Too much or not enough crimes? On the ambiguous effects of repression

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Abstract

The purpose of this paper is to investigate the optimal enforcement of the penal code when criminals invest in a specific class of avoidance activities termed *dissembling activities* (i.e. self-protection efforts undertaken by criminals to hedge their illegal gains in case of detection and arrestation). We show that the penal law has two screening effects: it separates the population of potential criminals between those who commit the crime and those who do not, and in the former group, between those who undertake dissembling efforts and those who do not. Then, we show that it is never optimal to use less than the maximal fine in contrast to what may occur with avoidance detection (i.e. efforts undertaken in order to reduce the probability of arrestation: Malik (1990)); and furthermore, that the optimal penal code may imply overdeterrence. Finally, we show that any reform of the penal code has ambiguous effects when criminals undertake dissembling activities which are a by-product of illegal activities, since increasing the maximum possible fine may increase or decrease the number of crimes committed and may increase or decrease the proportion of illegal gains hedged by criminals.

Keywords: deterrence, avoidance detection, dissembling activities, optimal enforcement of law.

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1 Introduction

The canonical economic literature on crime and punishment initiated by Becker (1968) has provided two classical results. On the one hand, the best trade-off between probability and penalty is achieved when monetary penalties are set to their maximum possible level, because fines are most of the time costless, allowing the enforcement authority to set them as high as possible. On the other hand, it is not optimal to completely deter individuals from engaging in an illegal activity, since for at least some individuals, the gains from engaging in the proscribed activity may be sometimes larger than the external costs it imposes on the rest of the society. The first result has prompted a large body of literature (see Garoupa (1997) or Polinsky and Shavell (2000) for surveys) discussing cases where fines are costly resources for enforcers or for the criminals, hence justifying that less than maximum fines be used. In contrast, the second result is a common by-product of the former, and it has been shown that whenever enforcement authorities have imperfect information about criminals' activities and/or their characteristics, the optimal design of the penal code allows some level of underdeterrence to exist.

Following this line, we tackle in this note two commonly acknowledged results: on the one hand the fact that avoidance activities undertaken by criminals are a major reason justifying the optimality of less than maximum fine (Malik (1990), Bebchuk and Kaplow (1993)), and on the other hand, that such activities aggravate the issue of criminals' underdeterrence (Sanchirico (2006)). In contrast, we will prove first that for the specific class of avoidance activities that we term dissembling activities, it is never optimal to use less than maximum fines. Second, we will also show that public policies designed to prevent criminal behavior may lead to overdeterrence, in the sense that some offenders are deterred from engaging in the illegal activity although their private benefit is larger than the external cost they impose on the rest of the society.

Avoidance activities encompass various expenditures engaged by criminals in order to reduce their exposure to the risk of punishment. It comprises installing radar detectors to avoid speeding tickets, lobbying politicians to relax the enforcement of regulations, bribing an enforcement agent to let go free a culprit, destroying or covering up incriminating evidences, or investing in long and costly litigations and so on. Thus, we suggest a basic albeit more comprehensive typology similar to the distinction made in the economics of insurance markets, between self-protection and self-insurance. In fact, some avoidance activities are undertaken in order to lower the probability of apprehension, conviction and/or punishment. Typically, this is the case for example with radar detectors. Note that such expenditures may be understood as self-protection investments from the point of view of criminals (they are more specifically termed avoidance detection by Sanchirico (2006)). But the rationale for others kind of avoidance activities is in contrast to reduce the impact of the arrestation and punishment on the wealth or welfare of the criminals: typically, it occurs when criminals are strategically bankrupt or non solvable, as it is the case when they render non seizable the benefits of the crime. In this case, it corresponds for the criminals to a kind of self-insurance behavior that will be termed dissembling activities in the paper.

In fact, the existing literature on avoidance activities focuses on the case of detection avoidance. Sanchirico (2006) has recently suggested that it is a serious limit to the effectiveness of public policies in the area of crime deterrence. He argues that it implies the unfortunate but unavoidable result that any increase in public monitoring expenditures leads to an increase in criminals' avoidance activities, which in turn has an adverse feedback effect on the effectiveness and efficiency of public detection, thruly leading to a high level of underdeterrence. Nevertheless, Sanchirico does not address the issue of the optimal probability/fine trade-off. Such an analysis has been earlier provided by Malik (1990) and Bebchuk and Kaplow (1992) who have shown that avoidance detection may justify that less than maximum fines are optimal[1]. Here, we focus on dissembling activities, assuming that criminals' investments in order to avoid the risk of punishment enable them to hedge their illegal benefits in case of arrestation, allowing the enforcer to seize only a small amount of those outcomes.

