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Abstract

We present an analytically tractable general equilibrium business cycle model that features micro-level investment lumpiness. We prove an exact irrelevance proposition which provides sufficient conditions on preferences, technology, and the fixed cost distribution such that any positive upper support of the fixed cost distribution yields identical equilibrium dynamics of the aggregate quantities normalized by their deterministic steady state values. We also give two conditions for the fixed cost distribution, under which lumpy investment can be important to a first-order approximation: (i) The steady-state elasticity of the adjustment rate is large so that the extensive margin effect is large. (ii) More mass is on low fixed costs so that the general equilibrium price feedback effect is small. Our theoretical results may reconcile some debate and some numerical findings in the literature.

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Keywords: generalized (S,s) rule, lumpy investment, general equilibrium, business cycles, marginal Q, exact irrelevance proposition

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1 Introduction

In this paper, we present an analytically tractable general equilibrium business cycle model that incorporates micro-level convex and nonconvex adjustment costs. Recent empirical studies have documented that nonconvexities of microeconomic capital adjustment are widespread phenomena. Examining a 17-year sample of large, continuing US manufacturing plants, Doms and Dunne (1998) find that typically more than half of a plant's cumulative investment occurs in a single episode. In addition, they find that long periods of relatively small changes are interrupted by investment spikes. Using the Longitudinal Research Database, Cooper and Haltiwanger (2006) find that about 8 percent of observations entail an investment rate near zero. These observations of inaction are complemented by periods of rather intensive adjustment of the capital stock. Cooper and Haltiwanger (2006) also estimate structural parameters of a rich specification of convex and nonconvex adjustment costs.

Given the above evidence, an important question is whether micro-level nonconvexities matter for aggregate macroeconomic dynamics.[1] This question is under significant debate in the literature. In a seminal study, Thomas (2002) challenges the previous partial equilibrium analyses (e.g., Caballero et al. (1995), Caballero and Engel (1999)) by providing a general equilibrium model with lumpy investment.[2] She applies the Dotsey et al. (1999) method and shows quantitatively that lumpy investment is irrelevant for business cycles. Subsequently, Khan and Thomas (2003, 2008) build more general models and use a different numerical method (Krusell and Smith (1998)) to solve the models. They still obtain a similar finding. Their key insight is that the general equilibrium price feedback effect offsets changes in aggregate investment demand.[3] However, some researchers remain unconvinced by the Khan-Thomas finding. Bachmann et al. (2008) and Gourio and Kashyap (2007) argue that both fixed adjustment costs and general equilibrium price movements are important for business cycle analysis. The relative importance of these two effects is sensitive to calibration. Both Bachmann et al. (2008) and Gourio and Kashyap (2007) calibrate a larger size of fixed adjustment costs. Gourio and Kashyap also argue that for the extensive margin effect to be large, the fixed cost distribution

[1] Embedding a partial equilibrium model similar to Abel and Eberly (1998) in a continuous-time general equilibrium framework, Miao (2008) studies the effect of corporate tax policy on long-run equilibrium in the presence of fixed costs and irreversibility.

[2] Veracierto (2002) embeds the partial equilibrium costly irreversibility model of Abel and Eberly (1996) in a general equilibrium business cycles model. Wang and Wen (2009) present a general equilibrium model with irreversible investment to study aggregate and firm-level volatility.

[3] House (2008) finds an approximate irrelevance result numerically in a different setup. In his model, the source of the irrelevance result is not the general equilibrium price movements, but is the nearly infinite intertemporal substitution for the timing of investment resulting from long-lived capital.

must be compressed in the sense that many firms face roughly the same sized fixed costs.

One reason causing the debate is due to the complexity of the general equilibrium models with heterogeneous firms in this literature. Researchers have to apply complicated numerical methods to solve these models. There is no general theoretical result for comparison. In this paper, we propose a benchmark analytically tractable general equilibrium model to understand the debate in the literature. Our model features both convex and nonconvex adjustment costs. Firms face aggregate labor-augmenting technology shocks and investment-specific technology shocks. In addition, firms face idiosyncratic fixed cost shocks, resulting in a generalized (S,s) investment rule as in Caballero and Engel (1999).

Our model is similar to the Khan and Thomas (2003) model with two main differences. First, we assume that the production function has constant returns to scale rather than decreasing returns to scale. Second, we assume that a firm's fixed costs are proportional to its existing capital stock rather than labor costs. These two assumptions allow us to exploit the homogeneity property of firm value to derive a closed-form solution for the generalized (S,s) investment rule. They also allow us to derive exact aggregation so that we can represent aggregate equilibrium dynamics by a system of nonlinear difference equations as in the real business cycle (RBC) literature. In particular, the distribution of capital matters only to the extent of its mean. The benefit of our modelling is that we do not need to use a complicated numerical method (e.g., Krusell and Smith (1998)) to approximate the distribution of capital. The cost is that our model cannot address distributional asymmetry and nonlinearity emphasized by Caballero et al. (1995). Nevertheless, our model is still rich enough for us to analyze business cycles with the essential feature of micro-level lumpiness, but also is tractable enough for us to analyze theoretically the effects of intensive margin, extensive margin, and general equilibrium price movements, which are the most important elements emphasized in the literature.

We derive the following main results. First, we prove an exact irrelevance proposition: If the production function is Cobb-Douglas, preferences are represented by a time-additive expected utility function consistent with balanced-growth path, and the idiosyncratic fixed cost shocks are drawn independently and identically from a power function distribution, then any positive upper support of the fixed cost distribution yields identical equilibrium dynamics of the aggregate quantities normalized by their deterministic steady-state values.

Second, we derive conditions under which lumpy investment is important for aggregate dynamics to a first order approximation. Essentially, we need the extensive margin effect to be large and the general equilibrium price feedback effect to be small. We show that the extensive margin effect is determined by the steady-state elasticity of the adjustment rate with respect to the investment trigger. The larger is this elasticity, the larger is the extensive margin effect. The general equilibrium price feedback effect is determined by preferences and the steady-state ratio of the option value of waiting to the price of capital. When the elasticity of intertemporal substitution is large, the interest rate feedback effect is small. When the fixed cost distribution is more right skewed (i.e. more firms have small fixed costs), the option value of waiting is larger, leading to a weaker general equilibrium wage feedback effect.

Third, we show numerically that introducing fixed costs to a model with convex adjustment costs raises business cycle volatility, but reduces persistence of output, consumption, investment, and hours. In addition, when lumpy investment becomes more important, it brings business cycle moments closer to those in the standard frictionless RBC model.

Our theoretical results may reconcile some of the debate and some of the numerical findings in the literature. For example, Khan and Thomas (2003, 2008) find that when they increase the maximal fixed cost by 10 folds, the equilibrium dynamics nearly have no change. This could be due to the fact that they assume a uniform distribution of fixed costs and a nearly constant-returns-to-scale production function (their calibrated value of returns to scale is 0.905 or 0.896). For the maximal size of fixed costs to matter, we need the production function to have high curvature as shown numerically by Gourio and Kashyap (2007) and Caballero et al. (2008). Gourio and Kashyap (2007) also argue that the fixed cost distribution must be compressed. We show that this feature of the distribution is not essential. What is essential is that the fixed cost distribution must be right skewed and must have a high steady-state elasticity of the adjustment rate.

We emphasize that the size of total fixed costs is not essential for the lumpy investment to be important. Gourio and Kashyap (2007) and Caballero et al. (2008) argue that Khan and Thomas calibrated fixed costs are too small and that raising the size of total fixed costs will make lumpy investment more important. By contrast, we use numerical examples to show that even the size of fixed costs is smaller, lumpy investment can be more important for the reason discussed before.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 analyzes equilibrium properties. Section 4 provides numerical results. Section 5 introduces idiosyncratic investment-specific technology shocks in order to generate differences in target investment levels across firms, thereby helping match the spike rate observed in the data. For tractability, we assume fixed costs are constant. In this case, we still obtain an exact irrelevance result: Given the preceding assumptions on preferences and technology, if the idiosyncratic investment-specific shocks are drawn independently and identically from a Pareto distribution, then any positive constant fixed costs yield identical equilibrium dynamics of the normalized aggregate quantities. Section 6 concludes. An appendix contains all proofs.

2 The Model

We consider an infinite horizon economy. Time is discrete and indexed by $t = 0, 1, 2, ...$ There is a continuum of heterogeneous production units, indexed by j and distributed uniformly over $[0, 1]$. We identify a production unit with a firm or a plant. There is a continuum of identical households, who trade all firms' shares. Each firm is subject to aggregate labor-augmenting productivity shocks and investment-specific technology shocks. In addition, each firm is subject to idiosyncratic shocks to fixed adjustment costs of investments. To focus on the implications of fixed costs for business cycles, we abstract from long-run growth. It is straightforward to incorporate growth because our model assumptions are consistent with balanced growth.

2.1 Firms

All firms have identical production technology that combines labor and capital to produce output. Specifically, if firm j owns capital K_t^j and hires labor N_t^j, it produces output Y_t^j according to the production function:

$$Y_t^j = F\left(K_t^j, A_t N_t^j\right), \tag{1}$$

where A_t represents aggregate labor-augmenting technology shocks and follows a Markov process given by:

$$\ln A_{t+1} = \rho_A \ln A_t + \sqrt{1-\rho_A^2}\,\sigma_A e_{A,t+1}.$$

Here, $\rho_A \in (0, 1)$, $\sigma_A > 0$ and $e_{A,t}$ is an identically and independently distributed (iid) standard normal random variable. Assume that F is strictly increasing, strictly concave, continuously differentiable, and satisfies the usual Inada conditions. In addition, it has constant returns to scale.

Each firm j may make investments I_t^j to increase its existing capital stock K_t^j. Investment incurs both nonconvex and convex adjustment costs. As in Uzawa (1969), Baxter and Crucini (1993), and Jermann (1998), capital accumulation follows the law of motion:

$$K_{t+1}^j = (1-\delta)K_t^j + K_t^j \Phi\left(\frac{I_t^j}{K_t^j}\right),\ K_0^j \text{ given}, \tag{2}$$

where δ is the depreciation rate and Φ represents convex adjustment costs. To facilitate analytical solutions, we follow Jermann (1998) and specify the convex adjustment cost function as:

$$\Phi(x) = \frac{\psi}{1-\theta} x^{1-\theta} + \varsigma, \tag{3}$$

where $\psi > 0$ and $\theta \in (0,1)$. Nonconvex adjustment costs are fixed costs that must be paid if and only if the firm chooses to invest. As in Cooper and Haltiwanger (2006), we measure these costs as a fraction of the firm's capital stock.[4] That is, if firm j makes new investment, then it pays fixed costs $\xi_t^j K_t^j$, which is independent of the amount of investment. As will be clear later, this modeling of fixed costs is important to ensure that firm value is linearly homogenous. Following Caballero and Engel (1999), we assume that ξ_t^j is identically and independently drawn from a distribution with density ϕ over $[0, \xi_{\max}]$ across firms and across time.

Each firm j pays dividends to households who are shareholders of the firm. Dividends are given by:

$$D_t^j = Y_t^j - w_t N_t^j - \frac{I_t^j}{z_t} - \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} \tag{4}$$

where w_t is the wage rate, and z_t represents aggregate investment-specific technology shocks. Here $\mathbf{1}_{I_t^j \neq 0}$ is an indicator function taking value 1 if $I_t^j \neq 0$, and value 0, otherwise. Assume z_t follows a Markov process given by:

$$\ln z_{t+1} = \rho_z \ln z_t + \sqrt{1-\rho_z^2}\ \sigma_z e_{z,t+1},$$

where $\rho_z \in (0,1)$, $\sigma_z > 0$, and $e_{z,t}$ is an iid standard normal random variable. All random variables A_t, z_t and ξ_t^j are mutually independent.

Firm j's objective is to maximize cum-dividends market value of equity P_t^j:

$$\max\ P_t^j \equiv E_t \left[\sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s}^j \right], \tag{5}$$

subject to (2) and (4). Here, $\beta^s \Lambda_{t+s}/\Lambda_t$ is the stochastic discount factor between period t and $t+s$. We will show later that Λ_{t+s} is a household's marginal utility in period $t+s$.

2.2 Households

All households are identical and have the same utility function:

$$E\left[\beta^t \sum_{t=0}^{\infty} U(C_t, 1-N_t)\right], \tag{6}$$

[4] There are several ways to model fixed adjustment costs in the literature. Fixed costs may be proportional to the demand shock (Abel and Eberly (1998)), profits (Caballero and Engel (1999) and Cooper and Haltiwanger (2006)), or labor costs (Thomas (2002) and Khan and Thomas (2003, 2008)).

where $\beta \in (0,1)$ is the discount factor, and U is a strictly increasing, strictly concave and continuously differentiable function that satisfies the usual Inada conditions. Each household chooses consumption C_t, labor supply N_t, and share holdings α_{t+1}^j to maximize utility (6) subject to the budget constraint:

$$C_t + \int \alpha_{t+1}^j \left(P_t^j - D_t^j \right) dj = \int \alpha_t^j P_t^j dj + w_t N_t. \tag{7}$$

The first-order conditions are given by:

$$\Lambda_t \left(P_t^j - D_t^j \right) = E_t \beta \Lambda_{t+1} P_{t+1}^j, \tag{8}$$

$$U_1 \left(C_t, 1 - N_t \right) = \Lambda_t, \tag{9}$$

$$U_2 \left(C_t, 1 - N_t \right) = \Lambda_t w_t. \tag{10}$$

Equations (8)-(9) imply that the stock price P_t^j is given by the discounted present value of dividends as in equation (5). In addition, Λ_t is equal to the marginal utility of consumption.

2.3 Competitive Equilibrium

The sequences of quantities $\{I_t^j, N_t^j, K_t^j\}_{t\geq 0}$, $\{C_t, N_t\}_{t\geq 0}$, and prices $\{w_t, P_t^j\}_{t\geq 0}$ for $j \in [0,1]$ constitute a competitive equilibrium if the following conditions are satisfied:

(i) Given prices $\{w_t\}_{t\geq 0}$, $\{I_t^j, N_t^j\}_{t\geq 0}$ solves firm j's problem (5) subject to the law of motion (2).

(ii) Given prices $\left\{ w_t, P_t^j \right\}_{t\geq 0}$, $\{C_t, N_t, \alpha_{t+1}^j\}_{t\geq 0}$ maximizes utility in (6) subject to the budget constraint (7).

