

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[1]The following two authoritative quotations (reproduced as epigraph at the beginning of Garegnani (2007)), describe the situation eloquently:

> "But I fear that when the economic theorist turns to the general problem of wage determination and labour economics, his voice becomes muted and his speech halting. If he is honest with himself, he must confess to a tremendous amount of uncertainty and self doubt concerning even the most basic and elementary parts of the subject" Samuelson (1956, p. 312).

> "In principle [in mainstream economics today], we ask about allocation among individuals or among owners of different factors of production, but it must be recognised that distributional questions are not asked very loudly or answered very well" Arrow (1991, p. 74).

tion between prices and income distribution under an alternative perspective, the Classical one, based on the notion of surplus; yet he refrained himself to take a definite position on *what determines* income distribution; his well-known indication to find the determinants of the profit rate in the monetary rates of interest sounds just as an hint to close the system, rather than a definitive word.

One reason to understand this apparent lack of theoretical developments on the topic could be the gradual acknowledgment that functional distribution could be hardly explained on the basis of a once-and-for-all univocal principle, having the same degree of generality of other relationships, like, for example, those determining prices once income distribution has been fixed from outside. This point is clearly stated, for example, by Garegnani (1983), (1984) and (2007) or Pasinetti (1988) and Pasinetti (2007).

The great proliferation also within the neoclassical approach of extremely specific models and contributions on the fixation of wages, or on wage rigidities confirms that this tendency is a common feeling. This entails that probably we have to renounce to an overall positive vision on income distribution, like those expressed by the first Marginalist economists, and to return to the Classical perspective, where social and institutional aspects deeply interact with technical ones.

In this work we will take another different perspective. We start from Sraffa (1960) framework and follow a course of analysis marked out by the works of Kaldor (1957) and Pasinetti (1962), (1981). Within his framework of structural change Pasinetti developed an analytical device to separate the study of the characteristics of an economic system concerning the relations that necessarily *have to be* satisfied in order to guarantee a "good working" of the system (in a way that will be specified later) from the analysis of the institutional aspects of the system itself. This goal is pursued by enucleating a particular configuration from his system, called 'natural system', which represents a particular lens through which to look at actual systems, in order to evaluate their performance and to imagine possible changes of some aspect of the institutional setting. In other words the natural configuration needs not to be accomplished by actual—capitalist—economic systems in which we live; notwithstanding it provides a benchmark to develop normative considerations concerning the various aspects of the working of actual systems.

In this work we will focus this lens on the aspects concerning income distribution. The main normative indication coming from the natural system on this aspect is that a *differentiated* set of natural profit rates is prescribed for adapting the structure of the system to the evolution of final demand. This makes the 'natural' distributive configuration not compatible in principle with what is normally considered the actual outcome of capitalist economic systems, that is, a tendency towards a *uniform* rate of profit among industries,

as a consequence of capital mobility. In this work after making explicit the main characteristics of the 'natural' determination of wages, profit and interest, we will try to propose a sort of 'bridge' between the ideal ('natural') configuration and the actual possibilities of capitalistic systems through the notion of the natural average rate of profit. This solves, on one side, the problem of non-uniformity of natural rates, but opens immediately another important issue, that of the necessity of a suitable system of interrelated loans and borrowings that have to take place among industries in order to cover the financial needs of the sectors characterized by a natural profit rate higher than the average and to invest the profits exceeding the natural levels coming from the other sectors.[2]

The work is organized as follows. Sections 2, 3 and 4 depict the framework within which we move: no substantially new material is presented here, but rather a attempt to make explicit several results and consequences implicitly contained in some of the original contributions listed above, with the purpose of opening a discussion on these topics. In section 5 a bridge between the natural distributive configuration and the actual system is provided through the notion of average natural rate of profit. A final Section concludes

2   The basic framework

The starting point is the reformulation of the framework of Classical political economy proposed by Sraffa (1960). It is known that the price system outlined by Sraffa displays one degree of freedom—once a numeraire has been chosen—concerning one of the two distributive variables: or the wage rate or the profit rate has to be fixed *outside of the price equations*. Thus there is a non-univocal configuration of income distribution compatible with the price equations, i.e. compatible with the possibility for the system of repeating the production activity on an at least unchanged scale of production: infinitely many alternative configurations of income distribution are thus compatible with the reproduction of the system. By using a suitable numeraire for prices and the wage rate—the Standard commodity—all the points of the segment of the graph of figure 1 represent these configurations. Paradoxically we could fix the profit rate equal to zero (attributing thus the whole net product to workers) without jeopardizing the reproducibility of the system. This is obviously an extreme case, but it is instructive to grasp the very nature of profits as the surplus of the system: a gain that *is not necessary* to guarantee the replication of the pro-

