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Adriani, Fabrizio and Deidda, Luca

School of Oriental and African Studies, University of London, Universita' di Sassari

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Fabrizio Adriani

CeFiMS - School of Oriental and African Studies

Luca G. Deidda

University of Sassari, CeFiMS and CRENoS

Abstract

In a market where sellers are heterogeneous with respect of the quality of their good and are more informed than buyers, high quality sellers' chances to trade might depend on their ability to inform buyers about the quality of the goods they offer. We study how the strength of competition among sellers affects the ability of sellers of high quality goods to achieve communication by means of appropriate pricing decisions in the context of a market populated by a large number of strategic price setting sellers and a large number of buyers. When competition among sellers is weak high quality sellers are able to use prices as a signaling device and this enables them to trade. By contrast, strong competition among sellers inhibits the role of prices as signals of high quality, and high quality sellers are driven out of the market.

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1 Introduction

A buyer interested in a specific digital camera could find out the list of retailers' price quotes at shopper.com in just a few seconds. For any model, such a list would invariably contain substantially dispersed prices.[1] Why would these price quotes not obey the law of one price? After all, if consumers have access to market prices at a negligible cost, one would expect all cameras of the same model and brand to trade approximately at the same price.

The Internet allows consumers to observe prices of any specific camera at almost no cost. Whether the Internet is as informative about other relevant characteristics of the product they are interested in, is far less clear. For example, Lin and Scholten (2005) document that not all the firms selling electronic products are explicit on whether they sell brand new or refurbished or open box products. Information about delivery, assistance, and customer care in general is also much less available than price quotes, and is rather opaque anyway. Since the availability of hard information is typically limited in online markets, sellers may attempt to signal quality through the choice of price.

When sellers are more informed than buyers, the ability of sellers endowed with high quality goods to inform buyers about the quality of their goods might indeed be crucial in keeping these sellers from being wiped out by price competition. However, is this ability independent of the competitive pressure faced by the sellers? Does price competition alter the effectiveness of price as a signaling device?

The signaling role of sellers' strategic price decisions in the presence of asymmetric information has been the subject of extensive research and a summary of the related literature is clearly beyond the scope of this paper. Within this literature, important contributions including Milgrom and Roberts (1986), Laffont and Maskin (1987), Bagwell and Riordan (1991) Bagwell (1991), Overgard (1987) and Ellingsen (1997) have focused on the case of monopoly.[2] Bagwell (1991) finds that, with a downward sloping

[1] Various studies document the presence of persistent significant price dispersion in internet markets for final goods. See, for instance, Clay, Krishnan and Wolff (2001) for the case of electronic bookstores and Baye, Morgan and Scholten (2004) for the case of consumer electronic products.

[2] Representative contributions focusing on other market structures include Laffont and Maskin (1989) for the case of oligopoly and Cooper and Ross (1982) for the case of free entry, Wolinski

demand, the only equilibrium which satisfies the Intuitive Criterion (Cho and Kreps 1987) is a separating equilibrium in which the high quality is traded at a higher price but sells less than the low quality. Ellingsen (1997), in a model with one seller and one buyer with inelastic demand, finds that there is a unique equilibrium surviving D1 (Cho and Kreps 1987). The equilibrium is separating: the seller sells with probability one at the low price and with probability less than one (but positive) at the high price. Hence, the general consensus is that, in the absence of competition, a high quality seller is able to signal quality by distorting his price upwards and reducing the volume of trade relative to the first best.

The main contribution of the present paper is to understand to what extent (and under what conditions) this conclusion applies when sellers might face competition. We identify two regimes: *weak competition* (buyers outnumber sellers of low quality goods), and *strong competition* (buyers are outnumbered by low quality sellers). When competition is weak, different qualities trade at different prices and in different amounts. This matches the standard result obtained with monopoly. By converse, when competition is strong, only the low quality is traded in any robust equilibrium. In this case, prices do not serve as signals of quality and sellers of high quality goods are driven out of the market. We thus establish a causal link between the competitive pressure faced by sellers and the information conveyed through prices, as well as with the associated volume of trade and its quality. This is, to our knowledge, a novel result within the literature on the signaling role of prices.

The result is established in a model where there are two qualities of the same good, and therefore two types of sellers. However, as shown in the appendix, the result holds in the more general case of an arbitrary number of qualities. The stronger the competitive pressure faced by the sellers, the lower the number of qualities that will be traded and the associated degree of price heterogeneity observed in the market.

Prices serve as signals only when different types of sellers have different incentives to announce a particular price. How could this depend on the competitive pressure faced by sellers? Consider the strong competition regime. Low quality sellers undercut each other until, in equilibrium, they announce the zero profit price. So long as sellers'

(1983) on search costs, and Jansen and Roy (2002) on durable goods.

valuations are increasing in the quality of the good, sellers of high quality goods cannot afford to sell at the price that gives zero profits to low quality sellers. Yet, in an equilibrium in which low quality sellers make zero profits, no trade can occur at higher prices. High quality sellers are thus unable to trade. Since both types of sellers make zero profits, they have identical incentives to deviate to any price at which high quality sellers would be willing to trade. Off-equilibrium prices cannot be used to signal the quality of the good. As a result, high quality sellers are driven out of the market.

Consider now the weak competition regime. Buyers compete and, therefore, low quality sellers make positive profits in equilibrium. Prices thus become an effective communication device. Consider, for instance, a candidate equilibrium in which only the low quality is traded. High quality sellers, who in equilibrium are out of the market, have an incentive to announce any off-equilibrium price greater than their reservation utility, whenever there is a positive chance to sell at those prices. Low quality sellers would announce such prices only if the chances to sell were sufficiently good, since they would make strictly positive profits by announcing the equilibrium price. Therefore, sellers of high quality goods are more seemly to benefit from announcing such prices than sellers of low quality goods. High quality sellers can thus use these prices to signal the quality of their goods. As a result, high quality sellers are never driven out of the market when competition is weak. In this case, the robust equilibrium is one in which both qualities are traded and the higher quality trades at a higher price.

Interestingly, incentive compatibility for the low quality sellers requires that the probability to sell at the higher price should be lower than one. Independently of the strength of demand, some high quality sellers will always be unable to sell, even when the price of high quality goods exceeds sellers' reservation price. The price should fall to equate demand and supply, but imperfect information inhibits such a market-clearing role of prices. Thus, sellers of high quality could be rationed in a sense similar to Stiglitz and Weiss (1981).[3] On the other hand, the upward pressure on prices induced by signaling reduces the downward pressure on prices resulting from competition among sellers. This is consistent with Daughety and Reinganum (2005)

[3]See also Hellmann and Stiglitz (2000) on how rationing can emerge in the presence of inter-markets competition.

and (2007) who find that duopolists might benefit from the upward distortion on prices of high quality goods due to signaling. Intuitively, a side effect of the upward distortion is that prices become closer to cartel prices.

Following the seminal work by Akerlof (1970), the case of a market with competition between price setting sellers more informed than buyers has been considered by Wilson (1979 and 1980). He documents how price dispersion can emerge as the result of a separating equilibrium. However, "the absence of restrictions on the expectations of agents outside the set of [equilibrium] prices actually announced" [Wilson, 1980, page 126] implies a huge degree of indeterminacy. Many types of equilibria could actually exist, each associated with a particular degree of price dispersion.[4] Subsequent works have have exploited the predictive power of forward induction refinements to address this issue. Consistent with the rest of the literature, we use D1 to restrict off-equilibrium beliefs.[5] The result is a set of robust equilibria which share all the same unique outcome in terms of prices, quantity and quality of trade.

Although the issue of equilibrium selection is a delicate one, the obvious advantage of using a strong solution concept is that the predictive power of the theory is greatly enhanced – whether high quality sellers are driven out of the market or not only depends on the competitive pressure faced by sellers. Since different qualities always trade at different prices in robust equilibria, price dispersion is also uniquely determined by the strength of competition.[6]

Competition also determines the magnitude of price distortions due to the use of prices as signals. In the strong competition regime, the upward distortion is extreme: high quality sellers can only reveal their type by announcing prices so high that no trade can take place. By contrast, when competition is weak, high quality sellers are able to separate themselves with little or no distortion on prices.

Recent contributions use the mechanism design methodology to study the maximum level of welfare achievable in an economy characterized by asymmetric infor-

[4]As already mentioned, Laffont and Maskin (1989) study an oligopolistic market where sellers can signal quality through their prices. They conclude that, even in the case of just two firms, the problem of characterizing all perfect Bayesian equilibria seems intractable.

[5]See Banks and Sobel (1987) and Cho and Kreps (1987).

[6]Villeneuve (2003) considers privately informed insurers competing to insure a single consumer. Differently from our framework, the type – which is determined by the consumer's risk profile – is the same for all insurers.

mation. Gul and Postlewaite (1992) study the conditions under which an economy characterized by asymmetric information can achieve efficiency as it becomes large. Muthoo and Mutuswami (2005) characterize the second best solution in markets with quality uncertainty where sellers are more informed than buyers.

Our approach is complementary to theirs to the extent that we study the prevailing equilibrium associated with a specific price convention. This enables us to find a relationship between competition and observable features of the market such as price dispersion. Moreover, we can assess how the use of prices as a communication device could affect the level of welfare and the distribution of surplus between buyers and sellers. In particular, while such a role of price could help high quality sellers to trade, it does not always lead to a welfare improvement. Equilibria in which prices are uninformative (pooling) might generate higher welfare. However, we show that when sellers are free to set their prices, these equilibria fail D1.

In many cases, policy makers have argued that regulations restricting competition serve the purpose of maintaining the quality of products and services and protecting customers from malpractice. This is the case, for instance, of professional services defined as services that require practitioners to display a high level of technical knowledge which consumers might not have. Our results suggest that while limiting competition does raise the average quality traded in the market, it also reduces the surplus available to the buyers. Accordingly, when there are positive gains from trading low quality goods, buyers (customers) are unambiguously better off in the strong competition regime, in spite of the adverse effect on the quality of goods.

Similarly, we show that, when high quality sellers are driven out of the market by competition, imposing a price floor might help to restore trade of the high quality. This is broadly consistent with recent evidence by Huck et al. (2007) (although their environment is slightly different from the one considered here). Intuitively, a price floor would prevent the profits of low quality sellers from falling to zero when competition is strong. This would allow high quality sellers to separate themselves by charging prices above the price floor. In particular cases, price controls might thus increase efficiency. On the other hand, price controls have relevant redistributive effects. Any gain in overall efficiency necessarily comes at the cost of a lower consumer surplus.

The paper is organized as follows. In section two we present the model. In Section three we describe the equilibrium concept and its refinements, and we characterize the set of robust equilibria. Section four describes the features of the equilibrium outcome and discusses potential policy interventions. Section five analyzes the robustness of the results to changes in the information structure. A final section concludes the paper.

2 The Model

We consider a large market populated by B buyers, and S sellers. The set $\mathcal{S}$ of sellers is indexed by $s = 1, ..., S$; $s \in \mathcal{S}$. Each seller is endowed with one unit of good. Goods come in two different qualities, $q \in \{h, l\}$, where l (h) stands for *low* (*high*). The general case of a finite number of qualities is analyzed in the appendix. We refer to sellers endowed with quality q as sellers of quality (or type) q. The quality of each seller is decided by nature: each seller has a probability λ to be of quality h and probability $1 - \lambda$ to be of quality l. The distribution of qualities is common knowledge. However, buyers cannot observe individual qualities. Moreover, quality is not verifiable ex post. The monetary utility that individual sellers of type q derive from their good is $v(q) > 0$, with $v(h) > v(l)$.

The set $\mathcal{B}$ of buyers is indexed by $b = 1, ..., B$; $b \in \mathcal{B}$. Each buyer consumes either one unit of good or nothing. Buyers share identical preferences defined by the monetary utility function $u(q) > 0$, with $u(h) > u(l)$. We impose $u(q) > v(q)$ for all $q \in \{h, l\}$, which implies that under full information there are always gains from trade to be realized. For expositional purposes, we also impose $u(l) < v(h)$: buyers are never willing to buy a low quality good at any price that is profitable for a high quality seller.

We are mainly interested in characterizing the behavior of agents in a large market. Accordingly, we consider the case in which both the number of buyers and the number of sellers go to infinity and their ratio, B/S, converges to some value $\theta \in \mathbb{R}^+$.[7]

The market functions as follows.

[7]As it is well known (see Judd 1985), the use of the law of large numbers with a continuum of agents may be inappropriate.

Pricing

At stage zero, each seller s observes his quality. Endowed with this piece of information, sellers move first, by simultaneously choosing their action, while buyers do nothing. The action p_s played by seller s consists in announcing a price $p \in [0, \overline{p}]$, where $\overline{p}$ is finite and strictly greater than $u(h)$, so that in equilibrium trade never occurs at $\overline{p}$. For simplicity, we adopt the convention that sellers who choose not to trade always announce $\overline{p}$. A strategy for seller s is a map from $\{l, h\}$ into the set $\mathbb{A}$ of probability distributions over $[0, \overline{p}]$. An action profile for the sellers is a collection $\mathbf{p} \equiv \{p_1, ..., p_S\}$, with $p_s \in [0, \overline{p}]$.

Beliefs

At stage 1, buyers observe the prices announced by sellers at stage zero and choose whether to buy and at what prices. Buyers' prior beliefs assign a probability λ to the event that an individual seller s is of type h. Upon observing the price p announced by a seller s, and given the sellers' action profile $\mathbf{p}$, buyers' (posterior) beliefs that the seller announcing p is of type q are denoted with the conditional probability function $\sigma(q|p, \mathbf{p})$.

Demand

We do not model explicitly strategic interaction between buyers since this would require the choice of a mechanism matching buyers and sellers. Since our results are to some extent independent of the specific market mechanism, we choose to simplify the analysis by specifying only the minimum requirements that the demand side must satisfy for the results. As will be made clear in the next section, this reduces to assuming that the essential properties of a market à la Bertrand are preserved.

