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Relevance or irrelevance of retention for dividend policy irrelevance

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Abstract. In an interesting recent paper, DeAngelo and DeAngelo (2006) highlight that Miller and Modigliani's (1961) proof of dividend irrelevance is based on the assumption that the amount of dividends distributed to shareholders is equal or greater than the free cash flow generated by the fixed investment policy. They claim that, if retention is allowed, dividend policy is not irrelevant. This paper shows that the dividend irrelevance proposition holds even in case of retention. The key assumption has not to do with retention but with the NPV of the extra funds (either retained or raised): if NPV is zero, dividend irrelevance applies. Yet, the dichotomy retention/no-retention is useful, because if agency problems are present, managers tend to retain funds and invest them in negative-NPV projects, and therefore the zero-NPV assumption must be removed, so that dividend irrelevance does not apply any more.

Keywords. Dividend policy, irrelevance, retention, zero-NPV, epistemology, modelling, agency theory.

JEL codes. B41, G12, G30, G31, G35.

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Introduction

A firm's value is given by the sum of the present value of forecasted cash flows. Resting on Miller and Modigliani's (1961) dividend irrelevance proposition, practitioners and some academics do not use actual cash flows; rather, they discount potential dividends, also known as free cash flows or free cash flows to firm (e.g. Damodaran, 2006a,b; Colepand, Koller and Murrin, 2000). Magni and Vélez-Pareja (2009) support the idea that only actual cash flows should be discounted, whereas potential dividends distort valuation in all cases where excess cash retained is not invested in NPV-neutral investments. In an interesting recent paper, DeAngelo and DeAngelo (2006) revisit Miller and Modigliani's (1961) paper on dividend policy irrelevance and claim that dividend policy is not irrelevant (see also DeAngelo and DeAngelo, 2007). In the 2006 paper, DeAngelo and DeAngelo (DD) underline the fact that Miller and Modigliani (MM) assume that 100% or more of the free cash flow is distributed to shareholders, thus shunting aside the possibility of retention. According to DD, the assumption of no-retention made by MM makes dividend irrelevance a "meaningless tautology" (p. 306). If retention is allowed, then dividend policy is relevant, because managers could choose suboptimal policies by investing in non-zero NPV projects.

This paper shows that relevance or irrelevance of dividend policy has not to do with retention; it has to do with the rate of return of the extra funds (excess cash) used for reinvestment or financing: dividend policy is irrelevant if and only if zero-NPV activities are undertaken, with or without assumption of retention. The dichotomy retention/no-retention is nevertheless useful, if it is reinterpreted as a regard for agency problems.

The paper is structured as follows.

- Section 1

  - A formal unambiguous definition of dividend policy irrelevance is given
  - It is proved that dividend policy irrelevance holds if and only if the extra activities are zero-NPV activities. Therefore, MM's thesis extends to the retention case
  - It is shown that MM's assumption of fairly priced stocks is (sufficient but) not necessary for dividend irrelevance
  - It is underlined that the case of retention give managers full control on shareholders' wealth, as opposed to the case of no-retention

- Section 2

  - DD's definition of dividend irrelevance is shown to be unnecessarily rigid and based on semantic conventions. While not all feasible dividend policies are optimal, all feasible dividend policies are optimal *if the zero-NPV assumption is made* (regardless of retention)
  - DD's correct stance about misconceptions of dividend irrelevance is underlined: not only investment policy matters, dividend policy is an important determinant as well

- Section 3

  - DD's worry about assumptions that constrain MM's thesis to be valid is shown to be unwarranted: the role of assumptions in deductive logic is just to constrain the thesis to be valid
  - DD's concern on retention is fruitfully reinterpreted as a regard for agency problems. If agency problems are present, then managers are inclined to retain funds and undertake negative-NPV projects The concern for agency problems means that the zero-NPV assumption is inappropriate and should be dismissed. In this sense, DD's stance is agreeable.

