Independent inflation-targeting regime versus monetary union: An analysis of dynamic stability under endogenous inflation expectations

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Independent inflation-targeting regime *versus* monetary union:

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Abstract: Some countries may face choice between targeting inflation independently and entering a monetary union that targets inflation. This paper shows that the choice of a country in favour of monetary union may be motivated by asymmetrical supply shocks. The demand shocks are neutralised under these regimes and don't explain the choice of joining a monetary union. Further, before or after the construction of the union, monetary authorities must keep a minimum concern for stabilising output around its potential in order to guarantee the dynamic stability of the economy in a framework where the central bank is assumed to be unable to perfectly control, through the manipulation of the repo interest rate, the interest rate at which the private financial and non-financial agents lend and borrow. The disappearance of national currencies can render the economy of the union unstable.

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1. Introduction

Inflation targeting has become explicitly or implicitly the principal framework for the conduct of monetary policy in many industrialised and emerging countries. Earlier researches have been focused on issues in closed economies. More and more attention is now being given to the studies of inflation targeting in open economies[1].

Most of the studies examine the inflation-targeting issues in small open economy. There are fewer studies of inflation targeting within a multi-country framework. Persson and Tabellini (1996), focusing on the relationship between the "ins" and the "outs" at Stage III of the EMU within a two-country framework, do not consider entering a monetary union that targets inflation as an alternative to independent inflation targeting.

Canzoneri, Nolan, and Yates (1997) compare inflation targeting with the ERM in a two-country model. They consider the case where one of the countries has low inflation and an optimal degree of stabilisation and the other country lacks the credibility to implement the optimal monetary policy rule. Their focus is on credibility of Central Bank rather than stabilisation of the economy.

Roseland and Torvik (2003) go further in establishing a bridge between the inflation targeting and the optima currency area (OCA) literature. In their view, a monetary union that targets inflation is the most relevant alternative to independent inflation targeting for many countries. They extend the theory of OCA to deal with the choice for some countries between targeting inflation independently and entering a monetary union that targets inflation. In contrast to the conventional theory, countries might form more of an optimum currency area the more asymmetrical supply shocks are. They consider only the static comparatives of alternative monetary regimes. The dynamics of the economy is evacuated in setting the economy at stationary state and in assuming that the expected inflation rate is equal to the

[1] See Rodseth (1996), Dueker and Fischer (1996), Ball (1998), Batini and Haldane (1999), Svensson (2000), Gali and Monacelli (2005), and Clarida, Gali, and Gertler (2001), Berger, Jensen, and Schelderup (2001), Leitemo and Roisland (2003).

inflation target of the central bank whatever is the nature of the shocks (persistent or transitory).

Benigno (2004), Benigno and López-Salido (2006) investigate in a two-region general equilibrium model with monopolistic competition and price stickiness, the optimal conduct of monetary policy in a currency area characterised by asymmetric shocks across regions. They show that monetary policy should follow a particular inflation-targeting policy in which higher weight is given to the inflation rate in the region with higher degree of nominal rigidity.

The present study extends that of Roseland and Torvik (2003) in giving a thorough dynamic treatment of independent inflation targeting and monetary union regimes while granting a role to the money markets that are neglected in the traditional inflation-targeting literature.[2] In effect, by controlling the repo interest rate which is a rate at which the commercial banks can refinance at the central bank, the central bank cannot control directly and tenuously the interest rates at which financial and non-financial agents lend and borrow. In this respect, even if a part of money supply is entirely determined by the market participants to the money market, private agents will check the state of liquidity on the money market to find out if there is too much or too little liquidity, hence generating inflationary or deflationary pressure. In a context of financial crisis or boom, central banks will be more likely to intervene, by injecting or withdrawing the liquidity, on the money market to regulate the money market interest rate which may deviate from the repo interest rate. In this context, the private inflation expectations will be governed by a dynamics determined by all equations of the model rather than just by the Phillips curve, IS curve and the optimal interest rate rule.

[2] According to King (2000), there are two conventional strategies to use LM in the literature relating to the interest rate rule. The first uses LM to specify money supply rules and to compare them with interest rate rules. The second uses interest rate rule to describe the monetary strategy, LM is used to determine the endogenous money supply. Romer (2000) proposes the teaching of macroeconomics without LM. Within a static framework of analysis adopting a interest rate rule, the absence of LM or money market has not any great inconvenience. The matter is different in a dynamic setting as shown by Dai and Sidiropoulos (2003) and this paper: the money markets described by LM can play the role of co-ordinating the inflation expectations of private agents.

The money market plays a central role as the coordination device for the formation of expectations about future variables (Dai and Sidiropoulos, 2003).

Our paper shows that, whatever is the regime adopted, monetary authorities must not place too much weight on inflation target relative to output target for the economy to be dynamically stable. Some difficulty in stabilising the economy of monetary union may arise with the disappearance of nominal exchange rate between national currencies. Even the present model is more traditional and not micro-founded like that of Benigno (2004) and Benigno and López-Salido (2006), it gains in tractability and gives rise to interesting insights in the analysis of dynamic stability.

Section 2 lays down the model. Section 3 studies the long-run effects of shocks under the two alternative monetary regimes. The dynamic stability of these regimes and the dynamic adjustment paths of different variables are examined in sections 4. Section 5 concludes.

2. The Model

Home country and foreign country are of the same size. Each country has specialised in producing a single good. Inflation in home country and foreign country (indexed by *f*) is respectively governed by an expectation-augmented Phillips curve:

$$\pi = \pi^{e} + \alpha\,(y - y^{*}) + \varepsilon_{\pi}, \qquad \alpha > 0, \tag{1}$$

$$\pi^{f} = \pi^{fe} + \alpha\,(y^{f} - y^{f*}) + \varepsilon_{\pi}^{f}. \tag{2}$$

where π $(\equiv dp/dt)$ and π^{f} $(\equiv dp^{f}/dt)$ are respectively home and foreign inflation rates, π^{e} and π^{fe} the expected inflation rates, y and y^{f} the actual outputs, y^{*} and y^{f*} the natural levels of output and ε_{π} and ε_{π}^{f} white noise inflationary (or supply) shocks. The aggregate demand in each country, which is equal to output, depends on the expected real interest rate, $(i - \pi^{e})$ or $(i^{f} - \pi^{fe})$, and the real exchange rate, s, as follows:

$$y = -\beta(i - \pi^e) + \gamma s + \varepsilon_d, \qquad \beta, \gamma > 0, \tag{3}$$

$$y^f = -\beta(i^f - \pi^{fe}) - \gamma s + \varepsilon_d^f, \tag{4}$$

where i and i^f are nominal interest rates, $s \equiv p^f + e - p$, with p denoting the domestic price level in home currency, p^f the foreign price level in foreign currency, e the nominal exchange rate, and ε_d and ε_d^f are white noise demand shocks.

The foreign exchange market is characterised by uncovered interest rate parity adjusted for a stochastic risk premium shock ε_e, i.e.,

$$\begin{aligned} i &= i^f + \dot{e}^e + \varepsilon_e \\ &= i^f + \dot{s}^e + \pi^e - \pi^{fe} + \varepsilon_e. \end{aligned} \tag{5}$$

with $\dot{e}^e$ and $\dot{s}^e$ denoting respectively the expected rate of change of nominal and real exchange rates.

The money market equilibrium are characterised by

$$m - p = l_1 y - l_2 i + \varepsilon_m, \qquad l_1, l_2 > 0, \tag{6}$$

$$m^f - p^f = l_1 y^f - l_2 i^f + \varepsilon_m^f, \tag{7}$$

where m and m^f represent respectively home and foreign money supplies. ε_m and ε_m^f are white noise shocks affecting respectively home and foreign money demands. The differentiation of (6)-(7) relative to time gives:

$$\dot{m} - \dot{p} = \mu - \pi = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_m, \qquad l_1, l_2 > 0. \tag{6'}$$

$$\dot{m}^f - \dot{p}^f = \mu^f - \pi^f = l_1 \dot{y}^f - l_2 \dot{i}^f + \dot{\varepsilon}_m^f. \tag{7'}$$

where $\dot{m} = dm/dt = \mu$ and $\dot{m}^f = dm^f/dt = \mu^f$ are respectively home and foreign money supply growth rates. It is assumed that $\dot{\varepsilon}_m = \dot{\varepsilon}_m^f = 0$. Since money supply is endogenous in the inflation-targeting framework, (6')-(7') imply in the long run that money growth rate will be equal to current and expected inflation rates, i.e., $\bar{\mu} = \bar{\pi} = \bar{\pi}^e$ and $\bar{\mu}^f = \bar{\pi}^f = \bar{\pi}^{fe}$.

2. Alternative monetary policy regimes

In the inflation-targeting literature, the main rationale for adopting explicit inflation targets is to enhance credibility of monetary policy. The central bank uses the repo interest rate as instrument in order to achieve the inflation target. It is generally argued that, at least in a transition period until credibility is fully obtained, monetary policy must give higher priority to achieving the inflation target than is the case with a discretionary policy. As the dynamics of the monetary policy transmission mechanism is generally not modelled, the inflation-targeting regime is interpreted as like the central bank targets inflation at a horizon where most of the intermediate dynamics has taken place.

The focus of this paper is the dynamics of the monetary policy transmission mechanism. Two alternatives monetary regimes are considered to close the model: independent inflation targeting or monetary union targeting inflation. Under these two regimes, only flexible inflation targeting is examined[3]. In fact, the strict inflation-targeting regime can be considered as a limit case where the central bank attributes a very weak weight to its output target. The advantage of this approach is that it permits the study of the dynamics of the monetary policy transmission mechanism and the limit to the priority that the central bank can place on inflation target. In the present dynamic setting, a special role, other than determining passively the money supply growth rate, is given to monetary markets and money growth rates. The latter play the role of indicators.

2.1 Independent inflation targeting

Before forming a monetary union, home and foreign countries target inflation independently. The independent inflation-targeting regime, where home and foreign countries

[3] Roisland and Torvik (2003) follow the approach of strict inflation targeting regime used by Persson and Tabellini (1996), Frankel and Chinn (1995). They consider also shortly the flexible inflation targeting regime.

target their respective inflation rate (π and π^f) and output (y and y^f), can thus be specified as Central Banks minimise the present discounted value of the following loss functions[4]:

$$L = \int_0^\infty \frac{1}{2}\left[\lambda(y - y^*)^2 + \kappa(\pi - \pi^T)^2\right]\exp(-\theta t)dt\,, \qquad \lambda, \kappa, \theta > 0\,, \tag{8}$$

$$L^f = \int_0^\infty \frac{1}{2}\left[\lambda(y^f - y^{f*})^2 + \kappa(\pi^f - \pi^{fT})^2\right]\exp(-\theta t)dt\,, \tag{9}$$

where preference parameters λ and κ denote respectively the weight that monetary authorities place on the output and on the inflation stabilisation, and θ is the discount factor. The home and foreign Central Banks minimise fluctuations of output around the potential output (respectively y^* and y^{f*}) and that of inflation around the inflation target (respectively π^T and π^{fT}), in using the nominal interest rates as the instrument of monetary policy. However, the direct instrument of the central bank is the repo interest rate. By manipulating the repo interest rate, the central bank is not always able to control the interest rates on the money and financial markets at which the private agents lend and borrow. To attain its target interest rate, the central bank may intervene in a discretionary manner on the money market in order to bring the market interest rates as close as possible to the optimal target interest rate defined in the following. The solution of central banks' optimisation programmes leads to (Appendix A.1):

$$i = \pi^e + \frac{1}{\beta}\left\{\gamma s + \frac{\kappa\alpha}{\lambda}(\pi - \pi^T) - y^* + \varepsilon_d\right\}, \tag{12}$$

$$i^f = \pi^{fe} + \frac{1}{\beta}\left\{-\gamma s + \frac{\kappa\alpha}{\lambda}(\pi^f - \pi^{fT}) - y^{f*} + \varepsilon_d^f\right\}. \tag{13}$$

Equations (12)-(13) represent the central banks' optimal interest rate rules. The nominal interest rate in each country increases to fully reflect expected inflation. It increases also whenever output (due to a positive demand shock) or current inflation rises above the central

[4] For the sake of simplicity, CPIs and GDP deflators are not distinguished here.

bank's target. Home (foreign) interest rate reacts positively (respectively negatively) to the real exchange rate. The reaction of nominal interest rate to current inflation, contrarily to the case of other variables, depends on the ratio κ/λ. When this ratio increases, it means that the central bank will give more weight for the achievement of inflation target. The presence of the real exchange rate (*s*) in the these rules makes it possible for the central bank to react indirectly to shocks affecting the financial markets which integrate information in the exchange rate through the purchase and sale of home and foreign assets.

