A Dual-Solovian Measure of Productivity Increase and its Early Antecedents

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3 June 2009

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Abstract. The duality between a production function and the cost function generated by it implies that a Solovian 'growth accounting' measure of productivity increase, as referred to the industry, has an equivalent dual measure, based on what may be called 'price accounting'. It is argued in this paper that the dual measure provides a coherent framework for considering productivity increase in relation to inflation/deflation, earnings dispersion, long-run variations in domestic relative prices and in external terms of trade. Even though the theoretical interest in measures based on real input prices dates back to the late 1960s, few or no attempts have been made thereafter to adopt it in practice. Curiously enough, the practical adoption of some kind of 'price accounting' dates to much earlier. We argue in this paper that, during the 19th century, distinguished statisticians and economic commentators such as G.R. Porter and R. Giffen based their evaluations on the comparative change in prices, wages and profits and in so doing they followed a logic that remarkably resembles that of a dual-Solovian measure.

Keywords: productivity increase, TFP, cost function, real wages, income distribution
JEL classification: D24, D33, B16.

May 2009

Acknowledgements. I thank Ian Steedman for his helpful comments on a previous draft.

1. Introduction

Solow's growth accounting model (Solow, 1957) has been continuously refined in half a century of applications: industrial heterogeneity, explicit aggregation, diverse labour skills, capital heterogeneity etc. have all been considered, and their consequences have been worked out to a maximum of precision (e.g. Kendrik, 1961; Denison, 1962; Jorgenson and Griliches, 1967; Hulten, 1978; Jorgenson, Gollop and Fraumeni, 1987; Jorgenson and Stiroh 2000; and OECD, 2001). This method, in one modern specification or another, gained such a consensus that no current macroeconomic analysis of potential growth in actual economies can do without it. Its enormous success is due not only to the irresistible charm of simplicity, but also to the fact that during the past fifty years (or more) the relevant macroeconomic aspect of technical change was its contribution to long-run, potential economic growth.

Yet productivity increase also has another and symmetrical set of macroeconomic effects, in which there is an increasing interest. Not only aggregate output, but also real earnings *rates* increase along with productivity increase and they do so in very uneven ways, leading to changes in factor shares and earnings dispersion. In particular, earnings dispersion has increased very much in recent years and the more so in Anglo-Saxon countries, in which the institutional setup of the economy is most conducive to productivity increase (e.g. Katz L.F. and Autor D.H., 1999; World Bank, 2006, pp. 28-54; Atkinson, 2007; Checchi D. and C. García-Peñalosa, 2008, pp. 603-606).

Even though output growth relative to inputs and price fall relative to earnings are two sides of the same coin, we have an agreed method for fitting productivity data for the first side, but not for the second. An observed change in, say, the money wages of unskilled workers in a certain industry, is in principle the result of a series of simultaneous changes concerning the industry's output price, other industries' (or imported) produced input prices, industrial productivity and the distribution of the industrial value added. It seems to be of some interest to fit all this into a simple and coherent accounting model of price change.

This paper presents such a measure of productivity increase and finds it to be the precise dual of a Solow-Jorgenson industrial measure (as based on gross output), and to be numerically identical to it. Using the dual, the same information can be processed in a different way, thus capturing other aspects of the same phenomenon. The redistribution of gross output between the industry under consideration and the input-providing industries and of the industrial value added among 'internal' input services, are important collateral aspects which are mixed together when a time series of 'real' input prices is considered: they need to be separated from each other and from technological change. At an aggregate level, the effect of a permanent change in the terms of trade must also be distinguished from technical change proper. These aspects can be coordinated on the basis of the dual.

Our general point of view on productivity change is not new, of course. The 1960s capital theory debates raised doubts on the possibility of fitting a non-arbitrary real capital index into Solow's formula and this opened the way to measures looking at prices. Hall (1968) presented some analysis of price-based measures of technical progress embodied in durable machines. Lydall (1969) proposed a statistically meaningful aggregate measure based on a simple weighted average of output deflated incomes. Steedman (1983) proposed theoretically sound Harrodian, Hicksian and Solovian measures by formalizing the shift of a the real wages-rate of profit (interest) frontier in an input output system.

We know that the emergence of theoretical alternatives to growth accounting did not have a significant impact on empirical analysis. This is surprising, because price-based measures have similar, if not more accessible statistical requirements, and they do not involve additional theoretical assumptions. To say the least, they are a good complement to growth accounting measures and offer a cross check for statistical accuracy and in this respect alone they would provide a clear gain.

We do not propose, in the present paper, any application of the dual measure to actual data. Rather, we show, in the second part of the paper, that some kind of 'price accounting' have been implicitly adopted much earlier than growth accounting. In fact, the 19th century analysts and commentators (economists, statisticians, public servants, journalists) typically analyzed the effects of the industrial take-off and the very pace of technical improvements using price and wage series. Since the 1830s, in fact, there has been in England a flourishing of statistical-economic studies and among them the analysis of price series was central (partly because price and wage records were available on a larger and more systematic scale than other data): Tooke's *History of Prices* and Porter's *Progress of the Nation* are certainly the two best known and most monumental studies. In the second half of the century, the leading personality of the field was certainly Robert Giffen. One can find in Porter's and in Giffen's studies an increasing awareness of the fact that price and wage series over long periods of time reflected a variety of lasting phenomena which needed to be distinguished by some kind of price accounting. In particular, gold appreciation or depreciation and productivity increase were the two main sources of long-run price change that they considered and tried to distinguish (redistribution was explicitly considered of negligible empirical relevance). It will be shown that Giffen, in particular, came very close to a general and coherent method of price accounting. Even though his argument concerned the economy as a whole and not the individual industry, the logical structure of his argument is quite similar to the formal analysis that we propose in the first part of the paper.

