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Abstract. This paper investigates household decisions, and optimal taxation in an overlapping generations model in which individual utility depends on a weighted average of consumption of ones peers — a "keeping up with the Joneses" consumption externality. In contrast to representative agent economies, the consumption externality *generally* affects steady state savings and growth rates. The nature of the externality's impact, however, critically depends on the rate at which labor productivity declines with age. For a (strongly enough) declining labor productivity (or when people gradually retire), the consumption externality *lowers* the steady state propensity to consume out of total wealth. The opposite holds for a constant labor productivity. The market economy can be decentralized by a (reverse) unfunded social security system if the rate of labor productivity decline is high (low). In contrast to previous results, the *optimal* steady state capital income tax is zero, in spite of the consumption externality.

Keywords and Phrases: Consumption externality, labor productivity, gradual retirement, overlapping generations, keeping up with the Joneses, optimal taxation, capital taxation.

JEL Classification Numbers: D91, E21, O40

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1 Introduction

Psychologists have often pointed to the fact that individuals experience happiness by doing well *relative to some reference group*. In economic terms, this observation refers to what is called *conspicuous consumption* or the desire to *keep up with the Joneses*.[1] The effects of which for household behavior in a market economy and for distortions are considered in this paper.

Economists have frequently argued that conspicuous consumption is one source for the low savings rates observed in developed countries. As everyone aims at keeping up with the Joneses, there is "overconsumption," and the savings rate is lower than optimal. But is this a valid argument? This paper will conclude that, in general, this is *not* a valid argument.

The phenomenon of conspicuous consumption is not a new one. In the past, many classical economists assumed that conspicuous consumption or the quest for status — a consumption externality, in modern terms — is an important component of the pursuit of self-interest (Kern, 2001). In *The Theory of Moral Sentiments*, Adam Smith notes:

> Though it is in order to supply the necessities and conveniences of the body that the advantages of external fortune are originally recommended to us, yet we cannot live long in the world without perceiving that the respect of our equals, our credit and rank in the society we live in, depend very much upon the degree in which we possess, or are supposed to possess those advantages. The desire of becoming the proper objects of this respect ... is perhaps the strongest of all our desires (Smith 1759, pp. 348–349).

More recently, the previous literature offers strong evidence of the *existence* of conspicuous consumption. Important contributions include Brekke and Howarth (2002), Frank (1985, 1999), Johansson-Stenman et al. (2002, 2006), Luttmer (2005), and Solnick and Hemenway (1998, 2005). The existence of externalities has been successful in

[1] In the following, we use the terms *conspicuous consumption* and *(keeping up with the Joneses) consumption externality* synonymously.

explaining a number of stylized facts. For example, externalities shed important light on the implications of status effects for happiness (Easterlin 1995, Frank 1985, Frank 1999, Scitovsky 1992), asset pricing (Abel 1999, Campbell and Cochrane 1999, Dupor and Liu 2003), optimal tax policy over the business cycle (Ljungqvist and Uhlig 2000), and optimal redistributive taxation (Boskin and Sheshinski 1978).

This evidence motivates the analysis of the theoretical effects of conspicuous consumption on consumption and savings decisions. As the framework for analysis, we employ a continuous time overlapping generations (OLG) model, in which individual labor productivity decreases with age. The representative agent model emerges as a special case of the employed framework.

This paper offers three main contributions with respect to the prior literature related to conspicuous consumption. First, conspicuous consumption generally changes the steady state propensity to consume, consumption (growth) and capital accumulation. The nature of the effects, however, critically depends on the rate at which labor productivity declines with age. If the rate of decline of the labor productivity is small, conspicuous consumption raises the steady state propensity to consume, and it lowers the steady state consumption and capital levels. The opposite is true when the rate of decline of the labor productivity is high. This result differs from what was shown for representative agent economies. Rauscher (1997), Brekke and Howarth (2002), Liu and Turnovsky (2005), and Turnovsky and Monteiro (2007) demonstrate that in a representative agent economy with exogenous labor supply, a consumption externality has *no impact* on the steady state equilibrium. In their settings, the steady state capital stock is fully determined by the Keynes-Ramsey rule, independently of a consumption externality. In this paper, we show that a consumption externality always has effects on the steady state equilibrium *in an overlapping generations framework.* This has been observed already by Garriga (2006), Wendner (2007) and Fisher et al. (2009). What has not been observed, however, is the fact that conspicuous consumption may either rise or lower the propensity to consume, depending on the rate at which labor productivity declines with age. In an overlapping generations model individual consumption levels change with age. As a consequence of the continuous inflow of new generations, individual and average consumption levels generally differ from each other. This opens a channel for the externality to have an impact on consumption and capital, as the steady state capital stock is not determined anymore by the Keynes-Ramsey rule. The rate at which labor productivity declines with age affects the differences between individual and average consumption growth rates, and thereby, the effects of conspicuous consumption.

Second, the socially optimal allocation can be decentralized by a tax on capital income and an unfunded social security system. In contrast to the results of Abel (2005) and Garriga (2006), however, the optimal steady state capital income tax rate equals zero, in spite of the presence of a consumption externality. The difference in results is caused by the fact that the propensity to consume is age-dependent in the two-period OLG model, while it is independent of age in the continuous time OLG model. Implementation of an optimal social security system affects a household's propensity to consume — thereby the intergenerational consumption growth rate — in the two-period OLG model. This effect, which is corrected for by a tax on capital income, is not seen in the continuous time OLG model.

Third, conspicuous consumption always introduces a distortion in the continuous time OLG model. If the rate at which labor productivity declines is high (low) an unfunded social security system (a reverse unfunded social security system) is capable of decentralizing the optimal allocation. With a high (low) rate of labor productivity decline, conspicuous consumption causes steady state overconsumption (underconsumption) which requires the optimal transfer scheme to increase (decrease) with age.

The results emphasize the significance of the rate at which labor productivity declines with age for the effects of conspicuous consumption. One direct implication of the above results is the following. Conspicuous consumption *either* raises the propensity to consume (in case of a low rate of labor productivity decline), *or* it yields overconsumption (in case of a high rate of labor productivity decline). But it cannot imply both effects at the same time.

Section 2 of the paper presents the economy's structure. Section 3 considers the steady state effects of conspicuous consumption in a market economy. Section 4 sets up a command optimum and shows that the optimal allocation can be decentralized by a (reverse) unfunded social security system. It discusses the impact of conspicuous consumption on the optimal social security system. Section 5 concludes the paper. The appendix contains a number of derivations and proofs that were distracting when placed in the main text.

2 The Economy's Structure

In this section, we augment the standard continuous time overlapping generations model by a "keeping up with the Joneses" consumption externality (conspicuous consumption). Individual utility not only depends on own consumption but also on a weighted average of consumption by others.

Population. An individual born at time v ("vintage") is uncertain about the length of his or her life. As in Blanchard (1985) and Buiter (1988), both the instantaneous probability of death of a cohort (the death or mortality rate), d, and the birth rate, b, are age-independent and constant over time.

At time t, the population size is $L(t)$. At each instant of time, a new cohort is born, the size of which is $b\,L(t)$. Also, at time t, the mass of people who die is $d\,L(t)$. Accordingly, for a large population size, the rate of population growth is

$$n = b - d\,. \tag{1}$$

Population at some date t_1 is given by: $L(t_1) = L(t_0)\,e^{n\,(t1-t0)}$. Without loss of generality, $L(0) = 1$. Consequently, $L(t) = e^{n\,t}$.

Denote the size of a vintage-v cohort at time t by $L(v,t)$. Under this population structure $L(v,t) = L(v,v)\,e^{-d\,(t-v)} = b\,L(v)\,e^{-d\,(t-v)} = b\,e^{n\,v}\,e^{-d\,(t-v)} = b\,e^{b\,v-d\,t}$. Similarly, the share of a vintage-v cohort in total population at time t is:

$$l(v,t) \equiv L(v,t)/L(t) = [b\,e^{b\,v}\,e^{-d\,t}]/e^{(b-d)\,t} = b\,e^{-b\,(t-v)}\,. \tag{2}$$

The expected remaining lifetime of any agent is: d^{-1}.[2] In the following, we focus on the case without population growth:

$$n = 0\,, \quad L(t) = 1\,. \tag{3}$$

As we conceptually distinguish the birth rate from the death rate, we will clarify which of the results are driven by the perpetual inflow of cohorts ($b > 0$) and which are driven by the finiteness of lifetime ($d > 0$).

Households. Time-t utility of a vintage-v household is a function $u(.)$ of consumption $c(v,t)$. The first argument in $c(.)$ refers to the birth date, and the second argument refers to time. At time t, an individual household not only cares about its *own* consumption, but also about how own consumption compares to some consumption reference level, $x(v,t)$, which is discussed below. Instantaneous utility is then given by $u(c(v,t), x(v,t))$.

In this paper, we consider the standard case of a CRRA utility function. We follow Dupor and Liu (2003) in specifying the felicity function[3] as:

$$u(c(v,t), x(v,t)) = \frac{\left[c(v,t)^{\frac{1}{1-\eta}}\, x(v,t)^{-\frac{\eta}{1-\eta}}\right]^{1-\sigma} - 1}{1-\sigma} = \frac{\left[c(v,t)\, \left(\frac{c(v,t)}{x(v,t)}\right)^{\frac{\eta}{1-\eta}}\right]^{1-\sigma} - 1}{1-\sigma}\,, \tag{4}$$

where $0 \leq \eta < 1$ is called the "reference parameter," which measures the importance of the consumption reference level. If $\eta = 0$, utility depends only on own consumption, and the model reduces to the usual model with interpersonally separable utility. If the reference parameter is strictly positive, η introduces a keeping up with the Joneses consumption externality (conspicuous consumption). The externality is reflected by the fact that utility is also derived from own consumption relative to a reference level (roughly, relative to others). The reference parameter represents the fraction of marginal utility of consumption stemming from a rise in this fraction, $c(v,t)/x(v,t)$.[4]

[2] As a special case, the representative-agent model emerges from the perpetual youth model by setting $b = 0$, and $d = -n$.

[3] This case is often referred to as the "multiplicative specification" (Galí, 1994).

[4] Johansson-Stenman et al. (2002) call this fraction the "marginal degree of positionality." Formally, define $r = c/x$, and $\tilde{u}(c,r) \equiv u(c,x)$. Then $\eta = [\partial\,\tilde{u}/\partial\, r\ \partial\, r/\partial\, c]/[(\partial\,\tilde{u}/\partial\, c) + (\partial\,\tilde{u}/\partial\, r\ \partial\, r/\partial\, c)]$.

If, say, $\eta = 0.2$, then 20% of marginal utility of consumption stems from a rise in $c(v,t)/x(v,t)$, while the remaining 80% directly come from a rise in own consumption $c(v,t)$, holding constant the fraction $c(v,t)/x(v,t)$.

Parameter σ governs the intertemporal elasticity of substitution. If $\eta = 0$, the intertemporal elasticity of substitution is given by σ^{-1}. If, however, $\eta > 0$, all parameters, σ, η, and ε determine the elasticity of substitution between consumption at any two points in time.