When shocks are stochastic and temporary, the expected inflation rates can be estimated *ex ante* to be equal to the inflation targets of the central banks, i.e. $\pi^e = \pi^T$ and $\pi^{fe} = \pi^{fT}$. However, if shocks are persistent or permanent, the private agents will not continue to believe in the announced targets. In contrast, private agents will take account, ex *post,* of these shocks as well as the economic conditions to modify rationally their inflation expectations. In effect, the liquidity in the economy is not always well regulated by market participants themselves and the central bank may intervene in an imperfect manner. That implies the inflation may have greater probability of deviating from the inflation target announced by the central bank. Consequently, the information from the money and financial markets is very useful for formation of inflation expectations by the private agents.

The long-run equilibrium ($\bar{\pi} = \bar{\pi}^e = \bar{\mu}, \bar{\pi}^f = \bar{\pi}^{fe} = \bar{\mu}^f, \bar{\pi} = \bar{e} + \bar{\pi}^f, \quad \bar{y}, \bar{y}^f, \bar{i}, \bar{i}^f, \bar{s}$) is characterised as follows (Appendix B.1):

$$\bar{y} = y^* - \frac{\varepsilon_\pi}{\alpha}, \qquad \text{and} \qquad \bar{y}^f = y^{f*} - \frac{\varepsilon_\pi^f}{\alpha}, \tag{14}$$

$$\bar{\pi} = \bar{\pi}^e = \bar{\mu} = \pi^T + \frac{\lambda}{\kappa\alpha^2}\varepsilon_\pi, \text{ and } \bar{\pi}^f = \bar{\pi}^{fe} = \bar{\mu}^f = \pi^{fT} + \frac{\lambda}{\kappa\alpha^2}\varepsilon_\pi^f, \tag{15}$$

$$\bar{s} = \frac{1}{2\gamma}(y^* - y^{f*} - \frac{\varepsilon_\pi}{\alpha} + \frac{\varepsilon_\pi^f}{\alpha} - \varepsilon_d + \varepsilon_d^f), \tag{16}$$

$$\bar{i} = \pi^T - \frac{(y^* + y^{f*})}{2\beta} + (\frac{\kappa\alpha + 2\beta\lambda}{2\beta\kappa\alpha^2})\varepsilon_\pi + \frac{1}{2\alpha\beta}\varepsilon_\pi^f + \frac{\varepsilon_d}{2\beta} + \frac{\varepsilon_d^f}{2\beta}, \tag{17}$$

$$\bar{i}^f = \pi^{fT} - \frac{(y^* + y^{f*})}{2\beta} + (\frac{\kappa\alpha + 2\beta\lambda}{2\beta\kappa\alpha^2})\varepsilon_\pi^f + \frac{1}{2\alpha\beta}\varepsilon_\pi + \frac{\varepsilon_d}{2\beta} + \frac{\varepsilon_d^f}{2\beta}, \tag{18}$$

$$\bar{\dot{e}} = \bar{i} - \bar{i}^f - \varepsilon_e = \pi^T - \pi^{fT} + \frac{\lambda}{\kappa\alpha^2}\varepsilon_\pi - \frac{\lambda}{\kappa\alpha^2}\varepsilon_\pi^f - \varepsilon_e, \tag{19}$$

$$\bar{i} - \bar{\pi} = \bar{i}^f - \bar{\pi}^f = -\frac{(y^* + y^{f*})}{2\beta} + \frac{\varepsilon_\pi + \varepsilon_\pi^f}{2\beta\alpha} + \frac{\varepsilon_d + \varepsilon_d^f}{2\beta}. \tag{20}$$

The interest rate rules formulated in (12)-(13) permit inflation rate and output in each country to be only sensible to national supply shocks[5]. Nominal interest rate reacts more strongly to national supply shocks than that from other country, but equally to demand shocks in each country due to the transmission channel of real exchange rate. The home (foreign) supply and demand shocks have negative (respectively positive) effect on the real exchange rate, i.e. improvement (respectively deterioration) of terms of exchange. As s is stationary at equilibrium, the nominal exchange rate can evolve in time at a constant rate, $\bar{\dot{e}}$. The latter is determined by the gap between home and foreign inflation targets, the supply shocks in each country and the exchange rate risk premium shock. The effect of supply shocks on the rate of change of nominal exchange rate ($\bar{\dot{e}}$) depends on the slope of Phillips curve and the central banks' preferences relative to inflation and output targets. The less the central banks cares about output targets (smaller ratio λ/κ), the less the supply shocks have effect on $\bar{\dot{e}}$.

2.2 Inflation targeting in monetary union

In a monetary union, home and foreign countries have a common currency and monetary policy. The central bank targets the average inflation rate of the union. The monetary policy regime is specified as the minimisation of the following loss function:

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[5] In Roisland and Torvik (2003), output in each country is affected by a weighted average of home and foreign supply shocks under the strict inflation targeting regime. The difference with the present result is due to two facts. First, they assume that the expected inflation rates are given and equal to the inflation targets. Second, in assuming that the central banks realise their CPI targets, they do not allow the nominal interest rate to compensate the exchange rate effect on the aggregate demands in each country.

$$L^U = \int_0^\infty \frac{1}{2}\left[\lambda(y^U - y^{U*})^2 + \kappa(\pi^U - \pi^{UT})^2\right]\exp(-\theta t)dt. \tag{21}$$

with $y^{U*} = (y^* + y^{*f})/2$, $\pi^U = (\pi + \pi^f)/2$. π^{UT} can be equal to $(\pi^T + \pi^{fT})/2$ or to other value chosen by the central bank. The solution of (21) leads to (Appendix A.2)

$$i^U = \pi^{Ue} + \frac{\alpha\kappa}{\lambda\beta}(\pi^U - \pi^{UT}) - \frac{y^{U*}}{\beta} + \frac{\varepsilon_d + \varepsilon_d^f}{2\beta}. \tag{22}$$

As the central bank stabilises the union's average inflation rate and output, the real exchange rate does not play any role in the interest rate rule. In fact, its contradictory effects on the average inflation rate and output are neutralised since the two countries are of same size. A reaction to it does not permit to control the average inflation rate and output of the union.

The solution of long-term equilibrium (where $\dot{s} = \pi^f - \pi = 0$, $\overline{\pi} = \overline{\pi}^e = \overline{\pi}^f = \overline{\pi}^{fe}$ $= \overline{\pi}^U = \overline{\pi}^{Ue} = \overline{\mu}^U$) is (Appendix B.2):

$$\overline{y} = y^* - \frac{\varepsilon_\pi}{\alpha} \quad \text{and} \quad \overline{y}^f = y^{f*} - \frac{\varepsilon_\pi^f}{\alpha}, \tag{23}$$

$$\overline{\pi}^U = \pi^{UT} + \frac{\lambda}{2\alpha^2\kappa}(\varepsilon_\pi + \varepsilon_\pi^f), \tag{24}$$

$$\overline{s} = \frac{1}{2\gamma}(y^* - y^{f*} - \frac{\varepsilon_\pi}{\alpha} + \frac{\varepsilon_\pi^f}{\alpha} - \varepsilon_d + \varepsilon_d^f), \tag{25}$$

$$\overline{i}^U = \pi^{UT} + \frac{\lambda\beta + \alpha\kappa}{2\beta\kappa\alpha^2}(\varepsilon_\pi + \varepsilon_\pi^f) + \frac{\varepsilon_d + \varepsilon_d^f}{2\beta} - \frac{y^{U*}}{\beta}, \tag{26}$$

$$\overline{i}^U - \overline{\pi}^U = -\frac{y^{U*}}{\beta} + \frac{\varepsilon_\pi + \varepsilon_\pi^f}{2\beta\alpha} + \frac{\varepsilon_d + \varepsilon_d^f}{2\beta}. \tag{27}$$

In comparing (14) and (23), the inflation-targeting union does not do better than independently inflation-targeting national central bank in stabilizing output. This is due to the fact that the expected inflation rates adjust fully to any shock after its realisation[6]. In examining (15) and (24), one finds that the difference between the two monetary policy

[6] The result here is quite different from the result of Roisland and Torvik (2003). In fact, it is assumed here that the expected inflation rates adjust after the (persistent) shocks are realized, so that the output in each country will not be influenced by inflationary (or supply shock) at national level.

regimes is only reflected in the level and variance of inflation rates. In the union, as the nominal exchange rate disappears and the nominal interest rate is unique, the equalisation of real return of financial assets (bonds, equities etc.) means that the inflation rates of the member countries can not be different in the long run. It is easy then to analyse the welfare effect of the alternative monetary regimes in using central banks' loss functions. As outputs depend only on internal supply shocks in each country, the analysis can be done in comparing only the variances of inflation rates. Using (15) and (24), one has:

$$\mathrm{var}(\overline{\pi}^U) = (\frac{\lambda}{2\alpha^2\kappa})^2[\sigma_\pi^2 + 2\mathrm{cov}(\varepsilon_\pi, \varepsilon_\pi^f) + \sigma_\pi^{f2}], \tag{28}$$

$$\mathrm{var}(\overline{\pi}) = (\frac{\lambda}{\kappa\alpha^2})^2\sigma_\pi^2, \quad \text{and} \quad \mathrm{var}(\overline{\pi}^f) = (\frac{\lambda}{\kappa\alpha^2})^2\sigma_\pi^{f2}. \tag{29}$$