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[2]These relationships go beyond those analysed by Pasinetti in the natural system, which are limited to loans and borrowings among *individuals*, the natural rates of profit providing the appropriate levels of financial resources to each (hyper-vertically integrated) sector (see Pasinetti (1981, chap. VIII)). The detachment from the natural structure of profit rates raises the need of financial relations *among industries*.
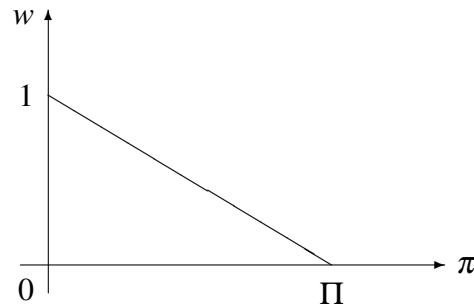
Figure 1: Relationship between the distributive variables in Sraffa's system

duction process on an unchanged scale: this possibility is assured by the price equations *for each configuration of income distribution lying on the straight line of figure 1.*

Far form saying that profits *must* be null, this result represents a real change of perspective with respect to the traditional neoclassical theory of distribution, where *one particular* profit rate (which is supposed to coincide with the interest rate) is univocally individuated by the intersection of the investment schedule (derived from the marginal productivity of capital) and the saving schedule, that is, by the equilibrium of the capital market.[3]

The above result has been interpreted in different ways. To simplify the more recurrent interpretations are of two kinds. On one side this degree of freedom is seen as a phenomenon of *indeterminacy*, mostly due to the lack of the demand side in the Sraffa's system (for example Burmeister (1984, Sections V–VII), Mandler (1999) Samuelson (2007)). On the other side (see, for example Pasinetti (1988)) the attitude is that of looking for the closure of the system *outside* the price equations, that is, outside the set of technical or "mechanical" relations: according to this interpretation the determination of income distribution belongs to a sphere of analysis where sociological, historical, political, in brief, institutional aspects do matter principally. Sraffa's hint to find the determinants of the rate of profit in the money rates of interest obviously goes in this direction. Garegnani very clearly argued in a recent work that:

> "On the one hand, we have the necessary quantitative relations, which competition entails between commodity prices and distributive variables and, which, in their comparative simplicity, are of a nature allowing for a mainly deductive treatment

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[3]We do not consider here the logical difficulties that the literature has emphasized in obtaining these schedules within the neoclassical approach.

[the 'core' of the theory, *added by E. B.*]. On the other, we have the circumstances determining what we have described as the intermediate data: the subsistence or, more generally, the wage, the outputs, the technical conditions of production. These circumstances were seen to be closely related to institutional and historical factors, which, because of their complexity and variability according to circumstances, prevented deducing the corresponding variables from a few basic principles as was possible for prices and profits in the core. Those intermediate data rather required, for their study, methods of a more inductive kind" (Garegnani (2007, section 5)).

(On this see also Garegnani (1983, in particular section II) or Garegnani (1984, in particular sections 4 and 5)).

Such a separation between the relations that take place among economic variables with a degree of 'necessity' and the other that can change in relation to the social and institutional context, is a very important point. But such an 'opening' is something that can be, or *should* be, filled, *at least in part*, by economic analysis. The opposite risk is, I think, that old and more traditional views return to prevail in the practical vision of income distribution.

3   Profits 'justified' by economic growth: the Cambridge equation

Theoretical analysis can provide, in effect, some material to move towards a more definite vision on income distribution, alternative to the mainstream. A first hint can come from the post-Keynesian theories on income distribution.[4] As known their crucial element is condensed in the so-called 'Cambridge equation':

$$\pi = \frac{1}{s_c} \cdot g_n, \tag{1}$$

where π is the profit rate, s_c is the capitalists' propensity to save and g_n is the natural rate of growth of the system (given by the sum of the growth rate of population, n, and the growth rate of productivity, λ, i.e. $g_n = n + \lambda$).

As known equation (1) selects that particular rate of profit, and thus that particular configuration of income distribution, that guarantees the maintenance of full employment of productive capacity and of labour force along the balanced growth path. According to the vision originally proposed by Kaldor (see Kaldor (1955-56, Section IV)) under appropriate conditions this result is the outcome of a process that tends to take place

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[4]These theories were elaborated by Kaldor (1955-56) and (1957), Joan Robinson (1956), Champernowne (1958) Kahn (1959) and Pasinetti (1962).

spontaneously in the economic system. According to a less pretentious argument (see Pasinetti (1962, in particular Section 9)) equation (1) individuates just the condition that *has to be satisfied* in order to ensure the maintenance of full employment as time goes by.