Consumption Reference Level. The consumption reference level of a household is a weighted geometric mean of others' consumptions of its own cohort, $\bar{c}(v,t)$, and average consumption of society, $c(t)$:

$$x(v,t) = \bar{c}(v,t)^{\varepsilon}\, c(t)^{1-\varepsilon}\,, \quad 0 \le \varepsilon < 1\,, \tag{5}$$

where ε and $(1-\varepsilon)$ represent the weights attached to consumption of one's own cohort and average consumption of society, $c(t) \equiv \int_{-\infty}^{t} l(v,t)\, c(v,t)\, dv$ respectively. In the special case in which ε approaches unity (in which $\varepsilon = 0$) consumption of one's own cohort (of society) represents the only frame of reference for consumption.

Sign Restrictions. First, $0 \le \eta < 1$ ensures positive marginal utility of consumption and quasiconcavity of the utility function. Second, $\sigma > 1$, which is overwhelmingly suggested by the literature, ensures decreasing marginal utility and strict concavity of $u(.)|_{\bar{c}(v,t)=c(v,t)}$ in $c(v,t)$. In particular, these sign restrictions imply:

$$\tilde{\sigma} \equiv \frac{(\sigma - \eta) - \varepsilon\, \eta(\sigma - 1)}{1 - \eta} \ge \sigma > 1\,. \tag{6}$$

At time t, expected remaining lifetime utility of a cohort born at date v is:

$$U(v,t) = \int_{t}^{\infty} u(c(v,s), x(v,s))\, e^{-(\rho+d)\,(s-t)}\, ds\,, \tag{7}$$

where ρ is the household's pure rate of time preference. The possibility of death $(d > 0)$ leads to a subjective discount rate $(\rho + d)$ higher than the pure rate of time preference.

Production. There is a large number of competitive, identical firms. The representative firm produces a homogeneous output, Y, according to

$$Y(t) = A\,K(t)^{\alpha}\,N(t)^{1-\alpha}\,, \quad A > 0,\, 0 < \alpha < 1\,, \tag{8}$$

where K and N are capital and *effective* labor services, and A is total factor productivity. Any *individual* laborer's productivity depends on her age. In particular, age-dependent productivity, $\pi(t-v)$, develops according to

$$\pi(t-v) = e^{-\lambda\,(t-v)}\,, \quad \lambda(\tilde{\sigma}-1) < \rho + d\,, \tag{9}$$

where we refer to λ as the rate at which individual labor productivity declines with age — although one might also think of λ as the rate of (exogenous) gradual retirement. The formulation can be extended to accommodate more complex paths of productivity profiles (e.g., an inverse U-shaped pattern). In the following, however, the main purpose of considering nonconstant productivity paths is to make the present value of future wage payments dependent on the profile of individual labor productivity. The simplest formulation of which is given by (9).

Labor supply, $L(t)$, and effective labor supply, $N(t)$, are related as follows:

$$N(t) = \int_{-\infty}^{t} \pi(t-v)\,L(v,t)\,dv = L(t)\int_{-\infty}^{t} b\,e^{-(b+\lambda)(t-v)}\,dv = L(t)\,\frac{b}{b+\lambda}\,, \tag{10}$$

that is, aggregate effective labor supply declines in λ. Let $y(t) \equiv Y(t)/L(t)$, and $k(t) \equiv K(t)/L(t)$ denote the average product and the capital labor ratio respectively:

$$y(t) = A\,k(t)^{\alpha}\left[\frac{b}{b+\lambda}\right]^{1-\alpha}. \tag{11}$$

Firms maximize profits and hire factors from households on competitive factor markets:

$$r(t) + \delta = \alpha\,A\;k(t)^{(\alpha-1)}\left[\frac{b}{b+\lambda}\right]^{1-\alpha}, \tag{12}$$

$$w(v,t) = (1-\alpha)\,A\,k(t)^{\alpha}\left[\frac{b}{b+\lambda}\right]^{-\alpha} e^{-\lambda\,(t-v)}\,, \tag{13}$$

$$w(t) = \int_{-\infty}^{t} l(v,t)\,w(v,t)\,dv = (1-\alpha)\,A\,k(t)^{\alpha}\left[\frac{b}{b+\lambda}\right]^{1-\alpha}, \tag{14}$$

where $r(t)$ is the rate of interest, $w(v,t)$ is the wage rate, $w(t)$ is the wage rate per effective unit of labor, and δ is the rate of depreciation of capital. According to the resource constraint, the average stock of capital evolves according to:

$$\dot{k}(t) = y(t) - c(t) - \delta\, k(t)\,, \tag{15}$$

where $y(t)$ is negatively affected by the productivity parameter λ, as seen in (11).

Consumption. Households do not have a bequest motive. They can buy fair life annuity contracts from life insurance companies, for which they pay or receive the annuity rate of interest $r^A(t)$. The contracts are canceled upon death of an individual. Actuarial fairness requires $r^A(t) = r(t) + d$. The annuity interest factor is given by: $R^A(t_0,\, t_1) \equiv \int_{t_0}^{t_1} [r(s) + d]\, d\, s$.

Every household inelastically supplies labor services and chooses consumption at all $t \geq v$ such as to maximize expected lifetime utility (7) subject to its intertemporal budget constraint:[5]

$$a(v,t) + h(v,t) - \int_t^\infty c(v,s)\, e^{-R^A(t,s)}\, ds = 0\,, \tag{16}$$

where $a(v,t)$ stands for time-t assets (accumulated wealth) of a vintage-v household, and human wealth $h(v,t) \equiv \int_t^\infty w(v,s)\, e^{-R^A(t,s)} ds$ is the discounted integral of present and future wage payments. In the market framework, a household does *not* consider the impact of its individual consumption on the consumption reference level.

Individual consumption levels are derived by applying Pontryagin's maximum principle, in the appendix. Define:

$$\Delta(t) \equiv \int_t^\infty e^{\int_t^s \frac{r(\tau)-\rho}{\tilde{\sigma}} + \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}}\frac{\dot{c}(\tau)}{c(\tau)}\, d\,\tau - R^A(t,s)} d\, s\,. \tag{17}$$

Then:

$$c(v,t) = \Delta^{-1}\, [a(v,t) + h(v,t)]\,, \quad c(t,t) = \Delta^{-1}\, h(t,t)\,, \tag{18}$$

[5] The transversality condition required to prevent households from running Ponzi schemes is: $\lim_{s\to\infty} e^{-R^A(t,s)}\, a(v,s) = 0$, or, equivalently, $\lim_{s\to\infty} \mu_a(s)\, e^{-(\rho+d)s}\, a(v,s) = 0$, where μ_a is the shadow price of accumulated wealth. Budget constraint (16) follows from combining the flow budget constraint, $\dot{a}(v,t) = r^A(t)\, a(v,t) + w(v,t) - c(v,t)$, with the transversality condition.

where the second expression follows from the fact that there is no operative bequest motive: $a(t,t)=0$. Consumption levels are proportional to (accumulated and human) wealth, with the *age-independent* factor of proportionality given by: $\Delta^{-1}(t)$, which can be interpreted as the propensity to consume out of total wealth. Notice that at any given point in time, consumption *levels* are *not* equal across cohorts.

Individual consumption growth rates are given by:

$$g_{c(v,t)} \equiv \frac{\dot{c}(v,t)}{c(v,t)} = \frac{[r(t)-\rho]}{\tilde{\sigma}} + \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}}\frac{\dot{c}(t)}{c(t)}\,. \tag{19}$$

Consumption growth rates are equal across cohorts. Individual consumption growth rates do not only depend on the rate of interest and the pure rate of time preference, but they also depend *positively* on the growth rate of average consumption.

Average accumulated wealth, $a(t)$, is given by $a(t) \equiv \int_{-\infty}^{t} l(v,t)\,a(v,t)\,dv$. Capital market clearing requires:

$$a(t) = k(t)\,. \tag{20}$$

Finally, average human wealth, $h(t)$, is:

$$h(t) = \int_{-\infty}^{t} l(v,t)\,h(v,t)\,dv = \frac{b}{b+\lambda}\,h(t,t) = \frac{b}{b+\lambda}\int_{t}^{\infty} w(s)\,e^{-R^A(t,s)}\,e^{-\lambda(s-t)}\,d\,s\,. \tag{21}$$

We are now ready to represent a perfect foresight equilibrium by a series of three differential equations in the variables c, k, Δ:

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\sigma} + \frac{\tilde{\sigma}}{\sigma}\left[\lambda - (b+\lambda)\,\Delta^{-1}\,\frac{k(t)}{c(t)}\right] \tag{M.1}$$

$$\dot{k}(t) = A\,k(t)^{\alpha}\left[\frac{b}{b+\lambda}\right]^{1-\alpha} - c(t) - \delta\,k(t)\,, \tag{M.2}$$

$$\dot{\Delta}(t) = -1 + \Delta(t)\left[r(t) + d - \frac{r(t)-\rho}{\tilde{\sigma}} - \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}}\frac{\dot{c}(t)}{c(t)}\right]\,, \tag{M.3}$$

where $r(t) = \alpha\,A\;k(t)^{(\alpha-1)}\;\left[\frac{b}{b+\lambda}\right]^{1-\alpha} - \delta$. Equation (M.1) is derived by differentiation of average consumption with respect to time and using (19).[6] Equation (M.2) restates the resource constraint. Equation (M.3) follows right from differentiation of (17).

[6]Denote individual consumption growth by $g_{c(v,t)}$. Here, we consider the fact $\dot{c}(t) = b\,c(t,t) - b\,c(t) + g_{c(v,t)}\,c(t)$, where $[b\,c(t,t) - b\,c(t)] = b\,\Delta^{-1}\,h(t,t) - b\,c(t) = b\,\Delta^{-1}\,(b+\lambda)/b\,h(t) - b\,c(t)$. Considering both $\Delta^{-1}\,h(t) = [c(t) - \Delta^{-1}\,k(t)]$ and $\Delta^{-1}(t) = c(t)/[k(t)+h(t)]$, yields: $b\,c(t,t) - b\,c(t) = \lambda\,c(t) - (b+\lambda)\,c(t)\,k(t)/[k(t)+h(t)]$. Equation (M.1) follows.

If $b = \lambda = d = 0$, the standard Ramsey model emerges. In a steady state – if $\dot{c} = 0$ – equation (M.1) represents the Keynes-Ramsey rule: $r(k) = \rho$. If, however, $(b, \lambda) \gg 0$, there is a continuous inflow of new cohorts without accumulated wealth ($b > 0$), and there is a continuous decay of human wealth of any existing cohort over time ($\lambda > 0$). As a consequence, the wealth (human and accumulated) of a new cohort may differ from the average wealth, which gives rise to the generation replacement effect.

Generation Replacement Effect. A household's individual consumption growth rate is independent of age. The generation replacement effect (GRE) refers to the difference between average and individual consumption growth rates, which is based on the fact that individual consumption levels change with age: $c(v, t) \neq c(t)$. Analytically, the GRE — captured by the term $\Gamma(t)$ — is determined by:

Definition 1 $\Gamma(t) \equiv \lambda - (b + \lambda) \frac{k(t)}{k(t)+h(t)} = \lambda - (b + \lambda) \Delta^{-1}(t) \frac{k(t)}{c(t)}$.

Employing Definition 1,

$$g_c(t) \equiv \frac{\dot{c}(t)}{c(t)} = g_{c(v,t)}(t) + \Gamma(t) \, . \tag{22}$$

In particular, $g_c(t) \gtreqless g_{c(v,t)}(t) \Leftrightarrow \Gamma(t) \gtreqless 0$. If $\Gamma(t) \neq 0$, there exists a GRE. The GRE is not present in the Ramsey model, where $b = 0$.