We therefore assume that, after observing $\mathbf{p}$, buyers play an underlying subgame in which each buyer chooses an action (or a set of actions) which may or may not result in a purchase. For instance, buyers can compare prices, inquire about availability, and then choose whether to buy at some price. The outcome of this game is a function $J(p; \mathbf{p})$, $J : [0, \overline{p}] \rightarrow [0, 1]$ which, for any $\mathbf{p}$, specifies the probability to sell at some price p. Given that all sellers are ex-ante identical and they move simultaneously, the function $J(p; \mathbf{p})$ will be the same to all sellers. Finally, we denote with K the share of buyers who are able to obtain a good. The values of $J(p; \mathbf{p})$ and K are connected by the restriction that the number of goods sold must equal the number of goods bought. Denote with $s(p)$ the fraction of sellers announcing p. Using the law of large numbers, this restriction can be stated as

$$\sum_p s(p)J(p; \mathbf{p}) = K\theta \tag{1}$$

where both sides are scaled by the total number of goods in the economy (S).

3 Equilibrium

We base our equilibrium analysis on the concept of perfect Bayesian equilibrium (PBE). We now discuss the restrictions that sellers' strategies, beliefs σ, and buyers' behavior (summarized by the function J) must satisfy.

Sellers' strategies

The expected payoff of a seller of type $q \in \{l, h\}$ announcing price p when prices announced by all sellers are summarized by $\mathbf{p}$ is $J(p; \mathbf{p})[p - v(q)]$. In equilibrium, the sellers' strategy profile must satisfy the following two restrictions

R1 All sellers play best replies given other sellers' strategies, beliefs σ, and $J(.;.)$.

R2 Strategies are symmetric: all sellers of the same quality announce the same prices with the same probabilities.

Condition R1 is entirely standard and requires no explanation. Condition R2 is commonly invoked when dealing with many agents. Symmetric strategies are usually imposed to simplify the equilibrium analysis. With price-setting sellers, this particular condition is further motivated by the fact that buyers' beliefs are derived from sellers' strategies. If strategies were not symmetric, buyers could assign different probabilities to be of a given type to sellers announcing the same price. This is at odds with the conventional idea of a large market in which trade is not affected by the identity of individuals. Moreover, since we allow for mixed strategies, symmetric strategies do not rule out asymmetric actions as an equilibrium outcome. Thus, imposing symmetric strategies does not imply a great loss of generality in terms of agents' behavior.

We also restrict our attention to equilibria in which sellers randomize over prices by using distributions with finite support. This restriction, together with the assumption that all sellers adopt the same strategies, permits to exploit the law of large numbers (see below).

Beliefs

R3 All buyers share the same beliefs and these are derived from sellers' strategies using Bayes rule where possible;

R4 Buyers' beliefs about a seller announcing a given price are not affected by the price announced by another seller, even in the presence of deviations.[8]

Condition [R3] is again standard. Symmetry, together with the assumption that buyers' beliefs obey Bayes' rule, imply that buyers should assign the same probability to be high quality to any pair of sellers taking the same action. Condition R4 implies that beliefs about seller s are independent of other sellers' actions, even in the presence of deviations (Fudenberg and Tirole, 1991, p. 332). This condition follows from the fact that sellers different from s have no information about s's type that is not also available to the buyers.

Buyers' behavior

As already mentioned, rather than assuming a specific market structure, we consider a broad range of possible interactions. We require however that the function $J(p, \mathbf{p})$ satisfy some familiar properties of competitive behavior,

R5 For all $\mathbf{p}$, and beliefs σ, the function $J(p; \mathbf{p})$ satisfies:

i) $J(p; \mathbf{p}) = 0$ for all p such that $\sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) < p$.

ii) If at some p $J(p; \mathbf{p}) > 0$ and there exists some p' such that

$$\sigma(h|p', \mathbf{p})u(h) + [1 - \sigma(h|p', \mathbf{p})]u(l) - p' > \sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) - p \quad (2)$$

[8]Formally, this is equivalent to saying that, for all p, $\hat{p}$, $\mathbf{p}$, $\hat{\mathbf{p}}$, and q, $\sigma(q|p, \mathbf{p}) = \sigma(q|\hat{p}, \hat{\mathbf{p}})$ if $p = \hat{p}$.

then $J(p'; \mathbf{p}) = 1$.

iii) If $K < 1$, then $J(p, \mathbf{p}) = 1$ for all p such that

$$\sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) > p \tag{3}$$

Condition [i] is the result of a standard participation constraint. If, given beliefs, buyers expect to make a loss at p, no buyer would buy at p. Hence, the probability to sell must be zero. Condition [ii] says that if at p the probability to sell is positive and, given beliefs, there is another price p' at which buyers make higher surplus, then the probability to sell at p' must be one. Intuitively, it is possible to sell at p only if at all prices that guarantee a better deal to the buyer there is no excess supply. This is akin to assuming Bertrand competition when supply is inelastic. Condition [iii] says that if there are buyers who are unable to obtain a good, then it must be possible to sell with probability one at all prices that, given beliefs, leave the buyer with a positive surplus. Again, the intuition is obvious. Notice also that conditions [i]-[iii] do not impose any restriction on how beliefs should vary according to the observed price. As it usually happens in the presence of adverse selection, the prices that leave the highest expected surplus to the buyer are not necessarily the lowest.

Conditions [i]-[iii] are natural when search costs are not particularly high. As observed by Bester (1993), this is exactly the case when we should expect posted prices as opposed to bargaining. Our results thus apply to any specific market setting where sellers post prices and buyers interact in a way that is compatible with **R5**.

For an immediate example, consider a situation in which buyers arrive sequentially and choose at which price to buy and then select at random among the sellers who have announced that price and have not sold to previous buyers. This is reminiscent of customers visiting a price comparison site listing sellers' quotes. Another example is provided in the working paper version of this paper. There, symmetric buyers simultaneously choose the price at which to buy. If at some price there is excess demand (supply), the purchase (sale) is allocated through a lottery. This is also the approach taken in Wilson (1980). A third example is a situation in which, after observing the prices posted by the sellers, buyers simultaneously submit a ranking of the prices at which they accept to buy. A seller is then matched with a buyer only when all sellers offering prices that are better placed in the buyers' rankings have been able to sell their goods.

As already mentioned, we are interested in the behavior of a market characterized by a large number of agents. The law of large numbers works on two levels here. First, the individual realizations of Nature's draws are irrelevant: the fraction of type h (resp. l) sellers in the market is always equal to λ (resp. $1 - \lambda$). Second, given symmetry and the assumption that agents randomize over distributions with finite support, individual realizations of agents' randomization are also irrelevant. In equilibrium, the fraction of sellers announcing a given price is certain. Given R1-R5, in the remainder of the paper, we will denote simply with $\sigma(q|p)$ the posterior probability that a seller announcing price p is of type q (or, equivalently, the fraction of type q sellers among sellers announcing p). Also, action profiles will be omitted when referring the probability to make a sale. This will be denoted simply as $J(p)$.

Equilibria can take different forms:

a. *Separating equilibria*, in which, by definition, different seller-types take different actions;

b. *Pooling equilibria*, in which all seller-types take the same action;

c. *Partially separating* or *Hybrid equilibria*, in which heterogenous poolings of sellers take different actions.

Associated with this variety of equilibria is a great deal of indeterminacy with respect to the market's outcome in terms of prices, traded quantities and qualities, as well as with respect to the associated expected payoffs of market's participants. For this very reason it is important to investigate how a robustness analysis helps restricting the set of possible equilibria.

3.1 Restrictions on off-equilibrium beliefs

The high degree of indeterminacy is due to a typical "unsent message" problem: if a seller deviates to a price p that is announced with probability zero in equilibrium, Bayes' rule cannot determine the posterior beliefs of the buyers. Thus, upon observing a profile $\mathbf{p}$ containing the deviation p, buyers could hold arbitrary beliefs about the quality of the seller who is announcing p. Therefore, we impose that buyers' off-equilibrium beliefs be consistent with a commonly used equilibrium refinement.

R6 Buyers' off-equilibrium beliefs satisfy D1 (Cho and Kreps 1987).

In this section we discuss how this restriction can be used to eliminate equilibria in the case of many buyers and sellers.[9] If seller s deviates and announces a new price p, his probability to sell at p depends on whether p is more or less appealing to buyers than the prices announced by other sellers. Buyers' beliefs about the seller who deviated are determined by D1. What beliefs do they hold about sellers who did not deviate? If a seller sticks to his equilibrium strategy, beliefs about him are not affected by the deviation of another seller. This follows from restriction R4.[10] As a result, buyers' beliefs about sellers who did not deviate are the same as in the candidate equilibrium.

We now describe how the refinement works in practice. Given a candidate equilibrium, we want to determine the extent to which a deviation can signal to the buyer that the seller who is deviating is of type h. Clearly enough, a type h seller would never deviate to a price $p < v(h)$ since he could only lose from this action. Intuitively, given a deviation $p \geq v(h)$, if type h sellers strictly benefit from deviating to p whenever type l sellers weakly benefit from the same deviation, then buyers assign probability zero to the event that p is announced by type l. More precisely, denote with J^l and J^h the critical values of the probability to sell when announcing p such that sellers of type l and h respectively would be indifferent between deviating by announcing p and playing their equilibrium strategy. Denoting with $\pi_s^*(q)$ the equilibrium value of type

[9] Mailath et al. (1993) point out that the D1-robust outcome may not converge to the full information outcome as buyers' information becomes "almost" perfect (e.g. when λ approaches one or to zero in our setup). In a monopolistic framework, Adriani and Deidda (2009) show that this may be problematic. As will become clear, the discontinuity affects the D1-robust outcome in our game when λ is close to one. However, allowing for "continuous" equilibria when λ is close to one would change some quantitative results, but not the main qualitative results (i.e. what qualities are traded).

[10] The intuition is that the seller who deviated does not possess any information about the quality of sellers who did not, that is not available also to the buyers.

q's expected payoff, J^l and J^h are implicitly defined by

$$\pi_s^*(l) = J^l[p - v(l)] \tag{4}$$

$$\pi_s^*(h) = J^h[p - v(h)] \tag{5}$$

Consider a deviation p such that $p - v(h) > \pi_s^*(h)$. According to R6, if $J^l > J^h$, buyers conclude that the deviating seller is of type l with probability zero, that is $\sigma(l|p) = 0$.[11] In words, if the critical value of the probability to sell such that a seller is indifferent between deviating or not is lower for a high quality seller than for a low quality one, a high quality seller is in a sense "more seemly to benefit" from the deviation.

Therefore, an equilibrium fails D1 if, based on beliefs refined in such a way, a seller would profit from deviating given that all other sellers stick to their equilibrium strategies. Conversely, an *equilibrium is robust to D1* if either of the two is true: 1) There is no deviation p at which buyers' refined beliefs are such that the seller announcing p is of low quality with probability zero, 2) If such p exist(s), then no seller must profit from deviating to p.

Given D1, buyers' beliefs assign probability zero to type l whenever the condition $J^l \leq J^h$ is violated. Another way to interpret R6 becomes apparent when substituting (4) and (5) into $J^l \leq J^h$,

$$\frac{\pi_s^*(l)}{p - v(l)} \leq \frac{\pi_s^*(h)}{p - v(h)} \tag{8}$$

In words, the opportunity cost ($\pi^*(q)$) of a deviation p relative to the potential gain ($p - v(q)$) must be smaller for a low quality seller than for a high quality one.

[11] More precisely, let $\mathbf{p}$ denote an action profile for the sellers comprising a deviation p. Let α^B denote a strategy profile for the buyers in the buyers' subgame. Let $MBRP(\mathbf{p})$ denote the set of profiles comprising only mixed strategies for which it is possible to find beliefs and some profile of strategy for other buyers such that they are best replies given $\mathbf{p}$. Finally, let

$$R_1(l|p) \equiv \{\alpha^B \in MBRP(\mathbf{p}) : \; J(p, \mathbf{p}) \geq J^l\} \tag{6}$$

and

$$R_2(h|p) \equiv \{\alpha^B \in MBRP(\mathbf{p}) : J(p, \mathbf{p}) > J^h\}. \tag{7}$$

According to D1, if $R_1(l|p) \subset R_2(h|p)$, then $\sigma(l|p) = 0$. Throughout the paper, we will consider deviations p such that $u(h) - p > \sigma(h|p')[u(h) - p'] + (1 - \sigma(h|p'))[u(l) - p']$, where p' is some price such that $J(p') > 0$. Hence, one can always find beliefs such that buying at p with any probability between 0 and 1 is consistent with a best reply for a buyer (given some strategy profile for other buyers). It is then immediate to build profiles such that, for any $j \in [0, 1]$, there exists $\alpha^B \in MBRP(\mathbf{p})$ such that $J(p, \mathbf{p}) = j$. This in turn ensures that $J^l < J^h$ is necessary and sufficient for $R_1(l|p) \subset R_2(h|p)$.

For the purposes of the robustness analysis we distinguish the candidate equilibria into two broad categories on the basis of how many qualities are traded:

Definition 1. *A type I equilibrium is a PBE where both qualities are traded. A Type II equilibrium is a PBE where only the low quality is traded.*

It is worth noting that, in general, there would be a third category, which includes those equilibria in which no quality is traded. However, as it turns out, the low quality is always traded given our assumptions.[12]

3.2 Type I equilibria: both qualities are traded

Type I equilibria can take two forms: a. Separating equilibria (SE); b. Pooling equilibria (PE) and Hybrid equilibria (HE). Rather than characterizing all equilibria and then discard those which fail D1, we only characterize equilibria that pass D1.

Let us analyze PE and HE, first. In any PE, by definition, there is a single equilibrium price p^* at which both high and low qualities are traded. In HE sellers of the same type may announce different prices and there is at least one price that is announced with positive probability by both types.

It is well known that D1 tends to select SE (see Cho and Sobel, 1990). The next lemma shows that also in the present model, within the set of type I equilibria, D1 discards PE and HE:

Lemma 1. *No pooling/hybrid equilibrium of type I survives D1.*

Proof. See Appendix.