In any case it is possible to fully appreciate the Keynesian flavour of this conclusion: equation (1) individuates that level of capitalists' saving, and hence of profits, that are necessary to finance the equilibrium growth of the system.

Whichever interpretation is attributed to equation (1), it identifies a level of the profit rate that is necessary from the technical point of view, and that can be considered 'desirable' from the social point of view: it guarantees a full-employment equilibrium growth path. A "social justification" of profits is what equation (1) supports.

4   A full generalization: the 'natural' rates of profit

A generalization and a reinforcement of this ideas has been provided by Pasinetti in his framework of structural change (see Pasinetti (1981, chap. VII, in particular Sections 3 and 4)), a reinforcement that remained almost unnoticed, probably because its main implications were not drawn in detail.[5] Let us recall the basic structure of the model in its most general formulation, where final commodities and capital goods are both produced by means labour and capital goods. Let $m = 1, \ldots, M$ the index of final goods. Each final good m requires for its production a_{Nm} units of labour and a specific set of capital goods; we can consider each of these sets as a single (composite) capital good, that we could call 'productive capacity of final good m', and we could chose as measure unit of each of these productive capacities that quantity that is necessary to produce exactly 1 unit of the final good. Suppose that in each period, a *constant* proportion, $1/T_m$, of these productive capacities wears out. Moreover suppose that the production of one unit of productive capacity of good m requires a quantity a_{Nk_m} of labour and a fraction γ_m of itself; let $1/T_{k_m}$ be the depreciation rate of each unit of productive capacity in producing itself. Let Q_N be the existing population, that is supposed to coincide, for simplicity, with labour force, and let a_{mN} and $a_{k_m N}$ be the units of final good m and of productive capacity of good m required by each individual for final purposes and for investment purposes, respectively. Let Q_m and Q_{k_m} be the quantities produced of final good m and of its productive capacity, k_m; let p_m and p_{k_m} be the corresponding prices and π_m and π_{k_m} the corresponding rates of profit (that, for the moment, are not assumed to be uniform among sectors). The production relationships can be represented by a couple of equation systems, one for quantities and

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[5]More emphasis on this point will be given in Pasinetti (2009, forthcoming).

one for prices:

$$
\begin{cases}
Q_m = a_{mN}Q_N, \quad m = 1,\ldots,M, \\[2mm]
Q_{k_m} = \dfrac{1}{T_m} \cdot Q_m + \dfrac{\gamma_m}{T_{k_m}} \cdot Q_{k_m} + a_{k_mN}Q_N, \quad m = 1,\ldots,M, \\[2mm]
\displaystyle\sum_{m=1}^{M} a_{Nm}Q_m + \sum_{m=1}^{M} a_{Nk_m}Q_{k_m} = Q_N,
\end{cases}
\tag{2}
$$

and

$$
\begin{cases}
p_m = (\pi_m + 1/T_m)p_{k_m} + a_{Nm}w, \quad m = 1,\ldots,M, \\[2mm]
p_{k_m} = p_{k_m} \cdot \dfrac{\gamma_m}{T_{k_m}} + \pi_{k_m}\gamma_m p_{k_m} + a_{Nk_m}w, \quad m = 1,\ldots,M, \\[2mm]
\displaystyle\sum_{m=1}^{M} a_{mN}p_m + \sum_{m=1}^{M} [a_{k_mN} - (\pi_m + \pi_{k_m}a_{mN})]p_{k_m} = w.
\end{cases}
\tag{3}
$$

The first $2M$ equation of system (2) determine the quantities produced of each good according to the corresponding effective demand; the first M equations concern final goods, the second M equations concern the productive capacity of each final good, and explain the three components of demand for productive capacity: the replacement of productive capacity worn out in producing the final good (Q_m/T_m), in producing itself ($\gamma_m Q_{km}/T_{km}$) and new investments ($a_{k_mN}Q_N$). The last equation of system (2) establishes the equality between labour requirements (in producing final goods and their productive capacities) and the existing labour force.

The first $2M$ equations of system (3) determine prices of final goods and of their productive capacities. The last equation of this system states that wages plus profits equal the value of final goods and investment goods demanded.[6]

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[6]For further details see Pasinetti (1981).