Lemma 1 (Generation Replacement Effect)
(i) $\{h(t, t) = [a(t) + h(t)]\} \vee \{b = 0\} \Leftrightarrow \Gamma(t) = 0$.
(ii) $(b > 0) \wedge (\lambda = 0) \Rightarrow \Gamma(t) < 0 \Leftrightarrow g_c(t) < g_{c(v,t)}$.
(iii) Suppose $b > 0$. *Then there exists* $\tilde{\lambda}(t) > 0$, *such that* $\Gamma(t) = 0$, *where* $\tilde{\lambda}(t) = b\, k(t)/h(t)$. *Moreover, for* $\lambda \lesseqqgtr \tilde{\lambda}(t) \Leftrightarrow \Gamma(t) \lesseqqgtr 0$.

Proof. See appendix. ||

In the model of perpetual youth, the average consumption growth rate may differ from the individual consumption growth rate. With $b > 0$, newborn cohorts *without accumulated wealth* continuously enter the economy. That is, a newborn cohort's accumulated wealth is lower than average accumulated wealth: $0 = a(t,t) < a(t)$. If $\lambda > 0$, however, a newborn cohort's human wealth is greater than average human wealth: $h(t,t) = h(t)\,(b+\lambda)/b > h(t)$. The rate of labor productivity decline decides on whether the total (human and accumulated) wealth of a newborn cohort is lower or greater than average total wealth. If $\lambda < \tilde{\lambda}$, $h(t,t) < h(t) + a(t)$, in which case $c(t,t) < c(t)$. If $\lambda > \tilde{\lambda}$, $h(t,t) > h(t) + a(t)$, in which case $c(t,t) > c(t)$.

At the same time, individual consumption growth rates are independent of age and equal among all cohorts at any given point in time. By the very fact that newborn cohorts — with consumption levels different from the average consumption levels — continuously enter the economy, the average consumption growth rate differs from the individual consumption growth rate. Only in the special case in which $h(t,t) = h(t) + a(t)$, there is no GRE: $\Gamma(t) = 0$, and $g_c(t) = g_{c(v,t)}$.

Strict positivity of the instantaneous birth rate (but not of the death rate) is necessary for the GRE to occur. The GRE occurs even in the setting of a perpetual youth model with infinitely-lived agents, that is, with $b > 0$, $d = 0$, as in Weil (1989). It is the inflow of newborn cohorts whose total wealth is different from the average wealth, which causes the GRE.

3 The Effects of Externalities

In the following section, we consider the effects of conspicuous consumption on balanced growth paths (steady state equilibria). For steady state values, we omit the time indexes as of here. In the market economy, a steady state equilibrium is given by $\dot{c} = \dot{k} = \dot{\Delta} = 0$:

$$0 = \frac{r(k)-\rho}{\sigma} + \frac{\tilde{\sigma}}{\sigma}\left[\lambda - (b+\lambda)\,\Delta^{-1}\,\frac{k}{c}\right] = \frac{r(k)-\rho}{\tilde{\sigma}} + \Gamma\,, \tag{M.SS.1}$$

$$c = y(k) - \delta\,k\,, \tag{M.SS.2}$$

$$\Delta^{-1} = r(k) + d - \frac{r(k)-\rho}{\tilde{\sigma}}\,. \tag{M.SS.3}$$

From (M.SS.1)–(M.SS.3) it follows that a steady state equilibrium can be represented by the two variables c, k, as shown in Figure 1.

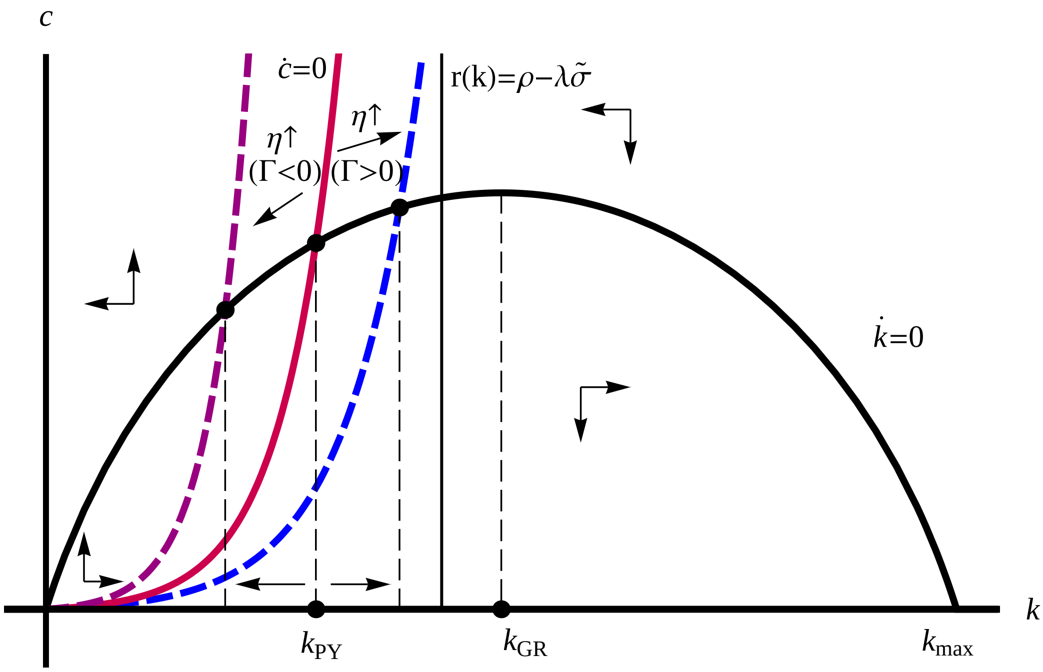

Figure 1: Impact of the Consumption Externality

Figure 1 displays the $\dot{k} = 0$– and $\dot{c} = 0$–lines in (k, c) space. The point of intersection (with $k > 0$, $c > 0$) shows the nontrivial steady state equilibrium.[7] The properties of the $\dot{k} = 0$– and $\dot{c} = 0$–lines, as displayed in the figure, are discussed in the appendix. Arrows indicate the movements of trajectories.

Figure 1 also shows an asymptote for the $\dot{c} = 0$-line at: $r(k) = \rho - \lambda\,\tilde{\sigma}$. The asymptote implies an important fact. At a steady state equilibrium, $r(k) > \rho - \lambda\,\tilde{\sigma}$. If $\lambda > 0$, $r(k)$ is allowed to assume a value *lower* than ρ in a steady state. In particular, we note:

Lemma 2 *In a steady state equilibrium,*

(i) $[r(k) - \rho] \gtreqless 0 \Leftrightarrow \Gamma \lesseqgtr 0\,,$

(ii)there exists $\tilde{\lambda} > 0$ *such that* $\Gamma(\tilde{\lambda}) = 0$,
where $\tilde{\lambda} = b\left[\frac{(1-\alpha)}{\alpha}\,\frac{\rho+\delta}{\rho+d+\lambda}\right]^{-1} > 0\,,$

(iii) $\Gamma(\lambda) \gtrless 0 \Leftrightarrow \lambda \gtrless \tilde{\lambda}\,.$

[7] It can readily be verified that the equilibrium is a saddle point.

Proof. See appendix. ||

Clearly, $\Gamma > 0$ does *not* imply dynamic inefficiency. Noticing that capital increases in λ, we define $\bar{\lambda}$ to be the value of λ, which implies the golden rule capital stock: $r(k(\bar{\lambda})) = 0$. From above, $r(k(\tilde{\lambda})) = \rho > 0$. Thus, $\bar{\lambda} > \tilde{\lambda}$. Dynamic *efficiency* occurs for all $\lambda \in [0, \bar{\lambda}]$. If, in addition $\lambda \in [\tilde{\lambda}, \bar{\lambda}]$, then $\Gamma > 0$.[8]

We can now analyze the impact of externalities on steady state equilibria. Figure 1 indicates that (a rise in the strength of) conspicuous consumption tilts the $\dot{c} = 0$-line either anticlockwise or clockwise. The effects of the consumption externality are ambiguous, depending on the sign of Γ.

Proposition 1 (Effects of the Consumption Externality) *Suppose* $0 \leq \lambda < \bar{\lambda}$. *In the market economy, the consumption externality has an ambiguous impact on the steady state allocation. In particular:*

$$\Gamma \lesseqgtr 0 \Leftrightarrow \frac{\partial c}{\partial \eta} \lesseqgtr 0 \,, \quad \frac{\partial k}{\partial \eta} \lesseqgtr 0 \,, \quad \frac{\partial (c/k)}{\partial \eta} \gtreqless 0 \,.$$

Proof. See appendix.||

Proposition 1 displays four main insights. First, given $\Gamma \neq 0$, a consumption externality — even with exogenous labor supply — does have a steady state impact on consumption, capital, and the propensity to consume out of accumulated wealth, c/k. The nature of the impact of a consumption externality, however, depends on the sign of Γ. If $\Gamma < 0$, the consumption externality raises the propensity to consume out of wealth. If $\Gamma > 0$, however conspicuous consumption lowers the steady state propensity to consume out of wealth, as explained below.

This result is in tension with the previous literature (Liu and Turnovsky 2005, Turnovsky and Monteiro 2007) that shows that consumption externalities, with exogenous labor supply, do *not* have an impact on the steady state allocation *in representative agent models.* This seeming contradiction can be rectified, however. The representative agent model is a special case of the present framework, with $b = d = 0$.

[8]The transversality conditions require $a(v,t)$ to increase at a rate lower than $r + d$, in a steady state. They are satisfied for $\Gamma > 0$, in which case $a(v,t)$ grows at a negative rate.

By Lemma 1(i), $\Gamma = 0$ in the representative agent model. As a consequence, Lemma 2(i) implies $r(k) = \rho$ whenever $\Gamma = 0$, in which case the consumption externality does not have an impact on the steady state equilibrium indeed.

In the framework of overlapping generation economies, both Wendner (2007) and Fisher et al. (2009) notice that consumption externalities raise the propensity to consume out of accumulated wealth, in spite of exogenous labor supply. Both papers, however, fail to recognize that the effects of conspicuous consumption are ambiguous and — depending on the rate of labor productivity decline — can go either way.

Second, if $\lambda < \tilde{\lambda}$ then $\Gamma < 0$, according to Lemma 1(iii). For this case, Proposition 1 shows that the keeping up with the Joneses consumption externality raises the steady state propensity to consume out of accumulated wealth, c/k. Intuitively, consumption is a positional good. A household not only derives utility from its own consumption level but also from the consumption-to-reference level ratio (roughly, from above-average consumption). The consumption externality provides an incentive to raise individual consumption relative to the reference level.

Initially, for a given level k, a rise in η raises individual consumption levels and lowers the individual consumption growth rate. This reaction is seen by restating the steady state version of Euler equation (19) as follows:

$$r(k) = \rho + \tilde{\sigma}\, g_{c(v,.)}\,. \tag{23}$$

For $\bar{c}(v,t) = c(v,t)$, the term $\tilde{\sigma}$ represents the (absolute) consumption elasticity of marginal utility. A rise in η raises the elasticity of marginal utility ($\tilde{\sigma}_\eta > 0$). That is, for a *given positive* growth rate of individual consumption, a rise in η induces the marginal utility to decline too strongly over time (the right hand side of the Euler equation exceeds the left hand side), and households will aim to smooth their consumption paths. Consequently, households will bring some future consumption forward to the present and, according to (23), lower the consumption growth rate.

