Defense RD: Effects on Economic Growth and Social Welfare

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Abstract

In the US, defense R&D share of GDP has decreased significantly since 1960. To analyze the implications on economic growth and welfare, we develop an R&D-based growth model that features the commonly discussed *crowding-out* and *spillover* effects of defense R&D on civilian R&D. The model also captures the important effects of defense technology on (a) national security and (b) aggregate productivity via the spin-off effect resembling consumption public goods and productive public goods respectively. In this framework, economic growth is driven by market-based civilian R&D as in standard R&D growth models and government-financed public goods (i.e. defense R&D) as in Barro (1990). We find that defense R&D has an inverted-U-shape effect on growth, and the growth-maximizing level of defense R&D is increasing in the spillover effect and in the spin-off effect. Also, there is a welfare-maximizing level of defense R&D that is increasing in the security effect of national defense, and there exists a critical degree of this security effect below (above) which the welfare-maximizing level of defense R&D is below (above) the growth-maximizing level.

***Keywords*:** defense R&D, economic growth, public goods, social welfare

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1. Introduction

In the US, defense R&D as a percentage of GDP decreased from 1.29% in 1961 to 0.57% in 2008 (see Figure 1). This phenomenon has ignited economists' interest in the effects of defense R&D on economic growth. For example, a recent empirical study by Goel *et al.* (2008) finds an interesting result that defense R&D has a positive and significant effect on growth in the US. While it is valuable to analyze the effects of defense R&D on growth, it is also important to consider the effects on social welfare. After all, growth maximization may not be equivalent to welfare maximization. To explore this issue, we develop a growth-theoretic framework to derive the different channels through which defense R&D affects economic growth and welfare.

Specifically, we develop an R&D-based growth model that formalizes the commonly discussed *crowding-out* and *spillover* effects of defense R&D on civilian R&D.[1] The crowding-out effect refers to the case in which an increase in defense R&D reduces the factor inputs available for civilian R&D and hence has a negative effect on the growth of civilian technology. For example, Hartley (2006) notes that "[d]efence R&D has obvious opportunity costs through the use of scarce scientific personnel and assets that could be used on civilian research." Also, Gullec and van Pottelsberghe (2003) analyze a group of OECD countries and find that defense R&D indeed has a crowding-out effect on civilian R&D. The spillover effect refers to the case in which defense R&D contributes to the performance of civilian R&D and hence has a positive effect on growth.[2] For example, Chakrabarti and Anyanwu (1993) find that in the US, defense R&D has an indirect positive effect on the growth rate of civilian output through technological

[1] See, for example, Cowan and Foray (1995) and Dunne and Braddon (2008) for a discussion.

[2] For example, James (2004, p. 37-38) suggests that defense R&D can affect civilian R&D via the following ways. First, defense R&D spending is partly devoted to the training of graduate students in science and engineering, thereby providing a favorable circumstance to foster civilian R&D talent. Second, defense R&D spending provides seed investment in new technology companies, and thus, can be viewed as a crucial source of early-stage seed funding for civilian technology companies.

change using the number of patents as a proxy, and they argue that this empirical finding supports the presence of a spillover effect from defense R&D to the civilian economy.

In addition to the crowding-out and spillover effects of defense R&D, the growth model also features two important effects of defense technology (which is accumulated by investment in defense R&D). Firstly, higher defense technology improves national security and increases the utility of households resembling consumption public goods. For example, Hartley (2006) argues that "[d]efense R&D increases a nation's military capability so improving its national security through using technology (quality) rather than increasing the quantity of arms." Secondly, like productive public goods, defense technology improves aggregate productivity through the development of general-purpose technologies (GPTs) that have civil applications. This effect is referred to as the spin-off effect in the defense literature. For example, Ruttan (2006) argues that research in defense has played a very important role in the development of some major GPTs, such as (a) interchangeable parts and mass production, (b) military and commercial aircraft, (c) nuclear energy and electric power, (d) computers and semiconductors and (e) the internet.

Within this framework, growth is driven by market-based civilian R&D as in standard R&D growth models and government-financed public goods (i.e. defense R&D) as in Barro (1990). We find that a fall in defense R&D has an ambiguous effect on the growth of output and consumption. In particular, starting at a high (low) level of defense R&D, reducing defense R&D has a positive (negative) effect on growth. Therefore, there exists a growth-maximizing level of defense R&D that is increasing in the spillover effect and in the spin-off effect. There also exists a welfare-maximizing level of defense R&D that is increasing in the security effect of national defense, and there is a critical degree of this security effect below (above) which the welfare-maximizing level of defense R&D is below (above) the growth-maximizing level.

