Robustness of Bayesian results for Inverse Gaussian distribution under ML-II epsilon-contaminated and Edgeworth Series class of prior distributions

Sinha, Pankaj and Jayaraman, Prabha

Faculty of Management Studies, University of Delhi

17 May 2009

Robustness of Bayesian results for Inverse Gaussian distribution under ML-II ε-contaminated and Edgeworth Series class of prior distributions

Pankaj Sinha and J.Prabha
Faculty of Management Studies
University of Delhi

Abstract

This paper aims to study the sensitivity of Bayes estimate of location parameter of an Inverse Gaussian (IG) distribution to misspecification in the prior distribution. It also studies the effect of misspecification of the prior distribution on two-sided predictive limits for a future observation from IG population. Two prior distributions, a class ML-II ε-contaminated and Edgeworth Series (ESD), are employed for the location parameter of an IG distribution, to investigate the effect of misspecification in the priors. The numerical illustrations suggest that moderate amount of misspecification in prior distributions belonging to the class of ML-II ε-contaminated and ESD does not affect the Bayesian results.

1. Introduction

The two-parameter inverse Gaussian (IG) distribution, as a first passage time distribution in Brownian motion, found a variety of applications in the life testing, reliability and financial modeling problems. It has statistical properties analogous to normal distribution. Banerjee and Bhattacharyya (1976) applied the IG distribution to consumer panel data on toothpaste purchase incidence for the assessment of consumer heterogeneity. Whitemore (1976, 1986) discusses the potential applications of IG distribution in the management sciences and illustrates the advantages of IG distribution for right-skewed positive valued responses and its applicability in stochastic model for many real settings. Aase (2000) showed that IG distribution fits the economic indices remarkably well in empirical investigations. Nadarajah and Kotz (2007) gave the distribution of ratio of two economic indices each having IG distribution for comparing the consumer price indices of six major economies.

The probability density function (pdf) of IG distribution is expressed as

$$p(x \mid m,\lambda) = \left(\frac{\lambda}{2\pi}\right)^{1/2} x^{-3/2} \exp\left[-\frac{\lambda\left(x-m\right)^2}{2m^2 x}\right],\ x>0,\ \theta>0,\ \lambda>0 \qquad (1)$$

where m and λ are the mean and shape parameters respectively.

Tweedie expressed equation (1) in terms of an alternative parameterization, making $\theta = 1/m$, as

$$p(x \mid \theta, \lambda) = \left(\frac{\lambda}{2\pi}\right)^{1/2} x^{-3/2} \exp\left[-\frac{\lambda x}{2}\left(\theta - \frac{1}{x}\right)^2\right],\ x > 0,\ \theta > 0,\ \lambda > 0 \qquad (2)$$

we designate equation (2) by IG(θ,λ).

Excellent monograph by Chhikara and Folks (1989) and Seshadri (1999) contain bibliographies and survey of the literature on IG distribution. Banerjee & Bhattacharyya (1979) considered the normal distribution, truncated at zero, as a natural conjugate prior for the parameter θ of IG(θ,λ), while exploring the Bayesian results for IG distribution. Though the MCMC methods freed the analysts from using conjugate prior distributions for mathematical convenience, the advantage of conjugate prior is that it treats the prior information as if it were a previous sample of the same process. However, subjectivity involved in choosing a prior distribution has drawn severe criticism of Bayesian methodology. Berger (1984) discussed several approaches for examining the sensitivity of Bayes' actions to possible misspecification of the prior distribution. A reasonable approach is to consider a family of plausible priors that are close to a specific assessed approximation to 'true' prior and study sensitivity of the decision as the prior varies over this class (cf. Berger 1984, 1985, 1990 and 1994).

The ε-contaminated class of prior distributions has attracted attention of a number of authors to model uncertainty in the prior distribution. Berger and Berliner (1986) used type II maximum likelihood technique (cf. Good, 1965) to select a robust prior from ε-contaminated class of prior distributions having the form:

$$\Gamma = \{\pi(\theta) = (1-\varepsilon)\,\pi_o + \varepsilon\, q\ ,\ q \in Q\}$$

Here, π_o is the true assessed prior and q, being a contamination, belongs to the class Q of all distributions. Q determines the allowed contaminations that are mixed with π_o, and $\varepsilon \in [0,1]$ reflects the amount of uncertainty in the 'true' prior π_o. ML-II technique naturally selects a prior with a large tail which will be robust against all plausible deviations. Sinha and Bansal (2008) used ε-contaminated class of prior for the problem of optimization of a regression nature in the decisive prediction framework.

The class of Edgeworth Series distributions (ESD) has been considered as a class of prior distributions by Bansal (1978), Chakravarti and Bansal (1988) and Bansal and Sinha (1992) to investigate the effects of non-normal prior on Bayes decisions and forecasts. The ESD as a class of priors for unknown mean of the normal population provides unimodal and proper moderately non-normal prior distributions in the Barton and Dennis (1952) regions. It is a neighborhood class of priors with normal as one of its members. Draper and Tierney (1972) reexamined the results given by Barton and Dennis (1952) on regions of positive definiteand unimodal expansion of ESD.

In the subsequent sections, we employ ML-II ε-contaminated class and Edgeworth Series (ESD) prior distributions (both truncated at zero) for the location parameter θ of IG(θ,λ), shape parameter λ known, to study sensitivity of Bayes decisions to misspecification in both the prior distributions. We further find the predictive density function of a future observation from IG distribution, given the outcomes of an informative experiment, under both the class of prior distributions to study sensitivity of the predictive decisions.

2. Robustness under ML-II ε-contaminated class of prior

Let $\underset{\sim}{x} = (x_1,...,x_n)$ be n independent observations from IG(θ, λ) with mean $\theta = 1/m$ and known shape parameter λ (>0). The likelihood function is given by

$$L(\theta \mid \underset{\sim}{x}, \lambda) = \left(\frac{\lambda}{2\pi}\right)^{\frac{n}{2}} \prod_{i=1}^{n} x_i^{-\frac{3}{2}} \exp\left[-\frac{\lambda v}{2} - \frac{n\lambda \bar{x}}{2}\left(\theta - \frac{1}{\bar{x}}\right)^2\right] \tag{3}$$

where $v = \sum_{i=1}^{n}\left(\frac{1}{x_i} - \frac{1}{\bar{x}}\right)$ and $\bar{x} = \sum_{i=1}^{n} x_i / n.$

The selection of the maximum likelihood type-II technique requires a robust prior π in the class Γ of priors, which maximizes the marginal $m(\underset{\sim}{x} \mid a)$. Thus, for

$$\pi(\theta) = (1-\varepsilon)\pi_o(\theta) + \varepsilon q(\theta) \;\; ; \hat{q} \in \text{Q}$$

the marginal of $\underset{\sim}{x}$

$$m(\underset{\sim}{x} \mid \pi) = (1-\varepsilon) m(\underset{\sim}{x} \mid \pi_o) + \varepsilon m(\underset{\sim}{x} \mid q)$$

can be maximized by maximizing it over Q. Let the maximum of $m(\underset{\sim}{x} \mid q)$ be attained at unique $\hat{q} \in Q$. Thus an estimated ML-II prior $\hat{\pi}(\theta)$ is given by

$$\hat{\pi}(\theta) = (1-\varepsilon)\pi_o(\theta) + \varepsilon \hat{q}(\theta) \tag{4}$$

Suppose θ has a prior distribution belonging ML-II ε-contaminated class of priors. Following Berger and Berliner (1986), we have $\pi_o(\theta)$ as N(μ,τ), truncated at zero, with pdf

$$\pi_o(\theta) = \frac{1}{G}\sqrt{\frac{\tau}{2\pi}} \exp\left[-\frac{\tau}{2}(\theta-\mu)^2\right] \; ; \; \theta \ge 0, \; \text{G} = \Phi(-\text{p}), \; \text{p} = -\mu\sqrt{\tau}$$

A

nd $\hat{q}(\theta)$ as $uniform(\mu - \hat{a}, \mu + \hat{a})$, $\hat{a}$ being the value of 'a' which maximizes

$$m(\underset{\sim}{x} \mid a) = \begin{cases} \frac{1}{2a} \int_{\mu-\hat{a}}^{\mu+\hat{a}} L(\theta \mid \underset{\sim}{x}, \lambda) d\theta & a > 0 \\ L(\mu \mid \underset{\sim}{x}, \lambda) & a = 0 \end{cases}$$

$m(\underset{\sim}{x} \mid a)$ is an upper bound on $m(\underset{\sim}{x} \mid q)$.

