Two-Sided Market with Spillover - Modeling a City

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Two-Sided Market with Spillover: Modeling a City *

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Abstract

The paper explores the analogy between city and two-sided market. It generalizes the results on the pricing strategies of the platform in the two-sided markets for the case when concentration spillover plays an important role. The two-sided market framework is applied to model a city. The paper highlights the importance of the network effect and labor market structure for city size, governance and agglomeration formation. The cases of an isolated city and competing cities are considered.

Keywords: Two-sided markets, Industrial organization, Urban economics, Concentration spillover, City, Labor matching market.

JEL Classification Numbers: D42, D43, H71, L12, L13, R12, R30

1 Introduction

The paper explores analogy between city and two-sided market and develops a model of city management.

The contribution of the paper is two-fold. On the one hand, it generalizes some of the results on the pricing strategies of the platform in two-sided market, obtained by Rochet and Tirole [2004] and Armstrong [2006]. The model of two-sided market, developed in this paper, is applied to the analysis of city management.

On the other hand, the paper contributes to the urban economics literature as it highlights the importance of the multi-sided nature of the city, whereas urban economics typically concentrates on the production side of a city.

I consider the monopolistic platform and the competing platforms cases, or, in words of urban economics, an isolated city and a city, competing for the firms and the citizens with another city[1]. The earnings of the firms and the citizens form a tax base, which is an important objective for the city manager.

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[1]The world "city" is used here in a broad context. It can be understood as a region or even as a country.

A particular attention is paid to the concentration spillover and labor matching market structure. From the two-sided market prospective, they determine the structure of the benefits of the platform users and the particular form of interaction on the platform. I show that they have an important influence on the size of the cities, urban agglomeration and taxation strategy of the city manager.

The particularities of the interaction between firms and workers-citizens[2] motivates the generalization of the benchmark models of two-sided market.

I assume that the city manager is aimed at maximizing the tax revenues in the city. This objective is closely related to the political power of the city manager. For instance, the large tax revenues collected in a city or a region can serve as an important argument in political bargaining with higher level authorities as it can be used as an evidence of importance of the city (region) for the national economy. The cite manager can also be interested in increasing the tax revenues because his earnings from managing the city can be directly related to the collected taxes.

For the case of a monopoly platform, which corresponds to an isolated city, I analyze, first, the case of the active city management, i.e. when the city manager acts as a platform in two-sided market, "actively" changing the taxes (access fees for the platform users). Second, I consider the passive city management (free entry, or self-organization of a city), when a site for forming a city (platform) serves merely as a focal point for agglomeration of economic activity. The two cases are compared to the social optimum.

For the case of an isolated city under active city management, I show that the the taxes are set at the level which leads to the suboptimal city size (in terms of population and the level of production). I characterize the tax level which implements the optimal size of the city. I also show that under free entry, the city size will be distorted compared to the optimum, but the direction of the distortion depends on the determinants of the benefits of the firms and the citizens from connecting to the city.

For the case of competing platforms (cities), I characterize the taxation strategies and analyze the sustainability of the configuration with competing cities rather than one agglomerated city.

I also show the importance of the labor market structure for the stability of the competing cities configuration and the taxation strategies of the city manager.

The model of the paper is based on the main approaches in modeling the two-sided markets and follows the standard framework in the urban economic literature.

The survey Fujita and Thisse [1996] pointed out that the main factors of concentration which lead to city creation: externalities related to knowledge spillover, increasing returns (they authors link it to the increasing variety) and spatial competition.

The idea that the spillover effects play an important role for the city formation and the production in the city is well established in the literature. Often the spillover is related to knowledge and ideas creation, accumulation and diffusion. Jacobs [1969] argues that knowledge generation plays an important role. The importance of knowledge accumulation is highlighted in Romer [1986]. The

[2]This can be generalized to the interaction on others markets for the production inputs

model based on knowledge and ideas diffusion is developed in Jovanovic and Rob [1989]. In general, the role of information spillover is discussed in Fujita and Ogawa [2005], Imai [2005]. The "creative" side of the cities (as opposed to the production) is becoming more and more important, as argued in Florida [2003].

Another important component of the city is labor matching market. The importance of the network effects related to the matching are discussed in Duranton and Puga [2003]. The literature points that not only the chances for matching but also quality of matching influences firms' production - see the surveys of Mortensen and Pissarides [1999] and Petrongolo and Pissarides [2001].

Finally, spatial aspects are important. Hotelling [1929] and Lerner and Singer [1937] have shown that space competition for the market areas leads to firms concentration[3]. At the same time, price competition is shown to be centrifugal force.

Glaeser et al. [2001] argues that the consumption side of the city is important together with the production side. Despite traditional viewpoint that cities create advantages for production and disadvantages in consumption, they demonstrate that attractiveness of the cities for consumers became more important especially when firms are highly mobile.

Put generally, the size of the city is determined through interaction between centrifugal and centripetal forces on both sides of a city - production and consumption.

The paper proceeds as follows. In section 2 I describe the model and present a motivating example which is used throughout the paper for illustrations of the general results. In section 3 I consider the case of an isolated city which corresponds to the monopolistic platform. In section 4 I consider the case of the two competing cities (platforms). Section 5 concludes.

2 Model setup

2.1 A Model Framework

Every city comprises firms and citizens. To start production, the firms need some infrastructure provided by the city, such as public facilities, labor (matching) market, institutions etc. The better quality of such kinds of infrastructure leads to higher production. I will call this infrastructure as production amenities.

The citizens need the city infrastructure as well to find a job, go for shopping and finally enjoy consumption. The citizens benefit from better quality of such kinds of infrastructure. It will be referred to as consumption-oriented amenities.

Firms and citizens can use the same pieces of infrastructure, e.g. labor market inside the city, water and electricity supply or routes. However, most of amenities are usage-specific, i.e. they are either production or consumption-oriented. For example, industrial power supply is used in production only; parks, entertainment facilities etc. are the consumption amenities. In the model, I assume that the two sorts of amenities don't overlap.

I denote by A the city amenities related to production, and by B the city consumption-oriented amenities.

[3]the famous result is that the unique Nash equilibrium of the Hotelling two ice-cream sellers problem is location at the middle of the Hotelling line

The city is governed by City Manager. In the model the city is considered as a platform, firms and citizens - as platform users.

Suppose that there is a population of firms and population of (potential) citizens which may (or may not) be located in the city. Let n_1 be the number of firms located in the city and n_2 be the number of citizens.

I adopt the standard assumption of the urban economics that the citizens commute from their living place inside the city to the working place (the Central Business District model).

Denote by φ_1 the profit of a firm located in a city, and by φ_2 the utility derived from being in the city for a citizen. They represent the benefits of the two sides of the platform from joining it. The benefits depend on the number of firms and citizens n_1, n_2 as well as on the amenities A, B provided in the city.

The City Manager decides on lump-sum taxes (entry fees) T_1 and T_2, imposed on firms and citizens correspondingly[4]. The taxes are not restricted to be positive. In terms of Industrial Organization of the two-sided markets, these are entry fees. Amenities A and B are considered as exogenously given

The objective of the city Manager is to maximize the overall tax revenues[5]. Such objective can reflect the fact that the City Manager cares about his political power - the more taxes is collected, the higher is the influence of the City Manager inside the city and outside it, for instance, in a region or in a country. For example, the big sum of the tax revenues collected in the city can serve as an important argument in the political bargaining with the higher level authorities in the federal state. Alternatively, the City Manager can be interested in maximizing the tax revenues because his earnings from performing the managing job depends on the tax revenues in the city.

2.2 A Motivating Example

In this part of the paper I develop a stylized model of a city following the standard approach of urban economics, see, e.g. Black and Henderson [1999] and consider its variants.

I will argue that the cornerstone models of the two-sided market literature of Rochet and Tirole [2004] and Armstrong [2006] are too stylized to capture the important particularities of the city. Because of this, I will consider a more general model of two-sided market and generalize the results on the pricing strategies of the platform, obtained in the above-mentioned papers.

The model will also be used to motivate the assumptions on the functions, describing the benefits of the platform users (see subsection 2.3). It will also be used throughout the paper for illustrations of the general results.

Assume that there is a city with n_1 firms and n_2 citizens (workers). All production occurs in the center of the city[6]. The firms don't have commuting cost whereas the citizens do.

Let labor be the only input in production and the firm's production function is given by

$$y = EL^{\delta}$$

[4]The tax levels can be considered as deviations from the "benchmark" tax (say, determined by higher level authority). So, taxes can in fact be tax deductions or subsidies to the firms or other benefits such as preferential terms in land allocation, public infrastructure use etc.

[5]Different objectives may also be considered

[6]So, there is CBD - Central Business District.

where $\delta < 1$ and E is total factor productivity.

There is a local industry-specific externality which increases the total factor productivity E when there are more workers located in the city[7]:

$$E = An_2^{\varepsilon} \tag{1}$$

where A is city production-oriented amenities. However, E is considered as constant by each firm because firms are small.

The agglomeration effect, modeled here by (1) is an important driving force for agglomeration of the production activity in the cities. With this modeling, it is assumed that there are no increasing returns at the firm level. However, increasing returns emerge as a local externality due to concentration of production activity. The local externality can be attributed to knowledge spillover.

The price for firms' production is normalized to 1. The firm's profit is then

$$\pi = EL^{\delta} - wL$$

Each firm's labor demand is then given by

$$L^D(w) = \left(\frac{E\delta}{w}\right)^{\frac{1}{1-\delta}}$$

and labor demand in the city is $n_1 L^D$.

Labor supply in the city is n_2.

Consider now the two variants of the model. First, assume that labor market is perfectly competitive. Equilibrium in the labor market gives

$$w = E\delta \left(\frac{n_1}{n_2}\right)^{1-\delta}$$

The resulting firm's profit is equal to

$$\varphi_1(n_1, n_2) = (1-\delta)An_1^{-\delta}n_2^{\varepsilon+\delta} \tag{2}$$

The workers (citizens) get utility from consumption and have to bear commuting cost.