(iii) Markets clear in that:

$$\alpha_t^j = 1,$$

$$N_t = \int N_t^j dj,$$

$$C_t + \int \frac{I_t^j}{z_t} dj + \int \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} dj = \int F\left(K_t^j, A_t N_t^j \right) dj. \tag{11}$$

3 Equilibrium Properties

We start by analyzing a single firm's optimal investment policy, holding prices fixed. We then conduct aggregation and characterize equilibrium aggregate dynamics by a system of nonlinear difference equations. We show that the equilibrium is constrained efficient. Next, we analyze steady state and prove an exact irrelevance result. Finally, we log-linearize the equilibrium dynamic system and examine the conditions under which lumpy investment can be important.

3.1 Optimal Investment Policy

To simplify problem (5), we first solve a firm's static labor choice decision. Let $n_t^j = N_t^j / K_t^j$. The first-order condition with respect to labor yields:

$$f'\left(A_t n_t^j\right) A_t = w_t, \tag{12}$$

where we define $f(\cdot) = F(1, \cdot)$. This equation reveals that all firms choose the same labor-capital ratio in that $n_t^j = n_t = n(w_t, A_t)$ for all j. We can then derive firm j's operating profits:

$$\max_{N_t^j} F\left(K_t^j, A_t N_t^j\right) - w_t N_t^j = R_t K_t^j,$$

where $R_t = f(A_t n_t) - w_t n_t$ is independent of j. Note that R_t also represents the marginal product of capital because F has constant returns to scale. Let $i_t^j = I_t^j / K_t^j$ denote firm j's investment rate. We can then express dividends in (4) as:

$$D_t^j = \left[R_t - \frac{i_t^j}{z_t} - \xi_t^j \mathbf{1}_{i_t^j \neq 0}\right] K_t^j,$$

and rewrite (2) as

$$K_{t+1}^j = \left[(1-\delta) + \Phi(i_t^j)\right] K_t^j. \tag{13}$$

The above two equations imply that equity value or firm value are linear in capital K_t^j. We can then write firm value as $V_t^j K_t^j$ and rewrite problem (5) by dynamic programming:

$$V_t^j K_t^j = \max_{i_t^j} \left[R_t - \frac{i_t^j}{z_t} - \xi_t^j \mathbf{1}_{i_t^j \neq 0}\right] K_t^j + E_t\left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j K_{t+1}^j\right], \tag{14}$$

subject to (13). Substituting equation (13) into equation (14), we rewrite problem (14) as:

$$V_t^j = \max_{i_t^j} \ R_t - \frac{i_t^j}{z_t} - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j) E_t\left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\right], \tag{15}$$

where we define:

$$g(i_t^j) = 1 - \delta + \Phi(i_t^j). \tag{16}$$

Note that R_t and Λ_t depend on current aggregate state (K_t, A_t, z_t) only. Suppose the equilibrium law of motion for aggregate capital is given by:

$$K_{t+1} = G\left(K_t, A_t, z_t\right). \tag{17}$$

Given this law of motion, Λ_{t+1} is also a function of (K_t, A_t, z_t). Thus, the state variable for V_t^j is (K_t, A_t, z_t, ξ_t^j). We can write it as

$$V_t^j = V\left(K_t, A_t, z_t, \xi_t^j\right),$$

for some function V. We aggregate each firm's price of capital V_t^j and define the aggregate value of the firm per unit of capital conditioned on aggregate state (K_t, A_t, z_t) as:

$$\bar{V}_t = \bar{V}(K_t, A_t, z_t) = \int_0^{\xi_{\max}} V(K_t, A_t, z_t, \xi)\,\phi(\xi)d\xi, \tag{18}$$

for some function $\bar{V}$. Because ξ_t^j is iid across both time and firms and is independent of aggregate shocks, we obtain:

$$E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}V_{t+1}^j\right] = E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\int_0^{\xi_{\max}} V(K_{t+1}, A_{t+1}, z_{t+1}, \xi)\,\phi(\xi)d\xi\right] = E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}\right], \tag{19}$$

We can now rewrite problem (15) as:

$$V\left(K_t, A_t, z_t, \xi_t^j\right) = \max_{i_t^j}\ R_t - \frac{i_t^j}{z_t} - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j)E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}\right], \tag{20}$$

where R_t, Λ_t, Λ_{t+1}, and $\bar{V}_{t+1}$ depend on the aggregate state (K_t, A_t, z_t) and the aggregate capital stock follows the law of motion (17). Firm j takes these variables and the aggregate law of motion as given. From problem (20), we can characterize a firm's optimal investment policy by a generalized (S,s) rule (Caballero and Engel (1999)). In so doing, we first define (aggregate) marginal Q as the (risk-adjusted) present value of a marginal unit of investment:

$$Q_t = E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}\right]. \tag{21}$$

Since investment becomes productive with a one period delay, marginal Q is equal to the discounted expected value of the firm of an additional unit of capital in the next period. In continuous time, the difference between marginal Q and the aggregate price of capital $\bar{V}_{t+1}$ disappears. Because firm value is linearly homogeneous in capital, Tobin's average Q is equal to the marginal Q (Hayashi (1982)).

Proposition 1 *Firm j's optimal investment policy is characterized by the (S, s) policy in that there is a unique trigger value $\xi_t^* > 0$ such that the firm invests if and only if $\xi_t^j \leq \min\{\xi_t^*, \xi_{\max}\}$. The trigger value ξ_t^* satisfies the equation:*

$$\frac{\theta}{1-\theta} z_t^{\frac{1-\theta}{\theta}} (\psi Q_t)^{\frac{1}{\theta}} = \xi_t^*. \tag{22}$$

The optimal target investment level is given by:

$$i_t^j = (\psi z_t Q_t)^{\frac{1}{\theta}}. \tag{23}$$

When $\xi_t^* \leq \xi_{\max}$, *marginal* Q *satisfies:*

$$Q_t = E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left\{R_{t+1} + (1-\delta+\varsigma)\,Q_{t+1} + \int_0^{\xi_{t+1}^*}\left[\xi_{t+1}^* - \xi\right]\phi(\xi)d\xi\right\}\right]. \tag{24}$$

Equation (22) says that, at the value ξ_t^*, the benefit from investment is equal to the fixed cost of investment. The benefit from investment increases with Q_t and z_t. Thus, the investment trigger ξ_t^* also increases with Q_t and z_t. If $\xi_t^* \geq \xi_{\max}$, then the firm always invests. In the aggregate with a cross section of firms, this means that all firms decide to invest. In the analysis below, we will focus on an interior solution for which $\xi_t^* < \xi_{\max}$.

Note that the investment trigger ξ_t^* depends on the aggregate state (K_t, A_t, z_t) only. It does not depend on the firm-specific state (K_t^j, ξ_t^j). This observation implies that conditioned on the aggregate state, the adjustment hazard, $\int_0^{\xi_t^*}\phi(\xi)\,d\xi$, is a constant. This result is due to our assumptions of competitive markets, constant-returns-to-scale production function, and the iid distribution of ξ_t^j. When the production function has decreasing returns to scale or there is monopoly power, the investment trigger ξ_t^* and the adjustment hazard will depend on the firm-specific capital stock, as discussed in Caballero et al. (1995), Caballero and Engel (1999), and Khan and Thomas (2003, 2008).

Equation (23) implies that all firms choose identical target investment level, which is inconsistent with empirical evidence on investment spikes (Cooper and Haltiwanger (2006)). One way to make investment targets depend on firm-specific characteristics is to introduce persistent idiosyncratic productivity shock (Khan and Thomas (2008)). This extension will complicate our analysis significantly, and is left for future research. An alternative way is to introduce idiosyncratic investment-specific shocks. We will study this setup in Section 5.

Equation (23) shows that the optimal investment level is positively related to marginal Q if and only if the firm's idiosyncratic shock ξ_t^j is lower than the trigger value ξ_t^*, conditioned on the aggregate state (K_t, A_t, z_t). When $\xi_t^j > \xi_t^*$, firm j chooses not to invest. This zero investment is unrelated to marginal Q. As a result, investment may not be related to marginal Q in the presence of fixed adjustment costs, a point made by Caballero and Leahy (1996).

Equation (24) is a type of asset-pricing equation. Ignoring the integration term inside the conditional expectation operator in equation (24), this equation states that the expected price of capital or marginal Q is equal to the risk-adjusted present value of marginal product of capital. The integration term in (24) reflects the option value of waiting because of the fixed adjustment costs. When the shock $\xi_t^j > \xi_t^*$, it is not optimal to pay the fixed costs to make investment. Firms will wait to invest until $\xi_t^j \leq \xi_t^*$ and there is an option value of waiting.

3.2 Aggregation and Equilibrium Characterization

Given the linear homogeneity feature of firm value, we can conduct aggregation tractably. We define aggregate capital $K_t = \int K_t^j dj$, aggregate labor demand $N_t = \int N_t^j dj$, aggregate output $Y_t = \int Y_t^j dj$, and aggregate investment expenditure in consumption units $I_t = \int I_t^j / z_t dj$.

Proposition 2 *The aggregate equilibrium sequences $\{Y_t,\ N_t,\ C_t,\ I_t,\ K_t, Q_t, \xi_t^*\}_{t\geq 0}$ are characterized by the following system of difference equations:*

$$\xi_t^* = \frac{\theta}{1-\theta} z_t^{\frac{1-\theta}{\theta}} \left(\psi Q_t\right)^{\frac{1}{\theta}}, \tag{25}$$

$$I_t = \left(\psi Q_t\right)^{\frac{1}{\theta}} z_t^{\frac{1-\theta}{\theta}} \left[\int_0^{\xi_t^*} \phi(\xi) d\xi\right] K_t, \tag{26}$$

$$K_{t+1} = (1-\delta+\varsigma) K_t + \frac{\psi}{1-\theta} K_t \left(z_t I_t / K_t\right)^{1-\theta} \left[\int_0^{\xi_t^*} \phi(\xi) d\xi\right]^{\theta}, \tag{27}$$

$$Y_t = F\left(K_t, A_t N_t\right) = I_t + C_t + K_t \int_0^{\xi_t^*} \xi \phi(\xi) d\xi, \tag{28}$$

$$\frac{U_2\left(C_t, 1-N_t\right)}{U_1\left(C_t, 1-N_t\right)} = A_t F_2\left(K_t, A_t N_t\right), \tag{29}$$

$$\begin{aligned} Q_t \quad &= \quad E_t \left\{ \frac{\beta U_1\left(C_{t+1}, 1-N_{t+1}\right)}{U_1\left(C_t, 1-N_t\right)} \left[F_1\left(K_{t+1}, A_{t+1} N_{t+1}\right) + \left(1-\delta+\varsigma\right) Q_{t+1} \right.\right. \\ &\quad \left.\left. + \int_0^{\xi_{t+1}^*} \left(\xi_{t+1}^* - \xi\right) \phi(\xi) d\xi \right] \right\}. \end{aligned} \tag{30}$$

Equation (25) is identical to (22). We derive equations (26) and (27) by aggregating equations (2) and (23). Equation (26) shows that aggregate investment rate I_t/K_t is positively related to marginal Q as predicted by the standard Q-theory. However, unlike this theory, marginal Q is not a sufficient statistic for the investment rate. In particular, the aggregate state (K_t, A_t, z_t) also helps explain the aggregate investment rate besides marginal Q, via its effect on ξ_t^*.

Equation (28) is the resource constraint. The last term in the equation represents the aggregate fixed adjustment costs. The first equality of equation (28) gives the aggregate output using a single firm's production function F. This result is primarily due to the constant returns to scale property of F. The representative household's consumption/leisure choice gives equation (29). Equation (30) is an asset pricing for the price of capital Q. It is obtained from equation (24). Note that by equations (26) and (25), we can show that the option value of waiting in the second line of (30) is equal to $\frac{\theta}{1-\theta}\frac{I_{t+1}}{K_{t+1}} - \int_0^{\xi_t^*} \xi\phi(\xi)d\xi$.

3.3 Constrained Efficiency

Is the competitive equilibrium we studied efficient? To answer this question, we consider a social planner's problem in which he faces the same investment frictions as individual firms. Suppose the planner selects an investment trigger ξ_t^* such that all firms make investments when the idiosyncratic fixed adjustment cost shock $\xi_t \le \xi_t^*$. We can then aggregate individual firms' capital and investments to obtain the resource constraint (28) and the capital accumulation equation (27). The social planner's problem is to maximize the representative agent's utility (6) subject to these two equations.

Proposition 3 *The competitive equilibrium allocation and the investment trigger characterized in Proposition 2 are constrained efficient in the sense that they are identical to those obtained by solving a social planner's problem.*

3.4 Steady State

We consider a deterministic steady state in which there is no aggregate shock to labor augmenting technology and no aggregate shock to investment-specific technology, but there is still idiosyncratic fixed costs shock. In this case, steady-state aggregate variables $(Y, C, N, K, I, Q, \xi^*)$ are deterministic constants by a law of large numbers.

Proposition 4 *Consider the lumpy investment model. Suppose $\delta > \varsigma$ and*

$$\delta - \varsigma < \frac{\xi_{\max}^{1-\theta}\psi}{(1-\theta)^{\theta}\,\theta^{(1-\theta)}} \int_0^{\xi_{\max}} \phi(\xi)d\xi.$$

Then the steady-state investment trigger $\xi^* \in (0, \xi_{\max})$ is the unique solution to the equation:

$$\delta - \varsigma = \frac{\xi^{*1-\theta}\psi}{(1-\theta)^{\theta}\,\theta^{(1-\theta)}} \int_0^{\xi^*} \phi(\xi)d\xi. \tag{31}$$

Given this value ξ^*, the steady-state value of Q is given by:

$$Q = \frac{1}{\psi}\left(\frac{\xi^*(1-\theta)}{\theta}\right)^{\theta}. \tag{32}$$

The other steady-state values (I, K, C, N) satisfy:

$$\frac{I}{K} = (\delta - \varsigma)(1 - \theta) Q, \tag{33}$$

$$F(K, N) = I + C + K \int_0^{\xi^*} \xi \phi(\xi) \, d\xi, \tag{34}$$

$$\frac{U_2(C, 1-N)}{U_1(C, 1-N)} = F_2(K, N), \tag{35}$$

$$Q = \frac{\beta}{1 - \beta(1 - \delta + \varsigma)} \left\{ F_1(K, N) + \int_0^{\xi^*} (\xi^* - \xi) \phi(\xi) d\xi \right\}. \tag{36}$$

The investment trigger ξ^* is uniquely determined by equation (31), which states that, for the aggregate capital stock to be constant over time, new investment must offset capital depreciation. The steady-state aggregate price of capital is determined by equation (32), which follows from equation (25). At this price, a firm is just willing to pay the fixed cost to invest if the shock to its new investment just hits the trigger value ξ^*.