$$m(\underset{\sim}{x} \mid a) = \left(\frac{\lambda}{2\pi}\right)^{\frac{n}{2}} \prod_{i=1}^{n} x_i^{-\frac{3}{2}} e^{-\frac{\lambda v}{2}} \sqrt{\frac{2\pi}{n\lambda\bar{x}}} \frac{1}{2a} \int_{\mu-\hat{a}}^{\mu+\hat{a}} \sqrt{\frac{n\lambda\bar{x}}{2\pi}} \exp\left[-\frac{n\lambda\bar{x}}{2}\left(\theta - \frac{1}{\bar{x}}\right)^2\right] d\theta$$

$$= \frac{S}{2a}\left\{\Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu + a - \frac{1}{\bar{x}}\right)\right] - \Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu - a - \frac{1}{\bar{x}}\right)\right]\right\} \quad (5)$$

where $S = \left(\frac{\lambda}{2\pi}\right)^{\frac{n}{2}} \prod_{i=1}^{n} x_i^{-\frac{3}{2}} e^{-\frac{\lambda v}{2}} \sqrt{\frac{2\pi}{n\lambda\bar{x}}}$ and $\Phi(\cdot)$ denotes standard normal cdf. On differentiating (5) with respect to a, we have

$$\frac{d}{da} m(\underset{\sim}{x} \mid a) = -\frac{S}{2a^2}\left\{\Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu + a - \frac{1}{\bar{x}}\right)\right] - \Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu - a - \frac{1}{\bar{x}}\right)\right]\right\} + \frac{S\sqrt{n\lambda\bar{x}}}{2a}\left\{\phi\left[\sqrt{n\lambda\bar{x}}\left(\mu + a - \frac{1}{\bar{x}}\right)\right] + \phi\left[\sqrt{n\lambda\bar{x}}\left(\mu - a - \frac{1}{\bar{x}}\right)\right]\right\}$$

where $\phi(\cdot)$ denotes standard normal pdf. (6)

Now we substitute $z = \sqrt{n\lambda\bar{x}}\left|\frac{1}{\bar{x}} - \mu\right|$ and $a^* = a\sqrt{n\lambda\bar{x}}$ in (6) and equate to zero. The equation becomes

$$\Phi(a^* - z) - \Phi[-(a^* + z)] = a^*\left\{\phi(a^* - z) + \phi[-(a^* + z)]\right\}$$

which can be written as

$$a^* = z + \left\{-2\log\left[\sqrt{2\pi}\left(\frac{1}{a^*}\left[\Phi(a^* - z) - \Phi[-(a^* + z)]\right] - \phi[-(a^* + z)]\right)\right]\right\}^{\frac{1}{2}} \quad (7)$$

We solve (7) by standard fixed-point iteration, set $a^* = z$ on the right-hand side, which gives

$$\hat{a} = \begin{cases} 0 & \text{if } z \le 1.65 \\ \dfrac{a^*}{\sqrt{n\lambda\bar{x}}} & \text{if } z > 1.65 \end{cases}$$

Following Berger and Sellke (1987), we make $\hat{a}$ equal to zero when $\bar{x}$ is close to μ.

The posterior distribution of parameter θ with respect to prior π(θ) is given by

$$\pi(\theta \mid \underset{\sim}{x}, \lambda) = \frac{L(\theta \mid \underset{\sim}{x}, \lambda)\pi(\theta)}{\lambda(\underset{\sim}{x})\int_{\Theta} L(\theta \mid \underset{\sim}{x}, \lambda)\pi_o(\theta)d\theta + (1 - \lambda(\underset{\sim}{x}))\int_{\Theta} L(\theta \mid \underset{\sim}{x}, \lambda)q(\theta)d\theta}$$

$$= \frac{L(\theta \mid \underset{\sim}{x}, \lambda)\pi(\theta)}{\lambda(\underset{\sim}{x})m(\underset{\sim}{x} \mid \pi_o) + (1 - \lambda(\underset{\sim}{x}))m(\underset{\sim}{x} \mid q)}$$

$$= \lambda(\underset{\sim}{x})\pi_o(\theta \mid \underset{\sim}{x}) + (1 - \lambda(\underset{\sim}{x}))q(\theta \mid \underset{\sim}{x}) \quad (8)$$

where

$$m(\underset{\sim}{x}\mid\pi_o)=S'\frac{G_1}{G}e^{-\beta}\quad;\quad S'=\left(\frac{\lambda}{2\pi}\right)^{\frac{n}{2}}\prod_{i=1}^{n}x_i^{-\frac{3}{2}}\sqrt{\frac{\tau}{\tau'}}\ ,\ \mathrm{G}_1=\Phi\left(-p'\right),p'=-\mu'\sqrt{\tau'},$$

$$m(\underset{\sim}{x}\mid q)\ =\frac{S}{2\hat{a}}\hat{\phi}_1\ ;\ \hat{\phi}_1=\Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu+\hat{a}-\frac{1}{\bar{x}}\right)\right]-\Phi\left[\sqrt{n\lambda\bar{x}}\left(\mu-\hat{a}-\frac{1}{\bar{x}}\right)\right],$$

$$\pi_o(\theta\mid\underset{\sim}{x})\ =\frac{L(\theta\mid\underset{\sim}{x},\lambda)\pi_o(\theta)}{m(\underset{\sim}{x}\mid\pi_o)}=\frac{1}{G_1}\sqrt{\frac{\tau'}{2\pi}}\exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right]\ ;\ \mu'=\frac{\tau\mu+n\lambda}{\tau'},\ \tau'=\tau+n\lambda\bar{x},$$

$$q(\theta\mid\underset{\sim}{x})\ \ =\frac{L(\theta\mid\underset{\sim}{x},\lambda)q(\theta)}{m(\underset{\sim}{x}\mid q)}=\frac{1}{\hat{\phi}_1}\sqrt{\frac{n\lambda\bar{x}}{2\pi}}\exp\left[-\frac{n\lambda\bar{x}}{2}\left(\theta-\frac{1}{\bar{x}}\right)^2\right],$$

$$\lambda(\underset{\sim}{x})=\left[1+\frac{\varepsilon m(\underset{\sim}{x}\mid q)}{(1-\varepsilon)m(\underset{\sim}{x}\mid\pi_o)}\right]^{-1}=\left[1+\frac{\varepsilon}{(1-\varepsilon)}\frac{G}{G_1}\left(\frac{n\lambda\tau\bar{x}}{2\pi\tau'}\right)^{\frac{-1}{2}}\frac{\hat{\phi}_1e^{\beta'}}{2\hat{a}}\right]^{-1},$$

$$\beta=\beta'+\frac{\lambda v}{2}\ \text{and}\ \beta'=\frac{n\lambda\tau\bar{x}}{2\tau'}\left(\mu-\frac{1}{\bar{x}}\right)^2\ .$$

2.1 Bayes Estimator and Bayes Risk

Under the quadratic loss function, $L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2$, the Bayes estimator $\xi(\underset{\sim}{x})$ and Bayes risk $\delta(\underset{\sim}{x})$ for θ are given as

and

$$\xi(\underset{\sim}{x})=\int_{\Theta}\theta\ \pi(\theta\mid\underset{\sim}{x},\lambda)d\theta=E_o^{\pi_o(\theta|\underset{\sim}{x})}(\theta)+E_q^{q(\theta|\underset{\sim}{x})}(\theta)$$

$$=\lambda(\underset{\sim}{x})\left(\frac{\phi(\mu'\sqrt{\tau'})}{G_1\sqrt{\tau'}}+\mu'\right)+(1-\lambda(\underset{\sim}{x}))\left(\frac{\phi(h)-\phi(h')}{\hat{\phi}_1\sqrt{n\lambda\bar{x}}}+\frac{1}{\bar{x}}\right)\qquad(9)$$

$$\delta(\underset{\sim}{x})=\int_{\Theta}\theta^2\ \pi(\theta\mid\underset{\sim}{x},\lambda)d\theta-\left(\xi(\underset{\sim}{x})\right)^2$$

$$=\lambda(\underset{\sim}{x})\left(\frac{\mu'\phi(\mu'\sqrt{\tau'})}{G_1\sqrt{\tau'}}+\frac{1}{\tau'}+\mu'^2\right)+(1-\lambda(\underset{\sim}{x}))\left(\frac{w\phi(h)-w'\phi(h')}{\hat{\phi}_1\sqrt{n\lambda\bar{x}}}+\frac{1}{\bar{x}^2}+\frac{1}{n\lambda\bar{x}}\right)-\left(\xi(\underset{\sim}{x})\right)^2\qquad(10)$$

where $w=\mu-\hat{a}-\frac{1}{\bar{x}},w'=\mu+\hat{a}-\frac{1}{\bar{x}},h=w\sqrt{n\lambda\bar{x}}$ and $h'=w'\sqrt{n\lambda\bar{x}}$