In any PE or HE where both qualities are traded, sellers of type h, who face a higher opportunity cost of selling ($v(h) > v(l)$), make lower equilibrium profits than sellers of type l. Hence, sellers of type h have a lower opportunity cost of deviating. Being aware of this, buyers infer that any deviation to a higher price must come from a high quality seller. Therefore, sellers of type h would find it optimal to stand out of the crowd by announcing a price that is slightly higher than the equilibrium price at which both qualities are traded. Accordingly, no PE or HE of type I is ever robust: equilibria of type I that are robust to D1 could only include SE.

[12]Consider an equilibrium where all sellers announce prices at which trade does not occur. Given $K < 1$, $J(p) = 1$ for all $p < u(l)$ independently of off-equilibrium beliefs. Then, deviating and announcing $p \in (v(l), u(l))$ is always profitable for a seller of type l.

In a separating equilibrium, low quality sellers announce a price p_l, while high quality sellers announce a different price $p_h \neq p_l$. Prices constitute a perfect signal of quality: $\sigma(h|p_h) = 1$, and $\sigma(h|p_l) = 0$. When a good is exchanged at p_q, $q \in \{l, h\}$, the buyer obtains $u(q) - p_q$. In any SE, p_h and p_l satisfy

$$[p_l - v(l)]J(p_l) \geq [p_h - v(l)]J(p_h) \tag{9}$$

$$[p_h - v(h)]\, J(p_h) \geq [p_l - v(h)]J(p_l) \tag{10}$$

These two inequalities represent the Incentive Compatibility Constraints (ICC) for low and high quality sellers, respectively. Any SE in which quality $q \in \{l, h\}$ is traded must also: 1) satisfy the participation constraint of sellers ($p_q \geq v(q)$) and buyers ($p_q \leq u(q)$); 2) ensure that if buyers obtain higher surplus from buying at p_q than at $p_{q'}$, then $J(p_q) = 1$. These conditions imply that any SE is characterized by $p_h > p_l$ and $J(p_h) < J(p_l)$.[13]

Whenever the equilibrium is separating, the robustness condition (8) can be rewritten as

$$\frac{[p_l - v(l)]\, J(p_l)}{p - v(l)} \leq \frac{[p_h - v(h)]\, J(p_h)}{p - v(h)} \tag{13}$$

If (13) is violated for some p that is potentially appealing to type h, then $\sigma(l|p) = 0$. The next lemma illustrates how D1 helps to restrict the set of separating equilibria.

Lemma 2. *In any D1-robust SE of type I, the ICC of low quality sellers is satisfied with equality unless $p_h = v(h)$.*

Proof. See appendix.

In any SE of type I the probability to sell at p_h must be less than 1; otherwise, low quality sellers would mimic. Therefore, a high quality seller who is announcing p_h would be willing to deviate and announce a price p slightly lower than p_h whenever

[13] If $J(p_h) \geq J(p_l)$, then, from type l ICC, $p_h < p_l$. Type l ICC can be rewritten as

$$\frac{p_l - v(l)}{p_h - v(l)} \geq \frac{J(p_h)}{J(p_l)} \tag{11}$$

But then, given $p_h < p_l$ and $v(h) > v(l)$,

$$\frac{p_l - v(h)}{p_h - v(h)} > \frac{p_l - v(l)}{p_h - v(l)} \geq \frac{J(p_h)}{J(p_l)} \tag{12}$$

so that type h ICC is violated. By converse, if $J(p_h) < J(p_l)$, then $p_h > p_l$ follows from **R5**(ii).

the gains from the increase in the probability to sell outweigh the loss due to the small reduction in the price. If the ICC of type l does not hold with equality, low quality sellers strictly prefer p_l to p_h. Therefore, they are not willing to deviate unless the chances to sell at p become relatively high. Buyers accordingly infer that the deviation p must come from a high quality seller, which in turn gives sellers the incentive to deviate. By contrast, when type l ICC holds with equality, low quality sellers are indifferent between p_l and p_h. Therefore, they are willing to deviate whenever the high quality are, which implies that buyers' off-equilibrium beliefs cannot be restricted.[14]

We are interested in market conditions under which a SE of type I is robust to D1. A key parameter in our discussion is the ratio, θ, between potential demand, given by the number of buyers, and potential supply, given by the number of sellers. This is a measure of the competitive pressure faced by buyers and sellers.

The next illustrates under what conditions separating equilibria of type I may emerge.

Lemma 3. *If $1-\lambda \geq \theta$ there is no SE of type I. If $1-\lambda < \theta$ D1-robust SE must be of type I.*

Proof. See appendix.

Consider a SE of type I: sellers of type l announce p_l and sellers of type h announce $p_h \geq v(h)$. If $1-\lambda$ exceeds θ, low quality sellers are relatively more numerous than buyers (i.e. they are the long side of the market). Sellers compete to sell, while buyers face no competitive pressure. Accordingly, competition among low quality sellers implies that, in equilibrium, $p_l = v(l)$.[15] However, if $p_l = v(l)$, the ICC of type l sellers can never be satisfied for any $p_h \geq v(h)$ unless the probability to trade at p_h were equal to zero, which would contradict the hypothesis of a type I SE.

Consider now the case $1-\lambda < \theta$. Low quality sellers are on the short side of the market. Buyers, on the other hand, face competitive pressure. If prices are such that buyers are making a positive surplus, then all buyers should be willing to buy. However, if $1-\lambda < \theta$, low quality sellers are not enough to satisfy the whole demand.

[14]Interestingly, type l sellers' ICC needs to be binding for a second best (see Muthoo and Mutuswami 2005).

[15]Price competition works here because the worst belief that buyers can assign to a seller announcing a price lower than p_l is that he is of type l with probability 1, and buyers are buying quality l at p_l.

Therefore, they will raise their prices until some of the buyers will be willing to buy (from high quality sellers) at a price $p_h \geq v(h)$. If, on the other hand, buyers obtain zero surplus, then $p_h > v(h)$. Lemma 2 thus ensures that the high quality is traded. Yet, the probability to sell at p_h must be sufficiently low to ensure that low quality sellers do not have incentive to announce p_h.

We now turn to the full characterization of robust type I equilibria. Define

$$\delta \equiv \frac{GFT_l}{u(h) - v(l)} \in (0, 1) \tag{14}$$

and

$$\gamma \equiv \frac{\Delta GFT}{v(h) - v(l)} \tag{15}$$

where, for one unit of quality q, $GFT_q \equiv u(q) - v(q)$ measures the gains from trade, and $\Delta GFT = GFT_l - GFT_h$. Note that δ represents the overall gains from trading the low quality scaled by the range of feasible prices $u(h) - v(l)$, while γ is the difference in the gains from trade between the two qualities, ΔGFT, over the difference in the seller's evaluation of the two qualities $v(h) - v(l)$. When $\gamma > (<)0$, the gains from trading the low quality are higher (lower) than those from trading the high quality.

Let $\hat{\theta} \equiv 1 - \lambda + \delta\lambda$, and $\theta_\gamma \equiv 1 - \lambda + \gamma\lambda I_{\{\gamma>0\}}$ where $I_{\{\gamma>0\}} : \mathbb{R} \to \{0, 1\}$ is an indicator function that takes value 1 if $\gamma > 0$ and zero otherwise. Note that since $\gamma < \delta$ holds, $\hat{\theta}$ is always strictly greater than θ_γ.

Proposition 1. *D1-robust equilibria of type I emerge if and only if* $1 - \lambda < \theta$. *In all these equilibria: i)* $J(p_l) = 1$, *and* $J(p_h) = \min\left[\frac{\theta-(1-\lambda)}{\lambda}, \delta\right]$, *ii)* p_l *and* p_h *are uniquely determined:*

i. $p_h = u(h)$, $p_l = u(l)$, *if* $\theta \in [\hat{\theta}, \infty)$;

ii. $p_h = v(l) + \frac{\lambda[u(h)-u(l)]}{1-\theta}$, $p_l = v(l) + \frac{\theta-(1-\lambda)[u(h)-u(l)]}{1-\theta}$ *if* $\theta \in (\theta_\gamma, \hat{\theta})$;

iii. $p_h = v(h)$, $p_l = u(l) - [u(h) - v(h)]$ *if* $\theta \in (1 - \lambda, \theta_\gamma]$.

Proof. See appendix.

Proposition 1 implies that the equilibrium outcome in terms of prices and traded quantities and qualities is uniquely determined and crucially depends on the buyers to sellers ratio, θ. If θ is very large, i.e. greater than $\hat{\theta}$, trade (of both qualities) occurs at buyers' reservation prices, $u(h)$ and $u(l)$ (case *i*). If θ is only moderately large, i.e. greater than $1 - \lambda$ but lower than $\hat{\theta}$, trade (of both qualities) occurs at prices that guarantee a positive surplus to the buyers (cases *ii* and *iii*). Notice that, provided that $\gamma \leq 0$, p_h will exceed type h sellers' reservation prices in any robust SE of type I. This occurs even though the probability to sell at p_h, $J(p_h)$, is always less than one, which would suggest that high quality sellers' profits should be competed away. This is a standard effect of asymmetric information. Price competition among high types is impaired by buyers' fear that low types may deviate and announce p_h if the probability to sell at p_h becomes too large. Hence, price competition among sellers of type h comes to a halt when the demand at p_h is such that low quality sellers are indifferent between announcing p_h and announcing p_l. Limited price competition causes high quality sellers' profits to remain positive, even if $J(p_h) < 1$.

Things change substantially if the gains from trading the low quality exceed those from trading the high quality ($\gamma > 0$) and $\theta \leq \theta_\gamma$ (if $\theta > \theta_\gamma$ the previous discussion applies). In this case, the ICC of low quality sellers does not hold with equality and the price announced by high quality sellers drops to $v(h)$. In other words, high quality sellers must forgo their profits in order to trade. Low quality sellers will then announce the highest possible price at which buyers (weakly) prefer to buy the low quality, given the option to buy the high quality at $v(h)$. Figure 1 illustrates the relationship between θ and the equilibrium prices.

3.3 Type II equilibria: Only the low quality is traded

In this section we turn our attention to the typical lemon-market situation in which the high quality is driven out of the market (type II equilibria). We will characterize the (unique) robust outcome of type II equilibria and show that such equilibria arise if and only if no robust type I equilibrium exists.

Proposition 2. *D1-robust equilibria of type II emerge if and only if* $1 - \lambda \geq \theta$. *In all these equilibria, the fraction of quality* l *traded is* $\theta/(1-\lambda) \leq 1$. *All trade occurs at a unique price* p^*, *which is equal to* $v(l)$ *if* $1 - \lambda > \theta$.[16]

Proof. See appendix.

In order to gather intuitions on proposition 2, notice that when $1 - \lambda > \theta$, (low quality) sellers face competitive pressure. They compete to sell their goods, which

[16]The analysis of the equilibrium price for the special case $1 - \lambda = \theta$ is presented in the proof. There, it is shown that a discontinuity arises when $\gamma > 0$.

drives their profits to zero ($p^* = v(l)$). If, on the other hand, $1 - \lambda < \theta$, sellers of low quality are on the short side and buyers face competitive pressure. Thus, low quality sellers would announce $p^* = u(l)$. At this price, they would extract all the surplus from the buyers and make strictly positive profits. It is easy to see why these equilibria fail D1. Whenever sellers of low quality make strictly positive profits, their opportunity cost of deviating, measured by $\pi^*(l)$, is larger than that of high quality sellers – which equals zero since they are not trading. Accordingly, while high quality sellers are never worse off when deviating, low quality sellers might be hurt. It follows that, upon observing a deviation $p > v(h)$, buyers should infer that the seller deviating is of high quality.

By contrast, given a type II equilibrium, low quality sellers make zero profits when they are the long side of the market. Hence, the opportunity cost of deviating is the same for low and high quality sellers, and both types of sellers are never worse off if deviating. Thus, deviating to prices at which no trade occurs in equilibrium is as cheap a way to signal quality for the low type as it is for the high type. Unsurprisingly, deviations to higher prices thus fail to signal higher quality and the equilibrium is robust.

Finally, notice that robust type II equilibria can be either SE or hybrid equilibria (HE). In SE all low quality sellers announce p^* while high quality sellers announce $\overline{p}$. In HE, all sellers of quality h and a fraction smaller than $1 - \lambda - \theta$ of low quality sellers announce $\overline{p}$ (and do not trade) while the rest announce p^*.[17]

4 Properties of the robust equilibrium outcome

We now discuss the properties of the equilibrium outcome and characterize the amounts of low and high quality goods traded and the distribution of surplus.

Cho and Sobel (1990) show that in signaling games that satisfy specific monotonicity and sorting conditions, D1 selects a unique equilibrium, which is a SE. In the model

[17] One might wonder whether D1 is necessary for the result that the high quality is traded whenever $1 - \lambda < \theta$. We considered robustness of type II equilibria to less powerful refinements such as Divinity (Banks and Sobel, 1987) and Sequential Perfection (Grossman and Perry, 1986). Both criteria give the same results. There is only a significant difference between these criteria and D1: when $1 - \lambda < \theta$, type II equilibria may pass Divinity and Sequential Perfection (but not D1) if the proportion of high quality sellers is sufficiently small.

we analyze, given any two prices p and $p' < p$ and associated probabilities to sell $J(p)$ and $J(p')$ their sorting condition would be

$$J(p)[p - v(l)] \geq J(p')[p' - v(l)] \Rightarrow J(p)[p - v(h)] > J(p')[p' - v(h)] \tag{16}$$

Whenever low quality sellers benefit from announcing a higher price, high quality sellers would strictly benefit from doing the same. Condition (16) is of course satisfied for all prices at which trade occurs, i.e. provided that $J(p) > 0$ and $J(p') > 0$. However, at prices at which the probability to sell is zero the net payoff is independent of the announced price and seller's type. Hence, at such prices, no sorting is possible and (16) is not satisfied. These observations help explaining why in the model we analyze the set of $D1$ robust equilibria does not include only separating equilibria and generally contains more than one equilibrium. Nevertheless, as it directly follows from the combination of proposition 2 and lemmata 1 and 3, $D1$ guarantees separation at prices at which trade occurs. Pooling can only occur at prices at which trade does not occur.[18] As a result, the equilibrium outcome is essentially unique in terms of quantities and prices at which trade occurs.