The other steady-state values (I, K, C, N) are determined by a system of four equations (33)-(36). In particular, equation (33) implies that the steady-state investment rate increases with the aggregate price of capital Q. Equation (36) shows that Q must satisfy a steady-state version of an asset-pricing equation, which states that it is equal to the present value of the marginal product of capital plus the option value of waiting.

We are unable to derive analytical comparative statics results for the steady sate values of (I, K, C, N) under general conditions because they are determined by a system of four nonlinear equations. If we make some specific assumptions on preferences and technology, we have the following sharp comparative statics results:

Corollary 1 *Consider the power function distribution with density $\phi(\xi) = \frac{\eta \xi^{\eta-1}}{(\xi_{\max})^{\eta}}$, $\eta > 0$. Assume that the parameter values are such that the inequality in (37) holds. Then the steady-state trigger value is given by:*

$$\xi^* = \left[\psi^{-1} (\delta - \varsigma)(1 - \theta)^{\theta} \theta^{(1-\theta)} \xi_{\max}^{\eta} \right]^{\frac{1}{\eta + 1 - \theta}} < \xi_{\max}. \tag{37}$$

In addition, consider the following specifications:

$$F(K, AN) = K^{\alpha} (AN)^{1-\alpha}, \ \alpha \in (0, 1), \tag{38}$$

$$U(C, 1-N) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} v(1-N) & \text{if } \gamma > 0, \neq 1 \\ \log(C) + v(1-N) & \text{if } \gamma = 1 \end{cases}, \tag{39}$$

where v is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada conditions. Then the steady-state values R/Q, $I/Y, C/Y$ and N are independent of $\xi_{\max}$. In addition,

$$\frac{\partial (I/K)}{\partial \xi_{\max}} > 0, \ \frac{\partial Q}{\partial \xi_{\max}} > 0, \ \frac{\partial R}{\partial \xi_{\max}} > 0, \ \frac{\partial w}{\partial \xi_{\max}} < 0, \tag{40}$$

$$\frac{\partial K}{\partial \xi_{\max}} < 0, \ \frac{\partial Y}{\partial \xi_{\max}} < 0, \ \frac{\partial C}{\partial \xi_{\max}} < 0, \ \textit{and} \ \frac{\partial I}{\partial \xi_{\max}} < 0. \tag{41}$$

As $\xi_{\max}$ increases, the power function distribution is more spread out. Thus, less firms will adjust capital for a given investment trigger. To raise the aggregate investment rate to compensate capital depreciation, the steady-state investment trigger ξ^* must rise, as shown in equation (37). As a result, the steady-state investment rate I/K and Q increase with $\xi_{\max}$. Under the additional assumptions on preferences and technology, both I and K decrease with $\xi_{\max}$, but K decreases faster than I. This in turn implies that the steady-state output Y and consumption C decrease with $\xi_{\max}$. In addition, the steady-state rental rate of capital R increases with $\xi_{\max}$, but the steady-state wage rate w decreases with $\xi_{\max}$ because capital becomes relatively scarce.

The surprising result is that the steady-state values of R/Q, $I/Y, C/Y$ and N are independent of $\xi_{\max}$. An important assumption for this result is that the distribution of the fixed costs is a power function, which has a homogeneity property. Our assumed functions for preferences and technology also have a homogeneity property. These two homogeneity properties are key to the independence result. We emphasize that the assumptions on preferences and technology in Corollary 1 are standard in macroeconomics and are consistent with balanced growth (e.g., King et al. (2002)). We next use Corollary 1 to study aggregate dynamics.

3.5 An Exact Irrelevance Result

We normalize an aggregate variable by its steady-state value characterized in Proposition 4. We let $\tilde{X}_t = X_t/X$ denote this normalized value of X_t when its deterministic steady-state value is X. We have the following irrelevance result:

Proposition 5 *Suppose the assumptions in Corollary 1 are satisfied. Then any maximal fixed cost $\xi_{\max} > 0$ does not affect the equilibrium system of nonlinear difference equations that characterizes aggregate dynamics of the normalized variables $\{\tilde{Y}_t, \tilde{N}_t, \tilde{C}_t, \tilde{I}_t, \tilde{K}_t, \tilde{Q}_t, \tilde{\xi}_t^*\}_{t\geq 0}$.*

Proposition 5 demonstrates that the maximal fixed cost $\xi_{\max} > 0$ matters for aggregate dynamics only to the extent that it affects the steady state. The system of difference equations that characterizes the normalized variables relative to their steady-state values do not depend on $\xi_{\max}$. As a result, $\xi_{\max}$ does not affect the second moment and impulse response properties of the normalized aggregate variables or the logarithms of these variables.

The intuition behind this proposition is that the system of nonlinear difference equations for the normalized equilibrium variables has a homogeneity property so that it is fully determined by the model parameters except for $\xi_{\max}$ and the steady-state values R/Q, $I/Y, C/Y$ and N. By Corollary 1, these steady state values are also independent of $\xi_{\max}$. Thus, the dynamic system is independent of $\xi_{\max}$. The key condition for this result is that the distribution of the idiosyncratic fixed cost shock is a power function. Other conditions are standard in the RBC literature.

We emphasize that this result does not imply that aggregate dynamics with fixed adjustment costs ($\xi_{\max} > 0$) are the same as those in a model without fixed adjustment costs ($\xi_{\max} = 0$), because the dynamic systems of the (normalized) aggregate variables in the two models are different. That is, there is discontinuity when $\xi_{\max}$ moves from 0 to a positive number. Importantly, the shape of the fixed cost distribution plays an important role in the lumpy adjustment model. To analyze this issue more transparently, we next consider a log-linearized equilibrium system.

3.6 Log-Linearized System

We first note that the equilibrium wage rate $w_t = A_t F_2 (K_t, A_t N_t)$ and the equilibrium gross interest rate r_{t+1} satisfies $U_1 (C_t, 1 - N_t) = E_t [\beta U_1 (C_{t+1}, 1 - N_{t+1}) r_{t+1}]$. Using these two equations, we log-linearize the dynamic system given in Proposition 2 around the deterministic steady state and obtain the following proposition after some tedious algebra. We use $\hat{X}_t = (X_t - X) / X$ to denote the deviation of a variable X_t from its steady state value X.

Proposition 6 *The log-linearized equilibrium dynamic system is given by:*

$$\hat{\xi}_t^* = \frac{1-\theta}{\theta} \hat{z}_t + \frac{1}{\theta} \hat{Q}_t, \tag{42}$$

$$\hat{I}_t - \hat{K}_t = \left(\frac{1}{\theta} \hat{Q}_t + \frac{1-\theta}{\theta} \hat{z}_t \right) + \frac{\xi^* \phi(\xi^*)}{\int_0^{\xi^*} \phi(\xi) d\xi} \hat{\xi}_t^*, \tag{43}$$

$$\hat{K}_{t+1} = (1 - \delta + \varsigma) \hat{K}_t + \theta \hat{K}_t + (1-\theta)(\hat{I}_t + \hat{z}_t) + \frac{\theta \xi^* \phi(\xi^*)}{\int_0^{\xi^*} \phi(\xi) d\xi} \hat{\xi}_t^*, \tag{44}$$

$$\hat{Y}_t = \frac{F_1 K}{Y} \hat{K}_t + \left(1 - \frac{F_1 K}{Y} \right) [\hat{A}_t + \hat{N}_t], \tag{45}$$

$$\hat{Y}_t = \frac{I}{Y}\hat{I}_t + \frac{C}{Y}\hat{C}_t + \left(1 - \frac{I}{Y} - \frac{C}{Y}\right)\left[\hat{K}_t + \frac{(\xi^*)^2\,\phi(\xi^*)}{\int_0^{\xi^*}\xi\phi(\xi)d\xi}\hat{\xi}_t^*\right], \tag{46}$$

$$\begin{aligned}\hat{Q}_t &= \beta E_t\hat{Q}_{t+1} - E_t\hat{r}_{t+1} + \beta(\delta - \varsigma)(1-\theta)E_t\hat{z}_{t+1} \\ &\quad + \frac{-F_{11}F_2}{F_{21}F_1}\frac{\beta R}{Q}E_t\left[\hat{A}_{t+1} - \hat{w}_{t+1}\right],\end{aligned} \tag{47}$$

$$E_t\hat{r}_{t+1} = u_{C,C}\hat{C}_t - u_{C,N}\hat{N}_t - E_t\left(u_{C,C}\hat{C}_{t+1} - u_{C,N}\hat{N}_{t+1}\right), \tag{48}$$

$$\begin{aligned}\hat{w}_t &= (u_{N,C} - u_{C,C})\hat{C}_t + (u_{C,N} - u_{N,N})\hat{N}_t \\ &= \frac{KF_{21}}{F_2}\hat{K}_t + \frac{NF_{22}}{F_2}\hat{N}_t + \left(1 + \frac{NF_{22}}{F_2}\right)\hat{A}_t,\end{aligned} \tag{49}$$

where we denote $u_{N,C} = \frac{CU_{21}(C,1-N)}{U_2(C,1-N)}$, $u_{N,N} = \frac{NU_{22}(C,1-N)}{U_2(C,1-N)}$, $u_{C,C} = \frac{CU_{11}(C,1-N)}{U_1(C,1-N)}$, *and* $u_{C,N} = \frac{NU_{12}(C,1-N)}{U_1(C,1-N)}$.

This proposition demonstrates explicitly how parameters for preferences, technology, and the fixed cost distribution determine the log-linearized equilibrium system. Equation (42) shows that changes in the investment-specific technology shock or in the price of capital determine changes in the investment trigger, and thus changes in the likelihood of capital adjustment and in the number of adjustors. This effect is often referred to as the *extensive margin effect*. Equation (43) shows that changes in the aggregate investment rate are determined by an *intensive margin effect* and an extensive margin effect. The intensive margin effect represented by the expression in the bracket on the right hand side of (43) determines the size of the aggregate investment rate. The magnitude of the extensive margin effect on the aggregate investment rate is determined by the steady-state elasticity the adjustment rate with respect to the investment trigger, $\xi^*\phi(\xi^*)/\int_0^{\xi^*}\phi(\xi)d\xi$. In order for lumpy investment to matter for business cycles, the extensive margin effect must be large. This requires the elasticity of the adjustment rate with respect to the investment trigger to be large. We will give some examples in the next section to illustrate this point.

Both the intensive margin and extensive margin effects are affected by the general equilibrium price movements because changes in the wage rate and in the interest rate affect the changes in the price of capital, as revealed by equation (47). The change in the interest rate is determined by preferences. As equation (48) shows, when the elasticity of intertemporal substitution is larger, the consumption smoothing incentive is stronger, leading to a smaller interest rate movement.

The magnitude of the wage movements is determined by the preferences and technology parameters as revealed by equation (49). Equation (47) shows that the wage feedback effect is magnified by the steady-state ratio of the marginal product of capital to Q or R/Q. Because the steady-state Q is equal to the present value of R and the option value of waiting as revealed by (36), the larger is the option value of waiting, the smaller is R/Q. We can show that holding the adjustment rate and the investment trigger fixed, if more low fixed costs have high probabilities or the fixed cost distribution is more right skewed, the option value of waiting is higher. In this case, R/Q is smaller and thus the wage feedback effect is smaller.

In summary, both the micro-level investment lumpiness and the general equilibrium price movements are important to determine aggregate dynamics. The relative importance of these two effects is determined by the preference and technology parameters and the distribution of the idiosyncratic fixed cost shock. In particular, holding preferences and technology fixed, if the steady-state elasticity of the adjustment rate is larger, then the extensive margin effect is stronger. If the fixed cost distribution is more right skewed, then the general equilibrium wage feedback effect is weaker.

4 Numerical Results

We evaluate our lumpy investment model quantitatively and compare this model with two benchmark models. The first one is a *frictionless RBC model*, obtained by removing all adjustment costs in the model presented in Section 2. In particular, we set $\xi_t^j = \theta = \varsigma = 0$ and $\psi = 1$. The second one is obtained by removing fixed adjustment costs only ($\xi_t^j = 0$). We call this model *partial adjustment model*.

In both benchmark models, all firms make identical decisions, and thus these models are equivalent to standard representative-firm RBC models (e.g., Fisher (2006) and Greenwood et al. (2000). Specifically, the equilibrium for the partial adjustment economy is characterized by equations (26)-(30), where we set $\xi_t^* = 0$. The equilibrium for the frictionless RBC economy is characterized by equations (27)-(30), where we set $Q_t = 1/z_t$, $\xi_t^* = 0$, $\theta = \varsigma = 0$, and $\psi = 1$. Because we have characterized the equilibria for all three models by systems of nonlinear difference equations as shown in the previous section, we can use the standard second-order approximation method to solve the models numerically.[5] To do so, we need first to calibrate the models.

[5]The Dynare code is available upon request.

4.1 Baseline Parametrization

For all model economies, we take the Cobb-Douglas production function, $F(K, AN) = K^{\alpha}(AN)^{1-\alpha}$, and the period utility function, $U(C, 1-N) = \log(C) - aN$, where $a > 0$ is a parameter. We fix the length of period to correspond to one year, as in Thomas (2002), and Khan and Thomas (2003, 2008). Annual frequency allows us to use empirical evidence on establishment-level investment in selecting parameters for the fixed adjustment costs and the distribution of idiosyncratic investment-specific shocks.

We first choose parameter values for preferences and technology to ensure that the steady-state of the frictionless RBC model is consistent with the long-run values of key postwar U.S. aggregates. Specifically, we set the subjective discount factor to $\beta = 0.96$, so that the implied annual real interest rate is 4 percent (Prescott (1986)). We choose the value of a so that the steady-state hours are about 1/3 of available time spent in market work. We set the capital share $\alpha = 0.36$, implying a labor share of 0.64, which is close to the labor income share in the NIPA. We take the depreciate rate $\delta = 0.1$, as in the literature on business cycles (e.g., Prescott (1986)).

It is often argued that convex adjustment costs are not observable directly and hence cannot be calibrated based on average data over the long run (e.g., Greenwood et al. (2000)). Thus, we impose the two restrictions:

$$\psi = \delta^{\theta} \text{ and } \varsigma = \frac{-\theta}{1-\theta}\delta, \tag{50}$$

so that the partial adjustment model and the frictionless RBC model give identical steady-state allocations.[6] As in our paper, Baxter and Crucini (1993), Jermann (1998), and Greenwood et al. (2000) make similar assumptions for the parameters in the adjustment cost function. We assume condition (50) throughout our numerical experiments below.