In matrix notation systems (2) and (3) become:

$$
\begin{bmatrix}
-1 & \cdots & 0 & 0 & \cdots & 0 & a_{1N} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & -1 & 0 & \cdots & 0 & a_{1M} \\
\hline
\frac{1}{T_1} & \cdots & 0 & -\left(1-\frac{1}{T_{k_1}}\gamma_1\right) & \cdots & 0 & a_{k_1N} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \frac{1}{T_M} & 0 & \cdots & -\left(1-\frac{1}{T_{k_M}}\gamma_M\right) & a_{k_mN} \\
\hline
a_{N1} & \cdots & a_{NM} & a_{N_k1} & \cdots & a_{N_kM} & -1
\end{bmatrix}
\begin{bmatrix}
Q_1 \\ \vdots \\ Q_M \\ \hline Q_{k_1} \\ \vdots \\ Q_{k_M} \\ \hline Q_N
\end{bmatrix}
=
\begin{bmatrix}
0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \\ \hline 0
\end{bmatrix}
\qquad (4)
$$

and

$$
\begin{bmatrix}
-1 & \cdots & 0 & \pi_1+\frac{1}{T_1} & \cdots & 0 & a_{N1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & -1 & 0 & \cdots & \pi_M+\frac{1}{T_M} & a_{MN} \\
\hline
0 & \cdots & 0 & \frac{\gamma_1}{T_{k_1}}+\gamma_1\pi_{k_1}-1 & \cdots & 0 & a_{Nk_1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & \frac{\gamma_M}{T_{k_M}}+\gamma_M\pi_{k_M}-1 & a_{Nk_m} \\
\hline
a_{1N} & \cdots & a_{MN} & a_{k_1N}-(\pi_1+\pi_{k_1}\gamma_1) & \cdots & a_{k_MN}-(\pi_M+\pi_{k_M}\gamma_M) & -1
\end{bmatrix}
\begin{bmatrix}
p_1 \\ \vdots \\ p_M \\ \hline p_{k_1} \\ \vdots \\ p_{k_M} \\ \hline w
\end{bmatrix}
=
\begin{bmatrix}
0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \\ \hline 0
\end{bmatrix}.
\qquad (5)
$$

(4) and (5) are two linear homogeneous systems; in order to exclude trivial solutions the determinants of the two coefficients matrices must be zero. It is possible to verify that for both systems this condition reduces to

$$
\sum_{m=1}^{M} a_{nm}a_{mn} + \sum_{m=1}^{M}\frac{1}{T_m}\frac{T_{k_m}}{T_{k_m}-\gamma_m}a_{nk_m}a_{mn} + \sum_{m=1}^{M}\frac{T_{k_m}}{T_{k_m}-\gamma_m}a_{nk_m}a_{k_mn} = 1.
\qquad (6)
$$

This condition may be interpreted, as known, as a macro-economic condition that guarantees full employment of labour force (see Pasinetti (1981, chap. II, Section 3)).

Once this condition is satisfied (we will return later on this point), systems (4) and (5) can be solved, respectively, with respect to Q_m and to p_m:

$$
\begin{cases}
Q_m = a_{mn}\bar{Q}_n, \quad m = 1,\ldots,M, \\
Q_{k_m} = \dfrac{T_{k_m}}{T_{k_m} - \gamma_m}\left(a_{k_m n} + \dfrac{1}{T_m}\cdot a_{mn}\right)\bar{Q}_N, \quad m = 1,\ldots,M
\end{cases}
\tag{7}
$$

$$
\begin{cases}
p_m = \left[\left(\pi_m + \dfrac{1}{T_m}\right)\dfrac{T_{k_m}}{T_{k_m}-\gamma_m-\pi_{k_m}\gamma_m T_{k_m}}\cdot a_{nk_m} + a_{nm}\right]w, \quad m = 1,\ldots,M, \\
p_{k_M} = \dfrac{T_{k_M}}{T_{k_M}-\gamma_M-\pi_{k_M}\gamma_M T_{k_M}}\cdot a_{nk_M}w, \quad m = 1,\ldots,M.
\end{cases}
\tag{8}
$$

The dynamic extension of this framework is straightforward. We suppose that population, technology and consumer's preferences change over time in a differentiated way. Following Pasinetti (1981, chapters III–IV) we introduce these changes by assuming the following time paths for Q_N and for coefficients a_{Nm}, a_{Nk_m} and a_{mN}:

$$
\begin{aligned}
Q_N(t) = Q_N(0)\mathrm{e}^{gt}, \quad a_{mN}(t) = a_{mN}(0)\mathrm{e}^{r_m t}, \\
a_{Nm}(t) = a_{Nm}(0)\mathrm{e}^{-\rho_m t} \quad \text{and} \quad a_{Nk_m}(t) = a_{Nk_m}(0)\mathrm{e}^{-\rho_{k_m} t}.
\end{aligned}
\tag{9}
$$

The first equation indicates that population growth at rate g; the second equation describes the evolution of final demand of each commodity; the third and the fourth equations describe how technical progress affects labour productivity of the various sectors.

