A Life-Cycle Analysis of Social Security with Housing

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Abstract

This paper incorporates two features of housing in a life-cycle analysis of social security: housing as a durable good and housing market frictions. We find that with housing as a durable good unfunded social security substantially crowds out housing consumption throughout the life cycle. By contrast, aggregate non-durable consumption is higher when social security is present, although it is postponed until late in life. Moreover, in the presence of housing market frictions, social security lowers the aggregate home ownership rate and reduces the average size of owner-occupied housing. The effects of social security on housing position, furthermore, exhibit substantial heterogeneity across households of different income levels.

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Keywords: Durable Goods, Housing Market Frictions, Housing Tenure Choice; Social Security

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1 Introduction

For most U.S. households, both housing and social security play key roles in their consumption and saving behavior over the life cycle. The durable feature of housing distinguishes housing services from non-durables by linking the cost of housing services to financial asset returns. Housing, moreover, constitutes the largest share in most homeowners' total assets and closely interacts with households' liabilities.[1] Social security, by contrast, discourages household savings and redistributes resources from one's working years to retirement. The distinctive features of housing and social security reopen the question of how social security, as mandatory savings for future retirement, crowds out private assets and, particularly, owner-occupied housing.

This paper incorporates housing and housing tenure decision into a life-cycle analysis of social security. Our analysis shows that unfunded social security substantially reduces housing consumption, both at the aggregate level and over the life cycle. Conversely, aggregate non-durable consumption is higher when social security is present, although it is postponed until late in life.[2] Social security, moreover, lowers the aggregate home-ownership rate and reduces the average size of owner-occupied housing. The effects of social security on housing position, furthermore, exhibit substantial heterogeneity across households of different income levels.

Our economy is a general equilibrium life-cycle model with heterogeneous agents that are subject to both idiosyncratic labor-income risks and uncertain lifetimes. Two features of housing are incorporated: First, housing services are explicitly valued by households, and housing is a durable good. Under a no-arbitrage condition between housing and financial assets, this feature builds a positive link between the cost of housing services in terms of non-durables and the interest rate. As a result, housing consumption is essentially a "current" consumption good relative to non-durables.

Second, housing markets are frictional. In particular, our model incorporates three types of housing market frictions that leads to a nontrivial housing tenure decision. The first market friction is rental-market friction, captured by the assumption that rental housing depreciates at a faster rate than owner-occupied housing.[3] The presence of rental-market frictions drives a wedge between the housing rental price and the user cost of housing.

[1] According to Bucks, Kennickell and Moore (2006), on average, primary residence accounts for 32.3 percent of total household assets, and home-secured debt accounts for 75.2 percent of total liabilities in the 2004 Survey of Consumer Finance. Moreover, the proportion of families with home-secured debt amounted to 47.9 percent in 2004.

[2] Non-durable consumption in this paper corresponds to expenditures on non-durable goods, market services, service flows from consumer durables, and government consumption.

[3] This assumption also captures in a very reduced form the (non-modeled) preferential tax treatments of owning a housing versus renting.

Accordingly, home ownership is a desirable choice for housing consumption. The second market friction is down-payment constraint. Under this friction, housing tenure choice is contingent on households' disposable income. This, in turn, constrains home ownership, especially for those with binding borrowing constraints. The third market friction is housing transaction cost. With housing transaction costs, households prefer to own a house in which the expected tenure is sufficiently long. This friction, again, tends to reduce home-ownership rates, given the hump shape of housing consumption over the life cycle.

We calibrate the economy to the U.S. data. Our calibrated economy can well capture housing tenure choices of U.S. households along several dimensions, such as home-ownership rates over the life cycle and the share of owner-occupied housing in households' net worth. This renders our model a useful benchmark to explore the impacts of social security reforms on consumption, housing tenure choice and portfolio allocation.

We then study the steady-state impact of elimination of social security.[4] Our major findings can be summarized as follows:

- The impacts of eliminating social security on the two types of consumption are drastically different. Specifically, this reform increases aggregate housing consumption by 47 percent, while *reducing* aggregate non-durable consumption by two percent. Housing consumption is crowded out by social security throughout most of the life cycle and, especially, during working years. On the other hand, the presence of social security merely postpones non-durable consumption until late in life. Accordingly, the proportional increase in housing stock is about 13-percent higher than physical capital (36.2 percent versus 23.2 percent) at the aggregate level when social security is eliminated.

The above result originates from the feature of housing as a durable good.[5] In our model, the presence of social security pushes up the price of housing services in terms of non-durable good since it discourages savings in financial assets and, thus, raises the interest rate. As a consequence, households tend to substitute non-durable consumption for housing consumption throughout the life cycle. Eliminating social security, conversely, reduces the price of housing consumption, which shows up as both a lower rental price and a lower cost of debt financing or opportunity cost of home equity for homeowners. Accordingly, a lower price of "current" consumption leads not only to a substitution of current consumption for future consumption, but also to a substitution of housing consumption for non-durable consumption throughout the life cycle. We show that in a partial equilibrium context, all

[4] A detailed exploration of the transitional path of eliminating social security in our settings is desirable. However, due to the computational burden it involves, we leave this interesting issue for future research.

[5] As a result, this result holds even in an environment without housing market frictions, as we show in Section 2.

these asymmetric impacts of social security on different types of consumption and capital are missing.

- Eliminating social security increases housing position at the aggregate level. In particular, this policy reform raises the average home-ownership rate by five percent, mostly around middle age. The average size of owner-occupied housing, moreover, is boosted by 33.0 percent.
- In terms of households' portfolio allocation, the proportional increase in owner-occupied housing is significantly larger than that of financial assets when social security is eliminated. Most of the increase in owner-occupied housing, moreover, happens during middle age. By contrast, the crowding-out effect of social security on financial assets is mostly around retirement age.
- The effects of social security on housing position exhibit substantial heterogeneity across households of different income levels. When social security is abolished, home-ownership rates rise among poor households and fall among the rich. On the other hand, the average size of housing increases among the rich, while shrinking for the poor.

Two key channels in our model underlie the above effects of social security on housing position. First, the payroll taxation of social security reduces households' disposable income. Given the presence of down-payment constraints, this forces many households to be renters, despite the fact that services of owner-occupied housing are cheaper. Accordingly, eliminating payroll taxation encourages home ownership, due to a relaxation of borrowing constraints. On the other hand, this channel pushes down the average size of owner-occupied housing, as new homeowners in the regime without social security tend to purchase smaller housing than those who would choose to be homeowners in both regimes. The second channel is through the effects of social security on the price of housing consumption. In the presence of rental-market frictions, this implies a large crowding-out of social security on owner-occupied housing. This is because, for homeowners, lower housing consumption is linked to smaller owner-occupied housing. Accordingly, eliminating social security raises the desired size of owner-occupied housing, especially at ages when the home-ownership rate is high. A desired larger house, however, encourages households to postpone home purchase under both housing transaction costs and down-payment constraints, so that they may accumulate enough financial assets to meet the down-payment requirement. Therefore, when housing market frictions are present, the second channel pushes down home-ownership rates.

The effects of social security on housing position for households of different income levels depend on which of the above two channels dominates for each individual. For poor households, the first channel dominates because elimination of the payroll tax has a larger impact on their disposable income. As a result, home-ownership rates increase significantly among the poor, with new homeowners buying smaller housing than those who would be homeowners in both regimes. For the rich, the second channel dominates, as they are less subject to borrowing constraints. Hence, home-ownership rates drop among the rich, while the average size of owner-occupied housing increases significantly when social security is abolished.

The mechanisms described in this paper for social security to affect housing consumption and housing tenure choice are consistent with the following empirical findings. Using household survey data, Ruprah and Marcano (2007) study the experience of Chilean housing affordability following Chile's privatization of social security system in 1981.[6] They find that, in 1990, 84 percent of households were unable to afford a house of market conditions, while by 2003, this statistics had fallen to 61 percent. Two thirds of this improvement in affordability, moreover, was due to the reduction in mortgage interest rates for a given change in the price of a house. Furthermore, their results indicate that the deepening of mortgage markets is driven mainly by an increase in savings by private pension funds.[7] Concerning housing tenure choice, Castles (1998) explores the relationship between rates of home ownership and various indices of public welfare in 20 OECD countries. His results show a significant cross-country negative correlation between the home-ownership rate and the size of public pension expenditures. Similarly, with panel regression, Conley and Gifford (2006) find that countries with a higher total social security benefit expenditures as a percentage of GDP have lower home-ownership rates.

This paper builds upon the literature on the life-cycle portfolio choice with housing. For example, Fernández-Villaverde and Krueger (2005) develop a model of durable consumption with collateral borrowing to explore the life-cycle patterns of consumption and saving. In their model, however, housing rental markets are shut down by assumption.[8] Our modeling strategy is close in spirit to Yao and Zhang (2005) and Li and Yao (2007). As in this paper, both papers incorporate housing tenure choice and the three types of housing market frictions. Their focus, nevertheless, is the life-cycle effects of housing-price risks on housing position and portfolio allocation.[9]

Moreover, our findings add to the extensive discussion on the desirability of social secu-

[6] In 1981, a new pension system was introduced in Chile to replace the pay-as-you-go system. The new system is based on personal capitalized accounts that are administered by private institutions.

[7] Their regression results indicate that an increase in the pension-fund balance of one percent of GDP results in an increase of 0.25 percent in mortgage debt-to-GDP ratio.

[8] Yang (2008) extends the framework of Fernández-Villaverde and Krueger (2005) to allow for housing tenure choice under frictionless rental markets.

[9] Other papers in this literature include Cocco (2005), Flavin and Yamashita (2002), Sinai and Souleles (2005), Ortalo-Magné and Rady (2006) and Chambers, Garriga and Schlagenhauf (2007).

rity.[10] This literature typically treats housing consumption and other consumption as perfect substitutes, rather than treating housing as a durable good.[11] Moreover, studies in this literature have abstracted from housing tenure choice, an important margin over households' life cycle. One contribution of our paper is to highlight the feature of housing consumption as "current" consumption relative to non-durables, under which unfunded social security inevitably substitutes nondurable consumption for housing consumption throughout the life cycle.

Our study also contributes to the emerging literature on the role of housing for a variety of macroeconomic issues. For example, Díaz and Luengo-Prado (2008) and Gruber and Martin (2003) explore the role of housing for wealth distribution in the U.S. Among the business-cycle studies, Davis and Heathcote (2005) explain both the volatility of residential investment and the co-movement of residential investment with other macro variables observed in the data. Kiyotaki, Michaelides and Nikolov (2007) and Ríos-Rull and Sánchez-Marcos (2008), moreover, study the dynamics of housing prices in response to various shocks. Nonetheless, few studies have been conducted to explore the implications of housing and housing tenure decision for various fiscal policies.[12] To our knowledge, this paper is the first to explore social security in a model with housing and housing tenure choice.

The paper is organized as follows. In Section 2, we construct an economy with rental housing, calibrate it to the U.S. economy and explore the effects of eliminating social security on consumption allocations. Section 3 performs a similar exercise in a economy with housing market frictions, which we use to explore the implications of eliminating social security for housing tenure decision and portfolio allocation. Section 4 concludes. The Appendix contains a definition of competitive equilibrium, the details of calibration, the computational details, and a welfare comparison.

2 An Economy with Rental Housing

In this section, we construct a model to explore the role of housing as a durable good. To this end, the economy abstracts from housing tenure choices and is referred to as "the benchmark economy". A full-blown model with both housing as a durable good and housing tenure choice will be provided in the following section.

[10] Contributors to this literature include, among others, Feldstein (1985), Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), İmrohoroğlu, İmrohoroğlu and Joines (1995), Huggett and Ventura (1999), Conesa and Krueger (1999), Storesletten, Telmer and Yaron (1999), Krueger and Kubler (2003) and Pries (2007).

[11] One main motivation for Hubbard and Judd (1987) to explore the interaction of payroll tax and borrowing restrictions is the observed collateral requirements. In their model, however, housing is not explicitly modeled.

[12] See Gervais (2002) on the roles of the preferential tax treatment of housing capital and Jeske and Krueger (2004) on the role of implicit guarantee for Government Sponsored Enterprises.

We consider a discrete-time dynamic general equilibrium life-cycle economy with both idiosyncratic income and lifetime uncertainty. Compared to a standard life-cycle economy, the model departs in two dimensions: Housing services are explicitly valued by households, and housing is a durable good. Rent is the only option to obtain housing services. Behind the abstraction of housing tenure choice are the following two implicit assumptions: First, rental markets are frictionless; second, all households are shut down from borrowing. Under these two assumptions, all households would prefer renting.

The setup of this economy serves two purposes: First, we would like to compare the effects of social security in this simple economy with those in a standard one-asset economy (sketched at the end of this section) to highlight the roles of housing as a durable good. Second, effects of social security in this economy serve as a benchmark to be compared later with their counterparts in a more realistic economy, with both housing as a durable good and housing market frictions.

2.1 The Model

2.1.1 Demographics

Assume that the demographic structure is stationary. In each period, the economy is inhabited by a continuum of *ex ante* identical individuals, with a constant population growth rate n. Each individual can live for a maximum of J periods, working only for the first $jr-1$ periods; the retirement age jr is exogenous. For each $j=1,...,J-1$, denote $\psi_j \in (0,1)$ as the probability of surviving onto age $j+1$ conditional on living at age j. Clearly, $\psi_0 = 1$ and $\psi_J = 0$. The probability of surviving through age s is then $\Pi_{j=1}^{s}\psi_j$. Denote $\{\mu_j\}_{j=1}^{J}$ as the fraction of individuals of age j in the whole population. Clearly, the fraction of newborns is $\mu_1 = \left(1+\sum_{j=1}^{J-1}(1+n)^{-j}\,\Pi_{i=1}^{j}\psi_i\right)^{-1}$ and the fraction of individuals for age $j=2,...,J-1$ can be computed recursively by $\mu_{j+1} = (1+n)^{-1}\,\psi_j\mu_j$.

Private annuity markets are assumed to be missing. In addition, accidental bequests are collected by the government and distributed uniformly as lump-sum transfers to all agents alive next period, after production takes place.