Proposition 3. *Given the values of λ, $\theta \neq 1 - \lambda$, $u(l)$, $v(l)$, $u(h)$, $v(h)$, all the resulting D1-robust equilibria yield the same unique outcome in terms of prices of traded goods, quality and quantity of trade. In particular: i. The fraction of quality l goods being traded (over the total supply of quality l) is $f(l) = \min[\theta/(1-\lambda), 1]$; ii. The fraction of quality h goods being traded (over the total supply of quality h) is $f(h) = \max\left[0, \min\left[\frac{\theta-(1-\lambda)}{\lambda}, \delta\right]\right]$.*

Proof. See appendix.

Because of this uniqueness property, the model implies a very precise relationship between the market conditions (as measured by θ) and:

a. Quantity and quality of trade;

b. Distribution of trade surplus;

c. Price dispersion and distortions;

[18] In particular, pooling survives $D1$ in equilibria of type II where only a fraction of low quality sellers announce $p^* = v(l)$ at which trade occurs, while high quality sellers and the rest of low quality sellers who decide not to trade announce $\overline{p}$. However, such HE yield the same equilibrium outcome in terms of quantities and qualities traded and agents' interim payoffs as the robust SE of type II in which all sellers of type l announce p^*.

d. Signaling role of prices.

a. Quantity and quality of trade.

If low quality sellers are relatively more numerous than buyers, the only D1-robust equilibrium is one in which only the low quality is traded; all buyers are able to buy. By converse, if buyers are relatively more numerous than low quality sellers, in the robust equilibrium both low quality and high quality sellers are able to sell their goods with positive probability. All low quality sellers are able to sell while only a fraction of sellers of high quality is able to find a buyer.

The fraction of high quality goods traded is a nondecreasing function of θ, as illustrated in Figure 2. For values of $\theta \leq 1 - \lambda$, the fraction of high quality traded, $f(h)$, is equal to zero, while the fraction of low quality, $f(l)$, increases in θ. If $\theta > 1-\lambda$, $f(h)$ linearly increases in θ until it reaches the value $\delta < 1$ where θ equals the critical value $\hat{\theta}$; $f(l)$ stays constant. Once θ has reached $\hat{\theta}$, further increases in θ do not affect $f(h)$ any longer. Notice that, while all buyers are able to buy one good if $\theta \leq \hat{\theta}$, a fraction $\lambda\delta$ of buyers do not obtain any good when the reverse (strict) inequality holds. This happens in spite of the fact that high quality sellers sell with probability $\delta < 1$. Although trade would be mutually beneficial, there might be buyers and sellers who are unable to trade. On the other hand, D1 selects the equilibrium where the amount of trade is maximized among all SE. Hence, the prevailing SE is the one in which the potential inefficiency related to the quantity of trade is minimized.

It is important to note that δ is decreasing in $u(h)$. When demand is sufficiently high, the higher is $u(h)$ the higher must be the price of high quality goods. However, the probability to sell a high quality good is bounded above by δ. As a result, the more buyers value high quality goods, the lower must be the maximum fraction of high quality sellers able to sell their good. The model thus displays a sort of curse on high quality sellers.

b. Distribution of trade surplus

The strength of competition among sellers has important redistributive implications. On the one hand, when competition is weak, the average quality traded is higher. On the other hand, sellers appropriate a larger share of the surplus. Our results show that, provided that there are potential gains from trading the low quality ($v(l) < u(l)$), buyers unambiguously benefit from strong competition. To see this, notice that if $1-\lambda$ exceeds θ the low quality is traded at $p = v(l)$. On the other hand, if θ exceeds $1-\lambda$, the low quality is always traded at a price higher than $v(l)$. In equilibrium, the surplus obtained from buying either quality is the same. Hence, buyers' surplus is always lower under weak competition than under strong competition. In other words, all the potential benefits from trading the high quality accrue to the sellers.

c. Price dispersion and distortions

Prices of traded goods are non decreasing functions of θ as described in figure 1. When $1-\lambda > \theta$, all trade occurs at price p_l (no price dispersion). Under reversed market conditions ($1-\lambda < \theta$), trade occurs at two different prices, p_l and p_h (price dispersion).

Wilson (1980) first argued that, in a market for lemons with price setting sellers, trade may occur at a distribution of prices rather than at a unique price. The results of our analysis have precise implications regarding the conditions under which a distribution of prices should arise in a lemon market. According to our model, the stronger the competition, the less price dispersion and variety of trade we should observe, and viceversa. This implication holds not just in the case of a market with two types of sellers and two qualities of goods, but also in the general case of any finite number N of qualities, which is analyzed in the appendix.

As in other models with adverse selection, signaling through prices might result in price distortions. Notably, the magnitude of such distortionary effects varies with market conditions. In particular, price distortion vanishes for sufficiently weak competition ($\theta > \hat{\theta}$), while it is extreme in the case of strong competition, where the high quality price is so high that no trade is possible.

d. Signaling role of prices

Off the equilibrium path, the information content of a deviation to a price higher than the equilibrium price changes according to whether sellers face weak competition or strong competition. If competition among sellers is weak, starting from an equilibrium in which only the low quality is traded, a deviation to a higher price allows high quality sellers to reveal themselves. As a consequence, the initial equilibrium unravels. By converse, in case of strong competition among sellers, both high and low quality sellers make zero profits and therefore have identical incentives to deviate. As a result, the deviation does not serve as a signal for high quality sellers and the initial equilibrium holds.

In equilibria that are robust to D1, the effectiveness of the price system at reflecting information along the equilibrium path also depends on the strength of competition among sellers. As we have shown, when competition is weak, both qualities are traded and each quality is traded at a different price. That is, prices are fully informative and enable high quality sellers to trade. However, when competition is strong, only low quality sellers are able to trade. The price system associated with this equilibrium is still fully informative, but the only credible way for high quality sellers to reveal their type is to decide not to trade (i.e. to announce a price at which trade does not take place). In other words, when competition is strong, there is no equilibrium price system that is both informative and would allow high quality goods to be traded.

These observations lead us to the the following conclusion. In lemon markets, whenever competition among sellers is strong, the traditional role of prices as a device for competing against rival sellers impairs the effectiveness of prices as a device for conveying information to the buyers.

4.1 Policy implications: price controls

We have shown that, when $1 - \lambda > \theta$, strong competition among sellers inhibits the signaling role of prices and drives high quality sellers out of the market. The resulting equilibrium outcome is inefficient whenever gains from trading quality h exceed those from trading quality l, i.e. $GFT_h > GFT_l$. We now show that the introduction of a price-floor – which forces sellers to set prices above a minimum price $\underline{p}$ – could help to restore trade of the high quality in these circumstances. Intuitively, the introduction of a price floor leaves positive profits to low quality sellers, thus making separation possible by reducing the incentive to mimic.

It is immediate to verify that Lemma 1 still applies, so that pooling/hybrid equilibria in which trade occurs at $p \geq \underline{p}$ are not robust to D1.[19] Therefore, we restrict attention to SE.

Assume that the price-floor $\underline{p}$ is set to satisfy $v(l) < \underline{p} < u(l)$. In a SE, low quality sellers are able to trade only if $p_l \in [\underline{p}, u(l)]$. Given $1 - \lambda > \theta$, there is always excess supply at p_l. Hence, price competition will drive the price of the low quality down to the price floor: $p_l = \underline{p}$. In contrast with the previous analysis, however, low quality sellers now make positive expected profits at $p_l = \underline{p}$. According to D1, if the high quality is not traded, any deviation to a price above $v(h)$ should be considered as emanating from a type h seller. It is then clear that, so long as $GFT_h > GFT_l$, the high quality must always be traded with positive probability in equilibrium. Otherwise, sellers could deviate to prices slightly above $v(h)$ and be able to attract buyers. Moreover, $GFT_h > GFT_l$ implies that p_h must exceed $v(h)$ in any separating equilibrium. Otherwise, buyers would not buy at p_l. This implies – see Lemma 2 – that the ICC of type l must be satisfied with equality in any D1-robust equilibrium

$$J(\underline{p})\left[\underline{p} - v(l)\right] = J(p_h)\left[p_h - v(l)\right] \tag{17}$$

Since both qualities are necessarily traded with probability less than one, buyers must be indifferent between buying the low quality at $\underline{p}$ and buying the high quality at p_h,

$$u(h) - p_h = u(l) - \underline{p} \tag{18}$$

Finally, the requirement that the number of goods bought equals the number of good sold (i.e. equation 1) implies

$$\lambda J(p_h) + (1 - \lambda) J(p_l) = \theta \tag{19}$$

Solving for p_h, $J(p_h)$, and $J(\underline{p})$ the system formed by equations (17), (18), and (19), yields the following equilibrium prices and the associated probabilities to trade

[19] In these equilibria, type l sellers make higher expected profits than type h sellers. Accordingly, sellers of high quality are more likely to benefit from deviating to a higher price, which undermines the equilibrium.

$$p_l = \underline{p};\ p_h = u(h) - u(l) + \underline{p} \tag{20}$$

$$J(\underline{p}) = \theta \frac{\underline{p} - v(l) + u(h) - u(l)}{\underline{p} - v(l) + (1-\lambda)\left[u(h) - u(l)\right]} \tag{21}$$

$$J(p_h) = \theta \frac{\underline{p} - v(l)}{\underline{p} - v(l) + (1-\lambda)\left[u(h) - u(l)\right]} \tag{22}$$

From (21) and (22), an increase in the price floor $\underline{p}$ within the interval $[v(l), u(l)]$ increases the probability to trade quality h and reduces the probability to trade quality l. When $GFT_h > GFT_l$, an increase in the price floor thus increases overall efficiency. However, notice that (18) implies that any increase in the price floor always reduces the surplus available to the buyers. As a result, any efficiency gain associated with a price floor comes at the cost of a lower buyers' surplus.

The introduction of price controls in the form of a price-floor can thus help to restore trade of the high quality under strong competition. Having said that, the price-floor only works if it is neither too low (i.e. lower than $v(l)$) nor too high (i.e. higher than $u(l)$). In the first case, it would simply be ineffective and undercutting by low quality sellers would drive the high quality out of the market. In the second case, complete market breakdown would be the only D1-robust equilibrium outcome. The argument for the second result is similar to the one developed by Adriani and Deidda (2009). If $\underline{p} > u(l)$, there is no separating equilibrium with trade. Since all pooling equilibria with trade fail D1, the unique D1-robust outcome must involve no trade.[20]

Finally, we note that the presence of a price-floor does not eliminate price-distortions. As a result, the probability of trading the high quality could be higher if the price control were to take the form of a fixed price. Forcing both qualities to stick to the same price would eliminate any scope for price signaling, thus implementing the outcome of a pooling equilibrium. A pooling price that is compatible with trade exists if[21]

$$\lambda[u(h) - v(h)] + (1-\lambda)[u(l) - v(h)] \geq 0 \tag{23}$$

[20]If $\underline{p} > u(l)$ there is always a D1-robust equilibrium with no trade. Intuitively, since both types make zero profits in equilibrium, D1 does not restrict buyers' beliefs in the presence of a deviation to any price above $\underline{p}$. As a result, beliefs that assign a sufficiently low probability to type h when observing an off-equilibrium price do not violate D1 and sustain the no trade equilibrium.

[21]A pooling price at which trade occurs must be at most equal to the expected quality $\lambda u(h) + (1-\lambda)u(l)$ and greater than or equal to the reservation price of type h, $v(h)$.

If trade occurs at the fixed price, the probability to trade the high quality is equal to θ, which is strictly greater than $J(p_h)$ as given by equation (22). Hence, if the fraction of high quality is sufficiently large or the gains from trading quality h are particularly large – so that (23) is satisfied – policy interventions aiming at maximizing efficiency should take the form of a fixed price. Otherwise, a price floor, which allows for separation, would be more effective.

Taken altogether, our results suggest that limiting the sellers' ability to choose their prices might be beneficial. This is broadly consistent with recent evidence by Huck et al. (2007). Although the problem that subjects face in their experiment is slightly different from the one considered here, the main effect at work is essentially the same.[22] Limiting the scope for Bertrand competition boosts trade of high quality goods.

An interesting question is why posted prices are so widespread, even though this price convention does not always attain the second best. While we believe that this issue deserves further scrutiny, Bester (1993) suggests a possible answer. A posted price convention tends to endogenously emerge in markets with asymmetric information, as opposed to bargaining for instance, when the search costs faced by the buyers become small.

5 Robustness and extensions

In the appendix, we extend the model to the case of an arbitrary number of qualities. Here, we focus on the information structure, by considering the possibility that buyers may have access to other sources of information in addition to the prices.

More specifically, we assume that buyers observe a noisy signal about the quality. With minor changes to the analysis carried forward in the previous sections, it is possible to show that this would not change the nature of the problem. Suppose that each buyer $b = 1, ..., B$ observes a private noisy signal x_b^s about seller s's quality after observing the seller's price but before choosing whether to buy.[23] Conditional on a

[22] In Huck et al. (2007), sellers choose ex-post the quality to be delivered.