The basic urban economic model (see, e.g.Black and Henderson [1999]) assumes that the city consists of the Central Business District (CBD), in which all production occurs, surrounded by the land, occupied by the citizens. The citizens commute to the CBD. In this model, in the city, populated by n_2 citizens, requiring 1 unit of land each, the total commuting costis equ al to $TCC = bn_2^{3/2}$, where $b = \frac{2}{3}\pi^{-1/2}\tau$, and τ is commuting cost per unit of length. Then, the average commuting cost is equal to $ACC = bn_2^{1/2}$.

The land rent can be easily accommodated in this setting, since the total land rent is also proportional to $n_2^{3/2}$, more precisely, $TR = (b/2)n_2^{3/2}$. So, the average land rent is $(b/2)n_2^{1/2}$. This means that taking into account the land

[7]Henderson and Becker [2000] assumed that the spillover is determined by the number of firms: $E = An_1^{\varepsilon}$. This doesn't change my results qualitatively. More complicated production function with capital, human capital, land etc. can be considered. To keep things simple, I consider a model with only one input.

rent on top of the commuting cost is just a matter of having coefficient $(3/2)b$ instead of b in the formulae for ACC.

The worker's utility is assumed to be linear in money[8] and is equal to

$$\varphi_2(n_1, n_2) = w(n_1, n_2) - bn_2^{1/2} = \delta A n_1^{1-\delta} n_2^{\varepsilon+\delta-1} - bn_2^{1/2} \tag{3}$$

Alternatively, I consider the case of rigid labor market, in which wage is fixed at the level $w = \overline{w}$ and labor demand is not equal to labor supply. There are two possibilities: either wage is fixed at the level below the equilibrium, and then labor supply is abundant; or wage is fixed at the level higher than equilibrium, which leads to the shortage of labor supply.

For the latter case, $n_1 L^D < n_2$, and I assume that all the vacancies are filled, and the firms get their desired amount of labor, whereas each worker gets a job with probability $(n_1 L^D)/n_2$. So, worker's (ex-ante) utility is equal to $\overline{w}(n_1 L^D)/n_2 - bn_2^{1/2}$.

This gives

$$\varphi_1(n_1, n_2) = (1-\delta) K n_2^{\frac{\varepsilon}{1-\delta}} \tag{4}$$

$$\varphi_2(n_1, n_2) = \delta K n_1 n_2^{\frac{\varepsilon}{1-\delta}-1} - bn_2^{1/2} \tag{5}$$

where

$$K = A \left(\frac{\delta A}{\overline{w}} \right)^{\frac{\delta}{1-\delta}} \tag{6}$$

For the case of lack of labor supply, $n_1 L^D > n_2$, I assume that labor is split equally among the firms so that each of them employs n_2/n_1 workers. The resulting profits and utilities are given by

$$\varphi_1(n_1, n_2) = A n_1^{-\delta} n_2^{\delta+\varepsilon} - \overline{w} \frac{n_2}{n_1} \tag{7}$$

$$\varphi_2(n_1, n_2) = \overline{w} - bn_2^{1/2} \tag{8}$$

The public good consumption, or, more generally the consumption amenities can easily be incorporated into the citizens' utility function. For example, for the case of the competitive labor market, one can consider $\varphi_2(n_1, n_2, A, B) = [w(n_1, n_2)]^{\alpha} B^{1-\alpha}$ or $\varphi_2(n_1, n_2, A, B) = w(n_1, n_2) + B$, which are the standard ways of modeling the public goods consumption.

Notice that the production amenities A influence worker's wage, and, consequently, citizens' benefits.

In the benchmark models of two-sided market of Rochet and Tirole [2004] and Armstrong [2006] the benefits of the actors from joining the platform are assumed to be linear in the number of the participants on the other side of platform, i.e. $\varphi_i = b_i n_j + a_i$ ($b_i > 0$). This functional form is based on assumption that each member on one side of platform interacts with each member on another side. These assumptions are too stylized for the purpose of modeling the city. In fact, the concentration spillover, represented here by ε, plays an important role in determining the benefits from joining the platform (city), as

[8] To justify this assumption, notice that the Cobb-Douglas preferences lead to indirect utility linear in income.

follows from the three cases, corresponding to the different structures of the labor market, considered as the motivating example above. On top of this, there is only one interaction between a firm and a worker on the platform. At the same time, there is a number of similarities. For instance, the benefit of the user on one side of platform is an increasing function of the number of users on another side. Because of this, a lot of intuition on the pricing strategies of the platform, presented in Rochet and Tirole [2004] and Armstrong [2006], is relevant for the model and the application on this paper.

2.3 Regularity Assumptions

I assume that following regularity conditions on the overall benefits of the platform users, i.e. firms' profit and citizens' wage net of commuting cost, $\varphi_1 n_1 + \varphi_2 n_2$, hold:

$$\frac{\partial}{\partial n_1}(\varphi_1 n_1 + \varphi_2 n_2) \geq 0, \quad \frac{\partial}{\partial n_2}(\varphi_1 n_1 + \varphi_2 n_2) \geq 0$$

$$\frac{\partial^2}{\partial n_1^2}(\varphi_1 n_1 + \varphi_2 n_2) \leq 0, \quad \frac{\partial^2}{\partial n_2^2}(\varphi_1 n_1 + \varphi_2 n_2) \leq 0, \quad \frac{\partial^2}{\partial n_1 n_2}(\varphi_1 n_1 + \varphi_2 n_2) \geq 0$$

For the examples considered above, for the competitive labor market and for the rigid labor market for the case of shortage of labor

$$\varphi_1 n_1 + \varphi_2 n_2 = A n_1^{1-\delta} n_2^{\delta+\varepsilon} - b n_2^{3/2}$$

for the rigid labor market when labor is abundant

$$\varphi_1 n_1 + \varphi_2 n_2 = K n_1 n_2^{\frac{\varepsilon}{1-\delta}} - b n_2^{3/2}$$

It's easy to check that with $\varepsilon + \delta < 1$ and $0 < \delta < 1$ all the regularity assumptions are satisfied for all n_1, n_2 for both functions. However, in general, the regularity assumptions are required only for n_1, n_2, satisfying some conditions (the first order conditions of the corresponding optimization problems). For instance, in the paper I don't require that $\varepsilon + \delta < 1$ holds for all n_1, n_2.

3 Monopoly Platform

In this section I analyze the case of an isolated city which is considered as a monopoly platform. I follow the general framework for the analysis of the two-sided market monopoly. However, it's impossible in the framework of a city to directly apply the general results of Rochet and Tirole [2004] or Armstrong [2006] because, as it is shown in the previous section, firms' and citizens' benefits from joining the city are influenced by the concentration externality and can't be taken of the forms assumed in these papers. Still, there are many analogies between the results of my paper and the results of the previous studies.

I start with setting the basic framework for the analysis and proceed then to the three regimes of city management. The first regime is the active city management under which the City Manager sets taxes, i.e. acts as an active platform in the two-sided market. The second regime is passive city management or self-organization of a city, when City Manager serves only to organize the site for a city (in particular, labor matching market) and doesn't intervene with taxes. In this case the size of the city is determined by free entry of firms and citizens. The third regime is the socially optimal city size. I show that it can be achieved with taxes set at particular level. After analyzing the three regimes, I compare the three regimes.

Assume that the firms are heterogenous and have an outside option - each of them may obtain profit $\widehat{\pi}$ outside the city. In other words, each firm requires the after-tax profit at least $\widehat{\pi}$ to operate in the city.

Let $F_1(\widehat{\pi})$ be the number of firms with required profit less than $\widehat{\pi}$. So, $F_1(\widehat{\pi})$ is analogous to CDF for the random variable $\widehat{\pi}$ but it takes values not necessarily from the interval $[0, 1]$.

The firms entry condition is

$$\varphi_1(n_1, n_2) - T_1 - \widehat{\pi} = 0 \tag{9}$$

$$n_1 = F_1(\widehat{\pi}) \tag{10}$$

The two equations determine the firms' quasi-demand[9] for the city (platform)

$$n_1 = D_1(T_1, n_2)$$

It follows from the definition of function F_1 that it is increasing. Consequently, the inverse function F_1^{-1} is increasing as well.

Additionally, I assume that the function $\frac{F_1(\widehat{\pi})}{F_1'(\widehat{\pi})}$ is increasing.

The assumptions for the citizens are similar to those for the firms. Each (potential) citizen has an outside option - the (indirect) utility level $\widehat{u}$ which he can obtain outside the city.

Let $F_2(\widehat{u})$ be the number of workers with required utility level less than $\widehat{u}$.

The workers entry condition is then

$$\varphi_2(n_1, n_2, B) - T_2 - \widehat{u} = 0 \tag{11}$$

$$n_2 = F_2(\widehat{u}) \tag{12}$$

The two equations determine the workers' quasi-demand for the city (platform)

$$n_2 = D_2(T_2, n_1)$$

It follows from the definition of function F_2 that it is increasing. Consequently, the inverse function F_2^{-1} is increasing as well.

Additionally, I assume that the function $\frac{F_2(\widehat{u})}{F_2'(\widehat{u})}$ is increasing.

The two quasi-demand equations can be solved together to give demand for the city from the firms and citizens:

$$n_1 = N_1(T_1, T_2)$$
$$n_2 = N_2(T_1, T_2)$$

I consider now three different regimes of city management.

[9] I use the term "quasi-demand" following Rochet and Tirole [2003]. The demand should depend on the taxes only.