Section 2 describes the basic set up used in the paper, and proves that the penal code has two different screening effects: on the one hand, it separates the population of offenders between those who commit the crime and those who do not; on the other, it also distinguishes among the active criminals between those who undertake dissembling efforts and those who do not. In section 3, we show that the beckerian result, namely the optimality of maximum fines, still holds here. However, and in contrast to what occurs both in Becker's paper or in Malik's paper, overdeterrence may now occur at the optimum. Section 4 focuses on the effectiveness of public interventions. We first show that monetary penalties and the probability of control may be either substitutable or complementary instruments. This implies that when enforcement policies become more repressive, criminals may take countervailing decisions which result in more crimes, more individuals making dissembling efforts and saving a larger proportion of their illegal benefits in case of arrestation. Finally, this means that the reform of the penal code has ambiguous effects on criminality: in the situation where underdeterrence exists at the optimum, the distortion from the first best may be reduced as the maximal level of fine increases (for example, with the seizable wealth or assets of criminals) since the optimal level of deterrence goes closer to the external cost of crimes: public policies become thus more efficient. On the contrary, in the case where overdeterrence occurs at optimum, then the distortion with respect to the first best may be aggravated as the maximal possible fine is raised, making the level of deterrence closer to full deterrence. Section 5 briefly concludes.

2 Criminals' behavior

Let us consider the case where the illegal activity allows the (risk neutral) criminal to obtain a benefit equal to b (and $b = 0$ if the illegal act is not undertaken) which will be called the type of the criminal. Public authorities do not observe the type b. They just know that b is distributed according to a uniform distribution function on $[0, B]$. On the other hand, the (external) loss to the rest of the society is $D < B$ in case of crime, whatever the private benefit for the criminal[2]. We consider here that public enforcers are endowed with two basic

[1] See also Nussim and Tabbach (2006).

[2] Thus as usual in the literature, the first best level of deterrence corresponds to the illegal benefit $b = D$ (assuming it can be obtained at a small enforcement cost). Given that the type

instruments, as is usual in the literature: monetary sanctions (penalty or fine) $f > 0$, and expenditures in the monitoring of criminals' behavior, defined for the sake of simplicity as the choice of a probability of control p (encompassing arrestation, conviction and punishment for an illegal behavior).

When he is caught, the offender has to pay the fine but the protective measures undertaken *ex ante* allow him to save only the fraction $\beta(x) \in]0,1[$ of his benefit b; x denotes the effort in the dissembling activity (caution) and we assume that $\beta(0) = 0$, $\beta' > 0$, $\beta'' < 0$. Furthermore, we assume that the monetary equivalent of the disutility cost of criminal's efforts is simply $v(x) = x$.The maximum expected benefit obtained by the criminal when he undertakes the illegal activity and makes the avoidance effort is equal to:

$$u \equiv \max_x \left(\pi(x,p)b - pf - x\right) \tag{1}$$

with $\pi(x,p) = 1 - p + p\beta(x)$, which may be understood as the *ex ante* total proportion of the illegal benefit saved by the offender[3]. The individually optimal behavior of a criminal is described by the following proposition, denoting $\hat{x}$ the efficient level of effort[4]:

Proposition 1 *The population of criminals separates in three different groups, defined according to two different thresholds of the benefits labelled $\bar{b}$ and b^*, such that:*

i) if the criminal's type is $b \in [0,\bar{b}]$, then he does not commit the crime and makes no dissembling efforts ($\hat{x} = 0$);

ii) if the criminal's type is $b \in]\bar{b}, b^*]$, then he does commit the crime but without undertaking any dissembling effort ($\hat{x} = 0$);

iii) if the criminal's type is $b \in]b^*, B]$, then he does commit the crime and undertakes a positive level of effort ($\hat{x} > 0$) which satisfies:

$$p\beta'(\hat{x})b = 1 \tag{2}$$

Proposition 1 means that any enforcement policy has in fact two distinct screening effects on the population of potential criminals. On the one hand, it leeds to the separation between those who become active criminals, and those who are deterred - this a basic effect. The threshold $\bar{b} \equiv \frac{p}{1-p}f$ corresponds to the level of deterrence under which no crime is committed (this threshold increases both with f and p). But there exists a second effect: among the active offenders, some of them will also invest in dissembling activities (make some efforts to hedge their benefits in case of arrestation), while the others will not. Namely, $b^* \equiv \frac{1}{p\beta'(0)}$ is the threshold over which any crime committed is accompanied by an effort in dissembling activities (and it decreases with p but

of the criminals is not observable, it is generally never attainable.

[3] With probability $1 - p$, the criminal saves the benefit b in proportion 1, although with probability p he saves only $\beta(x) < 1$.

[4] All the proofs are in the appendix.

is independant of f). It is easy to see[5] that for any $b \in]b^*, B]$, the optimal $\hat{x}$ is unambiguously increasing with p and b, but is independent from the fine. The value of the fine f matters only in the sense that it influences the decision to engage in the illegal activity or not, although it does not affect the decision to undertake or not the avoidance expenditures.

The rest of the paper studies the effects of the optimal enforcement of the law on this specific structure of the population of criminals.

3 Second best policies

The management costs associated with the monetary penalty are neglectable, but monitoring the criminal activity entails a cost equal to $m(p)$, with $m' > 0$ and $m'' > 0$. The government has to choose a fine f and a probability of control p in order to maximize the social welfare function:

$$S = \frac{1}{B}\int_{\bar{b}}^{b^*} ((1-p)b - D)db + \frac{1}{B}\int_{b^*}^{B} (\pi(x(p,b),p)b - x(p,b) - D)db - m(p)$$

under the constraint[6] $f \leq F$. The two first (integral) terms in S correspond to the expected private benefit associated with the illegal activity (the benefit of the criminal without dissembling efforts minus the external cost, plus his benefit when he commits the crime with an positive effort minus the cost of dissembling and the cost to the society). The last one is the cost of monitoring for public authorities. The fine is a mere transfer between the (risk neutral) criminal and the government, it does not appear in the social welfare function (it is not worth from a social point of view). It is obvious (see also Malik (1990)) that for small values of the external cost of crime and/or large values of the public cost of monitoring, the solution of this problem may be zero deterrence; and under the opposite conditions (large values of the external cost of crime and/or small values of the public cost of monitoring), we may obtain complete deterrence. Thus, we focus rather on the more powerful case with conditional deterrence hereafter.

If an interior solution (p, f) exists, it satisfies the first order conditions of maximization which are written:

$$\frac{1}{B}(D - pf)\frac{\bar{b}}{p(1-p)} = m' + \frac{1}{B}\left[\left(x^* - \frac{\beta^*}{\beta'(0)}\right)\frac{b^*}{p} + \int_{\bar{b}}^{b^*} bdb + \int_{b^*}^{B}(1-\hat{\beta})bdb\right] \tag{3}$$

[5] Applying the implicit function theorem to (2), one obtains: $\frac{\partial \hat{x}}{\partial p} = \frac{\beta'(\hat{x})}{-\beta''(\hat{x})p} > 0$ and $\frac{\partial \hat{x}}{\partial b} = \frac{\beta'(\hat{x})}{-\beta''(\hat{x})b} > 0$.

[6] This is the most natural specification when we consider that the cost of avoidance corresponds to the disutility of criminals' efforts, and F corresponds to the seizable wealth or assets of criminals.

$$\frac{1}{B}(D - pf)\frac{p}{1-p} = \lambda \qquad (4)$$

with $\lambda = 0$ if $f < F$ but $\lambda > 0$ otherwise, and denoting $\beta^* = \beta(x^*)$, $\hat{\beta} = \beta(\hat{x})$. More specifically, the LHS in (3) is the social marginal benefit from the control of illegal activities, while the RHS corresponds to the social marginal cost of controling which takes into account the enforcer's marginal cost of monitoring (first term) and the criminals' marginal cost of dissembling effort (last three terms). Similarly, the LHS in (4) is the social marginal benefit of fines, and the the RHS is their social maginal cost (which is simply the shadow price of the constraint, since fines are costless). In the appendix, we prove that the following results hold:

Proposition 2 *The solution with conditional deterrence has the following properties:*

i) The maximum fine $f = F$ is always optimal, and the probability p must be set as small as possible according to (3).

ii) We obtain that $pF < D$ and there may exist either over or underdeterrence at optimum ($\bar{b} \equiv \frac{p}{1-p}F \gtrless D$).