2.1.2 Production Technologies

Aggregate output of non-durable goods, denoted as Y, is produced using physical capital and labor input according to the production function $Y = ZF(K,N)$. K is the quantity of aggregate physical capital; N is aggregate efficient labor input. Z is the level of total factor productivity (TFP), which grows at a constant rate g. We assume that F is strictly increasing in both arguments and strictly concave. Furthermore, F satisfies the Inada conditions and is homogeneous of degree one. Without loss of generality, we assume that, in this economy, there is a representative firm that hires labor and physical capital from households to produce non-durable goods in each period. The output can be either consumed or invested in physical capital or housing on a one-to-one basis. We normalize the price of non-durable goods to one. Denote X_k as aggregate investment in physical capital, which by assumption, depreciates at a rate δ^k. The law of motion for physical capital can be written as

$$K' = \left(1 - \delta^k\right) K + X_k$$

where the superscript *prime* in this paper refers to end-of-period variables.

Housing stock depreciates at the beginning of each period at a rate δ^h.

2.1.3 Preferences and Endowments

Households enter into the economy with no assets and are endowed with one unit of time in each period. Individuals of different ages differ in their labor productivity. Denote $\{\epsilon_j\}_{j=1}^{J}$ as the deterministic age profile of average labor productivity. In addition, workers of the same age face idiosyncratic shocks to their labor productivity. The stochastic process of labor productivity is assumed to be identical across individual workers and follows a finite-state Markov process $\pi\left(\eta' \mid \eta\right)$ with the state space $\eta \in \mathbf{E} = \left\{\eta_{1,} ..., \eta_N\right\}$. Assume that π has a unique stationary distribution, denoted by Π.

Households derive utility from both non-durable goods and housing service flows. Leisure is not valued, and in each period labor supply is inelastic. The lifetime utility function can then be written as

$$E\left\{\sum_{j=1}^{J} \beta^j \Pi_{j=1}^{s} \psi_j u\left(c_j, h_j\right)\right\}$$

where β is the utility discount factor. c and h are non-durable and housing consumption, respectively. The period utility function u is assumed to be strictly increasing in both arguments, strictly concave and obeys the Inada conditions. Without loss of generality, we assume that one unit of housing capital generates one unit of housing service flow, regardless of whether it is owned or rented.

2.1.4 Social Security System

The social security system in the initial steady state is a Pay-As-You-Go ("PAYG" henceforth) system. In each period, the government levies a payroll tax on current workers and distributes the tax revenue uniformly across the current retirees. In the interest of computational tractability, we assume that the level of social security benefits received by a retiree is independent of her history of social security contributions. A more realistic assumption is that the level of social security benefits is a concave function of the accumulated contribution over the working years (see Storesletten *et al.*, 1999). However, under this assumption, the state variable will increase by one dimension, which will tremendously raise the computational costs. Our specification of the unfunded social security system implies an upper bound for the degree of intra-generational redistribution inherent in it.[13]

2.1.5 Market Arrangements

Households can save in one risk-free financial asset a. To highlight the roles of housing, we assume that no borrowing is allowed. This assumption is typically made in the social security literature for standard life-cycle economies and will also be followed for the one-asset economy constructed later in this section. We assume that in each period there exist two-period-lived financial intermediaries that pool individual households' deposit of financial assets. A financial intermediary can use deposit for two purposes: to purchase housing for renting out to individual renters at the end of the current period, and to purchase physical capital to supply to the representative firm in the next period. In the next period, after production of non-durable goods takes place, financial intermediaries sell off stocks of both housing and physical capital and pay back the principal and interest to households. The market for financial intermediaries is competitive.

2.1.6 Timing and Information

In each period, the events proceed as follows. At the beginning of each period, the idiosyncratic component of labor productivity is realized. Then, the housing tenure choice is made. Next, agents supply labor to the representative firm, which also rents physical capital for non-durable-good production. Agents then receive factor payments and transfers from the government and decide how much to consume and save in financial assets. Meanwhile, the good market opens. Next, the housing-rental market opens. Renters pay rent to the financial intermediary in return for rental housing services. Finally, uncertainty about early death is revealed. All information is publicly observed. The idiosyncratic labor productivity becomes common knowledge once realized.

2.1.7 The household's recursive problem

In a stationary equilibrium, the interest rate is constant at r and the wage rate w grows at a rate g. The household's state variables are given by (a, η, j), which denote the beginning-of-period financial asset, the stochastic component of labor productivity, and age, respectively.

[13] A larger intra-generational redistribution discourages poor households from saving in financial assets, which tends to drive up the Gini coefficient for wealth. However, as we discuss below, our results regarding the effects of social security on different types of consumption are robust to our specification of benefit formula.

The problem for the household can be written as

$$
\begin{aligned}
V(a,\eta,j) &= \max_{c,,a',s}\left\{u(c,d)+\beta\psi_j\sum_{\eta'}\pi\left(\eta'\mid\eta\right)V\left(a',\eta',j+1\right)\right\} \quad (1)\\
& s.t.\\
c+pd+a' &= (1+r)a+w(1-\tau)\epsilon_j\eta+I(j)\,b+T_r\\
c,a' &\geq 0
\end{aligned}
$$

where d is service flows generated by rental housing, and T_r is the lump-sum transfer from the government. b is social security benefit per retiree. $I(j)$ is an indicator function such that

$$
I(j)=\begin{cases} 0 \; if & 1\leq j\leq jr-1\\ 1 \; if & jr\leq j\leq J\end{cases}
$$

2.1.8 The Financial Intermediary's Problem

In each period, a financial intermediary takes in deposit A'. He then uses the fund to buy capital K' and rental housing. Since the rental payment is made at the end of each period, the cost of purchasing H' units of housing is $H'(1-p)$. In the next period, the financial intermediary collects the return on capital $K'(1+r')$ and the net-of-depreciation rental housing $H'\left(1-\delta^h\right)$ and pays back the deposit with interest $A'(1+r')$.[14]

The budget constraint for a financial intermediary is

$$
A'\geq H'(1-p)+K' \quad (2)
$$

where A' is the net deposit of financial assets by households, and H' is the aggregate stock of end-of-period housing for rental purposes.

The profit-maximizing problem for a financial intermediary is

$$
\max_{A',H^{r\prime},K'}\Pi=H'\left(1-\delta^h\right)-A'\left(1+r'\right)+K'\left(1+r'\right) \quad (3)
$$

subject to (2).

Since the financial market is competitive, each financial intermediary earns zero profit in equilibrium. Plugging (2) into (3) and using the zero-profit condition, we get the equilibrium rental price for housing.

$$
p=\frac{r'+\delta^h}{1+r'} \quad (4)
$$

Equation (4) shows that an increase in the return to financial assets will push up the price of housing consumption. Such a positive linkage between the price of housing and

[14] Alternatively, we may assume that rental housing accumulated today delivers housing services tomorrow. This implies a rental price for housing $p=r'+\delta^h$. Our results are robust to different timing specifications.

interest rates makes housing services essentially a "current" consumption good relative to non-durables. Clearly, what is behind this positive linkage is the feature of housing as a durable good.

2.1.9 One-Asset Economy

To better understand the role of housing as a durable good, we construct a standard life-cycle economy close to that in Conesa and Krueger (2002).[15]

In this economy, all households consume a single non-durable good, and save in one risk-free liquid asset. There is a single production sector that rents capital and labor from households to produce the single consumption good. Moreover, we assume that the borrowing market is closed. We call this economy a "one-asset economy." Alternatively, this economy could be interpreted as one in which housing services perfectly substitute for other types of consumption, and housing is part of the stock of productive capital.

2.2 Calibration and Computation

In this section, we first describe our calibration procedure. We then discuss our solution methods for the stationary equilibrium. After that, we explore how the benchmark model performs in matching the relevant U.S. data.

2.2.1 Demographics

One period in our model corresponds to one year of calendar time. The maximum number of periods that an agent is alive $J = 66$ and households retire at period $jr = 46$. This maps into an economy in which individuals enter the labor force at age 20 and retire at age 65, with the maximum age 85. The survival probabilities $\{\psi_i\}_{i=1}^{66}$ are taken from Bell and Miller (2005). Finally, the population growth rate n is set to be 0.01, a number in line with the U.S. long-run average.

2.2.2 Technology

We need to construct measures of output, capital, the stock of housing and their investment counterparts (Y, K, H, X_k, X_h). We use data from the 2003 revision of National Income and Product Accounts (NIPA) and Fixed Asset Tables (FAT) of Bureau of Economic Analysis (BEA) for the years 1954-2000. Physical capital K is measured as the sum of private fixed assets, consumer durables, inventory stock and net foreign assets, minus the stock of private residential structures. We exclude government capital from the definition of capital stock, as our interest is in the private sector. We measure housing H as the stock of private residential

[15] The only difference between our one-asset economy and that in Conesa and Krueger (2002) is that in our model, leisure is not valued by households, while it is in theirs.

structures. Output Y corresponds to GNP plus service flows from consumer durables, minus service flows from housing, which, in NIPA, is imputed as the rental value of both tenant-occupied and owner-occupied dwelling units.[16] The definitions of X_k and X_h correspond to the definition of K and H.

The production function for non-durable good takes the Cobb-Douglas form $Y = ZK^{\alpha}N^{1-\alpha}$. We then calibrate α so that the share of capital income in output Y matches the corresponding U.S. data (see Appendix 5.2 for details). This gives $\alpha = 0.2732$. Note that this number is different from the value used in the standard growth models since our measure of physical capital and the output it produces excludes housing and housing services, respectively. The productivity growth rate $g = 0.015$, which is consistent with the long-run average growth rate of U.S. real GNP per capita.[17]

We set the annual depreciation rates for physical capital to match an average investment-capital ratio, $\frac{X_k}{K}$, of 0.1201. The corresponding value of δ^k is 0.0951. We compute the depreciation rate for housing to match an average investment-capital ratio for private residential structures, which is 0.0455. This gives $\delta^h = 0.0205$.

2.2.3 Endowments

The deterministic age profiles of labor productivity $\{\epsilon_j\}_{j=1}^{J}$ are taken from Hansen (1993). For retirees, $\epsilon_j = 0$. We follow Huggett (1996) in parameterizing the idiosyncratic component of the labor-income process. Huggett (1996) uses the following AR(1) process for the log of labor income process:

$$\log \eta_{j+1} = \rho \log \eta_j + \varepsilon_{j+1}$$

where $\varepsilon \sim N\ (0, \sigma_{\varepsilon}^2)$ and $\log \eta_1 \sim N\ (0, \sigma_{\eta_1}^2)$. Following Huggett (1996), the auto-regressive coefficient ρ and variance of innovation σ_{ε}^2 are set to be 0.96 and 0.045, respectively. The variance of labor-income shocks at initial age $\sigma_{\eta_1}^2 = 0.38$. Using the method proposed by Tauchen (1986), we approximate the continuous AR(1) process with a seven-state Markov chain. This results in a value of Gini coefficient for labor income of 0.40, which is broadly consistent with the U.S. data (0.49, see Díaz and Luengo-Prado, 2008).[18] Table 1a reports the values of η in the seven states, together with the stationary distribution of the Markov chain.

[16] Accordingly, aggregate consumption, C, in our model corresponds to the sum of non-durables, services, and imputed service flows from consumer durables minus the service flows from housing.

[17] A positive growth for TFP tilts up households' life-cycle earning profile and, thus, tends to make their borrowing constraints more binding when young, compared with the case with no secular growth. Our main findings, however, are not sensitive to the value of g.

[18] Díaz and Luengo-Prado (2008) compute the Gini coefficient for earnings using data from the 1998 Survey of Consumer Finance. Using data from CPS, PSID and CEX, Heathcote, Perri and Violante (2009, Figure 16) obtain a value of around 0.40 for Gini coefficient for equivalent household earnings in 1998.

Table 1a: Parameter Values for the Markov Chain

	$Value$	Π
η_1	0.1838	0.0637
η_2	0.2948	0.1283
η_3	0.4728	0.1955
η_4	0.7583	0.2250
η_5	1.2163	0.1955
η_6	1.9509	0.1283
η_7	3.1290	0.0637

2.2.4 Preference

We parameterize the period utility function with the standard isoelastic specification.

$$u(c,d) = \frac{\left[\left(\theta c^{\upsilon} + (1-\theta) d^{\upsilon}\right)^{\frac{1}{\upsilon}}\right]^{1-\sigma} - 1}{1-\sigma}$$

Here, $\frac{1}{1-\upsilon}$ stands for the elasticity of substitution between housing services and non-durable consumption. The coefficient of relative risk aversion σ is set to 2, which is standard in macroeconomic literature. We set $\upsilon = 0$ following Fernández-Villaverde and Krueger (2005).[19] This implies a unit elasticity of substitution between the two types of consumption. We then calibrate the utility discount factor β and the share of the non-durable consumption in the utility function θ so that both the ratio $\frac{K}{Y}$ and the ratio $\frac{H}{Y}$ are consistent with the U.S. data. We choose these two ratios as our targets because, as we will show later, the effects of social security on the two types of consumption and portfolio allocations depend critically on the composition of aggregate wealth. According to our measures, the average $\frac{K}{Y}$ and $\frac{H}{Y}$ between 1954 and 2000 are 1.682 and 1.043, respectively. This gives $\beta = 0.9585$ and $\theta = 0.895$.[20] Our calibration implies an interest rate $r = \alpha \dfrac{Y}{K} - \delta^k = 0.0673$ at the initial steady state.

2.2.5 Social Security

In the initial steady state, we choose the replacement rate ϑ so that the payroll tax rate matches it empirical counterpart. Currently, the OASI (Old-Age and Survivors Insurance) rate is 10.7 percent.[21] This implies $\vartheta = 0.483$.

[19] Fernández-Villaverde and Krueger (2005) cite several empirical studies to argue that the hypothesis $\upsilon = 0$ cannot be rejected at the five-percent level.

[20] Since both types of consumption grow at a rate g at the steady state, the reported value of calibrated β is inflated by the growth component $1 + g$.