This study contributes to the literature on defense and economic growth by providing a tractable growth-theoretic framework that formalizes the commonly discussed crowding-out and spillover effects of defense R&D. To our knowledge, our study is the first attempt to model defense R&D within an innovation-driven growth model. Previous studies analyze the dynamic effects of defense spending either in an endowment economy or in a capital-accumulation-driven growth model.[3] For example, Shieh *et al.* (2002) perform a similar growth-welfare analysis on defense spending in an AK growth model and find that the welfare-maximizing level of defense spending is always above the growth-maximizing level. Comparing the different results from Shieh *et al.* (2002) and the present study shows that the welfare effects of defense spending through capital accumulation and innovation are quite different. In Shieh *et al.* (2002), defense spending also serves as a proxy for national security and productive public goods that have a positive effect on output. In our model, defense R&D has the additional spillover and crowding-out effects that are absent in Shieh *et al.* (2002).

Our study also relates to the literature on productive government spending and economic growth initiated by Barro (1990). While Barro (1990) models public inputs as a flow variable, our study follows the more realistic formulation in Futagami *et al.* (1993) to model public inputs as a stock variable.[4] The spin-off effect of defense technology gives rise to an extra channel of growth from productive public goods in addition to market-based civilian R&D. In addition to this spin-off effect, we also consider the role of defense technology as consumption public goods for enhancing national security as in Turnovsky (1996), Shieh *et al.* (2002) and others.

The theoretical implications of our study rationalize the previous empirical results that find an ambiguous growth effect of defense spending and R&D. In an early study, Benoit (1973)

[3] See, for example, Brito (1972), Deger and Sen (1983, 1984), Zou (1995), Chang *et al.* (1996, 2002), Shieh *et al.* (2002, 2007) and Aizenman and Glick (2006).
[4] See Irmen and Kuehnel (2008) for a recent survey on this literature.

finds that defense spending has a positive effect on growth in developing countries. However, upon surveying the follow-up studies, Ram (1995) concludes that defense spending has opposing effects on growth and the overall effect is ambiguous.[5] Similarly, Lichtenberg (1995) finds that defense R&D has opposing effects on growth and the net effect is ambiguous. In contrast, Goel *et al.* (2008) finds that defense R&D has a positive and significant effect on growth in the US, and surprisingly, this effect is even stronger than private industrial R&D.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and analyzes the properties of the balanced-growth path. Section 4 examines the growth and welfare effects of defense R&D. The final section concludes with some policy implications on the ongoing reduction in defense R&D.

2. A Quality-Ladder Growth Model with Defense R&D

We incorporate defense R&D into the Grossman-Helpman (1991) quality-ladder model. There are four effects of defense R&D in the model. Firstly, it has a positive spillover effect on civilian R&D. Secondly, it has a crowding-out effect on the factor inputs for civilian R&D and production. Thirdly, defense technology improves national security and has a positive effect on the utility of households as consumption public goods. Finally, like productive public goods, defense technology improves aggregate productivity through the spin-off effect. It is worth noting that defense R&D is a flow variable while defense technology is a stock variable. Although the quality-ladder model features only labor inputs, it is appropriate for our analysis because R&D scientists and engineers are the crucial inputs for innovation in civilian and

[5] For example, Macnair *et al.* (1995), Brumm (1997) and Murdoch *et al.* (1997) find a positive relationship between defense spending and growth as in Benoit (1973) while Deger and Smith (1983), Faini *et al.* (1984) and Deger (1986) find a negative relationship. Also, some studies, such as Biswas and Ram (1986) and Huand and Mintz (1990, 1991), find an insignificant effect of defense spending on growth.

defense technologies. Given that the quality-ladder model has been well-studied, the familiar components of the model will be briefly described while the new features will be described in more details below.