As, initially, every individual household raises its consumption level, average consumption rises as well. This reaction, in turn, implies a lowering in aggregate savings. Subsequently the capital stock declines, and the new steady state is characterized by a lower level of k. Consequently, average consumption declines as well, as shown in Figure 1. As the production function is strictly concave, the average product of capital declines in k. That is, $\partial\,(y/k)/(\partial\,k) = \partial\,(c/k)/(\partial\,k) < 0$. As k declines, the steady state propensity to consume out of accumulated wealth, c/k, rises.

Third, Lemma 1(iii) shows that $\lambda > \tilde{\lambda}$ implies $\Gamma > 0$. In this case, Proposition 1 demonstrates that the keeping up with the Joneses consumption externality *lowers* the propensity to consume out of accumulated wealth, and it raises average steady state consumption and capital levels. At first sight, this result does not square well with intuition. To gain insight, it is important to note that one's relative consumption $(c(v,t)/x(v,t))$ not only matters today but also in the future. Consuming more today rises one's relative consumption, ceteris paribus. This rise comes at a cost, however. Consuming more today lowers tomorrow's consumption level, thereby tomorrow's relative consumption, ceteris paribus. In contrast, lowering one's consumption today allows for a higher level of consumption (and a better relative position, other things being equal) in the future. If $\Gamma > 0$, households prefer the latter option.

If $\Gamma > 0$, it follows from Lemma 2(i) that $r(k) < \rho$. In this case, the human wealth of a newborn generation exceeds the average total wealth. Consequently, the individual steady state consumption growth rate is negative. That is, an individual's consumption level declines over time, and its marginal utility of consumption rises over time. As above, an increase in η raises the elasticity of marginal utility. As a consequence, for a given $g_{c(v,.)}$, marginal utility increases at too big a rate over time. Households will postpone consumption and, according to (23), lower the (negative) consumption growth rate.

Initially, for given levels of c and k, Euler equation (23) requires the individual consumption growth rate to increase ($g_{c(v,.)}$ to become *less* negative) as of a rise in η. This rise is achieved by initially lowering the individual consumption levels. As, initially, every individual household lowers its consumption level, average consumption declines as well. This lowering implies a rise in average savings. Subsequently the capital stock and average consumption rise. In the new steady state, the propensity to consume out of accumulated wealth is lower (by strict concavity of the production function).

Fourth, Proposition 1 also shows a special case: $b > 0$ and $\Gamma = 0$. In this "knife-edge" case, the optimal individual consumption growth rate is zero. While a rise in η still raises the elasticity of marginal utility, it implies no change on $g_{c(v,.)}$. Consequently, there is neither an initial nor a steady state response to the change in η. In this case, the consumption externality does not have an impact on the steady state equilibrium. It is important to note, however, that this special case can only occur if $\lambda = \tilde{\lambda} > 0$.

Corollary 1 *Suppose* $b > 0$ *and* $\lambda = 0$*. Then* $\frac{\partial(c/k)}{\partial\eta} > 0\,, \quad \frac{\partial k}{\partial\eta} < 0\,, \quad \frac{\partial c}{\partial\eta} < 0\,.$

Proof. $\lambda = 0 \Rightarrow \Gamma < 0$, by Lemma 1(ii). ||

In case individual labor productivity does not decline over lifetime, individual total (accumulated and human) wealth rises over time and so does individual consumption. The consumption smoothing effect of a rise in η, initially leads to an increase in average consumption, and to a rise in the propensity to consume out of accumulated wealth (initially and in the steady state). As discussed above, the effect of the consumption externality is quite different when $\lambda > \tilde{\lambda}$. In this case, the individual labor productivity parameter, λ, plays a key role in explaining the impact of the consumption externality on individual behavior.

Proposition 2 (Composition of the Consumption Reference Level)
Suppose $0 \leq \lambda < \bar{\lambda}$*. Then* $\Gamma \lesseqgtr 0 \Leftrightarrow \frac{\partial c}{\partial\varepsilon} \gtreqless 0\,, \quad \frac{\partial k}{\partial\varepsilon} \gtreqless 0\,, \quad \frac{\partial(c/k)}{\partial\varepsilon} \lesseqgtr 0\,.$

Proof. Noticing that $\tilde{\sigma}_\varepsilon = -\eta(\sigma-1)/(1-\eta) < 0$, the proof follows that of Proposition 1. ||

The proposition shows that not only the strength of the consumption externality, η, affects the steady state allocation, but so does also the nature of the consumption reference level. Consider the marginal utility of consumption, as displayed in (38) in the appendix:

$$u_{c(v,t)} = \frac{c(v,t)^{-\sigma}}{1-\eta} \left[\frac{c(t)}{c(v,t)}\right]^{\tilde{\sigma}-\sigma} .$$

The impact of the strength of the consumption externality is captured by the denominator $(1-\eta)$. The composition of the consumption reference level is captured by the exponent $(\tilde{\sigma}-\sigma)$, which declines in ε.

A rise in ε puts more weight on consumption of one's own cohort relative to average consumption of society. This affects the marginal utility of individual consumption via the rightmost term displayed above. If $\Gamma < 0$ then $c(t)/c(t,t) > 1$.[9] A rise in ε lowers the exponent $\tilde{\sigma}-\sigma$, thereby it lowers $[c(t)/c(t,t)]^{\tilde{\sigma}-\sigma}$. Other things being equal, marginal utility of own consumption declines, and so does the marginal rate of substitution of own $c(v,t)$ for $c(t)$.[10] The decline in the marginal rate of substitution requires a reduction in own consumption, initially. Thus, individual and thereby average savings rates increase. In the new steady state, both the average consumption and capital levels have increased and the propensity to consume out of accumulated wealth has been lowered as a consequence of the rise in ε.

If $\Gamma > 0$, $c(t)/c(t,t) < 1$, and a rise in ε raises the marginal rate of substitution of $c(v,t)$ for $c(t)$, initially. As a consequence, initial individual consumption is increased, and steady state consumption (capital) declines.

The main implication of the above discussion is that η and ε have opposing effects on the steady state allocation. If $\Gamma < 0$ (if $\Gamma > 0$), the effects of the "strength" of the conspicuous consumption externality, η, are the weaker, the more it is consumption of one's own cohort (average consumption of society) that constitutes the consumption reference level.[11]

Individual consumption growth. We now turn to the fact that — even in a steady state — the individual consumption growth rate is different from zero in the perpetual youth model, while the average consumption growth rate is zero.

[9] Clearly, $c(t)/c(v,t) > 1$ does not hold for all cohorts. Still, $c(t)/c(t,t) > 1$ is dominating, as the measure of young cohorts exceeds that of old cohorts for which $c(t)/c(v,t) < 1$.

[10] More precisely, the marginal rate of substitution of $c(v,t)$ for $x(v,t)$ at $x(v,t) = c(v,t)^{\varepsilon}\, c(t)^{(1-\varepsilon)}$.

[11] The impact of η on the marginal utility of consumption declines (rises) in ε if $\Gamma < 0$ (if $\Gamma > 0$).

Proposition 3 (Individual Consumption Growth)

Suppose $b > 0$. In the market economy, the impact of a rise in η on the individual consumption growth rate is ambiguous. In particular:

$$\frac{\partial\, g_{c(v,.)}}{\partial\, \eta} \gtreqless 0 \Leftrightarrow [r(k) - \rho]\,[\delta - (d + \lambda)] \gtreqless 0\,.$$

The impact is determined by two signs: the sign of Γ, and the sign of $[\delta - (d + \lambda)]$, the latter of which corresponds to the difference in the rates of depreciation of accumulated capital (δ) and human capital ($d + \lambda$).

Proposition 3 shows that the impact of the consumption externality on the individual consumption growth rate depends on two factors. First, the sign of Γ (thereby the sign of $[r - \rho]$), which determines whether conspicuous consumption lowers or raises k. Second, the sign of $[\delta - (d + \lambda)]$, which determines whether a rise (decrease) in k lowers or raises h/k, and thereby the individual consumption growth rate (which declines in h/k)[12].

The consumption externality, by changing k, exerts two effects on h/k (notice that $h/k = [w(k)/k]/[r(k) + d + \lambda]$). First, the externality changes h/k by a change in the undiscounted steady state wage stream to capital ratio $w(k)/k$ (wage stream effect). Second, the consumption externality changes the *present value* of any given wage stream (discount rate effect). If, e.g., $\Gamma < 0$, steady state capital declines upon a rise in η, and both the wage stream to capital ratio and the discount rate increase. The impact of conspicuous consumption on h/k is, therefore, ambiguous.

The discount rate effect dominates the wage stream effect if the rate at which accumulated capital depreciates (δ) exceeds the rate at which human capital effectively depreciates ($d + \lambda$).

If $\Gamma < 0$, conspicuous consumption lowers k. If the discount rate effect dominates the wage stream effect, then h/k declines, and the individual consumption growth rate rises. Otherwise, if $\Gamma > 0$, the consumption externality raises k, and h/k is increased, which implies a fall in the individual consumption growth rate.

[12] $\partial\, g_{c(v,.)}/\partial\,(h/k) = -(b + \lambda)/(1 + h/k)^2 < 0$.

It is important to notice that a *wealth effect* like the discount rate effect is absent in standard two period OLG models, in which all labor income accrues at the beginning of life. The discount rate effect is also absent in representative agent models. Once $b = 0$, the Keynes-Ramsey rule determines the steady level of state capital, which is independent of the present value of human wealth. In the perpetual youth model, if $d = \lambda = 0$, the discount rate effect *always* dominates the other effects. We therefore have:

Corollary 2 *Suppose $b > 0$. If $\delta > d = \lambda = 0$, the impact of a rise in η on the individual consumption growth rate depends on the sign of Γ only. In particular,*

$$\frac{\partial\, g_{c(v,.)}}{\partial\, \eta} \gtreqless 0 \Leftrightarrow \Gamma \lesseqgtr 0\,.$$

In Weil's (1989) model with overlapping families of infinitely-lived agents, e.g., the discount rate effect always dominates the wage stream effect. That is, if $[r(k) > \rho]$, a rise in η always lowers h/k. As a consequence, the consumption externality raises both individual consumption growth rates *and* the average propensity to consume out of accumulated wealth.

For the special case $\delta = d + \lambda$, the consumption externality may change consumption levels but has no impact on individual consumption growth rates, as the wage stream effect equals the discount rate effect.

Corollary 3 *(Average Consumption Level versus Individual Growth Rate)*
(i) Suppose $\lambda \in (\tilde{\lambda}, \bar{\lambda})$, and $[\delta - (d + \lambda)] < 0$. Then a rise in η increases both the average steady state consumption level and the individual steady state consumption growth rate.

(ii) If $\lambda = 0$, a rise in η always lowers the average steady state consumption level, while it either raises (if $\delta > d$) or lowers (if $\delta < d$) the individual steady state consumption growth rate.