[21] Social Security payroll-tax rate in the U.S. is 15.3 percent. Since we focus on retirement benefits, we subtract the part of the tax rate due to Medicare and Disability Insurance.

Table 1b summarizes the calibrated parameters.

Table 1b: Parameter Values for the Benchmark Economy

Symbol	Definition	Value
	Demographics	
J	Maximum age	66
jr	Retirement age	46
ψ_i	Survival probabilities	Bell et al. (2005)
n	Population growth rate	0.01
	Technology	
α	Capital share in non-durable good production function	0.2732
g	TFP growth rate	0.015
δ^k	Depreciation rate for physical capital	0.0951
δ^h	Depreciation rate for housing	0.0205
	Endowment	
ϵ_j	Deterministic productivity of agents in age j	Hansen (1993)
σ_ε^2	Variance of innovation to idiosyncratic shock	0.045
$\sigma_{\eta_1}^2$	Variance of income distribution at initial age	0.38
ρ	Autocorrelation coefficient in stochastic earning process	0.96
	Preference	
β	Discount factor in utility function	0.9585
θ	Share of non-durable consumption in utility function	0.895
σ	Coefficient of relative risk aversion	2
υ	Parameter governing elasticity of substitution in utility	0
	Market Arrangement	
$\underline{a}$	Lower bound for asset	0

2.2.6 One-Asset economy

For the one-asset economy, we adopt the same parameterization as the benchmark economy in terms of demographic features, endowment process, and the social security replacement rate. The capital K in this economy is defined as the sum of the private fixed assets, consumer durables, inventory stock and the net foreign assets. Accordingly, aggregate output, Y, corresponds to GNP plus service flows from consumer durables, and aggregate consumption, C, corresponds to the sum of non-durable goods, services, and service flows from a durable good.

The capital share in the final goods production, α, is chosen to 0.36, the average capital income share between 1954 and 2000. The period utility function is $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The coefficient of relative risk aversion, σ, is again set to 2 to be consistent with the benchmark economy. The depreciation rate for the capital stock, δ, is set to 0.0651 to match the investment-capital ratio in the U.S. data. According to NIPA, the average value of this ratio is 0.0902 between 1954 and 2000. We calibrate the utility discount factor β so that the capital-output ratio matches the wealth-income ratio in the U.S., which is 3.1. This gives $\beta = 0.9852$. Finally, the absence of borrowing opportunities implies $\underline{a} = 0$. Table 1c summarizes the parameter values specific to this economy.

Table 1c: Parameter Values for the One-Asset Economy

Symbol	Definition	Value
α	Capital share in output production function	0.360
δ	Depreciation rate for capital	0.0651
β	Discount factor in utility function	0.9852

2.2.7 Solution Methods

Since analytical solutions for this problem do not exist, we solve for the stationary equilibria of both economies by numerical methods. It is easy to show that a balanced growth path exists in this economy. On the balanced growth path, all aggregate variables grow at a constant rate $(1+g)(1+n)$. We detrend all aggregate variables so that the transformed variables are constant at the steady state.

We discretize the asset space and, for each grid point of end-of-period housing assets, use the Golden Section Search method to find the optimal level of end-of-period financial assets, which may not necessarily lie on the grid points.[22] Then, we find the optimal level of end-of-period housing assets by grid search.

In the tradition of computing general equilibrium overlapping-generations models, we solve for the households' problem by backwards induction. Appendix 5.3 summarizes the algorithm and the computational details.

2.2.8 General Features

To begin with, Table 2 reports some properties of the benchmark economy to compare with the U.S. data. The upper panel lists variables that are targeted by our calibration, while the bottom panel corresponds to non-targeted variables. Based on our calibration, the values for capital-output ratio K/Y and the value of aggregate housing-output ratio H/Y are consistent with the U.S. data. The Gini coefficient generated by our model is 0.73, which is in line with the U.S. data, 0.80 (see Budria *et. al*, 2002, Table 1). Our results shows that the large Gini coefficient generated by our model could be due to a larger fraction of agents in our model with zero wealth (22.0 percent) compared with the actual data (17.6 percent, see Wolff, 2004, Table 1). Several potential factors might explain this over-prediction. First, in our benchmark model, the young start their life with zero wealth

[22] See Chapter 10 of Press *et al.* (1992) for details of this method.

and face a growing income profile; second, the life-cycle consumption profile starts to tilt down when agents become old, due to a declining survival probability. This implies that they need less financial wealth to finance retirement consumption. Finally, the presence of social security discourages households from saving.

Table 2: General Features of The Benchmark Economy (Initial Steady State)

Variables	Ini. St. St. Values	U.S. Data
Targeted Variables		
Payroll tax rate, τ	0.107	0.107
r	6.75%	6.73%
K/Y	1.680	1.682
H/Y	1.042	1.043
Non-targeted Variables		
Gini coefficient for total wealth	0.73	0.80
Zero (or negative) wealth (%)	22.0	17.6

2.3 Policy Reform

This section explores the long-run effects of eliminating social security. Since our focus is on the long run, the policy experiment we conduct is to eliminate both social security benefits and contributions simultaneously and allow households to save through private asset markets for their retirement. We first report the aggregate statistics and explore the life-cycle profiles of consumption. We then analyze the welfare effects of this policy reform.

2.3.1 Aggregate Statistics

Table 3 summarizes the aggregate statistics. Eliminating social security leads to a 3.43-percent decrease in the interest rate. Accordingly, the capital-output ratio $\frac{K}{Y}$ increases by 23.2 percent. Interestingly, the housing-output ratio $\frac{H}{Y}$ rises by 36.2 percent, more than ten percent higher than its counterparts for physical capital. The fall in the interest rate also pushes down the price of housing consumption by 2.72 percent, under the no-arbitrage condition (4). As a result, we see that aggregate housing consumption (H) increases by 47.2 percent. Aggregate non-durable consumption, to the contrary, *decreases* by 1.6 percent. In addition, the total consumption expenditure ($C + pH$) falls by two percent. Part of this decline can be accounted for by a 1.4-percent fall in the expenditure on housing services, pH, under a lower price of housing services. Finally, the Gini coefficient for wealth drops by seven percent when social security is eliminated. Intuitively, absent social security, even poor households increase savings to prepare for retirement consumption.

In short, our aggregate statistics indicate that the effects of social security on the two types of consumption are drastically different. On the one hand, it substantially reduces aggregate housing consumption; on the other hand, aggregate non-durable consumption is larger when social security is present. Accordingly, the proportional increase in housing stock is much higher than that of physical capital when social security is eliminated.

Table 3a: Aggregate Effects of Social Security (The Benchmark Economy)

Variables	Ini. St. St.	Fin. St. St.
Replacement rate, ϑ	48.3%	0%
r	6.76%	3.69%
p	8.25%	5.53%
K/Y	1.680	2.070
H/Y	1.042	1.419
$(K+H)/Y$	2.722	3.489
C	1.27	1.25
H	1.76	2.59
$C+pH$	1.42	1.39
Gini coefficient for wealth	0.73	0.66

For comparison, we conduct the same policy reform for the one-asset economy. Note that in this economy, there is one single asset that corresponds to the sum of physical and housing capital in the data. As a result, housing services are implicitly incorporated in the aggregate output Y of the one-asset economy. The aggregate consumption, C, a non-durable good in the one-asset economy, now captures the level of all types of consumption in the real economy.

Table 3b reports the aggregate statistics for the one-asset economy. When socials security is eliminated, the interest rate drops by 2.41 percent. The aggregate wealth, as measured by the $\frac{K}{Y}$ ratio, increases by 25.8 percent. This number is close to a 28.2-percent increase in the aggregate capital-output ratio, $(K+H)/Y$, in the benchmark economy and ranges between its counterparts for physical capital and housing (23.2 percent and 36.2 percent), respectively. When social security is eliminated, we see that aggregate consumption, a non-durable good in this economy, increases by 2.1 percent. This result contrasts sharply with a slight decline in non-durable consumption in the benchmark economy.[23]

[23] The Gini coefficient for wealth at the initial steady state is lower for the one-asset economy than for the benchmark economy. This is because the calibrated β (and, thus, the value of $\beta(1+r)$) in the one-asset economy is larger than that in the benchmark economy. As a result, the fraction of the population with zero wealth in the one-asset economy is merely 12.1 percent.

Table 3b: Aggregate Effects of Social Security (One-Asset Economy)

Variables	Ini. St. St.	Fin. St. St.
Replacement rate, ϑ	48.3%	0%
r	5.22%	2.81%
K/Y	3.070	3.862
C	1.89	1.93
Gini coefficient for wealth	0.69	0.63

2.3.2 Life-Cycle Profiles

In this section, we explore the impact of eliminating social security on the life-cycle patterns of the two types of consumption.[24] Figure 1 plots the life-cycle profiles for both housing and non-durable consumption. Consistent with the data, both housing and non-durable consumption are hump-shaped. More importantly, the life-cycle impacts of social security on housing and on non-durable consumption are sharply different. Social security crowds out housing consumption throughout most of the life cycle, with a magnitude substantially larger during the working years. The effect of social security on the pattern of non-durable consumption, by contrast, is to merely postpone non-durable consumption until late in life.

Such a difference can be potentially explained by the substitution effects of a change in the price of housing consumption. As equation (4) indicates, the price of housing consumption—the rental price of housing in this economy—is positively linked to the interest rate. When social security is present, a higher interest rate makes housing consumption more expensive relative to non-durable consumption. As a result, households tend to substitute non-durable consumption for housing consumption throughout the life cycle. Eliminating social security has the opposite effect: lowering the interest rate and, thus, the price of "current" consumption. This encourages households not only to consume both types of goods earlier in life, but also to substitute housing services for non-durable consumption throughout the life cycle.

To highlight the above mechanism, we eliminate social security under the assumption that interest rates, pre-tax wages and transfers stay unchanged at their initial steady-state levels.[25] Figure 2 plots the life-cycle profiles of the two types of consumption, holding prices fixed. It is obvious that, now, the impacts of social security on the two types of consumption look very different from those in our benchmark case. First, without a change in the relative price of housing consumption, the increase in housing consumption over the

[24] The life-cycle pattern of assets and the impact of social security on it in the benchmark economy is qualitatively similar to those in the one-asset economy. The figures are available upon request.

[25] In this partial equilibrium, social security affects the life-cycle patterns of the two types of consumption mainly through its effects on lifetime wealth and the effects of payroll taxation on the tightness of borrowing constraint.

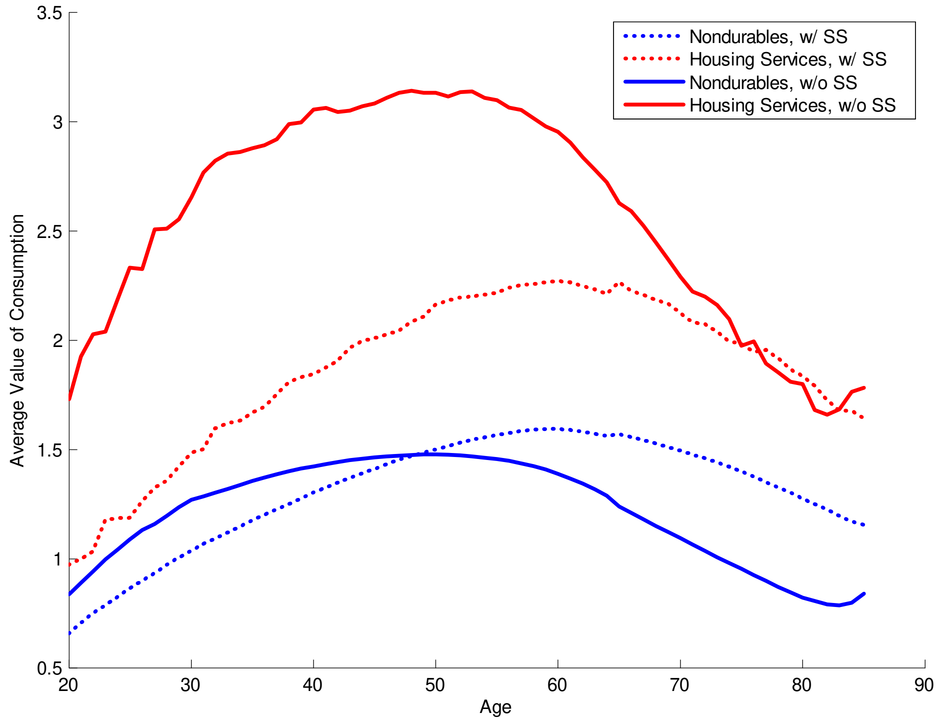

Figure 1: Housing and Non-durable Consumption over the Life Cycle, Benchmark Economy

life cycle becomes significantly dampened and similar in magnitude to that of non-durable consumption. Second, holding the price of current consumption fixed, the peaks of the two types of consumption in the final steady state are now similar to their counterparts in the initial steady states. Hence, our counterfactual experiment supports our argument that the change in the price of "current" consumption is the key mechanism by which social security affects the life-cycle profile of the two types of consumption.

In short, our life-cycle analysis reveals a sharp difference in the impacts of social security on the two types of consumption, both at the aggregate level and over the life cycle. This asymmetry results from the substitution effect of a higher price of housing consumption when social security is present. Note that this channel is missing in standard life-cycle models, which assume that all types of consumptions are non-durable and perfectly substitutable in utility. As a result, standard life-cycle economies suggest that all types of consumption will change in a similar fashion when social security is eliminated.