In the union, the inflation rate depends on the sum of inflationary (or supply) shocks. If the inflationary shocks in the two countries are of the same value ($\sigma_\pi^2 = \sigma_\pi^{f2}$) and asymmetrical (i.e. with opposite signs or negatively correlated), the variance of union's inflation rate (and so the variance of the inflation rates in each country of the union) will be less important than under independent inflation-targeting regime. The advantage of the union will not be clear-cut when the supply shocks have always the same sign (positively correlated) but different in value, the country having smaller shocks has less interest to join a monetary union. The home and foreign countries will join simultaneously the union only if the following conditions are satisfied:

$$\mathrm{var}(\overline{\pi}^U) - \mathrm{var}(\overline{\pi}) = (\frac{\lambda}{2\alpha^2\kappa})^2[2\mathrm{cov}(\varepsilon_\pi, \varepsilon_\pi^f) + \sigma_\pi^{f2} - 3\sigma_\pi^2] < 0, \tag{30}$$

$$\mathrm{var}(\overline{\pi}^U) - \mathrm{var}(\overline{\pi}^f) = (\frac{\lambda}{2\alpha^2\kappa})^2[2\mathrm{cov}(\varepsilon_\pi, \varepsilon_\pi^f) + \sigma_\pi^2 - 3\sigma_\pi^{f2}] < 0. \tag{31}$$

The nominal interest rate in the union will have a smaller variance[7] than under the independent inflation-targeting regime when the supply shocks in home and foreign countries have the same variance but negative covariance. It follows from (15), (17)-(18), (24) and (26):

$$\mathrm{var}(i^U) - \mathrm{var}(i) = (\frac{\lambda\beta}{2\beta\kappa\alpha^2})(\frac{2\kappa\alpha + 3\beta\lambda}{2\beta\kappa\alpha^2})(\sigma_\pi^{f2} - \sigma_\pi^2) + 2(\frac{\lambda\beta}{2\beta\kappa\alpha^2})^2 \mathrm{cov}(\varepsilon_\pi, \varepsilon_\pi^f), \quad (32)$$

$$\mathrm{var}(i^U) - \mathrm{var}(i^f) = (\frac{\lambda\beta}{2\beta\kappa\alpha^2})(\frac{2\kappa\alpha + 3\beta\lambda}{2\beta\kappa\alpha^2})(\sigma_\pi^2 - \sigma_\pi^{f2}) + 2(\frac{\lambda\beta}{2\beta\kappa\alpha^2})^2 \mathrm{cov}(\varepsilon_\pi, \varepsilon_\pi^f). \quad (33)$$

In contrast, the real interest rates have always the same variance under these two regimes.

Monetary union has another advantage. As the nominal exchange rate disappears, the risk premium shocks affecting the foreign exchange market will not influence any more the firms' import and export decisions and their international asset allocations. These gains are not reflected in the loss functions of the central banks.

The conventional wisdom regarding shocks and OCA can be summarised by saying that the more asymmetrical shocks countries face, the less of an optimum currency area they constitute. For Roisland and Torvik (2003), with inflation targeting, this conventional wisdom holds for demand shocks, but not for supply shocks. In the present model, the results of Roisland and Torvik concerning supply shocks are confirmed. The more asymmetrical are the supply shocks, the more interesting is the participation to a monetary union. In contrast, in this simple model where the expected inflation rates adjusted after the realisation of shocks, the demand shocks do not play any role in the decisions to form a union or not.

3. Stability analysis under alternative regimes

In the following, the intermediate dynamics under alternative inflation-targeting regimes will be studied. For Roisland and Torvik (2003), by assumption, the stationary equilibrium is always realisable and the economy will converge to it.

[7] A smaller variance of nominal interest rate may reduce risk premium incorporated in long term interest rate (not considered in this simple model) and stimulate capital investment in the long run.

The question is that, if this is the case, the inflation targeting will be a miracle receipt for all these countries suffering from chronic macro-economic instability. This might not be the case. It is important and interesting to discuss how the economy behaves dynamically when the inflation-targeting framework is adopted, and in particular, how the expected inflation rates adjust following the realisation of shocks.

The adjustment of expected inflation rates is justifiable when the stochastic shocks have persistent effects once realised and known with certainty. In this case, it is not reasonable to assume that private agents believe in central bank's inflation target and continue to take it as their expectation when all evidence indicates that the realised inflation rate is and will be higher or lower than the official target. As the money growth rates are endogenous, once announced by the central banks, they can be used as indicator by private agents in adjusting their expected inflation rates.

In fact, central banks decide in their open market operations not only the nominal interest rates but also the amounts of liquidity allocated to commercial banks. The private agents take simply the announced money growth rates as given and equalised to their expected ones. The use of the endogenous money growth rates as indicators leads to endogenous dynamic adjustments of expected inflation rates[8]. In addition to the role of endogenous determination of monetary growth rates, monetary markets are useful places in co-ordinating the inflation expectations of private agents, expressed through their buying and selling of financial assets.[9]

3.1. Independent flexible inflation targeting

The variation of differential equations of $\dot{s}$, $\dot{\pi}^e$ and $\dot{\pi}^{fe}$ around the steady state gives (Appendix C):

[8] This is the assumption adopted by Dai and Sidiropoulos (2003).
[9] That's the only role attributed to the money market since Taylor (1993) in the inflation targeting literature.

$$\begin{bmatrix} \dot{\pi}^e \\ \dot{\pi}^{fe} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\Omega} & 0 & 0 \\ 0 & -\frac{1}{\Omega} & 0 \\ \frac{\kappa\alpha}{\beta(\lambda+\kappa\alpha^2)} & -\frac{\kappa\alpha}{\beta(\lambda+\kappa\alpha^2)} & \frac{2\gamma}{\beta} \end{bmatrix} \begin{bmatrix} \pi^e - \overline{\pi}^e \\ \pi^{fe} - \overline{\pi}^{fe} \\ s - \overline{s} \end{bmatrix} \text{ with } \Omega = \frac{l_1}{\alpha} - \frac{l_2}{\beta}\frac{\kappa\alpha}{\lambda}. \qquad (34)$$

There are two stables eigenvalues ($E_1 = E_2 = -\frac{1}{\Omega}$) under the condition $\Omega > 0$, i.e. $\frac{\kappa}{\lambda} < \frac{l_1\beta}{l_2\alpha^2}$, and one unstable eignevalue $E_3 = \frac{2\gamma}{\beta}$. That is the case when the central banks do not attach too strong weight to their inflation targets and are interested quite much in stabilising output. As the expected inflation rates are variables that adjust more slowly[10], they can be considered as predetermined variables. That means that they do not react immediately but later to news. The flexible nominal exchange rate and so the real exchange rate plays the role of non-predetermined variable. In fact, the transactions on the exchange market are much faster than on the goods markets and hence the adjustment of nominal exchange rate is not subjected to menu costs as in the case of many goods. With two stable eigenvalues and two predetermined variables, the system is characterised by saddle-point equilibrium. There is only one stable converging path to the steady state under the condition that central banks of two countries attribute a reasonable weight to their output target. If central banks give too much attention to inflation rates, the economy may not be stable and can take any path from the equilibrium after any small shock changing temporarily or permanently the steady state of the economy. In this case, there is more risk to see the appearance of hyperinflation, hyper-deflation or other disequilibrium situations.

[10] In the present model, the expected inflation rate in the Phillips curve is formed before the arrival of shocks. The empirical studies (Gordon, 1997) show an inertia of the adjustment of inflation rate. In assuming the expected inflation rate as a predetermined variable, reacting to economic news slowly without jumps, one admits that the prices adjust with lags due to menu costs or other rigidities. In this paper, output and realised inflation rates can jumps initially to share the adjustment due to an inflationary shock or the jumps of interest rates or nominal exchange rate. See also Buiter and Panigirtzoglou (2003) for a similar assumption concerning inflation rate. This behaviour corresponds well to the situation in a low inflation period in contrast to high inflation or hyperinflation experiences.

3.2. Monetary union

In taking its variation around the steady state, the dynamic system can be written as (Appendix D):

$$\begin{bmatrix} \dot{\pi}^{e} \\ \dot{\pi}^{fe} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \Psi_1 & \Psi_2 & -\Phi \\ \Psi_2 & \Psi_1 & \Phi \\ -(1+\alpha\beta) & (1+\alpha\beta) & -2\alpha\gamma \end{bmatrix} \begin{bmatrix} \pi^{e} - \overline{\pi}^{e} \\ \pi^{fe} - \pi^{fe} \\ s - \overline{s} \end{bmatrix} \tag{35}$$

with $\Psi_1 = \frac{1}{2}(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} - \frac{\alpha}{l_1}) + \frac{\gamma(1+\alpha\beta)}{\beta}$; $\Psi_2 = \frac{1}{2}(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} + \frac{\alpha}{l_1}) - \frac{\gamma(1+\alpha\beta)}{\beta}$;

$\Phi = \frac{\alpha\gamma}{l_1\beta} - \frac{2\alpha\gamma^2}{\beta}$. The characteristic polynomial of the stability matrix is:

$$(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} - E)(-\frac{\alpha}{l_1} - E)(\frac{2\gamma}{\beta} - E) = 0. \tag{36}$$

The system has two stable eigenvalues and one unstable eigenvalue:

$$\mathrm{E}_1 = \frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} < 0, \text{ if } \frac{\kappa}{\lambda} < \frac{l_1\beta}{l_2\alpha^2},$$

$$\mathrm{E}_2 = -\frac{\alpha}{l_1} < 0,$$

$$\mathrm{E}_3 = \frac{2\gamma}{\beta} > 0.$$

If one admits like previously that the expected inflation rates are predetermined variables and the real exchange rate a non-predetermined variable, the economy has saddle-point equilibrium with a unique converging path. The necessary condition guaranteeing the existence of this path is the same as under the independent inflation-targeting regime where the central banks must moderate their preference for the stabilisation of inflation relative to that of output.

In the monetary union regime, the introduction of a unique currency does not confer to the central bank the power to attribute a stronger relative weight in favour of inflation target than national central banks, even though the member countries may be characterised by asymmetrical shocks. The central bank of the union being less conservative is then beneficial for the macro-economic stability of the union. This result seems uncomfortable for the position taken by the European central bank since it focuses excessively on its inflation target.

Nevertheless, the stability conditions in a monetary union may be questioned on the basis of the asymmetry of shocks and the assumption about the adjustment speed of the real exchange rate.

Firstly, when the shocks are asymmetrical, the economy of member countries may goes in different directions as the nominal interest rate of the union reacts to the average level of inflation in the union. If the central bank of the union has the same preferences than national central banks, the reaction of the nominal interest rate of the union will be smaller than that of individual country at origin of shocks under independent inflation targeting. Insufficient reaction means then a too low or too high real interest rate in one country and inversely in another. There is a risk that the economy of a member country enters into an unsustainable boom or a prolonged depression with the other country in an opposite situation.

Secondly, for the real exchange rate, the story in the monetary union is quite different from that under floating exchange rate regime, where the quick adjustment of the nominal exchange rate guarantees that of the real exchange rate. In a monetary union, as the foreign exchange market disappears, the adjustment of the real exchange rate is subjected to the same constraints as other goods prices; since its adjustment is resulting form that of nominal goods prices. If the adjustment of goods prices is submitted to menu costs and other rigidities, it is more reasonable to assume the real exchange rate as a predetermined variable. So with two stable roots and three predetermined variables, the economy of the union might not find a converging path to the stationary equilibrium. One may relative this scepticism in arguing that the nominal prices of tradable goods are more flexible than that of non-tradable goods. Then, if one redefines the real exchange rate as the relative price of tradable goods, it can be thought as a quite flexible and non-predetermined.