Proof. The corollary follows from Propositions 1, 3 and the fact that $\Gamma > 0 \Leftrightarrow [r(k) < \rho]$, by Lemma 2(i). The restriction $\lambda < \bar{\lambda}$ ensures dynamic efficiency. ||

The corollary emphasizes the observation that *average* consumption levels and *individual* consumption growth rates are not, in general, (indirectly) proportional to each other.

Table 1 displays the effects of conspicuous consumption for important economic variables.

TABLE 1

Steady State Effects of Conspicuous Consumption, and the Generation Replacement Effect

$\eta \uparrow$		$\Gamma < 0$	$\Gamma = 0$	$\Gamma > 0$
c/k		$\uparrow$	$=$	$\downarrow$
k		$\downarrow$	$=$	$\uparrow$
c		$\downarrow$	$=$	$\uparrow$
$g_{c(v,.)}$:	$\delta < d + \lambda$	$\downarrow$	$=$	$\uparrow$
$g_{c(v,.)}$:	$\delta = d + \lambda$	$=$	$=$	$=$
$g_{c(v,.)}$:	$\delta > d + \lambda$	$\uparrow$	$=$	$\downarrow$

It has been frequently argued that one factor causing the low savings rates seen in developed countries may be "overconsumption" resulting from a keeping up with the Joneses consumption externality. This argument is not supported by representative agent models, as argued above. The results of Wendner (2007) and Fisher et al. (2009) are consistent with this argument, in the sense that they find conspicuous consumption to raise the average propensity to consume out of accumulated wealth. In their models, labor productivity is constant with age. Considering a declining labor productivity (or gradual retirement), however, reveals that the impact of conspicuous consumption on the propensity to consume (on savings) is ambiguous. Thus, on theoretical grounds, as displayed by Table 1, there is *no* reason to believe that the consumption externality has no impact on the steady state allocation. At the same time, there is no reason to believe that the consumption externality would necessarily rise the propensity to consume out of total wealth. In this sense, conspicuous consumption does not explain the observed high propensity to consume (or low savings rates) on grounds of economic theory. It is an empirical matter whether conspicuous consumption lowers or raises the propensity to consume.

There are two implications. First, conspicuous consumption is a candidate for explaining low savings rates observed in developed countries. The matter, however, is an empirical one. Second, it is not clear at all that the propensity to consume, implied by conspicuous consumption, is higher than *optimal*. Whether or not conspicuous consumption leads to overconsumption is a different matter and is discussed in the proceeding section.

4 Distortionary Effects and Optimal Taxation

In the following, we first first characterize an optimal steady state allocation. Next, we show that the optimal allocation can be decentralized by an unfunded social security system. Finally, we investigate how conspicuous consumption affects the optimal tax scheme. Throughout, a tilde refers to an optimal value.

4.1 Optimal Allocation

In analogy to Calvo and Obstfeld (1988), the time-consistent utilitarian social welfare function must take the form:

$$W(t) = \int_{-\infty}^{t} L(v,t)\, U(v,t)\, e^{-\rho(t-v)}\, e^{-\tilde{\rho}(v-t)}\, d\,v \, + \int_{t}^{\infty} L(v,v)\, U(v,v)\, e^{-\tilde{\rho}(v-t)}\, d\,v \ .$$

The planner's objective, at time t, is the sum of two components. First, the integral of the expected remaining lifetime utilities of all cohorts alive at time t, measured from the perspective of his and her birthdate.[13] Second, the integral of the lifetime expected utilities of each of the generations to be born, as measured from the respective time of birth. The planner's discount rate, $\tilde{\rho}$, needs not equal an individual's pure rate of time preference.

Consider $L(v,t) = b\, e^{b\,v - d\,t}$, $L(v,v) = b\, e^{n\,v}$, $n = b - d = 0$, and change the order

[13] The factor $e^{-\rho(t-v)}$ discounts time-t-lifetime utility back to the birthdate. Clearly, once a cohort has survived up to time t, the appropriate discount rate is ρ, not $(\rho + d)$. As shown by Calvo and Obstfeld (1988), discounting back to birthdates ensures time-consistency.

of integration:

$$W(t) = \int_t^\infty \left\{ \int_{-\infty}^s b\, u[c(v,s), x(v,s)]\, e^{-[b+(\rho-\tilde{\rho})]\,(s-v)}\, d\, v \right\} e^{-\tilde{\rho}\,(s-t)}\, d\, s\,. \qquad (24)$$

The planning problem consists of maximizing (24) by choosing $c(v,s)$ and $c(s)$ subject to $c(s) \equiv \int_{-\infty}^s b\, e^{-b\,(s-v)}\, c(v,s)\, dv$ and (15), where the planner considers the impact of consumption on the reference level according to (5). The (constrained) optimal control problem is discussed in the appendix. Differential equations (43)–(46) yield the following optimal steady state allocation, which is given by $\dot{c} = \dot{k} = 0$:

$$\tilde{r}(k) = \tilde{\rho}\,, \qquad (25)$$

$$c = y(k) - \delta\, k\,. \qquad (26)$$

$$\frac{\dot{c}(v,.)}{c(v,.)} = \frac{\tilde{\rho} - \rho}{\tilde{\sigma}}\,, \quad \frac{c_v(v,t)}{c(v,t)} = -\frac{\tilde{\rho} - \rho}{\tilde{\sigma}}\,. \qquad (27)$$

Equation (25) represents the Keynes-Ramsey rule for capital accumulation, and 26 restates the steady state resource constraint. Equations (27) show both the optimal intragenerational and intergenerational consumption growth rates. In case the private and social discount rates are equal, it is optimal to implement an egalitarian plan, according to which every generation receives the same amount of consumption at any given point in time. If, however, the private discount rate exceeds the social discount rate, it is optimal to shift resources (consumption) towards the young cohorts, and the intergenerational growth rate, $c_v(.)/c(.)$ is positive. If the private discount rate is lower than the social one, the intergenerational growth rate is negative. As a consequence, according to (27), individual optimal consumption rises (declines) with age when private discount rate is lower (higher) than the social discount rate.

Two properties of the optimal allocation are particularly noteworthy. First, conspicuous consumption does not affect optimal average consumption and capital levels. But the consumption externality affects the intergenerational and the intragenerational consumption growth rates.

Second, in contrast to the market equilibrium, *optimal* consumption levels and growth rates are independent of both the birth and the death rate. Consequently, there is no GRE, and — in contrast to the market framework — the consumption externality does not have an impact on the optimal allocation *via the generation replacement effect.*

4.2 Decentralization and the Optimal Tax Scheme

In this and the proceeding subsections we show that the optimal steady state allocation can be decentralized. The government applies two instruments: a constant tax on capital income, τ_k, and lump sum transfers $\tau(v,t) > 0$ and taxes $\tau(v,t) < 0$. The tax on capital income is motivated by two facts. First, conspicuous consumption affects the propensity to consume out of accumulated wealth (the propensity to save) in the market framework, while it does not affect the propensity to consume out of accumulated wealth in the command optimum. Second, in the presence of conspicuous consumption, the previous literature shows that a capital income tax is required to decentralize the optimal allocation, once $\tilde{\rho} \neq \rho$ (Abel, 2005).

Augmenting the market framework of Section 2 with the two instruments makes necessary the following two modifications. First, the rate of interest is replaced by the after tax rate of interest $\hat{r}(t) = r(t)\,(1-\tau_k)$. Second, the flow budget constraint becomes:

$$\dot{a}(v,t) = \hat{r}^A(t)\,a(v,t) + w(v,t) + \tau(v,t) - c(v,t)\,, \tag{28}$$

where $\hat{r}^A(t) \equiv \hat{r}(t) + d$. Define the present value of the future lump sum transfer-/tax stream by: $\beta(v,t) \equiv \int_t^\infty \tau(v,t)\,e^{-\hat{R}^A(t,s)}$, where $\hat{R}^A(t_0,t_1) \equiv \int_{t_0}^{t_1} \hat{r}^A(s)\,d\,s$. In the following, we refer such a lump sum scheme as a "transfer stream." It follows:

$$\int_t^\infty c(v,s)\,e^{-\hat{R}^A(t,s)}\,ds = a(v,t) + h(v,t) + \beta(v,t)\,, \tag{29}$$

$$c(v,t) = \Delta^{-1}\left[a(v,t) + h(v,t) + \beta(v,t)\right], \tag{30}$$

$$c(t) = \Delta^{-1}\left[a(t) + h(t) + \beta(t)\right], \quad \beta(t) \equiv \int_{-\infty}^t l(v,t)\,b(v,t)\,d\,v\,, \tag{31}$$

where the propensity to consume, Δ^{-1}, now involves the after tax rate of interest.[14]

[14]Likewise, the transversality condition shown in footnote 5 also involves the after tax rate of interest.

Following the same procedure as in Section 2 yields:

$$\begin{aligned}
\frac{\dot{c}(t)}{c(t)} &= \frac{\hat{r}(t)-\rho}{\sigma} + \frac{\tilde{\sigma}}{\sigma}\left[\lambda - (b+\lambda)\,\Delta^{-1}\,\frac{k(t)+\beta(t)-\frac{b}{b+\lambda}\beta(t,t)}{c(t)}\right] \\
&= \frac{\hat{r}(t)-\rho}{\sigma} + \frac{\tilde{\sigma}}{\sigma}\left[\lambda - (b+\lambda)\,\frac{k(t)+\beta(t)-\frac{b}{b+\lambda}\beta(t,t)}{k(t)+h(t)+\beta(t)}\right]
\end{aligned} \tag{32}$$

$$\dot{k}(t) = y(t) - c(t) - \delta\,k(t)\,, \tag{33}$$

$$\frac{\dot{c}(v,t)}{c(v,t)} = \frac{\hat{r}(t)-\rho}{\tilde{\sigma}} + \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}}\,\frac{\dot{c}(t)}{c(t)}\,, \qquad \frac{c_v(v,t)}{c(v,t)} = -\frac{\hat{r}(t)-\rho}{\tilde{\sigma}}\,. \tag{34}$$

Except for the public sector's instruments, these equations of motion are identical to those given in Section 2. The capital accumulation equation (33), however, deserves a comment.

In general, a household's flow budget constraint involves transfers and taxes, $\tau(v,t)$, which show up at the aggregate capital accumulation equation if and only if the government does *not* run a balanced budget. Here, we argue that the government can run every *feasible* transfer scheme such that the government budget is balanced period by period. A balanced budget at date t implies: $\int_{-\infty}^{t} l(v,t)\,\tau(v,t)\,d\,v = 0$. Consequently, the transfer scheme does not affect the capital accumulation equation, once the budget is balanced period by period.

To see that every feasible transfer scheme can be implemented with a balanced government budget, notice that — with no initial debt at t — the intertemporal government budget constraint requires:

$$\int_{-\infty}^{t} l(v,t)\,\beta(v,t)\,d\,v + \int_{t}^{\infty} b\,\beta(s,s)\,e^{-\hat{R}(s,t)}\,d\,s = 0\,. \tag{35}$$

It can easily be shown that, as preferences are monotone, the intertemporal government budget constraint holds as a consequence of Walras law (Calvo and Obstfeld, 1988). Its main interpretation is that the present value of all present and future primary deficits equals zero (otherwise the transfer scheme is not feasible).