2.2 Predictive Density under ML-II ε-contaminated prior

Let y be an independent potential future observation from IG(θ,λ) population. The predictive density function of y, given a random sample $\underset{\sim}{x}$, is defined as

$$p(y\mid \underset{\sim}{x})=\int_0^\infty p(y\mid\theta,\lambda)\ \pi(\theta\mid \underset{\sim}{x},\lambda)d\theta = \lambda(\underset{\sim}{x})p_o(y\mid \underset{\sim}{x},\lambda)+(1-\lambda(\underset{\sim}{x}))q(y\mid \underset{\sim}{x},\lambda) \tag{11}$$

the right-hand side terms are

$$p_o(y\mid \underset{\sim}{x},\lambda)=\int_0^\infty p(y\mid\theta,\lambda)\ \pi_o(\theta\mid \underset{\sim}{x},\lambda)d\theta = \frac{G_2}{G_1}y^{-2}\sqrt{\frac{\tau_1}{2\pi}}\exp\left[-\frac{\tau_1}{2}\left(\mu'-\frac{1}{y}\right)^2\right]$$

$$q(y\mid \underset{\sim}{x},\lambda) \quad =\int_0^\infty p(y\mid\theta,\lambda)\ q(\theta\mid \underset{\sim}{x},\lambda)d\theta \ = \frac{\hat{\phi}_2}{\hat{\phi}_1}y^{-2}\sqrt{\frac{\tau''}{2\pi}}\exp\left[-\frac{\tau''}{2}\left(\frac{1}{y}-\frac{1}{\bar{x}}\right)^2\right]$$

where

$$\mu_1=\frac{\lambda+\tau'\mu'}{\lambda\tau' y}, \tau_1=\frac{\lambda\tau' y}{\tau'+\lambda y},$$

$$\mu''=\frac{n+1}{n\bar{x}+y},\ \ \tau''=\frac{n\lambda\bar{x}y}{n\bar{x}+y},$$

$$\hat{\phi}_2=\Phi\left[\sqrt{\lambda(y+n\bar{x})}\left(\mu+\hat{a}-\mu''\right)\right]-\Phi\left[\sqrt{\lambda(y+n\bar{x})}\left(\mu-\hat{a}-\mu''\right)\right],$$

$$\text{and}\ \ G_2=\Phi\left(-p''\right),\ \ p''=-\mu_1\sqrt{\tau'+\lambda y}.$$

3. Robustness under truncated Edgeworth Series prior distribution (ESD)

Suppose the prior distribution of θ is expressed by the first four terms of Edgeworth series truncated at zero given by

$$\pi(\theta)=\frac{1}{G}\sqrt{\frac{\tau}{2\pi}}\exp\left[-\frac{\tau}{2}(\theta-\mu)^2\right]H(\theta)\ ;\ \theta\geq 0 \tag{12}$$

where

$$H(\theta)=\left\{1+\frac{\lambda_3}{6}H_3\left[\sqrt{\tau}(\theta-\mu)\right]+\frac{\lambda_4}{24}H_4\left[\sqrt{\tau}(\theta-\mu)\right]+\frac{\lambda_3^2}{72}H_6\left[\sqrt{\tau}(\theta-\mu)\right]\right\}$$

$$G=\int_0^\infty\sqrt{\frac{\tau}{2\pi}}\exp\left[-\frac{\tau}{2}(\theta-\mu)^2\right]H(\theta)d\theta$$

$$=\left\{P_0+\frac{\lambda_3}{6}\sqrt{\tau}\left(\tau P_3-3P_1\right)+\frac{\lambda_4}{24}\left(\tau^2P_4-6\tau P_2+3P_0\right)+\frac{\lambda_3^2}{72}\left(\tau^3P_6-15\tau^2P_4+45\tau P_2-15P_0\right)\right\}$$

$$P_k=\int_0^\infty(\theta-\mu)^k\sqrt{\frac{\tau}{2\pi}}\exp\left[-\frac{\tau}{2}(\theta-\mu)^2\right]d\theta\ =\left(\sqrt{\tau}\right)^{-j}\int_q^\infty z^j\phi(z)dz=\left(\sqrt{\tau}\right)^{-j}I_j(q),\ \text{q}=-\mu\sqrt{\tau}$$

where $I_j(\cdot)$, j = 0,1,2,3,4,6 , is the jth incomplete moment of standard normal variate; $\phi(\cdot)$ is the pdf of SNV ; $H_k(\cdot)$ is a Hermite polynomial[1] of degree k ; $\lambda_3 = \sqrt{\beta_1}$ and $\lambda_4 = \beta_2$-3 are the measures of skewness and kurtosis respectively .With varying values of coefficients $\lambda_3 \in [0, 0.5]$ and $\lambda_4 \in [0, 2.4]$ in the limits of Barton-Dennis (1952) region, the Edgeworth series prior $\pi(\theta)$ represents a class of prior distributions. It gives a variety of moderately non-normal unimodal proper pdfs truncated at zero, taking both skewness and kurtosis into consideration. The normal distribution truncated at zero is a member of this class for $\lambda_3 = \lambda_4 = 0$.

The posterior distribution of parameter θ with respect to prior $\pi(\theta)$ can be shown to be

$$\pi(\theta \mid \underset{\sim}{x}, \lambda) = \frac{L(\theta \mid \underset{\sim}{x}, \lambda)\pi(\theta)}{\int_\Theta L(\theta \mid \underset{\sim}{x}, \lambda)\pi(\theta)d\theta} = \frac{1}{G_1}\sqrt{\frac{\tau'}{2\pi}} \exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right] H(\theta) \qquad (13)$$

where

$$\mu' = \frac{\tau\mu + n\lambda}{\tau'} \ , \ \tau' = \tau + n\lambda\bar{x}$$

$$G_1 = \int_0^\infty \sqrt{\frac{\tau'}{2\pi}} \exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right] H(\theta)d\theta$$

$$= \left\{C_0 + \frac{\lambda_3}{6}\sqrt{\tau}\left(\tau C_3 - 3C_1\right) + \frac{\lambda_4}{24}\left(\tau^2 C_4 - 6\tau C_2 + 3C_0\right) + \frac{\lambda_3^2}{72}\left(\tau^3 C_6 - 15\tau^2 C_4 + 45\tau C_2 - 15C_0\right)\right\}$$

$$C_k = \int_0^\infty (\theta-\mu)^k \sqrt{\frac{\tau'}{2\pi}} \exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right] d\theta = \sum_{j=0}^{k} \binom{k}{j} (\mu' - \mu)^{k-j} \left(\sqrt{\tau}\right)^{-j} I_j(q'), \ \ q' = -\mu'\sqrt{\tau'}$$

3.1 Bayes Estimator and Bayes Risk

We consider the quadratic loss function, $L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$, which gives the Bayes estimator and Bayes risk for θ as posterior mean and variance respectively. Now using the integrals, represented as a recurrence relation in terms of the constants obtained for the posterior distribution,

$$L_k = \int_0^\infty \theta(\theta-\mu)^k \sqrt{\frac{\tau'}{2\pi}} \exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right] H(\theta)d\theta = C_{k+1} + \mu C_k \ ; \ k = 0,1,...$$

$$V_k = \int_0^\infty \theta^2(\theta-\mu)^k \sqrt{\frac{\tau'}{2\pi}} \exp\left[-\frac{\tau'}{2}(\theta-\mu')^2\right] H(\theta)d\theta = C_{k+2} + 2\mu C_{k+1} + \mu^2 C_k \ ; \ k = 0,1,...$$

---

[1] *In particular* $H_o(\theta) = 1$, $H_1(\theta) = \theta$, , $H_3(\theta) = \theta^3 - 3\theta$, $H_4(\theta) = \theta^4 - 6\theta^2 + 3$ *and* $H_4(\theta) = \theta^6 - 15\theta^4 + 45\theta^2 - 15$.

.

the Bayes estimator becomes

$$\xi(\underset{\sim}{x}) = E^{\pi(\theta|\underset{\sim}{x},\lambda)}\left(\theta\right) = \int_0^\infty \theta \ \pi(\theta \mid \underset{\sim}{x}, \lambda) d\theta$$

$$= \frac{1}{G_1}\left\{ L_0 + \frac{\lambda_3}{6}\sqrt{\tau}\left(\tau L_3 - 3L_1\right) + \frac{\lambda_4}{24}\left(\tau^2 L_4 - 6\tau L_2 + 3L_0\right) + \frac{\lambda_3^2}{72}\left(\tau^3 L_6 - 15\tau^2 L_4 + 45\tau L_2 - 15L_0\right)\right\} \tag{14}$$

and the corresponding Bayes risk is

$$\delta(\underset{\sim}{x}) = Var^{\pi(\theta|\underset{\sim}{x},\lambda)}\left(\theta\right) = \int_0^\infty \theta^2 \pi(\theta \mid \underset{\sim}{x}, \lambda) d\theta - \left(\xi(\underset{\sim}{x})\right)^2$$

$$= \frac{1}{G_1}\left\{ V_0 + \frac{\lambda_3}{6}\sqrt{\tau}\left(\tau V_3 - 3V_1\right) + \frac{\lambda_4}{24}\left(\tau^2 V_4 - 6\tau V_2 + 3V_0\right) + \frac{\lambda_3^2}{72}\left(\tau^3 V_6 - 15\tau^2 V_4 + 45\tau V_2 - 15V_0\right)\right\} - \left(\xi(\underset{\sim}{x})\right)^2. \tag{15}$$