[23] To keep things simple, we also assume that a buyer's decision to buy or not is not observable by other buyers.

given seller s's quality, the signals about seller s $(x_1^s, ..., x_B^s)$ are independently drawn from the same distribution. To simplify the discussion, we assume that x_b^s can only take two values; $x_b^s \in \{L, H\}$. This is however not necessary for the point we want to make. The crucial assumption is that the conditional distribution of x has the same support independently of whether the seller's type is h or l – so that there is no realization of x_b^s that perfectly reveals the seller's quality. We denote with ρ_h the probability to observe a high signal ($x_b^s = H$) when the seller is of type h and with ρ_l the same probability when the seller is of type l. We also assume $1 > \rho_h > \rho_l > 0$, so that a buyer is more likely to observe H when the seller is of type h than when the seller is of type l. Since we are adding a further stage to the game – the stage in which buyers observe their signals – we need to distinguish between buyer's beliefs after observing the price (but not the signal) and beliefs after observing both price and signal. We refer to the first as "price-induced beliefs" and to the second simply as posterior beliefs.[24]

The crucial difference with the previous analysis is that, if the signal is informative, the probability to sell at a given price could now depend on the seller's type. Intuitively, given $\rho_h > \rho_l$, each buyer is more likely to observe H when the seller is of type h than when he is of type l. As a result, when deviating to an off-equilibrium price p, the probability to sell at p for a type h cannot be lower (and is possibly higher) than the same probability for a type l, independently of the beliefs induced by the deviation p.[25] In other words, denoting with $J(p, q)$ the probability to sell at price p for a type q, in the previous sections we had $J(p, h) = J(p, l)$ for all p. If we are to maintain full generality with regard to the market structure, we can now only impose the weaker restriction $J(p, h) \geq J(p, l)$ for all p. The problem analyzed in the previous sections

[24] Formally, denoting the price-induced beliefs with $\sigma(q|p)$ as in the previous sections, the posterior beliefs are now

$$\Pr(q|p, x = H) = \frac{\sigma(q|p)\rho_q}{\sigma(q|p)\rho_q + \sigma(q'|p)\rho_{q'}}, \; \Pr(q|p, x = L) = \frac{\sigma(q|p)(1-\rho_q)}{\sigma(q|p)(1-\rho_q) + \sigma(q'|p)(1-\rho_{q'})} \quad (24)$$

for $q, q' \in \{l, h\}$, $q \neq q'$.

[25] More precisely, for any price-induced beliefs, the expected quality for a buyer observing H is (weakly) higher than the expected quality for a buyer observing L. Hence, a buyer who finds it optimal to buy at a price p when observing L must necessarily find it optimal to buy when observing H. Since the probability that a buyer observes H is larger when the seller is of type h, the probability to sell at any price for a type h seller cannot be larger than the same probability for a type l.

is thus a special case of the problem we analyze here.

Nevertheless, it is still easy to establish that, when $1 - \lambda > \theta$, there exists a D1-robust equilibrium in which only the low quality is traded. All types make zero profits in this equilibrium (i.e. the unique price at which trade occurs is equal to $v(l)$). If a seller announces an off-equilibrium price, the price-induced beliefs cannot be determined by Bayes rule. It is however easy to find price-induced off-equilibrium beliefs that sustain the equilibrium. If these assign a sufficiently high probability to the seller being of type l, buyers will choose not to buy at the off-equilibrium price independently of the signal realization. As a result, sellers have no incentive to deviate from the candidate equilibrium.

These out of equilibrium beliefs are robust to D1. Type l would deviate to any $p \geq v(h)$ so long as the probability to sell at p were positive. If the signal is not perfectly informative, the probability to sell at p for a type l is positive whenever the same probability is positive for a type h (although the probability to sell for a type h might be larger). Intuitively, even if a buyer's strategy prescribes buying at the out of equilibrium price only when observing H, there is always a chance of observing H when the seller's type is l. As a result, if type h expects to sell at p with positive probability, type l expects the same. A type l would thus deviate to any price at which the high quality can be traded whenever a type h would want to deviate. Hence, beliefs that assign a sufficiently high probability to type l are robust to D1.

By the same token, we argue that the high quality is traded with positive probability in any D1-robust equilibrium if $1 - \lambda < \theta$. Suppose that the high quality is not traded. In this case, the equilibrium necessarily involves type l making strictly positive profits, leaving no surplus to the buyers. Consider then a deviation to a price $p > v(h)$. A type h would strictly benefit from deviating to p whenever the probability to sell at p is positive. By converse, a type l would deviate to p only if the probability to sell were sufficiently high. As already argued, the probability to sell at p for a type h cannot be lower than the probability to sell for a type l. Hence, a type h would strictly benefit whenever a type l weakly benefits from the deviation. It follows that no equilibrium in which the high quality is driven out of the market would survive D1.

To summarize, when competition is strong, there is still a D1-robust equilibrium in which only the low quality is traded. When competition is weak, the high quality is necessarily traded in any D1-robust equilibrium. This illustrates that the effect of competition we described in the previous sections is unaffected by the introduction of noisy signals. However, if we want to conclude that competition *necessarily* drives the high quality out of the market, we need to rule out equilibria in which the high quality is traded when competition in strong. This basically reduces to ruling out pooling and hybrid equilibria with trade. Unfortunately, ruling out pooling equilibria would require adding more structure to the model, i.e. explicitly modelling how buyers and sellers are matched. Rather than following this route and assume an explicit market structure, we turn the problem on its head and show that there are intuitive restrictions on the probability to sell that are sufficient to rule out pooling equilibria under D1. These boil down to a weak monotonicity condition for the ratio $J(p,h)/J(p,l)$. The following lemma extends Lemma 1.

Lemma 4. *If $J(p,h)/J(p,l)$ is non-decreasing in p, then no pooling/hybrid equilibrium survives D1.*

Proof. See appendix.

Intuitively, the requirement that $J(p,h)/J(p,l)$ is non-decreasing is equivalent to saying that buyers' information matters (weakly) more when the price is high than when the price is low.

In summary, under a monotonicity condition for the ratio $J(p,h)/J(p,l)$, strong competition always drives the high quality out of the market. The previous analysis can be extended without further complications to the case in which the signal is public, so that, for any seller s, all buyers observe the same (noisy) realization of x^s. This may be relevant for e-commerce applications, since many web sites list measures of the seller's quality (e.g. ratings) in addition to the sellers' prices.

6 Conclusions

This paper tackled the issue of strategic pricing in a competitive market for lemons where the potential gains from trade are always positive. Sellers' pricing decisions are affected by two types of considerations. On the one hand, sellers want to maximize the chance to find a buyer. On the other hand, they may want to use prices to conceal/reveal their true quality. Thus, in markets for lemons, pricing decisions retain a double function. Sellers may lower prices to undercut competitors or increase them to signal high quality. We argue that these two roles of prices may be at odds with each other. When competition among sellers is strong, the use of prices to compete with other sellers prevails. Announcing a price that is higher than the price at which trade occurs in the market does not help to be recognized as a high quality seller by the buyers. The reason is that profits of all sellers are driven to zero by competition. Hence, all sellers, irrespectively of their quality, have no opportunity cost of deviating and announcing a higher price.

By contrast, when competition is weak, announcing a price higher than the lowest price at which trade occurs conveys relevant information to the buyers. The rationale is that in this case at least some of the sellers make positive profits. Thus, announcing a higher price (which harms the seller by reducing the likelihood of making a sale) is relatively more costly for low quality sellers.

The model generates various predictions, some of which are empirically relevant. First, the degree of price dispersion is inversely related to the degree of competition among sellers, as measured by the sellers to buyers ratio. We should observe concentration of trade around few low prices when competition is strong, whereas trade should spread upon a distribution of relatively dispersed prices when competition is weak. Second, the average quality traded in the market should increase (although in a nonlinear fashion) as competition decreases. This is because weak competition allows high quality sellers (who would be driven out of the market in the presence of fierce competition) to sell their goods.

Finally, the paper suggests that, in some circumstances, imposing some degree of price control might increase efficiency by restoring a positive amount of trade of the high quality. However, any efficiency gain would come at the expenses of lower buyers' surplus.

A Appendix

Omitted Proofs

Proof of lemma 1

We first consider pooling equilibria (PE), then Hybrid Equilibria (HE). In any PE of type I there is a single equilibrium price p^* at which both high and low qualities are traded. Hence, all sellers have the same probability to sell, $J(p^*)$. Equilibrium profits are $\pi_s^*(q) = J(p^*)[p^* - v(q)]$, with $q \in \{l, h\}$. The payoff of a buyer obtaining the good at p^* is $\lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*]$. Since buyers' payoff must be non-negative a necessary condition for a PE is $p^* \leq \lambda u(h) + (1 - \lambda)u(l)$.

Condition (8) can be rewritten as

$$\frac{p^* - v(l)}{p - v(l)} \leq \frac{p^* - v(h)}{p - v(h)} \Rightarrow v(h)(p - p^*) \leq v(l)(p - p^*) \tag{A.1}$$

We note that p^* must always be strictly lower than $u(h)$, otherwise buyers would make a loss. Condition (A.1) is never verified for $p > p^*$ so that, for deviations above the pooling equilibrium price, beliefs are such that the seller who deviated is of high quality. From **R5** (ii), if

$$u(h) - p > \lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*] \tag{A.2}$$

then $J(p) = 1$. The above inequality is satisfied for all p lower than $p^* + \eta$, where:

$$\eta \equiv (1 - \lambda)[u(h) - u(l)] > 0. \tag{A.3}$$

Hence, a seller deviating to $p \in (p^*, p^* + \eta)$ would be able to sell with probability one at a higher price. The candidate equilibrium thus fails D1.

The same argument given for pooling equilibria applies to hybrid equilibria (HE). In any HE, there is always a type $q \in \{l, h\}$ who announces at least two different prices with positive probability. This implies that, given a seller's type q, the expected payoff, $\pi^*(q)$, must be the same at all prices announced by type q.

By definition, in any HE there is always a price p^* that is announced by both types of sellers. Note also that, in a HE of type I, $J(p^*) > 0$ must hold, since a necessary condition for quality h to be traded is that type l sellers make positive profits. Since $\pi^*(q)$ is the same at all prices announced by type q, in order to asses the robustness, one can just focus on the incentives to deviate from p^*. Condition (A.1) stays unchanged. From **R5** (ii), if

$$u(h) - p > \sigma(h|p^*)[u(h) - p^*] + (1 - \sigma(h|p^*))[u(l) - p^*] \tag{A.4}$$

then $J(p) = 1$. Since pooling occurs at p^*, $\sigma(h|p^*)$ must be strictly lower than one. But then, any $p \in (p^*, p^* + \tilde{\eta})$, with

$$\tilde{\eta} \equiv (1 - \sigma(h|p^*))[u(h) - u(l)] > 0 \tag{A.5}$$

would cause the equilibrium to unravel. $\square$

Proof of lemma 2

Consider a deviation $p > v(h)$. Rewrite condition (13) as:

$$\begin{aligned} p[J(p_l)[p_l - v(l)] - J(p_h)[p_h - v(h)]] \leq \\ v(h)J(p_l)[p_l - v(l)] - v(l)J(p_h)[p_h - v(h)] \end{aligned} \tag{A.6}$$

and the ICC of low quality sellers as

$$J(p_l)[p_l - v(l)] = J(p_h)[p_h - v(l)] + \xi \tag{A.7}$$

where $\xi \geq 0$. In any type I equilibrium, $0 < J(p_h) < 1$ ($J(p_h) < 1$ follows from $J(p_h) < J(p_l)$). If (A.6) were violated for p slightly lower than p_h, a seller could profit from deviating to p. Given $p < p_h$, condition **R5** (ii) would imply $J(p) = 1$ which, for p sufficiently close to p_h, would induce sellers to deviate. Hence, we need to check that (A.6) holds for prices slightly lower than p_h. From low quality sellers ICC, the LHS of (A.6) is nondecreasing in p. Thus, it is sufficient to check whether (A.6) holds for $p = p_h$:

$$\begin{aligned} p_h[J(p_l)[p_l - v(l)] - J(p_h)[p_h - v(h)]] \leq \\ v(h)J(p_l)[p_l - v(l)] - v(l)J(p_h)[p_h - v(h)] \end{aligned} \tag{A.8}$$

Inspection of (A.8) and (A.7) shows that $\xi = 0$ unless $p_h = v(h)$. □

Proof of lemma 3

Assume $1 - \lambda \geq \theta$ and consider a SE of type I, i.e. both $J(p_h)$ and $J(p_l)$ are greater than zero. If $1 - \lambda \geq \theta$ there is at least a low quality seller for every buyer. Hence, $J(p_h) > 0$ implies $J(p_l) < 1$ (see equation 1). However, from condition **R5** (ii), if $p_l > v(l)$, low quality sellers could sell with probability one by deviating to a slightly lower price. Hence, $p_l = v(l)$. The ICC of type l sellers then implies $J(p_h) = 0$, i.e. the high quality is never traded in equilibrium, so there is no SE of type I.

Let us now consider the case $1 - \lambda < \theta$. From condition **R5** (ii), it is clear that, if $J(p_l) < 1$, then $p_l = v(l)$. However, from type l ICC, this would imply $J(p_h) = 0$. From equation (1) and $1 - \lambda < \theta$, it follows that $K < 1$. But then, condition **R5** (iii) ensures that it is possible to sell with probability one at p_l. Hence, $J(p_l)$ must be one. We first show that in any robust SE with $p_q < u(q)\ \forall q \in \{h, l\}$, $J(p_h) > 0$ must hold [*Step 1*]; i.e. SE with $p_q < u(q)$ are of type I. Then, using lemma 2, we prove that $J(p_h) > 0$ also holds in SE where, for some $q \in \{l, h\}$, $p_q = u(q)$, i.e. these SE are also of type I [*Step 2*].

Step 1. Given $J(p_l) = 1$, equation (1) implies

$$\lambda J(p_h) = \frac{\theta K - (1 - \lambda)}{\lambda}. \tag{A.9}$$

Notice that, from **R5** (iii), $p_l < u(l)$ can only occur if $K = 1$. (Otherwise type l sellers would rise the price). Substituting for $K = 1$ into (A.9) shows that the RHS of (A.9) is strictly greater than zero for $\theta > (1 - \lambda)$.