The theoretical framework we are presenting individuates thus a particular structure for quantities produced and prices, that, according to Pasinetti, constitutes the 'natural' configuration, of the system; in consequence of (8) this configuration evolves as time goes by. The conception of a 'natural' system was firstly put forward by Pasinetti in (1981, Part II). The same author developed this concept in various subsequent works; a more complete account of this concept and of the idea to separate economic analysis into a 'natural' and into an 'institutional' level is developed in Pasinetti (2007, chapters IX and X). The present work itself aims to clarify and to analyse some parts of this notion.

The analytical structure introduced so far and this double level through which to look at economic systems permits to enucleate a particular configuration for income distribution, which is directly linked to the dynamic characteristics of the system we are dealing with. It concerns the determination of a set of sectoral profit rates, non necessarily uniform among the sectors, that, if fulfilled, would guarantee the replication of the system

and its expansion according to the evolution of the final demand of each commodity. Remember that r_m is the percentage rate of increase of final demand of commodity m; if population increases at rate g then the final demand of commodity m will increase at rate $g + r_m$, $m = 1, \ldots, M$. The profit rate that would permit to the vertically integrated sector of commodity m to finance this evolution of final demand is thus

$$\pi_m^* = \pi_{k_m}^* = g + r_m, \quad m = 1, \ldots, M. \tag{10}$$

Equations (10) determine an 'ideal' structure of profit rates, that, if realized, would put each (vertically integrated) sector in the conditions to expand their production according to the evolution of final wants. A configuration of sectoral profit rates that would permit a sectoral allocation of (a part of) the surplus of the system according to the evolution of social needs.

The adoption of this particular distribution configuration has a peculiar consequence for the resulting price system. To begin consider a simplified case of the model considered so far, where final goods are produced by means of labour and capital goods, while capital goods are produced by labour only. This can be obtained from the previous model by setting $\gamma_m = T_{k_m} = 0$ for $m = 1, \ldots, M$ in equations (3); then solve with respect to prices and insert conditions (10); the solutions for the price system become:

$$\begin{cases} p_m = w[a_{Nm} + (1/T_m)a_{Nk_m} + (g + r_m)a_{Nk_m}], & m = 1, \cdots, M \\ p_{k_m} = wa_{Nk_m} & m = 1, \cdots, M. \end{cases} \tag{11}$$

Prices are thus entirely determined by labour quantities: this appears immediately from the prices of capital goods; the price of each final good m is the sum of three components: the labour employed in its production (*direct labour*); the labour employed in the production of the fraction of productive capacity consumed in the production of final good m (*indirect labour*); and the labour that the system must devote to produce the additional productive capacity necessary to expand the supply of final good m in line with the expansion of its final demand (*hyper-indirect labour*, according to Pasinetti (1981, chap. VII, Section 4)). A renewed form of *labour theory of value* comes thus to hold, though under a different interpretation.[7]

This is a relevant result, as it appears in a context where no particular assumption has

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[7]This result can be easily extended to the general case considered in the previous pages, where capital goods were produced by means of labour and capital goods. In that case, after substitution of equations (10) in the price equations of the various capital good k_m (see system (8)), we can write

$$p_{k_m}[1 - \gamma_m/T_{k_m} - (r_m + g)\gamma_m] = a_{Nk_m}w, \quad m = 1, \ldots, M. \tag{$8k_m$}$$

been made on technology; the very responsible of such a result is the fixation of sectoral profit rates at their natural levels: a situation that—as we will see in a moment—do not *describe* what actually happens in capitalist economies; it is just an ideal situation. Labour quantities involved in prices (11) will not reflect thus *actual* exchange ratios.

Besides the resurrection of a particular form of *labour theory of value* the present construction allows us to individuate a set of sectoral profit rates that make possible the expansion of the production of each final good according to the evolution of its final demand. This provide a 'social' justification for profits.