Table 1. Main Notational conventions

a_t	free cash flow
d_t	Dividends
$h_t\ (= d_t - a_t)$	extra funds (excess cash)
ρ	cost of capital
FCF	free cash flow
n	liquidation date
DD	DeAngelo and DeAngelo
MM	Miller and Modigliani
$\vec{h}$	vector of extra funds
$\vec{0}$	null vector
$V(\vec{h})$	value of the firm as a function of dividend policy
ZA	Zero-NPV Assumption
EA	Each-ness Assumption
h_t^n	compound amount of h_t at time n
r^t	internal rate of return of extra activity
N	number of outstanding shares at time 0
ΔN	new shares issued at time 1

1. The Zero-NPV assumption and the irrelevance of retention

DeAngelo and DeAngelo (2006) (DD) shed light on a neglected issue: Miller and Modigliani (MM) prove the dividend irrelevance theorem by excluding the possibility of retaining part of the free cash flow (FCF) generated by the investment policy. That is, MM focus on the case where a firm distributes a fraction of FCF equal or greater than one. According to DD, it is just this assumption that enables MM to prove dividend irrelevance. If retention is allowed,

i.e. if managers may choose to distribute only a fraction of the FCF smaller than one, then, according to DD, dividend policy is relevant.

Our analysis maintains MM's assumptions of a frictionless market (no taxes, no flotations costs, no transaction costs), and it is assumed that a firm is incorporated at time 0 and liquidated at time n.[1] The firm is assumed to be unlevered and to follow a fixed (optimal) investment policy, which generates periodic free cash flows equal to a_t, available for distribution to shareholders at time t, $t = 1, 2, \ldots, n$.

Because this paper aims at showing that relevance or irrelevance has not to do with retention or no-retention, we first provide a formal unambiguous definition of dividend irrelevance, to avoid any semantic misunderstanding (see next section). Letting d_t be the dividend distributed at time t, we give the following

Definition 1. *The dividend policy is said to be irrelevant if and only if*

$$\sum_{t=1}^{n} \frac{a_t}{(1+\rho)^t} = \sum_{t=1}^{n} \frac{d_t}{(1+\rho)^t} \qquad \text{for any } d_t,\ t = 1, 2, \ldots, n-1. \tag{1}$$

Let $h_t \in R$ be the extra funds, i.e. $h_t = d_t - a_t$, and let $\vec{h} := (h_1, h_2, \ldots, h_{n-1})$. Then, the above definition states that dividend policy is irrelevant if, whatever the dividends, the firm value does not change: In symbols, letting $V(\vec{h}) = \sum_{t=1}^{n} \frac{a_t + h_t}{(1+\rho)^t}$, the dividend policy is irrelevant if $V(\vec{h}) = V(\vec{0})$ for any choice of $\vec{h}$.[2] Note that h_t may be positive (dividends are greater than FCF) or negative (FCF are not entirely distributed). In the former case, financing is needed to raise funds, in the latter case projects must be chosen where the excess cash retained is invested.

[1] The assumption of a finite-time horizon is by no means restrictive, but we believe it is conceptually more salient and, strictly speaking, more realistic than the assumption of a firm that is *everlasting*. DeAngelo and DeAngelo themselves use this assumption for a better illustration.

[2] The final h_n (and therefore, the final d_n) is not arbitrary, because it is determined by the previous extra funds (see Remark 1).

The case $h_t > 0$ is assumed by MM, whereas DD focus on the case $h_t < 0$.

Zero-NPV assumption (ZA). *Dividend policy is such that*

$$\sum_{t=1}^{n} \frac{h_t}{(1+\rho)^t} = 0 \tag{2}$$

Proposition 1. *Dividend policy is irrelevant if and only if ZA holds*.

Proof. Eq. (1) holds if and only if eq. (2) holds.□

The above Proposition says that, current stockholders' wealth does not change if and only if the use of extra funds is value-neutral. Stated equivalently, it says that dividend policy is irrelevant if the full present value of FCF is distributed to shareholders.

Remark 1. It is worth noting that $\sum_{t=1}^{n} h_t (1+\rho)^{-t} = 0$ is equivalent to

$$h_n = -\sum_{t=1}^{n-1} h_t (1+\rho)^{n-t} \tag{3}$$

which highlights the dependence of h_n on the previous extra funds. The amount h_n may then be seen as the compound amount of all previous extra activities (reinvestments and/or financing of extra funds).