2.3.3 Welfare Implications

We now explore the welfare effects of eliminating social security. A caveat to our welfare analysis is that since we focus on steady-state comparison, we abstract from costs of transition associated with such a policy reform. Accordingly, our welfare analysis is, at best, suggestive

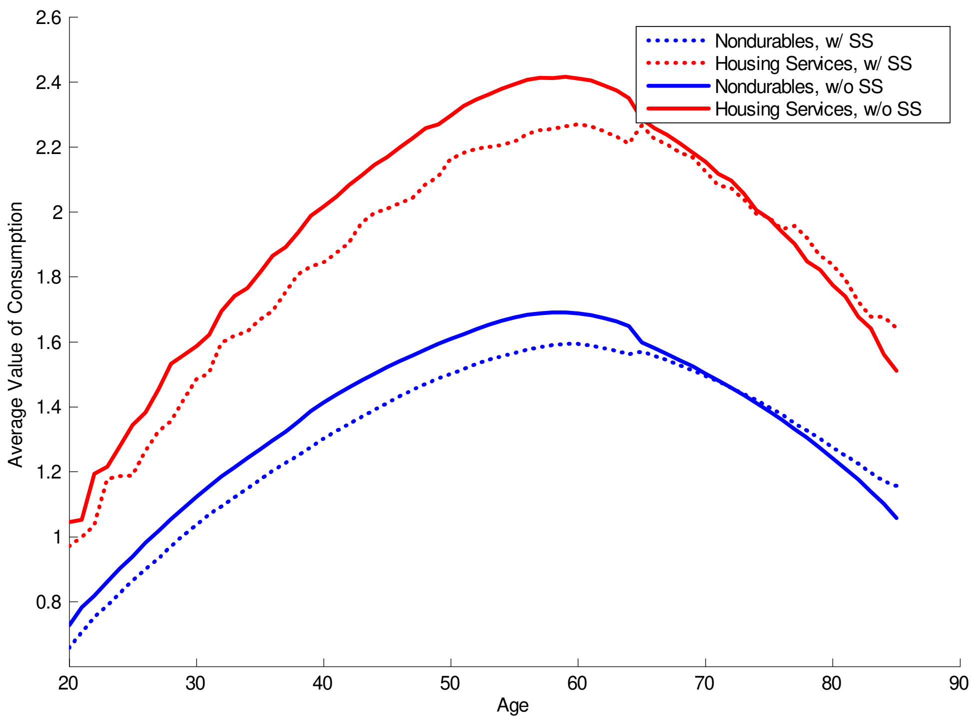

Figure 2: Effects of Social Security on Household Consumption, Fixed Prices

of how the full welfare results would change in the presence of housing.

Following the literature, the welfare effects of eliminating social security are measured by the compensating variations, denoted as CV. The compensating variations measure how much (in percent) the consumption index, defined as $c^{\theta}h^{1-\theta}$, must be increased at each period and each contingency in the economy with social security so that a given type of agent is indifferent between the two economies. The welfare gain of eliminating social security for an unborn agent (before the realization of all contingencies), denoted as w_0, is

$$w_0 = \left(\frac{\sum_{\eta \in E} \Pi \left(\eta \right) V_f(0, 0, \eta, 1)}{\sum_{\eta \in E} \Pi \left(\eta \right) V_p(0, 0, \eta, 1)} \right)^{\frac{1}{1-\sigma}} \tag{5}$$

where V_p (V_f) refers to the value in the economy with (without) social security. To better understand the welfare effects for different individuals, we classify all agents by the types of shocks to labor productivity they receive at the beginning of the first age. Agents who receive initial productivity shock η_i, $i = 1, .., 7$, are referred to as type-i agents. For a newborn type-i agent, the welfare gain of eliminating social security, denoted as w_i, is

$$w_i = \left(\frac{V_f(0, 0, \eta_i, 1)}{V_p(0, 0, \eta_i, 1)} \right)^{\frac{1}{1-\sigma}} \tag{6}$$

Note that if a given agent experiences welfare loss under the privatized system, this number is negative.

Table 4 reports the welfare effects of eliminating social security in the benchmark economy. The first column reports the welfare results for the general equilibrium. As its first row shows, an unborn agent would experience a welfare gain of about 20 percent of consumption at each state if she were born into an economy without social security. This welfare gain, moreover, holds robustly for each type of agents. Another noteworthy result is that the magnitude of the welfare gain is larger for poorer households.

Table 4: Welfare Effects of Eliminating Social Security in the Benchmark Economy

CV (%)	General Equi.	Fixed Prices	Difference
Aggregate Welfare Gain			
w_0	20.03	7.64	12.39
Welfare Gain Across Types of Agents			
w_1	25.06	7.47	17.59
w_2	23.25	7.53	15.72
w_3	21.24	7.54	13.70
w_4	19.22	7.58	11.64
w_5	17.12	7.78	9.34
w_6	14.62	8.28	6.34
w_7	11.65	8.89	2.76

We would like to understand what drives the above welfare effects, especially for the poor households. To this end, we compute the compensating variations, holding prices fixed at their initial steady-state level. The second column of Table 4 reports our results in this case. The difference from their counterparts in the general equilibrium, as listed in the third column, captures the general equilibrium effects: welfare effects of eliminating social security due to a change in factor prices.

We see that holding prices fixed, the aggregate welfare gain of eliminating social security is reduced by more than half. This indicates that the general equilibrium effect is the main channel for the welfare gain from eliminating social security. Moreover, in this partial equilibrium, richer households tend to benefit more from abolishing social security. Correspondingly, the magnitude of the general equilibrium effect declines with initial productivity types. Our counterfactual exercise, therefore, suggests that the general equilibrium effect is one key reason why poorer households tend to benefit more from eliminating social security.[26] The intuition is simple: A fall in the interest rate and an increase in wage rates

[26] Another key factor for poorer households to benefit more from eliminating social security is the presence of borrowing constraints. When borrowing constraints are missing, we find that the welfare gain for poor households becomes much smaller. This leads to a more significantly increasing pattern of welfare gain across household types.

create a larger welfare benefit for households with a higher fraction of labor earnings in total household income. This effect is missing in a partial equilibrium, in which an abolishment of redistribution from the rich to the poor tends to generate higher welfare benefits for richer households.

In Appendix 5.4.2, we compare the welfare consequences of eliminating social security among the benchmark economy, the one-asset economy, and the economy with housing tenure choice (to be constructed in the next section). Also, we decompose the overall welfare effects into several channels to understand the welfare differences among these economies.

3 An Economy with Housing-tenure Choice

Our benchmark economy abstracts away several realistic frictions in the housing markets. Accordingly, the model is silent on the observed housing tenure choice and portfolio allocation and the effects of social security along these margins. In this section, we construct a model that attempts to capture these two missing margins. In particular, we add three types of housing frictions into the benchmark model: First, there are rental-market frictions. Second, there is a down-payment requirement for housing purchase. Third, selling a house involves transaction cost. The presence of these frictions leads to a nontrivial role of housing tenure choice and portfolio allocation over a household's life cycle. We call this economy "the economy with housing tenure."

3.1 The Model

The economy shares many similarities with the benchmark model. In this economy, households derive utility from both housing services and non-durable consumption. In each period, households are subject to uninsurable idiosyncratic shocks to labor productivity and mortality risks. The only financial instrument available for households is a risk-free bond. Moreover, a constant returns-to-scale production technology turns physical capital and labor into aggregate output, which can be consumed or invested in physical capital or housing on a one-to-one basis. An unfunded social security system taxes labor earnings and redistributes them as retirement benefits uniformly across current retirees. In each period, the government collects the accidental bequest and distributes it uniformly to households alive in next period, after production takes place.[27]

The detailed difference in modeling is as follows. For consistency, the notations of all variables in this economy follow those in the benchmark economy, unless we explicitly state otherwise.

[27] The accidental bequests now include both housing and financial assets.

3.1.1 Housing Capital and Housing Services

In each period, households can either rent housing at a rental price p or own a house to obtain housing service flows. A homeowner can hold housing assets $h \in (0, \overline{h}]$, where $\overline{h}$ is a sufficiently large number so that it never binds. A non-convex transaction cost $\tau(h, h')$, in addition, is incurred each time a household changes its holdings of housing stock. The non-convex adjustment cost function ensures that the adjustment of housing assets is lumpy and infrequent. Renters, on the other hand, can change the quantity of their housing services without paying any transaction cost.

In order to purchase housing, the household must make a down payment. More specifically, a homeowner can borrow only up to a fraction $1 - \gamma$ of the value of her end-of-period housing stock, where γ is the down-payment ratio.

We model the rental-market frictions by assuming that owner-occupied housing and rental housing depreciate at different rates. Specifically, owner-occupied housing (rental housing) depreciates at a rate δ^o (δ^r) at the beginning of each period. This difference may capture the idea that, in practice, rental housing typically involves larger maintenance costs due to the moral hazard problem inherent in housing rental.

We again introduce financial institutions for simplicity. A financial intermediary can now use households' deposit for three purposes: to purchase housing for renting, to finance loans to homeowners, or to use it as physical capital to rent to the producers of non-durable good in the next period. With all borrowing and lending being intermediated and all rental-housing units being owned by financial institutions, the housing stock held by households corresponds to owner-occupied housing only. As a result, there is no need to keep track of the fraction of owner-occupied housing in a household's total housing stock.

3.1.2 The Household's Problem

Households were born into the economy with neither financial assets nor housing. With housing tenure choice, the beginning-of-period owner-occupied housing, h, enters the household's state variables. In each period, a household's problem can break down into two sub-problems. First, households choose housing tenure. Second, given housing tenure, a homeowner purchases housing, non-durable consumption, and saves or borrows into risk-free financial asset. A renter, on the other hand, chooses housing services to be purchased from financial intermediaries, non-durable consumption and savings in the risk-free financial asset.

The problem for a homeowner can be written recursively as

$$\begin{aligned} V^o(a,h,\eta,j) &= \max_{c,a',h'} \left\{ u\left(c,h'\right) + \beta\psi_j \sum_{\eta'} \pi\left(\eta' \mid \eta\right) V\left(a',h',\eta',j+1\right) \right\} && (7) \\ & s.t. \\ c + h' + a' &= (1+r)a + w(1-\tau)\epsilon_j \eta + (1-\delta^o)h - \tau(h,h') + I(j)b + T_r && (8) \\ a' &\geq -(1-\gamma)h' && (9) \\ c, h' &\geq 0 && (10) \end{aligned}$$

Equation (8) is the budget constraint for a homeowner; equation (9) is the down-payment constraint and equation (10) is the non-negative constraint for non-durables and owner-occupied housing.

The problem for a renter is similar to its counterpart in the benchmark economy.

$$\begin{aligned} V^r(a,h,\eta,j) &= \max_{c,,a',s} \left\{ u(c,d) + \beta\psi_j \sum_{\eta'} \pi\left(\eta' \mid \eta\right) V\left(a',0,\eta',j+1\right) \right\} \\ & s.t. \\ c + pd + a' &= (1+r)a + w(1-\tau)\epsilon_j \eta + (1-\delta^o)h - \tau(h,0) + I(j)b \\ c, a' &\geq 0 \end{aligned}$$

Different from a homeowner, a renter faces a non-borrowing constraint, because she has no collateral to borrow against.

For simplicity, there is no difference between interest rates for borrowing and for lending, and renegotiation of debt involves no cost. Clearly, under this assumption, a household's allocation between financial assets and debt is indeterminate given a. Therefore, we interpret a as net financial assets.

Households make housing tenure choice at the beginning of each period, by comparing the value of being a homeowner with that of being a renter.

$$V(a,h,\eta,j) = \max\{V^o, V^r\} \quad (11)$$

In our economy, housing tenure choice involves a comparison between the price of rental housing services and the shadow price of owner-occupied housing. Several factors determine the wedge between the two. First, as discussed below, the higher is the depreciation rate for rental housing compared to that of owner-occupied housing, the larger this wedge tends to be and, thus, the more likely a household is to choose to be a homeowner. Second, a household tends to choose to rent housing if the borrowing constraint it faces is tight. This is because when the borrowing constraint is binding, renting can afford more housing services. Third, the presence of housing transaction costs increases the user cost of owner-occupied housing relative to rental housing and discourages households from being homeowners.

To understand housing tenure decision, we compute the user cost of owner-occupied housing in the absence of borrowing constraints and transaction costs. One natural way to interpret the user cost of housing is from the perspective of a homeowner with a full down payment (equity finance). Since housing tenure choice is made at the beginning of each period t, the period-t cost of purchasing one unit of housing is 1, as it represents the opportunity cost of giving up one unit of deposit. At the beginning of period $t+1$, a homeowner can sell housing and, for each unit of housing, get back $1-\delta^o$, the present discounted value of which at time t is $\frac{1-\delta^o}{1+r'}$. Therefore, the net period-t cost of acquiring one unit of owner-occupied housing service is $1-\frac{1-\delta^o}{1+r'}=\frac{r'+\delta^o}{1+r'}$.

In practice, many households purchase housing with mortgage debt. Hence, it is more intuitive to reinterpret the user cost of housing as the cost of acquiring one unit of housing services with full debt financing: Suppose that, today, a household purchases one unit of housing without a down payment. Tomorrow, the homeowner can sell the housing after repaying all her outstanding debt and interest. Since the total amount of debt repayment is $1+r'$ and the revenue from selling housing is $1-\delta^o$, the cost of acquiring one unit of housing with debt financing today is $\frac{1+r'-(1-\delta^o)}{1+r'}=\frac{r'+\delta^o}{1+r'}$.

In both cases, a reduction in the shadow price of housing services due to a fall in interest rates leads to a lower user cost of housing. For a homeowner with full equity finance, this represents a reduction in the opportunity cost of home equity. For a homeowner with debt financing, this implies a lower interest payment and, thus, a lower cost of debt financing.[28] As a result, both types of homeowners tend to consume more housing services. Note that a fall in the cost of debt financing corresponds to a fall in the mortgage interest payments in the real economy, where mortgage contracts are widely used.

3.1.3 The Financial Intermediary's Problem

The financial intermediaries' problem is similar to their counterparts' in the benchmark economy. Again, we assume that the market for financial intermediaries is competitive, so that at equilibrium, each financial intermediary earns zero profit. This implies that the equilibrium rental price of housing is

$$p=\frac{r'+\delta^r}{1+r'}$$

Note that if $\delta^r>\delta^o$, the rental price is higher than the user cost of owner-occupied housing. As a result, in the absence of other housing market frictions, a household would prefer to be a homeowner.