PART I: Productivity increase and the industrial cost functions

2. A dual measure of industrial productivity increase

Let us consider an individual industry. In accordance with the EU Klems classification[1], we assume that production requires the use of many kinds of labour (characterized, say, by different skills), many kinds of capital goods (such as ICT capital, machinery, buildings, …), intermediate materials and energy. The service of each input is paid a certain rental rate, which we (provisionally) assume given to the industry. Let us denote by $\mathbf{w}$ the vector of wages and by $\boldsymbol{\pi}$ the vector of rental rates for the services of all other inputs. These rental rates may include an interest payment at rate r. For instance, in the case of a raw material bought from another industry at price p_j, we have $\pi_j = (1+r)p_j$; in the case of energy the rental rate may correspond to the un-augmented price; in the case of 'fixed capital' items yet another formula should be used. We do not need at this stage to enter into details, and just assume, if not otherwise stated, the correct π to be calculated for each item. In accordance to the Solow-Jorgenson assumptions, the industry has constant returns to scale[2] and is in a full long-run equilibrium, in the sense that pure profits are maximized *and* equal to zero.

The *unit* cost function, denoted by *c*, will suffice to fully describe technical conditions. In order to take (exogenous) productivity change into account, *c* will be a function not only of $\mathbf{w}$ and $\boldsymbol{\pi}$, but also of (logical) time, t. Let then

$$c = c(\mathbf{w}, \boldsymbol{\pi}, t) \qquad (1)$$

Since maximum profits are zero, we have

$$p = c(\mathbf{w}, \boldsymbol{\pi}, t) \qquad (2)$$

Now the rate of productivity increase, $\boldsymbol{\gamma}$, can be sensibly defined as

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[1] EU Klems is an European project that aims to create a database on measures of economic growth, productivity, employment creation, capital formation and technological change at the industry level for all European Union member states from 1970 onwards. See website.

[2] It would not be difficult to show that locally constant returns, at the bottom of a U-shaped average cost curve, would be fully consistent with the argument that follows.

$$\gamma \equiv -\frac{\dot{c}}{c} \qquad (3)$$

Since all prices generally change through time along with productivity change, we need to calculate $\boldsymbol{\gamma}$ on the basis of the observed price changes. This can be done by totally differentiating equation (2), obtaining

$$\dot{p} = \sum_i \frac{\partial c}{\partial w_i} \dot{w}_i + \sum_j \frac{\partial c}{\partial \pi_j} \dot{\pi}_j - \dot{c} \qquad (4)$$

Let now $(\mathbf{l}, \mathbf{k})$ be the cost minimizing input bundle per unit of output, where $\mathbf{l}$ denotes the vector of labour inputs and $\mathbf{k}$ denotes the vector of all other inputs. By Shephard's Lemma, we have

$$\frac{\partial c}{\partial w_i} = l_i; \quad \frac{\partial c}{\partial \pi_j} = k_j$$

Multiplying each input use by its rental price and dividing through by the output price, we obtain the input shares

$$\sigma_i = \frac{w_i l_i}{p}; \quad \sigma_j = \frac{\pi_j k_j}{p}$$

Equation (4) can thus be re-written as

$$\hat{p} = \sum_i \sigma_i \hat{w}_i + \sum_j \sigma_j \hat{\pi}_j - \gamma$$

or

$$\boldsymbol{\gamma} = \sum_i \boldsymbol{\sigma}_i (\hat{w}_i - \hat{p}) + \sum_j \boldsymbol{\sigma}_j (\hat{\boldsymbol{\pi}}_j - \hat{p}) \qquad (5)$$

The industrial rate of productivity increase is therefore a weighted sum of the rates of change in input rentals, *in terms* of the industrial output, using input shares as weights[3].

There is a clear symmetry with Solow's growth accounting formula, as referred to an industry characterized by the presence of heterogeneous labour and a variety of produced inputs. For the reader's convenience, let us write in obvious notation Solow's growth accounting equation, as referred to the industry

$$\hat{Y} = \hat{A} + \sum_i \sigma_i \hat{L}_i + \sum_j \sigma_j \hat{K}_j$$

A slight rearrangement gives

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[3] Our equation can be considered as a micro-funded and industry-level version of the aggregate index formulated by Lydall (1969), in his equation (8), p. 6.

$$\hat{A} = \sum_i \boldsymbol{\sigma}_i \left(\hat{Y} - \hat{L}_i \right) + \sum_j \boldsymbol{\sigma}_j \left(\hat{Y} - \hat{K}_j \right) \qquad (6)$$

It is clear that our equation (5) is the precise *dual* of the more conventional equation (6)[4], and that the two equations give in principle the same result. To see this, let us first note that, by homogeneity of degree one of c in $\mathbf{w}, \boldsymbol{\pi}$, and by Shephard's lemma, we have $\mathbf{wl} + \boldsymbol{\pi}\mathbf{k} = c$, and $\mathbf{wL} + \boldsymbol{\pi}\mathbf{K} = cY$, where $\mathbf{L} = \mathbf{l}Y, \mathbf{K} = \mathbf{k}Y$. By equation (2), the following must always hold

$$\sum_i \boldsymbol{\sigma}_i \left(\hat{w}_i + \hat{L}_i \right) + \sum_j \boldsymbol{\sigma}_j \left(\hat{\boldsymbol{\pi}}_j + \hat{K}_j \right) = \hat{p} + \hat{Y}$$

which clearly implies $\boldsymbol{\gamma} = \hat{A}$.

No information is lost or gained by moving from the traditional 'growth accounting' method to our proposed 'price accounting' method. Since information is elaborated differently, however, one may expect that some further aspects may come to light.

3. The distribution of the industrial gains from productivity increase

As we remarked in the Introduction, empirical evidence shows that in the past few decades the overall earnings dispersion tended to rise and the overall wages share tended to fall. It should be presumed, then, that at the industry level the rates of change in input rentals (in terms of the output) normally rise at different proportional rates and some of them even fall, along with productivity increase, and it would be of some interest to split each observed proportional change into two components: productivity increase and redistribution.

A preliminary distinction must be made here between value added and gross output. It is customary to make growth accounting calculations in terms of value added, when they refer to the economy as a whole; while one can find either value added or gross output (as in equation (6) above) in industry-level calculations (see OECD, 2001, p. 13). In the latter case, the effect of a change in relative commodity prices is incorporated into the measure of industrial output itself[5]. If, say, the price of cupper permanently falls relative to the price of electric wires, the 'real' value added in the electric wires industry automatically rises. There is of course a clear logic in accounting for such a change 'as if' it were due to productivity increase. Using price accounting,

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[4] In equation (6), of course, Total Factor Productivity change, $\hat{A}$, is the difference between of (average) proportional change of labour productivity and (average) capital deepening.