Consider *any* given feasible transfer scheme, and define the date-s-primary deficit $\pi(s) \equiv \int_{-\infty}^{s} l(v,s)\,\tau(v,s)\,d\,v$, where possibly $\{\pi(s)\}_{s=t}^{\infty} \neq 0$. Consider the expected future transfer stream of an agent born at date t, $\{\tau(t,s)\}_{s=t}^{\infty}$. Add $\{-\pi(s)\}_{s=t}^{\infty}$ to her transfer stream. The expected value of the transfer stream does not change due to the fact that the present value of primary deficits is zero. Thus, $\beta(t, v)$ is not affected, but the transfer scheme now involves a balanced budget period by period (Calvo and Obstfeld, 1988). In the following we only consider transfer schemes that involve balanced budgets.

The two government instruments serve specific purposes. The capital income tax is capable of correcting the intergenerational consumption growth rate. The lump sum tax/transfer-scheme is capable of correcting the average capital *level*.

4.3 Capital Income Tax

It is shown below, that the transfer scheme is indeed capable of correcting for the average capital level. Given that the optimal capital level can be decentralized by the transfer scheme, the function of the capital income tax is to adjust the intergenerational consumption growth rate, $c_v(.)/c(.)$.

For the following proposition, we assume that, in the steady state, $k = \tilde{k}$ resulting from an optimal transfer scheme. That such a tax/transfer system exists (and which properties it has) is shown in the following subsection.

Proposition 4 (Optimal Capital Taxation) *Suppose $k = \tilde{k}$ by an optimal transfer scheme. Then it is* not *optimal to tax capital income in the steady state, in spite of a consumption externality.*

Proof. Considering (27) and (34), optimality requires $\hat{r} = \tilde{\rho}$. That is $\hat{r} = r(\tilde{k})(1 - \tau_k) = \tilde{\rho}\,(1 - \tau_k)$, which equals $\tilde{\rho}$ only with $\tau_k = 0$. ||

The fact that the consumption externality affects the intergenerational consumption growth rate does *not* justify a tax on capital income in the long run.[15] This result is in contrast to Abel (2005), who shows that, in general, conspicuous consumption

[15]Erosa et al. (2002), Mathieu-Bolh (2006), and Spataro et al. (2008) show that optimal steady state capital income tax rates are generally nonzero in overlapping generations economies. This result is derived, however, for frameworks with elastic labor supply. It does not hold for overlapping generations economies with inelastic labor supply, such as the present one.

gives rise to a nonzero optimal tax rate on capital income. The capital income tax is required to adjust the intergenerational consumption growth rate.

The fact that Proposition 4 differs from the results shown by Abel (2005) is explained as follows. The propensity to consume is age-dependent in the two-period OLG model of Abel (2005), while it is independent of age in the continuous time OLG model. Decentralization of the optimal allocation requires lump sum taxes and transfers (as discussed below). Implementation of an optimal lump sum transfer scheme transfers wealth across generations, which affects a household's propensity to consume — thereby the intergenerational consumption growth rate — in the two-period OLG model. As a consequence, an additional instrument (the capital income tax) is required to restore the optimal intergenerational consumption growth rate. This mechanism is not at work in a continuous time OLG economy. Implementation of an optimal lump sum transfer scheme transfers wealth across generations, but it has no impact on the propensity to consume. Therefore no further instrument to correct for the impact of a lump sum transfer scheme on the intergenerational consumption growth rate is needed.

4.4 Unfunded Social Security System

We now show that the optimal allocation can be decentralized by a lump-sum transfer scheme with the capital income tax rate being at its optimum ($\tau_k = 0$). Considering the equations of motion describing a market equilibrium, (32) – (34), we focus on a characterization of $\beta(t,t)$ and $\beta(t)$ in the steady state. To simplify notation, let $\beta_0 \equiv \beta(t,t)$, and $\beta \equiv \beta(t)$, in a steady state.

Without loss of generality, we consider balanced budget lump-sum transfer schemes only. It is important to note that a balanced budget scheme does *not* imply $\beta = 0$. We will distinguish schemes according to the following definition.

Definition 2 (Unfunded Social Security) *A balanced budget lump-sum transfer scheme is said to be an unfunded social security system (U) if* $\beta_0 < 0$ *and* $\beta > 0$*. A balanced budget lump-sum transfer scheme is said to be a reverse unfunded social* *security system (RU) if* $\beta_0 > 0$ *and* $\beta < 0$.

If the optimal lump-sum transfer scheme distributes wealth away from the elder cohorts towards the young cohorts we refer to the scheme as a reverse unfunded social security system.

Walras law implies the intertemporal government budget constraint, which can be rewritten as: $\beta + \beta_0 \int_t^\infty b\, e^{-R(t,v)} d\, v = 0$. In a steady state:

$$\beta_0 = -\frac{\tilde{\rho}}{b}\, \beta\,. \tag{36}$$

Thus, any balanced budget lump-sum transfer scheme is either an U scheme or a RU scheme. In the appendix, β is derived. It follows:

$$\frac{b}{\tilde{\rho}}\, \beta_0 = \left(\frac{1}{\tilde{\rho} + d + \lambda} - \frac{1}{\Delta^{-1}} \right) (y - \delta\, k) + \frac{d + \lambda}{\tilde{\rho} + d + \lambda}\, k\,, \tag{37}$$

which allows to (i) distinguish a U scheme from a RU scheme, and (ii) find the effects of conspicuous consumption on the optimal transfer scheme.

The first result of this subsection concerns to the sign of β_0.

Proposition 5 (Unfunded Social Security scheme) *The optimal social security scheme is a* U *scheme (is a* RU *scheme) if:*

$$\operatorname{sgn}\left[\Gamma + \frac{\tilde{\rho} - \rho}{\tilde{\sigma}} \right] > 0 \quad \left(\operatorname{sgn}\left[\Gamma + \frac{\tilde{\rho} - \rho}{\tilde{\sigma}} \right] < 0 \right) .$$

Proof. See appendix.||

If the pure rate of time preference equals the social discount factor, the sign of β_0 is fully determined by the sign of the GRE: $\operatorname{sgn} \beta_0 = -\operatorname{sgn} \Gamma$. In the standard case with $\lambda = 0$ (that is, $\Gamma < 0$), the proposition implies: $\beta_0 > 0$. Consequently, it is optimal to implement a *reverse* unfunded social security scheme. Cohorts are faced, at birth, with a declining transfer stream. Thus, consumption smoothing requires them to (initially) lower consumption levels and rise savings. Consequently, the RU system raises capital accumulation to the point at which $k = \tilde{k}$.

Proposition 5 also identifies two necessary conditions for a U transfer scheme (redistribution towards the old cohorts) to be optimal. First, if $\rho = \tilde{\rho}$, $\Gamma > 0$. It is optimal to redistribute towards the old cohorts if their labor productivity declines strongly enough with age ($\lambda > \tilde{\lambda}$). Second, if $\Gamma \leq 0$, $\tilde{\rho} > \rho$ by a sufficient amount. In this case, it is optimal to implement a transfer of resources towards the old cohorts, as their socially optimal consumption is higher than that of the young cohorts.[16]

Effects of Conspicuous Consumption. If $\Gamma \neq 0$, conspicuous consumption affects k in the market economy (according to Proposition 1), while it leaves unaffected k in the command optimum. Based on this observation, one could expect conspicuous consumption to rise β_0 if $\Gamma < 0$, and to lower β_0 if $\Gamma > 0$. The opposite, however, is the case, as shown by Proposition 6. In the following we restrict our analysis to the case: $\tilde{\rho} = \rho$.

Proposition 6 (Unfunded Social Security and Conspicuous Consumption) *If* $\Gamma \neq 0$, *conspicuous consumption affects the optimal unfunded social security system in the following way:*

$$\Gamma \gtrless 0 \Rightarrow \frac{\partial \beta_0}{\partial \eta} \gtrless 0 \,.$$

If $\Gamma = 0$, *conspicuous consumption has no impact on the optimal unfunded social security system.*

Proof. See appendix. ||

Proposition 6 says that conspicuous consumption always reduces the decline or increase (with age) of a cohort's transfer stream. For example, if $\Gamma < 0$, $\beta_0 > 0$ but $\beta < 0$. That is, the transfer stream decreases with age (and becomes negative, eventually). According to the proposition, conspicuous consumption lowers the rate of decline in the transfer stream.

This counterintuitive "smoothing effect" has the following intuition. Consider a transfer scheme which fully decentralizes the optimal allocation. A rise in the strength of conspicuous consumption (of η) has no effects on the socially *optimal* average consumption and capital levels. If, for example, $\Gamma < 0$, both the average consumption

[16] A special case is given by $\Gamma = 0$, in which case $\operatorname{sgn} \beta_0 = -\operatorname{sgn}(\tilde{\rho} - \rho)$.

and capital levels decline in a market economy. As c is directly proportional to the average transfer stream, β has to be raised in order to restore optimality. The rise in β requires a decline in β_0, by (36). Intuitively, the consumption externality leads to a decline in the laissez-faire average consumption level. A lowering in β_0 lowers the "initial" consumption level of a newborn cohort, and it raises its savings. Eventually, in a steady state, the lowering in β_0 (the rise in β) leads to increases in individual and average consumption levels.

Proposition 6 has two important implications. First, a rise in the strength of the consumption externality implies a "smoothening" (a lower rise or decline with age) of the optimal transfer scheme. Second, the proposition shows that conspicuous consumption introduces a distortion whenever $\Gamma \neq 0$. The result is noteworthy, as the consumption externality introduces a steady state distortion even for the case of exogenous labor supply. Clearly, this result does not hold in a representative agent economy, where — by definition — $\Gamma = 0$.

Whether or not the distortion leads to overconsumption, however, is a different matter. We showed that the consumption externality implies steady state overconsumption only if $\Gamma > 0$, and it implies underconsumption if $\Gamma < 0$.

5 Conclusions

Economists have been interested in *conspicuous consumption* for centuries. Frequently, economists refer to conspicuous consumption as one factor for explaining the low savings rates faced by many developed countries. Motivated by this argumentation, this paper addresses two main questions. First, how does a "keeping up with the Joneses" consumption externality affect consumption and savings decisions? In more coarse terms, do people consume too much as a consequence of the desire to keep up with the Joneses? Second, compared to a social optimum, does conspicuous consumption lead to a distortion (to overconsumption)?

In the paper, we show that conspicuous consumption does *not*, in general, give rise to either an increased propensity to consume out of one's wealth, or to overconsumption according to a utilitarian social welfare function. For both questions raised, the sign of the *generation replacement effect* (present in an OLG model but absent in an infinitely lived agent model) is identified to play a decisive role. The generation replacement effect refers to the difference between average and individual consumption growth rates, which is based on the fact that individual consumption levels change with age.

In the absence of the generation replacement effect (GRE), conspicuous consumption does not have an impact on consumption or savings decisions. In its presence, however, conspicuous consumption raises (lowers) the steady state propensity to consume out of total wealth if individual consumption growth is positive (negative). Individual consumption growth is positive if individual labor productivity does not decline "by much" with age (in which case the GRE has a negative sign). Individual consumption growth is negative if individual labor productivity declines "strongly" with age (in which case the GRE has a positive sign).

Along the same line, the steady state effects of a consumption externality are shown to be distortionary only in the presence of a GRE. If the GRE is negative the keeping up with the Joneses consumption externality implies *steady state underconsumption*. Only if the GRE is positive (the rate of individual labor productivity decline is large enough), does the consumption externality lead to steady state overconsumption.