3.1 The Results for the Monopoly Platform

The City Manager (platform) program is

$$
\begin{aligned}
& T_1 n_1 + T_2 n_2 \rightarrow \max_{T_j} \\
s.t. \qquad & n_1 = N_1(T_1, T_2) \\
& n_2 = N_2(T_1, T_2)
\end{aligned}
$$

It is convenient to change variables to the threshold utilities $\widehat{u}$ and $\widehat{\pi}$. Substituting T_1 and T_2 from the entry conditions (9) and (11) and taking into account that $n_1 = F_1(\widehat{\pi})$, $n_2 = F_2(\widehat{u})$ gives an equivalent program

$$
(\varphi_1(F_1(\widehat{\pi}), F_2(\widehat{u})) - \widehat{\pi})F_1(\widehat{\pi}) + (\varphi_2(F_1(\widehat{\pi}), F_2(\widehat{u})) - \widehat{u})F_2(\widehat{u}) \longrightarrow \max_{\widehat{\pi}, \widehat{u}}
$$

The F.O.C for this program are:

$$
\begin{aligned}
(\varphi'_{11}(\cdot)F'_1 - 1)F_1(\widehat{\pi}) + (\varphi_1(\cdot) - \widehat{\pi})F'_1 + \varphi'_{21}F'_1F_2(\widehat{u}) = 0 \\
\varphi'_{12}(\cdot)F'_2F_1(\widehat{\pi}) + (\varphi'_{22}F'_2 - 1)F_2(\widehat{u}) + (\varphi_2(\cdot) - \widehat{u})F'_2(\widehat{u}) = 0
\end{aligned}
$$

Rearrangements and changing variables to n_1 and n_2 leads to the following result.

Proposition 1. *The number of monopoly platform users (city size under active city management) is determined from the system*

$$
\frac{\partial}{\partial n_1}(\varphi_1 n_1 + \varphi_2 n_2) = F_1^{-1}(n_1) + \frac{n_1}{F'_1(F_1^{-1}(n_1)} \tag{13}
$$

$$
\frac{\partial}{\partial n_2}(\varphi_1 n_1 + \varphi_2 n_2) = F_2^{-1}(n_2) + F_2^{-1}(n_2) + \frac{n_2}{F'_2(F_2^{-1}(n_2)} \tag{14}
$$

The entry fees (taxes) are given by

$$
\frac{T_i + \varphi'_{ji} n_j}{T_i} = \frac{1}{\eta_i} \tag{15}
$$

where $i = 1, 2$, $j \neq i$, $\varphi'_{ji} = \frac{\partial \varphi_j}{\partial n_i}$ *and* $\eta_i = -\frac{T_i D'_i}{D_i}$ *are the elasticities of the quasi-demands.*

The formula for taxes (15) is analogous to the standard Lerner formula for the price structure for the two-sided markets which can be written as (see Rochet and Tirole [2004])

$$
\frac{p_i - (c_i - b_j)}{p_i} = \frac{1}{\eta_i} \tag{16}
$$

where p_i is the per-transaction price for side i of the market, c_i is platform's transaction cost,[10] b_j is the other side per-transaction benefit. This benefit can be taxed out by the platform, so $b_j n_j$ represents the platform's marginal profit obtained on side j from a marginal member on side i.

[10] I assume $c_i = 0$ but non-zero cost can easily be accommodated. Zero transaction cost are relevant for modeling the city.

For the considered setting, $p_i = \frac{T_i}{n_j}$ if "transaction" is broadly understood. In fact, there is only one transaction between a firm and a worker, resulted from the job market matching. However, the higher number of, say, workers, may be thought of leading to more "transactions"-bargaining about wage between a firm and all the workers which results in lower wage.

The marginal benefit on side j resulted from the additional member on side i, represented by φ'_{ji} in (15), is a generalization of the per-transaction benefit b_j in (16). So, the formula (15), which describes the price structure for the more general setting, analyzed in this paper, is a generalization of the "standard" formulae (16) for the price structure in the two-sided markets.

Notice that the one-side monopoly with zero cost would set price (T_i) at the point with demand elasticity equal to 1. The additional term $\varphi'_{ji} n_j$ in (15) emphasizes that the two-sided nature of the city plays an important role for the tax setting of the city manager, and, consequently, affects the size of the city.

3.2 Social Optimum

Social welfare of the firms and citizens located in the city (net of foregone opportunities) is given by

$$SW = \int_0^{F_1^{-1}(n_1)} \left(\varphi_1(n_1, n_2) - \pi\right) F_1'(\pi) d\pi + \int_0^{F_2^{-1}(n_2)} \left(\varphi_2(n_1, n_2) - u\right) F_2'(u) du$$

The change of variables to $(\widehat{\pi}, \widehat{u})$ leads to a simpler program. Changing then the variables back to (n_1, n_2) gives the following result:

Proposition 2. *The socially optimal numbers of the backyard firms and citizens in the city is given by*

$$\frac{\partial}{\partial n_1}(\varphi_1 n_1 + \varphi_2 n_2) = F_1^{-1}(n_1) \tag{17}$$

$$\frac{\partial}{\partial n_2}(\varphi_1 n_1 + \varphi_2 n_2) = F_2^{-1}(n_2) \tag{18}$$

Proof is given in the Appendix.

Proposition 2 has a clear intuition. The left-hand side of (17) represents the marginal benefit of all firms and citizens in the city resulting from the marginal firm joining the city. Note that it's different from the marginal firm's own benefit due to presence of the network externality.

The right-hand side of (17) represents the foregone opportunities of the marginal firm which is equal to the social foregone opportunities. At the optimum, the marginal benefits should equalize the foregone opportunities.

Equation (18) represents the similar cost-benefit relation for the marginal citizen.

The optimal city size can be achieved through taxation. The taxes which lead to the optimal city size can be represented by the following formulae:

$$\frac{T_i + \varphi'_{ji} n_j + \frac{D_i}{F_i'}}{T_i} = \frac{1}{\eta_i} \qquad (i = 1, 2) \tag{19}$$

The taxes determined by the system (19) can be considered as Ramsey taxes for the city. The comparison of (15) and (19) leads to the following

Claim 1. *The monopolistic platform (city manager) imposes the entry fees (taxes) which lead to the socially optimal city size iff the populations of both firms and citizens are homogenous, i.e. have the same outside options.*

3.3 Self-organization of the City

Under self-organization, the city size is determined by the free entry conditions with zero taxes (9) and (11). The following Proposition can easily be established.

Proposition 3. *The city size under the free entry is given by*

$$\varphi_1(n_1, n_2) = F_1^{-1}(n_1) \tag{20}$$

$$\varphi_2(n_1, n_2) = F_2^{-1}(n_2) \tag{21}$$

The two equations represent the cost-benefit relations, as for the optimal city size. However, now these benefits are limited to the own benefits of the marginal firm and the marginal citizen joined the city and don't include externalities for the other firms and citizens. This leads to the deviation from the socially optimal city size, which are considered below.

3.4 Comparison of the City Management Regimes

First, compare the optimal city size with the one resulting from the active city management.

Claim 1 states that the city size is optimal iff the firms and the citizens are homogenous. The higher is heterogeneity in the outside opportunities of the firms and/or citizens, the higher is the city size distortions. Notice that in (19) $D_i = D_i(T_i, n_j)$, so that heterogeneity even in one side of the city will lead to distortion on this side which can be translated to the other side.

The direction of the distortion is easier to see by comparing the city size characterizations in Propositions 1 and 2.

First, there is an extra term $\frac{F_i}{F_i'}$ in the right-hand sides of the two equations (13) and (14) for the city size under the active city management. These terms are related to the monopoly position of the city Manager on each side of the platform. The City Manager wants the marginal firm and the marginal citizen to have higher benefits from joining to the city to be able to impose higher taxes. This is the standard monopoly deviation from the efficient outcome: the monopoly prefers to serve thinner market but impose higher prices, compared to the social optimum. This effect will be referred to as *monopolization effect.*

Second, despite the functional forms of the left-hand sides as function of two variable - n_1, n_2 are the same, their functional forms as functions of the own side size n_i change because the left-hand sides contain the size of the other side of the city n_j as a parameter, e.g. the left-hand side of the equation for firms (13) depends on the number of citizens, which changes in equilibrium with active City Manager, compared to the social optimum. As a consequence, the number of, say, firms changes not only as a response to the monopolization of the market for the firms by the City Manager but also as a response to the change in the number of the actors on the other side of the market - the citizens. This latter effect will be referred to as *adjustment effect*.

So, the overall distortion imposed by the City Manager may be thought of as a sum of the two effects - the monopolization and the subsequent adjustment[11].

To sum up the above discussion, under the active city management, the monopolization effect appears on each side of the city (platform) and it is related to the "own" side of the platform. The adjustment effect arises as a reaction to the changes on another side of the platform.

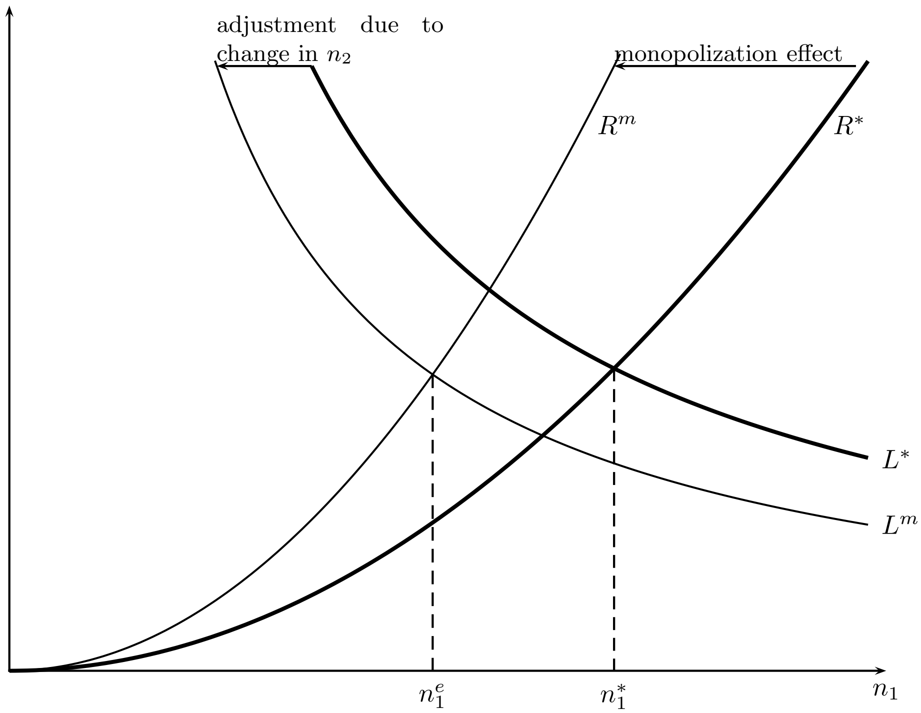

Figure 1: Distortion in the city size under active city management.