[5] This aspect has been brought to my attention by Robert Solow in the occasion of the XIII Eshet conference held in Thessaloniki, 23-26 April 2009.

however, we can do better than that: we can *separate* this effect from productivity increase in a proper sense.

For illustrative purposes, it will suffice to consider the very simple case in which all inputs other than labour have the nature of circulating capital, so that $\boldsymbol{\pi} = (1+r)\boldsymbol{p}$, and the rate of interest is constant. Denoting by v the value added per unit of output, the output price can be decomposed as

$$p = v + \sum_j \frac{\partial c}{\partial \pi_j} p_j \qquad (7)$$

Differentiating totally with respect to (logical) time, we obtain

$$\dot{p} = \dot{v} + \sum_j \frac{\partial c}{\partial \pi_j} \dot{p}_j \text{ , and}$$

$$\hat{p} = \frac{v}{p}\hat{v} + \sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)} \hat{p}_j$$

Now it is easy to see that $\frac{v}{p} + \sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)} = 1$ and therefore

$$(\hat{v} - \hat{p}) = \sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)} \frac{p}{v} (\hat{p} - \hat{p}_j) \qquad (8)$$

Equation (8) provides a measure of the aggregate change in real industrial earnings, due to the sole change in relative commodity prices (in terms of the industrial output): it measures, therefore, an inter-industry real transfer, whatever its economic sources (perhaps productivity-related) may be. We can now relate the change in value added to the change in individual earnings rates and to productivity increase as defined in equation (3). Substituting equation (2) into equation (7) and differentiating, one gets

$$\hat{v}\frac{v}{p} = \sum_i \boldsymbol{\sigma}_i \hat{w}_i + \frac{r}{(1+r)} \sum_j \boldsymbol{\sigma}_j \hat{p}_j - \boldsymbol{\gamma}$$

Since

$$\sum_i \boldsymbol{\sigma}_i + \frac{r}{(1+r)} \sum_j \boldsymbol{\sigma}_j = \frac{\sum_i w_i l_i + r \sum_j p_j k_j}{p} = \frac{v}{p}$$

We have

$$(\hat{v} - \hat{p}) = \frac{p}{v}\left[\sum_i \boldsymbol{\sigma}_i (\hat{w}_i - \hat{p}) + \frac{r}{(1+r)} \sum_j \boldsymbol{\sigma}_j (\hat{p}_j - \hat{p}) - \boldsymbol{\gamma}\right] \qquad (9)$$

It will be clear that $\frac{p}{v}\boldsymbol{\sigma}_i, \frac{p}{v}\frac{r}{(1+r)}\boldsymbol{\sigma}_j$ are the shares of wage and profit earnings *on value added*. Setting $\frac{p}{v}\boldsymbol{\sigma}_i = \boldsymbol{\upsilon}_i$, and $\frac{p}{v}\frac{r}{(1+r)}\boldsymbol{\sigma}_j = \boldsymbol{\upsilon}_j$, with $\sum_i \boldsymbol{\upsilon}_i + \sum_j \boldsymbol{\upsilon}_j = 1$, and combining equation (9) with equation (8), we finally get

$$\sum_i \boldsymbol{\upsilon}_i(\hat{w}_i - \hat{p}) + \sum_j \boldsymbol{\upsilon}_j(\hat{p}_j - \hat{p}) = \frac{p}{v}\boldsymbol{\gamma} + \sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)}\frac{p}{v}(\hat{p} - \hat{p}_j) \qquad (10)$$

In equation (10) the proportional changes in the real rental rates to each input[6] are related to productivity increase on one side and the relative change in commodity prices on the other.

Some remarks are in order. The first is that the effect of productivity increase alone on the real earnings *internal* to the industry is so to speak, amplified by the ratio of price to value added, and this confirms, from the point of view of the dual, the findings that Hulten (1978) referred to the primal[7] . Moreover, at a constant rate of interest, and in the absence of relative price changes, this effect is restricted to wages (and is therefore further amplified). If, say, the value added is 83% of the price and wages of the different kinds of labour are 99% of value added (these data being consistent with a rate of interest of 5% and a wages share on gross output of about 81%), a 2% rate of industrial productivity increase would determine an average increase in real wages of about 2,42%.

The second remark concerns the specific effect of relative price changes. Suppose that the output price falls relative to the prices of commodity inputs (say, because of differential rates of productivity increase). At a constant rate of interest, the real wages have, so to speak, a double burden: the diminution of value added per unit of gross output and the increase in real profits, as expressed by the second sum on the left hand side of equation (10). A relative *increase* of the output price has, of course, symmetric effects.

This leads us to the problem of isolating redistribution of value added within the industry. On the basis of equation (10), we can define the internal redistribution in favour (or to the expense) of labour of kind i , D_i , as the difference between the observed proportional change in the real wage of labour *i* on one side and the proportional change in the real value added on the other. Weighting this difference by the share of labour of kind *i* on value added, we get, by equation (10)

[6] It goes without saying that when the rate of interest is constant we have $\hat{\boldsymbol{\pi}}_j = \hat{p}_j$.

[7] This 'magnified' rate of productivity increase is called by Hulten 'effective rate' of sectoral productivity change. See Hulten, 1978, pp. 514-5.

$$D_i = \boldsymbol{\upsilon}_i \left[(\hat{w}_i - \hat{p}) - \frac{p}{v}\boldsymbol{\gamma} - \frac{p}{v}\sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)}(\hat{p} - \hat{p}_j) \right] \tag{11}$$

Similarly, sticking to our assumption of a constant rate of interest, redistribution in favour (or to the expense) of the real profit on produced input *j* is

$$D_j = \boldsymbol{\upsilon}_j \left[(\hat{p}_j - \hat{p}) - \frac{p}{v}\boldsymbol{\gamma} - \frac{p}{v}\sum_j \frac{\boldsymbol{\sigma}_j}{(1+r)}(\hat{p} - \hat{p}_j) \right] \tag{12}$$

Of course, we have

$$\sum_i D_i + \sum_j D_j = 0$$

The observed change in a real rental rate, say the real wage rate of labour *i*, can at this point be split into three components (productivity increase, change in relative commodity prices, redistribution of value added) just by rearranging equation (11) in an obvious way.

The *D*s define the ranking of inputs in the scale of redistribution within the industry. Moreover, their dispersion defines, in comparison with other industries, a ranking in the industrial 'degree' of internal redistribution.

