Strategic Allocation of Liquidity in the InterBank Money Market

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ABSTRACT

This paper focuses on an ex post trading problem in inter-bank money markets. An "over the counter" inter-bank market is modeled in this paper. Relationship banking leads to private proprietary information that causes bargaining failure in such markets with positive probability. Both independent and interdependent bargaining games are studied. It is shown that the allocation is not constrained efficient under bargaining games without monetary intervention. Monetary intervention is characterized as state contingent market making by the Central Bank. Such intervention is shown to dominate under a variety of informational and bargaining assumptions. The literature on monetary policy design is thus extended in the present paper by providing a micro-rationale for Central Bank intervention and by characterizing the solution of state contingent market making in liquidity.

KEY WORDS

Liquidity, Incomplete Information, Bargaining Failure, Central Bank, Monetary Intervention, Market Making

JEL Classification

E500, G 100, G 200

1. Introduction

The inter-bank money market is a cooperative arrangement that allows lending or deposit holding between banks in order to tide over short-term and long- term liquidity shortages and surpluses experienced by individual banks. Thus it works both as a risk sharing and a liquidity allocation mechanism between banks that face uncertain liquidity needs. The present paper examines trading problems in such a mechanism and role of monetary policy design to improve the allocation of liquidity.

Extending the model of Diamond and Dybvig [1] to address the issue of efficiency of the inter-bank market, Bhattacharya and Gale [2] showed that inter-bank market worked like a reciprocal insurance arrangement between banks that obviated the need to liquidate long-term illiquid assets at a heavy discount. At the same time they pointed out that the market suffered from a free rider or moral hazard problem: the liquidity insurance of the inter-bank arrangement provided an incentive to the individual banks to invest more in high return and illiquid securities and less in low yielding liquid assets so as to put the risk sharing arrangement in jeopardy. Bhattacharya and Padilla [3] show that the presence of inter-bank certificate in deposits together with a no arbitrage condition interferes with efficient term structure of interest rate and makes allocative efficiency difficult to achieve in a system of decentralized financial intermediation. Allen and Gale [4] consider the efficiency of cross regional deposit holdings by banks when the aggregate demand for liquidity is constant. They show that due to incomplete markets even small aggregate shocks manifesting in one region can lead to contagion of bank failures. Frexias and Parigi[5] study the credit risk implications of inter-bank payments problems show that there exists mechanisms which could lead to the possibility of contagion. Rochet and Tirole [6] model the systemic risk in the inter-bank market. Banks face liquidity risk as consumers are uncertain about the timing of their consumption needs. The inter-bank market fails when the failure of one bank signals that other banks have not been monitored properly causing withdrawal behavior to become correlated. Thus the Central Bank has a coordinating role and the justification of the too big to fail policy is investigated.