3.2 Predictive Density under ESD

Let y be an independent potential future observation from IG(θ,λ) population. The predictive density function of y, given a random sample $\underset{\sim}{x}$, is defined as

$$p(y \mid \underset{\sim}{x}) = \int_0^\infty p(y \mid \theta, \lambda) \ \pi(\theta \mid \underset{\sim}{x}, \lambda) d\theta$$

$$= \left(\frac{\lambda}{2\pi}\right)^{1/2} y^{-3/2} \sqrt{\frac{\tau'}{\tau_1}} \exp\left[-\frac{\lambda \tau' y}{2\tau_1}\left(\frac{1}{y} - \mu'\right)^2 \right] \frac{G_2}{G_1} \tag{16}$$

where

$$\mu_1 = \frac{\tau'\mu' + \lambda}{\tau_1}, \ \ \tau_1 = \tau' + \lambda y,$$

$$G_2 = \int_0^\infty \sqrt{\frac{\tau_1}{2\pi}} \exp\left[-\frac{\tau_1}{2}(\theta - \mu_1)^2 \right] H(\theta) d\theta$$

$$= \left\{ D_0 + \frac{\lambda_3}{6}\sqrt{\tau}\left(\tau D_3 - 3D_1\right) + \frac{\lambda_4}{24}\left(\tau^2 D_4 - 6\tau D_2 + 3D_0\right) + \frac{\lambda_3^2}{72}\left(\tau^3 D_6 - 15\tau^2 D_4 + 45\tau D_2 - 15D_0\right)\right\},$$

$$D_k = \int_0^\infty (\theta - \mu)^k \sqrt{\frac{\tau_1}{2\pi}} \exp\left[-\frac{\tau_1}{2}(\theta - \mu_1)^2 \right] d\theta = \sum_{j=0}^{k} \binom{k}{j} (\mu_1 - \mu)^{k-j} \left(\sqrt{\tau}\right)^{-j} I_j(q''), \ \ q'' = -\mu_1\sqrt{\tau_1} .$$

4. Illustration

In order to study sensitivity of the Bayes estimator and risk to the misspecification in the prior distributions, we consider the maintenance data on 46 repair time for an airborne communication transceiver given in Chhikara and Folks (1977). The data on active repair time (hours) are

Data-Set 1
0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0,1.0, 1.1, 1.3, 1.5, 1.5, 1.5, 1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3, 22.0, 24.5.

To study the effect of misspecification of prior distributions for small sample size we draw a random sample of size n = 10 from the above data.

Data-Set 2
1, 0.8, 0.6, 3.3, 4.5, 0.2, 1.3, 5.4, 8.8, 1.5.

We further simulate a random sample of size n = 20 from IG population using algorithm given in Devrorye (1986, page 149).

Data-Set 3
0.49, 1.15, 9.42, 1.24, 1.93, 2.85, 0.98, 5.65, 1.04, 0.67, 5.50, 4.69, 0.57, 0.18, 13.28, 3.57, 2.31, 4.40, 6.16, 0.50.

The Kolmogorov-Smirnov test statistic for the above three data-sets and the graphs of empirical and the theoretical curves are given in Appendix 1.1. The results show that IG is a good fit for all the above data-sets.

The prior parameter μ has been taken to be approximately equal to the reciprocal of median of the IG(θ,λ) and precision τ equal to the reciprocal of the ML estimate of the variance. The value of known shape parameter λ is taken to be the ML estimate of $\hat{\lambda} = \left(\frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{1}{x_i} - \frac{1}{\bar{x}} \right) \right)^{-1}$.

Under ε-contaminated prior

Tables 1-3 suggest that the increase in the contamination in the prior does not affect the Bayes estimate and risk for the data-sets n = {46, 10, 20}. We observe little increase in the Bayes estimate and little decrease in the Bayes risk with increase in the contamination in the prior at various precision levels (τ). The graphs 1-6 in Appendix 1.2 substantiate the above interpretation.

ML-II ε-contaminated
Comparative values of Bayes estimate and risk (underlined)
for varying τ, ε

Table 1
n = 46

ε / τ	0	0.05	0.2	0.5	0.9
0.0354	0.27704658 0.00361253	0.28004513 0.00334688	0.28356147 0.00301246	0.28555762 0.00281161	0.28640744 0.00272368
0.05	0.27706207 0.00361234	0.27974407 0.00337518	0.28323806 0.00304465	0.28541422 0.00282644	0.28638785 0.00272574
0.5	0.27753980 0.00360648	0.27872680 0.00349750	0.28131031 0.00325054	0.28421681 0.00295676	0.28619326 0.00274733

Table 2
n = 10

ε / τ	0	0.05	0.2	0.5	0.9
0.05	0.37342141 0.02751703	0.38485602 0.02589084	0.40349143 0.02268022	0.41803221 0.01969264	0.42549292 0.01799560
0.0581	0.37349401 0.02751404	0.38433232 0.02597622	0.40265791 0.02284160	0.41755913 0.01979761	0.42542035 0.01801279
0.5	0.37741175 0.02734800	0.38228795 0.02661103	0.39429716 0.02459325	0.41068767 0.02137384	0.42412214 0.01833436

Table 3
n = 20

ε / τ	0	0.05	0.2	0.5	0.9
0.039	0.30100257 0.01028180	0.31644883 0.00901021	0.33224115 0.00721680	0.34027073 0.00611366	0.34350795 0.00563245
0.05	0.30102170 0.01028077	0.31533943 0.00911761	0.33123849 0.00734556	0.33986521 0.00617249	0.34345465 0.00564054
0.5	0.30179830 0.01023883	0.30840187 0.00974138	0.32155732 0.00849043	0.33464430 0.00690257	0.34266403 0.00576025

Under ESD Prior

Tables 4-12 show that the Bayes estimate and risk are robust against the changes in the ESD prior, for the data-sets n = {46, 10 and 20}. For n = {10 and 20}, it is observed that Bayes estimate and Bayes risk both decrease with the increase in skewness (λ_3) in the prior. Bayes estimate increases whereas Bayes risk decreases with the increase in kurtosis (λ_4) in the prior. However, for n=46, Bayes estimate decreases with increase in skewness (λ_3) and increases with the increase in kurtosis (λ_4). Further Bayes risk increases with the increase in λ_3 and decreases with the increase in λ_4. *The above observed increase or decrease in the Bayes estimate and risks are insignificant.* The graphs 10-15 in Appendix 1.2 confirm that there is no effect of misspecification in the prior.

Truncated Edgeworth Series
Comparative values of Bayes estimate and risk (underlined) for varying τ, λ_3, λ_4.

Table 4

n = 46

τ = 0.0354

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.27704658 0.00361253	0.27701224 0.00361253	0.27697673 0.00361254	0.27693958 0.00361256	0.27690029 0.00361259
0.5	0.27705543 0.00361242	0.27702310 0.00361242	0.27698970 0.00361243	0.27695484 0.00361245	0.27691807 0.00361248
0.8	0.27706027 0.00361236	0.27702903 0.00361236	0.27699678 0.00361237	0.27696316 0.00361239	0.27692775 0.00361242
1.2	0.27706623 0.00361229	0.27703633 0.00361229	0.27700551 0.00361230	0.27697341 0.00361232	0.27693967 0.00361234
1.6	0.27707169 0.00361222	0.27704303 0.00361222	0.27701351 0.00361223	0.27698280 0.00361225	0.27695058 0.00361227
2.0	0.27707672 0.00361216	0.27704919 0.00361216	0.27702087 0.00361217	0.27699144 0.00361219	0.27696060 0.00361221

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.27706207 0.00361234	0.27702126 0.00361234	0.27697888 0.00361235	0.27693439 0.00361238	0.27688720 0.00361242
0.5	0.27707457 0.00361218	0.27703612 0.00361219	0.27699625 0.00361220	0.27695450 0.00361223	0.27691033 0.00361226
0.8	0.27708139 0.00361210	0.27704423 0.00361210	0.27700574 0.00361212	0.27696547 0.00361214	0.27692293 0.00361218
1.2	0.27708980 0.00361200	0.27705423 0.00361200	0.27701742 0.00361202	0.27697898 0.00361204	0.27693844 0.00361208
1.6	0.27709751 0.00361190	0.27706340 0.00361191	0.27702815 0.00361192	0.27699136 0.00361195	0.27695265 0.00361198
2.0	0.27710461 0.00361182	0.27707185 0.00361182	0.27703801 0.00361183	0.27700276 0.00361186	0.27696571 0.00361189