We next follow Khan and Thomas (2003) to select parameters for the aggregate shocks. They use Stock and Watson (1999) data set to estimate the persistence and volatility of the Solow residuals equal to 0.9225 and 0.0134, respectively. Transforming the total factor productivity shocks to our labor-augmenting technology shocks, we set $\rho_A = 0.9225$ and $\sigma_A = 0.0134/0.64 = 0.02\,1$. As in Khan and Thomas (2003), we set $\rho_z = 0.706$ and $\sigma_z = 0.017$ in the investment-specific technology shock process. Following Kiyotaki and West (1996), Thomas (2002), and Khan and Thomas (2003), we set $\theta = 1/5.98$, implying that the Q-elasticity of the investment rate is 5.98.

[6] Under the log-linear approximation method, only the curvature parameter θ in the convex adjustment cost function matters for the approximated equilibrium dynamics.

We adopt the power function distribution for the idiosyncratic fixed cost shock. We need to calibrate two parameters $\xi_{\max}$ and η. We try to match micro-level evidence on the investment lumpiness reported by Cooper and Haltiwanger (2006). Cooper and Haltiwanger (2006) find that the inaction rate is 0.081 and the positive spike rate is about 0.186. A positive investment spike is defined as the investment rate exceeding 0.2. For the power function distribution, the steady-state inaction rate is given by $1-(\xi^*/\xi_{\max})^\eta$ and the steady-state investment rate is given by equation (33). Because our model implies that the target investment rate I/K is identical for all firms, our model cannot match the spike rate. Therefore, there are many combinations of η and $\xi_{\max}$ to match the inaction rate. As baseline values, we follow Khan and Thomas (2003, 2008) and take a uniform distribution ($\eta = 1$). This implies that $\xi_{\max} = 0.0242$. In this case, total fixed adjustment costs account for 2.4 percent of output, 10 percent of total investment spending and 1.0 percent of capital stock, which are reasonable according to the estimation by Cooper and Haltiwanger (2006).

We summarize the baseline parameter values in Table 1.

Table 1. Baseline Parameter Vales

β	a	α	δ	ρ_A	σ_A	ρ_z	σ_z	θ	$\xi_{\max}$	η
0.9615	2.5843	0.36	0.1	0.9225	0.021	0.706	0.017	1/5.98	0.0242	1

4.2 Partial Equilibrium Dynamics

In order to understand the general equilibrium effects of fixed costs on business cycles, we start with a partial equilibrium analysis by fixing the wage rate and the interest rate at their steady state values. For the power function distribution, we can show that the elasticity of the adjustment rate is equal to η. Using assumption (50), the specification of the utility function and the production function, and setting $\hat{w}_t = \hat{r}_t = 0$, we can rewrite equations (43) and (47) as:

$$\hat{I}_t - \hat{K}_t = \underbrace{\frac{1}{\theta}\hat{Q}_t + \frac{1-\theta}{\theta}\hat{z}_t}_{\text{intensive}} + \underbrace{\eta\hat{\xi}_t^*}_{\text{extensive}}, \tag{51}$$

$$\hat{Q}_t = \beta E_t\hat{Q}_{t+1} + \delta\beta E_t\hat{z}_{t+1} + \left[1 - \left(1 - \frac{\delta}{1-\theta}\right)\beta - \frac{\beta\theta\delta}{1-\theta}\frac{1}{1+\eta}\right]\frac{1-\alpha}{\alpha}E_t\hat{A}_{t+1}. \tag{52}$$

The last term in the square bracket in equation (52) represents the option value of waiting in the presence of fixed costs. The log-linearized system for the partial adjustment model with fixed prices is obtained by setting $\eta = 0$ and ignoring equation (42).

We now analyze the impulse response properties based on the above log-linearized system. Figure 1 plots the impulse responses to a positive one standard deviation shock to the labor-augmenting technology (N-shock). Following this shock, the marginal product capital rises. Thus, the price of capital or the marginal Q rises. Because there is an option value of waiting, the increase in Q is higher in the lumpy investment model than in the partial adjustment model. The increase in Q has both an intensive and extensive margin effects in the lumpy investment model as revealed by equation (43). In particular, it raises the adjustment rate by 11 percent in the lumpy investment model. Due to this extensive margin effect, the increase in the investment rate in the lumpy investment model is also higher than that in the partial adjustment model (22 percent versus 10 percent).

[Insert Figures 1-2 Here.]

Figure 2 plots the impulse responses to a positive one standard deviation shock to the investment-specific technology (I-shock). Following this shock, the marginal Q rises by the same magnitude in both the lumpy investment and in the partial adjustment model because these two models deliver an identical coefficient of $\hat{z}_t$ in (52). Even though the increase in marginal Q is identical, the investment rate increases much more in the lumpy investment model than in the partial adjustment model (15 percent versus 8 percent). The reason is that the investment-specific technology shock has a direct extensive margin effect by raising the adjustment rate (see equation (42)). In particular, the adjustment rate rises by about 8 percent.

4.3 General Equilibrium Dynamics

We now turn to general equilibrium dynamics by endogenizing the prices. In this case, the general equilibrium price movements play an important role in shaping aggregate dynamics. To see this, we write the log-linearized equation for the marginal Q as:

$$\begin{aligned}\hat{Q}_t &= \beta E_t \hat{Q}_{t+1} + \beta\delta E_t \hat{z}_{t+1} - E_t\left[\hat{r}_{t+1}\right] \\ &\quad + \left[1 - \left(1 - \frac{\delta}{1-\theta}\right)\beta - \frac{\beta\theta\delta}{1-\theta}\frac{1}{1+\eta}\right]\frac{1-\alpha}{\alpha} E_t[A_{t+1} - \hat{w}_{t+1}],\end{aligned} \tag{53}$$

where the equilibrium interest rate and wage rate satisfy

$$E_t\left[\hat{r}_{t+1}\right] = E_t\left[\hat{C}_{t+1}\right] - \hat{C}_t,$$

$$\hat{w}_t = \hat{C}_t = (1-\alpha)\hat{A}_t + \alpha\left(\hat{K}_t - \hat{N}_t\right).$$

In general equilibrium, a positive N-shock or I-shock raises the interest rate and the wage rate, and thus dampens the increases in the marginal Q or the price of capital, as revealed by equation (53). As a result, both the extensive and intensive margin effects are weakened in general equilibrium. Thomas (2002) and Khan and Thomas (2003, 2008) emphasize this general equilibrium effect. They also find that movements in interest rates and wages yield quantity dynamics that are virtually indistinguishable from a standard RBC model without fixed adjustment costs.

[Insert Figures 3-4 Here.]

Figures 3-4 plot impulse responses to a positive N-shock. Compared to Figure 1, the increase in the investment rate is about 10 times smaller in general equilibrium for the lumpy investment and partial adjustment models than that in partial equilibrium. In addition, the responses in the lumpy investment and partial adjustment models are similar, but the lumpy investment model brings predictions closer to those of the frictionless RBC model. The intuition is that the partial adjustment model implies too sluggish responses of investment due to convex adjustment costs. The extensive margin effect in the lumpy investment model raises the responses of investment to shocks. But the price feedback effect partially offsets this extensive margin effect. Figure 4 shows that both the interest rate and the wage rate rise. As a result, the increase in marginal Q in the lumpy investment model is much smaller in general equilibrium than in partial equilibrium (0.1 percent versus 1.8 percent). This in turn causes the adjustment rate to rise by less than 1 percent as revealed in Figure 3, compared to 11 percent in partial equilibrium.

[Insert Figures 5-6 Here.]

Figures 5-6 plot the impulse responses to a positive I-shock. Comparing with Figure 2, we find that the effects on the investment rate is much smaller in general equilibrium than in partial equilibrium. In addition, the impulse responses in the lumpy investment and the partial adjustment model are similar. In contrast to the partial equilibrium case, a positive I-shock lowers marginal Q in both the lumpy investment and partial adjustment models. The intuition follows from equation (53) and Figure 6. The increase in the interest rate and the wage rate lowers the profitability of the firm and hence raises the cost of investment. This effect dominates the positive effect of investment-specific technology shock on Q. Why do the investment rate and the adjustment rate still rise? The reason is that the increase in the I-shock decreases the price of new investment. Thus, it has a direct positive effect on the investment trigger and the investment rate as revealed by equations (42) and (43), respectively.[7] However, the effect is smaller than that in partial equilibrium, due to the powerful general equilibrium price feedback effect. Figure 5 shows that the adjustment rate rises by 1.5 percent only, which is much smaller than 8 percent in partial equilibrium.

Next, we turn to the business cycle moments properties. Table 2 presents standard deviations, autocorrelations, and contemporaneous correlations for several model economies. We first consider the result for the frictionless RBC and partial adjustment models. It is well known that the partial adjustment model delivers less volatile and more persistent equilibrium quantities and prices than the frictionless RBC model because of the smoothing role of the convex adjustment costs. We then introduce fixed costs into the partial adjustment model. Rows labelled "Lumpy1" in Table 2 present the result for this lumpy investment model with the baseline parameter values. They reveal that although impulse responses in the partial adjustment model and the lumpy investment model are similar, the difference in the model predicted second moments is non-negligible. The lumpy investment model delivers higher volatility in all quantities and prices than the partial adjustment model as revealed in Panel A. In particular, aggregate investment, the investment rate, and hours are 16, 13, and 28 percent, respectively, more volatile in the lumpy investment model than in the partial adjustment model. Panel B of Table 2 shows that the lumpy investment model predicts less persistent equilibrium quantities and prices, which are closer to the predictions of the frictionless RBC model. Panel C of Table 2 presents contemporaneous correlations with output. Marginal Q is negatively correlated with output for all models because a positive investment-specific technology shock lowers the price of capital directly. All other quantities and prices move positively with output. In summary, Table 2 demonstrates that the predictions of the lumpy investment model are closer to those of the standard frictionless RBC model. Thus, it also suffers from a number of difficulties in matching the US business cycle facts, as in the standard frictionless RBC model. Thomas (2002) reports a similar finding.

So far, we have shown that under the baseline calibration, the general equilibrium price movements dampen the extensive margin effect significantly, making predictions of the lumpy investment model and the partial adjustment model similar. We now illustrate that the shape parameter of the distribution function of the idiosyncratic shock is important for the extensive margin effect. We set $\eta = 20$ and re-calibrate $\xi_{\max} = 0.02232$ such that the inaction rate is equal

[7] In contrast to the N-shock, the initial response of consumption is negative because investment crowds out consumption as typical in models with investment-specific technology shocks.

Table 2. Business Cycle Moments

	Y	C	I	N	Q	I/K	r	w
	A. Standard deviations (percentage)							
PA	2.27	2.03	4.09	0.87	1.18	3.74	0.28	2.03
Lumpy1	2.41	2.12	4.74	1.11	1.22	4.41	0.36	2.12
Lumpy2	2.60	2.24	5.72	1.45	1.36	5.45	0.48	2.24
LumpyGK	2.59	2.22	5.44	1.46	1.32	5.11	0.46	2.22
RBC	2.81	2.36	7.58	1.95	1.67	7.46	0.60	2.36
	B. Autocorrelations							
PA	0.94	0.96	0.77	0.63	0.83	0.68	0.63	0.96
Lumpy1	0.93	0.96	0.73	0.60	0.78	0.64	0.60	0.96
Lumpy2	0.91	0.95	0.67	0.56	0.70	0.58	0.56	0.95
LumpyGK	0.92	0.95	0.70	0.58	0.72	0.61	0.58	0.95
RBC	0.89	0.93	0.64	0.57	0.70	0.58	0.57	0.93
	C. Contemporaneous correlations with output							
PA	1	0.92	0.83	0.45	-0.21	0.33	0.09	0.92
Lumpy1	1	0.89	0.81	0.47	-0.32	0.34	0.20	0.89
Lumpy2	1	0.83	0.80	0.51	-0.43	0.37	0.32	0.83
LumpyGK	1	0.83	0.83	0.51	-0.42	0.39	0.32	0.83
RBC	1	0.73	0.78	0.56	-0.53	0.43	0.44	0.73

Notes: All variables are in logarithms. RBC: the standard real business cycle model. PA: the partial adjustment model. Lumpy1: our lumpy investment model under the baseline calibration in Table 1. Lumpy2: our lumpy investment model with power function distribution where $\eta = 20$ and $\xi_{\max} = 0.02232$. LumpyGK: our lumpy investment model with our calibrated Gourio and Kashyap (2007) distribution.

to 0.081. In this case, the elasticity of the adjustment rate is 20 times of that in the baseline calibration so that the extensive margin effect is much larger. Of course, this calibration is unreasonable because total fixed costs are too large, accounting for 4.3 percent of output, 19.1 percent of total investment spending, and 1.9 percent of capital stock.

Rows labelled "Lumpy2" in Table 2 present the result for this calibration. The result reveals that the difference between the lumpy investment model and the partial adjustment model becomes larger. In particular, aggregate investment in the lumpy investment model is 40 percent more volatile than in the partial adjustment model. The investment rate in the lumpy investment model is 46 percent more volatile than in the partial adjustment model. However, the differences in the autocorrelations and contemporaneous correlations across these two models are small.

Gourio and Kashyap (2007) argue that for the extensive margin effect to be large, the fixed cost distribution must be sufficiently compressed in the sense that many firms must face nearly identical fixed costs. We have argued in Proposition 6 that the key determinant of the extensive margin effect is the steady-state elasticity of the adjustment rate, but not the compression property. We now take Gourio and Kashyap (2007) distribution $H(\xi/\xi_{\max})$, where $H(x) = \frac{h(x)-h(0)}{h(1)-h(0)}$ and $h(x) = [\arctan(\sigma_1(x-\chi)) + \arctan(\sigma_2(x-1))] / (2\pi)$, for $\chi \in (0,1)$. This distribution has the property that most firms bunch around $\xi_{\max}$ and $\chi\xi_{\max}$. As in Gourio and Kashyap (2007), we set $\sigma_1 = 150$ and $\sigma_2 = 33.3$. Unlike their distribution with $\chi = 0.5$, we set $\chi = 0.05$ so that there are many firms having small fixed costs at the size of 5 percent of $\xi_{\max}$. We then set $\xi_{\max} = 0.022494$ to match the inaction rate of 0.081. In this case, total fixed costs are smaller than those in our baseline calibration. They account for 1.42 percent of output, 5.75 percent of total investment spending, and 0.58 percent of capital stock. However, the effect of lumpy investment is much larger than in the baseline calibration, as shown in Table 2.