The main advantage of price accounting is therefore that it provides a test of coherence for the measures of real earnings growth when productivity increase and redistribution are at work together.

4. Aggregation over industries

The current aggregation procedure of Solow's Total Factor Productivities in input-output systems is based on Domar (1961), later generalized by Hulten (1978). Aggregation, however, is also possible starting from the dual. A relatively neglected, but theoretically clear contribution in this direction is that of Steedman (1983). The main premise, apart from constant returns to scale, is that the wage rates (in a common numéraire) and the rate of interest tend to be uniform across industries. Moreover, commodities are paid by the input receiving industries their long-run competitive price. Under these premises, which are fully shared also by Domar-Hulten methods (see Jorgenson, 1990, p. 67), prices, wages and interest rates for the economy as a whole are tied together by a system of equations à la Sraffa-Leontief. The aggregate rate of productivity increase can at this point be expressed by a measure of the shift in the real wages-rate of profit (interest) frontier. Steedman proposed, among other measures, a 'Harrodian' measure, consisting in the common potential increase in the real wage rates (all expressed in terms of the same composite numéraire) associated with a constant rate of interest. He assumed a given number of (joint production) processes[8] to be known at each time and measured the rate of productivity increase in each process by a price-weighted sum of rates of change of output coefficients minus a price weighted sum of input coefficients. His main finding is that the aggregate rate of productivity increase ‘can be far greater than the average process-level improvement rate, due to the role of produced means of production’ (Steedman, 1983, p. 232), and even greater than that expected on the basis of Domar’s weights.

We adopt here the same logic of Steedman’s measure. In order to preserve industry-level duality, however, we translate his argument in terms of industries (rather than processes) and of unit cost functions (rather than input output coefficients). The simple case with two industries will suffice to illustrate the main rationale of our method of aggregation.

Assuming, once again, a constant rate of interest, and that the inputs to each industry are skilled labour (index *S*), unskilled labour (index *U*), and the other industry’s output, we may write equation (5) for each industry as

$$\hat{p}_1 = \sigma_{S1}\hat{w}_S + \sigma_{U1}\hat{w}_U + \sigma_{21}\hat{p}_2 - \gamma_1$$
$$\hat{p}_2 = \sigma_{S2}\hat{w}_S + \sigma_{U2}\hat{w}_U + \sigma_{12}\hat{p}_1 - \gamma_2$$

Let the numéraire be a composite commodity formed by n_1 units of commodity 1 and n_2 units of commodity 2, so that $n_1 p_1 + n_2 p_2 = 1$. Moreover let $s_i = n_i p_i, i = 1, 2$ be the shares of commodity i in the numéraire, at the time to which our analysis refers. Needless to say, we have

$$\hat{p}_1 = -\frac{s_2}{s_1}\hat{p}_2$$

Substituting $\hat{p}_1$ into the pair of equations above and eliminating $\hat{p}_2$, we obtain

$$(s_1 + \sigma_{12}s_2)\gamma_1 + (s_2 + \sigma_{21}s_1)\gamma_2 = (s_2 + \sigma_{21}s_1)(\sigma_{S2}\hat{w}_S + \sigma_{U2}\hat{w}_U) + (s_1 + \sigma_{12}s_2)(\sigma_{S1}\hat{w}_S + \sigma_{U1}\hat{w}_U)$$

Now the aggregate rate of productivity increase, Γ, can sensibly be defined, as in the case of Steedman’s ‘Harrodian’ aggregate, as the *potential* common rate of growth of the real wages of skilled labour and of unskilled labour (see Steedman, 1983, p. 226). Setting $\hat{w}_S = \hat{w}_U = \hat{w}$, and recalling that $s_1 + s_2 = 1$, we have

$$(s_1 + \sigma_{12}s_2)\gamma_1 + (s_2 + \sigma_{21}s_1)\gamma_2 = (1 - \sigma_{12}\sigma_{21})\hat{w}$$

and

$$\Gamma = \frac{(s_1 + \sigma_{12}s_2)}{(1 - \sigma_{12}\sigma_{21})}\gamma_1 + \frac{(s_2 + \sigma_{21}s_1)}{(1 - \sigma_{12}\sigma_{21})}\gamma_2$$

[8] Note that in a joint production system it is customary to speak of ‘processes’ rather than ‘industries’ or ‘sectors’.

As in Domar's and Steedman's model, the aggregate rate of productivity increase is a weighted sum of industrial rates. In the absence of produced inputs, the weights would simply be (s_1, s_2) and the aggregate rate would be a weighted *average* of industrial rates. Positive produced input shares 'increase' the weights and the more so the higher they are.

Our weights have some relation to Domar's ratios of price to value added. Having replaced Domar's capital with a second kind of labour, a comparison requires that the rate of interest be null. In this case the ratios prfice to value addede are simply: $1/(1-\sigma_{12})$, and $1/(1-\sigma_{21})$. Our weights are directly comparable to Domar's if we assume $\sigma_{12} = \sigma_{21} = \sigma$, obtaining:

$$\Gamma = \frac{(s_1 + \sigma s_2)}{(1-\sigma)(1+\sigma)}\gamma_1 + \frac{(s_2 + \sigma s_1)}{(1-\sigma)(1+\sigma)}\gamma_2$$

Now if the two commodities have equal shares in the numéraire ($s_1 = s_2 = 1/2$), or there are equal rates of productivity increase ($\gamma_1 = \gamma_2 = \gamma$), then the weights correspond to Domar's, and Γ reduces, respectively, to

$$\Gamma = \frac{1}{2(1-\sigma)}(\gamma_1 + \gamma_2) \text{ and } \Gamma = \frac{1}{(1-\sigma)}\gamma$$

It should also be noted that the aggregate rate depends on the choice of numéraire, apart from special cases[9] and there are clear economic reasons for that: productivity increase in the various industries normally determines a change in the proportions of output prices (and they do so even in the case of equal γ s) . Now, in general, the real wages, and therefore the aggregate rate of productivity increase, naturally increase at different proportional rates as expressed in terms of different commodities.