2.1. Households

There is a unit continuum of identical households, who have a life-time utility function

$$U = \int_0^\infty e^{-\rho t} (\ln C_t + \delta \ln D_t) dt \,, \tag{1}$$

where $\rho > 0$ is the discount rate and C_t is the consumption of final goods. D_t is the level of defense technology and its law of motion is $\dot{D}_t = D_t f(L_{d,t})$ with D_0 normalized to unity. The improvement of defense technology is increasing in defense R&D labor $L_{d,t}$ to be discussed later. As mentioned above, we follow previous studies to assume that households derive utility from national security for which the level of defense technology D_t serves as a proxy.[6] In other words, defense technology resembles consumption public goods, and $\delta \geq 0$ is a parameter that captures *this security effect of defense technology*. Households maximize utility subject to

$$\dot{V}_t = R_t V_t + W_t - C_t - T_t \,. \tag{2}$$

V_t is the value of assets owned by households, and R_t is the real rate of return. Each household is endowed with one unit of labor to be allocated between production, civilian R&D and defense

[6] This formulation captures Hartley's (2006) argument (quoted in the Introduction) that higher defense technology improves a country's ability in defending its national interests against the threat of foreign rivals and hence increases the utility of households.

R&D. The market wage rate is W_t. The government levies a lump-sum tax T_t on the households to finance defense R&D.[7] The familiar Euler equation is

$$\frac{\dot{C}_t}{C_t} = R_t - \rho . \tag{3}$$

2.2. Final Goods

Final goods Y_t are produced by a standard Cobb-Douglas aggregator over a continuum of differentiated intermediates goods $X_t(i)$ for $i \in [0,1]$ given by

$$Y_t = \exp\left(\int_0^1 \ln X_t(i) di \right). \tag{4}$$

This sector is perfectly competitive, and the producers take the output and input prices as given.

2.3. Intermediate Goods

There is a unit continuum of industries indexed by $i \in [0,1]$ producing the differentiated intermediate goods. In each industry *i*, there is a temporary monopolistic leader, who holds a patent on the latest invention and dominates the market until the next invention occurs. The production function for the leader in industry *i* is

$$X_t(i) = z^{n_t(i)} D_t^{\alpha} L_{x,t}(i) . \tag{5}$$

$L_{x,t}(i)$ is the number of production workers in industry *i*. As discussed in Ruttan (2006), defense technology D_t facilitates the development of GPTs and hence increases aggregate productivity like productive public goods in Barro (1990). $\alpha \in (0,1)$ is a parameter that determines *this spin-*

[7] We focus on lump-sum tax to highlight the crowding-out effect of defense R&D. In the case of distortionary taxes, increasing defense R&D would unsurprisingly lead to other distortionary effects on the economy in addition to the crowding-out effect.

off effect of defense technology. As for technological progress from civilian R&D, $z > 1$ is the exogenous size of quality improvement from each invention, and $n_t(i)$ is the number of inventions occurred in industry *i* as of time *t*. Given $z^{n_t(i)}$, the marginal cost of production for the leader in industry *i* is $MC_t(i) = W_t / (z^{n_t(i)} D_t^{\alpha})$. As is standard in the literature, the current and former industry leaders engage in Bertrand competition.[8] The familiar profit-maximizing price for the current industry leader is a constant markup z over the marginal cost given by

(6) $$P_t(i) = z MC_t(i).$$[9]

2.4. Civilian R&D

Denote the value of an invention in industry i as $\widetilde{V}_t(i)$. Due to the Cobb-Douglas specification in (4), the amount of profit is the same across industries. As a result, $\widetilde{V}_t(i) = \widetilde{V}_t$ for $i \in [0,1]$. Because inventions are the only assets in the economy, their market value equals the value of assets owned by households (i.e. $\widetilde{V}_t = V_t$). The familiar no-arbitrage condition for V_t is

(7) $$R_t V_t = \Pi_{x,t} + \dot{V}_t - \lambda_t V_t .$$

The left-hand side of (7) is the return on this asset. The right-hand side of (7) is the sum of (a) the monopolistic profit $\Pi_{x,t}$ generated by this asset, (b) the potential capital gain $\dot{V}_t$ and (c) the expected capital loss due to creative destruction for which λ_t is the aggregate Poisson arrival rate of inventions.

[8] Grossman and Helpman (1991) show that in this model, an industry leader has no incentive to invest in her own industry, and hence, the next invention must come from another inventor.
[9] Li (2001) considers a CES production function. In this case, the monopolistic markup can be determined by either the quality step size or the elasticity of substitution depending on whether innovations are drastic or non-drastic.