Step 2. Assume first $p_l = u(l)$. Then, if $p_h > v(h)$, the ICC of low quality sellers must hold with equality. But then $J(p_h) > 0$ must be satisfied since $J(p_l)$ is positive in any equilibrium. Condition **R5** (ii) then implies that p_h equals $u(h)$. (For $p_h < u(h)$, $J(p_h)$ would be one and the ICC of type l would be violated.) For analogous reasons,

$p_h = v(h)$ can never be a SE if $p_l = u(l)$ as buyers would always prefer p_h. Assume now $p_h = u(h)$. If $J(p_h)$ were zero, lemma 2 would ensure either $p_l = v(l)$ or $J(p_l) = 0$. Clearly, the second never holds. As for $p_l = v(l)$, it is never an equilibrium if low quality sellers are the short side of the market $(1-\lambda < \theta)$. From (1), $J(p_h) = 0$ would imply $K < 1$. From condition **R5** (iii), any type l seller could deviate and announce a price slightly higher than p_l and still be able to make a sale with probability one. Therefore, $J(p_h) > 0$ must hold. $\square$

Proof of proposition 1

Lemmata 1 and 3 imply that $1-\lambda < \theta$ is necessary for a robust type I equilibrium. From lemma 1, robust equilibria of type I are separating. From lemma 3, there is no SE of type I if $1-\lambda \geq \theta$. The following characterization of D1-robust equilibria of type I shows that $1-\lambda < \theta$ is also sufficient. When $1-\lambda < \theta$, the following statements must be true in any robust SE of type I:

Statement 1. $J(p_l)$ is equal to one. This has already been established in the proof of lemma 2.

Statement 2. $p_l = u(l)$ and $p_h = u(h)$ hold whenever $K < 1$. If $K < 1$ and $p_l < u(l)$, then, from **R5** (iii), a type l seller could profit from slightly raising his price. Given $p_l = u(l)$, $p_h = u(h)$ follows from **R5** (ii).

Statement 3. $J(p_h)$ satisfies

$$J(p_h) = \frac{\theta K - (1-\lambda)}{\lambda}. \tag{A.10}$$

The result follows from *statement 1* and equation (1).

Statement 4.p_l and p_h are such that

$$u(l) - p_l \geq u(h) - p_h \tag{A.11}$$

Otherwise, $J(p_h)$ would be one – see **R5** (ii) – and type l ICC would be violated.

Endowed with these results, we turn to the particular cases:

Case 1: $\theta > \hat{\theta}$. Buyers can make positive surplus only if $K = 1$ (*statement 2*). Assume then $K = 1$. From *statement 3*,

$$J(p_h) = \frac{\theta - (1-\lambda)}{\lambda}. \tag{A.12}$$

Using (A.11), and type l sellers ICC one obtains

$$J(p_h) \leq \frac{p_l - v(l)}{u(h) - v(l) - (u(l) - p_l)}, \tag{A.13}$$

which implies $J(p_h) \leq \delta$ for $p_l \leq u(l)$. By combining equation (A.12) with $J(p_h) \leq \delta$, one would obtain $\theta - (1-\lambda) \leq \lambda\delta$ or $\theta \leq \hat{\theta}$, which would be a contradiction. Thus, $K < 1$ and buyers make zero surplus. It follows that $p_l = u(l)$ and $p_h = u(h)$ must hold and lemma 2 implies $J(p_h) = \delta$.

Case 2: $\theta_\gamma < \theta \leq \hat{\theta}$. Assume $K < 1$. Then $p_q = u(q)\ \forall q$ follows from *statement 2*. Lemma 2 implies $J(p_h) = \delta$. But then *statement 3* requires:

$$\theta K - \lambda\delta \geq 1 - \lambda \tag{A.14}$$

which would contradict $\theta \leq \hat{\theta}$. Therefore, $K = 1$. We now show that $p_q = u(q)$ for all q is impossible unless θ is exactly equal to $\hat{\theta}$. Consider first the case $\theta = \hat{\theta}$. If $K = 1$, (A.12) must hold. Substituting $\theta = \hat{\theta}$ in equation (A.12) yields $J(p_h) = \delta$. Substituting p_l from type l ICC into (A.11) – *statement 4* – shows that $p_h \geq u(h)$. Thus, buyers must make zero surplus ($p_q = u(q)$). This in turn requires $J(p_h) = \delta$ from lemma 2 and type l ICC.

Consider now the case $\theta < \hat{\theta}$. If $p_q = u(q)$, then the ICC of type l holding with equality (lemma 2) implies $J(p_h) = \delta$. However, one can use (A.12) to show that this would contradict $\theta < \hat{\theta}$. Thus, $p_q < u(q)$, $\forall q$. Given $K = 1$, (A.11) must hold with equality. Otherwise type l sellers could slightly raise their prices and still be able to sell with probability one (**R5** (ii)). If $p_h > v(h)$, then prices can be found by replacing (A.12) into the following equations:

$$p_h = \frac{u(h) - [u(l) - v(l) + v(l)J(p_h)]}{1 - J(p_h)} \tag{A.15}$$

$$p_l = \frac{v(l) + J(p_h)[u(h) - v(l) - u(l)]}{1 - J(p_h)}. \tag{A.16}$$

These follow from (A.11) and type l ICC both holding with equality (lemma 2). Simple algebra shows that they do not violate any participation constraint when $J(p_h)$ is given by (A.12) and $\theta_\gamma < \theta \leq \hat{\theta}$. What remains to show is that $p_h > v(h)$ for $\theta > \theta_\gamma$. Assume $p_h = v(h)$, then, given (A.11) holding with equality, $p_l = u(l) - GFT_h$. It is then immediate to check that type l sellers ICC and (A.12) would imply $\theta \leq \theta_\gamma$ which is a contradiction.

Case 3: $\theta \leq \theta_\gamma$. The same arguments as in case 2 can be used to claim that $K = 1$ and to rule out $p_q = u(q)$. The only difference here is that $p_h = v(h)$ must hold. To see this, assume $p_h > v(h)$. Equation (A.11) holding with equality implies $p_l > u(l) - GFT_h$. At the same time equation (A.12) and lemma 2 imply that type l ICC can be written as:

$$p_l = v(l) + \frac{\theta - (1 - \lambda)}{\lambda}[v(h) - v(l)] \tag{A.17}$$

Substituting this expression for p_l into $p_l > u(l) - GFT_h$ and solving for θ yields $\theta > \theta_\gamma$ which is a contradiction. Therefore, $p_h = v(h)$, which implies $p_l = u(l) - [u(h) - v(h)]$. *Statement 3* implies that $J(p_h)$ is given by (A.12).

To complete the characterization, note that, by construction, the equilibrium outcome is sustained by robust off-equilibrium beliefs. Therefore, $1 - \lambda < \theta$ is sufficient for a D1-robust equilibrium. $\square$

Proof of proposition 2

We start by showing that all trade occurs at a unique price p^* in all type II equilibria. Then, we characterize the equilibrium outcomes of all type II equilibria in terms of the price at which trade occurs and analyze beliefs that support the existence of these equilibria. Next, we show that equilibria of type II always fail D1 if $1 - \lambda < \theta$ and that there is an equilibrium outcome that passes D1 if $1 - \lambda > \theta$. The special case $1 - \lambda = \theta$ is then considered. Finally, we characterize the amount of trade.

a. Uniqueness of price. Suppose that trade occurs at more than one price. We prove the result for the case of two different equilibrium prices p' and p'' with $p'' > p'$.

The same argument applies to any number of prices higher than one. Clearly, p', p'' could be a pair of equilibrium prices if and only if profits for type l sellers were the same at the two prices, which would imply that $J(p'') < J(p')$. It follows that $J(p'')$ must be lower than 1. But then, any seller announcing p'' would profit from deviating to $p'' - \epsilon$, where ϵ is greater than zero but sufficiently small. From **R5**(ii), he could sell with probability one at $p'' - \epsilon$ since off-equilibrium beliefs cannot assign him a quality lower than l. Therefore, the only possible equilibrium outcome implies a single price p^*.

b. Equilibrium price. Notice first that in every type II equilibrium p^* necessarily lies in the interval $[v(l), u(l)]$. If $(1-\lambda) > \theta$, sellers of type l are the long side of the market, and $J(p^*) = \theta K/(1-\lambda) < 1$. In this case, no price p^* greater than $v(l)$ can emerge in equilibrium. If $p^* > v(l)$ sellers would profit from slightly reducing their price and sell with probability one (**R5**(ii)). As a result, $p^* = v(l)$ must hold. Clearly, type h sellers must announce a price such that buyers prefer p^*.

Consider now the case $(1-\lambda) < \theta$. We start by showing that $p^* = u(l)$ and $J(p^*) = 1$. If $p^* < u(l)$, buyers make positive surplus and, therefore, are all willing to buy at p^*. Given $1-\lambda < \theta$, K must be less than one. But then (**R5**(iii)) implies that type l sellers could announce a slightly higher price and be able to sell with probability one. Hence, $p^* = u(l)$. At this price, type l sellers must have no incentive to cut their prices. Hence, $J(p^*) = 1$ must hold. Again, from **R5**(ii), type h sellers must announce a price $p_h \geq u(h)$.

Finally, in both cases, there must be off-equilibrium beliefs such that no one has any incentive to deviate. It is easy to verify that these beliefs exist. For instance, beliefs assigning $\sigma^(h|p) = 0$ to any seller announcing an off-equilibrium price p sustain the equilibrium. The next step shows that beliefs which sustain equilibria of type II are robust if and only if $1-\lambda \geq \theta$.

c. Robustness. It is immediate to show that type II equilibria satisfy D1 when $1-\lambda > \theta$. Both the RHS and the LHS of (8) are zero for all p. By contrast, type II equilibria where $1-\lambda < \theta$ always fail D1. In equilibrium, low quality sellers make profits $\pi^*(l) = u(l) - v(l)$ while high quality sellers make zero profits. Condition (8) becomes, for any $p > v(h)$:

$$\frac{u(l) - v(l)}{p - v(l)} \leq 0, \tag{A.18}$$

which does not hold for any $p > v(h) > v(l)$. Since buyers make zero surplus, $J(p) = 1$ for all $p > v(h)$ such that $u(h) - p > 0$. Hence, any deviation $p \in (v(h), u(h))$ would destabilize the candidate equilibrium.

d. Case $1-\lambda = \theta$. Notice that also in this case the equilibrium is characterized by a unique price $p^* \in [v(l), u(l)]$. For a deviation $p > v(h)$, condition (8) becomes

$$\frac{p^* - v(l)}{p - v(l)} \leq 0. \tag{A.19}$$

From **R5**(ii), $J(p) = 1$ for all p such that

$$u(h) - p > u(l) - p^*. \tag{A.20}$$

In this special case, it is necessary to distinguish between $\gamma \leq 0$ and $\gamma > 0$, as in proposition 1. Consider first the case $\gamma \leq 0$ (which implies $u(l) - v(l) \leq u(h) - v(h)$). The only D1 robust equilibrium of type II is such that $p^* = v(l)$. To show this, notice that condition (A.19) is violated whenever $p^* > v(l)$. Given $\gamma \leq 0$, for any

$p^* > v(l)$ it is possible to find a deviation $p > v(h)$ such that (A.20) is satisfied. Hence, $p^* = v(l)$ is the only possible equilibrium price. Consider now the case $\gamma > 0$. Condition (A.19) is still violated whenever $p^* > v(l)$. However, whether there is any $p > v(h)$ such that (A.20) holds now depends on p^*. A deviation $p > v(h)$ such that (A.20) holds is possible only if $p^* > u(l) - [u(h) - v(h)]$. Hence, robustness to D1 requires $p^* \leq u(l) - [u(h) - v(h)]$. In principle, any $p^* \in [v(l), u(l) - [u(h) - v(h)]$ can be an equilibrium price when $\gamma > 0$. The reason why D1 does not permit to select a price in this case is that, when $\gamma > 0$, a discontinuity arises at $\theta = 1 - \lambda$. To see this, consider the limit of a robust type I equilibrium for $\theta \to 1 - \lambda$. When $\gamma > 0$, the right limit of the expression for p_l given in proposition 1 selects $p^* = u(l) - [u(h) - v(h)]$. On the other hand, the left limit for $\theta \to 1 - \lambda$ of the price of a type II robust equilibrium selects $p^* = v(l)$.

e. Fraction of quality l traded. It is immediate to check that $K = 1$ whenever $1 - \lambda \geq \theta$. Therefore, each buyer is able to obtain a unit of a quality l good. It follows that the fraction of quality l goods traded is $\frac{\theta}{1-\lambda}$. □

Proof of proposition 3

Case 1. $1 - \lambda > \theta$. From Proposition 2, there exist D1-robust equilibria of type II. In all of these equilibria, the price, p^*, at which trade occurs and the fraction of quality traded, $f(l)$, are uniquely determined by the model's exogenous parameters. Moreover, according to lemma 3 no equilibrium of type I exists. Hence, we conclude that for $1 - \lambda > \theta$ the equilibrium outcome is unique.
Case 2. $1 - \lambda < \theta$. From Proposition 2 there is no D1-robust equilibria of type II. On the other hand, according to proposition 1 there exist D1-robust equilibria of type I. Again, in all these equilibria, the prices at which trade occurs, p_q, and the fractions of high and low quality traded ($f(h)$ and $f(l)$ respectively) are uniquely determined by parameters. Thus, the equilibrium outcome is unique.
For completeness, we discuss the special case $1 - \lambda = \theta$. The proof of proposition 2 implies that, if $\gamma \leq 0$, the results under case 1 also apply to $1 - \lambda = \theta$. If $\gamma > 0$, a discontinuity arises at $1 - \lambda = \theta$. While traded quantities are still uniquely determined, the price p^* experiences a jump from $v(l)$ to $u(l) - [u(h) - v(h)]$. □

Proof of Lemma 4

Suppose that a pooling occurs at price $p^* \geq v(h)$ and consider a deviation to a higher price $p' > p^*$. A type l would weakly benefit from deviating to p' if

$$J(p', l)[p' - v(l)] \geq J(p^*, l)[p^* - v(l)] \tag{A.21}$$

where the LHS of the expression is the expected profit from the deviation and the RHS is the expected equilibrium payoff for type l. A type h seller would strictly benefit if

$$J(p', h)[p' - v(h)] > J(p^*, h)[p^* - v(h)] \tag{A.22}$$

Assume now that, for all price-induced beliefs, $J(p, h)/J(p, l)$ is non-decreasing in p. Given (A.21), it is immediate to check that

$$\frac{p' - v(h)}{p^* - v(h)} > \frac{p' - v(l)}{p^* - v(l)} \geq \frac{J(p^*, l)}{J(p', l)} \geq \frac{J(p^*, h)}{J(p', h)} \tag{A.23}$$

where the first inequality comes from $p' > p^*$ and $v(h) > v(l)$, the second inequality comes from (A.21) and the third comes from the monotonicity condition on $J(p,h)/J(p,l)$. The above result thus implies that (A.22) is satisfied whenever (A.21) is satisfied. In words, type h strictly benefits from the deviation whenever type l weakly benefits. According to D1, buyers should then assign probability zero to type l when observing a deviation to any $p' > p^*$. Notice that, so long as the signal x does not allow to perfectly assess the quality of the good, buyers always attach a strictly positive probability to type l when the price is p^*. As a result, a deviation to a price slightly above p^* would produce a discrete jump in the expected quality. From **R5** (ii), the probability to sell at p' must be one. This implies that sellers have incentive to deviate to higher prices when there is a pooling or a hybrid equilibrium. Hence, any pooling or hybrid equilibrium would fail D1. □

Extension to any finite number of qualities

This section generalizes the results concerning the set of equilibria robust to D1 to the case of a finite number $(N+1)$ of qualities. We show that, as in the case of two qualities, D1 guarantees separation at all prices at which trade occurs. The comparative statics for the general case are also consistent with the results obtained in the two qualities case. When θ is so low that buyers are relatively more numerous than sellers of the lowest quality, no quality other than the lowest is traded. Increases in the value of θ allow higher qualities to be traded until, for θ sufficiently large, all qualities are traded.