While it is undoubtedly true that ex ante risk sharing against bank specific liquidity shocks is the primary motive for the existence of an inter-bank market, and that the potential problems in such risk sharing exist and warrant theoretical modeling, the extant research has focused exclusively on this, neglecting another important problem. The neglected problem has to do with the potential incentive problem in trading, namely, the incentives faced by a bank with surplus in providing liquidity to a bank with deficit. Transactions in the inter-bank market are typically “over the counter” or “otc”, and this implies that trading only occurs through successful bargaining. Bargaining may fail in some cases since a liquidity-surplus bank does not know the return that the deficit bank gets from borrowing and can therefore overcharge. The probability of bargaining failure is increased by the fact that banks may be competitors in the same credit market. Mallick [7] discusses this in the context of an isolated bilateral bargaining game and extends it to a context of repeated games to show that in a dynamic context there may be too much liquidity in the market rather than too little. However, it is an open question as to what happens under more complex games with more than two banks, different information structures and different degrees of interdependence between the bargaining games. Without examining these possibilities and their implications it may be premature to judge the "interbank market" since the "market outcome" caused by the different bargaining games going on simultaneously may or may not simply be an amplifying picture of a "single bargaining game". To move from the concept of a single "bargaining failure" to that of a complete taxonomy of "market failure" is the first objective of this paper. The next objective is to identify the nature of the "superior" liquidity allocation mechanism both under a situation where a single bargaining game is a good representation of the market as well as where it need not be. The literature on bargaining and competition (see Osborne and Rubinstein [8]) is relevant this context. Typically bargaining under incomplete information leads to too little trade on average if valuation distributions do not have disjoint supports. In such cases there does not exist any social choice mechanism which is ex post efficient. However, if the supports are disjoint there still may be bargaining failure but there may be a role for policy intervention. The assumption of disjoint support in the context of banking is a useful approximation to start with. Our assumption of disjoint support essentially stems from the empirics of banking industries. The first important observation is that mergers take place if two banks are sufficiently similar in terms of returns, assets, costs and there exist synergies (captured by the maximum distance between their supports) between themselves. Such mergers "eat away" the overlapping domains. Thus banks which are "clustered in a small neighborhood" are good candidates for mergers". The second supporting fact is that banks which are "stand alone" are "outliers" compared to their right hand as well as their left hand sides. The third and very important fact is that efficiency ranking of banks are often quite unambiguous in the sense that the lowest possible return for a more efficient bank usually dominates the upper bound on the return of a less efficient bank (here we are talking about net returns on assets).

2. Two Bargaining Games

We consider two different scenarios. In the first one, individual bargaining games are independent of each other and the Central Bank is informationally constrained. Despite that we show the superiority of monetary intervention. In the second, bargaining games are interdependent and there is differential information. In the second case also intervention yields superior liquidity allocation ex post.

2A. Independent Bargaining and Constrained Inefficiency

We consider a model with three banks (the analysis can be extended straightforwardly to the general n bank case). Initially, in state 0, three clients are matched to each bank where each client needs θ units of liquidity for project finance. The initial matching is assumed to be optimal for clients. In other words, the clients are matched with banks in a way such that the value of the relationship to the client is maximized. Let the maximum value of such relationship to each client be V. If a client is compelled to switch their projects across banks because the client's original bank has liquidity constraints, then the value for the client falls to v where V is greater than v.

Return to bank i (i =1-3) from the client-project it is originally matched with is R_i. It is private information to bank i only. To others it is known (with common knowledge) that it has cumulative distribution function $F_i(R_i)$ over the interval $[R_{imin}, R_{imax}]$. It is assumed that F_i is twice continuously differentiable for all i. The assumption of private information and relationship maximization is in line with the empirical and theoretical foundations of banking: namely, opacity of bank assets due to proprietary information and value maximizing relationships in banking.

In the model, subsequent to state 0, there is a purely distributive liquidity shock to the system such that the aggregate liquidity is unchanged but individual bank specific shocks are realized that are negatively correlated. The state of liquidity is common knowledge. Table 1 below illustrates[1].

Insert Table 1 here

We consider a market setting where simultaneous bargaining takes place. Each bank with a surplus offers a price at which it is willing to lend and the bank with shortage agrees or

[1] We consider a distribution of shocks where there is only one bank with deficit in any state of nature. It is shown that bargaining problems arise even with one deficit bank. Therefore, it is obvious that problems would be greater when there are many banks with deficits. Keeping this in mind the simplest shock structure is considered for analytical simplicity.

disagrees. In state i (denoted by L_i), bank i is in need of additional funds 2θ. If it is state 1, then bank 1 goes to borrow from bank 2 and bank 3 (Similarly for bank 2 in state 2 and bank 3 in state 3). Bargaining takes place as follows: the bank with surplus funds asks for an interest rate - in case of L_1, bank 2 offers a loan with an interest rate r_{21} to bank 1 and bank 3 offers a loan with an interest rate r_{31} to bank 1, and it can be accepted or rejected with no further bargaining taking place. If bargaining fails then one project has to be released by the bank with liquidity shortage and the client switches over to the bank with surplus liquidity. *Banks with surplus funds cannot collude or interact in any other way.* Therefore each bargaining game is independent of other games. Later we shall relax the assumption of independence and examine the consequences.