Rows labelled "LumpyGK" in Table 2 present the result for the Gourio-Kashyap distribution. We find that our calibrated Gourio-Kashyap distribution and the power function distribution with $\eta = 20$ deliver similar second moments, but the former distribution gives slightly less volatile investment and investment rate. To see the intuition, we compute the steady-state elasticities of the adjustment rate with respect to the investment trigger for the Gourio-Kashyap distribution and for the power function distribution. We find they are equal to 6.28 and 20 respectively. As a result, the extensive margin effect for the Gourio-Kashyap distribution is smaller, justifying less volatile investment. But why is the difference in equilibrium second moments for the two distributions so small? The intuition comes from the general equilibrium price feedback effect. As Proposition 6 shows, the magnitude of the wage feedback effect is determined by the state-steady ratio R/Q. We find that it is equal to 0.145 for the Gourio-Kashyap distribution, which is smaller than the value 0.159 for the power function distribution with $\eta = 20$, because the Gourio-Kashyap distribution is more right skewed than the power function distribution. Thus, the price feedback effect is smaller for the Gourio-Kashyap distribution, which makes the powerful dampening effect on investment much smaller.

5 Idiosyncratic Investment-Specific Technology Shocks

So far, we have assumed that idiosyncratic shocks are to fixed costs and are iid. In this case, all firms choose the same target investment level, and thus the model cannot generate the spike rate observed in the data. To allow different firms choose different target investment levels, we introduce idiosyncratic investment-specific technology shocks.[8] To keep the model tractable, we shut down idiosyncratic fixed cost shocks. That is, we assume each firm must pay a fixed fraction ξ of capital if it decides to make investments.

We assume that the idiosyncratic investment-specific technology shock ε_t^j is iid and drawn from a distribution with density ϕ. In this case, firm j's dividends are given by:

$$D_t^j = Y_t^j - w_t N_t^j - \frac{I_t^j}{z_t \varepsilon_t^j} - \xi K_t^j \mathbf{1}_{I_t^j \neq 0}. \tag{54}$$

We can follow similar steps to derive that firm j's optimal investment policy is characterized by the (S, s) policy in that there is a unique trigger value $\varepsilon_t^* > 0$ such that the firm invests if and only if $\varepsilon_t^j \geq \varepsilon_t^*$. The trigger value ε_t^* satisfies the equation:

$$\frac{\theta}{1-\theta} (z_t \varepsilon_t^*)^{\frac{1-\theta}{\theta}} (\psi Q_t)^{\frac{1}{\theta}} = \xi.$$

The optimal target investment level is given by:

$$i_t^j = \left(\psi z_t \varepsilon_t^j Q_t\right)^{\frac{1}{\theta}}. \tag{55}$$

The marginal Q satisfies:

$$\begin{aligned} Q_t \quad = \quad & E_t \left\{ \frac{\beta U_1 (C_{t+1}, 1 - N_{t+1})}{U_1 (C_t, 1 - N_t)} \left[F_1 (K_{t+1}, A_{t+1} N_{t+1}) + (1 - \delta + \varsigma) Q_{t+1} \right.\right. \\ & \left.\left. + \xi \int_{\varepsilon_{t+1}^*}^{\infty} \left[\left(\varepsilon / \varepsilon_{t+1}^*\right)^{\frac{1-\theta}{\theta}} - 1 \right] \phi(\varepsilon) d\varepsilon \right] \right\}, \end{aligned} \tag{56}$$

[8]Khan and Thomas (2008) introduce persistent idiosyncratic productivity shocks.

where the expression in the second line represents the option value of waiting. Equation (55) reveals that the target investment level depends on the idiosyncratic investment-specific technology shock ε_t^j. Thus, different firms have different target investment levels. This property helps generate the spike rate observed in the data.

Given an individual firm's decision rules, we can conduct aggregation as in Section 3 and show that the aggregate equilibrium sequences $\{Y_t, N_t, C_t, I_t, K_t, Q_t, \varepsilon_t^*\}_{t\geq 0}$ are characterized by a system of difference equation similar to that in Proposition 2 with equations (26)-(28) being replaced by:

$$I_t = (\psi Q_t)^{\frac{1}{\theta}} \left(\int_{\varepsilon_t^*}^{\infty} (z_t \varepsilon)^{\frac{1-\theta}{\theta}} \phi(\varepsilon) d\varepsilon \right) K_t, \tag{57}$$

$$K_{t+1} = (1 - \delta + \varsigma) K_t + \frac{\psi}{1-\theta} K_t \left(\frac{z_t I_t}{K_t} \right)^{1-\theta} \left(\int_{\varepsilon_t^*}^{\infty} \varepsilon^{\frac{1-\theta}{\theta}} \phi(\varepsilon) d\varepsilon \right)^{\theta}, \tag{58}$$

$$\xi = \frac{\theta}{1-\theta} \left(z_t \varepsilon_t^* \right)^{\frac{1-\theta}{\theta}} (\psi Q_t)^{\frac{1}{\theta}}, \tag{59}$$

$$Y_t = F(K_t, A_t N_t) = I_t + C_t + \xi K_t \int_{\varepsilon_t^*}^{\infty} \phi(\varepsilon) \, d\varepsilon. \tag{60}$$

The equilibrium dynamics are determined by parameters in preferences, technology, and the distribution of idiosyncratic shocks. Under some specific assumptions on these parameters, we can prove an irrelevance result similar to Proposition 5.

Proposition 7 *Consider the Pareto distribution with density $\phi(\varepsilon) = \eta \varepsilon^{-\eta - 1}$, $\varepsilon > 1$, $\eta > (1-\theta)/\theta$ and specifications (38)-(39). Then any positive fixed costs do not matter for the aggregate dynamics of the normalized variables $\{\tilde{Y}_t, \tilde{N}_t, \tilde{C}_t, \tilde{I}_t, \tilde{K}_t, \tilde{Q}_t, \tilde{\varepsilon}_t^*\}_{t\geq 0}$ in the sense that the fixed cost parameter ξ does not affect the system of difference equations that characterizes these normalized variables.*

The intuition behind this proposition is similar to that behind Proposition 5. The Pareto distribution has a homogeneity property making the steady state values R/Q, I/Y, C/Y and N independent of ξ. Given the homogeneity of the preferences and technology, the dynamic system for the normalized variables is fully determined by these steady state values and other structural parameters except for ξ. We thus obtain the irrelevance result.

6 Conclusion

We have presented an analytically tractable general equilibrium business cycle model that features micro-level investment lumpiness. We prove an exact irrelevance proposition, consistent with numerical findings in Thomas (2002) and Khan and Thomas (2003, 2008). We also give conditions under which lumpy investment can be important to a first-order approximation. Essentially, we need the fixed cost distribution satisfies two conditions: (i) The steady-state elasticity of the adjustment rate is large so that the extensive margin effect is large. (ii) The fixed cost distribution is right skewed so that the general equilibrium price feedback is small. We also show numerically that introducing fixed costs to a model with convex adjustment costs raises business cycle volatility, but reduces persistence of output, consumption, investment, and hours. In addition, when lumpy investment becomes more important, it brings business cycle moments closer to those in the standard frictionless RBC model.

Our model serves as a theoretical benchmark for understanding the general equilibrium effect of lumpy investment. It is useful for reconciling some debate and some numerical findings in the literature. One limitation of our model is that it is not suitable for addressing distributional asymmetry and aggregate nonlinearity. To address this issue, it is necessary to relax the assumption of constant returns to scale. In this case, the distribution of capital is a state variable and there is no analytical solution available. One has to use a numerical method to approximate the distribution of capital.

Appendix

A Proofs

Proof of Proposition 1: From (20), we can show that the target investment level i_t^j satisfies the first-order condition:

$$\frac{1}{z_t} = g'\left(i_t^j\right) E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}\right]. \tag{A.1}$$

By equations (3), (16) and (21), we can derive equation (23). Using this equation, we define $V^a\left(K_t, A_t, z_t, \xi_t^j\right)$ as the price of capital when the firm chooses to invest. It is given by:

$$\begin{aligned} V^a\left(K_t, A_t, z_t, \xi_t^j\right) &= R_t - \frac{i_t^j}{z_t} - \xi_t^j + g(i_t^j)Q_t \\ &= R_t + (1 - \delta + \varsigma)\, Q_t \\ &\quad + \frac{\theta}{1-\theta} z_t^{\frac{1-\theta}{\theta}} \left(\psi Q_t\right)^{\frac{1}{\theta}} - \xi_t^j. \end{aligned} \tag{A.2}$$

Define $V^n\left(K_t, A_t, z_t\right)$ as the price of capital when the firm chooses not to invest. It satisfies:

$$V^n\left(K_t, A_t, z_t\right) = R_t + (1 - \delta + \varsigma)\, Q_t, \tag{A.3}$$

which is independent of ξ_t^j. We can then rewrite problem (20) as:

$$V\left(K_t, A_t, z_t, \xi_t^j\right) = \left\{V^a\left(K_t, A_t, z_t, \xi_t^j\right), V^n\left(K_t, A_t, z_t\right)\right\}. \tag{A.4}$$

Clearly, there is a unique cutoff value ξ_t^* given in (22) satisfying the condition:

$$V^a\left(K_t, A_t, z_t, \xi_t^*\right) = V^n\left(K_t, A_t, z_t\right), \tag{A.5}$$

$$V^a\left(K_t, A_t, z_t, \xi_t^j\right) \geq V^n\left(K_t, A_t, z_t\right) \text{ if and only if } \xi_t^j \leq \xi_t^*. \tag{A.6}$$

Because the support of ξ_t^j is $[0, \xi_{\max}]$, the investment trigger is given by $\min\left\{\xi_t^*, \xi_{\max}\right\}$.

When $\xi_t^* \leq \xi_{\max}$, we show that:

$$\begin{aligned} \bar{V}_t &= \int_0^{\xi_{\max}} V\left(K_t, A_t, z_t, \xi\right) \phi(\xi) d\xi \\ &= \int_{\xi_t^*}^{\xi_{\max}} V^n\left(K_t, A_t, z_t\right) \phi(\xi) d\xi + \int_0^{\xi_t^*} V^a\left(K_t, A_t, z_t, \xi\right) \phi(\xi) d\xi \\ &= V^n\left(K_t, A_t, z_t\right) + \int_0^{\xi_t^*} \left[V^a\left(K_t, A_t, z_t, \xi\right) - V^n\left(K_t, A_t, z_t\right)\right] \phi(\xi) d\xi. \end{aligned}$$

We use equations (A.2), (A.3) and (22) to derive

$$\begin{aligned} V^a(K_t, A_t, z_t, \xi) - V^n(K_t, A_t, z_t) &= \frac{\theta}{1-\theta} z_t^{\frac{1-\theta}{\theta}} (\psi Q_t)^{\frac{1}{\theta}} - \xi \\ &= \xi_t^* - \xi. \end{aligned} \tag{A.7}$$

Using the above two equations, (A.3), and (21), we obtain (24). Q.E.D.

Proof of Proposition 2: From (12), we deduce that all firms choose the same labor-capital ratio n_t. We thus obtain $N_t = n_t K_t$. We then derive

$$\begin{aligned} Y_t &= \int Y_t^j dj = \int F\left(K_t^j, A_t N_t^j\right) dj = \int F\left(1, A_t n_t^j\right) K_t^j dj \\ &= F(1, A_t n_t) \int K_t^j dj = F(1, A_t n_t) K_t = F(K_t, A_t N_t), \end{aligned}$$

which gives the first equality in equation (28). As a result, we use equation (12) and $n_t^j = n_t$ to show:

$$A_t F_2(K_t, A_t N_t) = w_t. \tag{A.8}$$

By the constant return to scale property of F, we also have:

$$R_t = F_1(K_t, A_t N_t). \tag{A.9}$$

Equation (25) follows from equation (22) and (9). We next derive aggregate investment:

$$I_t = \int \frac{I_t^j}{z_t} dj = \int \frac{i_t^j}{z_t} K_t^j dj = K_t \int_0^{\xi_t^*} z_t^{\frac{1}{\theta}-1} (\psi Q_t)^{\frac{1}{\theta}} \phi(\xi) d\xi,$$

where the second equality uses the definition of i_t^j, the third equality uses a law of large numbers and the optimal investment rule (23). We thus obtain (26).

We turn to the law of motion for capital. By definition,

$$K_{t+1} = \int_0^1 \left[(1-\delta) + g(i_t^j)\right] K_t^j dj.$$

Substituting optimal investment in equation (23) and using equation (26), we obtain (27).

Equation (30) follows from substituting equations (9), (26), (25), and (A.9) into equation (24). Equation (29) follows from equations (9), (10) and (A.8). Finally, equation (28) follows from a law of large number, the market clearing condition (11), and Proposition 1. Q.E.D.

Proof of Proposition 3: Let λ_t and $\lambda_t Q_t$ be the Lagrange multipliers associated with (28) and (27). We derive the following first-order conditions:

$$C_t : U_1 \left(C_t, 1 - N_t\right) = \lambda_t, \tag{A.10}$$

$$N_t : U_2 \left(C_t, 1 - N_t\right) = \lambda_t A_t F_2 \left(K_t, A_t N_t\right), \tag{A.11}$$

$$I_t : 1 = Q_t \psi K_t^{\theta} z_t^{1-\theta} I_t^{-\theta} \left[\int_0^{\xi_t^*} \phi(\xi) d\xi\right]^{\theta}, \tag{A.12}$$

$$\begin{aligned} K_{t+1} \quad : \quad \lambda_t Q_t = E_t \beta \lambda_{t+1} & \left[F_1 \left(K_{t+1}, A_{t+1} N_{t+1}\right) + \left(1 - \delta + \varsigma\right) Q_{t+1} - \int_0^{\xi_{t+1}^*} \xi \phi(\xi) d\xi\right] \\ + E_t \beta \lambda_{t+1} Q_{t+1} & \left[\frac{\theta \psi}{1-\theta} \left(\frac{I_{t+1} z_{t+1}}{K_{t+1}}\right)^{1-\theta} \left(\int_0^{\xi_{t+1}^*} \phi(\xi) d\xi\right)^{\theta}\right], \end{aligned} \tag{A.13}$$

$$\xi_t^* : \lambda_t \xi_t^* \phi\left(\xi_t^*\right) K_t = \lambda_t Q_t \frac{\psi \theta}{1-\theta} K_t \left(z_t I_t / K_t\right)^{1-\theta} \left[\int_0^{\xi_t^*} \phi(\xi) d\xi\right]^{\theta-1} \phi\left(\xi_t^*\right). \tag{A.14}$$

Equation (A.12) gives (26). Equations (A.10) and (A.11) together give equation (29). Using equations (A.12) and (A.14), we can derive equation (25). From (25) and (26), we can derive

$$\frac{\theta}{1-\theta} \frac{I_t}{K_t} = \int_0^{\xi_t^*} \xi_t^* \phi(\xi) d\xi. \tag{A.15}$$

Using this equation and equations (A.12) and (A.13), we can derive (30). Q.E.D.