There is a unit continuum of R&D entrepreneurs indexed by $j \in [0,1]$, and they hire workers to create inventions. The expected profit for R&D entrepreneur *j* is

(8) $$\Pi_{r,t}(j) = V_t \lambda_t(j) - W_t L_{r,t}(j).$$

$L_{r,t}(j)$ is the number of civilian R&D workers hired by entrepreneur *j*, and the arrival rate of inventions for entrepreneur *j* is $\lambda_t(j) = \overline{\varphi}_t L_{r,t}(j)$, where $\overline{\varphi}_t$ is the productivity of civilian R&D. Free entry leads to zero expected profit in the R&D sector and hence

(9) $$V_t \overline{\varphi}_t = W_t.$$

This condition determines the allocation of labor between civilian R&D and production. To formalize *the spillover effect of defense R&D* on civilian R&D, $\overline{\varphi}_t$ is assumed to be increasing in defense R&D labor $L_{d,t}$. For analytical tractability, we consider the following functional form

(10) $$\overline{\varphi}_t = \varphi L_{d,t}^{\phi}.$$

This functional form is tractable because the spillover effects of defense R&D is captured by a single parameter $\phi \in (0,1)$. When ϕ equals zero, the R&D sector reduces to the setup in the Grossman-Helpman model in which the productivity of civilian R&D labor is solely determined by the parameter φ.

2.5. Defense R&D

Government invests in defense R&D to improve defense technology according to

(11) $$g_{d,t} \equiv \dot{D}_t / D_t = f(L_{d,t}).$$

$g_{d,t}$ is the growth rate of defense technology, and the function $f(.)$ satisfies the following regularity conditions $f(0)=0$, $f'>0$ for $L_{d,t}\in[0,1)$, $f'(1)=0$ and $f''\leq 0$. The government's balanced-budget condition is

(12) $$T_t = W_t L_{d,t}.$$

This setup can be interpreted as the case in which defense R&D is performed by the government and (11) is the government's production function of defense technology. Alternatively, (12) can be viewed as cost-reimbursement contracts with defense firms. In this case, $f(.)$ is also affected by the incentives of defense firms in doing efficient R&D.[10] Under either interpretation, a higher level of defense R&D increases the tax burden and reduces the supply of labor for production and civilian R&D (i.e. *the crowding-out effect of defense R&D*).

3. Decentralized Equilibrium

This section defines the equilibrium and characterizes the balanced-growth path. The equilibrium is a sequence of allocations $\{C_t, Y_t, X_t(i), L_{x,t}(i), L_{r,t}(j), L_{d,t}\}_{t=0}^{\infty}$, a sequence of prices $\{W_t, R_t, V_t, P_t(i)\}_{t=0}^{\infty}$ and a sequence of tax policies $\{T_t\}_{t=0}^{\infty}$. Also, in each period,

a. households choose $\{C_t\}$ to maximize utility subject to (2) taking $\{W_t, R_t, T_t\}$ as given;

b. competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking $\{P_t(i)\}$ as given;

c. the leader in industry $i\in[0,1]$ produces $\{X_t(i)\}$ and chooses $\{P_t(i), L_{x,t}(i)\}$ to maximize profit according to the Bertrand competition and taking $\{W_t\}$ as given;

d. R&D entrepreneur $j\in[0,1]$ chooses $\{L_{r,t}(j)\}$ to maximize the expected profit taking $\{W_t, V_t\}$ as given;

[10] See, for example, Rogerson (1995) for a discussion on defense firms' incentives under cost-based contracts.

e. the market for final goods clears such that $C_t = Y_t$;

f. the labor market clears such that $L_{x,t} + L_{r,t} + L_{d,t} = 1$;

g. the government balances its budget constraint such that $T_t = W_t L_{d,t}$.

Lemma 1: *Given a constant L_d, the economy is on a unique and stable balanced-growth path.*

Proof: See Appendix A.□

Given that the economy is always on a balanced-growth path, we derive the steady-state equilibrium allocation of civilian R&D labor for a given L_d.