Figure 1 illustrates the changes in the number of firms. Similar figure for the citizens should be drawn to represent the full picture.

The analysis of the distortions created by the monopolistic platform (active city manager) can be illustrated by this figure.

In Figure 1, the curve L^* represents the marginal social benefit from the marginal firm joining the city. It is described by the left-hand side of (17) and is a decreasing functions of n_1 because $\frac{\partial^2}{\partial n_1^2}(\varphi_1 n_1 + \varphi_2 n_2) \leq 0$, according to the regularity assumptions. The curve R^* represents the foregone social opportunities (i.e. the outside option) of the marginal firm. It is described by the right-hand side of (17) and is an increasing function of n_1 since F_1^{-1} is increasing functions and $\frac{F_1}{F_1'}$ is assumed to be increasing.

[11]After the changes resulted from the monopolization effect, there will be the first round of adjustment. This will lead, however, to another change in, say, n_1, which will lead to the second round of adjustment in n_2, and so on. So, in fact there will be infinitely many rounds of adjustment with decreasing magnitude which will result in the overall adjustment after the reaction to the distortion initiated by the monopolization effect.

The optimal city size is determined by the intersection of the curves L^* and R^*, which gives the number of the firms n_1^*.

Similar figure for the citizens gives the socially optimal number of the citizens n_2^*.

The distortion created by the monopolistic platform (active city manager) can be illustrated in the following way. First, the curve R^* shifts upward to the position R^m due to the monopolization effect. This shift decreases n_1 at the intersection. However, similar shift takes place for the citizens and lead to the decrease in n_2. This, however, leads to the change in the left-hand side of (17), which is illustrated by the shift of the curve L^*. Since $\frac{\partial^2}{\partial n_1 n_2}(\varphi_1 n_1 + \varphi_2 n_2) \geq 0$, according to the regularity assumptions, the decrease in n_2 leads to the downward shift of the curve L^* in Figure 1. This first step of the adjustment for the firms will in turn affect the citizens which will lead to a decrease in n_2. This leads to the subsequent second step of adjustment for the firms - further downward shift of the curve L^* etc. After all (in fact, infinite number) steps of adjustment, the city sizes n_1, n_2 the curves L on the two graphs - Figure 1 and similar figure for the number of the citizens will reach their final positions L^m and the equilibrium city size under active city management will be obtained at the intersection of the curves L^m and R^m, which will lead to the city size n_1^m.

Notice that all the shifts lead to the decrease of n_i. So, for the active city management, the equilibrium city size will be suboptimal. The geometrical argument presented above can easily be formalized to the algebraic proof, so that the following claim can be established.

Claim 2. *The number of the firms and the citizens under the active city management is suboptimal if the firms or the citizens are heterogenous (in terms of the outside option).*

Second, compare the optimal city size with the free entry outcome.

The marginal firm and citizen own benefit from joining the city can be clearly distinguished from the social benefits if (17) and (18) will be rewritten as:

$$\varphi_i + \varphi'_{ii} + \varphi'_{ji} n_j = F_i^{-1}(n_i) \qquad (i = 1, 2) \tag{22}$$

The first term in the left-hand sides of (22) represents the own benefits of the marginal firm (marginal citizen), the two other terms in the left-had side represent the externalities.

Compare (22) with the free entry city size characterization in Proposition 3. The right-hand sides are the same and the shift in the left-hand sides is determined by the sign of $\varphi'_{ii} + \varphi'_{ji} n_j$ which is, however, unclear because the sign of the first term is unclear. The detailed analysis for the particular case of the rigid labor market is conducted in the subsection 3.5.

However, it's clear that the mechanisms of the deviation from the optimal city size are different for the active and passive city management.

For the active city management, the deviation emerges due to the monopolization effect and the subsequent adjustment. The externalities from joining an additional member on one side of the platform on another side of the platform are taken into account by the city manager. By contrast, for the free entry under the passive city management, the deviation from the optimal city size appears because of disregarding of the externalities. In the context of the two-sided market framework, this means that the entry decision of the platform members are shaped by the one-sided considerations only.

The result is different from the City Developer model of Henderson and Becker [2000]. In their model, the city size under the active city management is optimal whereas it is sub-optimal under free entry. However, unlike the City Manager framework of this paper, the City Developer framework is based on the maximization of the land rent collected in the city net of the subsidies payed to the firms and citizens.

3.5 Application

Consider the model of the city with competitive labor market, described in subsection 2.2. The socially optimal size of the city (n_1^{SO}, n_2^{SO}) is determined, according to Proposition 2 by the system of equations

$$\Psi_1(n_1, n_2) \equiv A(1-\delta)n_1^{-\delta}n_2^{\varepsilon+\delta} = F_1^{-1}(n_1) \tag{23}$$

$$\Psi_2(n_1, n_2) \equiv A(\varepsilon+\delta)n_1^{1-\delta}n_2^{\varepsilon+\delta-1} - \frac{3}{2}bn_2^{1/2} = F_2^{-1}(n_2) \tag{24}$$

Under the active city management, the size is determined by the system of equations with the same left-hand sides and disturbed right-hand sides, according to Proposition 1. However, if the firms and the citizens are homogenous, there is no disturbance and the size of the city under the active management is socially optimal. Otherwise, the city is suboptimal.

Under the free entry regime, for the case of the competitive labor market, the city size (n_1^{fe}, n_2^{fe}) is determined by Proposition 3 which gives the system of equations

$$\Phi_1(n_1, n_2) \equiv A(1-\delta)n_1^{-\delta}n_2^{\varepsilon+\delta} = F_1^{-1}(n_1) \tag{25}$$

$$\Phi_2(n_1, n_2) \equiv A\delta n_1^{1-\delta}n_2^{\varepsilon+\delta-1} - bn_2^{1/2} = F_2^{-1}(n_2) \tag{26}$$

Compare now the socially optimal city size and the city size under free entry.

The first equations of the two systems, (23) and (25), coincide since $\Psi_1(\cdot) = \Phi_1(\cdot)$ whereas the second equations, (24) and (26), differ. The comparison of the functions Ψ_2 and Φ_2 gives the following result:

Lemma 1. *For each n_1 there exists a threshold $\widehat{n}_2$ determined by*

$$\widehat{n}_2^{3/2-\varepsilon-\delta} = \frac{2A\varepsilon n_1^{1-\delta}}{b}$$

such that

$$\Phi_2(n_1, n_2) < \Psi_2(n_1, n_2) \quad \text{for} \quad n_2 < \widehat{n}_2$$
$$\Phi_2(n_1, n_2) > \Psi_2(n_1, n_2) \quad \text{for} \quad n_2 > \widehat{n}_2$$

Proof is given in the Appendix.

Restrict now attention to the case of the homogenous outside options for the firms and the citizens. In this case, the right-hand sides of (23)-(26) are constants.

The analysis is easier to do with the help of the graph. In Figure 2 the curves SO and fe represents $\Psi_2(n_1^{SO}, n_2)$ and $\Phi_2(n_1^{SO}, n_2)$ as functions of n_2.

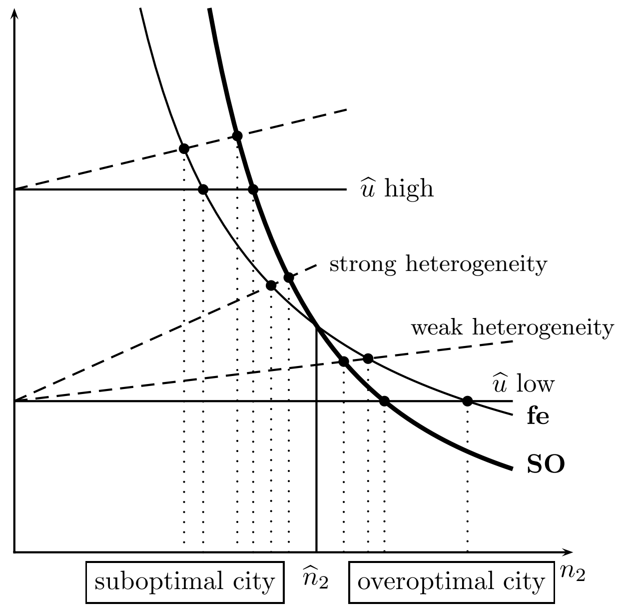

Figure 2: Distortion in the city size under passive city management with competitive labor market. Curve SO corresponds to the social optimum (function $\Psi_2(\cdot, n_2)$), curve fe corresponds to the the free entry (function $\Phi_2(\cdot, n_2)$)

According to Lemma 1, for small n_2 the curve fe lies below the curve SO; for large n_2 the relative position of the two curves changes. The two horizontal lines $\widehat{u}$ *low* and $\widehat{u}$ *high* represent the cases of low and high outside option for the firms.