PART II: Some early attempts at measuring productivity increase by 'price accounting'

5 Introduction

As we remarked, in the last fifty years, the growth accounting method dominated the scene, and the assignment of the rate of output growth to different 'sources' has been the main problem. It has not always been so, however. Since the time of the first industrial revolution through all the 19th century at least, *both* aspects have been considered, and if one of the two had originally a lead, this was the *price accounting* aspect. This had a theoretical counterpart in the fact that the classical and

[9] The interested reader may verify that numéraire *independence* requires $(1-\sigma_{21})\gamma_2 = (1-\sigma_{12})\gamma_1$.

early neoclassical economists from Smith to Marshall had a keen interest in the (potential and actual) contribution of productivity increase to real wage growth and to the progress of the working classes in general[10]. In this respect, they found it quite natural to establish a link between productivity increase and the 'cheapness' of commodities relative to nominal wage rates.

During the 1830s in England there was a flourishing of statistical studies: the 1834 foundation of the Statistical Society of London (LSS), the publication of Porter's 'Progress of the Nation' (in 1836, followed by further editions in 1846 and 1851), and of Tooke's 'History of Prices' (a monumental publication started in 1838 and thereafter widened and updated until 1857, with the collaboration of W. Newmarch), have been at the basis of a subsequent flow of statistical analyses of the many aspects of the late 18th century and 19th century industrial take-off.

The analysis of the sources of the variations of (money) prices was one of the most intriguing and controversial topics. According to Jevons, 'a true understanding of the course of prices can alone explain many facts in the statistical and commercial history of the country' (Jevons, 1865, p. 294). The tables of prices frequently embraced very long periods of time: Tooke's series ranged from 1782 to the 1850s, and Porter's series from the start to the middle of the century; likewise, the proceedings of the LSS frequently carried studies with series of fifty years or more.

The price series reflected at once a variety of temporary factors like bad harvests, commercial crises, taxation, speculation and wars, together with more lasting causes, such as the relative abundance or scarcity of gold and the relation of gold to paper currency, as well as a permanent paramount cause, which was 'the continuous progress of invention and production' (Jevons, 1865, p. 308). Hence the need to separate, first of all, the local peaks and troughs from more lasting variations: this was typically done by averaging across prices and times, thus obtaining something similar to what we now call a trend variation of prices. Secondly, a further and more difficult problem was to distinguish between two sources of the 'trend' in money prices: currency appreciation or depreciation and productivity increase.

6 The effect of productivity increase on prices: Porter and Jevons

Even though Tooke's work soon became the main source and reference for further studies, his mode of presentation of data has been criticized on the grounds that his narrative presented a succession of heterogeneous facts and the tables failed to distinguish amongst them. According to Jevons,

[10] A comparison between J.S. Mill's and Marshall's conceptions of the actual and prospective progress of the working classes can be found in Opocher (2010).

> large tables of figures are but a mass of confused information for those causally looking into them. They will probably be the source of error to those who pick out a few figures only; a systematic (…) course of *calculation and reduction* is necessary to their safe and complete use. (Jevons, 1865, p. 294; emphasis added).

Some elementary tools for 'eliciting the general facts contained in them [the tables of prices]' (Jevons, *ibidem*) and in particular the role of currency appreciations or depreciations have been proposed by Porter and consisted of the calculation of an aggregate of prices by means of index numbers:

> There is perhaps no single circumstance more pregnant with instruction on this subject [whether the currency be at any time redundant or otherwise] than a *general* rise or fall of prices when viewed and adjusted in combination with local or temporary causes of disturbances. (Porter, 1851 [1834], p. 431)

The basis of the index numbers should be a period

> in which prices were considered to be at or near their natural level, and in which the mercantile community in this kingdom were believed to be principally engaged in their regular and legitimate business; a period, in fact, which should be free from any undue depression on the one hand, and without the excitement of speculation on the other. (Porter, 1851 [1834], p. 427)

This double distinction between individual and general changes, and between temporary and permanent changes, marked a considerable progress in the interpretation of the statistical evidence of prices and all the local and temporary factors which were at the heart of Tooke's narrative could be considered as disturbances of more general and lasting factors.

Along these lines, Jevons distinguished between three long alternating periods of elevation and fall of general prices: some thirty years of elevation around the turn of the 18th century, followed by a period of about the same length of a general fall, until the middle of the new century, and then again a prolonged elevation until 1865 (and which was to last until the mid 1870s).

Abstracting from the local peaks and troughs within each period, due to temporary factors, these secular movements were, according to Jevons, the result of two fundamental forces which may act in accordance or in opposition with each another: 'the production of gold' and 'the progress of invention'(Jevons, 1865, p. 303).

He argued that the 'great fall, proceeding from 1818 to 1830, and reaching its lowest point as yet in 1849' was not difficult to understand:

> The production of almost all articles has been improved, extended, and cheapened during this period, and all the imported articles must, too, have been affected by improvements in navigation, while there was no corresponding improvement in the production of precious

metals, from the derangement of the American mines in 1810 to the Californian discoveries in 1849. (Jevons, 1865, p. 303)

Jevons's analysis followed here closely in the steps of Porter's book. In the chapters which provide minutely detailed evidence of the improvements in the cotton, linen, silk, wool, iron and steel industries and in mining, during the first half of the 19th century, Porter invariably provided a scalar, synthetic measure of such improvements by the individual price reduction that they allowed. His comment on the spectacular fall in the price of cotton wool in the thirty years from 1820 to 1849 is worth quoting at some length, because it shows that Porter was quite close to conceiving of the measure of productivity increase as the outcome of a price accounting exercise:

> the average price [of cotton wool] per yard, which in 1820 was $12_{2/3d}$, [fell] in 1849 to $3_{2/5d}$. The average price of twist in 1820 was $2s.5_{1/2d}$, and in 1849 was little more than $10_{3/4d}$ per pound. If, in addition to these values, we take account of the reduction that has occurred in the price of raw cotton, we may be enabled to *form some judgment* as to the economy which has been introduced into the process of manufacture during the last 30 years, and be besides able to *apportion* the degrees of that economy which appertain to the spinning and to the weaving branches of the manufacture respectively. (Porter, 1851 [1834], pp. 179-81; emphasis added)

Interpreting 'judgment' in the sense of 'measure', we have here a clear statement of the idea, developed in the first part of our paper, that the industry-specific rate of productivity increase can be measured using the algebraic differences between the changes in the output price and the input prices. In the above case, explicit attention has been limited to the comparative evolution of produced input prices. One may wonder at this point why he did not mention in the quoted passage the comparative change in wages as well. There is a simple reason for that. His tables never fail to record also *wage* changes, and they show that during the same period (money) wages have been roughly *constant* (see e.g. p. 184 for weavers, p. 194 for spinners), so that the recorded fall in the output price had to be 'corrected' only for produced inputs, whose prices, too, were cheapened by technological improvements in related industries. In other words, the price of manufactures decidedly fell *relative to wages* and this was the 'real' change, detecting technological improvement. With constant wages, the nominal fall in prices (as averaged in long periods of time) was a fairly good measure of productivity increase.