Lemma 2: *The equilibrium allocation of civilian R&D labor is stationary and given by*

(13)
$$L_r = (1 - L_d)\left(\frac{z-1}{z}\right) - \frac{\rho}{z\varphi L_d^{\phi}}.$$

Proof: See Appendix A.□

The properties of L_r are quite intuitive. A larger markup z increases the amount of monopolistic profit and the incentives for civilian R&D. Therefore, L_r is increasing in z. A larger discount rate decreases the present value of an invention. Therefore, L_r is decreasing in ρ. (13) shows that increasing defense R&D leads to contrasting effects on civilian R&D. On one hand, a larger L_d reduces the supply of labor in the market (i.e. the crowding-out effect) and hence decreases R&D labor. On the other hand, a larger L_d raises R&D productivity $\overline{\varphi} = \varphi L_{d,t}^{\phi}$ (i.e. the spillover effect) and increases R&D labor. We assume φ to be sufficiently large such that equilibrium R&D labor is non-negative.

The Cobb-Douglas specification in (4) implies that each industry i employs an equal number of workers. Substituting (5) into (4) yields aggregate production function $Y_t = Z_t D_t^{\alpha} L_x$, where the aggregate level of civilian technology is defined as

$$Z_t \equiv \exp\left(\int_0^1 n_t(i) di \ln z\right) = \exp\left(\int_0^t \lambda_s ds \ln z\right), \tag{14}$$

where the last equality uses the law of large numbers. Using (14), the balanced-growth rate of civilian technology is

$$g_z \equiv \dot{Z}_t / Z_t = \lambda \ln z, \tag{15}$$

where $\lambda = \overline{\varphi} L_r$ is the aggregate arrival rate of inventions.

4. Growth and Welfare Effects of Defense R&D

This section firstly analyzes the growth effect of defense R&D labor that approximately equals defense R&D spending as a share of GDP.[11] The balanced-growth rate of consumption denoted by $g_c \equiv \dot{C}_t / C_t = \dot{Y}_t / Y_t$ is

$$g_c = g_z + \alpha g_d = \left(\frac{\varphi L_d^{\phi}(1-L_d)(z-1)-\rho}{z}\right) \ln z + \alpha f(L_d). \tag{16}$$

(16) shows that economic growth is driven by g_z (i.e. market-based civilian R&D as in standard R&D growth models) and g_d (i.e. government-financed public goods as in Barro (1990)).

[11] In the model, defense R&D as a share of GDP is given by $WL_d / (C + WL_d) = WL_d / (WL_x + \Pi_x + WL_d) \approx L_d$, where the approximation holds because $\Pi_x \approx \Pi_x \lambda / (\rho + \lambda) = V\lambda = WL_r$, where the last equality follows from (9).

Proposition 1: *There exists a growth-maximizing level of defense R&D L_d^* that is increasing in ϕ (the spillover effect of defense R&D) and α (the spin-off effect of defense technology). A decrease in defense R&D leads to (a) a positive effect on the growth rate of consumption if $L_d > L_d^*$ and (b) a negative effect if $L_d < L_d^*$.*

Proof: See Appendix A.□

Proposition 1 suggests that the fall in defense R&D in the US should have an ambiguous effect on growth. This ambiguous effect arises from the opposing forces of the crowding-out effect versus the spillover effect and the spin-off effect. Therefore, the growth-maximizing defense R&D is increasing in ϕ (i.e. the spillover effect of defense R&D) and α (i.e. the spin-off effect of defense technology). Figure 2 plots consumption growth as a function of L_d.

[Insert Figure 2 here]

We next evaluate the effects of defense R&D on social welfare. Imposing the balanced-growth condition on (1) simplifies the lifetime utility of households to

$$U = \frac{1}{\rho}\left(\ln C_0 + \frac{g_c}{\rho}\right) + \frac{\delta}{\rho}\left(\ln D_0 + \frac{g_d}{\rho}\right) = \frac{1}{\rho}\left(\ln L_x + \frac{g_z + \alpha\, g_d}{\rho}\right) + \frac{\delta}{\rho}\left(\frac{g_d}{\rho}\right), \tag{17}$$

where the last equality is obtained by normalizing the exogenous Z_0 and D_0 to one without loss of generality. We restrict the parameter space of φ / ρ to ensure that U is well-behaved and concave in L_d.[12] Differentiating (17) with respect to L_d yields

$$\frac{\partial \rho U}{\partial L_d} = \frac{1}{L_x}\left(\frac{\partial L_x}{\partial L_d}\right) + \frac{1}{\rho}\left(\frac{\partial g_z}{\partial L_d} + \alpha \frac{\partial g_d}{\partial L_d}\right) + \frac{\delta}{\rho}\left(\frac{\partial g_d}{\partial L_d}\right). \tag{18}$$

[12] See the proof of Proposition 2 in Appendix A.