It is evident that conditions (10) cannot be satisfied in capitalist economic systems, where rates of profits tend to be uniform among sectors. Notwithstanding conditions (10) discover a very relevant point, that is, which is the very role of profits for a growing system: financing the growth of the system. This—we could say—represents the *natural* justification for positive profits and fixes at the same time the sectoral levels that achieve the purpose. We will return on this point in the following section.

Three observations may be done here.

1. In the natural configuration profits appear just as the source for financing investments, and not as an income of some class, typically capitalists. These latter or, more in general, the promoters and the coordinators of production activities, perceive another income for their work: wages, and their consumption comes from these wages. In other words there is no strictly technical (i.e. 'natural') reason for the working of the system that profits are perceived by capitalists to finance their consumptions.

2. The wage rate is *uniform* among sectors and among the different kinds of labour that are employed; it would be obviously possible to take into account the differences in the remunerations of labour activities by adopting, for example, the device suggested by Sraffa to reduce any difference in quality of labour to differences in quantity (Sraffa (1960, chapter II, § 9)); however this is not a necessity in the natural configuration. One hour work of the engineer is paid as an hour work of the baker or

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Each of these equations refers to a process that produces 1 unit of capital good k_m as gross product. The magnitude in brackets on the left-hand side is the corresponding net quantity produced of capital good k_m; it is obtained by deducing from the unit of capital good produced (the gross product) the depreciation (γ_m/T_{k_m}) of the capital employed to produce this unit, and the quantity of it that—for each unit of gross production of capital good k_m—it is necessary to expand the productive capacity of commodity m according to the evolution of its final demand, $(r_m + g)\gamma_m$. Equation $(8k_m)$ affirms that the value of the net product of such a process is proportional to the quantity of labour employed in it: the same hyper-integrated labour theory of value obtained in (11).

of the mason. Each job is thus as important as the other ones: the highly diffused division of labour that characterizes industrialized economies put on the same ground each worker, as he contributes to produce, directly or indirectly, all the commodities produced by the system. This for what concerns the natural 'core' of the system; the institutional setting of actual systems may obviously establish more detailed and articulated wage structures, whose ultimate purpose lies outside the natural level of analysis. Observe how the different perspective laying behind the present approach do not put any emphasis on incentive mechanisms that are often included in wages: this is, in fact, an issue attaining to the *institutional* sphere; it can be differently shaped and tailored from time to time and may be differently solved from system to system, in connection with the cultural and sociological characteristics of the individuals belonging to the system. Although important within actual situations, it is an issue that is excluded from the *fundamental* level of analysis, where only the *necessary* relationships are taken into consideration.

3. A further and crucial element that characterizes profits within the natural system emerges from the fact that in this configuration the price of each commodity resolves entirely in wages: this entails that in this case all profits, being totally reinvested, are immediately re-introduced in the system in the form of wages, the wages that go to pay the hyper-indirect labour.

These net conclusions emerge from—we must remember—the natural configuration, which probably is never realized in actual economic systems. Yet, the role of the natural configuration is to discriminate between what is *necessary* for the best working of the system and what is the result of social, political, historical, that is, the institutional setting of a society. These distinction appears very useful for all those institutions of the economic system that aim to design, or to re-design, the actual set up of the system.

For what concerns income distribution, for example, which is the main focus of the present work, from the natural system we deduce that profits are determined by the growth requirements of the system, even in individuating their sectoral levels. Actual rates of profits higher than their natural levels may be accepted on the basis of some other 'institutional' justification (for example to stand up the competition on international capital markets), but they have not the same 'degree of necessity': their existence could be questioned in connection with other institutional questions, without jeopardizing the reproduction and the desired growth of the system.

5 'Natural' income distribution in capitalist economies

An obvious critical point concerning the natural configuration of income distribution described in the previous section is that a differentiated structure of natural rates of profit (10) is not compatible with capitalist systems, where, on the contrary, the working of capital mobility tends to make the profit rates uniform among sectors. The situation is further complicated by the fact that rates of profits indicated in relations (10) refer to vertically integrated sectors and not to industries: thus each commodity should yield a different rate of profit according to the vertically integrated sector it is employed in: a situation impossible to be realized by any actual economic system. All these observations constitute a true obstacle in recognizing to the natural configuration of income distribution an actual meaning, even under the normative perspective suggested before. In this section we propose a device to avoid these difficulties and to keep the normative content of natural profits in connection with capitalist economic systems.