[28] In our model, the interest rates for borrowing and for lending are the same. If we extend the current model to allow borrowing and lending rates to differ, a fall in the lending rate due to eliminating social security will trigger a fall in the borrowing rate at equilibrium. Still, this implies a lower user cost of owner-occupied housing for homeowners with both equity and debt financing.

A formal definition of stationary equilibrium is provided in Appendix 7.1.[29]

3.2 Calibration

The calibration of the demography, technology, endowment, preference and social security system follows their counterparts in the benchmark economy. The detailed difference is as follows.

The annual depreciation rate for rental housing is chosen to match the corresponding data for rental properties. Using data on actual real estate transactions for single-family houses, Shilling, Sirmans and Dombrow (1991) estimate that the average first-year depreciation rate for rental properties is 0.0254. Therefore, we choose $\delta^r = 0.0254$. We then choose values for δ^o, the discount factor, β and the share of non-durable consumption in the utility function, θ to jointly match the long-run home-ownership rate, the $\frac{K}{Y}$ ratio and the $\frac{H}{Y}$ ratio in the U.S. data. According to the American Housing Survey, the home-ownership rate in the period 1982-1994 was 64.1 percent.[30] Accordingly, $\delta^o = 0.013$, $\beta = 0.9578$ and $\theta = 0.8954$.

The transaction cost function for selling housing is set as $\tau(h, h') = I_{h'}\varphi h$, where

$$I_{h'} = \begin{cases} 1\, if \quad h' \neq h \\ 0\, if \quad h' = h \end{cases}$$

Implicit in the above function is that each time a homeowner changes her housing stock, she needs to sell her current housing assets first. This selling incurs a loss proportional to the selling price. We choose $\varphi = 0.05$, the typical fee charged by real estate brokers in the U.S. The down-payment ratio γ is set to 20 percent, which is the average down-payment ratio of primary mortgage loans in the U.S.

Table 5 summarizes the parameter values specific to this economy.

Table 5: Parameter Values for the Economy w/ Housing Tenure

Symbol	Definition	Value
δ^r	Depreciation rate for rental housing	0.0254
δ^o	Depreciation rate for owner-occupied housing	0.0130
φ	Transaction cost	0.05
β	Discount factor in utility function	0.9578
θ	Share of non-durable consumption in utility function	0.8954
γ	Down payment ratio	0.20

[29] The timing of events in each period is similar to that in the benchmark economy.

[30] After a long period of stability, the home-ownership rate in the U.S. took off in 1995, increasing from 64.7 percent in 1995 to 69 percent in 2004. Since our model is intended to capture the housing tenure choice in the long run, we ignore the period from 1995 on when computing the average home ownership rate. A larger targeted home-ownership rate requires a further decrease in δ^o, which, as we will show below, strengthens the incentive to be homeowners and our major results.

3.2.1 General Features

As a starting point, we would like to explore how well the model is able to replicate housing tenure choices, portfolio allocation and wealth inequality observed in the data. Figure 3 plots the model-generated age profile of home-ownership rates, as well as its data counterpart.[31] We see that our model is able to replicate the life-cycle profile of home ownership reasonably well. Both the model and the data feature a hump shape in home-ownership rates with a peak around age 60. The reason for this hump shape is as follows: Though rental-market frictions encourage households to purchase housing, down-payment constraints prevent households, especially the poor, from being homeowners early in life. In addition, the presence of housing transaction costs encourages households to postpone home purchase, since they would like to reduce the frequency of moving. On the other hand, as households approach the terminal period, the expected tenure in the housing becomes shorter. To avoid mandatory housing-selling cost upon death, some households will tend to be renters. We conclude that, overall, our model is able to capture the life-cycle housing tenure decision for U.S. households reasonably well.

We now ask how the model is able to match households' portfolio choice. Table 6 shows that the model generates an average share of owner-occupied housing in total net worth that is very close to its data counterpart.[32] Regarding the leverage, the model generates an average loan-to-housing-value ratio of 47.8 percent among borrowers.[33] This value is higher than its counterpart in the data (33.4 percent), measured as the mean ratio of principal residence debt to the value of the primary residence (see Wolff, 2004, Table 4). The high loan-to-value ratio in our model could result from our assumption that the minimum size of owner-occupied housing is zero. As a result, even very poor households, who tend to have high leverage, will try to use debt financing to buy owner-occupied housing.[34]

[31] The data source of the home-ownership rate is the U.S. Census Bureau. For each year between 1982 and 1999, the home-ownership rate is provided in 5-year age intervals up to age 80. We take the average of home-ownership rates across different years and linearly extrapolate the home-ownership rate for the age interval 81-85. Correspondingly, to compare with the data we also average the model-generated home ownership for each of the five-year age intervals. For example, in Figure 3, home ownership at age 20 corresponds to the average home ownership between ages 20 and 24.

[32] The data are computed by the author according to Table 4 of Wolff (2004). According to Wolff (2004), in 2001 data for the Survey of Consumer Finance, the share of principal residence in gross assets is 28.2 percent, and the share of liabilities in gross assets is 12.5 percent. Therefore, the share of principal residence in net worth (the difference between gross asset and liabilities) is 32.2 percent.

[33] Since, in our model, a household's allocation between financial assets and debt is indeterminate given a, the model-generated loan-to-value ratio constitutes a lower bound for the ratio of mortgage debt to housing value.

[34] A larger minimum size of owner-occupied housing discourages households from being homeowners due to the presence of borrowing constraints.

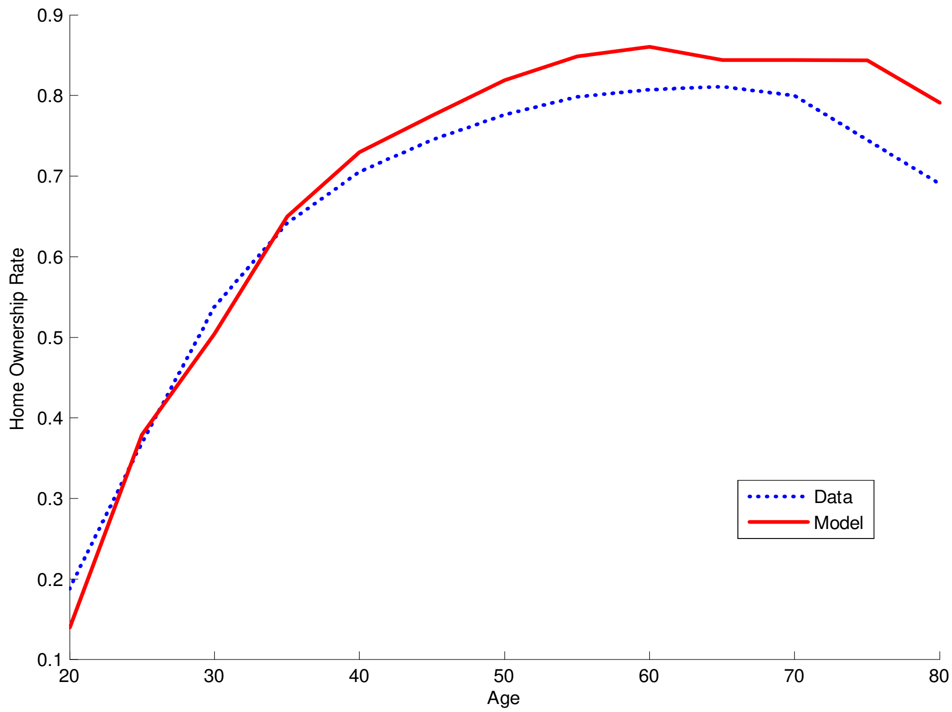

Figure 3: Home-ownership Rate over the Life Cycle, Model and Data

Table 6: General Features of The Economy with Housing Tenure (Initial Steady State)

Variables	Ini. St. St. Values	U.S. Data
Targeted Variables		
Payroll tax rate, τ	0.107	0.107
r	6.73%	6.73%
K/Y	1.682	1.682
H/Y	1.042	1.043
Home ownership rate	64%	64.1%
Non-targeted Variables		
$H^o/(A+H^o)$	32.2%	32.2%
Mean loan-to-value ratio (for borrowers)	47.8%	33.4%
Gini coefficient for total wealth	0.73	0.80
Gini coefficient for financial wealth	0.93	0.94
Gini coefficient for housing	0.53	0.64

The model also delivers implications on wealth inequality. Interestingly, our Gini coefficient for wealth, 0.73, is close in magnitude to its counterpart in the benchmark economy. The last two rows of Table 6 show that this is jointly determined by both a high Gini coefficient of financial wealth (0.93) and a relatively low Gini coefficient for housing (0.53), though our Gini coefficient for housing is lower than its data counterpart.[35] In our model, while the presence of collateral borrowing raises the inequality for financial assets, the presence of housing as a durable good pushes down inequality for net worth.

In short, our model can well capture housing tenure choice, portfolio allocation, and wealth distribution. This renders our model a useful benchmark to explore the impacts of social security reforms on housing tenure and portfolio choice and, in particular, for households with different income levels.

3.3 Policy Reform

We now consider the effects of eliminating social security in this economy. We first report the aggregate statistics and compare them with their counterparts in the benchmark economy. We then explore the implications of social security on life-cycle housing tenure choice and portfolio allocation. Finally, we investigate the effects of social security on housing position for households of different income levels.

3.3.1 Aggregate Statistics

Table 7 shows the effects of social security on various economic aggregates. Despite the sharp difference in the degrees of housing market frictions, the aggregate effects of eliminating social security are very similar in the two economies. As shown by the first two columns, in the economy with housing tenure, the proportional increase in the housing-output ratio, H/Y, (36.7 percent) is much higher than that for physical capital (23.2 percent). Similarly, aggregate housing consumption increases by 44.3 percent, while aggregate non-housing consumption slightly decreases. Housing rental prices again drop by 2.69 percent, indicating that the above asymmetric impacts result from the substitution effects of a fall in the cost of housing consumption. Note that, in this economy, a fall in the interest rate not only lowers the market price of rental housing, but also reduces the shadow price of owner-occupied housing by the same magnitude.

[35] The data for Gini coefficient on housing and financial assets are from Diaz and Luengo-Prado (2008, Table 1) and are computed using the data from the 1998 Survey of Consumer Finance.

Table 7: Aggregate Effects of Social Security in The Two Economies with Housing

	Economy w/ Housing Tenure		Benchmark Economy	
Variables	Ini. St. St.	Fin. St. St.	Ini. St. St.	Fin. St. St.
Replacement rate, ϑ	48.3%	0	48.3%	0
r	6.73%	3.68%	6.76%	3.69%
p	8.69%	6.00%	8.25%	5.53%
K/Y	1.682	2.072	1.680	2.070
H/Y	1.042	1.382	1.042	1.419
C	1.27	1.26	1.27	1.25
H	1.76	2.54	1.76	2.59
Gini coefficient for wealth	0.73	0.68	0.73	0.66
H^o	1.47	2.08	-	-
$H^o/(A+H^o)$	32.2%	32.9%	-	-
Home ownership rate	64%	67.9%	-	-
Average age of first-time home-buyers	29	28	-	-
Average size of owner-occupied housing	2.36	3.14	-	-
Mean loan-to-value ratio (for borrowers)	47.8%	45.9%	-	-
Aggregate debt-to-output ratio	8.67%	10.96%	-	-

What explains the quantitative similarity of the aggregate effects of eliminating social security? In the economy with housing tenure, the introduction of rental-market frictions encourages households to borrow to finance home purchase. Many of these households are poor and, therefore, hold a negligible fraction of aggregate physical capital and a very small fraction of aggregate housing. This is evidenced by a high Gini coefficient for financial assets and owner-occupied housing in the initial steady state. As a result, they are not important for the changes in aggregate physical capital and housing stock following social security reform. The quantitative impacts of social security on aggregate physical capital and housing are, therefore, very similar across these two economies. Accordingly, the magnitudes of a fall in the interest rate and, thus, the price of housing consumption are very close to each other.

Despite the quantitative similarities of the aggregate effects, the economy with housing tenure delivers unique implications for the effects of social security on households' portfolio allocation and housing tenure choice. First, we see that an increase in housing consumption at the aggregate level leads to a significant increase in owner-occupied housing, H^o (41.5 percent). By contrast, the percentage increase in financial asset, A, is only 36.9 percent. As a result, the share of owner-occupied housing in households' net worth slightly increases.

The increase in H^o can be further decomposed into changes in the home-ownership rate and changes in the average size of owner-occupied housing.[36] Home ownership rate, as Table 7 shows, increases by 3.9 percent when social security is privatized. Moreover, the average size of owner-occupied housing, computed as the total size of owner-occupied housing divided by the measure of homeowners, increases by 33.1 percent.

An increase in housing position also encourages more households to borrow when they become homeowners. Accordingly, the aggregate debt-to-output ratio increases by more than two percent when social security is privatized.[37] The mean loan-to-value ratio for borrowers, however, is reduced by about two percent. Intuitively, when social security is eliminated, old households have to rely on their own savings in financial assets to finance non-durable consumption. As a result, most retirees will pay off their mortgage debt and switch to net savers in financial assets (Figure not shown).

3.3.2 Life-Cycle Profile

We now investigate the life-cycle effects of social security. As Figure 4 shows, the impacts of social security on the two types of consumption are quantitatively very similar to those in the benchmark economy (Figure 1). The reason is simple: As our benchmark analysis shows, the main driving force for the asymmetric impacts of social security on the two types of consumption is the change in the relative price of housing consumption. Due to the quantitative similarity in aggregate impacts of social security, it is not surprising that its effects on the life-cycle consumption profiles are similar in the two economies.

Figure 5 illustrates the impact of eliminating social security on the life-cycle profile of home-ownership rates. We see that this reform encourages more households above age 35 to be homeowners. In particular, it boosts the home-ownership rate by about 10 percent for households between ages 40 and 60, ages when housing consumption peaks in the final steady state. The average home ownership rate for households below age 35, however, declines, and for households beyond age 75, it barely changes.