For example, if n=2, eq. (2) boils down to

$$\frac{h_1}{1+\rho} + \frac{h_2}{(1+\rho)^2} = 0$$

which may be rewritten as

$$h_2 = -h_1(1+\rho).$$

If $h_1 < 0$, i.e. retention is chosen, then dividend policy is irrelevant if the amount $-h_1$ is invested in a zero-NPV project. In such a case, the reduction in the first dividend is

compensated by the distribution of an extra dividend $h_2 > 0$ at the final date, such that the previous reduction is neutralized. If $h_1 > 0$, i.e. more than 100% of a_1 is distributed, then dividend policy is irrelevant if the extra funds are raised from a zero-NPV financing. In their 1961 paper, MM assume that the extra funds are raised by selling new stocks, and that the new shareholders expect a rate of return equal to the cost of capital ρ (stocks are fairly priced). In other words, it is assumed that the firm finances with a zero-NPV financing. In such a case, the extra distribution to current shareholders is compensated later by a distribution of a smaller dividend to those shareholders, which exactly offsets the extra cash previously received. Irrelevance of dividend policy is then based on the coincidence of the cost of capital with the expected rate of return of the extra funds, which means that the activities undertaken (whether investment or financing) are zero-NPV activities.

Let us have the following

Each-ness Assumption (EA). *Each extra fund h_t is used for a zero-NPV activity*.

The above assumption means that if $h_t < 0$, excess cash is invested at the cost of capital, and if $h_t > 0$, funds are raised at the cost of capital. Miller and Modigliani assume EA.

Proposition 2. *EA is a sufficient condition for dividend policy irrelevance*.

Proof. Let h_t^n be the compound amount, at time n, of $-h_t$. EA is formalized as

$$h_t^n = -h_t\,(1+\rho)^{n-t} \text{ for each } t,\ t=1, 2, \ldots, n-1.$$

Therefore, h_n is determined as

$$h_n = \sum_{t=1}^{n-1} h_t^n = -\sum_{t=1}^{n-1} h_t (1+\rho)^{n-t},$$

which is just eq. (3), or, equivalently, eq. (2). Using Proposition 1 the result is obtained.□

Remark 2. It is worth noting that EA is sufficient but not necessary: EA implies eq. (2) but is not implied. Dividend irrelevance may hold even if each extra activity has nonzero NPV.

Suppose the internal rate of return for each activity is $r^{(t)}$, with $r^{(t)} \neq \rho$. Then, $-h_t(1+r^{(t)})^{n-t} = h_t^n$ for each t=1, 2, … , $n-1$, which implies

$$h_n = \sum_{t=1}^{n-1} h_t^n = -\sum_{t=1}^{n-1} h_t(1+r^{(t)})^{n-t},$$

whence $\sum_{t=1}^{n} h_t(1+r^{(t)})^{-t} = 0$. However, dividend irrelevance still holds, if

$$\sum_{t=1}^{n} h_t(1+r^{(t)})^{-t} = \sum_{t=1}^{n} h_t(1+\rho)^{-t},$$

because the latter equality implies $\sum_{t=1}^{n} h_t(1+\rho)^{-t} = 0$. Therefore, dividend policy holds if EA does not hold, *provided* that ρ is the internal rate of return (not of each extra activity but) of the portfolio of all extra activities. In other terms, it is not necessary that each and every NPV be zero, but only that the NPV of the *portfolio* be zero (see Table 2). As a result, EA implies ZA, whereas ZA does not imply EA.[3]

Table 2. Sufficiency and necessity

	Each-ness Assumption	Zero-NPV Assumption
Sufficient	YES	YES
Necessary	NO	YES

DeAngelo and DeAngelo draw attention to the fact that Miller and Modigliani do not allow retention policy, so that their irrelevance theorem is "an *automatic* by-product of the investment choice" (p. 300). In fact, dividend irrelevance in MM is not automatic, because it rests on a crucial assumption that is not adequately underlined in DD's analysis: MM assume that the firm issues new stocks whose expected rate of return is ρ. This means that h_t is raised at the cost of capital. Putting it differently, Miller and Modigliani assume EA (in addition to the assumption of frictionless market).[4] According to DD, the major reason why

[3] All the other assumptions are obviously taken as valid: No assumption, separately taken, is sufficient to achieve MM's thesis.