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.27753980 0.00360648	0.27741268 0.00360640	0.27727412 0.00360644	0.27712279 0.00360656	0.27695711 0.00360677
0.5	0.27766312 0.00360500	0.27754182 0.00360497	0.27741005 0.00360504	0.27726661 0.00360520	0.27711008 0.00360544
0.8	0.27773093 0.00360417	0.27761288 0.00360416	0.27748488 0.00360425	0.27734579 0.00360443	0.27719428 0.00360469
1.2	0.27781501 0.00360313	0.27770104 0.00360315	0.27757774 0.00360326	0.27744406 0.00360346	0.27729876 0.00360374
1.6	0.27789261 0.00360216	0.27778245 0.00360220	0.27766352 0.00360234	0.27753484 0.00360255	0.27739528 0.00360284
2.0	0.27796446 0.00360125	0.27785787 0.00360131	0.27774301 0.00360146	0.27761897 0.00360169	0.27748470 0.00360199

Table 5
n=10

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.37342141 0.02751703	0.37311077 0.02750710	0.37278757 0.02749737	0.37244775 0.02748769	0.37208676 0.02747791
0.5	0.37352632 0.02751273	0.37323349 0.02750345	0.37292933 0.02749436	0.37261022 0.02748532	0.37227216 0.02747621
0.8	0.37358361 0.02751037	0.37330054 0.02750144	0.37300675 0.02749269	0.37269890 0.02748401	0.37237326 0.02747526
1.2	0.37365427 0.02750745	0.37338322 0.02749895	0.37310222 0.02749063	0.37280821 0.02748236	0.37249777 0.02747406
1.6	0.37371911 0.02750477	0.37345910 0.02749666	0.37318983 0.02748871	0.37290846 0.02748084	0.37261188 0.02747293
2.0	0.37377882 0.02750229	0.37352900 0.02749453	0.37327050 0.02748694	0.37300074 0.02747941	0.37271684 0.02747186

τ = 0.0581

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.37349401 0.02751404	0.37315910 0.02750334	0.37280996 0.02749287	0.37244222 0.02748248	0.37205097 0.02747200
0.5	0.37361589 0.02750904	0.37330010 0.02749905	0.37297144 0.02748928	0.37262604 0.02747960	0.37225958 0.02746985
0.8	0.37368247 0.02750630	0.37337714 0.02749669	0.37305965 0.02748730	0.37272640 0.02747799	0.37237337 0.02746864
1.2	0.37376459 0.02750290	0.37347217 0.02749376	0.37316845 0.02748483	0.37285013 0.02747599	0.37251355 0.02746711
1.6	0.37383996 0.02749978	0.37355940 0.02749106	0.37326831 0.02748254	0.37296364 0.02747412	0.37264205 0.02746568
2.0	0.37390938 0.02749688	0.37363976 0.02748856	0.37336028 0.02748042	0.37306815 0.02747238	0.37276028 0.02746433

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.37741175 0.02734800	0.37646454 0.02731516	0.37542759 0.02728462	0.37429249 0.02725541	0.37304908 0.02722649
0.5	0.37842482 0.02730487	0.37751618 0.02727596	0.37652498 0.02724931	0.37544354 0.02722409	0.37426263 0.02719941
0.8	0.37898443 0.02728017	0.37809754 0.02725333	0.37713194 0.02722874	0.37608034 0.02720563	0.37493401 0.02718317
1.2	0.37968074 0.02724855	0.37882138 0.02722422	0.37788796 0.02720210	0.37687368 0.02718149	0.37577041 0.02716167
1.6	0.38032587 0.02721840	0.37949244 0.02719630	0.37858917 0.02717636	0.37760966 0.02715797	0.37654633 0.02714048
2.0	0.38092525 0.02718964	0.38011631 0.02716953	0.37924132 0.02715154	0.37829428 0.02713512	0.37726811 0.02711968

Table 6

n=20

τ = 0.039

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.30100257 0.01028180	0.30090036 0.01028139	0.30079553 0.01028105	0.30068672 0.01028077	0.30057242 0.01028054
0.5	0.30101845 0.01028094	0.30092225 0.01028056	0.30082369 0.01028024	0.30072157 0.01027998	0.30061458 0.01027976
0.8	0.30102711 0.01028047	0.30093419 0.01028011	0.30083904 0.01027980	0.30074056 0.01027954	0.30063753 0.01027934
1.2	0.30103779 0.01027990	0.30094890 0.01027955	0.30085795 0.01027926	0.30076394 0.01027901	0.30066574 0.01027881
1.6	0.30104758 0.01027937	0.30096238 0.01027904	0.30087529 0.01027876	0.30078535 0.01027852	0.30069156 0.01027833
2.0	0.30105659 0.01027888	0.30097480 0.01027857	0.30089123 0.01027830	0.30080504 0.01027807	0.30071528 0.01027789

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.30102170 0.01028076	0.30090588 0.01028030	0.30078688 0.01027993	0.30066317 0.01027963	0.30053305 0.01027941
0.5	0.30104207 0.01027967	0.30093304 0.01027924	0.30082116 0.01027889	0.30070507 0.01027861	0.30058327 0.01027840
0.8	0.30105319 0.01027907	0.30094787 0.01027866	0.30083986 0.01027832	0.30072790 0.01027805	0.30061060 0.01027785
1.2	0.30106689 0.01027833	0.30096613 0.01027794	0.30086289 0.01027762	0.30075601 0.01027736	0.30064422 0.01027717
1.6	0.30107945 0.01027765	0.30098288 0.01027728	0.30088400 0.01027698	0.30078176 0.01027673	0.30067499 0.01027654
2.0	0.30109101 0.01027703	0.30099829 0.01027668	0.30090342 0.01027638	0.30080543 0.01027615	0.30070325 0.01027597

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.30179830 0.01023883	0.30143412 0.01023723	0.30104899 0.01023646	0.30063838 0.01023644	0.30019712 0.01023712
0.5	0.30199902 0.01022803	0.30165464 0.01022671	0.30129124 0.01022614	0.30090478 0.01022629	0.30049068 0.01022708
0.8	0.30210889 0.01022208	0.30177540 0.01022090	0.30142389 0.01022045	0.30105061 0.01022067	0.30065127 0.01022152
1.2	0.30224462 0.01021470	0.30192461 0.01021369	0.30158781 0.01021336	0.30123076 0.01021367	0.30084953 0.01021458
1.6	0.30236940 0.01020788	0.30206184 0.01020701	0.30173856 0.01020679	0.30139638 0.01020717	0.30103172 0.01020814
2.0	0.30248452 0.01020156	0.30218846 0.01020082	0.30187766 0.01020069	0.30154918 0.01020114	0.30119969 0.01020214

4.1 Prediction Interval

In order to study the effects of ML-II ε-contaminated and ESD prior on the prediction interval we consider the mean $\mu = 1/x_{med}$ and various values of ε, τ, λ_3 and λ_4. For earlier considered samples of size n = {46, 10, 20}, we compute the values of α as given in Chhikara and Folks (1982)

$$P\left[0.26 < y < 20.4 \mid \underset{\sim}{x}\right] = \alpha$$

The comparative values of α for ML-II ε-contaminated are given in the Table 7-9 and for ESD in Table 10-12. Tables 7-9 indicate insignificant effect of contaminations in prior on the probability content α of prediction interval, for the given samples sizes. Further Table 10-12 shows insignificant effect of ESD prior on the probability content with increase in λ_3 and λ_4 values. The change in τ, the precision of prior distribution, also does not bring any significant change in the probability content. The graphs 7-9 and 16-18 in Appendix 1.2 validate the results.

ML-II ε-contaminated

Comparative values of $P\left[0.26 < y < 20.4 \mid \underset{\sim}{x}\right]$ for varying τ, ε

Table 7

n = 46

ε / τ	0	0.05	0.2	0.5	0.9
0.0354	0.95807379	0.95914667	0.96040509	0.96111944	0.96142336
0.05	0.95807777	0.95903793	0.96028877	0.96106767	0.96141630
0.5	0.95819806	0.95862972	0.95956919	0.96062596	0.96134486

Table 8

n = 10

ε / τ	0	0.05	0.2	0.5	0.9
0.05	0.92541816	0.92784392	0.93179734	0.93488201	0.93646496
0.0581	0.92542944	0.92772991	0.93161852	0.93478126	0.93644941
0.5	0.92602989	0.92708737	0.92969213	0.93324767	0.93616189

Table 9

n = 20

ε / τ	0	0.05	0.2	0.5	0.9
0.039	0.94427885	0.94849021	0.95279522	0.95498344	0.95586622
0.05	0.94428396	0.94818686	0.95252160	0.95487270	0.95585106
0.5	0.94449047	0.94629195	0.94987995	0.95344842	0.95563617

Truncated Edgeworth

Comparative values of $P\left[0.26 < y < 20.4 \mid \underset{\sim}{x}\right]$ for varying τ, λ_3, λ_4