Result i) is in contrast to the one obtained by Malik (1990) in the case of detection avoidance *i.e.* when avoidance activities enable criminals to lower the probability of arrestation and punishment: whereas less than maximum fine may be optimal under detection avoidance, this never occurs under dissembling activities. These two different results are easily explained. Under dissembling activities, criminals effort are independent of the fine: raising the fine entails no additional costs on criminals (beyond the expected fine paid in case of arrestation), and thus has only the direct effect on deterrence. Hence, insufficient deterrence obtaines unless maximum fines are set. In contrast, with detection avoidance (Malik (1990)), the fines impose a private cost on criminals, over the expected fine paid in case of arrestation; depending on whether avoidance expenditures become more or less sensitive to the fine, then the enforcement autorities may use less than maximum fine or not.

Part ii) also challenges the usual result of the literature. In the canonical model of Becker, there is not enough deterrence at the optimum: some of the criminals for which the benefit of committing the crime is smaller than the external cost on the society, are not deterred. This is explained by the fact that the level of deterrence corresponds to the expected fine paid by criminals when they are arrested - and random detection is justified by the costly resources used to control criminal activities. In contrast, in the present set up the expected fine is always smaller than the external cost of crime but does not determine the level of deterrence: this latter is set at a threshold high enough to deter only those in the population of criminals who would never engage in dissembling activities; but on the other hand, as far as it is socially worth to deter also some of the active criminals who do make an effort (as soon as their benefit is lower than D), it may be necessary to set the probability of control at a high level given that these individuals are not sensitive to the level of the fine. As a result, the probability may be sometimes set at a level high enough to induce excessive deterrence at optimum. But depending on the properties both of the disutility cost of effort and of the technology of dissembling (see the three last terms in the RHS of (3)), the opposite result of underdeterrence may arise, as usually found both by Becker (1968), as well as Malik (1990) or Sanchirico (2006) under detection avoidance.

Note that it could be possible also that the first best level of crimes occurs (by chance). Nevertheless, in such a case, due to the asymmetric information the penal code always imposes an excessive cost to the society: among the criminals who are not deterred, some do make a dissembling effort although their activity is valuable (*i.e.* they would never be punished if their type were observable), and the distuility cost of their effort reduces the social welfare.

Finally, proposition 2 implies that although maximum fines are always optimal, they have an ambiguous effect on the number of crimes committed when criminals' type is not observable, and when some of them invest in dissembling activities. In the following, we investigate the consequences of this result more deeply.

4 Countervailing behaviors of repressive policies

As proven by Garoupa (2001), although the canonical result of Becker is usually understood as establishing the substituability between both instruments, this is not necessarily true. Let us focus here on the degree of substitutability/complementarity between fines and controls, *i.e.* whether the optimal probability decreases or increases with the maximum fine in the presence of dissembling activities. Applying the implicit function theorem to (3) with $f = F$, it is easy to verify that $sign\frac{dp}{dF} = signS_{pF}$ with:

$$S_{pF} = \frac{1}{B}\frac{D - 2pF}{(1-p)^2} \tag{5}$$

Hence the following result is straightforward:

Proposition 3 *When the fine increases, the optimal probability of control may either decrease or increase.*

This is essentially the same result as the one obtained in the canonical model without avoidance activity; see Garoupa (2001) for a more detailed discussion of its intuitive meaning: when the maximal fine is high, the level of deterrence is also high (and there may exist overdeterrence); thus, raising F, the enforcer has the opportunity to decrease the probability in order to reduce the enforcements costs. In contrast, when the maximal fine is small (to the limit, close to zero), the level of deterrence is also small, and it may be worth in this case to raise both F and the probability in order to reach enough deterrence.

This first finding has several implications. The next proposition focuses on the impact of the monetary sanctions on the distorsion to the first best number of crimes, on the number of criminals undertaking an effort and on the proportion of illegal benefits they can save in case of detection.