It is easy to see from Figure 2 that if outside option for the citizens is low, there will be upward distortion in the number of citizens. This will lead to the upward shift for the function Φ_1, as compared to Ψ_1. Put formally, $\Phi_1(n_1, n_2) > \Psi_1(n_1, n_2^{SO})$ for $n_2 > n_2^{SO}$. As a result, the distortion in n_2 will lead to the distortion in n_1, oriented in the same direction. This, in turn, will strength the distortion in n_2 etc. At the end, the distorted city of the size n_1^{fe}, n_2^{fe} will emerge.

The argument presented above means that the direction of the overall distortion, i.e. the direction of shift from n_i^{SO} to n_i^{fe} is determined by the direction of the initial shift in n_2.

This, in turn, leads to the conclusion that for low outside option city size will be overoptimal, whereas the high outside option leads to suboptimal city size.

The argument for the homogenous outside option can be generalized to the case of the heterogenous outside option. It's easy to see from Figure 2 that both overoptimality and suboptimality of the city size (if this is the case) become less pronounced.

Finally, since an increase in the concentration spillover ε increases the threshold $\widehat{n}_2$, according to Lemma 1, it's more likely to have suboptimal city size for the cities based on the industries with high concentration spillover.

4 Competing Platforms (Cities)

In this section I consider the case when the two cities (platforms) compete for the firms and citizens.

I consider the tax setting of the City Managers and generalize the results of Armstrong [2006]. I also consider stability of the two-cities configuration. When this configuration isn't stable, there will be formed one agglomerated city[12].

Consider the two cities[13] located at two ends of $[0, 1]$ Hotelling line. The cities will be referred to as "Left" and "Right". All the values related to the Left city will be indexed by "L". The ones for the Right city - by R. The transportation costs for producers and citizens are t_j $(j = 1, 2)$.

Let the firms and the workers be distributed uniformly along $[0, 1]$. As before, each city manager decides on two tax levels - for firms and for citizens.

4.1 Competing Platforms equilibrium

The firm located at x chooses to be in city L rather than R iff

$$\varphi_1(n_1^L, n_2^L; A^L, B^L) - T_1^L - t_1 x \geq \varphi_1(n_1^R, n_2^R; A^R, B^R) - T_1^R - t_1(1 - x)$$

Similarly, the condition for the citizens is

$$\varphi_2(n_1^L, n_2^L; A^L, B^L) - T_2^L - t_2 x \geq \varphi_2(n_1^R, n_2^R; A^R, B^R) - T_2^R - t_2(1 - x)$$

Consider the market sharing case[14]. Then, $n_j^R = 1 - n_j^L$ and demand for the city L is determined by

$$n_1^L = \frac{1}{2} + \frac{\varphi_1(L) - \varphi_1(R)}{2t_1} - \frac{T_1^L - T_1^R}{2t_1} \tag{27}$$

$$n_2^L = \frac{1}{2} + \frac{\varphi_2(L) - \varphi_2(R)}{2t_2} - \frac{T_2^L - T_2^R}{2t_2} \tag{28}$$

where

$$\varphi_j(L) = \varphi_j(n_1^L, n_2^L; A^L, B^L)$$
$$\varphi_j(R) = \varphi_j(n_1^R, n_2^R; A^R, B^R) = \varphi_j(1 - n_1^L, 1 - n_2^L; A^R, B^R)$$

Equations (27) and (28) determine the demand functions for the Left city $n_j^L = D_j^L(T_1^L, T_2^L; T_1^R, T_2^R)$ in an implicit way. Similarly, they determine the demand for the Right city.

For the future purposes it is useful to compute the derivatives of the quasi-demand functions $N_i^L(T_1^L, T_2^L)$, i.e. the demands $D_i^L(T_1^L, T_2^L; T_1^R, T_2^R)$ where the Right city's decisions on taxes T_j^R are taken as given.

Denote by N'_{ij} the partial derivative of the function $N_i(T_1, T_2)$ with respect to T_j:

$$N'_{ij} = \frac{\partial N_i}{\partial T_j} = \frac{\partial D_i(T_1^L, T_2^L; T_1^R, T_2^R)}{\partial T_j^L}$$

[12] Of course, spillover shouldn't be too large compared to the commuting cost to avoid city explosion.

[13] To be more precise, I consider the two sites, on which the two cities can emerge.

[14] The conditions for such equilibrium to exist will be given below for the case of the cities with identical amenities.

and by φ'_{ij} the partial derivative of the function φ_i with respect to n_j:

$$\varphi'_{ij} = \frac{\partial \varphi_i}{\partial n_j}$$

The derivatives N'_{ij} can be obtained by differentiating the system of equations (27) – (28) with respect to T_1^L and T_2^L.

Lemma 2. *The derivatives of the quasidemand functions N_1, N_2 are given by*

$$N'_{11} = \frac{d}{D}, \qquad N'_{21} = -\frac{c}{D}, \qquad N'_{12} = -\frac{b}{D}, \qquad N'_{22} = \frac{a}{D}$$

where

$$a = a(N_1, N_2) = \varphi'_{11}(L) + \varphi'_{11}(R) - 2t_1; \qquad b = b(N_1, N_2) = \varphi'_{12}(L) + \varphi'_{12}(R)$$
$$c = c(N_1, N_2) = \varphi'_{21}(L) + \varphi'_{21}(R); \qquad d = d(N_1, N_2) = \varphi'_{22}(L) + \varphi'_{22}(R) - 2t_2$$
$$D = ad - bc$$

Proof is given in the Appendix.

Notice that the coefficient a, b, c, d depend on the city size but are symmetric, i.e. the same for the two cities, even if the cities are asymmetric which can be the case if they have different amenities.

Consider the decision making of the city L manager, taken city R taxes as given. In what follows the index L will be omitted to simplify notation whereas index R will be kept.

The City Manager program is

$$T_1 N_1(T_1, T_2) + T_2 N_2(T_1, T_2) \rightarrow \max_{T_1, T_2}$$

which gives the first order conditions

$$N'_{11} T_1 + N_1 + N'_{21} T_2 = 0 \tag{29}$$
$$N'_{22} T_2 + N_2 + N'_{12} T_1 = 0 \tag{30}$$

Substituting here N'_{ij} from Lemma 2, we obtain the Best Response functions in implicit way, since N_j are functions of T_1, T_2:

$$\begin{aligned} dT_1 - cT_2 &= -N_1 D \\ -bT_1 + aT_2 &= -N_2 D \end{aligned}$$

The solution of this system gives the Best Response taxes as implicit functions in a more structured way:

$$T_1 = -aN_1 - cN_2 \tag{31}$$
$$T_2 = -bN_1 - dN_2 \tag{32}$$

where N_j are considered as functions of taxes in own and another city: $N_j = D_j(T_1, T_2; T_1^R, T_2^R)$.

Clearly, the first order conditions (29) – (30) and the formulae for the taxes (31) – (32) are only necessary conditions. To make sure that the solution to the system (29) − (30) will in fact maximize the objective function, one should check the corresponding second order conditions.

Notice, that up to this point it wasn't assumed that the two cities have identical amenities, so the formulae for the cities sizes (27) and (28) and for taxes (31) and (32) hold even for the case of cities with different amenities.

I restrict now the analysis to the case of the cities with identical amenities.

Consider the two sites for the cities with the same amenities. In other words, the two sites are ex-ante identical. In general, the are two possibilities. Either the two sites will be developed into the cities. In this case the two symmetric cities emerge[15] and there will be symmetric equilibrium. Or one site will corner all the firms and workers and one agglomerated city will emerge. This equilibrium can be called "agglomeration equilibrium".

The following Proposition generalizes the results of Armstrong [2006] on the existence of the market-sharing equilibrium and the pricing strategies of the platforms.

Proposition 4. *The symmetric market sharing equilibrium exists iff*[16]

$$t_1 > \varphi'_{11} \tag{33}$$

$$t_2 > \varphi'_{22} \tag{34}$$

$$4\left(t_1 - \varphi'_{11}\right)\left(t_2 - \varphi'_{22}\right) > \left(\varphi'_{12} + \varphi'_{21}\right)^2 \tag{35}$$

Equilibrium entry fees (taxes) are given by

$$T_1 = t_1 - \varphi'_{11} - \varphi'_{21} \tag{36}$$

$$T_2 = t_2 - \varphi'_{12} - \varphi'_{22} \tag{37}$$

where $\varphi'_{ij} \equiv \varphi'_{ij}(\frac{1}{2}, \frac{1}{2})$.

Proof is given in the Appendix.

Clearly, we need only one of the conditions (33) or (34) together with (35) because if one of (33) or (34) holds, another holds as well given (35).

Armstrong [2006] considered a similar model which is motivated by application to such two-sided markets as "nightclubs, shopping malls and newspapers". In the single-homing case he considered the functions φ_j of a simpler form, compared to my model, for instance φ_1 depends only on n_2 and φ_2 depends only on n_1 and the two functions are linear. On the other hand, Armstrong assumed that there are costs for the platform of serving the users at the platform (firms and workers in my case, in Armstrong - e.g. supermarket buyers and suppliers) which are assumed to be zero in my analysis. The assumption of zero cost is not crucial for my results and can be easily relaxed. Though, it is relevant for modeling the city. His result on the equilibrium taxes (prices in his model) is then a particular case of my analysis. Moreover, considering the simpler functions requires much less technique. So, my analysis in this part is a generalization of the Armstrong's analysis of the single-homing case.

My results have clear economic intuition. As in standard Hotelling model, t_j represents market power. The derivative φ'_{11} is negative or zero in the examples

[15] The possibility for asymmetric equilibrium in which both cities have non-infinitesimal size can also studied

[16] Despite these are second order conditions, there is no second order derivatives of the functions φ_j because they cancel out due to symmetricity of the considered equilibrium.