Table 10

n = 46

τ = 0.0354

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.95807379	0.95806438	0.95805589	0.95804748	0.95803854
0.5	0.95807348	0.95806461	0.95805664	0.95804873	0.95804032
0.8	0.95807477	0.95806620	0.95805850	0.95805087	0.95804278
1.2	0.95807654	0.95806834	0.95806098	0.95805370	0.95804600
1.6	0.95807875	0.95807089	0.95806384	0.95805689	0.95804954
2.0	0.95808015	0.95807261	0.95806585	0.95805919	0.95805216

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.95807777	0.95806673	0.95805659	0.95804612	0.95803520
0.5	0.95807839	0.95806801	0.95805846	0.95804862	0.95803838
0.8	0.95808019	0.95807015	0.95806093	0.95805144	0.95804158
1.2	0.95808259	0.95807297	0.95806416	0.95805510	0.95804572
1.6	0.95808536	0.95807615	0.95806771	0.95805905	0.95805009
2.0	0.95808764	0.95807879	0.95807069	0.95806240	0.95805383

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.95819806	0.95816513	0.95813171	0.95809554	0.95805587
0.5	0.95822735	0.95819594	0.95816417	0.95812989	0.95809241
0.8	0.95824415	0.95821360	0.95818272	0.95814949	0.95811321
1.2	0.95826565	0.95823616	0.95820642	0.95817447	0.95813968
1.6	0.95828620	0.95825768	0.95822899	0.95819824	0.95816482
2.0	0.95830492	0.95827733	0.95824961	0.95821997	0.95818782

Table 11
n=10

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.92541816	0.92538220	0.92534436	0.92530413	0.92526099
0.5	0.92543453	0.92543453	0.92536502	0.92532727	0.92528691
0.8	0.92544347	0.92541069	0.92537630	0.92533990	0.92530103
1.2	0.92545449	0.92542311	0.92539022	0.92535547	0.92531844
1.6	0.92546460	0.92543450	0.92540299	0.92536974	0.92533439
2.0	0.92547392	0.92544499	0.92541475	0.92538288	0.92534906

τ = 0.0581

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.92542944	0.92539070	0.92534980	0.92530623	0.92525940
0.5	0.92544842	0.92541189	0.92537340	0.92533250	0.92528868
0.8	0.92545880	0.92542347	0.92538629	0.92534685	0.92530465
1.2	0.92547160	0.92543776	0.92540219	0.92536453	0.92532433
1.6	0.92548334	0.92545087	0.92541679	0.92538076	0.92534237
2.0	0.92549415	0.92546294	0.92543023	0.92539642	0.92539569

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.92602989	0.92592636	0.92580904	0.92567706	0.92552925
0.5	0.92618174	0.92608204	0.92596968	0.92584384	0.92570346
0.8	0.92626562	0.92616811	0.92605853	0.92593610	0.92579983
1.2	0.92636999	0.92627527	0.92616920	0.92605105	0.92591991
1.6	0.92646670	0.92637462	0.92627185	0.92615769	0.92603129
2.0	0.92655654	0.92646697	0.92636731	0.92625689	0.92613491

Table 12
n = 20

τ = 0.039

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.94427885	0.94425725	0.94423499	0.94421184	0.94418739
0.5	0.94428308	0.94426275	0.94424183	0.94422010	0.94419723
0.8	0.94428539	0.94426575	0.94424556	0.94422461	0.94420259
1.2	0.94428824	0.94426945	0.94425015	0.94423016	0.94420918
1.6	0.94429085	0.94427284	0.94425437	0.94423524	0.94421521
2.0	0.94429325	0.94427597	0.94425824	0.94423991	0.94422074

τ = 0.05

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.94428396	0.94425946	0.94423416	0.94420779	0.94417990
0.5	0.94428938	0.94426632	0.94424255	0.94421781	0.94419172
0.8	0.94429235	0.94427008	0.94424712	0.94422327	0.94419815
1.2	0.94429601	0.94427470	0.94425277	0.94423000	0.94420607
1.6	0.94429936	0.94427893	0.94425793	0.94423615	0.94421331
2.0	0.94430245	0.94428284	0.94426268	0.94424182	0.94421996

τ = 0.5

λ_4 \ λ_3	0	0.1	0.2	0.3	0.4
0	0.94449047	0.94441419	0.94433236	0.94424213	0.94414468
0.5	0.94454357	0.94447139	0.94439418	0.94430928	0.94421790
0.8	0.94457263	0.94450272	0.94442804	0.94434605	0.94425795
1.2	0.94460855	0.94454143	0.94446988	0.94439147	0.94430740
1.6	0.94464156	0.94457703	0.94450835	0.94443323	0.94435285
2.0	0.94467202	0.94460988	0.94454385	0.94447175	0.94439474

5. Conclusion

The numerical illustrations suggest that moderate amount of misspecification in prior distributions belonging to the class of ML-II ε-contaminated and ESD does not affect the Bayesian results. The mathematical results obtained in Section 2 and 3 play down the effect of subjective choice of prior for the location parameter of an inverse Gaussian distribution.

References

Aase, K.K. (2000). An equilibrium asset pricing model based on Levy processes: relations to stochastic volatility and the survival hypothesis. *Insurance Mathematics and Economics*, 27, 345-363.

Banerjee, A.K. and Bhattacharyya, G.K. (1976). A purchase incidence model with inverse Gaussian interpurchase times. *J. Amer. Statist. Assoc.*, 71, 823-829

Banerjee, A.K. and Bhattacharyya, G.K. (1979). Bayesian results for the inverse Gaussian distribution with an application. *Techonometrics*, 21, 247-251

Bansal, A.K.(1978). Robustness of Bayes estimator for the mean of a normal population with non-normal prior. *Commun. Statist.-Theor.Meth.*, A7(5), 453-460.

Bansal, A.K. and Sinha, P. (1992). Sensitivity of Bayesian sampling inspection schemes to a non-normal prior distribution. *Journal of Applied Statistics*, *19*, 103-109.

Bansal, A.K. and Sinha, P. (2008). Bayesian optimization analysis with ML-II ϵ contaminated prior. *Journal of Applied Statistics*, 35, 203-211

Barton, D.E. and Dennis, K.E. (1952). The condition under which Gram-Charlier and Edgeworth curves are positive definite and unimodal. *Biometrica*, 39, 425-427.

Berger, J.O. (1984). The robust Bayesian viewpoint (with discussion). In *Robustness of Bayesian Analysis,* J. Kadane (Ed.), North Holland, Amsterdam, 63-124.

Berger, J.O. (1985). *Statistical Decision Theory and Bayesian Analysis.* Springer-Verlag, New York.

Berger, J.O. (1990). Robust Bayesian analysis: sensitivity to the prior. *Journal of Statistical Planning and Inference*, 25, 303-323.

Berger, J.O. (1994). An overview of robust Bayesian analysis. *Test*, 5-59.

Berger, J.O. and Berlinear, M. (1986). Robust Bayes and empirical Bayes analysis with ϵ contaminated priors. *Annals of Statistics*, 14, 461-486.

Berger, J.O. and Sellke, T. (1987). Testing a point null hypothesis: The irreconcilability of p values and evidence. *J. Amer. Statist. Assoc.*, 82,112-139.

Chakravarti, S. and Bansal, A.K. (1988). Effect of non-normal prior for regression parameter on Bayes decisions and forecasts. *Journal of Quantitative Economics*, 4, 247-259.

Chhikara, R.S. and Folks, J.L. (1989). *The Inverse Gaussian distribution*. Marcel Decker, Inc., New York.

Chhikara, R.S. and Folks, J.L. (1977). The Inverse Gaussian distribution as a life-time model. *Technometrics*, 19, 461–468.

Chhikara, R.S, and Guttman, I. (1982). Prediction limits for the Inverse Gaussian distribution. *Techonometrics*, 24, 319–314.

Devroye, L. (1986). *Non-Uniform Random Variate Generation*. Springer-Verlag, New York.

Draper, N.R. and Tierney, D.E. (1972). Regions of positive and unimodal series expansion of the Edgeworth and Gram-Charlier approximations. Biometrika, 59, 463-465.

Good, I.J. (1965). *The Estimation of Probabilities*. MIT Press, Cambridge, MA.

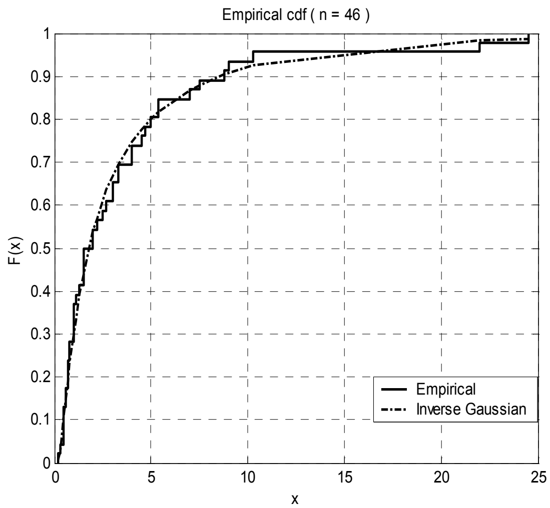
Nadarajha, S. and Kotz, S. (2007). Inverse Gaussian random variables with application to price indices. Applied Economics Letters, 14, 673-677.