The last term in (18) captures the positive effect of growth in defense technology on national security. The first term in (18) captures the negative crowding-out effect of defense R&D on production labor that results into a lower initial consumption $C_0 = L_x$. Combining (13) and the labor-market clearing condition yields

$$L_x = 1 - L_d - \left((1 - L_d)\left(\frac{z-1}{z}\right) - \frac{\rho}{z\varphi L_d^{\phi}}\right) = \frac{1}{z}\left(1 - L_d + \frac{\rho}{\varphi L_d^{\phi}}\right), \tag{19}$$

which is decreasing in L_d.

To derive the value of δ at which the growth-maximizing and welfare-maximizing levels of defense R&D coincide, we firstly set (18) equal zero to derive the first-order condition for welfare maximization. Then, we substitute the growth-maximizing L_d^* into this condition to set $\partial g_z / \partial L_d + \alpha \partial g_d / \partial L_d$ equal to zero. Finally, we rearrange terms to obtain

$$\bar{\delta}(L_d^*) \equiv - \frac{\partial L_x}{\partial L_d}\left(\frac{\rho}{L_x f'}\right)\Bigg|_{L_d = L_d^*} > 0. \tag{20}$$

Proposition 2: *The welfare-maximizing level of defense R&D is increasing in δ. Also, there exists a critical value $\bar{\delta}$ below (above) which the welfare-maximizing level is below (above) the growth-maximizing level, and $\bar{\delta}$ is increasing in the growth-maximizing level of defense R&D.*

Proof: See Appendix A.□

Proposition 2 shows that there is a welfare-maximizing level of defense R&D denoted by L_d^{**} that is increasing in δ, and there exists a critical value $\bar{\delta}$ below (above) which L_d^{**} is below (above) the growth-maximizing level L_d^* as illustrated in Figure 3. Intuitively, in addition to the effects on consumption growth, defense R&D has the following two additional effects on welfare (a) a negative effect on initial consumption and (b) a positive effect on national security. If national security is not very important to households (i.e. $\delta < \overline{\delta}$), then (a) dominates (b) such that the welfare-maximizing defense R&D is below the growth-maximizing level. Otherwise, (b) dominates (a) such that the opposite is true. Also, $\overline{\delta}$ is increasing in L_d^*.[13] As L_d increases, $C_0 = L_x$ decreases; consequently, the value of δ at which $L_d^{**} = L_d^*$ must increase.

[Insert Figure 3 here]

5. Conclusion

This paper develops a simple R&D-based growth model to analyze the effects of defense R&D on economic growth and welfare. We find that the growth effect of defense R&D follows an inverted-U shape reflecting the opposing forces of the crowding-out effect versus the spillover effect and the spin-off effect. Also, whether or not the fall in defense R&D in the US should have improved growth (welfare) depends on the level of defense R&D in the economy relative to its growth-maximizing (welfare-maximizing) level, which in turn is determined by the spillover and spin-off effects (the security effect). We also find that the welfare-maximizing level of defense R&D can be above or below the growth-maximizing level. These theoretical results imply that even if defense R&D contributes to growth as suggested by recent empirical evidence, reducing defense R&D can still be consistent with either a positive or negative effect on social welfare. This finding suggests the importance of investigating beyond the growth effects when policymakers perform a cost-benefit analysis on reducing defense R&D.

[13] It is worth noting that L_d^* is an endogenous variable. In other words, the comparative static of L_d^* on $\overline{\delta}$ applies to underlying exogenous parameter changes (e.g. α) that affect welfare only through consumption growth.

Finally, the canonical quality-ladder model is a first-generation R&D growth model that exhibits counterfactual scale effects (i.e. an economy that has a growing population experiences an increasing growth rate rather than a balanced-growth path). In our model, scale effects are eliminated by normalizing population size to unity. The literature has two other ways to remove scale effects (a) the semi-endogenous growth model and (b) the second-generation endogenous-growth model that combines quality improvement and variety expansion.[14] On one hand, Jones (1999) and Li (2000) provide theoretical support for the semi-endogenous growth model by showing that the second-generation model consists of two knife-edge parameter conditions. On the other hand, a number of empirical studies, such as Laincz and Peretto (2006) and Madsen (2008), tend to provide empirical support for the second-generation model and reject the semi-endogenous growth model. Our simple model's implication that devoting a larger *share* of labor to R&D would increase growth is consistent with the second-generation model.