The other two periods considered by Jevons, however, have been characterized by a general *increase* of prices. This raised a clear problem of interpretation: 'The progress of our industry (…) has been *continuous*, and its only change that of acceleration in recent years. There is nothing in such constant progress that can account for a great rise in price' (Jevons, 1865, p. 303). Evidently, Jevons argued, 'if the progress of invention causes a fall of price, then we need even more potent causes to raise prices in opposition to it' (Jevons, 1865, p. 303). These causes were concerned with the availability of gold. The 'current of gold' has been 'considerable' at the turn of the 18th century, thus making for rising prices. The proof of this relationship, according to Jevons, was a sort of price-specie flow mechanism involving England and India, in which prices were comparatively higher in the country with comparatively more precious metals (England), thus determining a compensating flow of gold towards India (Cf. Jevons, 1865, p. 304). The abundance of gold determined an elevation of prices of greater force than the downward pressure due to technical improvements. As the current of gold 'greatly fell off' in correspondence to the Mexican War of Independence (1810-21) and remained low in the next thirty years or so, such a downward pressure could operate undisturbed. In the third period, the price fall has been interrupted and reverted into a general rise, by 'the Californian and Australian discoveries of gold, which were followed almost immediately by the great drain, unremitted to the present time' (Jevons, 1865, p. 305): similarly to what had happened in the years around the turn of the 18th century, the abundance of gold more than compensated the effect on prices of technical improvements.

The merit of Jevons's analysis is to draw attention to general and secular movements in nominal prices and in so doing he could make it clear that in a long-run perspective the importance of gold production could hardly be overstated. It also has a fundamental limit, however, which consists in the failure to *separate* the effect of 'gold' from the effect of technical improvements. In fact, according to his argument, only in the absence of productivity change would the trend variation in general nominal prices reflect 'gold' alone. And, conversely, only if there were no comparative change in 'gold', would the change in nominal prices reflect only productivity change. But in general, he presented no criteria for assessing the individual contributions to price change. Even worse, he appeared to think that such criteria did not exist: he maintained, in fact, that the measure of 'the fall in prices which *might have been expected* from the continuous progress of invention and production (…) is *necessarily* unknown (Jevons, 1865, p. 308; emphasis added).

Yet, as we have seen, one can find in Porter's comments on prices and wages in the first half of the 19th century some distinct elements for a method of analysis of nominal changes which can provide a measure of productivity change and this method is based on a *relative* change in input and output prices. Porter did not develop his own argument any further because during that period the nominal price of the main input, labour, did not show a systematic direction of change so that he was content with saying that (nominal) 'cheapening', in the sense of the observed systematic fall in price, was a fair approximation of the 'real' rate of cost reduction in any industry.

7 Towards a real measure of 'cheapness': Giffen

It was not until Giffen's series of articles for the *Journal of the Statistical Society of London* (later *Journal of the Royal Statistical Society*) from 1879 to 1888 that a method for separating technical change from 'gold' components in the long-run variations of prices was proposed. Giffen's argument is based on two main premises.

The first premise is that the question of the existence and measure of appreciation or depreciation of gold should be deprived of the abstract prejudices of the time, and dealt with in practical and conventional terms. It was all a matter of relative change, according to Giffen. An appreciation of gold *in terms of commodities* is *by definition* a general and lasting fall in commodity prices as expressed in gold and the question of appreciation or depreciation can only be settled not in general terms, but with reference to a *specific* set of 'things' in terms of which gold is evaluated:

> It is of the utmost importance (…) that the question of the appreciation of money at the present time [1888] should be discussed for its own sake as a question of fact merely, and as a purely statistical rather than an economic question (…). It is convenient to employ the phrases appreciation of money and depreciation of money, (…) when the expressions are used scientifically, as the mere equivalents of the fall or rise of the prices of those articles or groups of articles with which money is compared. (Giffen, 1888, p. 714)

The second and complementary premise consisted of the observation that the proportions among prices and between prices and wages/incomes normally change through time. Only in a 'stationary community, which goes on from year to year with the same population, producing and consuming the same things' would 'the fall or rise of prices (…) extend to all commodities equally, and to wages and incomes also' (Giffen, 1888, p. 715). In such a case, 'nothing would be easier apparently than to ascertain appreciation or depreciation' (Giffen, 1888, p. 716). But this was contrary to historical experience. Most 19th century communities were all but stationary. Giffen dismissed the case of 'retrograding communities' as 'a very rare one'. Therefore he concentrated on an 'advancing community' in which the 'average prices of commodities' (Giffen, 1888, p. 740) have a systematic tendency to *fall relative to* average 'wages and incomes per head' (Giffen, 1888, p.716): by the 'advance in the return to the industry of the community', there were in fact more 'real things to divide' (Ibid). This had nothing to do with money and was an entirely 'real' change. As a consequence, however, appreciation or depreciation of money was *automatically different* according to whether it was measured by commodities or by incomes and the analyst can only (and indeed must) make this very explicit and transparent.

On the basis of these two premises, Giffen was able to perform a price accounting analysis which separated money appreciation/depreciation from productivity increase. He distinguished between three cases of appreciation, three cases of depreciation and a further 'mixed' case. For brevity of exposition, let us denote by $\hat{P}$ the proportional average change of prices and by $\hat{W}$ the proportional average change of wages and incomes per head. The obvious cases of appreciation are

i) $\hat{P} < 0 = \hat{W}$

ii) $\hat{P} < \hat{W} < 0$

In the first case, $-\hat{P}$ is the rate of productivity increase, as in Porter's analysis: 'the fall of prices might be the measure of the increase of the return to the industry of the community, assuming that the labour employed in services improves generally as does the labour employed in the production of commodities' (Giffen, 1888, p. 716). At the same time, we also have *one form* of appreciation, still equal to $-\hat{P}$, if measured by commodities.