To complete the description of the model, we need the following definition and assumptions:

Definition 1: If client of bank i switches over to bank j, then return to bank j is R_{ij} where $i,j = 1\text{-}3$ and $i \neq j$

Assumption 1:

a) $R_{1min} > R_{12} > R_{13} > R_{2max} > R_{3max}$

b) $R_{21} > R_{2max}$, $R_{31} > R_{3max}$ and

$R_{i1} - R_{imax} > V - v$ for all $i = 2,3$

c) $R_{2min} > R_{23} > R_{3max}$

d) $R_{31} > R_{32} > R_{3max}$ and

$R_{32} - R_{3max} > V - v$

e) $1 / F'_i\{ (R_{imin} \} > (R_{imin} - R_{ij})$ where $i < j$

f) $F''_i \geq 0$

Assumption (a) says that if client of bank 1 switches her project to bank 2 or 3, the return to the banking system from that client falls. Assumption (b) says that if client of bank 2 or bank 3 switches over to bank 1, then the return to the banking system from that client increases. Further, it says that the gain to the banking system surplus due to such switching is greater than the loss to the client from lower project return. Assumption (c) says that if client of bank 2 switches to bank 3, the return to the banking system from that client falls. Assumption d) says that the gain to the banking system surplus due to client switching from bank 3 to bank 2 is greater than the loss to the client. Assumption (e) guarantees positive probability of bargaining failure while assumption (f) is a necessary second order condition.

Assumption 2 : It is further assumed that each bank has a capacity of $4\theta^2$.

Now let us consider the case without intervention. It is described as the following game of incomplete information : *Game without Intervention (GWOI)*

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[2] The above assumption reflects the empirical truth that banks operate within given overheads and with a fixed operational and management skills which cannot take business loads beyond a point without further restructuring or expansion. We consider such changes beyond the time frames of our discussion. The assumption is basically about a kind of diminishing returns in banking that allows the coexistent of different banks at the social optimum. Without such a capacity constraint, efficiency would dictate that only the most efficient bank be allowed to survive. In that case one would not only have to trade off monopoly distortion with the efficiency gains from allowing one bank to become big, but also face the fact that there would not be any inter-bank market for risk sharing in the first place.

At t = 0 banks and clients are matched

At t = 1 the liquidity shocks are realized

At t = 2 bargaining takes place

At t = 3 client switching takes place if any

The measure of social welfare gain or loss in this context is taken into account by adding the changes in banking system surplus and client profits. The first proposition that we get is basically that a more efficient bank will essentially refuse to lend to a less efficient one. This creates efficiency gains since allocation of liquidity becomes more concentrated on the relatively efficient banks.

Proposition 1 : (a) Suppose that the relatively inefficient bank has liquidity shortage. There does not exist any rate of interest at which the efficient bank will agree to lend which is agreeable to the inefficient bank. (b) Moreover this bargaining failure is also optimal. [Consider state 3: bargaining failure will result in efficient switching of clients.]

Proof : (a) Let i be the efficient and j be the relatively inefficient bank. Suppose there is a rate of interest at which at which there is mutual gains from trade. Now the maximum interest that bank j can pay is R_{jmax} ,but the gain to bank i from not lending is $R_{ji} > R_{jmax}$. Therefore, bank i will always disagree to lend at an interest that is profitable for bank j. Thus we arrive at a contradiction.

(b) The social gain from not lending is = $R_{ji} – R_j$ - (V – v) > $R_{ji} – R_{jmax}$ - (V – v) > 0 since $R_{ji} – R_{jmax}$ > (V – v) by assumption. Q.E.D.