Proof of Proposition 4: We first observe that the deterministic steady-state values of A_t and z_t are equal to 1. In steady state, equations (26) and (25) imply that:

$$\frac{I}{K} = \left(\psi Q\right)^{\frac{1}{\theta}} \int_0^{\xi^*} \phi(\xi) d\xi, \tag{A.16}$$

$$\xi^* = \frac{\theta}{1-\theta} \left(\psi Q\right)^{\frac{1}{\theta}}, \tag{A.17}$$

From these two equations, we obtain:

$$\frac{I}{K} = \xi^* \frac{1-\theta}{\theta} \int_0^{\xi^*} \phi(\xi) d\xi. \tag{A.18}$$

In steady state, equation (27) becomes:

$$\delta - \varsigma = \frac{\psi}{1-\theta} \left(I/K\right)^{1-\theta} \left[\int_0^{\xi^*} \phi(\xi) d\xi\right]^{\theta}. \tag{A.19}$$

Substituting equation (A.18) into the above equation yields equation (31). The expression on the right-hand side of this equation increases with ξ^*. Given the condition in this proposition, there is a unique interior solution $\xi^* \in [0, \xi_{\max}]$.

Equation (32) follows from (A.17). Equations (A.18) and (A.19) imply that:

$$\delta - \varsigma = \frac{\psi}{1-\theta} \frac{I}{K} \left(\frac{\xi^* (1-\theta)}{\theta} \right)^{-\theta}. \tag{A.20}$$

From this equation and equation (32), we obtain (33). The other equations in the proposition follow from the steady-state versions of equations (29)-(30). Q.E.D.

Proof of Corollary 1: For the power function distribution, we have $\int_0^{\xi} \phi(x)dx = \left[\frac{\xi}{\xi_{\max}}\right]^{\eta}$, and

$$\frac{\int_0^{\xi^*} \xi \phi(\xi) d\xi}{\xi^* \int_0^{\xi^*} \phi(\xi) d\xi} = \frac{\eta}{\eta + 1}. \tag{A.21}$$

We can then use equation (31) to derive equation (37). Equation (37) implies that the investment trigger ξ^* increases with $\xi_{\max}$. It follows from equation (32) and (33) that Q and I/K also increase with $\xi_{\max}$.

Using equation (A.18), we can compute the steady-state value of the ratio of option value of waiting to the investment rate:

$$\frac{\int_0^{\xi^*} [\xi^* - \xi] \phi(\xi) d\xi}{I/K} = \frac{\int_0^{\xi^*} [\xi^* - \xi] \phi(\xi) d\xi}{\frac{1-\theta}{\theta} \xi^* \int_0^{\xi^*} \phi(\xi) d\xi} = \frac{\theta}{1-\theta} \left[1 - \frac{\int_0^{\xi^*} \xi \phi(\xi) d\xi}{\xi^* \int_0^{\xi^*} \phi(\xi) d\xi} \right].$$

Using this equation and equation (33), we derive the steady-state value of the ratio of the option value to the price of capital:

$$\frac{\int_0^{\xi^*} [\xi^* - \xi] \phi(\xi) d\xi}{Q} = \frac{\int_0^{\xi^*} [\xi^* - \xi] \phi(\xi) d\xi}{I/K} \frac{I/K}{Q} = \theta (\delta - \varsigma) \left[1 - \frac{\int_0^{\xi^*} \xi \phi(\xi) d\xi}{\xi^* \int_0^{\xi^*} \phi(\xi) d\xi} \right].$$

Substituting it into equation (36), we obtain:

$$Q = \frac{\beta}{1 - \beta (1 - \delta + \varsigma)} \left\{ F_1 (K, N) + \theta (\delta - \varsigma) \left[1 - \frac{\int_0^{\xi^*} \xi \phi(\xi) d\xi}{\xi^* \int_0^{\xi^*} \phi(\xi) d\xi} \right] Q \right\}. \tag{A.22}$$

This equation implies that:

$$\begin{aligned} R &= F_1 (K, N) = \frac{\alpha Y}{K} \qquad \text{(A.23)} \\ &= Q \left\{ \frac{1}{\beta} - (1 - \delta + \varsigma) - \theta (\delta - \varsigma) \left[1 - \frac{\int_0^{\xi^*} \xi \phi(\xi) d\xi}{\xi^* \int_0^{\xi^*} \phi(\xi) d\xi} \right] \right\}. \end{aligned}$$

Thus, by (A.21), R/Q is independent of $\xi_{\max}$.

Using equations (A.21), (33) and (A.23), we derive the steady-state investment-output ratio:

$$\frac{I}{Y}=\frac{I/K}{Y/K}=\frac{\alpha\left(\delta-\varsigma\right)\left(1-\theta\right)}{\frac{1}{\beta}-\left(1-\delta+\varsigma\right)-\frac{\theta(\delta-\varsigma)}{\eta+1}}, \tag{A.24}$$

which is independent of $\xi_{\max}$. We next compute the ratio of total fixed costs to output using equations (A.24) and (33):

$$\begin{aligned}\frac{K\int_0^{\xi^*}\xi\phi(\xi)d\xi}{Y} &= \int_0^{\xi^*}\xi\phi(\xi)d\xi\frac{I/Y}{I/K}=\frac{\alpha\int_0^{\xi^*}\xi\phi(\xi)d\xi}{Q\left\{\frac{1}{\beta}-\left(1-\delta+\varsigma\right)-\frac{\theta(\delta-\varsigma)}{\eta+1}\right\}}\\ &= \frac{\alpha\eta/\left(\eta+1\right)}{\frac{1}{\beta}-\left(1-\delta+\varsigma\right)-\frac{\theta(\delta-\varsigma)}{\eta+1}}\frac{\xi^{*1+\eta-\theta}\psi\theta^{\theta}}{\xi_{\max}^{\eta}\left(1-\theta\right)^{\theta}}\\ &= \frac{\alpha\eta/\left(\eta+1\right)\left(\delta-\varsigma\right)\theta}{\frac{1}{\beta}-\left(1-\delta+\varsigma\right)-\frac{\theta(\delta-\varsigma)}{\eta+1}},\end{aligned}$$

which is independent of $\xi_{\max}$.

Using the resource constraint, we can compute the steady-state consumption-output ratio:

$$\frac{C}{Y}=1-\frac{I}{Y}-\frac{K\int_0^{\xi^*}\xi\phi(\xi)d\xi}{Y}.$$

Thus, C/Y is independent of $\xi_{\max}$.

To show N is independent of $\xi_{\max}$, we use the assumption on preferences and the steady-state version of equation (35) to derive:

$$\frac{\left(1-\alpha\right)Y}{N}=Cv'\left(1-N\right)\text{ or }\frac{Cv'\left(1-N\right)}{1-\gamma}. \tag{A.25}$$

We obtain the desired result because C/Y is independent of $\xi_{\max}$.

Because Q increases with $\xi_{\max}$ and R/Q is independent of $\xi_{\max}$, R must increase with $\xi_{\max}$. Since $R=f'(k)$ and $\partial R/\partial\xi_{\max}>0$, we must have $\partial k/\partial\xi_{\max}<0$, where $k=K/N$. Because N is independent of $\xi_{\max}$, we obtain $\partial K/\partial\xi_{\max}<0$. Since $w=f(k)-f'(k)k$, so we have $\partial w/\partial k>0$ and $\frac{\partial w}{\partial\xi_{\max}}=\frac{\partial w}{\partial k}\frac{\partial k}{\partial\xi_{\max}}<0$. Since $Y=F(K,N)$, we have $\partial Y/\partial\xi_{\max}<0$. Since C/Y and I/Y are independent of $\xi_{\max}$, we also have $\partial C/\partial\xi_{\max}<0$ and $\partial I/\partial\xi_{\max}<0$. Q.E.D.

Proof of Proposition 5: We focus on the utility function $U\left(C,1-N\right)=\frac{C^{1-\gamma}}{1-\gamma}v(1-N)$, where $\gamma>0,\neq 1$. The proof for the other utility function in the proposition is similar. We then have $\Lambda_t=C_t^{-\gamma}v(1-N_t)$, and

$$\frac{U_2\left(C_t,1-N_t\right)}{U_1\left(C_t,1-N_t\right)}=\frac{1}{\gamma-1}\frac{C_tv'(1-N_t)}{v(1-N_t)}.$$

Using Proposition 2 and the assumptions, we can characterize the equilibrium dynamics by the following system of difference equations:

$$I_t = (\psi Q_t)^{\frac{1}{\theta}} z_t^{\frac{1-\theta}{\theta}} K_t \left(\frac{\xi_t^*}{\xi_{\max}} \right)^{\eta}, \tag{A.26}$$

$$K_{t+1} = (1-\delta+\varsigma)K_t + \frac{\psi}{1-\theta} K_t \left(z_t I_t / K_t \right)^{1-\theta} \left(\frac{\xi_t^*}{\xi_{\max}} \right)^{\theta\eta}, \tag{A.27}$$

$$\xi_t^* = \frac{\theta}{1-\theta} z_t^{\frac{1-\theta}{\theta}} \left(\psi Q_t \right)^{\frac{1}{\theta}}, \tag{A.28}$$

$$\frac{1}{\gamma-1} \frac{C_t v'(1-N_t)}{v(1-N_t)} = \frac{(1-\alpha)Y_t}{N_t}, \tag{A.29}$$

$$Y_t = F\left(K_t, A_t N_t\right) = I_t + C_t + K_t \frac{\eta}{\eta+1} \frac{(\xi_t^*)^{\eta+1}}{(\xi_{\max})^{\eta}}, \tag{A.30}$$

$$Q_t = E_t \left\{ \frac{\beta C_{t+1}^{-\gamma} v(1-N_{t+1})}{C_t^{-\gamma} v(1-N_t)} \left[\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta+\varsigma) Q_{t+1} + \frac{1}{\eta+1} \frac{(\xi_t^*)^{\eta+1}}{(\xi_{\max})^{\eta}} \right] \right\}.$$

Using the steady-state equations from Corollary 1 and the definition of normalization, $X_t = X\tilde{X}_t$ for any variable X_t, we can rewrite the above system of difference equations as follows:

$$\tilde{I}_t = \tilde{Q}_t^{\frac{1}{\theta}} \tilde{z}_t^{\frac{1-\theta}{\theta}} \tilde{K}_t \left(\tilde{\xi}_t^* \right)^{\eta}, \tag{A.31}$$

$$\tilde{K}_{t+1} = (1-\delta+\varsigma)\tilde{K}_t + \frac{\psi \left[(\delta-\varsigma)(1-\theta) \right]^{1-\theta}}{1-\theta} \tilde{K}_t \left(\tilde{z}_t \tilde{I}_t / \tilde{K}_t \right)^{1-\theta} \left(\tilde{\xi}_t^* \right)^{\theta\eta}, \tag{A.32}$$

$$\tilde{\xi}_t^* = \tilde{z}_t^{\frac{1-\theta}{\theta}} \tilde{Q}_t^{\frac{1}{\theta}}, \tag{A.33}$$

$$\frac{1}{\gamma-1} \frac{\tilde{C}_t v'(1-N\tilde{N}_t)}{v(1-N\tilde{N}_t)} = \frac{Y}{C} \frac{(1-\alpha)\tilde{Y}_t}{N\tilde{N}_t}, \tag{A.34}$$

$$\tilde{Y}_t = \tilde{K}_t^{\alpha} (\tilde{A}_t \tilde{N}_t)^{1-\alpha} = \frac{I}{Y}\tilde{I}_t + \frac{C}{Y}\tilde{C}_t + \left(1 - \frac{I}{Y} - \frac{C}{Y} \right) \tilde{K}_t \left(\tilde{\xi}_t^* \right)^{\eta+1}, \tag{A.35}$$

$$\begin{aligned} \tilde{Q}_t \quad = \quad & E_t \left\{ \frac{\beta \tilde{C}_{t+1}^{-\gamma} v(1-N\tilde{N}_{t+1})}{\tilde{C}_t^{-\gamma} v(1-N\tilde{N}_t)} \left[\frac{R}{Q} \frac{\tilde{Y}_{t+1}}{\tilde{K}_{t+1}} + (1-\delta+\varsigma) \tilde{Q}_{t+1} \right. \right. \\ & \left. \left. \frac{\int_0^{\xi^*} (\xi^* - \xi)\, \phi(\xi) d\xi}{Q} \left(\tilde{\xi}_t^* \right)^{\eta+1} \right] \right\}. \end{aligned} \tag{A.36}$$

Note that equation (A.22) implies:

$$\frac{\int_0^{\xi^*} (\xi^* - \xi)\, \phi(\xi) d\xi}{Q} = \frac{1}{\beta} - (1-\delta+\varsigma) - \frac{R}{Q}.$$

The dynamics of the above system of nonlinear difference equations are fully determined by the steady-state ratios R/Q and C/Y, I/Y and the steady-state value N, structural parameters $\{\alpha, \beta, \gamma, \delta, \psi, \varsigma, \theta, \eta\}$, the function $v(1-N)$, and the process of exogenous technology shocks $\tilde{A}_t$ and $\tilde{z}_t$. By Corollary 1, R/Q, C/Y, I/Y and the steady-state value N are independent of the nonconvex adjustment costs parameter $\xi_{\max}$. Thus, we obtain the desired result. Q.E.D.