4. Dynamic adjustments after supply and demand shocks

Two kinds of shocks, i.e. the supply and demand shocks, are considered. The shocks affecting money and foreign exchange markets are not examined, as they do not change the real variables and the inflation rates.

4.1. Independent flexible inflation targeting

The general solution of the dynamic system (34) is written as follows:

$$\begin{bmatrix} \pi^e - \overline{\pi}^e \\ \pi^{fe} - \overline{\pi}^{fe} \\ s - \overline{s} \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} k_1 e^{E_1 t} \\ k_2 e^{E_2 t} \\ k_3 e^{E_3 t} \end{bmatrix}. \tag{37}$$

where $V_{ij}, i = 1,2,3,$ are eigenvectors which corresponds to eigenvalues E_j, $j = 1,2,3$. They are obtained using the following system:

$$\begin{bmatrix} -1/\Omega - E_j & 0 & 0 \\ 0 & -1/\Omega - E_j & 0 \\ \kappa\alpha/\beta(\lambda + \kappa\alpha^2) & -\kappa\alpha/\beta(\lambda + \kappa\alpha^2) & 2\gamma/\beta - E_j \end{bmatrix} \begin{bmatrix} V_{1j} \\ V_{2j} \\ V_{3j} \end{bmatrix} = 0. \tag{38}$$

The coefficients k_i in (37) can be determined given the initial conditions at the instant $t = 0$, namely $\pi^e(0)$ and $\pi^{fe}(0)$. As $E_3 > 0$, the dynamic adjustment along the unique convergent path will be described by (39) while imposing the restriction $k_3 = 0$. Thus, π^e and π^{fe} being predetermined by the previous evolution of the economy, s adjusts instantaneously and reaches the value $s(0^+)$ that ensures $k_3 = 0$ at $t = 0$. As $k_1 = \pi^e(0) - \overline{\pi}^e$, $k_2 = \pi^{fe}(0) - \overline{\pi}^{fe}$, the solution of (37) can be rewritten as

$$\begin{bmatrix} \pi^e - \bar{\pi}^e \\ \pi^{fe} - \bar{\pi}^{fe} \\ s - \bar{s} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\kappa\alpha/(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2) & \kappa\alpha/(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2) \end{bmatrix} \begin{bmatrix} (\pi_0^e - \bar{\pi}^e)e^{E_1 t} \\ (\pi_0^{fe} - \bar{\pi}^{fe})e^{E_2 t} \end{bmatrix}. \quad (39)$$

Knowing that $E_1 = E_2$, the instantaneous adjustment of s at $t = 0$ will satisfy the following relation:

$$s(0^+) - \bar{s} = \frac{\kappa\alpha[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]}{(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2)}. \quad (40)$$

For other variables, using (39), (A9)-(A.12), (6')-(7') and (12)-(13), it yields:

$$\pi - \bar{\pi} = \frac{\lambda}{\lambda + \kappa\alpha^2}(\pi^e - \bar{\pi}^e) = \frac{\lambda(\pi_0^e - \bar{\pi}^e)}{\lambda + \kappa\alpha^2} e^{E_1 t}, \quad (41)$$

$$y - \bar{y} = -\frac{\alpha\kappa}{\lambda + \kappa\alpha^2}(\pi^e - \bar{\pi}^e) = -\frac{\alpha\kappa(\pi_0^e - \bar{\pi}^e)}{\lambda + \kappa\alpha^2} e^{E_1 t}. \quad (42)$$

$$\pi^f - \bar{\pi}^f = \frac{\lambda}{\lambda + \kappa\alpha^2}(\pi^{fe} - \bar{\pi}^{fe}) = \frac{\lambda(\pi_0^{fe} - \bar{\pi}^{fe})}{\lambda + \kappa\alpha^2} e^{E_1 t}, \quad (43)$$

$$y^f - \bar{y}^f = -\frac{\alpha\kappa}{\lambda + \kappa\alpha^2}(\pi^{fe} - \bar{\pi}^{fe}) = -\frac{\alpha\kappa(\pi_0^{fe} - \bar{\pi}^{fe})}{\lambda + \kappa\alpha^2} e^{E_1 t}. \quad (44)$$

$$i - \bar{i} = \frac{[\beta(\lambda + \kappa\alpha^2) + \kappa\alpha](\pi_0^e - \bar{\pi}^e)}{\beta(\lambda + \kappa\alpha^2)} e^{E_1 t} + \frac{\gamma\kappa\alpha[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]}{\beta(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2)} e^{E_1 t}, \quad (45)$$

$$i^f - \bar{i}^f = \frac{[\beta(\lambda + \kappa\alpha^2) + \kappa\alpha](\pi_0^{fe} - \bar{\pi}^{fe})}{\beta(\lambda + \kappa\alpha^2)} e^{E_1 t} - \frac{\gamma\kappa\alpha[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]}{\beta(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2)} e^{E_1 t}, \quad (46)$$

$$\begin{aligned} \mu - \bar{\mu} = {} & \frac{\lambda(\pi_0^e - \bar{\pi}^e)}{\lambda + \kappa\alpha^2} e^{E_1 t} - \frac{l_1 E_1 \alpha\kappa(\pi_0^e - \bar{\pi}^e)}{\lambda + \kappa\alpha^2} e^{E_1 t} - \frac{l_2 E_1[\beta(\lambda + \kappa\alpha^2) + \kappa\alpha](\pi_0^e - \bar{\pi}^e)}{\beta(\lambda + \kappa\alpha^2)} e^{E_1 t} \\ & - \frac{l_2 E_1 \gamma\kappa\alpha[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]}{\beta(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2)} e^{E_1 t}, \end{aligned} \quad (47)$$

$$\begin{aligned} \mu^f - \bar{\mu}^f = {} & \frac{\lambda(\pi_0^{fe} - \bar{\pi}^{fe})}{\lambda + \kappa\alpha^2} e^{E_1 t} - \frac{l_1 E_1 \alpha\kappa(\pi_0^{fe} - \bar{\pi}^{fe})}{\lambda + \kappa\alpha^2} e^{E_1 t} - \frac{l_2 E_1[\beta(\lambda + \kappa\alpha^2) + \kappa\alpha](\pi_0^{fe} - \bar{\pi}^{fe})}{\beta(\lambda + \kappa\alpha^2)} e^{E_1 t} \\ & + \frac{l_2 E_1 \gamma\kappa\alpha[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]}{\beta(2\gamma - \beta E_1)(\lambda + \kappa\alpha^2)} e^{E_1 t}. \end{aligned} \quad (48)$$

An inflationary (or adverse supply) shock in the home country will in the long term increase the realised and expected inflation rates, reduce the output and the competitiveness in the home country. In the foreign country, only nominal and real interest rates react in order to neutralise the effects of the gain in its competitiveness. The real exchange rate jumps down initially to place the economy on the converging path and pursues its decrease until the steady state. The expected inflation rate adjusts smoothly, without any jump, to a higher level. Other variables (except the foreign output and inflation rates which will not adjust) follow increasing or decreasing path according to the case with an initial jump-up or jump-down[11] (Figure 1). The direction of initial movement of μ depends on the relative importance of three factors, i.e., the initial adjustments of inflation rate, output and interest rate according to equation (6'). As a consequence, the money growth rate in home country can initially jump up, contrary to what is illustrated in the Figure 1, where it jumps down.

A positive demand shock in home country has no long term effect on inflation rates and output levels, so the adjustment is entirely operated through the jumps of the nominal and real exchange rates as well as that of home and foreign nominal interest rates. The dynamic paths of other variables are stationary.

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[11] Using (14)-(15) and (A.9)-(A.10) into (41)-(42), one has $\pi_0 = \frac{\lambda}{\lambda+\kappa\alpha^2}(\pi^e - \overline{\pi}^e) = \pi^T + \frac{\lambda}{\lambda+\kappa\alpha^2}\varepsilon_\pi$ and $y_0 = y^* - \frac{\alpha\kappa(\lambda+\kappa\alpha^2)\varepsilon_\pi}{\kappa\alpha^2(\lambda+\kappa\alpha^2)} + \frac{\alpha\kappa}{\lambda+\kappa\alpha^2}\frac{\lambda}{\kappa\alpha^2}\varepsilon_\pi = y^* - \frac{\alpha\kappa}{\lambda+\kappa\alpha^2}\varepsilon_\pi$, home country's current inflation and output share the initial adjustment due to the supply shock by a jump-up and a jump-down respectively. From (43) and (44), foreign country's output and inflation rate will not be influenced by the home country supply shocks. The paths of home and foreign money supply growth rates and interest rates can be determined using (45)-(48) and (6')-(7'), (12)-(13) and (15).

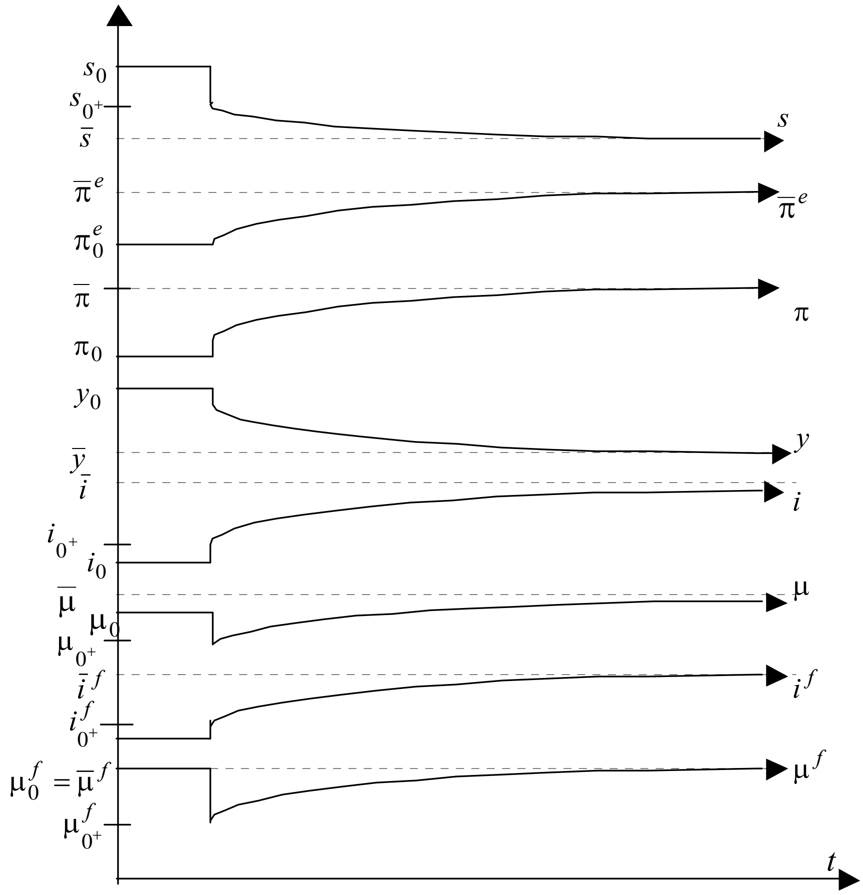

Figure 1. Adjustment paths of endogenous variables after a persistent adverse supply shock in the independent inflation-targeting regime.