In the presence of housing market frictions, two main channels underlie the above effects of social security on home-ownership rates. The first channel stems from the presence of down-payment constraint. The payroll taxation of social security reduces households' disposable income. With down-payment constraints, this forces many of them to be renters, despite the fact that owner-occupied housing is cheaper. Accordingly, an increase in disposable income due to the elimination of payroll taxation tends to increase the home-ownership rate.

The second channel is through the effects of social security on the prices of housing

[36] Since the measure of population in our model is 1, H^o = home ownership rate $\times$ average size of owner-occupied housing.

[37] Note, again, this represents a lower bound for the increase in mortgage debt–to-output ratio.

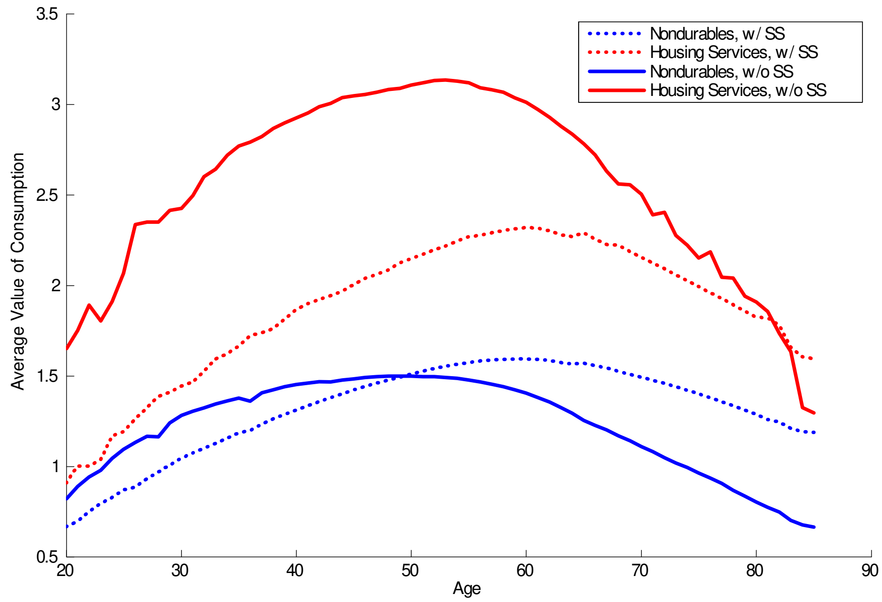

Figure 4: Housing and Non-durable Consumption over the Life Cycle, Economy with Housing Tenure

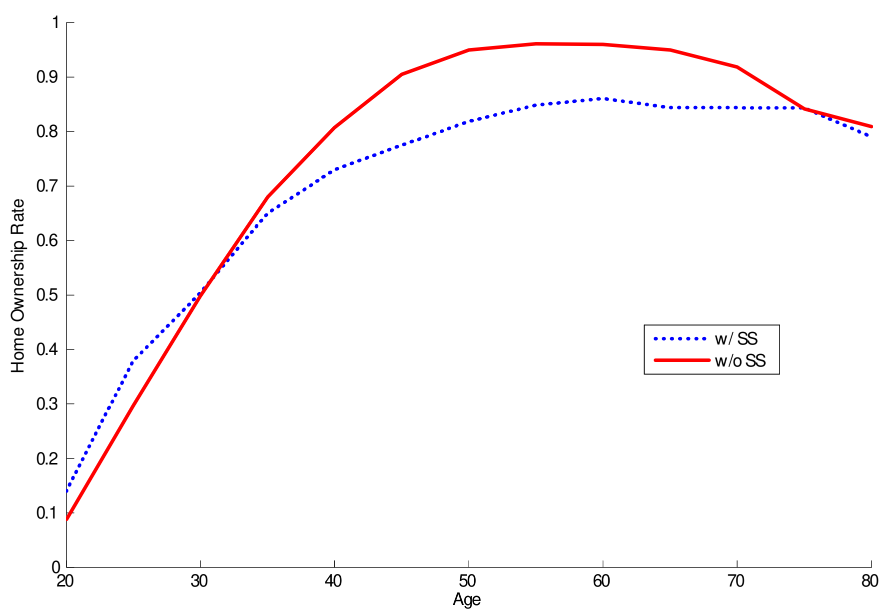

Figure 5: Home-ownership Rate Over the Life Cycle

consumption and, thus, housing consumption over the life cycle.[38] As shown in Figure 4, without social security housing consumption is much higher during the working years, with a peak at around age 50. Due to the presence of housing transaction costs, a household would prefer to own a house in which the expected tenure is sufficiently long. Eliminating social security, thus, encourages households to postpone home purchase, until they have accumulated enough financial assets to meet the down payment of a desired larger house. Moreover, housing consumption features a downward pattern along retirement ages, especially without social security. Since it is costly for households to downsize their housing frequently, eliminating social security also discourages the old from being homeowners. Note that the role of the second channel for home-ownership rates relies critically on the presence of housing transaction costs.

We now shut down the second channel by setting housing transaction costs to zero in both steady states. In this counterfactual economy, only the first channel—by relaxing the down-payment constraints—plays a role. This is because, without both borrowing constraints and housing transaction costs, social security would not affect home ownership at all: In both steady states, all households would choose to be homeowners throughout the life cycle, due to the presence of rental frictions.

Figure 6 plots the home-ownership rate between the two steady states in this counterfactual economy. Two importance differences exist between Figures 5 and 6. First, eliminating social security now increases home-ownership rates throughout the life cycle, including the youngest ages. The second noticeable difference is that, without social security, almost all retirees become homeowners except those approaching the terminal period. Accordingly, the increase in home-ownership rates tends to be higher for elderly households. These differences support our argument that the second channel is the key to understanding why home-ownership rates reduce for the young households and barely change for the old when social security is eliminated.

Figure 7 illustrates the impacts of social security on the life-cycle profiles of the two assets. We see that in each steady state, the life-cycle profiles of both assets are hump-shaped, though such a pattern is much more pronounced for financial assets than for owner-occupied housing. More importantly, the impacts of eliminating social security on the life-cycle patterns of the two assets exhibit a sharp difference. On the one hand, eliminating social security encourages households to accumulate more owner-occupied housing throughout the life cycle, especially between ages 40 and 60, when the home-ownership rate is high. The reason is simple: A higher housing consumption increases the demand for owner-occupied housing, especially during ages with a high home-ownership rate. On the other hand, this

[38] In addition, an increase in lifetime earnings due to eliminating social security will increase the desired size of owner-occupied housing. However, as our previous life-cycle analysis indicates, the quantitative importance of this effect is not large relative to the channel via changes in the price of housing services.

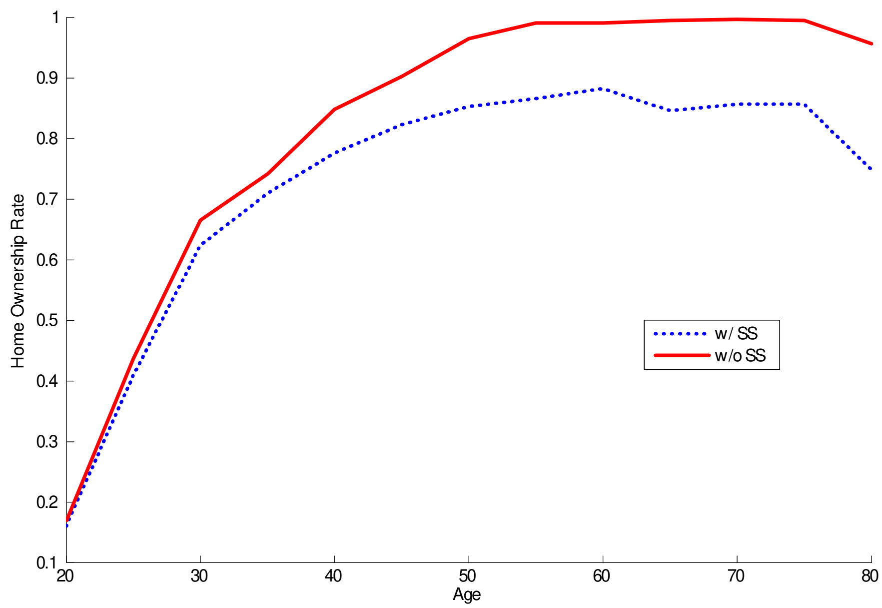

Figure 6: Effects of Social Security on Home-ownership Rate, without Housing Transaction Costs

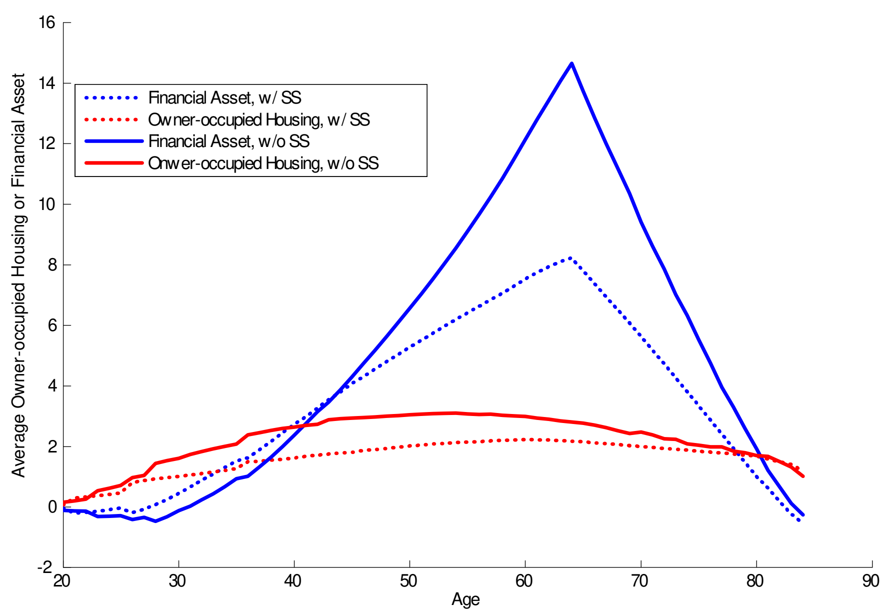

Figure 7: Households' Portfolio Allocation over the Life Cycle

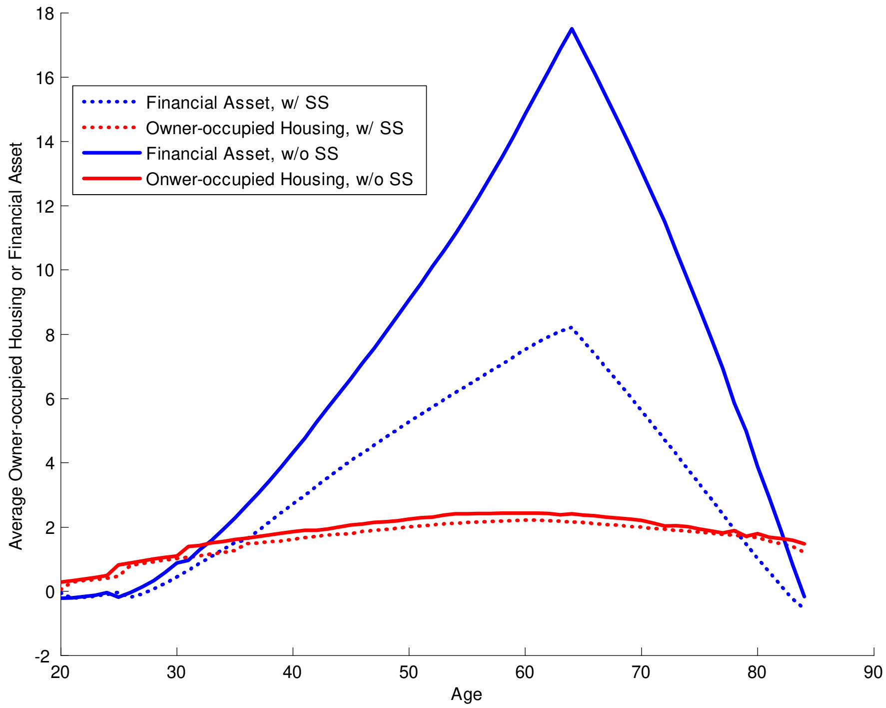

Figure 8: Effects of Social Security on Household Portfolio Allocation, Fixed Prices

policy reform leads to more borrowing and less savings in financial assets early in life. Note that the expansion in borrowing early in life is closely linked to the increased purchase of owner-occupied housing. Only around the retirement age, the crowding-out of financial assets by social security becomes significantly larger. This is because social security, as mandatory savings for retirement, serves as a substitute for financial assets.

Figure 8 plots the life-cycle profiles of the two types of assets, holding factor prices fixed as their initial steady-state levels. Under this counterfactual experiment, the life-cycle impact of social security on owner-occupied housing is mostly driven by the joint effect of a tighter borrowing constraint and a lower lifetime income. Consistent with a much smaller effect on housing consumption, we see that owner-occupied housing barely changes throughout the life cycle. This indicates that the substantial increase in owner-occupied housing in general equilibrium again is largely driven by a lower shadow price of housing consumption, or the user cost of housing.

In summary, we find that with both housing market frictions and the feature of housing as a durable good, eliminating social security significantly increases home-ownership rates, mostly around middle age. Moreover, under rental-market frictions, higher housing consumption leads to larger owner-occupied housing when social security is eliminated.

3.3.3 Heterogeneity of Impacts across Households

Our previous discussion indicates that eliminating social security increases housing positions along both the extensive margin (allowing more households to become homeowners) and the intensive margin (allowing homeowners to purchase larger housing). We now explore in more details the magnitude of these effects on households of different income levels.

Home-ownership rate Which types of agents account for the increases in the aggregate-level home-ownership rate? Table 8 reports the impacts of eliminating social security on home-ownership rate across households of different initial productivity. It is not surprising that due to down-payment constraints, home-ownership rates are higher among richer households in both steady states. More importantly, when social security is eliminated, home-ownership rates increase significantly for the poor, but fall for richer households. In other words, the increase in the home-ownership rate at the aggregate level is explained solely by the corresponding increase among the poor.