Qualities are indexed by $q = 0, ..., N$. Each seller's quality is drawn from a distribution $\lambda : \{0, 1, ..., N\} \to [0, 1]$, where λ_q, $\sum_{q=0}^{N} \lambda_q = 1$, denotes the probability associated with quality q. Buyers' posterior beliefs are denoted with $\sigma(q|p_s^0)$, $\sum_{q=0}^{N} \sigma(q|p_s^0) = 1$. We maintain the convention that agents who choose not to trade announce $\overline{p} > u(N)$ (sellers) or select $p = 0$ (buyers).

Let us concentrate first on pooling and hybrid equilibria in which two or more types of sellers trade at the same price. In order to assess the robustness of these equilibria, we need to state the equivalent of condition (8). In any of these equilibria there always exists a price p^* at which a non-singleton non-empty set of qualities, $M \subseteq \{0, ..., N\}$, is traded. Let q_M be the highest quality in M. Take any quality $q \in M$, $q \neq q_M$. Given pooling at p^*, condition (8) becomes:

$$\frac{p^* - v(q)}{p - v(q)} \leq \frac{p^* - v(q_M)}{p - v(q_M)}, \tag{A.24}$$

which, given $v(q) < v(q_M)$, is violated for any $p > p^*$. Thus, robust beliefs should assign probability 0 to a deviation $p > p^*$ by any quality in M except for q_M. As for qualities $q \notin M$, the following applies. Sellers of qualities $q < q_M$ who do not trade at p^* make at least the same profits they would make at p^* by charging a different price (since they could always announce p^*). Since $v(q) < v(q_M)$, they should be assigned probability zero. Sellers of qualities $q > q_M$ should also be assigned probability zero so long as $p < v(q_M+1)$. Thus, deviations $p^* < p < v(q_M+1)$ are attributed to sellers of quality q_M with probability 1. As for buyers, the probability to sell at p is one so long as:

$$u(q_M) - p > \sum_{q \in M} \sigma(q|p^*)[u(q) - p^*], \tag{A.25}$$

which is always true for p close enough to p^*. Thus deviating to a price slightly higher than p^* would allow sellers of type q_M to reveal their type and induce buyers to buy.

Therefore, neither pooling nor hybrid equilibria in which two or more types trade at the same price survive D1.

The set of robust equilibria therefore includes only separating equilibria and hybrid equilibria with the necessary condition that each price at which trade occurs is announced by only one type of seller (i.e. pooling only occurs at $\overline{p}$). We now focus on these equilibria. If $\theta \leq \lambda_0$, the discussion made in the previous sections leads to the immediate conclusion that only quality 0 is traded.

By converse, when $\theta > \lambda_0$, sellers of quality 0 make positive profits and, therefore, higher qualities must be traded. In order to characterize these equilibria, we analyze the properties of the ICC. We start by showing that when the "adjacent upward" ICC is satisfied, all the ICC with respect to all higher qualities are satisfied. The relevant ICC for sellers is:

$$J(p_{q-s})[p_{q-s} - v(q-s)] \geq J(p_q)[p_q - v(q-s)] \tag{A.26}$$

for all q and $s = 0, ...q$. It is immediate to check that $J(p_0) = 1$ holds. This, together with (A.26) yields $J(p_q) < J(p_{q-1}) < ... < J(p_1) < 1$, whenever $p_q > p_{q-1} > ... > p_0$. From equation (A.26):

$$J(p_{q-1})[p_{q-1} - v(q-1)] \geq J(p_q)[p_q - v(q-1)] \tag{A.27}$$

and

$$J(p_q)[p_q - v(q)] \geq J(p_{q+1})[p_{q+1} - v(q)]. \tag{A.28}$$

Then, by using $v(q) > v(q-1)$ and $J(p_q) > J(p_{q+1})$, it follows that

$$J(p_{q-1})[p_{q-1} - v(q-1)] \geq J(p_{q+1})[p_{q+1} - v(q-1)] \tag{A.29}$$

always holds. Applying the same reasoning to qualities higher than $q+1$ shows that when the "adjacent upward" ICC are satisfied, all the ICC with respect to all higher qualities are satisfied.

The next step is to show, by applying D1, that the "adjacent upward" ICC of a given quality must hold with equality whenever the "adjacent upward" quality is traded. It is immediate to check that **R5** (ii) ensures that buyers' surplus is constant across all quantities that are traded,

$$u(q) - p_q = k \ \forall q = 0, 1, ..., N, \tag{A.30}$$

where k is a constant.

For $q > 0$, buyers may only be attracted through deviations $p < p_q$. Hence, we restrict attention to deviations $p < p_q$. Notice that D1 requires that type q should be assigned probability zero of deviating to price p, if there exists q' such that:

$$\frac{J(p_q)[p_q - v(q)]}{p - v(q)} > \frac{J(p_{q'})[p_{q'} - v(q')]}{p - v(q')} \tag{A.31}$$

This is the equivalent of condition (8). The next lemma generalizes lemma 2.

Lemma 5. *For all $q = 1, ..., N$, in any robust equilibrium in which quality q is traded, the "adjacent upward" ICC of sellers of quality $q-1$ holds with equality unless $p_q = v(q)$.*

Proof. Consider a deviation $p \in (v(q), p_q)$. Notice that buyers are willing to buy at p if they think that the deviation comes from type q, since $p < p_q$. We argue that whenever it is possible to delete type $q-1$ from the deviation it is also possible to delete all types $q-s,\ s \geq 2$. To show this point, assume that type $q-1$ can be eliminated:

$$\frac{J(p_{q-1})[p_{q-1} - v(q-1)]}{p - v(q-1)} > \frac{J(p_q)[p_q - v(q)]}{p - v(q)}. \tag{A.32}$$

Consider now type $q-s$. From the incentive compatibility condition:

$$\frac{J(p_{q-s})[p_{q-s} - v(q-s)]}{p - v(q-s)} \geq \frac{J(p_{q-1})[p_{q-1} - v(q-s)]}{p - v(q-s)}. \tag{A.33}$$

But then, for any $p > v(q) > p_{q-1}$, the following relationship

$$\frac{J(p_{q-s})[p_{q-s} - v(q-s)]}{p - v(q-s)} > \frac{J(p_{q-1})[p_{q-1} - v(q-1)]}{p - v(q-1)} \tag{A.34}$$

holds, which implies that type $q-s$ can be deleted, whenever type $q-1$ can be deleted. Since $p < p_q < u(q) < v(q+1)$, sellers of type higher than q are never willing to deviate to p. This implies that if type $q-1$ can be deleted, beliefs should be that the deviation comes from q. As in the two-quality case, we show that whenever $p_q > v(q)$, a viable deviation $p \in (v(q), p_q)$, for which type $q-1$ can be deleted, exists so long as the incentive compatibility condition of type $q-1$ holds with inequality. Suppose then that the ICC holds with strict inequality. Assume that the deviation consists in a price $p = p_q - \epsilon,\ \epsilon > 0$ which is a small undercutting of price p_q. We want to show that there exists $\epsilon > 0$ such that:

$$\frac{J(p_{q-1})[p_{q-1} - v(q-1)]}{p_q - \epsilon - v(q-1)} > \frac{J(p_q)[p_q - v(q)]}{p_q - \epsilon - v(q)}. \tag{A.35}$$

Condition (A.35) can be rewritten as:

$$\frac{J(p_{q-1})[p_{q-1} - v(q-1)]}{J(p_q)[p_q - v(q-1)]} > \frac{p_q - v(q)}{p_q - v(q-1)} \frac{p_q - \epsilon - v(q-1)}{p_q - \epsilon - v(q)}. \tag{A.36}$$

For $p_q > v(q)$, the LHS (which does not depend on ϵ) is strictly greater than 1 whenever the ICC of type $q-1$ holds with strict inequality. On the other hand, the RHS goes to 1 as ϵ becomes small. Thus, there always exists ϵ such that type $q-1$ can be deleted unless either the ICC holds with equality or $p_q = v(q)$. In fact, in the case $p_q = v(q)$, undercutting is never profitable for type q. Hence, either the ICC holds with equality or $p_q = v(q)$. □

We are now ready to characterize the robust equilibria. We distinguish between the case in which all qualities are traded and the case in which a subset of qualities is traded.

a) All $N+1$ qualities traded:
If all qualities are traded, sellers' ICC ensure that $p_q > v(q)$ for all qualities except, possibly, quality N. Thus, for $q < N$, sellers' ICC and lemma 5 imply:

$$J(p_q) = J(p_{q-1}) \frac{u(q-1) - v(q-1) - k}{u(q) - v(q-1) - k}. \tag{A.37}$$

Unless quality N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, $p_N > v(N)$ must hold and (A.37) holds with equality for quality N as well. In the special case in which N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, $J(p_N)$ is between 0 and the value implied by (A.37).

Using the initial condition $J(p_0) = 1$, equation (A.37) yields:

$$J(p_q) = \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - k}{u(q-i) - v(q-i-1) - k}. \tag{A.38}$$

Since $k \geq 0$, the maximum value of $J(p_q)$ is achieved when $k = 0$. Thus, the probability to trade at p_q, for $q > 0$, is bounded above by:

$$\overline{J}_q = \prod_{i=0}^{q-1} \delta_{q-i}. \tag{A.39}$$

where:

$$\delta_q \equiv \frac{u(q-1) - v(q-1)}{u(q) - v(q-1)}. \tag{A.40}$$

Notice also that $k = 0$ implies $p_q = u(q)$ $\forall q = 0, ..., N$ (i.e. buyers make zero surplus). Assume $k > 0$, so that buyers make positive surplus at all prices. Then, all buyers want to trade. Clearly, $k > 0$ can only emerge if $K = 1$ – otherwise sellers of quality $q = 0$ could always profit from raising their price. Hence, the requirement that the number of goods sold is equal to the number of goods bought (1) can be restated as

$$\sum_{q=0}^{N} \lambda_q J(p_q) = \theta, \tag{A.41}$$

where $J(p_q)$ is given by (A.38) for $q < N$. $J(p_N)$ is also given by (A.38) unless N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, in which case $J(p_N)$ is between zero and the value implied by (A.38). For the case in which all qualities are traded, finding an equilibrium outcome is equivalent to finding a value k^* for which (A.41) holds. From (A.38), the LHS of equation (A.41) is monotonically decreasing in k, for $k \in [0, \hat{k}]$, where $\hat{k} \equiv \min_{q\in\{0,...,N\}} u(q) - v(q)$. We note that simultaneous satisfaction of all the participation constraints requires that k always lie in the interval $[0, \hat{k}]$. Given monotonicity, there always exists at most one value k^* in the above interval. This also implies that if there exists an equilibrium, its outcome must be unique. As for existence, consider the following. From (A.38), the LHS of expression (A.41) reaches its maximum in the relevant interval when $k = 0$ and its minimum when $k = \hat{k}$. Therefore, necessary and sufficient conditions for an interior solution are

$$\theta < \lambda_0 + \sum_{q=1}^{N} \lambda_q \prod_{i=0}^{q-1} \delta_{q-i} \tag{A.42}$$

and

$$\theta > \lambda_0 + \sum_{q=1}^{N-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - \hat{k}}{u(q-i) - v(q-i-1) - \hat{k}} + \lambda_N J_N^{min}. \tag{A.43}$$

where J_N^{min} is given by

$$J_N^{min} = \begin{cases} \prod_{i=0}^{N-1} \frac{u(N-i-1)-v(N-i-1)-\hat{k}}{u(N-i)-v(N-i-1)-\hat{k}} & \text{if } \arg\min_{q\in\{1,...,N\}} u(q)-v(q) \neq \{N\} \\ 0 & \text{if } \arg\min_{q\in\{1,...,N\}} u(q)-v(q) = \{N\} \end{cases} \tag{A.44}$$

Otherwise, if one of the above conditions is not satisfied, the robust equilibrium takes a different form. If condition (A.42) is not satisfied, then k must be equal to zero and the equilibrium is characterized by $p_q = u(q)$ for all $q = 0, 1, ..., N$ and probabilities $\overline{J}_q$ given by (A.39). As $\overline{J}_q > 0$ $\forall q = 0, ..., N$, all qualities are traded also in this case. Thus, a sufficient condition for all qualities being traded is that θ is high enough to ensure that (A.43) holds. Below, we show that this condition is also necessary. In order to gather intuitions on condition (A.43), notice that it is always satisfied when the gains from trade are nondecreasing in the quality since, in this case, $\hat{k} = u(0) - v(0)$ and the RHS is equal to λ_0. Therefore, as long as $\lambda_0 < \theta$, an internal solution in which all qualities are traded must exist.

b) More than one and less than $N+1$ qualities are traded:
Assume now that θ is relatively small so that condition (A.43) is not satisfied. If the value of the RHS of equation (A.43) for $k = \hat{k}$ is greater than or equal to θ, then some qualities are not traded. To see this, note that k cannot exceed $\hat{k}$. Let $\underline{J}_q$ be the probability to sell at p_q when k equals $\hat{k}$. Let also $\hat{q}$ be the lowest quality in $\arg\min_{q\in\{0,...,N\}} u(q) - v(q)$. Then, by definition, $u(\hat{q}) - v(\hat{q}) - \hat{k} = 0$ so that, from equation (A.38), $\underline{J}_q$ is zero for all $q > \hat{q}$. Thus, when $k = \hat{k}$, all qualities above the quality which provides the lowest gain from trade are not traded. Buyers' surplus k should increase, however it fails to increase because, by increasing, it would violate the participation constraint of sellers of type $\hat{q}$. Hence, the incentive compatibility for type $\hat{q}$ requires that no higher quality is traded. Thus, (A.43) is a necessary condition for all qualities being traded.