4.2. Monetary union

The dynamic system of the monetary union is solved as follows:

$$\begin{bmatrix} \pi^e - \bar{\pi}^e \\ \pi^{fe} - \bar{\pi}^{fe} \\ s - \bar{s} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 0 & \dfrac{2l_1(1+\alpha\beta)}{2\alpha l_1\gamma - \alpha} \end{bmatrix} \begin{bmatrix} k_1 e^{E_1^U t} \\ k_2 e^{E_2^U t} \end{bmatrix}. \tag{49}$$

where $E_1^U = \dfrac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} < 0$ (if $\dfrac{\beta l_1}{l_2\alpha^2} > \dfrac{\kappa}{\lambda}$) and $E_2^U = -\dfrac{\alpha}{l_1} < 0$. In setting $t = 0$ and taking the expected inflation rates as predetermined variables, one obtains from (49) $k_1 = \dfrac{1}{2}[(\pi_0^e - \bar{\pi}^e) + (\pi_0^{fe} - \bar{\pi}^{fe})]$ and $k_2 = \dfrac{1}{2}[(\pi_0^{fe} - \bar{\pi}^{fe}) - (\pi_0^e - \bar{\pi}^e)]$.

Using (49), (6')-(7'), (22) and (A.19)-(A.22), it leads to the following dynamic paths for other variables:

$$\begin{aligned}\pi-\bar{\pi}=&\frac{\lambda}{2(\alpha^2\kappa+\lambda)}[(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&-\frac{(\alpha+2l_1\alpha\gamma)(1+\alpha\beta)}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2^U t},\end{aligned}\tag{50}$$

$$\begin{aligned}y-\bar{y}=&-\frac{\alpha\kappa}{2(\alpha^2\kappa+\lambda)}[(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&-\frac{\alpha\beta+4\gamma l_1+2\gamma l_1\alpha\beta}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2^U t},\end{aligned}\tag{51}$$

$$\begin{aligned}\pi^f-\bar{\pi}^f=&\frac{\lambda}{2(\alpha^2\kappa+\lambda)}[(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&+\frac{(\alpha+2l_1\alpha\gamma)(1+\alpha\beta)}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2 t},\end{aligned}\tag{52}$$

$$\begin{aligned}y^f-\bar{y}^f=&-\frac{\alpha\kappa}{2(\alpha^2\kappa+\lambda)}[(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&+\frac{\alpha\beta+4\gamma l_1+2\gamma l_1\alpha\beta)}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2^U t},\end{aligned}\tag{53}$$

$$i^U-\bar{i}^U=\frac{1}{2}(1+\frac{\lambda\alpha\kappa}{\beta(\alpha^2\kappa+\lambda)})[(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1 t},\tag{53}$$

$$\begin{aligned}\mu-\bar{\mu}=&[\frac{\lambda}{2(\alpha^2\kappa+\lambda)}-\frac{l_1E_1\alpha\kappa}{2(\alpha^2\kappa+\lambda)}-l_2E_1\frac{1}{2}(1+\frac{\lambda\alpha\kappa}{\beta(\alpha^2\kappa+\lambda)})][(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&-\frac{(\alpha+2l_1\alpha\gamma)(1+\alpha\beta)+l_1E_2(\alpha\beta+4\gamma l_1+2\gamma l_1\alpha\beta)}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2^U t}.\end{aligned}\tag{54}$$

$$\begin{aligned}\mu^f-\bar{\mu}^f=&[\frac{\lambda}{2(\alpha^2\kappa+\lambda)}-\frac{l_1E_1\alpha\kappa}{2(\alpha^2\kappa+\lambda)}-l_2E_1\frac{1}{2}(1+\frac{\lambda\alpha\kappa}{\beta(\alpha^2\kappa+\lambda)})][(\pi_0^e-\bar{\pi}^e)+(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_1^U t}\\&+\frac{(\alpha+2l_1\alpha\gamma)(1+\alpha\beta)+l_1E_2(\alpha\beta+4\gamma l_1+2\gamma l_1\alpha\beta)}{2(2\alpha l_1\gamma-\alpha)}[(\pi_0^e-\bar{\pi}^e)-(\pi_0^{fe}-\bar{\pi}^{fe})]e^{E_2^U t}.\end{aligned}\tag{55}$$

Consider as before the case of an adverse supply shock in the home country. It will in the long term increase the realised and expected inflation rates in the union, reduce the output and the competitiveness in the home country. It has no effect on the output of the foreign country. As the union's nominal interest rate reacts to the home country's shock, this reaction is too weak to absorb the shock for the home country and too high for the foreign country that is not at the origin of the shock. The initial reaction of the real exchange rate (assumed to be flexible) is not clear in this situation. It depends on the sign of $2l_1\gamma-1$ according to (49).

If $2l_1\gamma - 1 > 0$, there will be an initial under-adjustment. In the contrary, there will be an over-adjustment (the case represented in Figure 2). Given that, output in foreign country reacts in the short and intermediate terms to home country's supply shock. The adjustment of different variables is influenced by the initial over- or under-adjustment of real exchange rate. The exact nature of their initial adjustment depends on the value of different parameters. After the initial jumps (except the expected inflation rates), the endogenous variables follow an increasing or decreasing paths according to the case. It is assumed, in Figure 2, that the positive effects dominate the negative ones, so the money growth rates in the two countries increase initially.

As under the independent inflation-targeting regime, a positive demand shock in the home country has no long-term effect on inflation rates and outputs, the adjustment is entirely operated through the initial jumps of the real exchange rate, and nominal and real interest rates. The dynamic paths of expected and realised inflation rates and outputs as well as money growth rates in the member countries are stationary.

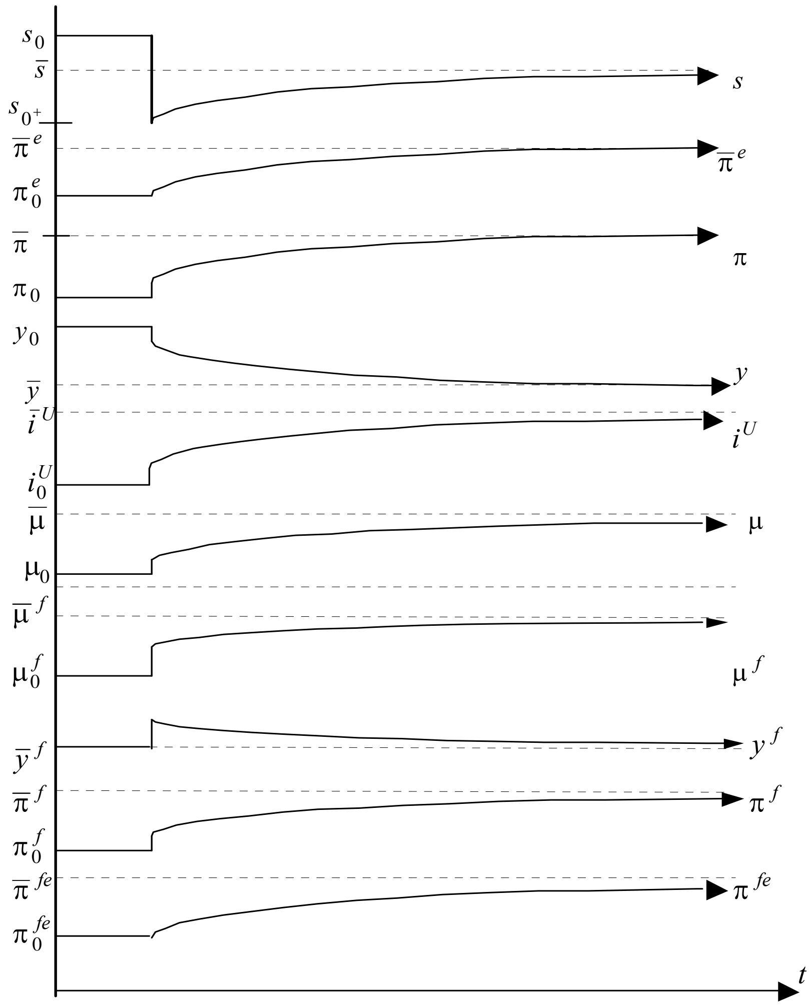

Figure 2. Adjustment paths of endogenous variables after a persistent adverse supply shock in the inflation-targeting monetary union.

5. Conclusion

Considering that the money supply is not perfectly regulated neither by the market participants to the money market nor by the central bank, this paper examine the implication of this hypothesis for inflation expectations dynamics in setting of economic union where member countries can adopt individually the inflation-targeting regime or abandon their national currency to the profit of unique currency with a common monetary policy conducted by a common inflation-targeting central bank.

This paper compares, in a dynamic setting, the independent inflation-targeting regime and a monetary union targeting inflation. Four main conclusions come out. Firstly, when the expected inflation rates of private agents are fully adjusted through the co-ordination of the monetary markets, only supply shocks influence the equilibrium inflation and output levels. Secondly, central banks' preferences for inflation and output stabilisations must be limited under these two alternative regimes. When central banks attribute a too strong weight for the inflation stabilisation relative to the output stabilisation, inflation and output may go out of control. Unsustainable boom and deflation become possible scenarios. Thirdly, contrary to the traditional OCA literature, demand shocks have not any influence on the decision of a country to join a monetary union. Only asymmetrical supply shocks can justify this choice. Finally, the central bank of the union, facing asymmetrical shocks, has difficulty to control the inflation rates in each country as it reacts to the average inflation rate. The nominal interest rate of the union may be too low for the country that is at the origin of the inflationary and demand shocks but too high for the other country.

The difficulty in guaranteeing the macro-economic stability becomes particularly critical with the disappearance of the floating foreign exchange market between the currencies of the countries forming the union. The real exchange rate may not adjust as quickly as when the foreign exchange market exists. The rigidities in the adjustments of expected inflation rates and real exchange rate might imply that a disparate union is more subject to risky adjustment paths conducting to undesirable situations.