[4] We conventionally view EA (or ZA) as an assumption that is *added* to the assumption of 'frictionless market' just to underline its importance. Evidently, one can consider the term 'frictionless market' as inclusive of all assumptions. In this sense, dividend irrelevance is indeed

dividend irrelevance holds in MM's analysis is their assumption of $h_t > 0$, which rules out retention: "Automatic optimization of payout policy does occur in Miller and Modigliani (1961), but only because they mandate 100% FCF payout" (DeAngelo and DeAngelo, 2006, p. 305). As seen, Propositions 1 and 2 above hold whatever the sign of h_t, i.e. they hold regardless of whether retention is allowed or not. Dividend irrelevance keeps on holding, as long as ZA is maintained. And, symmetrically, irrelevance does not hold if ZA does not hold, i.e. MM's result is not guaranteed by the no-retention assumption.

In the two-period example, suppose FCF is partially retained at time 1, so that $h_1 < 0$; if excess cash is invested in zero-NPV activities, then

$$h_1^2 = h_2 = -h_1(1+\rho) \tag{4}$$

which is equivalent to eq. (2) with n=2 Suppose, on the contrary, that an extra distribution of FCF is chosen which is financed with a sale of new stocks. If the rate of return expected by the new shareholders is the cost of capital ρ, this means that the new shareholders require their investment to be a zero-NPV activity. Letting N be the number of outstanding shares, the new shareholders spend h_1 for purchasing ΔN shares of the firm, and will receive dividends for a total of $[\Delta N/(N+\Delta N)]a_2$ at time 2. Given that the required rate of return is equal to the cost of capital ρ, the NPV is zero:

$$h_1 = \frac{\Delta N}{(N+\Delta N)(1+\rho)} a_2 . \tag{5}$$

The old shareholders will receive $a_2+h_2=[N/(N+\Delta N)]a_2$, which implies

$$h_2 = -[\Delta N/(N+\Delta N)]a_2.$$

Therefore, eq. (5) becomes $h_1 = -h_2/(1+\rho)$, which is equivalent to eq. (4).

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automatic given the set of assumptions. However, the thesis of any possible theorem is *automatic* in deductive logic, once the set of the assumptions is accepted (see Section 3).

In other words, the case of retention (assumed by DD) is mathematically equivalent to the case of no-retention (assumed by MM). The only difference resides in the sign of h_t: if positive, the activity is a financing; if negative, the activity is an investment. But the sign itself is immaterial to the thesis of Propositions 1 and 2; MM's dividend irrelevance theorem does not depend on the assumption of $h_t > 0$: it holds even if $h_t < 0$, i.e. if retention is assumed. And, symmetrically, it does not hold if the zero-NPV assumption does not hold, whether h_t is positive or negative. Therefore, while DD correctly state that managers may choose a suboptimal policy if retention is allowed, their drawing attention to the retention case as opposed to MM's assumption of no-retention is, theoretically, a bit misleading. The fundamental point is: does EA hold? Or, more precisely, does ZA hold or does it not?

Thus, it is true that MM prove irrelevance by assuming that the firm distributes more than 100% of FCF ($h_t > 0$), but it is also true that dividend irrelevance holds even in the case $h_t < 0$, with which MM do not deal. The key assumption regards the sign of the NPV, not the sign of h_t (see Table 3).

Table 3. Dividend policy is …

	Retention ($h_t < 0$)	No-retention ($h_t > 0$)
Zero-NPV	irrelevant	irrelevant
Nonzero-NPV	relevant	relevant

The emphasis of DD on the retention case make them neglect that, according to their very definition of irrelevance, even in the no-retention case irrelevance does not hold if the key assumption (ZA) is ruled out. That the latter is actually the *fundamental* assumption in the dividend irrelevance argument is explicitly stated by MM in their *fundamental principle of valuation* at p. 412, as well as in the following sentence: "Under our *basic* assumption, however, ρ must be the same for all investors, new as well as old. Consequently, the market value of the dividends diverted to the outsiders, which is both the value of their contribution and the reduction in terminal value of the existing shares, must always be precisely the same

as the increase in current dividends" (Miller and Modigliani, 1961, p. 420, italics added). Strictly speaking, they are just assuming EA.