These results have important implications. First, it is generally mistaken to associate conspicuous consumption with an increase in the propensity to consume or with overconsumption. Both might happen. But while the former happens in case the GRE is negative (for a low rate of individual labor decline), the latter happens in case of a positive GRE (for a high rate of individual labor decline). As a consequence, overconsumption requires a decline in the propensity to consume due to the consumption externality.

Second, the consumption externality does not give rise to a positive capital income tax rate in the long run. The main reason is that the propensity to consume is independent of age in the continuous time overlapping generations model.

Third, the market equilibrium can be decentralized by a social security system. The paper identifies the sign of the GRE to determine whether the scheme is an unfunded social security scheme or a reverse unfunded social security scheme (with transfers from the old to the young). Unless the labor productivity declines strongly enough with age, the optimal scheme turns out to be a *reverse* unfunded social security scheme. Finally, a rise in the strength of the consumption externality implies a "smoothening" (a lower rise or decline with age) of the optimal unfunded social security transfer scheme.

Given the theoretical findings of this study, it is important to empirically identify the sign of the GRE appropriately. In this sense, I hope that this study helps to clarify the effects of conspicuous consumption, and it will contribute to the future debate of the effects of consumption externalities in a dynamic framework.

Appendix

A.1 Derivation of Individual Consumption. For any individual, consider the current value Hamiltonian:[17]

$$H_c[c(v,s), a(v,s), \mu_a(s), s] = u(c(v,s), x(v,s)) + \mu_a(s)[(r(s)+d)\, a(v,s) + w(v,s) - c(v,s)]\,.$$

For all $s \geq t$, $c(v,s)$ is chosen such as to maximize expected utility (7), subject to:

$$c(v,s) \geq 0\,,$$
$$\dot{a}(v,t) = (r(s)+d)\, a(v,s) + w(v,s) - c(v,s)\,,$$
$$a(v,t) \text{ given}\,, \quad \lim_{s\to\infty} \mu_a(s)\, e^{-(\rho+d)s}\, a(v,s) = 0\,.$$

From $\partial H_c / \partial c(v,s) = 0$ and $-\partial H_c / \partial a(v,s) = \dot{\mu}_a - (\rho + d)\, \mu_a$, it follows:

$$\mu_a(s) = u_{c(v,s)} = \frac{c(v,s)^{-\sigma}}{1-\eta} \left[\frac{c(s)}{c(v,s)}\right]^{\tilde{\sigma}-\sigma}, \tag{38}$$
$$g_{c(v,s)} \equiv \frac{\dot{c}(v,s)}{c(v,s)} = \frac{[r(s)-\rho]}{\tilde{\sigma}} + \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}} \frac{\dot{c}(s)}{c(s)}\,,$$

[17]Notice that $\eta < \sigma$, which is sufficient for the Hamiltonian to be concave in the decision and state variables.

where no individual considers its impact of individual consumption on the reference level. Thus,

$$c(v,s) = c(v,t)\, e^{\int_t^s \frac{[r(s)-\rho]}{\tilde{\sigma}} + \frac{\tilde{\sigma}-\sigma}{\tilde{\sigma}} \frac{\dot{c}(s)}{c(s)}\, d\, s}\,.$$

Combining this equation with the intertemporal budget constraint (16), and considering the definition of $\Delta(t)$ in (17) yields: $c(v,t) = \Delta^{-1}(t)\,[a(v,t) + h(v,t)]\,.$ ||

A.2 Proof of Lemma 1

(i) Restate the average consumption growth rate as:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{c}(v,t)}{c(v,t)} - b\,\frac{c(t) - c(t,t)}{c(t)}\,,$$

where we consider the fact $\dot{c}(t) = b\,c(t,t) - b\,c(t) + g_{c(v,t)}\,c(t)$. Taking (22) into account: $\Gamma(t) = b\,[c(t,t) - c(t)]/c(t)$, that is, $\Gamma(t) = 0 \Leftarrow c(t,t) = c(t) \Leftrightarrow \Delta^{-1}\,h(t,t) = \Delta^{-1}\,[h(t) + a(t)] \Leftrightarrow h(t,t) = h(t) + a(t)$. Also, $\Gamma(t) = 0 \Leftarrow b = 0$. |

(ii) If $b > 0$ and $\lambda = 0$, $h(t,t) = h(t) < h(t) + a(t) \Leftrightarrow c(t,t) < c(t) \Leftrightarrow \Gamma(t) < 0$.|

(iii) From the definition of $\Gamma(t)$, it directly follows that $\tilde{\lambda}(t) = b\,k(t)/h(t)$. Ceteris paribus, $\partial\,\Gamma(t)/(\partial\,\lambda) = h(t)/[h(t) + k(t)] > 0$. ||

A.3 Figure 1. The figure displays two demarcation lines in (k,c) phase space: $\dot{k} = 0$, and $\dot{c} = 0$.

The $\dot{k} = 0$ line. We define the graph of $\dot{k} = 0$ by the set $kk \equiv \{(k,c) \in \mathbb{R}^2_+ \,|\, c = y(k) - \delta\,k\}$. As $y(0) = 0$, $(0,0) \in kk$. Next, $\partial\,c/\partial\,k > 0$ as long as $y'(k) - \delta = r(k) > 0$, and $\partial\,c/\partial\,k < 0$ when $r(k) < 0$. As $y(k)$ is strictly concave, and $(-\delta\,k)$ is weakly concave, $y(k) - \delta\,k$, that is, the $\dot{k} = 0$ line, is strictly concave. The capital stock for which $r(k) = 0$ is denoted the "golden rule" capital stock in Figure 1: $r(k_{GR}) = 0$. Finally, for the maximal attainable stock of capital, k_{max}, it holds: $y(k_{max}) = \delta\,k_{max}$.

For any given (k,c), a rise in λ clearly lowers y and tilts the $\dot{k} = 0$ line clockwise down. It lowers both k_{GR} and k_{max}.

The $\dot{c} = 0$ line. From (M.SS.1) it follows:

$$c = k\,\frac{(b+\lambda)[(r+d)\,\tilde{\sigma} - (r-\rho)]}{r - [\rho - \lambda\,\tilde{\sigma}]}\,. \qquad (39)$$

We define the graph of $\dot{c} = 0$ by the set: $cc \equiv \{(k, c) \in \mathbb{R}_+^2 \mid c = k \, \frac{(b+\lambda)[(r+d)\,\tilde{\sigma} - (r-\rho)]}{r-\rho+\lambda\,\tilde{\sigma}}\}$. As k approaches zero, $r(k)$ goes to infinity, and $[r(k)\,k]$ approaches zero. We therefore find: $(0, 0) \in cc$.[18]

Next, in (39), we find an asymptote at: $r(k) = \rho - \lambda\,\tilde{\sigma}$. To the left of the asymptote, $r(k) > \rho - \lambda\,\tilde{\sigma}$, and the slope of the $\dot{c} = 0$ curve is positive:

$$\begin{aligned} \frac{\partial c}{\partial k}|_{\dot{c}=0} =& \frac{(b+\lambda)[(r+d)\,\tilde{\sigma} - (r-\rho)]}{r - [\rho - \lambda\,\tilde{\sigma}]} \\ & - k\,\frac{(b+\lambda)\,\tilde{\sigma}}{[r-\rho+\lambda\,\tilde{\sigma}]^2}\,[\rho + d - (\tilde{\sigma}-1)\,\lambda]\,r'(k) > 0\,, \end{aligned}$$

where the positive sign follows from two facts. First, $r'(k) < 0$. Second, $[\rho + d - (\tilde{\sigma} - 1)\,\lambda] > 0$ follows from the sign restriction in (9). By the fact that there is an asymptote, the positive slope increases in k for $0 \leq k < r^{-1}(\rho - \lambda\,\tilde{\sigma})$.

We finally discuss the impact of (a change in) λ on the $\dot{c} = 0$ locus. A rise in λ tilts the $\dot{c} = 0$ line clockwise. To see this, consider (39):

$$\frac{\partial c}{\partial \lambda}|_{\{k \text{ fixed}\}} = k\,\frac{\rho + r(\tilde{\sigma}-1) + \tilde{\sigma}\,d}{[r-\rho+\lambda\,\tilde{\sigma}]^2}\,(r - \rho - b\,\tilde{\sigma}) < 0\,.$$

It remains to show that $(r - \rho - b\,\tilde{\sigma}) < 0$. In a steady state, I claim that $r(k) < \rho + b\,\tilde{\sigma}$. Suppose, to the contrary, $r(k) \geq \rho + b\,\tilde{\sigma}$. Then there exists some $\varepsilon \geq 0$ such that: $r(k) = \rho + b(1+\varepsilon)\,\tilde{\sigma}$, or, $[r(k) - \rho]/\tilde{\sigma} = b(1+\varepsilon)$. From (M.SS.1) and (M.SS.2):

$$\begin{aligned} \frac{f(k)}{k} - \delta &= \frac{(b+\lambda)[r + d - (r-\rho)/\tilde{\sigma}]}{\lambda + (r-\rho)/\tilde{\sigma}} \\ &= \frac{b+\lambda}{\lambda + b(1+\varepsilon)}\,[f'(k) - \delta + d - b - \varepsilon\,b] \\ &= \frac{b+\lambda}{\lambda + b(1+\varepsilon)}\,[f'(k) - \delta - \varepsilon\,b] \leq [f'(k) - \delta]\,, \end{aligned}$$

where the third line follows from $b = d$. The inequality, however cannot possibly be true by strict concavity of the production function. Thus, $r(k) < \rho + b\,\tilde{\sigma}$. ||

A.4 Proof of Lemma 2.

(i) Consider (M.SS.1) and Definition 1. Then, $0 = \frac{r(k)-\rho}{\tilde{\sigma}} + \Gamma$, and (i) directly follows.

[18] As $(0, 0) \in cc$ and $(0, 0) \in kk$, there exists a trivial steady state.

(ii) From (12) and (14), $(w/k) = (r+\delta)\,(1-\alpha)/\alpha$. Thus:

$$\frac{h}{k} = \frac{w/k}{r+d+\lambda} = \frac{(1-\alpha)}{\alpha}\,\frac{r+\delta}{r+d+\lambda}\,. \tag{40}$$

From $\Gamma(\lambda) = \lambda - (b+\lambda)/(1+h/k)$ and $\Gamma(\tilde{\lambda}) = 0$:

$$\tilde{\lambda} = b\left[\frac{h}{k}|_{r=\rho}\right]^{-1} = b\left[\frac{(1-\alpha)}{\alpha}\,\frac{\rho+\delta}{\rho+d+\lambda}\right]^{-1}.$$

(iii) As $\Gamma = -(r(k)-\rho)/\tilde{\sigma}$ it follows that $\partial\,\Gamma/(\partial\,\lambda) = -r'(k)/\tilde{\sigma}\,\partial\,k/(\partial\,\lambda) > 0$. ||

A.5 Proof of Proposition 1.