Appendix A. Derivation of the optimal monetary policy rule

A.1. Independent inflation-targeting regime

The first-order conditions of central banks' minimisation problems (12)-(13) are:

$$\partial L / \partial \pi = 0 \;\Rightarrow\; \lambda \frac{\partial y}{\partial \pi}(y - y^*) = -\kappa(\pi - \pi^T), \Rightarrow\; y = y^* - \frac{\kappa\alpha}{\lambda}(\pi - \pi^T), \qquad \text{(A.1)}$$

$$\partial L^f / \partial \pi^f = 0 \Rightarrow \lambda \frac{\partial y^f}{\partial \pi}(y^f - y^*) = -\kappa(\pi^f - \pi^{fT}), \Rightarrow y^f = y^* - \frac{\kappa\alpha}{\lambda}(\pi^f - \pi^{fT}), \qquad \text{(A.2)}$$

Using equations (3)-(4) and the partial derivatives, $\frac{\partial y}{\partial \pi}=\frac{1}{\alpha}$, $\frac{\partial y^f}{\partial \pi^f}=\frac{1}{\alpha}$, obtained from equations (1)-(2), it leads to

$$\frac{\lambda}{\alpha}[-\beta(i-\pi^e)+\gamma s-y^*+\varepsilon_d]+\kappa(\pi-\pi^T)=0. \tag{A.3}$$

$$\frac{\lambda}{\alpha}[-\beta(i^f-\pi^{fe})-\gamma s-y^{f*}+\varepsilon_d^f]+\kappa(\pi^f-\pi^{fT})=0. \tag{A.4}$$

Equations (A.3)-(A.4) give the following central banks' optimal monetary policy rules:

$$i=\pi^e+\frac{1}{\beta}[\gamma s+\frac{\kappa\alpha}{\lambda}(\pi-\pi^T)-y^*+\varepsilon_d], \tag{A.5}$$

$$i^f=\pi^{fe}+\frac{1}{\beta}[-\gamma s+\frac{\kappa\alpha}{\lambda}(\pi^f-\pi^{fT})-y^{f*}+\varepsilon_d^f]. \tag{A.6}$$

Substituting the expected real interest rates drawn from (A.5) and (A.6) respectively in equations (3) and (4), one has:

$$y=y^*-\frac{\kappa\alpha}{\lambda}(\pi-\pi^T). \tag{A 7}$$

$$y^f=y^{f*}-\frac{\kappa\alpha}{\lambda}(\pi^f-\pi^{fT}). \tag{A.8}$$

Combining equations (1)-(2) with (A.7)-(A.8) respectively, one has:

$$\pi=\frac{\lambda}{\lambda+\kappa\alpha^2}(\pi^e+\varepsilon_\pi+\frac{\kappa\alpha^2}{\lambda}\pi^T), \tag{A.9}$$

$$y=y^*+\frac{\alpha\kappa}{\lambda+\kappa\alpha^2}(-\pi^e-\varepsilon_\pi+\pi^T). \tag{A.10}$$

$$\pi^f=\frac{\lambda}{\lambda+\kappa\alpha^2}(\pi^{fe}+\varepsilon_\pi^f+\frac{\kappa\alpha^2}{\lambda}\pi^{fT}), \tag{A.11}$$

$$y^f=y^{f*}+\frac{\alpha\kappa}{\lambda+\kappa\alpha^2}(-\pi^{fe}-\varepsilon_\pi^f+\pi^{fT}). \tag{A.12}$$

A.2. Monetary union with inflation-targeting central bank

As home and foreign countries are assumed to have the same size, the average inflation rate of union is $\pi^U = \frac{\pi + \pi^f}{2}$, the average output $y^U = \frac{y + y^f}{2}$, the average expected inflation rate $\pi^{Ue} = \frac{\pi^e + \pi^{fe}}{2}$. The nominal interest rate is the same for the two countries, i.e. i^{Ue}. Dividing by two the sum of equations (1) and (2) and that of equations (3) and (4) yields the average Phillips curve and aggregate demand in the union:

$$\pi^U = \pi^{Ue} + \alpha\,(y^U - y^{U*}) + \frac{\varepsilon_\pi + \varepsilon_\pi^f}{2}. \tag{A.13}$$

$$y^U = -\beta\,(i^U - \pi^U) + \frac{\varepsilon_d + \varepsilon_d^f}{2}. \tag{A.14}$$

The minimisation of the loss of the union's central bank, (21), gives:

$$\frac{\partial L^U}{\partial \pi^U} = 0 \Rightarrow \lambda \frac{\partial y^U}{\partial \pi^U}(y^U - y^*) + \kappa(\pi^U - \pi^T) = 0. \tag{A.15}$$

From (A.13), one has: $\partial y^U / \partial \pi^U = 1/\alpha$. Inserting this result and (A.14) into (A .15) yields

$$\frac{\lambda}{\alpha}[-\beta\,(i^U - \pi^{Ue}) + \frac{\varepsilon_d + \varepsilon_d^f}{2} - y^{U*}] + \kappa(\pi^U - \pi^{UT}) = 0, \tag{A.16}$$

That gives the optimal nominal interest rate rule of the central bank in the union:

$$i^U - \pi^{Ue} = \frac{\varepsilon_d + \varepsilon_d^f}{2\beta} - \frac{y^{U*}}{\beta} + \frac{\alpha\kappa}{\lambda\beta}(\pi^U - \pi^{UT}). \tag{A.17}$$

Using the fact $i^U = i = i^f$ and equations (1)-(4) and (22), π, y, π^f and y^f can be solved in terms of dynamic variables (s, π and π^f), exogenous variables and shocks through the following system:

$$\begin{bmatrix} 1 & -\alpha & 0 & 0 \\ \frac{\alpha\kappa}{2\lambda} & 1 & \frac{\alpha\kappa}{2\lambda} & 0 \\ 0 & 0 & 1 & -\alpha \\ \frac{\alpha\kappa}{2\lambda} & 0 & \frac{\alpha\kappa}{2\lambda} & 1 \end{bmatrix} \begin{bmatrix} \pi \\ y \\ \pi^f \\ y^f \end{bmatrix} = \begin{bmatrix} \pi^e - \alpha\, y^* + \varepsilon_\pi \\ \frac{\beta(\pi^e - \pi^{fe})}{2} - \frac{\varepsilon_d + \varepsilon_d^f}{2} + y^{U*} + \frac{\alpha\kappa}{\lambda}\pi^{UT} + \gamma s + \varepsilon_d \\ \pi^{fe} - \alpha\, y^{f*} + \varepsilon_\pi^f \\ -\frac{\beta(\pi^e - \pi^{fe})}{2} - \frac{\varepsilon_d + \varepsilon_d^f}{2} + y^{U*} + \frac{\alpha\kappa}{\lambda}\pi^{UT} - \gamma s + \varepsilon_d^f \end{bmatrix}, \tag{A.18}$$

with the determinant of the matrix given by $\det = \dfrac{\alpha^2\kappa + \lambda}{\lambda}$, the solution of the system yields:

$$\pi = \frac{1}{\det}[(1+\frac{\alpha\beta}{2}+\frac{\alpha^2\kappa}{2\lambda}+\frac{\beta\alpha^3\kappa}{2\lambda})\pi^e - (\frac{\alpha\beta}{2}+\frac{\alpha^2\kappa}{2\lambda}+\frac{\alpha^3\beta\kappa}{2\lambda})\pi^{fe}$$
$$+ (\alpha+\frac{\alpha^3\kappa}{\lambda})\gamma\, s + \frac{\alpha^2\kappa}{\lambda}\pi^{UT} - \alpha(1+\frac{\alpha^2\kappa}{2\lambda})\, y^* + \alpha y^{U*} + \frac{\alpha^2\kappa}{2\lambda}\alpha\, y^{f*} \tag{A.19}$$
$$+ (1+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_\pi - \frac{\alpha^2\kappa}{2\lambda}\varepsilon_\pi^f + \alpha(\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d - \alpha(\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d^f],$$

$$y = \frac{1}{\det}[-(\frac{\beta}{2}+\frac{\alpha\kappa}{2\lambda}+\frac{\alpha^2\beta\kappa}{2\lambda})\pi^{fe} + (\frac{\beta}{2}-\frac{\alpha\kappa}{2\lambda}+\frac{\alpha^2\beta\kappa}{2\lambda})\pi^e$$
$$+ (\gamma + \frac{\alpha^2\gamma\kappa}{\lambda})s + y^{U*} + \frac{\alpha\kappa}{\lambda}\pi^{UT} + \frac{\alpha^2\kappa}{2\lambda}y^* + \frac{\alpha^2\kappa}{2\lambda}y^{f*} \tag{A.20}$$
$$- \frac{\alpha\kappa}{2\lambda}\varepsilon_\pi^f - \frac{\alpha\kappa}{2\lambda}\varepsilon_\pi - (\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d^f + (\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d],$$

$$\pi^f = \frac{1}{\det}[(1+\frac{\alpha^2\kappa}{2\lambda}+\frac{\alpha\beta}{2}+\frac{\alpha^3\beta\kappa}{2\lambda})\pi^{fe} - (\frac{\alpha^2\kappa}{2\lambda}+\frac{\alpha\beta}{2}+\frac{\alpha^3\beta\kappa}{2\lambda})\pi^e - (\alpha+\frac{\alpha^3\kappa}{\lambda})\gamma\, s$$
$$+ \alpha y^{U*} + \frac{\alpha\alpha\kappa}{\lambda}\pi^{UT} + \frac{\alpha^3\kappa}{2\lambda}y^* - \alpha(1+\frac{\alpha^2\kappa}{2\lambda})y^{f*} \tag{A.21}$$
$$+ (1+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_\pi^f - \frac{\alpha^2\kappa}{2\lambda}\varepsilon_\pi - \alpha(\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d + \alpha(\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d^f],$$

$$y^f = \frac{1}{\det}[-(\frac{\beta}{2}+\frac{\alpha\kappa}{2\lambda}+\frac{\alpha^2\beta\kappa}{2\lambda})\pi^e + (\frac{\beta}{2}-\frac{\alpha\kappa}{2\lambda}+\frac{\alpha^2\beta\kappa}{2\lambda})\pi^{fe}$$
$$- (\gamma + \frac{\alpha^2\gamma\kappa}{\lambda})s + y^{U*} + \frac{\alpha\kappa}{\lambda}\pi^{UT} + \frac{\alpha^2\kappa}{2\lambda}y^* + \frac{\alpha^2\kappa}{2\lambda}y^{f*} \tag{A.22}$$
$$- \frac{\alpha\kappa}{2\lambda}\varepsilon_\pi^f - \frac{\alpha\kappa}{2\lambda}\varepsilon_\pi - (\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d + (\frac{1}{2}+\frac{\alpha^2\kappa}{2\lambda})\varepsilon_d^f].$$

From (A.19) and (A.21), it is easy to obtain:

$$\pi + \pi^f = \frac{1}{\det}(\pi^e + \pi^{fe} + \frac{2\alpha^2\kappa}{\lambda}\pi^{UT} + \varepsilon_\pi + \varepsilon_\pi^f), \tag{A.23}$$

$$\pi - \pi^f = (1+\alpha\beta)(\pi^e - \pi^{fe}) + 2\,\alpha\gamma\, s - \alpha\, y^* + \alpha\, y^{f*} + \varepsilon_\pi - \varepsilon_\pi^f + \alpha\varepsilon_d - \alpha\varepsilon_d^f. \tag{A.24}$$

Appendix B. The solutions of long-run equilibrium

B.1. Independent inflation targeting with floating exchange rate

In the long term, one has: $\bar{\pi} = \bar{\pi}^e = \bar{\mu}$, $\bar{\pi}^f = \bar{\pi}^{fe} = \bar{\mu}^f$, $\dot{s} = \pi^f + \dot{e} - \pi = 0$, $\bar{\pi} = \bar{\dot{e}} + \bar{\pi}^f$, $\bar{i} = \bar{i}^f + \bar{\dot{e}}$, $\bar{i} - \bar{\pi} = \bar{i}^f - \bar{\pi}^f$. From equations (1) and (2), it results:

$$\bar{y} = y^* - \frac{\varepsilon_\pi}{\alpha}, \tag{B.1}$$

$$\bar{y}^f = y^{f*} - \frac{\varepsilon_\pi^f}{\alpha}. \tag{B.2}$$

To solve the other variables, one can rewrite (3)-(4) and (12)-(13) using (B.1)-(B.) as follows:

$$y^* - \frac{\varepsilon_\pi}{\alpha} = -\beta(\bar{i} - \bar{\pi}) + \gamma \bar{s} + \varepsilon_d, \tag{B.3}$$

$$y^{f*} - \frac{\varepsilon_\pi^f}{\alpha} = -\beta(\bar{i} - \bar{\pi}) - \gamma \bar{s} + \varepsilon_d^f, \tag{B.4}$$

$$\bar{i} - \bar{\pi} = \frac{1}{\beta}[\gamma s + \frac{\kappa\alpha}{\lambda}(\bar{\pi} - \pi^T) - y^* + \varepsilon_d], \tag{B.5}$$

$$\bar{i} - \bar{\pi} = \frac{1}{\beta}[-\gamma s + \frac{\kappa\alpha}{\lambda}(\bar{\pi}^f - \pi^{fT}) - y^{f*} + \varepsilon_d^f]. \tag{B.6}$$