But unfortunately, bargaining failure need not always be optimal. There is also the result that there can be bargaining failure between banks i and j when bank i ($i < j$) needs liquidity and is willing to pay a reasonable price. By assumption, this type of failure is inefficient. Moreover, gains from trade exist since $R_{imin} > R_{ij}$. However, trade may not take place. If the optimal value $r^*_{ji} > R_i$ then trade fails. To see that there is a positive probability of this happening, it has to be shown that $r^*_{21} > R_{1min}$. The next proposition shows this.

Proposition 2: Suppose the efficient bank has liquidity shortage. Bargaining may fail even though agreement and trade is always the efficient outcome.

Proof : For Bank j the problem is:

$$\max\ (r_{ji} - r_d)[1 - F_i\{ (r_{ji} \}] + (R_{ij} - r_d)F_i\{ (r_{ji} \}] \tag{1.1}$$

w.r.t r_{ji} (where r_d is the unit cost of funds to banks.)

$$\text{s.t. } r_{ji} \leq R_{imax} \tag{1.2}$$

$$r_{ji} \geq R_{imin} \tag{1.3}$$

The Kuhn-Tucker conditions are:

$$- (r^*_{ji} - r_d) F'_i\{ (r^*_{ji} \} + [1 - F_i\{ (r^*_{ji} \}] + (R_{ij} - r_d)[F_i'\{ (r^*_{ji} \}] \leq 0 ,$$

$$r^*_{ji} = R_{1min} \tag{1.4}$$

$$\text{or } - (r^*_{ji} - r_d) F'_i\{ (r^*_{ji} \} + [1 - F_i\{ (r^*_{ji} \}] + (R_{ij} - r_d)[F_i'\{ (r^*_{ji} \}] \geq 0 ,$$

$$r^*_{ji} = R_{1max} \tag{1.5}$$

$$\text{or } - (r^*_{ji} - r_d) F'_i\{ (r^*_{ji} \} + [1 - F_i\{ (r^*_{ji} \}] + (R_{ij} - r_d)[F_i'\{ (r^*_{ji} \}] = 0 ,$$

$$R_{1max} \geq r^*_{ji} \geq R_{1min} \tag{1.6}$$

Note that the second order condition is:

$- (r^*_{ji} - r_d) F''_i\{ (r^*_{ji})\} - 2F'_i\{ (r^*_{ji})] + (R_{ij} - r_d)[F_i''\{ (r^*_{ji})] < 0$ or $- (r^*_{ji} - R_{ij}) F''_i\{ (r^*_{ji}) - 2F'_i\{ (r^*_{21}\}] < 0$

The second order condition is satisfied given $F''_i \geq 0$ from assumption 1.(d).

It follows from the first order Kuhn-Tucker conditions that the required condition is the following:

$- (R_{imin} - R_{ij}) F'_i\{ (R_{imin}\} + [1 - F_i\{ (R_{imin}\}] > 0$

or $[1 - F_i\{ (R_{imin}\}] / F'_i\{ (R_{imin}\} > (R_{imin} - R_{ij})$

or $1 / F'i\{ (R_{imin}\} > (R_{imin} - R_{ij})$

Given assumption 3.1.(c) the required condition is satisfied. In that case $r^*_{21} > R_{1min}$ and it is then possible that bargaining may fail because there is a positive probability $r^*_{21} > R_1$.

Q.E.D.

As the preceding proposition illustrates, Social Welfare may not be maximized in this kind of bargaining setup due to incomplete information. Table below gives a complete taxonomy.

Insert Table 2 here

Note however, that we cannot characterize this situation with many banks as a market failure unless we can demonstrate, that a regulator with the same incomplete information about project returns as the market, could actually do better. Therefore, we need to compare the outcome of the game without intervention with that of intervention.