I consider. It reflects the fact that additional members on the own side decrease benefits from joining the city. This is attributed, in particular, to the worse matching opportunities. The term $(-\varphi'_{11})$, taken for the marginal firm, increases the tax T_1, making the city "less attractive" for the firms, located farther than the marginal one but keeping at the same time the city attractive enough for the firms located in the city as the increased tax controls the number of firms joined to the city. Finally, the terms $-\varphi'_{21}$ corrects the tax upward because the higher number of the firms makes the city more attractive for the citizens.

The interpretation of the components of the tax T_2 is similar. It's, however, worth to notice that the term φ'_{22} can be positive because the concentration spillover appears due to the concentration of the workers. So, the tax for the citizens is corrected downward twice. This is resulted from the fact that each City Manager is more prone to attract more workers-citizens as they create the spillover and the two City Managers compete for the citizens tougher than for the firms.

4.2 Application - the Role of Spillover and Labor Market Structure

I turn now to the application of the general result stated in Proposition 4 to the models of city with competitive and rigid labor market.

4.2.1 Competitive Labor Market

For the case of competitive labor market, substituting (2) and (3) into Proposition 4 gives the following results. The taxes in the city are

$$T_1 = t_1 \tag{38}$$

$$T_2 = \tau - A\varepsilon 2^{1-\varepsilon} \tag{39}$$

where $\tau = t_2 + \frac{b}{\sqrt{2}}$ can be thought of as general congestion cost for the citizens (a sum of transportation cost and land rent).

The conditions for the existence of the 2-city equilibrium are:

$$t_1 > -(1-\delta)\delta A \cdot 2^{1-\varepsilon} \tag{40}$$

$$\tau > -\delta(1-\varepsilon-\delta)A \cdot 2^{1-\varepsilon} \tag{41}$$

$$\begin{aligned} 4\left(t_1 + A(1-\delta)\delta \cdot 2^{1-\varepsilon}\right)\left(\tau + A\delta(1-\varepsilon-\delta) \cdot 2^{1-\varepsilon}\right) > \\ > \left(A(1-\delta)(\varepsilon+2\delta) \cdot 2^{1-\varepsilon}\right)^2 \end{aligned} \tag{42}$$

The inequality (40) holds, given that $\delta < 1$. So, the necessary and sufficient condition for the second order conditions to hold reduces to (42). After rearrangement it leads to

$$\begin{aligned} t_1\tau \cdot 2^{2(\varepsilon-1)} + A\delta\left((t_1+\tau)(1-\delta) - t_1\varepsilon\right)2^{\varepsilon-1} > \\ > A^2(1-\delta)\left(\frac{1-\delta}{4}\varepsilon^2 + \delta\varepsilon\right) \end{aligned} \tag{43}$$

So, we have established the following result.

Claim 3. *1. For the two competing cities with competitive labor market, the taxes are given by (38) and (39).*

2. The necessary and sufficient condition for the two-city equilibrium to exist is given by (43).

The spillover, the two-sided nature of the city and amenities all play an important role for both tax setting and existence of the 2-city configuration.

In fact, the "one-side" taxes are given by $T_i = t_i$. When the city is considered as the two-sided market, in the formula for tax on the firms, the additional components $-\varphi'_{11} - \varphi'_{21}$ cancel out, which is an artefact of the particular functional form of the production technology; for the tax on the citizens, the additional component $-\varphi'_{12} - \varphi'_{22}$ is non-trivial in (39).

The two-sided nature of the city is also important for the existence of the 2-city equilibrium. Notice that it's possible to have the two second order conditions $\frac{\partial^2 T}{\partial T_j^2} < 0$, given by (40) and (41), satisfied, but the hessian matrix still not to be negative-semidefinite. This means that the possible instability of the 2-city configuration is due to the two-sided nature of the city. In fact, if one of the taxes is kept constant, there is no profitable deviation through changing another tax only, because the individual second order derivatives are negative. The profitable deviation can, however, exist is the two taxes are changed simultaneously - tax for one side increases, and tax for another side decreases holding a particular ratio of the changes in the taxes[17].

The role of spillover should be clear from (39) and (43). For the spillover intensity ε, it's clear from (39) that stronger spillover intensity decreases the tax on the citizens[18]. In other words, spillover makes competition between city managers for the citizens (whose concentration creates the spillover) more tight.

The inspection of (43) shows that for $\varepsilon = 0$ the left-hand side is positive, and the right-hand side is equal to zero, so that (43) holds. As ε increases, the right-hand side can grow faster than the left-hand side, so that it's possible to have the inequality (43) violated for some values of ε. This is illustrated by Figure 3.

Finally, the role of production amenities A can be understood from (39) and (43). Higher A leads to lower taxes on the citizens and can destroy the 2-city equilibrium.

The tax revenues collected in each city, which can be considered as a characteristic of political power of the City Manager, is given by

$$T = \frac{t_1 + \tau}{2} - A\varepsilon \cdot 2^{-\varepsilon}$$

which a decreasing function of spillover intensity when it's small enough and is a decreasing function of amenities A.

The comparative static results are summarized in the following claim.

Claim 4. *For the competing cities with competitive labor market, governed by the active city manager*

[17] It can be shown that deviation from the taxes given by the first order conditions can be profitable only if it has a described structure.

[18] In fact, tax in (39) is a non-monotone function of ε. However, for ε small enough, which is the case for the existing industries, the function of ε in (39) is decreasing.

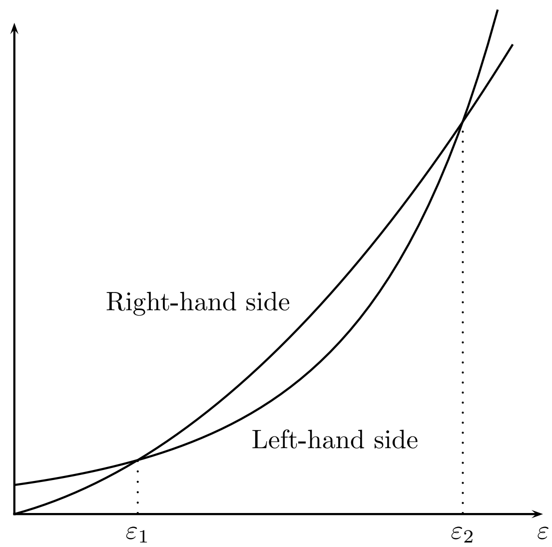

Figure 3: Inequality (43) with $A = 1.7$, $t1 = .1$, $\tau = 2$, $\delta = .25$

1. *The taxes on the (qualified) workers are lower for the cities, based on the industries with stronger concentration spillover.*

2. *The cities agglomerate to form a larger city more often when concentration spillover is stronger.*

4.2.2 Rigid Labor Market

I focus on the case when wage w is fixed on the higher than equilibrium level.

Then there will be excessive labor supply, $n_1 L^D < n_2$, which is equivalent to $A\delta/w > 2^{\varepsilon}$. Substitution of the benefits from joining the city for the firms and citizens given by (4) and (5) to Proposition 4 gives

$$T_1 = t_1 - 2K\delta \cdot 2^{-\frac{\varepsilon}{1-\delta}} \tag{44}$$

$$T_2 = \tau + 2K\left(\delta - \frac{\varepsilon}{1-\delta}\right) \cdot 2^{-\frac{\varepsilon}{1-\delta}} \tag{45}$$

The conditions for the existence of the 2-cities equilibrium are:

$$t_1 > 0$$
$$\tau > 2\delta K(\frac{\varepsilon}{1-\delta} - 1) \cdot 2^{-\frac{\varepsilon}{1-\delta}}$$
$$t_1\tau - 2t_1\delta K\left(\frac{\varepsilon}{1-\delta} - 1\right) \cdot 2^{-\frac{\varepsilon}{1-\delta}} > K^2(\varepsilon+\delta)^2 \cdot 2^{-\frac{2\varepsilon}{1-\delta}} \tag{46}$$

Clearly, the first inequality always holds. So, the necessary and sufficient condition for the second order conditions to hold reduces to the last inequality.

So, we have established following

Claim 5. *For the two competing cities with rigid labor market, when wage is fixed on the higher than equilibrium level, the taxes are given by* (44) *and* (45).

The necessary and sufficient condition for the two-city equilibrium to exist is given by (46).

Again, as for the competitive labor market, the two-sided nature of the city, spillover and amenities affect the tax setting and existence of the 2-city equilibrium.

Notice, however, that the rigidity of the labor market changes the impact of these factors.

The firms now take the whole benefit from the concentration spillover (for the competitive labor market, the benefit was shared with the workers through wage increase due to labor market competition). However, attracting more firms in the cities means increasing the chances for the citizens to be employed. This determines the competition of the city managers for the firms and explains the deviation from the "one-sided" tax: the tax on the firms decreases, which reflects the tighter competition.

The city manager isn't interested in attracting additional citizens in the considered case, because all the firms' vacancies are filled. So, attracting additional citizens will make worse off those who are already in the city, as a consequence, the taxes on the citizens should be lowered. So, in this case the citizens are heavily taxed.

The stronger spillover intensity increases the tax on the firms, as it increases the firms' profit, which isn't shared with the workers now. It decreases the tax on the citizens since the firms' labor demand increases and the citizens have more chances to be employed, so that the City Managers compete for attracting more citizens tighter.

Finally, the stronger spillover intensity can destroy the 2-city equilibrium, similar to the case of competitive labor market.

The overall city tax revenues when the 2-city equilibrium exists, is given by

$$T = \frac{t_1 + \tau}{2} - K\frac{\varepsilon}{1-\delta} \cdot 2^{-\frac{\varepsilon}{1-\delta}}$$

which is a decreasing function of the spillover intensity ε.

The following Claim summarizes the result of the comparative static analysis.