Proof of Proposition 6: We log-linearize the nonlinear dynamic system in Proposition 2 and obtain equations (45), (43), (42), (44), (46) and

$$\begin{aligned} \hat{Q}_t + u_{C,C}\hat{C}_t - u_{C,N}\hat{N}_t &= E_t\left(u_{C,C}\hat{C}_{t+1} - u_{C,N}\hat{N}_{t+1}\right) + \beta\left(1-\delta+\varsigma\right)E_t\hat{Q}_{t+1} \\ &\quad + \frac{\beta F_1}{Q}E_t\left[f_{KK}\hat{K}_{t+1} + f_{KN}(\hat{A}_{t+1}+\hat{N}_{t+1})\right] \\ &\quad + \beta\theta(\delta-\varsigma)E_t(\hat{I}_{t+1}-\hat{K}_{t+1}) - \frac{\beta\left(\xi^*\right)^2\phi(\xi^*)}{Q}E_t\hat{\xi}^*_{t+1}, \end{aligned} \tag{A.37}$$

where we have used (A.15) in (30) to derive the above equation. Following King, Plosser and Rebelo (2002), we denote the elasticities of marginal utility to its arguments by $u_{N,C} = \frac{CU_{21}(C,1-N)}{U_2(C,1-N)}$, $u_{N,N} = \frac{NU_{22}(C,1-N)}{U_2(C,1-N)}$, $u_{C,C} = \frac{CU_{11}(C,1-N)}{U_1(C,1-N)}$, $u_{C,N} = \frac{NU_{12}(C_t,1-N)}{U_1(C,1-N)}$. We then log-linearize equation (29) to obtain equation (49). We log-linearize the equation $U_1\left(C_t, 1-N_t\right) = \beta E_t\left[U\left(C_{t+1}, 1-N_{t+1}\right)r_{t+1}\right]$ to obtain (48).

We now use equation (43) to derive

$$\beta\theta(\delta-\varsigma)E_t(\hat{I}_{t+1}-\hat{K}_{t+1}) = \beta(\delta-\varsigma)E_t\left[\hat{Q}_{t+1} + (1-\theta)\hat{z}_{t+1} + \frac{\theta\xi^*\phi(\xi^*)}{\int_0^{\xi^*}\phi(\xi)d\xi}\hat{\xi}^*_{t+1}\right]. \tag{A.38}$$

By equations (A.18) and (33), we have:

$$\frac{\theta}{1-\theta}\frac{I}{K} = \xi^*\int_0^{\xi^*}\phi(\xi)d\theta = (\delta-\varsigma)\theta Q.$$

Thus,

$$\beta(\delta-\varsigma)\frac{\theta\xi^*\phi(\xi^*)}{\int_0^{\xi^*}\phi(\xi)d\xi} = \beta(\delta-\varsigma)\frac{\theta\left(\xi^*\right)^2\phi(\xi^*)}{(\delta-\varsigma)\theta Q} = \beta\frac{\left(\xi^*\right)^2\phi(\xi^*)}{Q}.$$

Using this equation and plugging equation (A.38) into (A.37), we obtain:

$$\begin{aligned} &\hat{Q}_t + u_{C,C}\hat{C}_t - u_{C,N}\hat{N}_t \\ = \;& E_t\left(u_{C,C}\hat{C}_{t+1} - u_{C,N}\hat{N}_{t+1}\right) + \beta E_t\hat{Q}_{t+1} + \beta(\delta-\varsigma)(1-\theta)E_t\hat{z}_{t+1} \\ &+ \frac{\beta F_1}{Q}E_t\left[\frac{KF_{11}(K,N)}{F_1(K,N)}\hat{K}_{t+1} + \frac{NF_{12}(K,N)}{F_1(K,N)}(\hat{A}_{t+1}+\hat{N}_{t+1})\right]. \end{aligned} \tag{A.39}$$

Because F is linearly homogenous, we have

$$NF_{22} + KF_{21} = 0, \ \ KF_{11} + NF_{12} = 0.$$

We can then derive:

$$\frac{KF_{11}(K,N)}{F_1(K,N)}\hat{K}_{t+1} + \frac{NF_{12}(K,N)}{F_1(K,N)}(\hat{A}_{t+1} + \hat{N}_{t+1}) = \frac{KF_{11}(K,N)}{F_1(K,N)}\left(\hat{K}_{t+1} - \hat{A}_{t+1} - \hat{N}_{t+1}\right),$$

$$\begin{aligned} \hat{w}_{t+1} &= \frac{KF_{21}}{F_2}\hat{K}_{t+1} + \frac{NF_{22}}{F_2}\hat{N}_{t+1} + \left(1 + \frac{NF_{22}}{F_2}\right)\hat{A}_{t+1} \\ &= \hat{A}_{t+1} + \frac{KF_{21}}{F_2}\left(\hat{K}_{t+1} - \hat{A}_{t+1} - \hat{N}_{t+1}\right). \end{aligned}$$

Using the above two equations, we can derive equation (47) from equation (A.39). Q.E.D.

Proof of Proposition 7: To prove this proposition, we need two lemmas:

Lemma 1 *Suppose $\delta > \varsigma$. Then the steady-state investment trigger ε^* is given by the unique solution to the equation:*

$$(\delta - \varsigma)(\varepsilon^*)^{\frac{(1-\theta)^2}{\theta}} = \frac{\xi^{1-\theta}\psi}{(1-\theta)^\theta \theta^{(1-\theta)}} \int_{\varepsilon^*}^{\infty} \varepsilon^{\frac{1-\theta}{\theta}} \phi(\varepsilon) d\varepsilon.$$

Given this value ε^*, the steady-state value of Q is given by:

$$Q = \frac{\xi^\theta}{\beta\psi}\left(\frac{1-\theta}{\theta}\right)^\theta (\varepsilon^*)^{\theta-1}.$$

The other steady-state values (I, K, C, N) satisfy:

$$\frac{I}{K} = \beta(\delta - \varsigma)(1-\theta)Q,$$

$$F(K,N) = I + C + \xi K \int_{\varepsilon^*}^{\infty} \phi(\varepsilon)\, d\varepsilon,$$

$$\frac{U_2(C, 1-N)}{U_1(C, 1-N)} = F_2(K,N),$$

$$Q = \frac{1}{1 - \beta(1 - \delta + \varsigma)}\left\{F_1(K,N) + \xi \int_{\varepsilon^*}^{\infty}\left[\left(\frac{\varepsilon}{\varepsilon^*}\right)^{\frac{1-\theta}{\theta}} - 1\right]\phi(\varepsilon)d\varepsilon\right\}.$$

Lemma 2 *Consider the Pareto distribution with density $\phi(\varepsilon) = \eta\varepsilon^{-\eta-1}$, $\varepsilon > 1$, $\eta > (1-\theta)/\theta$, and the specifications in (38)-(39). Assume $\delta > \varsigma$ and the parameter values are such that:*

$$\varepsilon^* \equiv \left[\frac{\xi^{1-\theta}\psi\eta}{(\delta - \varsigma)(\theta\eta + \theta - 1)}\left(\frac{\theta}{1-\theta}\right)^\theta\right]^{1/(\theta+\eta-1)} > 1.$$

Then the steady-state trigger value is ε^*, the steady-state values R/Q, $I/Y, C/Y$ and N are independent of ξ.

We omit the proofs, which follow from similar arguments in the proofs of Proposition 4 and Corollary 1. We now turn to the proof of Proposition 7. We focus on the utility function $U(C, 1-N) = \frac{C^{1-\gamma}}{1-\gamma}v(1-N)$, where $\gamma > 0, \neq 1$. The proof for the other utility function specification in the proposition is similar. We then have $\Lambda_t = C_t^{-\gamma}v(1-N_t)$, and

$$\frac{U_2(C_t, 1-N_t)}{U_1(C_t, 1-N_t)} = \frac{1}{\gamma - 1}\frac{C_t v'(1-N_t)}{v(1-N_t)}.$$

Using the Pareto distribution function and the equations in Section 5, we can characterize the equilibrium dynamics by the following system of difference equations:

$$I_t = \frac{\xi(1-\theta)}{\theta}\frac{\eta}{\eta - \frac{1-\theta}{\theta}}\varepsilon_t^{*-\eta}K_t,$$

$$K_{t+1} = (1-\delta+\varsigma)K_t + \frac{\psi}{1-\theta}K_t^{\theta}(z_t I)^{1-\theta}\left(\frac{\eta}{\eta - \frac{1-\theta}{\theta}}\varepsilon_t^{*\frac{1-\theta}{\theta}-\eta}\right)^{\theta},$$

$$\xi = \frac{\theta}{1-\theta}(z_t\varepsilon_t)^{\frac{1-\theta}{\theta}}\left(\beta\psi E_t\left[\frac{C_t^{\gamma}v(1-N_{t+1})}{C_{t+1}^{\gamma}v(1-N_t)}Q_{t+1}\right]\right)^{\frac{1}{\theta}},$$

$$Q_t = \frac{\alpha Y_t}{K_t} + \beta(1-\delta+\varsigma)E_t\left[\frac{C_t^{\gamma}v(1-N_{t+1})}{C_{t+1}^{\gamma}v(1-N_t)}Q_{t+1}\right] + \xi\frac{\frac{1-\theta}{\theta}}{\eta - \frac{1-\theta}{\theta}}\varepsilon_t^{*-\eta},$$

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} = I_t + C_t + \xi K_t \varepsilon_t^{*-\eta},$$

$$\frac{1}{\gamma-1}\frac{v'(1-N_t)}{v(1-N_t)} = \frac{1}{C_t}\frac{(1-\alpha)Y_t}{N_t}.$$

Using the definition of normalization, $X_t = X\tilde{X}_t$ for any variable X_t, we can rewrite the above system of equations as follows:

$$\tilde{I}_t = \tilde{\varepsilon}_t^{*-\eta}\tilde{K}_t,$$

$$\tilde{K}_{t+1} = (1-\delta+\varsigma)\tilde{K}_t + (\delta-\varsigma)\tilde{K}_t^{\theta}\left(\tilde{z}_t\tilde{I}_t\right)^{1-\theta}\tilde{\varepsilon}_t^{*1-\theta-\eta\theta},$$

$$1 = (\tilde{z}_t\tilde{\varepsilon}_t)^{\frac{1-\theta}{\theta}}\left(E_t\left[\frac{\tilde{C}_t^{\gamma}v(1-N\tilde{N}_{t+1})}{\tilde{C}_{t+1}^{\gamma}v(1-NN_t)}\tilde{Q}_{t+1}\right]\right)^{\frac{1}{\theta}},$$

$$\begin{aligned}\tilde{Q}_t \quad &= \quad \frac{R}{Q}\frac{\tilde{Y}_t}{\tilde{K}_t} + \beta(1-\delta+\varsigma)E_t\left[\frac{\tilde{C}_t^{\gamma}v(1-NN_{t+1})}{\tilde{C}_{t+1}^{\gamma}v(1-NN_t)}\tilde{Q}_{t+1}\right] \\ &\quad + [1-\beta(1-\delta+\varsigma) - R/Q]\tilde{\varepsilon}_t^{*-\eta},\end{aligned}$$

$$\tilde{Y}_t = \tilde{K}_t^{\alpha}(\tilde{A}_t\tilde{N}_t)^{1-\alpha} = \frac{I}{Y}\tilde{I}_t + \frac{C}{Y}\tilde{C}_t + \left(1 - \frac{I}{Y} - \frac{C}{Y}\right)\tilde{K}_t\tilde{\varepsilon}_t^{*-\eta},$$

$$\frac{1}{\gamma-1}\frac{Nv'(1-N\tilde{N}_t)}{v(1-NN_t)}=\frac{(1-\alpha)Y}{C}\frac{1}{\tilde{C}_t}\frac{\tilde{Y}_t}{\tilde{N}_t}.$$

The dynamics of the above system of nonlinear difference equations are fully determined by the steady-state ratios R/Q and C/Y, I/Y and steady-state value N, structural parameters $\{\alpha, \beta, \gamma, \eta, \delta, \theta, \varsigma, \psi\}$, the function $v(1-N)$, and the process of exogenous technology shocks $\tilde{A}_t$ and $\tilde{z}_t$. By Lemma 2, R/Q, C/Y, I/Y and the steady-state value N are independent of the nonconvex adjustment costs parameter ξ. Thus, we obtain the desired result. Q.E.D.

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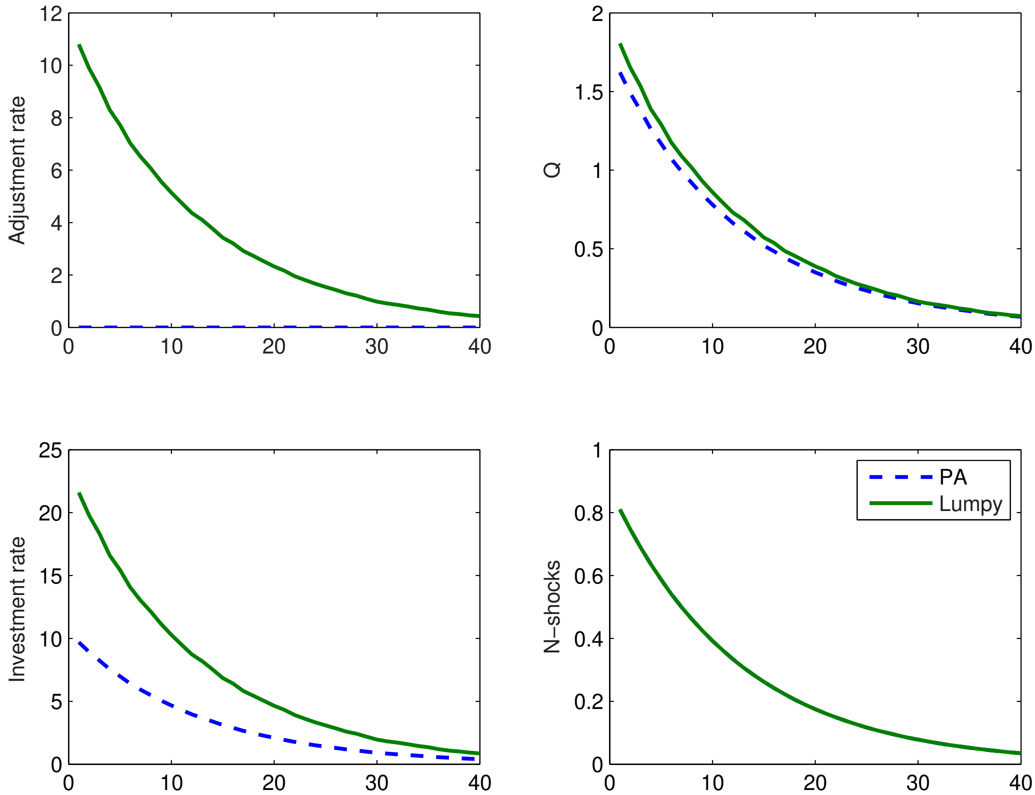

Figure 1: **Impulse responses to an N-shock in partial equilibrium.** This figure plots impulse responses (measured in percentage) to a standard deviation positive shock to the labor-augmenting technology in partial equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model.