The game with intervention is described in following way: *Game with Intervention (GWI)*

At t = 0 banks and clients are matched and the Central Bank announces the mechanism of monetary intervention

At t = 1 the liquidity shocks are realized

At t = 2 bargaining takes place

At t = 3 Central bank "intervenes" with the pre-announced mechanism with probability p

At t = 4 client switching takes place if any

What is monetary intervention? – It is defined in this context as a stochastic mechanism: at t = 0 the Central bank makes a credible commitment to intervene in a State Contingent and Transaction Specific manner at time t=3. The probability (exogenous) of intervention happening in the prescribed way is p. This is because the Central Bank has other goals that may dominate and sometimes come in conflict with its goals of efficient liquidity allocation in the inter-bank market.

The state of liquidity is as already shown, denoted by L_i where i =1,2 or 3. The other state variable of interest is number of transactions. We assume that the Central Bank can observe transactions (but not the interest rates charged) in the inter-bank market through the clearinghouse system. There can be two transactions, or one or zero. So the number of transactions T = 0,1, or 2. The intervention instruments are:

(a) a borrowing rate (conditional on state and transactions) : $B_R(L_i,T)$

(b) a lending rate (conditional on State and transactions) : $L_R(L_i,T)$

The borrowing rate is the rate at which the Central bank will borrow from different banks and the lending rate is the rate at which the Central Bank will lend to banks through the discount window or through open market operations. The intervention mechanism is

$I_M = [p, \{ L_R(L_i,T), B_R(L_i,T)\}] \quad \forall (L_i,T)$

Equilibrium with Intervention is defined as follows: Stage 1 game (which is a dynamic game itself with offer and accept/reject sequence)E_1 (after liquidity shock takes place) is a Subgame Perfect *Nash Equilibrium given policy* I_M ,

$$E_1 = [(r_{ji}) , D(r_{ji})] \quad \forall r_{ji} , j \neq i, i,j = 1 \text{ to } 3 \quad \text{in } t = 2 \tag{1.7}$$

such that

$$r_{ji} \in \arg\max \prod_j \mid [F_i , I_M] \;\forall j = 1 \text{ to } 3 \tag{1.8}$$

$$D(r_{ji}) \in \arg\max \prod_i \mid [R_i , I_M] \;\forall i = 1 \text{ to } 3 \tag{1.9}$$ where

D is a binary variable representing acceptance (1) or rejection (0) of the offered price in the bargaining game between j and i

$$\text{and } I_M \in \arg\max \{ L_R(L_i,T) - B_R(L_i,T)\} \;\forall (L_i,T) \tag{1.10}$$

$$\text{Subject to } I_M \in \arg\max \sum_j \prod_j + dS(V) \tag{1.11}$$ where

dS(V) is the change in surplus to the clients from switching.

Here the policy of the Central Bank is the maximization of Bank revenue subject to achieving efficiency in liquidity allocation. This lexicographic ordering seems a reasonable assumption since the Central bank is primarily concerned with Social Welfare maximization in this context, and subject to achieving that, it will try to maximize its income from borrowing and lending activities that relaxes its budget constraints in other policy contexts. Notice that such a policy function may also help to eliminate policy indeterminacy and help to identify the true equilibrium in the policy game. Also note that time consistency of policy is taken care of by the requirement of subgame perfectness of equilibrium. This leads to the next proposition:

Proposition1.1.3: A Complete characterization of Optimal Central bank Policy is as follows:

State 1: Partial Failure: ($L_R(L_1,1) = [R_{1min}]$, $B_R(L_1,1) = (1 / 3\theta)[R_{13} . \theta + R_{3max} .2\theta]$)

State 1: Complete Failure: ($L_R(L_1,0) = [R_{1min}]$, $B_R(L_1,0) = (1 / 3\theta) [R_{13} . \theta + R_{3max} .2\theta]$)

State 2: Complete Failure: ($L_R(L_2,0) = [R_{2min}]$, $B_R(L_2,0) = (1 / 3\theta)[R_{23} . \theta + R_{3max} .2\theta]$)

State 3: Complete Failure: ($L_R \in \Phi$, $B_R \in \Phi$)

Proof **:** From any given distribution of liquidity, the following is the Ex post optimal one :