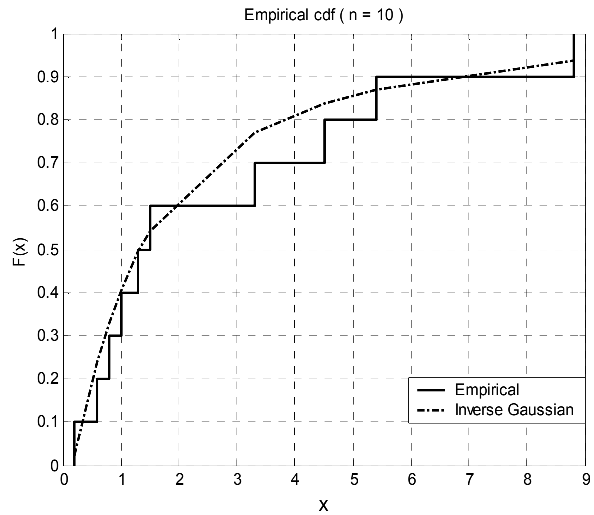
Seshadri, V. (1999). *The Inverse Gaussian Distribution, Statistical Theory and application*. Springer-Verlag, New York.

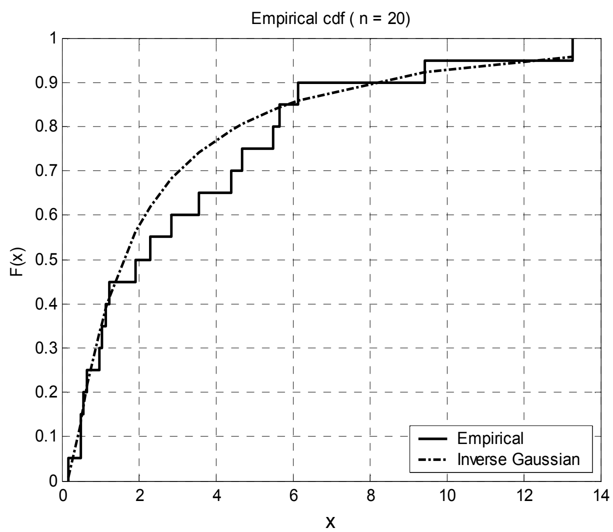
Whitemore, G.A. (1976). Management applications of the inverse Gaussian distributions. *Int. J. Manage. Sci.*, 4, 215-223.

Whitemore, G.A. (1986). Inverse Gaussian ratio estimation. Applied Statistics, 35, 8-15.

Appendix 1.1







	Kolmogorov –Smirnov Test and p sig. values		Decision at 5%
	k-s	p	0.05
n=10	0.1698	0.9089	Data fits IG
n=46	0.0694	0.9754	Data fits IG
n=20	0.1419	0.7840	Data fits IG

Appendix 1.2

ML-II Prior for varying ε

Fig.1: n = 46

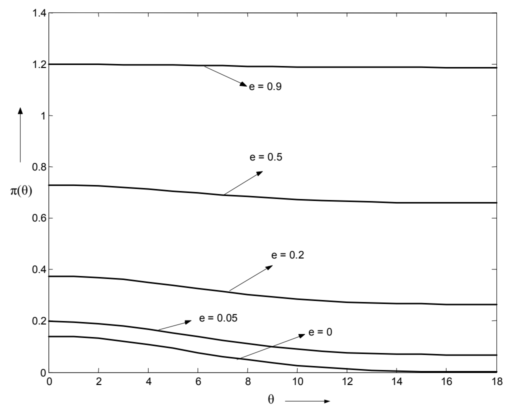


Fig.2: n = 10

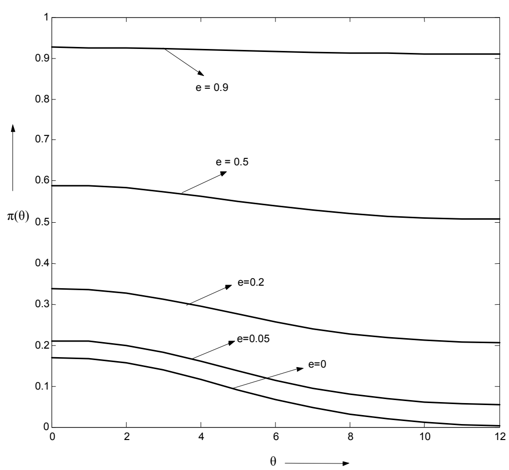
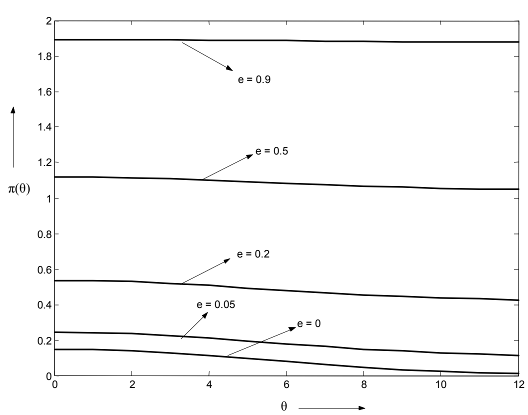


Fig.3: n = 20



ML-II Posterior for varying ε

Fig.4: n = 46

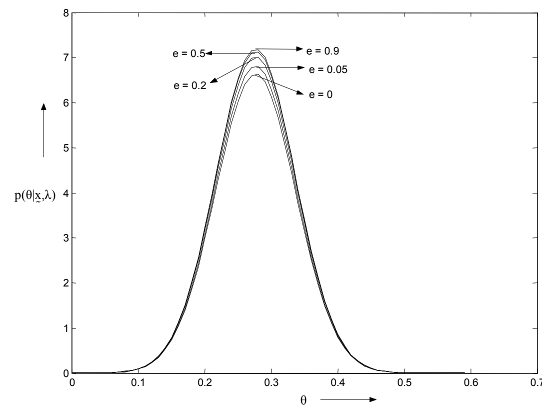


Fig.5: n = 10

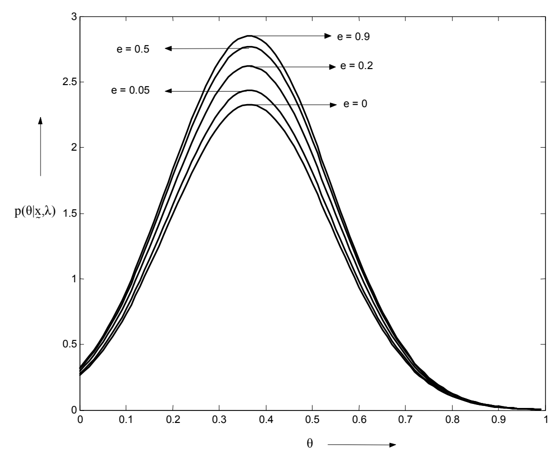
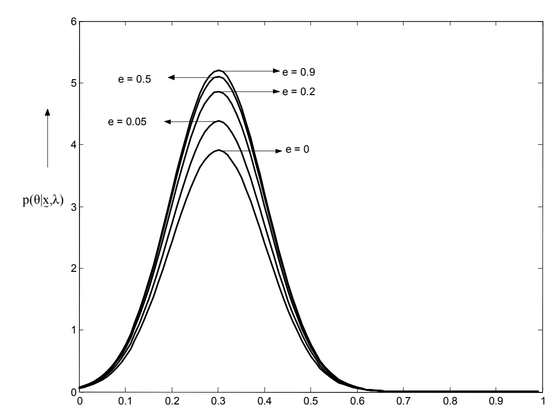