Proposition 4 *An increase in the maximum fine yields:*

i) an increase in the level of deterrence $\bar{b}$ *when the probability and the fine are complements, but an ambiguous effect when they are substitutes.*

ii) a decrease in the number of active criminals undertaking dissembling efforts $b^* - \bar{b}$*, if the probability and the fine are complements, but an ambiguous effect if they are substitutes.*

The ambiguity in part i) of proposition 4 is easily explained by the fact that an increase in F has a direct effect on $\bar{b}$ which is always positive, but also an indirect effect through the variation of p which is positive when p and F are supposed to be complementary, but negative when they are substitutable: thus, the total effect depends on whether the first or the second one dominates. The indirect effect also holds in part ii) since the threshold b^* does not directly depend on F but is sensitive only to the fequency of control.

An obvious implication of proposition 4 is that it is very uncertain whether the enforcement authority has the opportunity to reach a fine tuning of the level of criminality when criminals invest in dissembling activities. In fact, the following results generally hold:

Corollary 5 *An increase in the maximum fine:*

i) always yields a decrease (respectively, an increase) in the level of underdeterrence (overdeterrence) when the probability and the fine are complements at the optimum, whereas the effect is ambiguous when they are substitutes.

ii) has no effect on the benefits saved by the marginal criminal b^**.*

iii) implies a decrease (an increase) in the benefits saved by any criminal $b > b^*$ *when the probability and the fine are substitutes (respectively complements).*

Notice first that the ambiguity in i) arises only when the probability and the fine are substitutes. Secondly, when the probability and the fine are complements, increasing the sanction has favorable effects in case of underdeterrence, but adverse ones in case of overdeterrence. When underdeterrence occurs at the optimum, then the optimal level of deterrence goes closer to the external cost of crimes as the maximum fine grows up; in other words, the distortion to the first best level of deterrence is reduced, and public policies become more efficient. On the contrary, when overdeterrence occurs at the optimum, then the distortion with respect to the first best increases as the maximum fine is raised, making the level of deterrence closer to full deterrence.

Concerning ii) and iii), the results reflect that the proportion $\hat{\beta}$ for any $b \geq b^*$ do not directly depend on F but is sensitive only to the fequency of control. Thus for part ii), the increase in the fine has two indirect but opposite effects on the marginal criminal b^* through its dissembling efforts: one on the probability of control, the other on the illegal benefit b^*, but given that they have the same magnitude, the net effect is nul. In contrast, in iii) for any $b > b^*$ the effect is simply the one associated to the probability of control.

5 Final remarks

This paper provides a different view on the effects of the penal code when criminals have the opportunity to undertake avoidance activities. We have modified Malik (1990)'s model to incorporate a continuum of criminals and we assume that those criminals have the opportunity to invest in dissembling activities which allow them to hedge the benefits of the crime when they are arrested and punished (prevent that illegal assets be seized by the enforcer). In this set up, we show that the adoption by criminals of such self-protective measures has major consequences: specifically, we show that maximum fines are always optimal, and that overdeterrence may be optimal. This differs from the results previously obtained by Malik (1990) or Sanchirico (2006): avoidance activities are usually expected to justify the use of less than maximum fines, and to aggravate the problem of underdeterrence which initially appeared in the canonical world *à la* Becker.

More generally, it also challenges the common view which is to condition the design of law enforcement on the seizable wealth of criminals since the maximal possible fine is commonly interpreted as the individual wealth of criminals. For example, Garoupa (2001) concluded that the optimal probability is an inversed U-shaped function in criminals' wealth (both small and large criminals face a low probability of sanction) when wealth is a public information (observable before detection and prosecution). In contrast, when wealth is a private information (Polinsky and Shavell (1991)), the optimal probability is U-shaped with respect to criminal's wealth (both small and large criminals face a large probability of sanction). Our results suggest that in the presence of dissembling expenditures, which are an unavoidable by-product of illegal activities, things are less clear, and more restrictive policies may have counterintuitive and/or adverse effects.