Insert Table 3 here

The most efficient bank 1, should work in full capacity always since, by assumption 3.2, $R_{i1} - R_{imax} > V - v$, for all i =2,3 which means that it always increases social welfare to transfer resource from an inefficient bank to a more efficient bank. So 4θ units should thus go to bank 1 and we have 5θ more units to remain to be allocated among bank 2 and bank 3. But bank 2 is more efficient than bank 3 and by assumption 3.2, we have $R_{32} - R_{3max} > V - v$, which implies that bank 2 should be allocated 4θ units and allowed to work in full capacity. Thus the remaining θ unit should go to bank 3. Thus starting from any distribution that is not optimal, the Central bank's task is to ensure that funds are allocated in such way that it continues to borrow from relatively inefficient banks and lend them on to more efficient ones until the optimal allocation is reached. To make this feasible, the participation constraints of borrowing and lending banks have to be met.

In State 1 there are two possibilities that can require intervention: Partial Failure or Complete Failure. To achieve the optimal allocation when it is a complete failure, the surplus funds of bank 3 should be transferred to bank 1. Further, since bank 1 has unutilized capacity even after the transfer, more funds should be transferred until capacity is fully utilized. From the discussion above it is clear that it is optimal to transfer the necessary liquidity not from bank 2 but bank 3. Note that although borrower clients suffer due to breakdown in relationships, total surplus is greater since efficient banks do more business now: $R_{31} – R_{3max} > V – v$. Therefore under partial and complete failure it is only required to transfer 2θ and 3θ, respectively, from bank 3 to bank 1. Further, this should be done in a way that meets the participation constraints of both but such that it does not meet the participation constraint of bank 2. The Central Bank Net Revenue Maximizing solution is the following Open Market package:

(L_R , B_R) argmax. Є Max. $3\theta\ [L_R - B_R]$

s.t.

$L_R \le R_1$ (1.12)

$L_R > R_j$ (1.13)

$R_{13} . \theta + R_{3max} .2\theta \le B_R . 3\theta$ (1.14)

$R_{12} . \theta + R_{2min} .2\theta > B_R . 3\theta$ (1.15)

It is easy to see that at the optimum, constraints (1.12) and (1.14) will bind and the other two constraints are satisfied. So the optimal open market package under State 1 is:

Partial Failure: $(L_R(L_1,1) = [R_{1min}]\ ,\ B_R(L_1,1) = (1 / 3\theta)[R_{13}\,.\,\theta\ +\ R_{3max}\,.2\theta]\ )$

Complete Failure: $(L_R(L_1,0) =\ [R_{1min}]\ ,\ B_R(L_1,0)\ = (1 / 3\theta)\ [R_{13}\,.\,\theta\ +\ R_{3max}\,.2\theta\ ]\ )$

In State 2 intervention is required if the offer of bank 3 to bank 2 is not accepted. If accepted, 3 units will be borrowed by bank 2 from bank 3. This implies that two client-projects will have to switch from bank 3 to bank 2. If not accepted, the optimal Open Market package which satisfies the Ex post (after bargaining failure) incentive constraint of bank 3 is the following Central Bank Net Revenue Maximizing Open Market package:

(L_R, B_R) argmax. € Max. $3\theta\ [L_R - B_R]$ s.t.

$L_R \leq R_2$ (1.16)

$L_R > R_3$ (1.17)

$R_{23}\,.\,\theta\ +\ R_{3max}\,.2\theta\ \leq\ B_R\,.\,3\theta$ (1.18)

$R_{21}\,.\,\theta\ +\ R_{1min}\,.2\theta\ >\ B_R\,.\,3\theta$ (1.19)

Note that from constraints (1.16) and (1.17), it is made sure that the Central Bank lending rate is too high for bank 3 to make profit from it (bank 1 is at full capacity and will not ask for any loan), but, bank 2 can take a loan at this rate and almost always make a positive profit. Constraint (1.18) ensures that bank 3 will always find it incentive compatible to accept the borrowing rate of the Central Bank. On the right hand side of this constraint is shown the revenue from accepting the Central Bank rate and on the left hand side the maximum revenue from not accepting the rate is shown (costs are same on both sides and therefore irrelevant). Constraint (1.19) ensures that bank 1 will never find it incentive compatible to accept the borrowing rate of the Central Bank. On the right hand side the revenue from accepting the borrowing rate is shown and the left hand side shows the minimum revenue from not accepting.