The solutions are given by

$$\bar{i} = \pi^T - \frac{(y^* + y^{f*})}{2\beta} + (\frac{\kappa\alpha + 2\beta\lambda}{2\beta\kappa\alpha^2})\varepsilon_\pi + \frac{1}{2\beta}\frac{\varepsilon_\pi^f}{\alpha} + \frac{\varepsilon_d}{2\beta} + \frac{\varepsilon_d^f}{2\beta} \tag{B.7}$$

$$\bar{\pi} = \bar{\pi}^e = \bar{\mu} = \frac{\lambda}{\kappa\alpha}(\frac{\kappa\alpha}{\lambda}\pi^T + \frac{\varepsilon_\pi}{\alpha}), \tag{B.8}$$

$$\bar{s} = \frac{1}{2\gamma}(y^* - \frac{\varepsilon_\pi}{\alpha} - \varepsilon_d - y^{f*} + \frac{\varepsilon_\pi^f}{\alpha} + \varepsilon_d^f), \tag{B.9}$$

$$\bar{\pi}^f = \bar{\pi}^{fe} = \bar{\mu}^f = \frac{\lambda}{\kappa\alpha}(\frac{\varepsilon_\pi^f}{\alpha} + \frac{\kappa\alpha}{\lambda}\pi^{fT}). \tag{B.10}$$

Using the equalities $\bar{\pi} = \bar{\dot{e}} + \bar{\pi}^f$, $\bar{i} - \bar{\pi} = \bar{i}^f - \bar{\pi}^f$ and $\bar{i} = \bar{i}^f + \bar{\dot{e}}$, one obtains also:

$$\bar{\dot{e}} = \bar{\pi} - \bar{\pi}^f = \pi^T - \pi^{fT} + \frac{\lambda}{\kappa\alpha}(\frac{\varepsilon_\pi}{\alpha} - \frac{\varepsilon_\pi^f}{\alpha}), \tag{B.11}$$

$$\bar{i}^{f} = \pi^{fT} - \frac{(y^{*} + y^{f*})}{2\beta} + (\frac{\kappa\alpha + 2\beta\lambda}{2\beta\kappa\alpha^{2}})\varepsilon_{\pi}^{f} + (\frac{\kappa\alpha}{2\beta\kappa\alpha^{2}})\varepsilon_{\pi} + \frac{\varepsilon_{d}}{2\beta} + \frac{\varepsilon_{d}^{f}}{2\beta}) . \qquad (B.12)$$

B.2. Monetary union with inflation targeting

In the long run, one has: $\bar{\pi} = \bar{\pi}^{e} = \bar{\pi}^{f} = \bar{\pi}^{fe} = \bar{\pi}^{U} = \bar{\pi}^{Ue} = \bar{\mu}^{U}$, $\dot{s} = \pi^{f} - \pi = 0$. As before, from equations (1) and (2), it results:

$$\bar{y} = y^{*} - \frac{\varepsilon_{\pi}}{\alpha}, \qquad (B.13)$$

$$\bar{y}^{f} = y^{f*} - \frac{\varepsilon_{\pi}^{f}}{\alpha}. \qquad (B.14)$$

Using (B.14)-(B.15), equations (3)-(4) and (22) can be rewritten in the long term as

$$y^{*} - \frac{\varepsilon_{\pi}}{\alpha} = -\beta[\frac{\varepsilon_{d} + \varepsilon_{d}^{f}}{2\beta} - \frac{y^{U*}}{\beta} + \frac{\alpha\kappa}{\lambda\beta}(\bar{\pi}^{U} - \pi^{UT})] + \gamma\bar{s} + \varepsilon_{d}, \qquad (B.15)$$

$$y^{f*} - \frac{\varepsilon_{\pi}^{f}}{\alpha} = -\beta[\frac{\varepsilon_{d} + \varepsilon_{d}^{f}}{2\beta} - \frac{y^{U*}}{\beta} + \frac{\alpha\kappa}{\lambda\beta}(\bar{\pi}^{U} - \pi^{UT})] - \gamma\bar{s} + \varepsilon_{d}^{f}, \qquad (B.16)$$

$$\bar{i}^{U} = \bar{\pi}^{U} + \frac{\varepsilon_{d} + \varepsilon_{d}^{f}}{2\beta} - \frac{y^{U*}}{\beta} + \frac{\alpha\kappa}{\lambda\beta}(\bar{\pi}^{U} - \pi^{UT}). \qquad (B.17)$$

Solving (B.15)- (B.17) gives the solutions for $\bar{\pi}^{U}, \bar{s}$ and $\bar{i}$:

$$\bar{\pi}^{U} = \bar{\pi} = \bar{\pi}^{e} = \bar{\pi}^{f} = \bar{\pi}^{fe} = \bar{\pi}^{Ue} = \bar{\mu}^{U} = \pi^{UT} + \frac{\lambda}{2\alpha^{2}\kappa}(\varepsilon_{\pi} + \varepsilon_{\pi}^{f}), \qquad (B.18)$$

$$\bar{s} = \frac{1}{2\gamma}(y^{*} - y^{f*} + \varepsilon_{d}^{f} - \varepsilon_{d} + \frac{\varepsilon_{\pi}^{f}}{\alpha} - \frac{\varepsilon_{\pi}}{\alpha}) \qquad (B.19)$$

$$\bar{i}^{U} = \pi^{UT} + (\frac{\lambda}{2\alpha^{2}\kappa} + \frac{1}{2\alpha\beta})(\varepsilon_{\pi} + \varepsilon_{\pi}^{f}) + \frac{\varepsilon_{d} + \varepsilon_{d}^{f}}{2\beta} - \frac{y^{U*}}{\beta}. \qquad (B.20)$$

Appendix C. Differential equations of expected inflation rates and real exchange rate under independent inflation-targeting

C.1. The differential equations for expected inflation rates ($\dot{\pi}^{e}$ *and* $\dot{\pi}^{fe}$)

Consider first the case of home country. Given π, $\dot{y}$ and $\dot{i}$, equation (6') can be used to determine the money growth rate, with $\mu = \pi + l_1\dot{y} - l_2\dot{i}$. It permits also the private agents to reformulate their inflation expectations as the expected money growth rate satisfies:

$$\mu^e = \pi^e + l_1\dot{y}^e - l_2\dot{i}^e . \tag{C.1}$$

The equations (6') and (C.1) imply that at the stationary state $\overline{\pi}^e = \overline{\pi} = \overline{\mu}$. Knowing that, at every moment, information on μ is available through the announcement of the central bank, the private agents adjust simply their expectations of μ to the announced rate so that $\mu^e = \mu$ and thus:

$$\pi^e = \mu - l_1\dot{y}^e + l_2\dot{i}^e . \tag{C.2}$$

Equation (C.2) implies that the private agents can use, while adjusting their inflation expectations, the whole information concerning the conditions of supply and demand on the goods market as well as on the financial and monetary markets. Combining (6') and (C.2) in eliminating μ, it yields:

$$\pi^e = \pi + l_1\dot{y} - l_2\dot{i} - l_1\dot{y}^e + l_2\dot{i}^e . \tag{C.3}$$

In admitting $\dot{\varepsilon}_d = 0, \dot{\varepsilon}_\pi = 0$ (i.e. shocks without tendency), equation (1) can be derived to time to give $\dot{y} = \frac{1}{\alpha}(\dot{\pi} - \dot{\pi}^e)$ and $\dot{y}^e = \frac{1}{\alpha}(\dot{\pi}^e - \dot{\pi}^e) = 0$. In deriving (12) to time, one obtains $\dot{i} = \frac{1}{\beta}(\gamma\dot{s} + \frac{\kappa\alpha}{\lambda}\dot{\pi}) + \dot{\pi}^e$ and $\dot{i}^e = \frac{1}{\beta}(\gamma\dot{s}^e + \frac{\kappa\alpha}{\lambda}\dot{\pi}^e) + \dot{\pi}^e$. Using (A.9), one has $\dot{\pi} = \frac{\lambda}{\lambda + \kappa\alpha^2}\dot{\pi}^e$.

In using these results and the assumption $\dot{s} = \dot{s}^e$, equation (C.3) can be rewritten as:

$$\pi^e = \frac{\lambda}{\lambda + \kappa\alpha^2}(\pi^e + \varepsilon_\pi + \frac{\kappa\alpha^2}{\lambda}\pi^T) + \frac{l_1}{\alpha}(\frac{\lambda}{\lambda + \kappa\alpha^2}\dot{\pi}^e - \dot{\pi}^e) - \frac{l_2}{\beta}\frac{\kappa\alpha}{\lambda}(\frac{\lambda}{\lambda + \kappa\alpha^2}\dot{\pi}^e - \dot{\pi}^e), \tag{C.4}$$

or more simply:

$$\dot{\pi}^e = -\frac{1}{\Omega}\pi^e + \frac{\lambda}{\Omega\kappa\alpha^2}(\varepsilon_\pi + \frac{\kappa\alpha^2}{\lambda}\pi^T), \text{ with } \Omega = \frac{l_1}{\alpha} - \frac{l_2}{\beta}\frac{\kappa\alpha}{\lambda}. \tag{C.5}$$

Taking the variation of (C.5) around the long-run equilibrium, one has:

$$\dot{\pi}^e = -\frac{1}{\Omega}(\pi^e - \bar{\pi}^e). \tag{C.6}$$

Using (2), (4), (7') and (13) similarly leads to:

$$\dot{\pi}^{fe} = -\frac{1}{\Omega}(\pi^{fe} - \bar{\pi}^{fe}). \tag{C.7}$$

C.2. The differential equation for real exchange rate ($\dot{s}$)

Equations (5) and (2) can be rewritten as

$$\dot{s}^e = i - i^f - \pi^e + \pi^{fe} - \varepsilon_e, \tag{C.8}$$

As the foreign exchange market adjusts quickly, it is assumed simply $\dot{s} = \dot{s}^e$. Taking the variation of (C.8) around the steady state leads to:

$$\dot{s} = \dot{s}^e = i - \bar{i} - (i_f - \bar{i}_f) + (\pi^e_f - \bar{\pi}^e_f) - (\pi^e - \bar{\pi}^e). \tag{C.9}$$

Using then the optimal interest rate rules (12)-(13), one has:

$$i - \bar{i} = \pi^e - \bar{\pi}^e + \frac{\gamma}{\beta}(s - \bar{s}) + \frac{\kappa\alpha}{\lambda\beta}(\pi - \bar{\pi}), \tag{C.10}$$

$$i^f - \bar{i}^f = \pi^{fe} - \bar{\pi}^{fe} - \frac{\gamma}{\beta}(s - \bar{s}) + \frac{\kappa\alpha}{\lambda\beta}(\pi^f - \bar{\pi}^f). \tag{C.11}$$

From (A.9) and (A.11), one has:

$$\pi - \bar{\pi} = \frac{\lambda}{\lambda + \kappa\alpha^2}(\pi^e - \bar{\pi}^e), \tag{C.12}$$

$$\pi^f - \bar{\pi}^f = \frac{\lambda}{\lambda + \kappa\alpha^2}(\pi^{fe} - \bar{\pi}^{fe}). \tag{C.13}$$

Inserting (C.10)-(C.13) into (C.9) gives the following dynamic equation:

$$\dot{s} = \frac{2\gamma}{\beta}(s - \bar{s}) + \frac{\kappa\alpha}{\beta(\lambda + \kappa\alpha^2)}(\pi^e - \bar{\pi}^e) - \frac{\kappa\alpha}{\beta(\lambda + \kappa\alpha^2)}(\pi^{fe} - \bar{\pi}^{fe}). \tag{C.14}$$

Equations (C.6), (C.7) and (C.14) can be rewritten in matrix form as in the system (34).