(i) We use the properties of the $\dot{k} = 0$- and $\dot{c} = 0$-lines. A change in η has no impact on the strictly concave $\dot{k} = 0$-curve. Consider the $\dot{c} = 0$-line, as expressed by (39). To the left of the asymptote (for all positive $k : r(k) - [\rho - \lambda\,\tilde{\sigma}] > 0$) consumption rises (declines) at points above (below) the $\dot{c} = 0$-line. Hold k fixed:

$$\frac{\partial\,c}{\partial\,\eta}|_{\{\dot{c}=0,\ k\,\mathrm{fixed}\}} = (r-\rho)\,\frac{k\,(b+\lambda)(d+r+\lambda)\tilde{\sigma}_{\eta}}{[r-\rho+\lambda\,\tilde{\sigma}]^2}\,. \tag{41}$$

As $\tilde{\sigma}_{\eta} = (\sigma-1)(1-\varepsilon)/(1-\eta)^2 > 0$, the sign of (41) depends on the sign of $[r(k)-\rho]$.
(ii) If $\Gamma < 0$, $[r(k)-\rho] > 0$ by Lemma 2. From (41) it follows that $\frac{\partial c}{\partial \eta}|_{\dot{c}=0} > 0$, for a fixed k. That is, a rise in η induces the $\dot{c} = 0$-line to tilt up and to the left. As the slope of the $\dot{k} = 0$-line is positive and smaller than the slope of the $\dot{c} = 0$-line, $\partial\,k/\partial\,\eta < 0$.
(iii) As $\lambda < \bar{\lambda}$, $k_{PY} < k_{GR}$. Thus, $0 < r(k_{PY}) = \partial\,[y(k) - \delta\,k]/(\partial\,k) = \partial\,c/(\partial\,k) \Rightarrow \partial\,c/\partial\,\eta = (\partial\,c/\partial\,k)\,(\partial\,k/\partial\,\eta) < 0$.
$c/k = y/k - \delta \Rightarrow \partial\,(c/k)/(\partial\,k) = \partial\,(y/k)/(\partial\,k) < 0$, as the production function is strictly concave. As k declines, c/k rises.
Similar reasoning applies to the other cases, in which $[r(k)-\rho] < 0 \Leftrightarrow \Gamma > 0$ and $[r(k)-\rho] = 0 \Leftrightarrow \Gamma = 0$, respectively.||

A.6 Proof of Proposition 3.

From $g_{c(v,.)} = -\Gamma = -\lambda + (b+\lambda)/(1+h/k)$ it follows:

$$\frac{\partial\,g_{c(v,.)}}{\partial\,\eta} = -\frac{b+\lambda}{(1+h/k)^2}\,\frac{\partial\,(h/k)}{\partial\,k}\,\frac{\partial\,k}{\partial\,\eta}\,.$$

From (40) and $r'(k) < 0$, it follows: $\mathrm{sgn}[\partial\,(h/k)/\partial\,k] = \mathrm{sgn}[\delta-(d+\lambda)]$. Since $\mathrm{sgn}\,k_\eta = \mathrm{sgn}[-(r(k)-\rho)]$, it follows: $\mathrm{sgn}\left[(\partial\,g_{c(v,.)})/(\partial\,\eta)\right] = \mathrm{sgn}\,\{[\delta-(d+\lambda)]\,[r(k)-\rho]\}$. ||

A.7 Optimal Allocation. The current value Hamiltonian for the command optimum is given by:

$$H_c[c(v,s), c(s), k(s), \mu(s), \mu_k(s), s] = \int_{-\infty}^{s} b\,u[c(v,s), x(v,s)]\,e^{-[b+(\rho-\tilde{\rho})](s-v)}\,d\,v$$

$$+\,\mu(s)\,[c(s) - b\int_{-\infty}^{s} e^{-b\,(s-v)}\,c(v,s)\,d\,v\,] + \mu_k(s)\,\,[y(k(s)) - c(s) - \delta\,k(s)]\;, \qquad (42)$$

where $x(v,s) = c(v,s)^{\varepsilon}\,c(s)^{1-\varepsilon}$, $c(v,s)$ and $c(s)$ are decision variables, $k(.)$ is a state variable, $\mu_k(.)$ is a costate variable, and $\mu(.)$ is the Lagrange multiplier associated with the consumption constraint. Pontryagin's principle implies:

$$\begin{aligned}
&\mu(s) = u_{c(v,s)}\,e^{-(\rho-\tilde{\rho})(s-v)}\,,\\
&-\int_{-\infty}^{s} b\,u_{c(s)}\,e^{-[b+(\rho-\tilde{\rho})](s-v)}\,d\,v + \mu(s) - \mu_k(s) = 0\,,\\
&-\,\mu_k(s)\,(y'(k(s)) - \delta) = \dot{\mu}_k(s) - \mu_k(s)\,\tilde{\rho}\,.
\end{aligned}$$

We consider

$$u_{c(s)} = \frac{u_{c(s)}}{u_{c(v,s)}}\,u_{c(v,s)} = \frac{c(v,s)\,(1-\varepsilon)\eta}{c(s)\,(1-\varepsilon\,\eta)}\,\mu(s)\,e^{(\rho-\tilde{\rho})(s-v)}$$

to solve the above integral: $\mu_k(s) = \mu(s)\,(1-\eta)/(1-\varepsilon\,\eta)$. It follows:

$$\tilde{\rho} - (y'(k(s)) - \delta) = \frac{\dot{\mu}_k(s)}{\mu_k(s)} = \frac{\dot{\mu}(s)}{\mu(s)} = \frac{\dot{u}_{c(v,s)}}{u_{c(v,s)}} - (\rho - \tilde{\rho})\,,$$

thus,

$$\frac{\dot{u}_{c(v,s)}}{u_{c(v,s)}} = \rho - (y'(k(s)) - \delta)\,.$$

Let $\tilde{r}(s) \equiv (y'(\tilde{k}(s)) - \delta)$, and consider the utility function (4). We then conclude:

$$\frac{\dot{c}(s)}{c(s)} = \frac{\tilde{r}(s) - \tilde{\rho}}{\tilde{\sigma}}\,, \qquad (43)$$

$$\frac{\dot{c}(v,s)}{c(v,s)} = \frac{\tilde{r}(s) - \rho}{\tilde{\sigma}} + \frac{\tilde{\sigma} - \sigma}{\tilde{\sigma}}\,\frac{\dot{c}(s)}{c(s)}\,, \qquad (44)$$

$$\frac{c_v(v,s)}{c(v,s)} = \frac{\rho - \tilde{\rho}}{\tilde{\sigma}}\,, \qquad (45)$$

$$\dot{k} = y(k(s)) - c(s) - \delta\,k(s)\,. \qquad (46)$$

A.8 Derivation of β. We notice that $h(v,t) = \frac{w\, e^{-\lambda(t-v)}}{r+d+\lambda}\, \frac{b+\lambda}{b}$. Considering (30):

$$c(v,t) = \frac{\tilde{\rho}+d-\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}}{\tilde{\rho}+d+\lambda}\,(y-k\,y')\,e^{-\lambda(t-v)}\,\frac{b+\lambda}{b} + \left[\tilde{\rho}+d-\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}\right]\,[a(v,t)+\beta(v,t)]\,,$$

where we consider that $r=\tilde{\rho}$ needs to hold in the decentralized allocation. We next aggregate over cohorts, and observe the capital market clearing condition:

$$c(t) = \frac{\tilde{\rho}+d-\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}}{\tilde{\rho}+d+\lambda}\,(y-k\,y') + \left[\tilde{\rho}+d-\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}\right]\,[k(t)+\beta(t)]\,.$$

We next consider $y-k\,y' = y-\delta\,k-k(y'-\delta) = y-\delta\,k-\tilde{\rho}\,k$. Finally, rearranging terms yields:

$$\beta(t) = \left(\frac{1}{\Delta^{-1}} - \frac{1}{\tilde{\rho}+d+\lambda}\right)(y-\delta\,k) - \frac{d+\lambda}{\tilde{\rho}+d+\lambda}\,k\,.$$

A.9 Proof of Proposition 5. Considering the definition of the GRE:

$$\Gamma = \lambda-(b+\lambda)\,\Delta^{-1}\,\frac{k}{y-\delta\,k} \;\Rightarrow\; \frac{k}{y-\delta\,k} = \frac{\lambda-\Gamma}{(b+\lambda)\Delta^{-1}}\,.$$

Applying this relationship in (37) yields:

$$\frac{b}{\tilde{\rho}}\,\frac{\beta_0}{y-\delta\,k} = -\frac{1}{(\tilde{\rho}+d+\lambda)\Delta^{-1}}\,\left[\Gamma+\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}\right]\,.$$

It follows that

$$\text{sgn}\ \beta_0 = -\,\text{sgn}\left[\Gamma+\frac{\tilde{\rho}-\rho}{\tilde{\sigma}}\right]\,.$$

The proposition follows. ||

A.10 Proof of Proposition 6. We consider the impact of a marginal increase in η on $\beta_0\,b/\tilde{\rho}$ for the case $\rho=\tilde{\rho}$.

Step 1. We first reexpress β_0.

$$\begin{aligned}\frac{b}{\tilde{\rho}}\,\beta_0 &= \left[\frac{1}{\tilde{\rho}+d+\lambda} - \frac{1}{\tilde{\rho}+d}\right](y-\delta\,k) + \frac{d+\lambda}{\tilde{\rho}+d+\lambda}\,k \\ &= \frac{-\lambda}{(\tilde{\rho}+d+\lambda)(\tilde{\rho}+d)}\,(y-\delta\,k) + \frac{d+\lambda}{\tilde{\rho}+d+\lambda}\,k \\ &= k\left[\frac{-\lambda}{(\tilde{\rho}+d+\lambda)(\tilde{\rho}+d)}\,\frac{c}{k} + \frac{d+\lambda}{\tilde{\rho}+d+\lambda}\right],\end{aligned}$$

where we consider $(y - \delta\, k) = c$ in a steady state.

Step 2. Suppose $\Gamma < 0$. Differentiation (of the third line) with respect to η yields:

$$\frac{\partial}{\partial\,\eta}\left[\frac{b}{\tilde{\rho}}\,\beta_0\right] = \left[\frac{b}{\tilde{\rho}}\,\frac{\beta_0}{k}\right]\frac{\partial\,k}{\partial\,\eta} - k\,\frac{\lambda}{(\tilde{\rho}+d+\lambda)(\tilde{\rho}+d)}\,\frac{\partial\,(c/k)}{\partial\,\eta}\,.$$

If $\Gamma < 0$: $\beta_0 > 0$ (by Proposition 5), $\partial\,k/\partial\,\eta < 0$ and $\partial\,(c/k)/\partial\,\eta > 0$ (by Proposition 1). Thus $\partial\,\beta_0/\partial\,\eta < 0$. |

Step 2. Suppose $\Gamma > 0$. Differentiation (of line two) with respect to η yields:

$$\frac{\partial}{\partial\,\eta}\left[\frac{b}{\tilde{\rho}}\,\beta_0\right] = \frac{-\lambda(y'(k)-\delta)}{(\tilde{\rho}+d+\lambda)(\tilde{\rho}+d)} + \frac{d+\lambda}{\tilde{\rho}+d+\lambda} = \frac{d(\tilde{\rho}+d+\lambda)+\lambda(\tilde{\rho}-r(k))}{(\tilde{\rho}+d+\lambda)(\tilde{\rho}+d)} > 0\,,$$

where the inequality is implied by $(\tilde{\rho}-r(k) > 0$, because $\partial\,k/\partial\,\eta > 0 \Leftrightarrow \partial\,r(k)/\partial\,\eta < 0$ for $\Gamma > 0$. ||

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