The second case depicts a different and stronger form of appreciation which extends to wages. The rate of productivity increase is now measured by $-\left(\hat{P} - \hat{W}\right)$: 'the *difference* between it [the fall of prices] and the fall in wages and incomes might represent the advance in the return to the industry of the community' (Giffen, 1888, p. 716; emphasis in original).

A more extreme form of appreciation is distinguished by Giffen as a third case, and this occurs when not only the wage *rates* and incomes *per head* fall, but also the aggregate nominal income does, notwithstanding the increase in population. Denoting by $\hat{N}$ the rate of population growth, we have

iii) $\hat{P} < \left(\hat{W} + \hat{N}\right) < 0$

Symmetrically, there are two obvious forms of depreciation:

iv) $0 = \hat{P} < \hat{W}$

v) $0 < \hat{P} < \hat{W}$

In case iv), depreciation is mild and is detected only if measured by incomes. Moreover, the rate of productivity increase would be measured by $\hat{W}$ if productivity grew uniformly in manufacturing and non-manufacturing sectors: 'the increase in [wages and incomes per head] might correspond with the increase of the return to the industry of the community' (Giffen, 1888, p. 717); likewise, this rate also measures one special form of appreciation. In case v), depreciation is so strong that the 'natural' tendency of prices to fall is reverted into a general rise. The rate of productivity increase is still measured by $-\left(\hat{P} - \hat{W}\right)$: 'the improvement in the [return to the industry] might be measured by the *difference* between the rise in the prices of commodities and the rise in wages and incomes' (Giffen, 1888, p.717; emphasis in original).

Giffen's sense of symmetries generated a third, even stronger case of depreciation, in which there was 'absolute inflation in all prices along with a continued cheapening of production' (ibid), but it is unclear how this case

vi) $0 << \hat{P} < \hat{W}$

should be different from case v).

Rather, a seventh distinct and more interesting case is singled out:

vii) $\hat{P} < 0 < \hat{W}$

This case 'may be described as intermediate between the mildest types of appreciation and depreciation above specified' (ibid) (that is, cases i) and iv)). Notwithstanding a general fall in commodity prices, it would be inappropriate to speak of money appreciation (or depreciation either) in this case.

For our purposes, it may be interesting to note that, algebraically, the rate of productivity increase is measured by $-\left(\hat{P} - \hat{W}\right)$ *in all cases*: since an implicit aggregation of prices and wages/incomes was behind Giffen's averages, our equation (5) above provides a formal basis (at the level of the individual manufacturing industry) for Giffen's price accounting analysis. An even more explicit statement of this general rule is made by Giffen where he compared the recent evolutions (1876-1888) of prices and wages/incomes in different countries:

> Thus the phenomenon of falling prices of commodities and stationary or, at least, not greatly declining incomes and wages, appears to be very general in gold-using countries. It does not follow that the result should be the same in every country. We cannot assume that the rate of advance in material progress to be the same in each, or that the margin between the average prices of commodities and the average income should widen in the same way. But although the same result precisely is not to be looked for, if we could measure with the necessary degree of fineness, we cannot but assume that the communities of all the countries named [Germany, Belgium, France, Italy] are progressing to some extent. (Giffen, 1888, pp. 139-40)

The phrase 'average income', like 'wages and incomes per head' and similar phrases reported so far may seem ambiguous, and the reader may wonder what assumptions allowed Giffen to treat them as a single magnitude.

Giffen was a leading expert on wages: his 1883 inaugural address to the LSS was on 'The Progress of the Working Classes in the Last Century' and a few years later he published a long paper with some 'Further Notes' on the same subject. For the purpose of illustrating the economic basis of the marked progress in living standards made by the working classes in the fifty years round the middle of the century (which he, like Jevons, judged to be much more 'decisive' to what has taken place in the period covered by Porter's data: see Giffen, 1886, pp. 30-31), a series of concordant evolutions in wage rates in some relevant districts and for some 'typical' kinds of labour in leading sectors of the British economy was certainly enough:

> While no precise answer [on the degree of the improvement] is possible, I wish to point out that the reasons for believing in a very considerable degree of improvement, almost if not quite to the extent of enabling us to say that the working classes are twice as well off as they were fifty years ago, are so strong as to be beyond reasonable doubt. The data may be incomplete, but read with little care they show us that the minimum limit of the improvement must be a very high one. (Giffen, 1886, p. 32)

He evaluated that, by the general rise in *money* wages and average constancy or decline in prices over the same period (with the exception of house rents and 'meat': see Giffen, 1886, p. 47; Giffen, 1883, pp. 601-605; Giffen 1879, p. 39), the improvement was 'at least between 50 and 100 per cent., and with an allowance for the shortening of the hours of labour, may be placed nearer the 100 than the 50, if not over the 100' (Giffen, 1886, p. 33).

When confronted, however, with the problem of assessing the degree of productivity increase and of money appreciation/depreciation by comparing the change in prices and incomes, Giffen found an obstacle in the 'want of records of wages'. In principle, in fact, a weighted average of the change in a very wide variety of wages was needed. 'But no such records are in existence. Instead there are only records of isolated rates of wages, not weighted in any way' (Giffen, 1888, p.728). His strategy, at least for the twenty years from 1867 to 1887, was to consider a proxy of the overall income per head and he identified it in the 'income tax incomes', for which the record was 'tolerably complete' (ibid). Of course, they admittedly represented mainly 'the earnings of profit on capital' (ibid), but he maintained that 'what we do know of wages points in the same direction' (Giffen, 1888, p. 729). Thus, for instance, he evaluated that the average index number of commodity prices based on the average prices of imports and exports in the ten years 1878-1887, as compared with the average in the ten years 1868-77 fell by 16.5% (see table on p. 722; the table refers to the estimate of 'Economist', which was his own pseudonym). At the same time, income tax incomes per head rose by 13.3% (see table on p. 728). Under the assumption that also wages rose by about the same proportion (which would be realistic on the basis of the table on p. 731), the ten-year 'recent' rate of productivity increase was about 30%, which amounts to an yearly average rate of 2.66%.