It is easy to see that at the optimum, constraints (1.16) and (1.18) will bind and other two constraints are satisfied. So the optimal open market package under State 2 is :

$(L_R(L_2,0) = [R_{2min}]\ ,\ B_R(L_2,0) = (1 / 3\theta)\ [R_{23}\ .\ \theta\ +\ R_{3max}\ .2\theta]\ )$

In State 3 the optimal allocation is achieved by the distribution of shocks and no intervention is optimal. Q.E.D.

Section 2B. Interdependent Bargaining and Differential Information

In this section, we change some of the previous assumptions. First, we assume that the Central bank knows the returns from client projects with perfect certainty but does not know the state of liquidity. Now assume that each bank has one client who requires 3θ units of cash for financing its project. If the client gets 2θ units of cash it will not switch. In a sense, we are using an assumption about stronger relationship orientation in banking. Further, we assume that liquidity distribution is slightly different in the sense that banks have committed funds. This ensures that banks with surplus can trade the funds they have received by liquidity shocks only and nothing else. If a fund is committed beforehand to an existing client, then that fund cannot be sold through bargaining to a bank with shortage. Now consider the following liquidity distribution given in the table below.

Insert Table 4 here

Bargaining takes place in two rounds. Each bank with a surplus offers a price at which it is willing to lend and the bank with shortage agrees or disagrees. If both bargaining succeeds or fails then there is no further bargaining. But if one surplus bank is able to sell funds, while another is not, then the former goes to borrow from the latter in order to re-lend to the shortage bank at the original price.

The client switching assumption implies that either there is complete bargaining failure in which case (provided there is no policy intervention) client will switch to other banks or, there is no failure. The latter means that since a client will not switch in case of partial failure, the bank with surplus with whom bargaining failed in the first round will find it's extra funds lying idle and lend to the other surplus bank. We assume there is an outside rate of return for idle funds that will be applicable in the determination of the second round of bargaining.

The objective function of bank 2 in state 1 is :

$\Pi_2 = \theta\ (r_{21} - r_d)[1\text{-}F(r_{21})][1\text{-}F(r_{31})] + \theta\ (r_a - r_d)[F(r_{21})][1\text{-}F(r_{31})] + \theta\ (2r_{21} - r_d - r_a)[F(r_{31})][1\text{-}F(r_{21})]$

$+ \theta\ (r_{12} - r_d)[F(r_{21})][F(r_{31})](1\text{-}p) + \theta\ (r_a - r_d)[F(r_{21})][F(r_{31})](p)$

The first term shows what happens when both the first round bargaining games (between 1 and 2 and 1 and 3) succeed. The second term shows that when bank 2's ask price is rejected and that of bank 3 is accepted, bank 2 gives the surplus fund to bank 3 in the second round of bargaining. Since the outside option for bank 2 is r_a therefore the bank 3 only pays that amount. The third term shows that when bank 3's ask price is rejected and that of bank 2 is accepted, bank 3 gives the surplus fund to bank 2 in the second round of bargaining The fourth term shows the case when there is complete failure in the first round and the Central Bank cannot intervene, client switching takes place. Final term shows that in the event of complete bargaining failure, and Central Bank intervention, the optimal borrowing rate for the Central Bank is simply r_a. Similarly, one can set up the objective function for bank 3. Due to the possibility of client switching from one lender to another lender, the probability of bargaining failure is smaller if the outside option is not too attractive. However, with a low enough outside option, complete bargaining failure is a possibility if p is small, leading to inefficient client switching taking place in state 1 and 2. The question that arises is what is the optimal Central Bank policy under such circumstances?