In other words, the fundamental question is not:

Does the firm pay out more than FCF or does it pay out less than FCF?

which DeAngelo and DeAngelo focus on. The relevant question is:

Are extra activities value-neutral or not?

DD's paper is actually important because it warns against a common misconception: dividend policy does not count, only investment policy counts. On the contrary, investment policy is not the only determinant of shareholders' value; dividend policy is a first-order determinant as well. DD correctly underline the fact that if $h_t < 0$ managers could act so as to create agency problems (Jensen and Meckling, 1976; Jensen, 1986): DD's focus on retention just derives from their will of highlighting the fact that "retention increases managers' opportunities to expropriate stockholders" (p. 313) by selecting negative-NPV projects. Therefore, it is important to warn against misinterpretation of the notion of irrelevance. However, dividend relevance itself does not depend on retention, but on fulfilment of the zero-NPV assumption: dividend policy is relevant or irrelevant in the sense of Definition 1 depending on whether eq. (2) holds or not, *regardless of* whether dividends are greater or smaller than FCF. The fulfilment of eq. (2) means that the full present value of FCF is distributed to shareholders. And the distribution of the full present value of FCF is the crucial point, irrespective of h_t being positive or negative.

DD's insistence on the retention/no-retention dichotomy may therefore seem, from a mere theoretical point of view, a bit overstated. Yet, there is a sense in which their dichotomy is useful. It has to do with managers' control on the effects of their actions. If retention is chosen by managers ($h_t < 0$), the NPV of the reinvestment depends on whether they choose

zero- or nonzero-NPV activities. If, instead, managers choose to distribute dividends in excess of the FCF ($h_t > 0$), then extra funds are raised by issuing new stocks, whose NPV does not depend on managers' actions, but on the rate of return that purchasers of shares expect to receive in the future. This means that relevance or irrelevance of dividends may depend on investors' expectations. If retention is chosen, managers have full control on their actions and may be tempted to pursue personal objectives. If managers choose distribution of extra dividends and new shares are issued, shareholders' wealth is affected by investors' expectations. This is clearly stated by Jensen (1986, p. 323):

> Payouts to shareholders reduce the resources under managers' control, thereby reducing managers' power and making it more likely they will incur the monitoring of the capital markets which occurs when the firm must obtain new capital … The problem is how to motivate managers to disgorge the cash rather than investing it at below the cost of capital or wasting it on organization inefficiencies.

Retention has to do with the full control of managers on shareholders' wealth (Table 4).

Table 4. Shareholders' wealth is controlled by …

	Retention ($h_t < 0$)	No-retention ($h_t > 0$)
Manager's actions	YES	NO
Investors' expectations	NO	YES

2. A semantic ambiguity

DeAngelo and DeAngelo do not merely dwell on the retention case; they also become involved in what seems to be a semantic disquisition regarding the meaning of the term 'irrelevance'; they correctly write that

> provided that managers distribute the full present value of FCF, the timing of those payouts is a matter of indifference to stockholders. (p. 303)

In the light of Definition 1 and Propositions 1 and 2, their sentence just means that dividend policy is irrelevant if EA (or ZA) is assumed. Yet, to DD, "this is *not* 'payout policy irrelevance', since managers can also choose policies in the interior of Fig. 2" (pp. 303-4), i.e. they can choose suboptimal policies in the feasible set (negative-NPV activities). We notice that the possibility of choosing suboptimal policies does not invalidate the fact that managers' actions do not change shareholders' wealth *if the zero-NPV assumption holds*, in which case managers' actions are *irrelevant* in determining shareholders' wealth. DeAngelo and DeAngelo write that irrelevance is a property of the opportunity set and requires "a one-to-one correspondence between feasible and optimal policies" (p. 294). Therefore, irrelevance means that "all feasible decisions are optimal" (p. 294, p. 312). Suboptimal policies are possible with the assumption $h_t < 0$ if eq. (2) does not hold, because in this case not all feasible decisions are optimal (managers may choose negative-NPV projects). Therefore, DD conclude that dividend policy is not irrelevant. But this is a semantic convention. One may correctly claim that "all feasible decisions are optimal *if the zero-NPV assumption is assumed*". Whether or not managers' actions change shareholders' wealth is indeed relevant to shareholders, but, once established (by assumption) that any of the infinite possible dividend policies is value-neutral, then it is irrelevant what managers will do.[5] DD write that if zero-NPV activities are undertaken by managers, policies are indeterminate (p. 301) but not irrelevant. The fact that policies are indeterminate or not has nothing to do with the fact that policies are irrelevant: if it has been established by assumption that managers behave in a value-neutral way (i.e. EA or ZA), dividend policy is irrelevant. Choosing among zero-NPV projects may perhaps be indeterminate, but, nonetheless, it is of no relevance to shareholders' wealth (and, as seen, ZA can be met even with projects having different NPVs).