Table 8: Home-ownership Rate By Household Types

	Ini. St. St.	Final St. St.	Difference
Aggregate	64	67.9	3.9
Type 1	19.5	36.8	17.3
Type 2	27.3	45.5	18.2
Type 3	43.3	56.0	12.7
Type 4	70.1	68.1	-2.0
Type 5	85.4	81.7	-3.7
Type 6	96.5	91.7	-4.8
Type 7	99.1	98.5	-0.6

To explain such a heterogeneous pattern, recall that two opposite channels, as we discussed in the last section, govern the effects of social security on home-ownership rates for each types of households. We now shut down the second channel by setting housing transaction costs to zero. Table 9 reports the effects of social security on home-ownership rates in this counterfactual economy. Several points are worth mentioning. First, the increase in the aggregate home-ownership rate, 7.8 percent, is twice as much as its counterpart with housing transaction costs. This suggests that without housing transaction costs, households tend to become homeowners earlier in life to satisfy their increased demand for housing consumption when social security is eliminated. Second, the larger increase in the aggregate home-ownership rate in this counterfactual economy is attributed mainly to a larger corresponding increase among the rich. Combined with the results in Table 8, this suggests that with housing transaction costs, eliminating social security encourages richer households to postpone their first-time housing purchase, because they want to accumulate enough wealth to buy a larger house. Finally, the magnitude of the increase in the home-ownership rate now decreases monotonically along households' types. Obviously, what is behind this monotonic relationship is the first channel: The poorer are the households, the more binding are their borrowing constraints and the larger is the effect of eliminating social security on their disposable income. And the stronger is the impact of the first channel.

Table 9: Home-ownership Rate By Household Types (w/o Adjustment Cost)

	Ini. St. St.	Final St. St.	Difference
Aggregate	67.5	75.3	7.8
Type 1	23.1	39.8	16.7
Type 2	32.9	49.0	16.1
Type 3	47.9	63.6	15.7
Type 4	75.0	79.7	4.7
Type 5	87.9	89.4	1.5
Type 6	98.9	99.4	0.5
Type 7	99.1	99.4	0.3

Average size of owner-occupied housing Now we consider how eliminating social security affects the size of owner-occupied housing for different types of homeowners.[39] In our model, a change in the size of owner-occupied housing is driven mainly by two components: an increase in the size of housing by households who would choose to be homeowners in both steady states, referred to as the "incumbent effect"; and an effect capturing the difference between the size of housing chosen by new homeowners when social security is eliminated and those who would be homeowners in both steady states, referred to as the "entrant effect."

Through these two components, the same two key channels affect the average size of housing, apart from home-ownership rates. The first channel, through which eliminating payroll tax increases home ownership, leads to a negative entrant effect on the average size of owner-occupied housing. This is because in the final steady state, new homeowners, with a relaxed borrowing constraint, tend to buy a house smaller than those of households who would be homeowners even when social security is present. On the other hand, the second channel creates a positive incumbent effect on the average size of housing, via a lower price of housing services.

Table 10 reports the effects of social security on the average size of owner-occupied housing. As the last column shows, its increase in the average size of owner-occupied housing is concentrated among rich homeowners. For the poor, the average size of housing actually declines. This implies that the first channel dominates for the poor. Intuitively, eliminating

[39] A more careful approach to exploring the intensive margin is to track the change in the size of owner-occupied housing for existing homeowners following Social Security reform. This would involve an analysis on the transitional path, which we leave for future research.

social security encourages a large fraction of poor households who would be renters in the initial steady state to become homeowners. Since these new homeowners desire smaller housing, the average size of owner-occupied housing for poor households is pushed down in the new steady state.[40] Richer households, by contrast, are less subject to borrowing constraints. Accordingly, the effects of eliminating social security on the size of their housing works mainly through the second channel.

Table 10: Size of Owner-Occupied Housing By Household Type

	Ini. St. St.	Final St. St.	Difference
Aggregate	2.36	3.14	0.78
Type 1	1.40	1.32	-0.08
Type 2	1.64	1.60	-0.04
Type 3	1.80	2.07	0.27
Type 4	1.96	2.67	0.71
Type 5	2.33	3.40	1.07
Type 6	2.91	4.37	1.46
Type 7	3.76	5.58	1.62

In order to quantify the importance of the second channel for the size of housing belonging to different types of homeowners, we abolish social security, holding prices fixed as in the initial steady state. Table 11 reports the changes in the average size of housing under both fixed prices and general equilibrium. We see that under fixed prices, the average size of housing at the aggregate level is even smaller when social security is absent. The last column indicates that this is due mainly to a reduction of the average size of housing for rich households. Thus, our result suggests that when social security is eliminated, the significant increase in the size of housing among the rich is largely driven by the second channel, that is, via a lower user cost of housing.

Table 11: Effects of Eliminating Social Security on the Size of Owner-Occupied Housing

	Fixed Price	General Equi.	Difference
Aggregate	-0.01	0.78	-0.79
Type 1	-0.27	-0.08	-0.19
Type 2	-0.30	-0.04	-0.26
Type 3	-0.26	0.27	-0.53
Type 4	0.05	0.71	-0.66
Type 5	0.15	1.07	-0.92
Type 6	0.25	1.46	-1.21
Type 7	0.41	1.62	-1.21

[40] We find that in an economy without borrowing limits, the average sizes of housing increase for all types of households when Social Security is eliminated. The results are available upon request.

In summary, we find that the impacts of social security on housing position exhibit substantial heterogeneity among households of different income levels. For poor households, home-ownership rates rise significantly, while the average size of housing declines. By contrast, richer households tend to postpone their housing purchase to enjoy larger housing. As a consequence, the increase in the aggregate home-ownership rate is driven solely by the increase of the poor, while the rich account for the increase in the average size of housing.

4 Conclusion

In this paper, we incorporate two features of housing in a life-cycle analysis of social security: housing as a durable good and housing market frictions. We find that with housing as a durable good, unfunded social security substantially crowds out housing consumption throughout the life cycle. By contrast, aggregate non-durable consumption is higher when social security is present, although it is postponed until late in life. Moreover, in the presence of housing market frictions, social security lowers the aggregate home-ownership rate and reduces the average size of owner-occupied housing. The effects of social security on housing position, furthermore, exhibit substantial heterogeneity across households of different income levels.

It is important to note that our model leaves out several issues that warrant future research. First, in the current framework, it would be interesting to study the transitional path of eliminating social security. In particular, the key mechanism in this paper by which eliminating social security affects the price of housing consumption depends on an increase in private savings, which necessarily take time to be accumulated. Second, in our economy, bequest motives are absent and all agents were born without housing assets. Our model has the potential to distinguish the impacts of social security reform between two types of individuals: those who receive housing as bequests and those who do not. Moreover, our model abstracts from housing-price risks and the rent risk, as well as other risky assets, such as stocks. The incorporation of these uninsurable risks may reveal richer implications of the impacts of social security on portfolio allocations and welfare.

In addition, our life-cycle framework with housing tenure choice is suitable to address a variety of other macroeconomic and policy issues. Such issues may include the impact of demographics—say, the baby boom and population aging—on housing prices in the low frequency and the impacts of other retirement policies, such as the introduction of tax-deferred accounts, on households' consumption and savings.

5 Appendix

5.1 Definition of the Stationary Equilibrium

We now define the stationary equilibrium. Let $\mathbf{J}=\{1,...J\}$ and let $\mathbf{S}=\mathbf{R}\times\mathbf{R}_{+}\times\mathbf{E}\times\mathbf{J}$. Let $\mathcal{B}(\mathbf{R})$ and $\mathcal{B}(\mathbf{R}_{+})$ be the σ-algebra of $\mathbf{R}$ and $\mathbf{R}_{+}$, respectively, and $\mathcal{P}(\mathbf{E})$ and $\mathcal{P}(\mathbf{J})$ be the power set of $\mathbf{E}$ and $\mathbf{J}$, respectively. Let $\mathcal{S}=\mathcal{B}(\mathbf{R})\times\mathcal{B}(\mathbf{R}_{+})\times\mathcal{P}(\mathbf{E})\times\mathcal{P}(\mathbf{J})$ and let $\mathbf{M}$ be the set of finite measures over the measurable space $(\mathbf{S},\mathcal{S})$. Denote $s=\{a,h,\eta,j\}$ as the individual state variables. Let $\Phi(s)$ denote the measure of individuals with state s.

Definition 1 *Given a replacement rate ϑ, a stationary equilibrium consists of value function $\{V,V^{o},V^{r}\}$ for the households, a set of individual policy functions $\{c,d,a',h'\}$, production plan $\{Y,\ K,\ N\}$ for the representative firm, an allocation for financial intermediary $\left\{A',H^{r'},K'\right\}$, a set of prices $\{r,\ w,\ p\}$ and a finite measure $\Phi\in\mathbf{M}$, such that*

1. Given $\{r,w,p\}$, V, V^{o} and V^{r} solve the individual's problem (11), (7) and (1), respectively, with c, d,a', h' as the associated policy functions.

2. $\{r,w\}$ are such that the maximization problem of the representative firm is solved.

$$\begin{aligned} r &= ZF_{K}(K,N)-\delta^{k} \\ w &= ZF_{N}(K,N) \end{aligned} \tag{12}$$

3. p is such that the financial intermediary's problem is solved.

4. The social security policies satisfy

$$\begin{aligned} b &= \frac{\vartheta wN}{\int\Phi(da\times dh\times d\eta\times\{1,...,jr-1\})} \\ \tau wN &= b\int\Phi(da\times dh\times d\eta\times\{jr,...,J\}) \end{aligned}$$

5. Individual and aggregate behaviors are consistent

$$\begin{aligned} A' &= \int a'(s)\Phi(da\times dh\times d\eta\times dj) \\ N &= \int \epsilon_{j}\eta\Phi(da\times dh\times d\eta\times dj) \\ C &= \int c(s)\Phi(da\times dh\times d\eta\times dj) \\ H' &= \int h'(s)\Phi(da\times dh\times d\eta\times dj) \\ H^{r'} &= \int_{h'=0} d(s)\Phi(da\times dh\times d\eta\times dj) \end{aligned}$$

6. Transfer are given by

$$Tr = \left[\begin{array}{c} (1+r') \int \left(1-\psi_j\right) a'(s) \Phi\left(da \times dh \times d\eta \times dj\right) \\ +\left(1-\delta^o\right) \int_{h'>0} h'(s)\, \Phi\left(da \times dh \times d\eta \times dj\right) \end{array} \right] / (1+n) \tag{13}$$

7. Markets clear

(a) Housing-rental market clears.

$$H^{r'} = H' - H^{o'} \tag{14}$$

where $H^{r'} = \int_{h'=0} d(s)\Phi\left(da \times dh \times d\eta \times dj\right),\ H^{o'} = \int_{h'>0} h'(s)\, \Phi\left(da \times dh \times d\eta \times dj\right).$

(b) Goods market clears

$$C + X_h + X_k + \int \tau\left(h, h'(s)\right) \Phi\left(da \times dh \times d\eta \times dj\right) = Y \tag{15}$$

where

$$X_k = K' - \left(1-\delta^k\right) K$$

$$X_h = X_r + X_o$$

$$\begin{aligned} X_r &= H^{r'} - (1-\delta^r) H^r \\ X_o &= H^{o'} - \left(1-\delta^h\right) H^o \end{aligned}$$

(c) Asset market clears.

$$K' + H^{r'}(1-p) = A'$$

(d) All factor markets clear.

8. The law of motion for Φ *is stationary*

$$T(\Phi) = \Phi$$

where the operator $T : \mathbf{M} \to \mathbf{M}$ *can be explicitly expressed as:*

a. for all J *such that* $1 \notin J$, *all* $A \times H \times E \in \mathcal{B}(\mathbf{R}) \times \mathcal{B}(\mathbf{R}_+) \times \mathcal{P}(\mathbf{E})$, *and all* $s = \{a, h, \eta, j\} \in \mathbf{S}$

$$T(\Phi)\left(A \times H \times E \times J\right) = \int P\left(s; A \times H \times E \times J\right) \Phi\left(da \times dh \times d\eta \times dj\right)$$

where

$$P\left(s; A \times H \times E \times J\right) = \begin{cases} \sum_{\eta' \in E} \psi_j \pi\left(\eta' \mid \eta\right) & \text{if } j+1 \in J,\ a'(s) \in A \text{ and } h'(s) \in H \\ 0 & \text{else} \end{cases}$$

b. for all $A \times H \times E \in \mathcal{B}(\mathbf{R}) \times \mathcal{B}(\mathbf{R}_{+}) \times \mathcal{P}(\mathbf{E})$

$$T(\Phi)(A \times H \times E \times 1) = \begin{cases} \sum_{\eta \in E} \mu_1 \Pi(\eta) & \textit{if } 0 \in A \textit{ and } 0 \in H \\ 0 & \textit{else} \end{cases}$$

5.2 Calibration of Capital Income Share in Economies with Housing

In this section, we describe the procedure of our calibration of the capital income share α for economies with housing. We use data from the 2003 revision of National Income and Product Accounts (NIPA) and Fixed Asset Tables (FAT) of the Bureau of Economic Analysis (BEA) for the years 1954-2000. The calibration procedure follows Cooley and Prescott (1995) and Díaz and Luengo-Prado (2008), with special attention to the following issues.

Denote Y_h as services flow from housing. Then the capital share α_p in private fixed capital (excluding consumer durables and residential structures) is computed as

$$\alpha_p = \frac{UCI - Y_h + DEP}{GNP - ACI - Y_h}$$

where $UCI = rental\ income + net\ interest + corporate\ profit$ refers to unambiguous capital income. DEP denotes consumption of fixed capital. And $ACI = proprietors'\ income + indirect\ business\ taxes$.

Denote $Y_{kp} = \alpha_p(GNP - Y_h)$ as the income of physical capital (excluding consumer durables) and Y_{sd} as the service flows from consumer durables, which is computed following Cooley and Prescott (1995). Then, the capital share in the output function α is computed as

$$\alpha = \frac{Y_{kp} + Y_{sd}}{GNP + Y_{sd} - Y_h}$$

This gives a value 0.2732 for α.