Fig.6: n = 20



ML-II Predictive for varying ε

Fig.7: n = 46

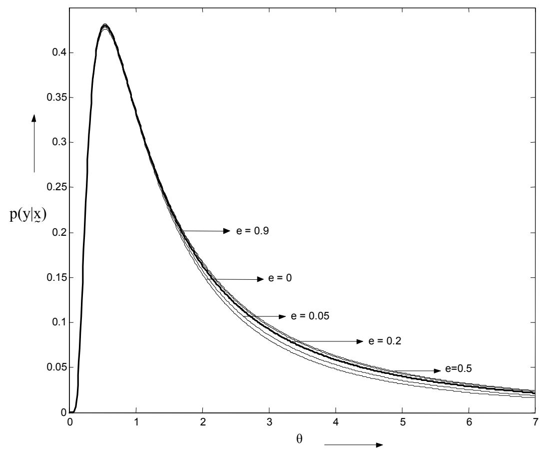


Fig.8: n = 10

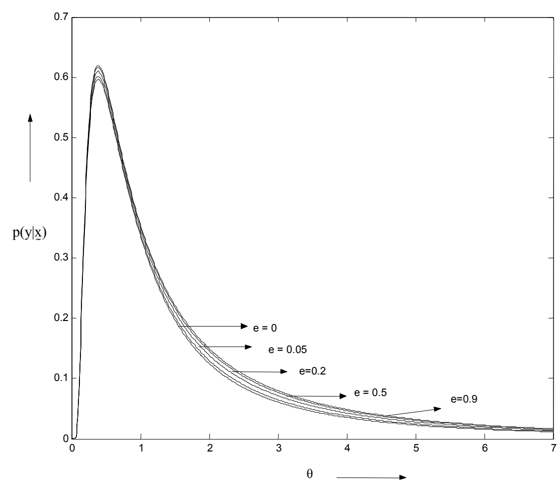
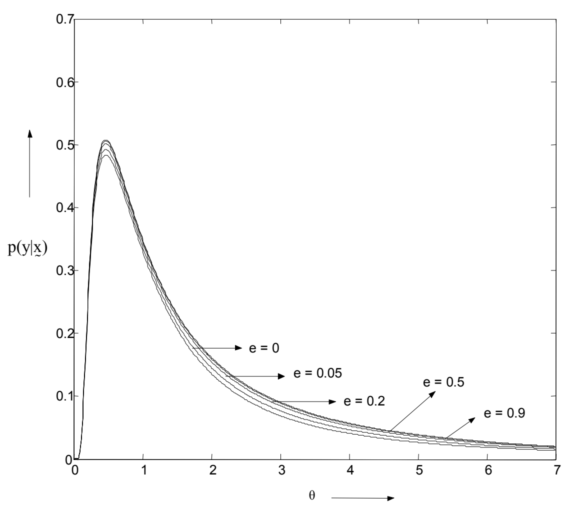


Fig.9: n = 20



ESD Prior for varying λ_3, λ_4

Fig.10: n = 46

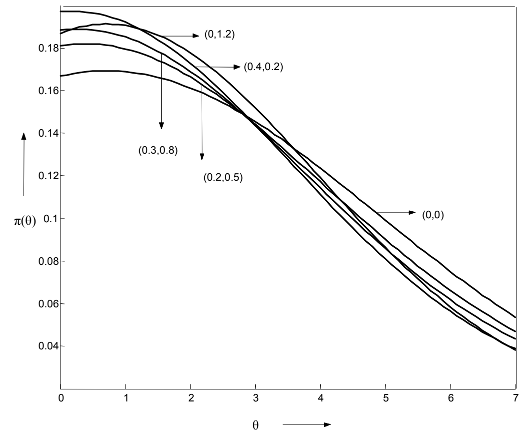


Fig.11: n = 10

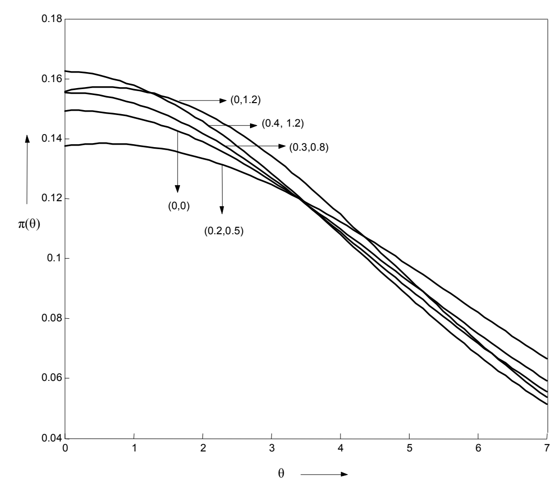
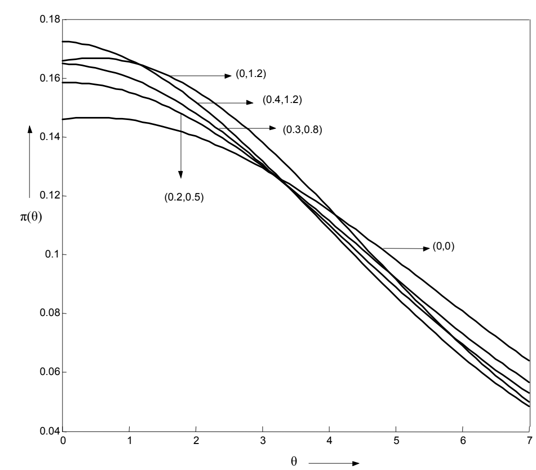


Fig.12: n = 20



ESD Posterior for varying λ_3, λ_4

Fig.13: n = 46

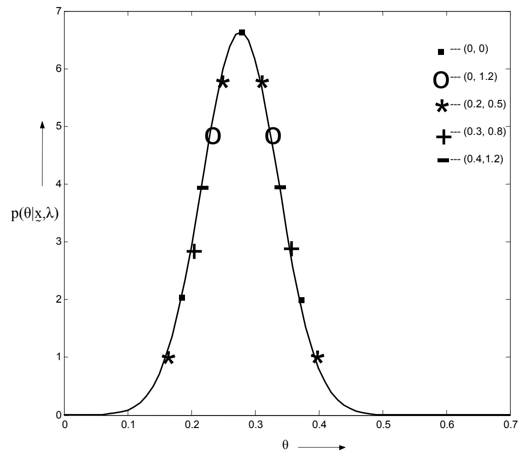


Fig.14: n = 10

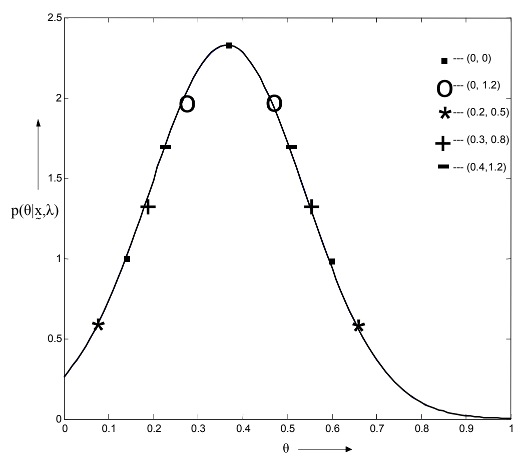
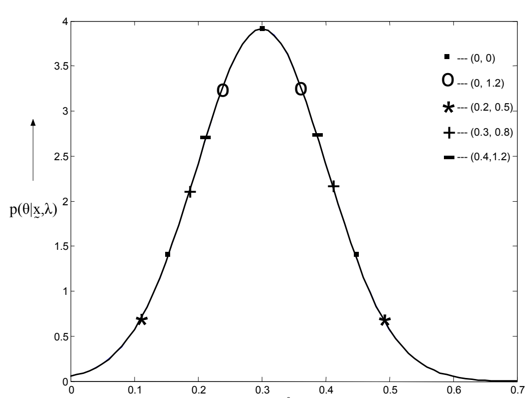


Fig.15: n = 20



ESD Predictive for varying λ_3, λ_4

Fig.16: n = 46

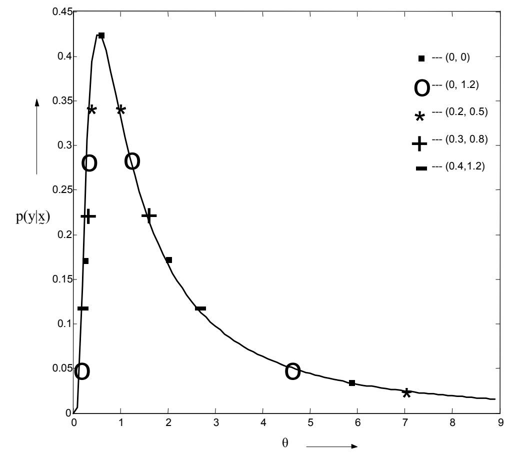


Fig.17: n = 10

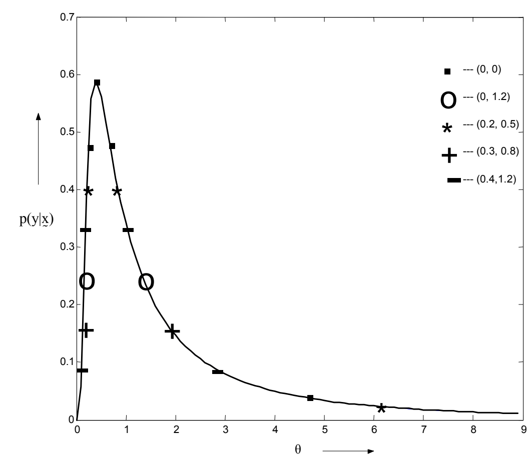


Fig.18 : n = 20