Claim 6. *For the competing cities with rigid labor market, governed by the active city manager*

1. *The taxes on the (qualified) workers are lower and the taxes on the firms are higher for the cities, based on the industries with stronger concentration spillover.*

2. *The cities agglomerate to form a larger city more often when concentration spillover is stronger.*

4.2.3 The Impact of the Labor Market Structure on Taxes and Agglomeration

In this part of the paper I compare the taxes set by the City Manager and the stability of the City configuration under the two structures of the labor market

- competitive and rigid. For the case of the rigid labor market I focus on the voluntary unemployment (higher than equilibrium wage) case.

First, compare the taxes on firms and on citizens. According to (38) and (44), the taxes on firms for the competitive and rigid labor market are given, correspondingly by

$$T_1^c = t_1, \qquad T_1^r = t_1 - 2\delta K \cdot 2^{-\frac{\varepsilon}{1-\delta}}$$

and the taxes on the citizens, according to (39) and (45) are

$$T_2^c = \tau - 2\varepsilon A \cdot 2^{-\varepsilon}, \qquad T_2^r = \tau + 2\left(\delta - \frac{\varepsilon}{1-\delta}\right) K \cdot 2^{-\frac{\varepsilon}{1-\delta}}$$

It's clear that $T_1^c > T_1^r$. In other words, the cities compete tighter for the firms under the rigid labor market. In fact, attracting an additional firm under the rigid labor market, creates the benefit for another side of the platform, since the citizens will have better chances to be employed. This makes competition for the firms between the cities tighter.

It's also easy to see that, at least for small ε, $T_2^c < T_2^r$. Put differently, under the competitive labor market, the cities' competition for the citizens (the factor creating the concentration spillover) is stronger, than under the rigid labor market.

As for the firms, the explanation is based on the analysis of the benefits for another side of the platform. An additional citizen under the competitive labor market increases the concentration spillover, i.e. all the workers become more productive, which benefits the firms. On top on this, an additional citizen increases labor supply and decreases wage, which creates an additional benefit for the firms, but only under competitive labor market. Finally, under rigid labor market, attracting an additional citizen would increase congestion (i.e. decrease the citizens' chances to be employed), which would weaken the citizens' benefit from joining the city. Because of this, city managers compete less tightly for the citizens under the rigid labor market.

So, we have established the following Claim.

Claim 7. *When the 2-city configuration is stable under both competitive and rigid labor market, the taxes on firms are higher under competitive labor market, whereas the taxes on human capital are higher under rigid labor market.*

Second, compare the stability of the 2-city equilibrium, or, looking from another angle, when the 2 cities agglomerate into one big city. It's based on the comparison of (43) and (46) and leads to the following result.

Claim 8. *Agglomeration appears more often under rigid labor market.*

Proof is given in the Appendix.

The conclusion on agglomeration seems to be inline with the evidence. The labor market is more rigid in the developing countries. And, at least for the last 40-50 years, the agglomeration process was most pronounced in the developing countries - see, e.g., Abdel-Rahman and Anas [2004].

5 Conclusion

It is generally accepted that increasing returns to scale are essential to explain the agglomeration of economic activities in the cities. In Krugman [1991] increasing returns play an important role in explaining production concentration. Scotchmer and Thisse [1992] state that the essential role of increasing returns to scale is the "Folk Theorem" of the economic geography. Increasing returns in production, given imperfect competition on the final good market, translates into increasing returns in the firms profits.

There can be different mechanisms behind increasing returns. They can emerge at individual firm level because of non-zero fixed cost - see Spence [1976], Dixit and Stiglitz [1977], Krugman [1991] or workers' increased productivity due to specialization as in Duranton [1998]. It is also possible that each firm has constant returns to scale but due to concentration, increasing returns emerge at the industry level. The underlying mechanism can be knowledge accumulation - see Romer [1986], knowledge generation - see Jacobs [1969], knowledge and ideas diffusion - see Jovanovic and Rob [1989] and information spillover - see Fujita and Ogawa [2005], Imai [2005]. The "creative" side of the cities (as opposed to the production) is becoming more and more important - see Florida [2003] for a survey.

This paper relies on the increasing returns, but it also demonstrates that the multi-sided nature of the city plays an important role in the decisions of the City Manager on taxation of the firms and citizens. Together with concentration spillover, the multi-sided nature of the city is important for the agglomeration of the economic activity.

The paper also demonstrates that the labor market structure in the city is crucial for both agglomeration of economic activities and taxation of the city manager.

The model of the paper is best suited for the analysis of the medium-size cities, which are typically based on only one industry (see Henderson [1997]). Clearly, the model of this paper has some limitations. For instance, it should be modified to capture the important distinctive characteristic of mega-cities, based on many industries and in which the interaction between these industries (through the markets of the production factors, or externalities) are crucial for the productions as well as for the citizens.

The framework of this paper can be further extended. For example, a multi-factor production technology with capital, human capital, raw labor, others inputs can be considered. An important characteristic of the firms is that they can multihome, i.e. operate in different cities, unlike most of the others production factors.

Appendix

Proof of Proposition 2

Proof. The number of firms in the city is related to the value of outside option of the marginal firm: $n_1 = F_1(\widehat{\pi})$. Similarly, the number of citizens is determined by $n_2 = F_2(\widehat{u})$. With the change of variables $(n_1, n_2) \to (\widehat{\pi}, \widehat{u})$ we obtain

$$
\begin{aligned}
SW = &\int_0^{\widehat{\pi}} \left(\varphi_1(F_1(\widehat{\pi}), F_2(\widehat{u})) - \pi\right) F_1'(\pi) d\pi + \\
&+ \int_0^{\widehat{u}} \left(\varphi_2(F_1(\widehat{\pi}), F_2(\widehat{u})) - u\right) F_2'(u) du = \\
= &\varphi_1\left(F_1(\widehat{\pi}), F_2(\widehat{u})\right) \cdot \left[F_1'(\pi)\right]_0^{\widehat{\pi}} - \int_0^{\widehat{\pi}} \pi F_1'(\pi) d\pi + \\
&+ \varphi_2\left(F_1(\widehat{\pi}), F_2(\widehat{u})\right) \cdot \left[F_2'(u)\right]_0^{\widehat{u}} - \int_0^{\widehat{u}} u F_2'(u) du
\end{aligned}
$$

So, the Socially optimal values for the outside options for the marginal firm and marginal citizen should solve the following program

$$
\begin{aligned}
&\varphi_1\left(F_1(\widehat{\pi}), F_2(\widehat{u})\right) F_1(\widehat{\pi}) - \int_0^{\widehat{\pi}} \pi F_1'(\pi) d\pi + \\
&+ \varphi_2\left(F_1(\widehat{\pi}), F_2(\widehat{u})\right) F_2(\widehat{u}) - \int_0^{\widehat{u}} u F_2'(u) du \longrightarrow \max_{\widehat{\pi}, \widehat{u}}
\end{aligned}
\tag{47}
$$

Written in this form, social welfare has clear economic interpretation. The first term $\varphi_1(\cdot)F_1(\widehat{\pi})\,(= \varphi_1(\cdot)n_1)$ represents the overall profit of the city firms. The term $\varphi_2(\cdot)F_2(\widehat{u})\,(= \varphi_2(\cdot)n_2)$ represents citizens' overall utility. The integrals represent the foregone outside opportunities. So, the whole expression represents the society net gains from joining to the city of the part of the populations of firms and citizens.

Maximization with respect to $(\widehat{\pi}, \widehat{u})$ gives

$$
\begin{aligned}
(\varphi_{11}'(\cdot)F_1'F_1(\widehat{\pi}) + \varphi_1(\cdot)F_1') - \widehat{\pi}F_1' + \varphi_{21}'F_1'F_2(\widehat{u}) = 0 \\
\varphi_{12}'(\cdot)F_2'F_1(\widehat{\pi}) + \varphi_{22}'F_2'F_2(\widehat{u}) + \varphi_2(\cdot)F_2'(\widehat{u}) - \widehat{u}F_2' = 0
\end{aligned}
$$

where $\varphi_{jk}' = \frac{\partial \varphi_j}{\partial n_k}$

After rearrangements this gives characterization of the socially optimal size of the city (number of firms and citizens).

□

Proof of Lemma 1

Proof. The inequality $\Phi_2(n_1, n_2) < \Psi_2(n_1, n_2)$ can be written as

$$A\delta n_1^{1-\delta} n_2^{\varepsilon+\delta-1} - bn_2^{1/2} < A(\varepsilon+\delta) n_1^{1-\delta} n_2^{\varepsilon+\delta-1} - \frac{3}{2} bn_2^{1/2}$$

which simplifies to $\frac{1}{2} bn_2^{1/2} < A\varepsilon n_1^{1-\delta} n_2^{\varepsilon+\delta-1}$, which gives after simple rearrangements the inequality of the Lemma. □

Solution to the program of subsection 3.5

The maximization program is

$$T(n_1, n_2) = An_1^{1-\delta} n_2^{\varepsilon+\delta} - bn_2^{3/2} - \pi n_1 - un_2 \to \max_{n_1, n_2}$$

The First Order Conditions:

$$T_1' = A(1-\delta) n_1^{-\delta} n_2^{\varepsilon+\delta} - \pi = 0$$
$$T_2' = A(\varepsilon+\delta)\delta n_1^{1-\delta} n_2^{(\varepsilon+\delta-1)} - \frac{3}{2} bn_2^{1/2} - u = 0$$

The 2-nd order derivatives:

$$T_{11}'' = -A(1-\delta)\delta n_1^{-\delta-1} n_2^{\varepsilon+\delta}$$
$$T_{22}'' = A(\varepsilon+\delta)(\varepsilon+\delta-1) n_1^{1-\delta} n_2^{(\varepsilon+\delta-2)} - \frac{3}{4} bn_2^{-1/2}$$
$$T_{12}'' = A(1-\delta)(\varepsilon+\delta)\delta n_1^{-\delta} n_2^{\varepsilon+\delta-1}$$

The second order conditions $T_{11}'' < 0$ always holds, given that $\delta < 1$; $T_{22}'' < 0$ holds for all n_1, n_2 if $\varepsilon+\delta < 1$. However, even if $\varepsilon+\delta > 1$, the condition $T_{22}'' < 0$ can hold for n_1, n_2 satisfying the first order conditions. Finally, $T_{11}'' T_{22}'' - (T_{12}'')^2 > 0$ leads to

$$\frac{3}{4} b\delta - A(\varepsilon+\delta)\varepsilon n_1^{1-\delta} n_2^{\varepsilon+\delta-3/2} > 0$$

which is the necessary and sufficient condition for the finite city to exist.