5.3 Algorithm

We solve the stationary equilibrium by the following steps:

1. Guess r and Tr.
2. Solve for the individual household's decision rules by backward recursion.
3. Use forward recursion to compute the distribution Φ, and then compute the aggregate K.
4. Use equations (12) and (13) to update r and Tr, respectively.
5. Iterate on r and Tr until convergence.

5.4 Welfare Comparison

This section provides a comparison of the welfare effects of eliminating social security in the three economies: the economy with housing tenure, the benchmark economy and the one-asset economy. To understand its difference (or similarity) among these three economies, we also decompose the overall welfare gain into several components according to the following method.

5.4.1 Method of Welfare Decomposition

In our models, the potential channels by which eliminating social security affects welfare include the following: 1) General equilibrium effects, that is, welfare effects of eliminating social security arising from changes in equilibrium factor prices. 2) Intertemporal consumption smoothing effects. When households are credit-constrained, the payroll tax as mandatory savings distorts the life-cycle consumption and saving behavior. 3) Imperfect annuity effects. Without private annuity markets, the annuity form of social security benefits provides partial insurance against mortality risks, leading to a reduction in precautionary savings and accidental bequests. 4) Income risk sharing effects. The nonlinear correlation between social security contributions and benefits provides within-cohort redistribution among retirees, thereby providing partial risk sharing against idiosyncratic income uncertainties. 5) Wealth effects. When the internal return on social security contributions, $g + n$, is not equivalent to the market return on capital, r, eliminating social security will directly change lifetime wealth, thereby affecting the welfare of households.

To decompose the overall welfare effect, we conduct several counterfactual experiments similar to those in Storesletten *et al.* (1999), in which progressively fewer candidates are at work.[41] Specifically, to isolate the general equilibrium effects, we hold prices fixed at their initial steady state level and compute the associated compensating variations, denoted as w_p. The difference of the welfare gain between this economy and the benchmark economy, $w_0 - w_p$, constitutes the magnitude of general equilibrium effects. Next, to identify the intertemporal consumption smoothing effect, we extend the borrowing limit for all households to a sufficiently large level so that essentially no households are borrowing constrained.[42] The compensating variations in this economy is denoted as w_i. The intertemporal consumption smoothing effect can then be measured as $w_p - w_i$. To compute the imperfect annuity

[41] In Storesletten *et al.* (1999), the welfare decomposition does not involve the welfare gain from intertemporal consumption smoothing.

[42] Note that it is crucial to isolate the intertemporal consumption smoothing effect before we shut down income heterogeneity. This is because, as argued by Hubbard and Judd (1987), the marginal welfare loss due to the borrowing constraints is convex in the tightness of the constraint. As a result, the welfare loss of Social Security arising from distorting intertemporal consumption smoothing is underestimated in an economy without within-cohort income heterogeneity.

effects of the PAYG system, we further allow perfect annuity markets and compute the resulting compensating variations, denoted as w_a. The imperfect annuity effect is measured as $w_i - w_a$. Finally, we shut down income uncertainty and compute the associated compensating variations, denoted as w_d. The difference $w_a - w_d$ measures the income risk sharing effects, and w_d measures the welfare gain attributable to the wealth effects.

5.4.2 Welfare Results

Table 12 summarizes the welfare results of eliminating social security for the three economies. The top panel of this table reports the compensating variations under alternative experiments, while the bottom panel reports our measures of the quantitative importance for difference channels. A comparison of the overall welfare effects highlights two results: First, the magnitude of compensating variations is similar between the benchmark economy and the economy with housing tenure, despite the incorporation of housing markets frictions in the latter. Second, this number is substantially larger for economies with housing than for the one-asset economy.

Table 12: A Comparison of Welfare Effects of Eliminating Social Security

CV (%)	w/ Tenure	Benchmark	One-Asset	One-Asset (lower β)
		Welfare Gains		
Overall Welfare Effects, w_0	20.14	20.03	12.12	20.71
Partial Equilibrium, w_p	7.24	7.64	2.95	8.11
No borrowing limit, w_i	2.78	3.78	.9994	5.24
Perfect Annuity, w_a	3.49	5.57	2.47	5.99
No Uncertainty, w_d	9.01	9.02	6.85	9.72
		Welfare Decomposition		
General Equi. Effects, $w_0 - w_p$	12.90	12.39	9.17	12.60
Con. Smoothing Effect, $w_p - w_i$	4.46	3.86	1.95	2.87
Imperfect Annuity Effects, $w_i - w_a$	-0.71	-1.79	-1.47	-0.75
Inc. Risk Sharing Effects, $w_a - w_d$	-5.52	-3.45	-4.38	-3.73
Wealth Effects, w_d	9.01	9.02	6.85	9.72

The reason for the similarity of welfare gain between the two economies with housing is as follows. In both economies, welfare gains arise mainly from two channels: the general equilibrium effect and the wealth effect (see the bottom panel of Table 12). The first depends on the magnitude of changes in the factor prices, while the second depends on the gap between financial asset returns and the implicit returns of social security, $(n + g)$. Since the changes in factor prices due to eliminating social security are very similar between the two economies, the magnitude of the general equilibrium effect is similar. Moreover, as we calibrate both economies to match the same targets, the initial rate of return for financial assets are the same, rendering the wealth effect quantitatively similar between the two economies.[43]

We now explain what drives the much larger welfare gain of eliminating social security in economies with housing than in the one-asset economy. For conciseness, we compare the welfare effects between the benchmark economy and the one-asset economy. Our decomposition results indicate that the difference of the overall welfare gains between the two economies (8.02) arises mainly from three main channels: the general equilibrium effect (3.22), the intertemporal consumption smooth effect (1.91) and the wealth effect (2.17). Note that the value of calibrated β is much smaller for the benchmark economy (0.9585) than the one-asset economy (0.9852). In other words, households in the one-asset economy tend to be more patient than their counterparts in the benchmark economy. Accordingly, when social security is eliminated, households in the one-asset economy increase savings more. This will result in a larger fall in the interest rate and, thus, a larger loss of interest income. Moreover, the more patience of households in the one-asset economy leads to less distortion of social security on the life-cycle consumption profile. Finally, a higher β in the one-asset economy implies a higher $\frac{K}{Y}$ and, thus, a lower return for savings in the initial steady state. Consequently, the welfare gain from the wealth effect is smaller.

To check the quantitative effect of the value of β for the welfare gain of eliminating social security, we set the value of β in the one-asset economy to be the same as that in the benchmark economy (0.9585), while keeping all other parameter values as before.[44] As the last column of Table 12 shows, with a lower β, the welfare gain of eliminating social security increases tremendously and becomes similar in magnitude to that of the benchmark economy. Moreover, the bottom panel indicates that the major sources of this increase in welfare gain are the general equilibrium effect (3.43) and the wealth effect (2.87). This confirms that a gap in the value of calibrated β is the key reason for the large difference in the welfare gain between the economies with housing and the one-asset economy.

[43] The two economies differ in the magnitudes of the intertemporal consumption smoothing effect and the income risk sharing effect. Intuitively, in the economy with housing tenure, social security crowds out owner occupied housing, a cheaper option for housing services, especially at young ages. Hence, the distortion on intertemporal consumption smoothing is higher. On the other hand, housing transaction costs make households locally more risk-averse. This leads to a larger effect of social security on income risk sharing.

[44] The interest rate and the aggregate $\frac{K}{Y}$ ratio in the initial steady state now become 7.40 percent and 2.59, respectively.

References

[1] Auerbach, A. J. and L. J. Kotlikoff (1987), "*Dynamic Fiscal Policy*", New York: Cambridge University Press.

[2] Bell, F.C. and M.L. Miller (2005), "Life Table for the United States Social Security Area 1900-2100", Office of the Chief Actuary, Socail Security Administration, Actuarial Studies 116

[3] Bucks, B. A. Kennickell and K. Moore (2006), "Recent Changes in U.S. Family Finances: Evidence from the 2001 and 2004 Survey of Consumer Finances," *Federal Reserve Bulletin*, March

[4] Santiago Budría, J. Díaz-Giménez, V. Quadrini, and J.-V. Ríos-Rull (2002), "Updated Facts on the U.S. Distributions of Earning Income and Wealth", *Federal Reserve Bank of Minneapolis Quarterly Review*, summer, 2-35

[5] Castles, F.G. (1998), "The really Big Trade-off: Home Onwership and the Welfare State in the New Work and the Old", *Acta Politica*, 33(1), 5-19

[6] Chambers, M., C. Garriga and D. Schlagenhauf (2007), "Accounting for Changes in Homeownership Rate", forthcoming, *International Economic Review*

[7] Cocco, J. (2005), "Portfolio Choice in the Presence of Housing," *Review of Financial Studies*, 18, 535-567.

[8] Conesa, J. C. and D. Krueger (1999), "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 2, 757-795.

[9] Conley, D. and Gifford, B. (2006), "Home Ownership, Social Insurance and the Welfare State," *Sociological Forum*, 21 (1), 55-82

[10] Cooley T. F. and E. C. Prescott (1995), "Economic Growth and the Business Cycles," in Cooley, T. F. (Ed.), *Frontiers of Business Cycle Research*, Chapter 1, Princeton: Princeton University Press.

[11] Davis, M. and J. Heathcote (2005), "Housing and the Business Cycle," *International Economic Review*, 46(3), 751-784.

[12] Díaz, A. and M. J. Luengo-Prado (2008), "The Wealth Distribution with Durable Goods", *forthcoming, International Economic Review.*

[13] Feldstein, M. (1985), "The Optimal Level of Social Security Benefits," *Quarterly Journal of Economics*, C, 302-320.

[14] Fernández-Villaverde, J. and D. Krueger (2005), "Consumption and Saving over the Life Cycle; How Important are Consumer Durables?," working paper.

[15] Fernández-Villaverde, J. and D. Krueger (2006), "Consumption over the Life Cycle: Some Facts from Consumer Expenditure Survey Data," *Review of Economics and Statistics*, 89(3), 552-565 .

[16] Flavin, M. and T. Yamashita (2002), "Owner-Occupied Housing and the Composition of the Household Portfolio," *American Economic Review*, 79, 345-362.

[17] Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu (2003), "A Welfare Analysis of Social Security in a Dynamic Framework," *International Economic Review*, 44, 1247-.

[18] Gervais, M. (2002), "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49, 1461-89.

[19] Gruber, J.W. and R.M. Martin (2003), "Does Housing Wealth Make Us Less Equal? The Role of Durable Goods in the Distribution of Wealth," Working Paper, Federal Reserve Board.

[20] Hansen, G. D. (1993), "The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8, 71-80.

[21] Heathcote, J., F. Perri and G. L. Violante (2009), "Unequal We Stand: A Empirical Analysis of Economic Inequality in the United States, 1967-2006", mimeo

[22] Hubbard, G. and K. Judd (1987), " Social Security and Individual Welfare: Precautionary Savings, Liquidity Constraints and the Payroll Tax," *American Economic Review*, 77, 630-646.

[23] Huggett, M. (1996), "Wealth Distribution in Life-cycle Economies," *Journal of Monetary Economies*, 38, 469-494

[24] Huggett, M. and G. Ventura (1999), "On the distributional Effects of Social Security Reform," *Review of Economic Dynamics*, 2, 498-531.

[25] İmrohoroğlu, A., S. İmrohoroğlu, and D.H. Joines (2003), "Time Inconsistent Preferences and Social Security", Quarterly Journal of Economics, 118, 745-784

[26] Jeske, K. and D. Krueger (2005), "Housing and the Macroeconomy: The Role of Implicit Guarantees for Government Sponsored Enterprises", unpublished manuscript, University of Pennsylvania

[27] Kiyotaki, N., A. Michaelides and K. Nikolov (2007), "Winners and Losers in Housing Market", *mimeo*

[28] Krueger, D. and F. Kubler (2003), "Pareto Improving Social Security When Financial Markets are Incomplete!?," *American Economic Review*, 96, 737-755.

[29] Li, W. and R. Yao (2006), "The Life-Cycle Effects of Housing Price Changes", *Journal of Money, Credit and Banking*, vol. 39, 1375-1409

[30] Ortalo-Magné, F. and S. Rady (2006), "Housing Market Dynamics: on the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studie*s, 73, 459-485

[31] Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery (1992), *Numerical Recipes in Fortran*, Cambridge, Cambridge University Press.

[32] Pries, M. J. (2007), "Social Security Reform and Intertemporal Smoothing", *Journal of Economic Dynamics and Control,* 31, 25-54

[33] Ríos-Rull, J.-V. and V. Sánchez-Marcos (2008), "Aggregate Shocks and the Volatility of Housing Prices", *mimeo*

[34] Ruprah, I.J. and L.T. Marcano (2007), "A Meta-Impact Evaluation of Social Housing Programs: The Chilean Case", working paper, Inter-American Development Bank

[35] Shilling, J., C.F. Sirmans and J. F. Dombrow (1991), "Measuring Depreciation in Single Family Rental and Owner-Occupied Housing," *Journal of Housing Economics*, 1, 368-383

[36] Sinai, T. and N. Souleles (2005), "Owner-Occupied Housing as A Hedge against Rent Risk," *Quarterly Journal of Economics*, 763-789

[37] Storesletten, K., C. I. Telmer, and A. Yaron (1999), "The Risk Sharing Implications of Alternative Social Security Arrangements,". *Carnegie-Rochester Conference Series on Public Policy*, 50, 213-259.

[38] US Census Bureau (2003), "Net Worth and Asset Ownership of Households: 1998 and 2000", May, 70-88

[39] Wolff, E.N. (2004), "Changes in Household Wealth in the 1980s and the 1990s in the U.S.", in Wolff, E.N. (ed.), *International Perspectives on Household Wealth*, Elgar Publishing Ltd.

[40] Yang, F. (2008), "Consumption Over the Life Cycle: How Different is Housing," *forthcoming*, Review of Economic Dynamics.

[41] Yao, R. and H. H. Zhang (2005), "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraint," *Review of Financial Studies,* 18, 197-239.