Consequently, there is nothing wrong in MM's result, even if retention is allowed, and while MM only cope with the assumption $h_t > 0$, their result is easily generalized to the case $h_t < 0$, as we have shown. And there is nothing semantically wrong in the sentence "dividend policy is irrelevant if the zero-NPV assumption holds": it has the same logical

[5] Note that if managers invest extra funds $-h_t$ in negative-NPV projects for some periods and in positive-NPV projects for the other periods, EA is not met; however, ZA is met, if the internal rate of return of the portfolio of all extra activities is ρ, which is sufficient for dividend policy irrelevance.

status as the sentence "the debt/equity ratio of a firm is irrelevant if the no-arbitrage assumption holds". Essentially, the no-arbitrage principle is to capital structure policy what ZA is to dividend policy. A formal definition such as Definition 1 may be a useful tool to avoid semantic ambiguity.

However, despite semantic ambiguities, DD's stance is rather clear and agreeable: MM's irrelevance theorem should not be intended as implying that only investment policy matters, while dividend policy is of no concern. Given that MM's theorem may be erroneously interpreted as stating that managers' decisions on dividends do not change shareholders' wealth, DD's paper is welcome. Dividend policy does count, because if eq. (2) does not hold, then stockholders' wealth does change under changes in dividend policies.

DeAngelo and DeAngelo correctly affirm that "the familiar NPV rule for investments … is not by itself sufficient to ensure stockholder wealth maximization; an NPV rule for payouts is also necessary: ('distribute the full PV of FCF to currently outstanding shares')" (p. 295): to assume ZA just boils down to adopting such a rule.

So, DeAngelo and DeAngelo correctly claim that "anything goes" is not true for dividend policy. However, armed with our Definition 1, we may claim: "Anything goes *if the zero-NPV assumption holds*".

3. Epistemological issues and agency costs

DeAngelo and DeAngelo seem to be particularly worried by the fact that "irrelevance is hard-wired into MM (1961) by assumptions that shrink the feasible set to optimal policies" (p. 294). MM's assumptions are such that managers are "constrained to choose optimal policies" (p. 304); in this way, "the firm's opportunity set is artificially constrained to payout policies that fully distribute free cash flow". Their worrying about an assumption that 'constrains' the firm to act in a way that is irrelevant for shareholders is theoretically unwarranted. Any mathematical theorem or proposition, any theoretical model that makes use of deduction is based on assumptions which *constrain* the thesis to be valid. The role of assumptions is just to force things so that the thesis is valid. MM's assumptions are such that their thesis holds. And we have proved that their thesis holds even with the assumption $h_t < 0$, which they do no cope with. To understand why no theoretical problems arise with

MM's dividend irrelevance theorem, just think of the well-known Extreme Value Theorem (EVT): *if a real-valued function is continuous in a closed, bounded interval [a, b], then such a function attains both a maximum and a minimum value in [a,b]*. Obviously, the assumptions here force the thesis to be valid. Let the set of all real-valued functions be the feasible set, and let a choice be defined optimal if a function is chosen which attains a maximum and minimum value in [*a*, *b*]. The assumptions of the EVT *force* the validity of the thesis because the choice of any function is optimal if assumptions are met. Therefore, the choice of any function fulfilling the assumptions is (perhaps indeterminate but nonetheless) irrelevant. It is evident that the original feasible set (real-valued functions) is now restricted to a narrower set (real-valued functions continuous in a closed bound interval) where choice is irrelevant. But this is just the epistemological role of assumptions. Without assumptions, there would be no significant result. So, MM's use of EA (which implies ZA) does restrict the feasible set, and so do the other assumptions about frictionless market.