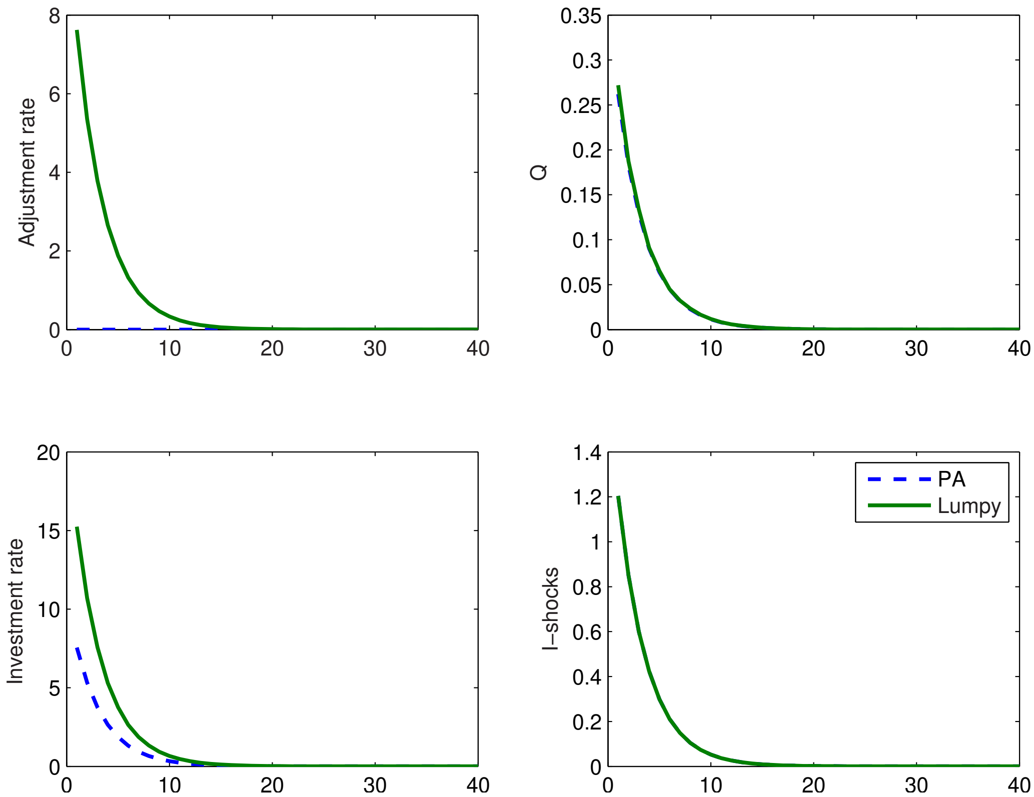

Figure 2: **Impulse responses to an I-shock in partial equilibrium.** This figure plots impulse responses (measured in percentage) to a standard deviation positive shock to the investment-specific technology in partial equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model.

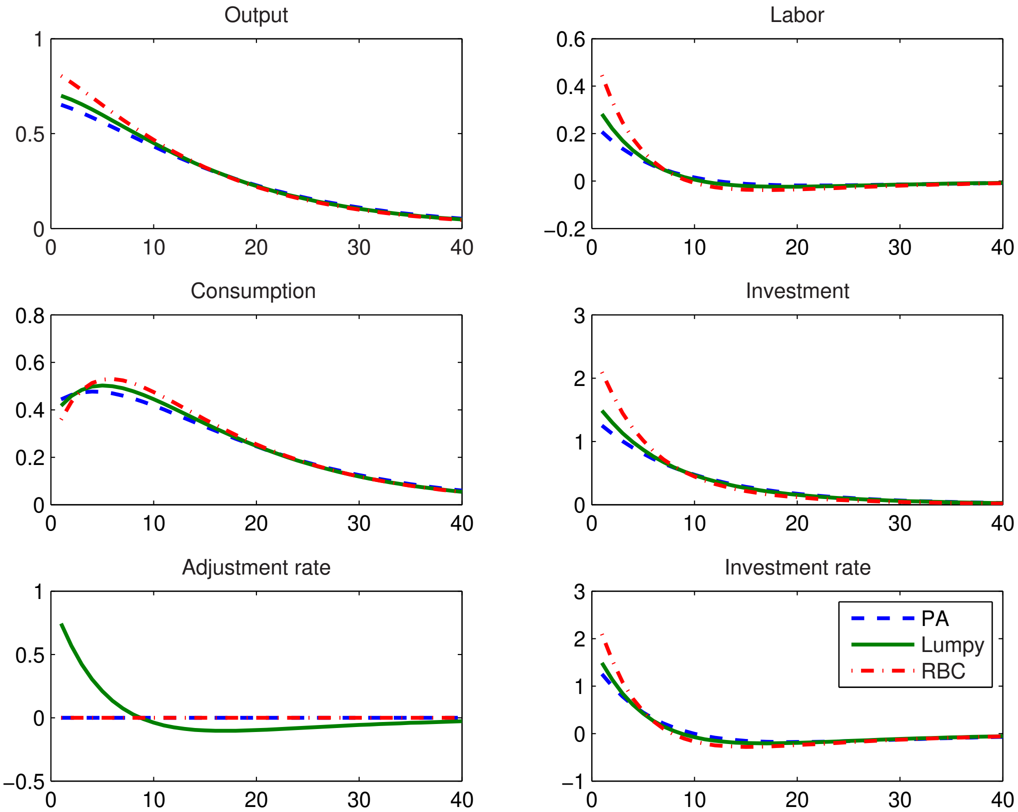

Figure 3: **Impulse responses to an N-shock in general equilibrium.** This figure plots impulse responses (measured in percentage) of quantities to a standard deviation positive shock to the labor-augmenting technology in general equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model. RBC: frictionless RBC model.

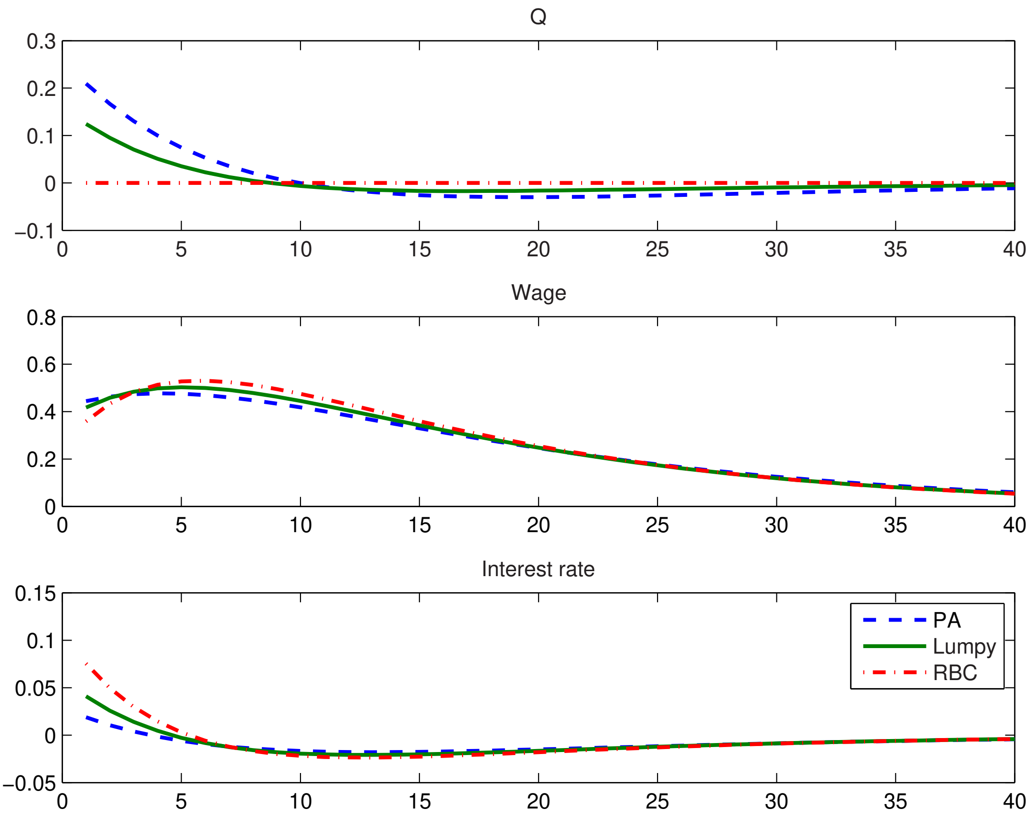

Figure 4: **Impulse responses to an N-shock in general equilibrium.** This figure plots impulse responses (measured in percentage) of prices to a standard deviation positive shock to the labor-augmenting technology in general equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model. RBC: frictionless RBC model.

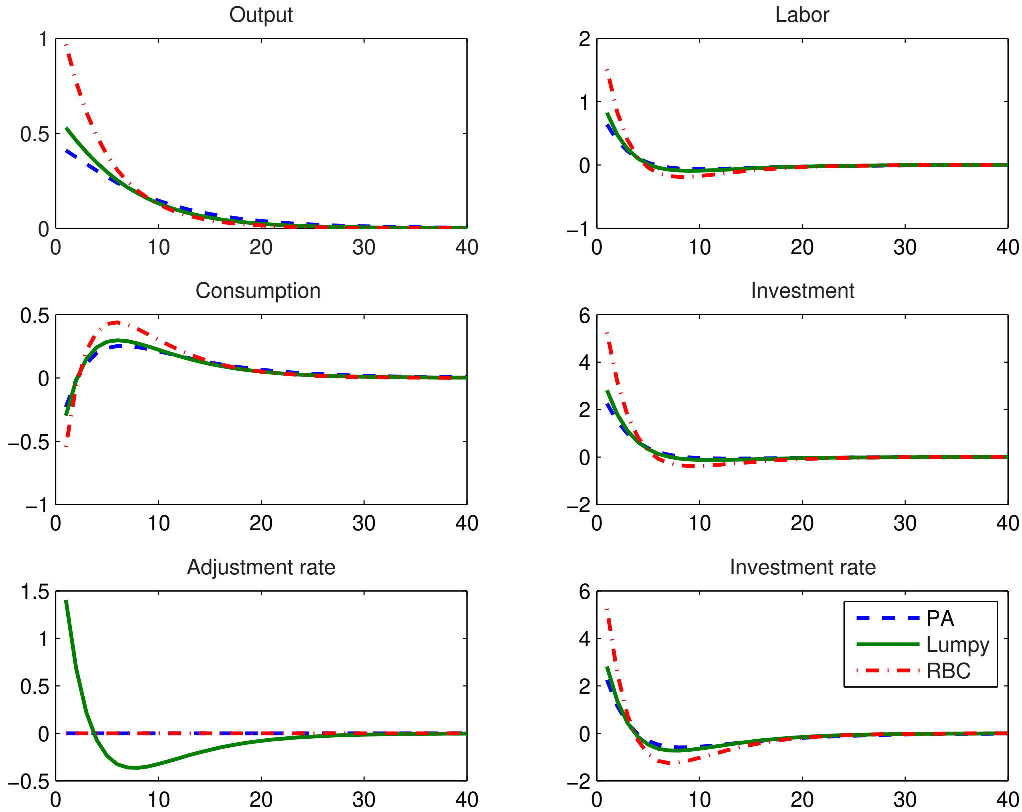

Figure 5: **Impulse responses of an I-shock in general equilibrium.** This figure plots impulse responses (measured in percentage) of quantities to a standard deviation positive shock to the investment-specific technology in general equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model. RBC: frictionless RBC model.

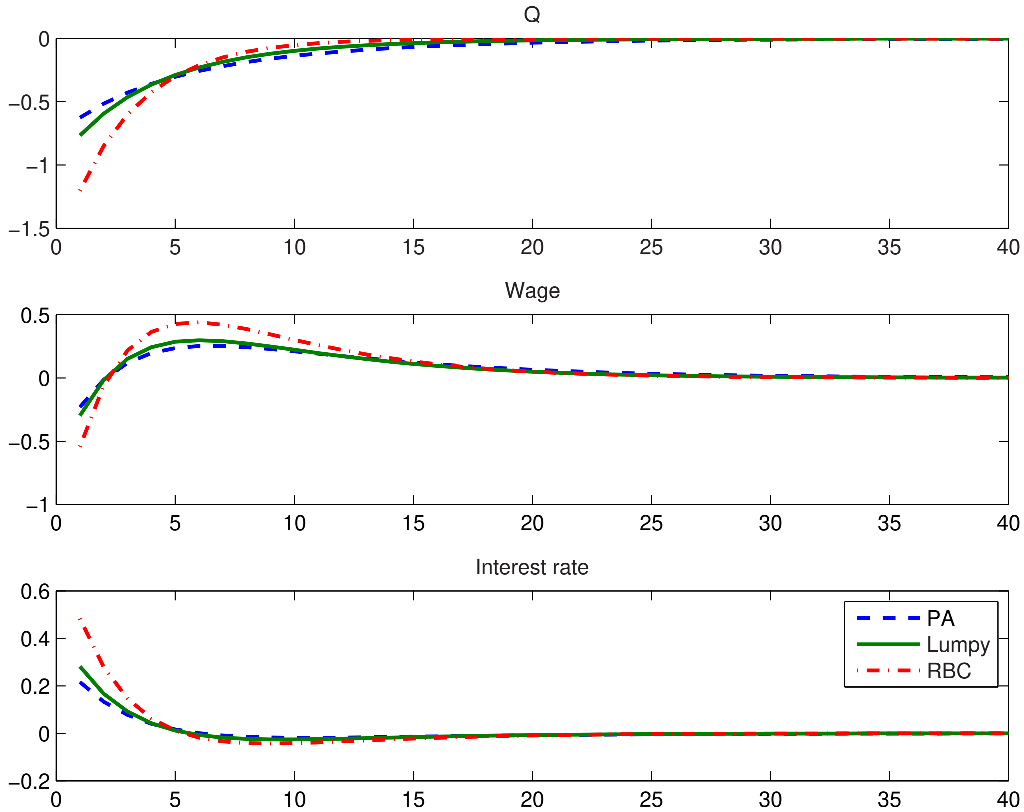

Figure 6: **Impulse responses of an I-shock in general equilibrium.** This figure plots impulse responses (measured in percentage) of prices to a standard deviation positive shock to the investment-specific technology in general equilibrium. PA: partial adjustment model. Lumpy: lumpy investment model. RBC: frictionless RBC model.