The Optimal Monetary Policy is such that 2θ units of liquidity is allocated to bank 1 through a contract that only bank 1 will accept in state 1, θ units of liquidity is allocated to bank 2 that only bank 2 will accept in state 2 and nothing is offered to bank 3. Note that, if the same amount of liquidity is offered to both banks 1 and 2, then one bank will mimic the other. Essentially, we face the following mechanism design problem:

$$\text{Max } (R_{1m} - r_a)2\theta + (R_{2m} - r_a)1\theta$$

With respect to (R_{1m}) , (R_{2m}).

Subject to

$(R_1 - R_{1m})2\theta \geq 0$ (3.1)

$(R_1 - R_{1m})2\theta \geq (R_1 - R_{2m})1\theta$ (3.2)

$(R_2 - R_{2m})1\theta \geq 0$ (3.3)

$(R_2 - R_{2m})1\theta \geq (R_2 - R_{1m})2\theta$ (3.4)

Solution: The incentive constraint for bank 1 binds and the participation constraint for bank 2 binds. Therefore, we get the following proposition:

Proposition 3.3.3.1 : The optimal lending rates are as follows : $R_{1m} = (R_1 + R_2) / 2$ and $R_{2m} = R_2$

Bank 1 is charged a price that bank 2 will not be able to afford, and it is efficient to lend more at that price to bank 1 while bank 2 gets a smaller package. This monetary mechanism can be implemented through differential pricing scheme in discount window lending or through open market operations involving nonlinear pricing of bonds.

3. Conclusion

The essential point about the present paper is that liquidity allocation will not be efficient and intervention by the Central Bank by market making in liquidity can improve the allocation. Due to incomplete information, bargaining may fail even if gains to trade exist. Naturally, the question arises as to what kind of intervention is needed which can improve social welfare defined as the sum of profits in the banking system and the profit from client-projects. When we consider constrained efficiency with independent bargaining we see that efficiency can be restored through a lending and borrowing package suitably designed to meet incentive compatibility and participation constraints of the lenders and borrowers. With interdependent bargaining, there is either full trade or no trade. When Central Bank knows the returns but not liquidity (differential information) again the first best can be attained by a mechanism design problem.

Thus the Central Bank efficiency persists in a variety of circumstances and it is quite a significant and reassuring result. The results suggest a micro-rationale behind monetary policy design: to reallocate liquidity to increase social welfare. However, it will be instructive to extend the present model to more complex scenarios with repeated interaction among heterogenous banks.

Tables

Table 1: Liquidity position of the banking system without aggregate shocks

	Bank 1	Bank 2	Bank 3
State 0 = L_0	3θ	3θ	3θ
State 1 = L_1	θ	4θ	4θ
State 2 = L_2	4θ	θ	4θ
State 3 = L_3	4θ	4θ	θ

Table 2: Efficiency Properties of Likely Outcomes

	Efficiency Properties of Likely Outcomes		
State 1	No Bargaining Failure (efficient)	Partial Bargaining Failure (inefficient)	Complete Bargaining Failure (inefficient)
State 2	No Bargaining Failure (impossible)	Partial Bargaining Failure (efficient)	Complete Bargaining Failure (inefficient)
State 3	No Bargaining Failure (impossible)	Partial Bargaining Failure (impossible)	Complete Bargaining Failure (efficient)

Table 3: Optimal Liquidity Distribution

	Bank 1	Bank 2	Bank 3
Liquidity	4θ	4θ	θ

Table 4: Deficits or surpluses with respect to committed funds

	Bank 1	Bank 2	Bank 3
State O	0	0	0
State 1	- 2θ	+ 1θ	+ 1θ
State 2	+ 1θ	- 2θ	+ 1θ
State 3	- 2θ	+ 1θ	- 2θ

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