The fact that MM's "proof makes no assumption about managerial objectives" (DeAngelo and DeAngelo, 2006, p. 296) does not invalidate MM's result logically, for the same reason why the fact that EVT makes no assumption about, say, monotonicity does not invalidate EVT. In MM's theorem, no room is left for conflict between managers and shareholders. If managerial objectives and agency problems are of particular concern, one should attack neither MM's thesis nor any of their assumptions on a *logical* or *semantic* level; one could explicitly refuse an assumption and set a different result. Possibly, DeAngelo and DeAngelo's intention is just to reject MM's thesis *empirically*, which implies (by *modus tollens*) that (at least) one of MM's assumptions is *empirically* unacceptable. But DD point the finger at $h_t > 0$, which is a redundant assumption, because MM's thesis holds with $h_t < 0$ as well. The key assumption is ZA: if this assumption is removed, MM's result is not implied by the other assumptions. In terms of the EVT, it is as if, for example, one considered non-monotonic functions: monotonicity has nothing to do with EVT's thesis, so the theorem holds for non-monotonic functions as well. From a logical point of view, no problem lies in the thesis nor in the (redundant) assumption of monotonicity.[6]

[6] One might say that comparing a mathematical theorem with a proposition bearing economic content is not admissible. However, we are just comparing the logical structures that the two Propositions share: Both are deductive arguments with one thesis which follows from a set of assumptions. This schema is rather obvious in any scientific field that makes use of deduction.

To claim that in MM's paper "irrelevance obtains, but only in an economically vacuous sense" (DeAngelo and DeAngelo, 2006, p. 293) is not correct if this means that the assumption of no-retention is restrictive. If, instead, one refers 'economically vacuous' to ZA and means that ZA is empirically refutable, then the sentence may be accepted, not as a logical truth, but as an empirical evidence. If MM's intention was to prove a thesis given a set of assumptions, the set of assumptions guarantee the thesis; and we have seen that this set of assumptions may be relaxed to allow for retention, which means that MM's result is untouched. Whether or not MM's assumptions hold, i.e. whether or not they are realistic, acceptable, empirically justifiable etc. is a totally different issue.

We stress that our stance should be intended as a reinterpretation and a clarification of DD's results, not as a critique. As anticipated, their dichotomy retention/no-retention is useful, because it is strictly connected with Jensen and Meckling's (1976) agency problems: if agency problems are assumed, ZA must be abandoned in order to provide a more realistic scenario. Managers may control shareholders' wealth with dividend policy, either by choosing retention of funds or by choosing extra distribution of dividends, but they have *full* control of shareholders' wealth only if they select retention rather than extra distribution of dividends. This is the reason why retention is important: DeAngelo and DeAngelo deserve credit for underlining that agency theory makes MM's assumptions inappropriate, although it is ZA, not retention, that has to be considered inappropriate. Agency problems conflict with ZA because managers are inclined to retain part of the FCF to pursue personal objectives, a problem that in MM's treatment of dividend policy does not arise. Removing the zero-NPV assumption, agency problems comes to the fore (or, vice versa, coming agency problems to the fore, ZA is removed) and the fact that full controllability of shareholders' wealth is possible only with retention make managers eager of distracting funds from distribution (see Table 5, where the first case refers to the MM's assumptions and thesis). Whether this is compensated or not by the need of increasing dividends in the attempt of manipulating price has to do with the introduction of a further assumption in the model: dividend policies bear (or do not bear) informational content (signalling theory).

In essence, the following sentences hold:

1. MM prove that dividend irrelevance holds with a set of assumptions, among which are EA and $h_t > 0$
2. MM's dividend irrelevance theorem holds even if EA is replaced by ZA, which does not require that each activity has a zero NPV
3. MM's dividend irrelevance theorem holds even if the assumption $h_t > 0$ is removed, allowing for any choice of $h_t \in R$, $t = 1, 2, \ldots, n-1$
4. MM does not hold if ZA is removed
5. the assumption of managers pursuing personal objectives is incompatible with ZA (unless personal objectives and shareholders' objectives coincide). Therefore
6. to assume that managers pursue personal objectives means that ZA is removed, because agency problems are present
7. if ZA is removed, dividend policy is relevant.