For earlier periods, Giffen did not venture to present data in aggregate index numbers, and therefore we cannot make similar calculations. He did, however, present a series of qualitative results (in the main consistent, of course, with Porter's and Jevons's) which allow us to fit each period into his classification:

a) ‘Towards the closing of last century, and the early part of the present century [that is, about 1775-1810], there was a remarkable rise in prices, and an equally remarkable, if not more remarkable, rise in incomes, indicating that, on the whole, the community was then advancing’ (Giffen, 1888, p. 747): this was case v) of his classification;
b) By contrast, in the following period, from the early 19th century through about 1850, there has been a ‘steady fall of general prices (…) [while] average money incomes increased very little’ (Giffen, 1888, p. 746), in accordance to Porter’s evidence: this was case i) or vii);
c) Then, in 1850-1873, there has been a ‘great rise in money incomes accompanied by a much less rise in commodities’ (Giffen, 1888, p. 746): case v) once again;
d) The next period, from the middle 1870s to the late 1880, has been finally characterized, by ‘stationary and almost slightly declining incomes, accompanied by a great fall in the prices of commodities’ (ibid): case i) or ii).

Despite these oscillations, during the whole period there has constantly been some productivity increase, and Giffen thought that in the two later periods it was ‘much the same’ (ibid), while he was ‘inclined to think that (…) before 1845 [it] was not so great as it has since been’ (Giffen, 1888, p.747).

Some broad technological facts hiding behind money prices and wages/incomes are unveiled by Giffen’s price accounting analysis, and at the same time, the much debated issues concerning alternating periods of money appreciation or depreciation are settled on a technically sound ground. What he calls ‘real cheapness’ (e.g. Giffen, 1888, p. 748), that is low prices in relation to incomes, coexisted ‘with any (…) range of money prices or any (…) change in that range’ (ibid). As he remarked with pride ‘much confusion has arisen from the neglect of this distinction’ (ibid), while, by his analysis, ‘the facts are all in harmony’ (Giffen, 1888, p.746).

9. Concluding remarks

We have shown in this paper that the familiar Solovian measure of productivity increase, based on growth accounting, has a dual, based on what may be called ‘price accounting’. At the level of the individual industry, the dual is of course equivalent to the primal, but it serves for different purposes. In particular, the price accounting is appropriate for the study of the sources of real earnings growth and we have proposed, in a simplified model, a method for distinguishing between technical improvements, inter-industry redistribution of gross output, and intra-industry redistribution of value added. We have also shown, by means of a simple illustration, that the dual industrial measures can be aggregated in a way similar to that of the primal measure. Even though some theoretical advantages of the dual point of view have been recognized since the late 1960, few or no attempts have been made thereafter to adopt it empirically. Curiously enough, this has been done much earlier, by leading analysts of the 19th century, like Porter and Giffen. The latter, in particular, very explicitly dealt with the question of separating money appreciation/depreciation from 'real cheapening' arising from technological improvements and provided the rationale for a general 'price accounting' measure of productivity increase. He did not pay any attention, however, to the redistribution of income going along with technological improvements which he considered of negligible empirical relevance.

References

Atkinson A.B. (2007), 'The Distribution of earnings in OECD Countries', *International Labour Review*, 146 (1-2): 41-60.

Checchi D. and C. García-Peñalosa (2008), 'Labour market institutions and income inequality', *Economic Policy*, October 2008: 601-649.

Denison E.F. (1962), *The Sources of Economic Growth in the United States and the Alternatives before Us*. Committee on Economic Development, New York.

Domar E.D. (1961), 'On the Measurement of Technological Change',*Economic Journal*, 71 (284): 709-729.

Giffen R. (1888), 'Recent Changes in Prices and Incomes Compared', *Journal of the Royal Statistical Society*, 51 (4): 713-815.

Giffen R. (1886), 'Further Notes on the Progress of the Working Classes in the Last Half Century', *Journal of the Statistical Society of London*, 49, (1): 28-100.

Giffen R. (1879), 'On the Fall of Prices of Commodities in Recent Years, *Journal of the Statistical Society of London*, 42, (1): 36-78.

Hall R.A. (1968), 'Technical Change and Capital from the Point of View of the Dual', *The Review of Economic Studies*, 35 (1): 35-46

Hulten C.R. (1978), 'Growth Accounting with Intermediate Inputs', *Review of Economic Studies*, 45 (3): 511-18.

Jevons W.S. (1865), 'On the Variation of Prices and the Value of the Currency since 1782', *Journal of the Statistical Society of London*, 28, (2): 294-320.

Jorgenson D.W. (1990), 'Productivity and Economic Growth', in E.R. Bernt and J.E. Triplett, *Fifty Years of Economic Measurement*, The University of Chicago Press, Chicago and London.

Jorgenson D.W. and Z. Griliches (1967), 'The Explanation of Productivity Change', *The Review of Economic Studies*, 34 (3): 249-283.

Jorgenson D.W., Gollop F.M., Fraumeni B.M. (1987), *Productivity and U.S. Economic Growth*, Harvard University Press, Cambridge, MA.

Jorgenson D.W. and K.J. Stiroh (2000), 'U.S. Economic growth at the industry level', *The American Economic Review*, 90 (2): 161-167.

Lydall H. (1969), 'On Measuring Technical Progress', *Australian Economic Papers*, 6 (12): 1-12.

Katz, L. F.; Autor, D. H. 1999. "Changes in the wage structure and earnings inequality", in Orley Ashenfelter and David Card (eds): *Handbook of labor economics*. Volume 3A. Amsterdam, Elsevier: 1463–1555.

Kendrik J.W. (1961), *Productivity Trends in the U.S.*, Princeton University Press.

OECD (2001), *Measuring Productivity*, OECD.

Opocher A. (2010), ‘The Future of the Working Classes: A Comparison between J.S. Mill and A. Marshall’ *European Journal of the History of Economic Thought*, 17(2), forthcoming.

Porter G.R. (1851) [1834], *The Progress of the Nation*, 3rd edition, London, John Murray.

Solow R.M. (1957), ‘Technical Change and the Aggregate Production Function’, *The Review of Economics and Statistics*, 39 (3): 312-320.

Steedman I. (1983), ‘On the Measurement and Aggregation of Productivity Increase’, *Metroeconomica*, 35 (3): 223-233.

World Bank (2006), ‘Equity and Development’, *World Development Report 2006*.