What are the characteristics of the equilibrium when (A.43) is violated? Note that if (A.43) is not satisfied we have:

$$\sum_{q=0}^{\hat{q}} \lambda_q \underline{J}_q = \sum_{q=0}^{N} \lambda_q \underline{J}_q > \theta, \tag{A.45}$$

where the equality comes from the fact that all qualities higher than $\hat{q}$ are not traded. This suggests that even if only $\hat{q}+1$ qualities are traded out of $N+1$, the quantity sold is still higher than the quantity bought. Then, quality $\hat{q}$ cannot be traded in equilibrium. Let us assume that quality $\hat{q} - 1$ is traded. It follows that its price, $p_{\hat{q}-1}$, must be compatible with D1. In other words, there must be no incentive to deviate for sellers of type $\hat{q}$ or above. Therefore, $p_{\hat{q}-1}$ must be such that $k_1 \equiv u(\hat{q}-1) - p_{\hat{q}-1} \geq u(\hat{q}) - v(\hat{q})$. If so, there is no price sellers of type $\hat{q}$ could possibly announce to attract buyers and still make no loss. Of course, buyers incentive compatibility implies $u(q) - p_q = k_1$ for all qualities that are traded, i.e. $q = 0, ..., \hat{q} - 1$. Now, let $\tilde{q}$ be the lowest quality in $\arg\min_{q\in\{0,...,\hat{q}-1\}} u(q) - v(q)$, i.e. the lowest quality among those which give minimum gain from trade when attention is restricted to qualities lower than $\hat{q}$. It is clear that k_1 should now satisfy $u(\hat{q}) - v(\hat{q}) \leq k_1 \leq u(\tilde{q}) - v(\tilde{q})$. Therefore, all that remains to be checked is whether there exists k_1^* such that:

$$\sum_{q=0}^{\hat{q}-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - k_1^*}{u(q-i) - v(q-i-1) - k_1^*} = \theta. \tag{A.46}$$

If it does, then the equilibrium is such that qualities $q = 0, ..., \hat{q} - 1$ are traded. If it does not, then all the process starts again by choosing $\tilde{q}$ as the first quality that is not traded. It should be noted that, since we are assuming $\theta > \lambda_0$, the process eventually leads to an equilibrium in which more than one quality is traded. In fact, as long as $\theta > \lambda_0$, qualities 0 and 1 are always traded. By iterating this process, one can show that the number of qualities traded in equilibrium decreases with θ. Thus, price dispersion increases as θ increases.

The extension to $N + 1$ qualities generalizes the result of an inverse relationship between price dispersion and the degree of competition (as measured by θ) derived for the case of two qualities. When competition among sellers is so strong that $\theta \leq \lambda_0$, only the lowest quality is traded and there is no price dispersion. When competition is weak (θ satisfies condition (A.43)) all qualities are traded and price dispersion is maximized. For intermediate values of θ such that $\theta > \lambda_0$ while (A.43) is not satisfied, the number of qualities which are traded and the degree of price dispersion (weakly) increase with θ.

References

[1] Adriani, F., and L. G. Deidda, 2006. Competition and the Signaling Role of Prices. DeFiMS discussion paper.

[2] Adriani, F., and L. G. Deidda, 2009. Price Signaling and the Strategic Benefits of Price Rigidities. Games and Economic Behavior, forthcoming.

[3] Akerlof, G. A. 1970. The Market for "Lemons": Quality Uncertainty and the Market Mechanism. Quarterly Journal of Economics. 84: 488-500.

[4] Bagwell, K. 1991. Optimal Export Policy for a New-Product Monopoly. American Economic Review. 81: 1156-1169.

[5] Bagwell, K. and M.H. Riordan 1991. High and Declining Prices Signal Product Quality. American Economic Review. 81: 224-239.

[6] Banks, J., and J. Sobel, 1987. Equilibrium Selection in Signaling Games. Econometrica. 55: 647-662.

[7] Baye, M.R., J. Morgan, and P. Scholten, 2004. Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site. Journal of Industrial Economics. 52: 463-96.

[8] Bester, H., 1993. Bargaining versus Price Competition in Markets with Quality Uncertainty. American Economic Review. 83: 278-288.

[9] Cho, I. K., and D. M. Kreps, 1987. Signaling Games and Stable Equilibria. Quarterly Journal of Economics. 102: 179-221.

[10] Cho, I. K., and J. Sobel, 1990. Strategic Stability and Uniqueness in Signaling Games. Journal of Economic Theory. 50: 381-413.

[11] Clay K.,R. Krishnan, and E. Wolff, 2001. Prices and Price Dispersion on the Web: Evidence from the Online Book Industry. Journal of Industrial Economics. 49:521-539.

[12] Cooper R., and T.W. Ross, 1984. Prices, Product Qualities, and Asymmetric Information: The Competitive Case. Review of Economic Studies 51: 197-207.

[13] Daughety, A. F, and J. F. Reinganum, 2005. Imperfect Competition and Quality Signaling. Rand Journal of Economics. 39: 163-183.

[14] Daughety, A. F, and J. F. Reinganum, 2007. Competition and confidentiality: Signaling quality in a duopoly when there is universal private information. Games and Economic Behaviour 58: 94-120.

[15] Ellingsen, T. 1997. Price Signals Quality: The Case of Perfectly Inelastic Demand. International Journal of Industrial Organization. 16: 43-61.

[16] Fudenberg, D., and J. Tirole, 1991. Game Theory. MIT Press.

[17] Grossman, S. J., and M. Perry, 1986. Perfect Sequential Equilibrium. Journal of Economic Theory. 39: 97-119.

[18] Gul F., and A. Postlewaite, 1993. Asymptotic Efficiency in Large Exchange Economies with Asymmetric Information. Econometrica. 60: 1273-1292.

[19] Hellmann, T., and J. Stiglitz, 2000. Credit and equity rationing in markets with adverse selection. European Economic Review Volume 44: 281-304.

[20] Huck, S., Lnser, G.K., and J.-R. Tyran, 2007. Pricing and Trust. Mimeo.

[21] Janssen, M. C. W., and S. Roy, 2002. Dynamic trading in a Durable Good Market with Asymmetric Information. International Economic Review. 43: 257-282.

[22] Judd, K. L., 1985. The law of large numbers with a continuum of IID random variables. Journal of Economic theory. 35: 19-25.

[23] Kohlberg E., and J.-F. Mertens, 1986. On the Strategic Stability of Equilibria. Econometrica. 54: 1003-1038.

[24] Laffont, J.- J., and E. Maskin. 1987. Monopoly with Asymmetric Information about Quality. European Economic Review. 31: 483-489.

[25] Laffont, J.-J., and E. Maskin. 1989. Rational Expectations with Imperfect Information. Economics Letters. 30: 269-274.

[26] Lin, Y.C., and P. Scholten. 2005. Pricing Behaviors of Firms on the Internet. Evidence from Price Comparison Sites Cnet and Nextag. Mimeo.

[27] Mailath, G. J., Okuno-Fujiwara M., and A. Postlewaite. 1993. Belief-Based Refinements in Signaling Games. Journal of Economic Theory. 60: 241-276.

[28] Milgrom, P., and J. Roberts. 1986. Price and Advertising Signals of Product Quality. Journal of Political Economy. 94: 796-821.

[29] Moreno, D., and J. Wooders. 2007. Decentralized Trade Mitigates the Lemons Problem. Universidad Carlos III.

[30] Muthoo, A., and S. Mutuswami. 2005. Competition and Efficiency in Markets with Quality Uncertainty. Mimeo.

[31] Overgaard P. B., 1993. Price as a Signal of Quality: a Discussion of Equilibrium Concepts in Signalling Games. European Journal of Political Economy. 9: 483-504.

[32] Stiglitz, J. and A. Weiss, 1981. Credit Rationing in Markets with Imperfect Information. American Economic Review. 71: 393-410.

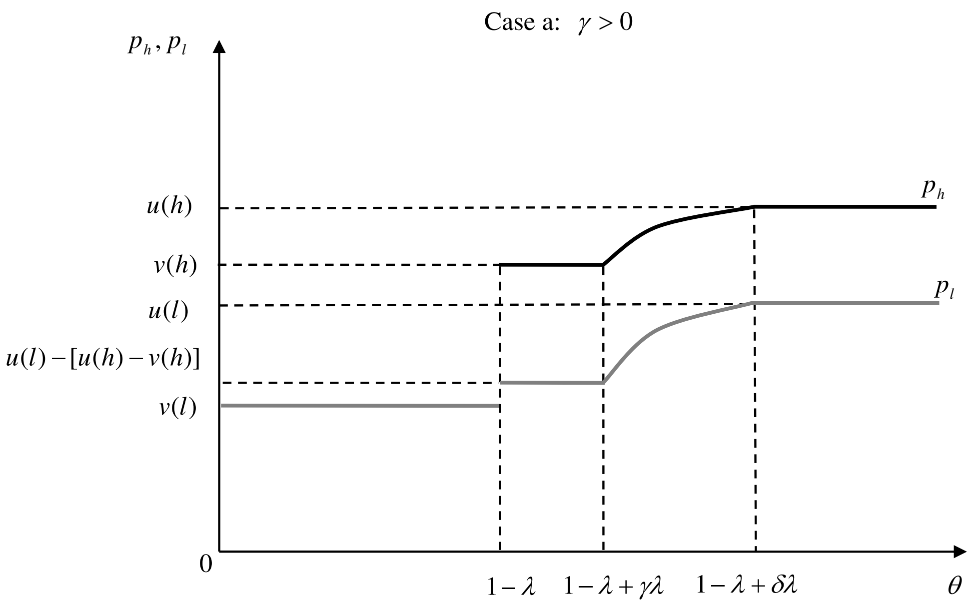
[33] Villeneuve, B., 2005. Competition Between Insurers with Superior Information. European Economic Review. 49: 321-340.

[34] Wilson, C., 1979. Equilibrium and Adverse Selection. American Economic Review. 69: 313-317.

[35] Wilson, C., 1980. The Nature of Equilibrium in Markets with Adverse Selection. Bell Journal of Economics. 11: 108-130.

[36] Wolinsky, A., 1983. Prices as Signals of Product Quality. Review of Economic Studies. 50: 647-658.

Figure 1: Equilibrium prices as a function of θ



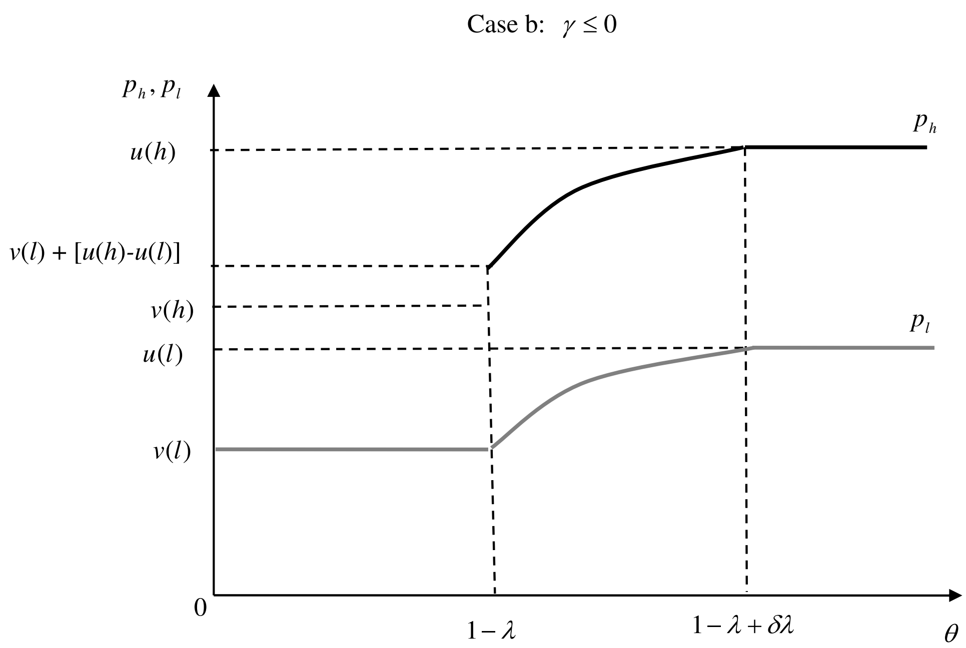


Figure 2: Quality of trade as a function of θ