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APPENDIX

Proof of proposition 1

Remark first that there exist at least some values of $b \in]0, B]$ such that: $p\beta'(0)b - 1 \leq 0 \Leftrightarrow b \leq b^* \equiv \frac{1}{p\beta'(0)}$: then it is individually efficient for such criminals to make no dissembling efforts otherwise their expected benefit would be decreasing with x. Thus, for any criminal of type $b \leq b^*$ there are two possibilities:

A/ either the condition $(1-p)b - pf \leq 0 \Leftrightarrow b \leq \bar{b} \equiv \frac{p}{1-p}f$ also holds and the criminal does not enter into the illegal activity; this proves i);

B/ or $b > \bar{b}$, and the criminal of type $b \leq b^*$ commits the crime but chooses a $\hat{x} = 0$. This proves ii).

On the other hand, for any $b > b^*$ if an interior solution $\hat{x} > 0$ exists, it must satisfy (2) which is necessary and sufficient given the assumptions put on β. Remark now that the associated u in (1) writes $u = [(1-p)b - pf] + pb\beta(\hat{x}) - \hat{x}$: given that the first bracketed term is positive as soon as $b > \bar{b}$, while the second one is also positive for any $b > b^*$ since $\beta(x)$ is increasing and concave, then it is obvious that any criminal $b > b^*$ engages in the illegal activity and makes a positive effort in the dissembling activity. This proves iii).

Proof of proposition 2

i) Let us consider a solution where the optimal fine satisfies $f < F$. According to (4), this implies that $pf = D$ and thus for any positive probability we obtain

$$S_p = -m' - \frac{1}{B}\left[\left(x^* - \frac{\beta^*}{\beta'(0)}\right)\frac{b^*}{p} + \int_{\bar{b}}^{b^*} bdb + \int_{b^*}^{B}(1-\hat{\beta})bdb\right] < 0$$

since according to the concavity property: $x^* - \frac{\beta^*}{\beta'(0)} > 0$, which is a contradiction to the assumption that it is optimal to control. As a result $f = F$ is optimal, and p must be set as low as possible according to the condition (3). It is easy to verify that the second order condition is satisfied as long as m has enough decreasing returns to scale (left to the reader).

ii) Finally, given that the RHS of (4) is positive, it must be that $D > pF$. Hence, there may exist either over or underdeterrence at optimum, given that $\bar{b} \equiv \frac{p}{1-p}F \gtrless D$.

Proof of proposition 4

It is straightforward to show that:

$$\frac{d\bar{b}}{dF} = \frac{p}{1-p} - \frac{F}{(1-p)^2}\frac{S_{pF}}{S_{pp}}$$

where $\frac{dp}{dF} = -\frac{S_{pF}}{S_{pp}}$, with $S_{pF} > 0$ when p and F are complements, but $S_{pF} < 0$ when p and F are substitutes. S_{pp} is negative (by the second order condition) but has several terms either positive or negative since the private marginal cost of effort in (3) is not monotonic in p (left to the reader), such that $FS_{pF} - p(1-p)S_{pp} \lesseqgtr 0$. Hence the result i).

The impact on the difference $b^* - \bar{b}$ is:

$$\frac{d}{dF}(b^* - \bar{b}) = -\frac{p}{1-p} - \left(\frac{1}{p^2\beta'(0)} + \frac{F}{(1-p)^2}\right)\frac{dp}{dF}$$

which is negative if $\frac{dp}{dF} > 0$ but has an ambiguous sign otherwise. This is result ii).

Proof of corollary 5

At the threshold b^* we have:

$$\frac{d\beta^*}{dF} = \beta^{*\prime}\frac{x^*}{p}\frac{dp}{dF}(e_p^{x^*} - e_{b^*}^{x^*})$$

where $e_p^{x^*} \equiv \frac{\partial x^*}{\partial p}\frac{p}{x^*} = \frac{\beta'(x^*)}{-\beta''(x^*)x^*} = e_{b^*}^{x^*} \equiv \frac{\partial x^*}{\partial b^*}\frac{b^*}{x^*}$: hence $\frac{d\beta^*}{dF} = 0$. This proves ii).

But for any $b > b^*$, we have:

$$\frac{d\hat{\beta}}{dF} = \hat{\beta}'\frac{\hat{x}}{p}\frac{dp}{dF}e_p^{\hat{x}}$$

which has the sign of $\frac{dp}{dF}$. Hence the result iii).