The natural rates of profits as defined by Pasinetti appear at a first sight as independent each other and on any technical magnitude of the system, as they reflect the growth of demand of each commodity. They depend directly by the evolution of consumers' preferences, which is taken as given for any commodity. But even if the final demand of each final commodity may evolve in a way that can be considered 'arbitrary', the overall expense for final commodities cannot but be linked to the source of growth of individual incomes, that is, the growth of productivity. At sectoral level we cannot envisage a precise relationship between each natural rate of profit (π_m^*) and the corresponding coefficient of productivity growth (ρ_m). But some sort of link between the natural rates of profit and the coefficients of productivity growth must be verified at least on average. We can have a more precise idea of this by re-writing the macroeconomic condition (6) after substitution of the laws of evolution of technical and demand coefficients:[8]

$$
\sum_{m=1}^{M} a_{Nm}(0)a_{mN}(0)e^{(r_m-\rho_m)t} +
$$
$$
+ \sum_{m=1}^{M} \left(\frac{1}{T_m} + g + r_m \right) \frac{T_{k_m}}{T_{k_m}-\gamma_m} a_{Nk_m}(0)a_{mN}(0)e^{(r_m-\rho_{k_m})t} = 1. \tag{12}
$$

---

[8]To obtain formula (12) the investment coefficients of each sector, $a_{km}(t)$, have been replaced by their long-run levels, $(g + r_m)a_{mn}(t)$, according to what obtained by Pasinetti (1981, chap. V, § 4), where a misprint in Pasinetti's original formula (V.4.1) has been corrected.

After defining coefficients ω_m as follows:

$$\omega_m = a_{Nm}(0)a_{mN}(0), \quad m = 1, \ldots, M$$

$$\omega_{k_m} = \left(\frac{1}{T_m} + g + r_m\right)\frac{T_{k_m}}{T_{k_m} - \gamma_m}a_{Nk_m}(0)a_{mN}(0), \quad m = 1, \ldots, M,$$

expression (12) is equivalent to:

$$\sum_{m=1}^{M}\omega_m e^{(r_m - \rho_m)t} + \sum_{m=1}^{M}\omega_{k_m}e^{(r_m - \rho_{k_m})t} = 1. \tag{12'}$$

By substituting equations (10) in (12') we obtain

$$\sum_{m=1}^{M}\omega_m e^{(\pi_m^* + g - \rho_m)t} + \sum_{m=1}^{M}\omega_{k_m}e^{(\pi_m + g - \rho_{k_m})t} = 1. \tag{13}$$

This relation must continue to hold if growth productivity coefficients (ρ_m and ρ_{k_m}) are substituted by their average value, ρ^*, also called the Standard growth rate of productivity:[9]

$$\sum_{m=1}^{M}\omega_m e^{(\pi_m^* + g - \rho^*)t} + \sum_{m=1}^{M}\omega_{k_m}e^{(\pi_m + g - \rho^*)t} = 1. \tag{14}$$

Assuming that condition (13) is satisfied at time, $t = 0$, then

$$\sum_{m=1}^{M}\omega_m + \sum_{m=1}^{M}\omega_{k_m} = 1. \tag{15}$$

---

[9] ρ^* is defined by Pasinetti as a weighted average of the rates of changes of labour productivity of the various sectors, ρ_m and ρ_{k_m} (see Pasinetti (1981, chap. V, § 14)); as argued by Pasinetti the weights to be attributed to each rate of change of productivity are to be found in the addenda of the effective demand equilibrium condition (6). With a slight generalization they are defined by

$$\omega_m(t) = a_{mN}(t)a_{Nm}(t), \quad m = 1, \cdots, M,$$

for each final commodity and by

$$\omega_{k_m}(t) = (g + r_m + 1/T_m)a_{mN}(t)a_{Nk_m}(t), \quad m = 1, \cdots, M,$$

for each capital good. The Standard growth rate of productivity can thus be defined by the following expression,

$$\rho^* = \sum_{m=1}^{M}[\rho_m\omega_m(t) + \rho_{k_m}\omega_{k_m}(t)].$$

Condition (14) will be kept satisfied along time if the various π_m^*s are substituted by their average:

$$\bar{\pi}^* = \rho^* + g. \tag{16}$$

This expression defines thus the average of the natural rates of profits. This is a notable result: the sectoral profit rates may be scattered according to the various levels of the growth rates of final demand of each commodity; nevertheless their average value is linked with the growth rate of productivity and the growth rate of population by the simple relation (16).