Appendix D. Differential equations of expected inflation rates and real exchange rate in inflation-targeting monetary union

D.1 The differential equations for expected inflation rates $(\dot{\pi}^e, \dot{\pi}^{fe})$

It is assumed that national money and financial markets are not fully integrated at the union's level, so that the private agents of each country take account of their national real interest rate (instead of union's real interest rate) to decide their consumption and investment. As under independent inflation-targeting, the national monetary markets give information for private agents to form their expectations of national inflation rate.

Examine first the expected inflation rate for the home country. Substituting $\dot{i}$ and $\dot{i}^f$ by $\dot{i}^U$, $\dot{i}^e$ and $\dot{i}^{fe}$ by $\dot{i}^{Ue}$ respectively in equations (6')-(7'), and using the same procedure as in Appendix C give:

$$\pi^e = \pi + l_1\dot{y} - l_2\dot{i}^U - l_1\dot{y}^e + l_2\dot{i}^{Ue}. \tag{D.1}$$

$$\pi^{fe} = \pi^f + l_1\dot{y}^f - l_2\dot{i}^U - l_1\dot{y}^{fe} + l_2\dot{i}^{Ue}. \tag{D.2}$$

At the steady state, one has $\bar{\pi}^e = \bar{\pi} = \bar{\mu} = \bar{\pi}^{fe} = \bar{\pi}^f = \bar{\mu}^f = \bar{\pi}^{Ue} = \bar{\pi}^U = \bar{\mu}^U$. The sum and the difference of (D.1) and (D.2) give respectively:

$$\pi^e + \pi^{fe} = \pi + \pi^f + l_1(\dot{y} + \dot{y}^f) - l_1(\dot{y}^e + \dot{y}^{fe}) - 2l_2\dot{i}^U + 2l_2\dot{i}^{Ue}, \tag{D.3}$$

$$\pi^e - \pi^{fe} = \pi + \pi^f + l_1(\dot{y} - \dot{y}^f) - l_1(\dot{y}^e - \dot{y}^{fe}). \tag{D.4}$$

In inserting $\dot{y} = \frac{1}{\alpha}(\dot{\pi} - \dot{\pi}^e)$ and $\dot{y}^e = \frac{1}{\alpha}(\dot{\pi}^e - \dot{\pi}^e) = 0$ derived from equation (1), $\dot{y}^f = \frac{1}{\alpha}(\dot{\pi}^f - \dot{\pi}^{fe})$ and $\dot{y}^{fe} = \frac{1}{\alpha}(\dot{\pi}^{fe} - \dot{\pi}^{fe}) = 0$ from equation (2), $\dot{i}^U = \dot{\pi}^{Ue} + \frac{\alpha\kappa}{\lambda\beta}\dot{\pi}^U$ and $\dot{i}^{Ue} = \dot{\pi}^{Ue} + \frac{\alpha\kappa}{\lambda\beta}\dot{\pi}^{Ue}$ from equation (22), $\dot{\pi}^U = \frac{\dot{\pi} + \dot{\pi}^f}{2}$ and $\dot{\pi}^{Ue} = \frac{\dot{\pi}^e + \dot{\pi}^{fe}}{2}$ (by definition),

$\pi + \pi^f = \dfrac{\lambda}{\alpha^2\kappa + \lambda}(\dot{\pi}^e + \dot{\pi}^{fe})$ from (A.23), $\dot{\pi} - \dot{\pi}^f = (1+\alpha\beta)(\dot{\pi}^e - \dot{\pi}^{fe}) + 2\alpha\gamma\dot{s}$ from (A.24),

as well as (A.23) and (A.24) into (D.3)-(D.4) leads to

$$\frac{\dot{\pi}^e + \dot{\pi}^{fe}}{2} = \frac{-\alpha\lambda\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\frac{(\pi^e + \pi^{fe})}{2} + \frac{\alpha\lambda\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\pi^{UT} + \frac{\lambda^2\beta}{2(\lambda\beta l_1\,\alpha\kappa - l_2\alpha^3\kappa^2)}(\varepsilon_\pi + \varepsilon_\pi^f), \quad \text{(D.5)}$$

$$\frac{\dot{\pi}^e - \dot{\pi}^{fe}}{2} = -\frac{\alpha}{2l_1}(\pi^e - \pi^{fe}) - \frac{\alpha\gamma}{l_1\beta}s - \frac{\gamma}{\beta}\dot{s} + \frac{1}{2l_1\beta}[\alpha(y^* - y^{f*}) - (\varepsilon_\pi - \varepsilon_\pi^f) - (\varepsilon_d - \varepsilon_d^f)]. \quad \text{(D.6)}$$

The average inflation rate follows an autonomous dynamic path independent of the real exchange rate. The stability condition is $\lambda\beta l_1 - l_2\alpha^2\kappa > 0$, or otherwise $\dfrac{\kappa}{\lambda} < \dfrac{l_1\beta}{l_2\alpha^2}$. That's the same as under independent inflation-targeting regime.

In the monetary union, the real exchange rate will evolve respecting: $\dot{e} = \dot{s} - \dot{p}^f + \dot{p} = \dot{s} - \pi^f + \pi = 0$. Using (A.24), that leads to

$$\dot{s} = \pi^f - \pi = -(1+\alpha\beta)(\pi^e - \pi^{fe}) - 2\alpha\gamma s + \varepsilon_\pi^f - \varepsilon_\pi - \alpha\varepsilon_d + \alpha\varepsilon_d^f. \quad \text{(D.7)}$$

From (D.5)-(D.6), it is easy to obtain:

$$\begin{aligned}\dot{\pi}^e = {} & \frac{1}{2}\left(\frac{\lambda\alpha\beta}{\lambda\beta l_1 - l_2\kappa\alpha^2} - \frac{\alpha}{l_1}\right)\pi^e - \frac{1}{2}\left(\frac{\lambda\beta\alpha}{\lambda\beta l_1 - l_2\kappa\alpha^2} - \frac{\alpha}{l_1}\right)\pi^{fe} - \frac{\alpha\gamma}{l_1\beta}s - \frac{\gamma}{\beta}\dot{s} \\ & + \frac{\alpha}{2l_1\beta}(y^* - y^{f*}) + \frac{\alpha\lambda\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\pi^{UT} - \frac{\alpha}{2l_1\beta}(\varepsilon_d - \varepsilon_d^f) \\ & + \frac{\lambda^2\beta}{2(\lambda\beta l_1\,\alpha\kappa - l_2\alpha^3\kappa^2)}(\varepsilon_\pi + \varepsilon_\pi^f) - \frac{1}{2l_1\beta}(\varepsilon_\pi - \varepsilon_\pi^f),\end{aligned} \quad \text{(D.8)}$$

$$\begin{aligned}\dot{\pi}^{fe} = {} & \frac{1}{2}\left(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} + \frac{\alpha}{l_1}\right)\pi^e - \frac{1}{2}\left(\frac{\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} + \frac{\alpha}{l_1}\right)\pi^{fe} + \frac{\alpha\gamma}{l_1\beta}s + \frac{\gamma}{\beta}\dot{s} \\ & - \frac{\alpha}{2l_1\beta}(y^* - y^{f*}) + \frac{\alpha\lambda\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\pi^{UT} + \frac{\alpha}{2l_1\beta}(\varepsilon_d - \varepsilon_d^f) \\ & + \frac{\lambda^2\beta}{2(\lambda\beta l_1\,\alpha\kappa - l_2\alpha^3\kappa^2)}(\varepsilon_\pi + \varepsilon_\pi^f) + \frac{1}{2l_1\beta}(\varepsilon_\pi - \varepsilon_\pi^f).\end{aligned} \quad \text{(D.9)}$$

The system (D.7)-(D .9) can be rewritten in matrix form as follows:

$$
\begin{bmatrix} \dot{\pi}^{e} \\ \dot{\pi}^{fe} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} - \frac{\alpha}{l_1}\right) + \frac{\gamma(1+\alpha\beta)}{\beta} & \frac{1}{2}\left(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} + \frac{\alpha}{l_1}\right) - \frac{\gamma(1+\alpha\beta)}{\beta} & -\frac{\alpha\gamma}{l_1\beta} + \frac{2\alpha\gamma^2}{\beta} \\ \frac{1}{2}\left(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} + \frac{\alpha}{l_1}\right) - \frac{\gamma(1+\alpha\beta)}{\beta} & \frac{1}{2}\left(\frac{-\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa} - \frac{\alpha}{l_1}\right) + \frac{\gamma(1+\alpha\beta)}{\beta} & \frac{\alpha\gamma}{l_1\beta} - \frac{2\alpha\gamma^2}{\beta} \\ -(1+\alpha\beta) & (1+\alpha\beta) & -2\alpha\gamma \end{bmatrix} \begin{bmatrix} \pi^{e} \\ \pi^{fe} \\ s \end{bmatrix}
$$

$$
+ \begin{bmatrix} +\frac{\alpha}{2l_1\beta}(y^{*} - y^{f*}) + \frac{\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\pi^{UT} - \frac{\alpha}{2l_1\beta}(\varepsilon_d - \varepsilon_d^f) + \frac{\lambda^2\beta}{2(\lambda\beta l_1\alpha\kappa - l_2\alpha^3\kappa^2)}(\varepsilon_\pi + \varepsilon_\pi^f) - \frac{1}{2l_1\beta}(\varepsilon_\pi - \varepsilon_\pi^f) \\ -\frac{\alpha}{2l_1\beta}(y^{*} - y^{f*}) + \frac{\lambda\alpha\beta}{\lambda\beta l_1 - l_2\alpha^2\kappa}\pi^{UT} + \frac{\alpha}{2l_1\beta}(\varepsilon_d - \varepsilon_d^f) + \frac{\lambda^2\beta}{2(\lambda\beta l_1\alpha\kappa - l_2\alpha^3\kappa^2)}(\varepsilon_\pi + \varepsilon_\pi^f) + \frac{1}{2l_1\beta}(\varepsilon_\pi - \varepsilon_\pi^f) \\ \varepsilon_\pi^f - \varepsilon_\pi - \alpha\varepsilon_d + \alpha\varepsilon_d^f \end{bmatrix}.
$$

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