Table 5. ZA out, agency problems in

Set of assumptions	F FIP EA $h_t > 0$	F FIP ZA	F FIP AP $h_t < 0$	F FIP AP $h_t > 0$ *
Dividends irrelevance	YES	YES	NO	NO
Dividend depends on	—	—	managers	new shareholders

F=*Frictionless market* FIP=*Fixed investment policy*

AP=*Agency problems* **The sign of* h_t *is a managers' choice*

Conclusions

This paper first sets aside any linguistic ambiguity of the expression "dividend irrelevance" by introducing the formal unambiguous Definition 1, according to which one may say that dividend policy is irrelevant if the following key assumption is made: the portfolio of extra activities has zero NPV. Therefore, relevance or irrelevance does not depend on the fact that retention is allowed or not. From a mathematical perspective, the major dichotomy is zero-NPV/nonzero-NPV, which enables one to extend Miller and Modigliani's (1961) result even in the retention case studied by DeAngelo and DeAngelo (2006). Therefore, MM's assumption of no-retention is irrelevant, while the zero-NPV assumption (ZA) is relevant, because if the assumption $h_t > 0$ is removed MM's dividend theorem still holds, whereas if one removes ZA, MM's theorem does not hold any more.

The dichotomy retention/no-retention is nonetheless useful: if retention is chosen by managers, then NPV is determined by managers' actions; if extra-distribution is chosen by managers, NPV is determined by expectations of new shareholders. In other words, shareholders' wealth is fully controllable by managers only in case of retention. Thus, retention has not to do with irrelevance (the zero-NPV assumption is the cardinal assumption) but has to do with the controllability of shareholders' wealth by managers. Managers have incentive in retaining funds because they can fully control shareholders' wealth. Therefore, one may certainly claim that retention bears a strong relation to dividend irrelevance issues if agency problems are assumed. Reinterpreting this way DeAngelo and DeAngelo's paper, their contribution is noteworthy in that it not only sweeps away any misbelief that dividend policy is, in general, irrelevant, but also allows us to ask for removal of ZA, which is in contrast with Jensen's (1986) agency theory: managers are inclined to retain funds, which makes the possibility of negative-NPV activities highly realistic.

Whether managers may have some power on shareholders' wealth even in the case of no-retention with issues of new stocks, depends on whether a further assumption is added: managers may influence shareholders' expectations.

References

Copeland, T.E., Koller, T. and Murrin, J. (2000) *Valuation: Measuring and Managing the Value of Companies*, 3rd Edition. John Wiley & Sons.

Damodaran, A. (2006a) *Damodaran on Valuation. Security Analysis for Investment and Corporate Finance***,** second edition. Hoboken, NJ: John Wiley & Sons.

Damodaran, A. (2006b) Valuation approaches and metrics: a survey of the theory and evidence. *Working paper* (November). Available at SSRN: <http://www.stern.nyu.edu/~adamodar/pdfiles/papers/valuesurvey.pdf>. Originally published in *Foundations and Trends® in Finance*, 1(8), 693-784, 2005.

DeAngelo, H. and DeAngelo, L. (2006). The irrelevance of the MM dividend irrelevance theorem. *Journal of Financial Economics*, 79(2), 293−315.

DeAngelo, H. and DeAngelo, L. (2007) Payout policy pedagogy: what matters and why. *European Financial Management*, 13(1), 11-27.

Jensen, M. (1986). Agency costs of free cash flow, corporate finance, and takeovers. *American Economic Review*, 76(2), 323−329.

Jensen, M. and Meckling, W. (1976). Theory of the firm: managerial behavior, agency theory, and ownership structure. *Journal of Financial Economics*, 3, 305−360.

Magni, C.A. and Vélez–Pareja, I. (2009). Potential dividends versus actual cash flows in firm valuation, *The ICFAI Journal of Applied Finance*, forthcoming.

Miller, M. and Modigliani, F. (1961). Dividend policy, growth, and the valuation of shares. *Journal of Business*, 34, 411−433.