It is to be observed that the average of natural rates of profit, $\bar{\pi}^*$, is that *uniform* rate of profit that, if realized, would permit to the system *as a whole* to recruit all financial resources necessary to sustain the growth of each (vertically integrated) sector according to its specific rates of growth, $\pi_m^* = r_m + g$. Through a suitable series of inflows and outflows of financial resources among the sectors, this *uniform* rate of profit makes possible to replicate, in principle, the same performance that in the natural system is sustained by natural profit rates. This is a normative indication with a sphere of application which is much wider than the natural configuration, as it is based on a *uniform* rate of profit, a situation which, in the absence of significant forms of market power is compatible with capitalism. Under this perspective a minimal financial system would be required in order to re-address the profits from sectors with potential growth rates lower than the average towards those sectors with potential growth rates higher than the average.

Obviously the average of natural rates of profits, $\bar{\pi}^*$, represents the *minimum* return necessary to accomplish the potentialities of the system concerning growth. Its *actual* level may be higher with respect to $\bar{\pi}^*$, for, at least, two reasons: i) in order to facilitate the flows of financial resources among the sectors; ii) to recognize to capitalists a further revenue, in addition to the wages obtained for the work of coordination and direction of production activities: an additional revenue that could be granted them in a measure to be determined at an institutional level (and not on a technical basis), essentially to compensate the burden of the risk connected with the undertaking of production activity. This reduces wages correspondingly.

It may be observed that relation (16) is not new in economic analysis. In the proportional growth model presented by von Neumann (1937) the equality between the growth rate and the rate of profit emerges as a consequence of the solution of a minimax problem (in this model the productivity growth rate is zero). The same relation has been obtained within neoclassical aggregate growth models: it selects that particular capital/labour ratio that yields the highest level of consumption per head (the so-called 'golden rule' of accumulation proposed by Phelps (1961)). Moreover the same equality becomes relevant in

the open dynamic Leontief model with alternative techniques: it ensures that the decentralized process of choice of technique (that for any given wage rate selects the technique that yields the highest rate of profit) selects a technique which is also optimal from the social point of view, that is, a technique that entails the highest consumption per head, for any given growth rate.

However different may be the meaning of the equality between the profit rate and the growth rate and the contexts where it has emerged, we have seen that this equality comes ever to have a normative role: roughly it selects the better way to satisfy the variations of final demand of the various commodities made possible by technical change.

6   Concluding remarks

In this work an ideal configuration of income distribution has been depicted for a growing model characterized by structural change. In this 'natural' configuration each sector yields profits exactly in proportion to the financial requirements induced by the evolution of demand of its final good. Under this distributive configuration it is possible to prove that the price of each commodity (final or intermediate) resolves entirely in wages: wages paid to the labour employed to produce the commodity (direct labour), wages paid to the labour employed to produce the means of production of the commodity (indirect labour), and wages paid to the labour that must be employed to expand the production of the commodity according to the evolution of its final demand (hyper-indirect labour). A particular kind of 'labour theory of value' holds thus under the natural configuration.

By consequence in the natural configuration workers would enjoy completely the fruits of their work. In fact, natural profits, once re-invested, resolve immediately into new wages: the wages paid to the hyper-indirect labour. The surplus of natural system is thus entirely attributed to workers (the reversal of the conclusions of old classical economists—in particular Smith, Ricardo and Marx—where the surplus of the system was constituted by profits).

But, unless we consider the unrealistic case of proportional growth, in cases characterized by structural change the natural configuration is characterized by a set of profit rates which are different among sectors. For this reason the natural configuration of income distribution has to be considered just an ideal situation, with no practical probability to take place spontaneously in capitalist economies. Nevertheless this configuration permits to understand the true function of profits in growing economies: that of financing growth. Moreover it could be seen as a sort of reference configuration of the economic system. Under this perspective the notion of average natural profit rate developed in Section 5

should help in individuating a benchmark from which actual profit rate(s) should not depart too much: a band over the average natural profit rate may be justified as reward to capitalists for bearing the risk connected with the undertaking of the production activity, in addition to wages perceived for their working activity. The excess with respect to such a band would be a surplus without economic or social justification. Obviously the amplitude of such a band is a magnitude whose determination belongs to the institutional sphere.

<div align="center">* * *</div>

The general aim of the analysis presented here was to raise a discussion on the emergent and crucial issues concerning income distribution nowadays, on the basis of a theoretical framework grounded within the Surplus approach and the Keynesian tradition. I do not know if the several souls of the schools of thought involved recognize themselves in some measure within the present pages. It is to be hoped that a reduction rather than a strengthening of the differences may result from the discussion of this work.

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