Proof of Lemma 2

Proof. Differentiate (27) and (28) on T_1^L. This gives the following equalities:

$$aN_{11}' + bN_{21}' \equiv 1 \quad (48)$$
$$cN_{11}' + dN_{21}' \equiv 0 \quad (49)$$

Its solution gives the two first formulas of the proposition.

By differentiating equations 27 and 28 with respect to T_2^L we obtain the system

$$aN_{12}' + bN_{22}' \equiv 0 \quad (50)$$
$$cN_{12}' + dN_{22}' \equiv 1 \quad (51)$$

The solution of this system gives the two last formulas of the Proposition.

□

Proof of Proposition 4

Proof. If the symmetric equilibrium exists, the taxes are determined by the first order conditions (31) and (32). In the symmetric case

$$N_1 = N_2 = \frac{1}{2} \tag{52}$$

$$a = 2A\varphi'_{11} - 2t_1 \quad , \quad b = 2A\varphi'_{12} \quad , \quad c = 2\varphi'_{21} \quad , \quad d = 2\varphi'_{22} - 2t_2 \tag{53}$$

where all the derivatives of functions φ are taken at $(N_1, N_2) = (\frac{1}{2}, \frac{1}{2})$. Substituting these expressions into (31) and (32), we obtain (36) and (37).

To obtain the existence condition, we should consider the second order conditions. The second order derivatives are computed from (29) and (30). Denote

$$T(T_1, T_2) = T_1 N_1(T_1, T_2) + T_2 N_2(T_1, T_2)$$

For the general case

$$\frac{\partial^2 T}{\partial T_1^2} = 2N'_{11} + T_1 N''_{111} + T_2 N_{211} \tag{54}$$

$$\frac{\partial^2 T}{\partial T_1 \partial T_2} = N'_{12} + N'_{21} + T_1 N''_{112} + T_2 N''_{212} \tag{55}$$

$$\frac{\partial^2 T}{\partial T_2^2} = 2N'_{22} + T_1 N''_{122} + T_2 N_{222} \tag{56}$$

For the symmetric case all the second order derivatives of N_j are equal to zero at $(1/2, 1/2)$.

To obtain this result, first notice that for all the functions a, b, c, d the derivatives on N_j at $(1/2, 1/2)$ are equal to zero:

$$\frac{\partial \omega(N_1, N_2)}{\partial N_j}|_{(\frac{1}{2}, \frac{1}{2})} = 0$$
$$\text{for } \omega(\cdot, \cdot) = a(\cdot, \cdot), b(\cdot, \cdot), c(\cdot, \cdot), d(\cdot, \cdot)$$

In fact, the derivatives can be computed from the definitions of the functions a, b, c, d in Lemma 2. Consider, for example the function $a(N_1, N_2)$:

$$a(N_1, N_2) = A^L \varphi'_{11}(L) + A^R \varphi'_{11}(R) - 2t_1 \equiv A^L \varphi'_{11}(N_1, N_2) + A^R \varphi'_{11}(1 - N_1, 1 - N_2) - 2t_1$$

Differentiating this with respect to N_1 gives

$$\frac{\partial a}{\partial N_1} = A^L \varphi''_{111}(L) - A^R \varphi''_{111}(R)$$

for the symmetric cities $A^L = A^R$ and $\varphi''_{111}(L) = \varphi''_{111}(R) = \varphi''_{111}(1/2, 1/2)$, so

$$\frac{\partial a}{\partial N_1}\left(\frac{1}{2}, \frac{1}{2}\right) = 0$$

All the other derivatives are computed in the same way.

Now consider the equalities (48) and (49). By differentiating them with respect to T_1 we obtain[19]

$$(a_1' N_{11}' + a_2' N_{21}')\, N_{11}' + aN_{111}'' + (b_1' N_{11}' + b_2' N_{21}')\, N_{21}' + bN_{211}'' \equiv 0$$
$$(c_1' N_{11}' + c_2' N_{21}')\, N_{11}' + cN_{111}'' + (d_1' N_{11}' + d_2' N_{21}')\, N_{21}' + dN_{211}'' \equiv 0$$

At the symmetric equilibrium point it is simplified to

$$aN_{111}''\left(\frac{1}{2},\frac{1}{2}\right) + bN_{211}''\left(\frac{1}{2},\frac{1}{2}\right) = 0$$
$$cN_{111}''\left(\frac{1}{2},\frac{1}{2}\right) + dN_{211}''\left(\frac{1}{2},\frac{1}{2}\right) = 0$$

The solution of this system is

$$N_{111}''\left(\frac{1}{2},\frac{1}{2}\right) = 0\ ,\ N_{211}''\left(\frac{1}{2},\frac{1}{2}\right) = 0$$

By differentiating the equalities (48) and (49) with respect to T_2 we obtain

$$N_{112}''\left(\frac{1}{2},\frac{1}{2}\right) = 0\ ,\ N_{212}''\left(\frac{1}{2},\frac{1}{2}\right) = 0$$

By considering in the same way the equalities (50) and (51) we obtain the four other second order derivatives:

$$N_{121}''\left(\frac{1}{2},\frac{1}{2}\right) = 0\ ,\ N_{221}''\left(\frac{1}{2},\frac{1}{2}\right) = 0\ ,\ N_{122}''\left(\frac{1}{2},\frac{1}{2}\right) = 0\ ,\ N_{222}''\left(\frac{1}{2},\frac{1}{2}\right) = 0$$

Go back now to the second derivatives of the objective function (54)-(56). They can be simplified to

$$\frac{\partial^2 T}{\partial T_1^2} = 2N_{11}'; \qquad \frac{\partial^2 T}{\partial T_1 \partial T_2} = N_{12}' + N_{21}'; \qquad \frac{\partial^2 T}{\partial T_2^2} = 2N_{22}'$$

and rewritten by using Lemma 2:

$$\frac{\partial^2 T}{\partial T_1^2} = 2\frac{d}{D}; \qquad \frac{\partial^2 T}{\partial T_1 \partial T_2} = -\frac{b+c}{D}; \qquad \frac{\partial^2 T}{\partial T_2^2} = 2\frac{a}{D}$$

The condition for the Hessian matrix to be negative semi-definite are:

$$\frac{d}{D} \le 0 \text{ or } \frac{a}{D} \le 0$$
$$4ad - (b+c)^2 > 0$$

It easy to see that by using $(b+c)^2 > 2bc$ we obtain $4ad > (b+c)^2 = b^2 + c^2 + 2bc > 4bc$ which means $D = ad - bc > 0$ So, the first two conditions for the Hessian matrix can be simplified to

$$d \le 0 \text{ or } a \le 0$$

which gives the statement of the Proposition after substituting the values of the functions a, b, c, d in symmetric equilibrium from (53).

□

[19] for the variable x, x_j' means the derivative with respect to the argument j, i.e. a_1' means $\frac{\partial a}{\partial N_1}$ whereas N_{11}' $\frac{\partial N_1}{\partial T_1}$ etc.

Proof of Claim 8

Proof. The conditions for stability of the 2-city configuration for the competitive and rigid labor market are given by (43) and (46), which can be rewritten as:

$$\begin{gathered} t_1\tau + 2\delta t_1(1-\delta-\varepsilon)\left[A\cdot 2^{-\varepsilon}\right] + 2A\delta\tau(1-\delta)\cdot 2^{-\varepsilon} > \\ > (1-\delta)(\varepsilon+\delta)^2\left[A\cdot 2^{-\varepsilon}\right]^2 - A^2(1-\delta)\delta(\delta-2\varepsilon+\varepsilon^2)\cdot 2^{-2\varepsilon} \end{gathered} \tag{57}$$

$$\begin{gathered} (1-\delta)t_1\tau + 2\delta t_1(1-\delta-\varepsilon)\left[K\cdot 2^{-\frac{\varepsilon}{1-\delta}}\right] > \\ > (1-\delta)(\varepsilon+\delta)^2\left[K\cdot 2^{-\frac{\varepsilon}{1-\delta}}\right]^2 \end{gathered} \tag{58}$$

Taking into account that for the case of wage higher than equilibrium on the rigid labor market, $w \geq \frac{A\delta}{2^{\varepsilon}}$, we have, according to (6), $K \leq A\cdot 2^{\frac{\varepsilon\delta}{1-\delta}}$, which gives

$$K\cdot 2^{-\frac{\varepsilon}{1-\delta}} \leq A\cdot 2^{-\varepsilon}$$

where the equality holds for the equilibrium wage.

This allows to compare the two conditions (57) and (58). First, the left-hand side of (57) is always greater than the one of (58). The right-hand side of 57 for the equilibrium wage is smaller than those of (58). If wage is not too much higher than the equilibrium one, the relation between the two right-hand sides continues to hold.

So, we can conclude that if (58) holds, then (57) holds as well. This means that if the symmetric equilibrium exists under rigid labor market, it also exists for the competitive labor market. However, it's possible to have (58) violated and (57) continue to hold. Consequently, we can conclude that agglomeration appears more often under rigid labor market.

□

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