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[14] See Jones (1999) for an excellent discussion on scale effects, the semi-endogenous growth model and the second-generation endogenous-growth model.

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Appendix A

Proof of Lemma 1: Given a constant L_d (and hence a constant $\overline{\varphi}$), the labor-market clearing condition is $1-L_d = L_{x,t} + L_{r,t}$. Production labor-income share of final goods is $W_t L_{x,t} = C_t / z$ and the profit share of final goods is $\Pi_{x,t} = C_t(z-1)/z$. The arrival rate of inventions is $\lambda_t = \overline{\varphi} L_{r,t}$. The R&D zero-expected-profit condition is $V_t \overline{\varphi} = W_t$. Substituting these conditions into $1-L_d = L_{x,t} + L_{r,t}$ yields

(A1) $$\lambda_t = \overline{\varphi}(1-L_d) - \theta_t / z,$$

where $\theta_t \equiv C_t / V_t$ is a transformed variable. The law of motion for θ_t is

(A2) $$\frac{\dot{\theta}_t}{\theta_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{V}_t}{V_t} = \frac{\Pi_{x,t}}{V_t} - \lambda_t - \rho,$$

which uses (3) and (7). Substituting $\Pi_{x,t} = C_t(z-1)/z$ and (A1) into (A2) yields

(A3) $$\frac{\dot{\theta}_t}{\theta_t} = \theta_t - \overline{\varphi}(1-L_d) - \rho.$$

The phase diagram for this simple differential equation is plotted in Figure 4. Figure 4 shows that θ_t must jump to its non-zero steady state given by $\theta^* = \overline{\varphi}(1-L_d) + \rho$.□

Proof of Lemma 2: Imposing the balanced-growth condition on (7) yields

(A4) $$V_t = \Pi_{x,t} / (\rho + \lambda).$$

Substituting (A4), $\Pi_{x,t} = C_t(z-1)/z$ and $\lambda = \overline{\varphi} L_r$ into $\theta^* = \overline{\varphi}(1-L_d) + \rho$ yields (13).□

Proof of Proposition 1: Recall that $\phi \in (0,1)$. Differentiating (16) with respect to L_d yields

(A5) $$\frac{\partial g_c}{\partial L_d} = L_d^{\phi}\left(\frac{\phi(1-L_d)}{L_d}-1\right)\left(\frac{(z-1)\varphi \ln z}{z}\right)+\alpha f'(L_d)=0,$$

(A6) $$\frac{\partial^2 g_c}{\partial L_d^2} = -\frac{\phi}{L_d^{1-\phi}}\left((1-\phi)\left(\frac{1-L_d}{L_d}\right)+2\right)\left(\frac{(z-1)\varphi \ln z}{z}\right)+\alpha f''(L_d)<0.$$

(A6) shows that g_c is strictly concave in L_d. Therefore, the solution to (A5) denoted by L_d^* is a global maximum. Also, L_d^* is an interior solution because (a) $\partial g_c / \partial L_d > 0$ at $L_d = 0$ and (b) $\partial g_c / \partial L_d < 0$ at $L_d = 1$. Rearranging (A5) yields

(A7) $$\left(\frac{\phi(1-L_d)}{L_d}-1\right)\left(\frac{(z-1)\varphi \ln z}{z}\right)+\frac{\alpha f'(L_d)}{L_d^{\phi}}=0.$$

Because the LHS of (A7) is decreasing in L_d and increasing in α and ϕ, L_d^* is also increasing in α and ϕ. Recall that $L_d \in (0,1]$ so that L_d^{ϕ} is weakly decreasing in ϕ.□

Proof of Proposition 2: We firstly characterize the first-order condition. From (16) and (19), we know that both L_x and g_c are independent of δ. Therefore, in (18), the marginal benefit of L_d is increasing in δ while the marginal cost of L_d is independent of δ. As a result, the welfare-maximizing level of defense R&D L_d^{**} must be increasing in δ. Also, $L_d^{**} < 1$ because at $L_d = 1$, consumption equals zero (i.e. $C_0 = L_x = 0$). At $\delta = 0$, $L_d^{**} < L_d^*$ because $\partial L_x / \partial L_d < 0$ in (18). Let's recall the definition of $\bar{\delta}(L_d^*)$ in (20). As δ increases to $\bar{\delta}$, we have $L_d^{**} = L_d^*$ because $\partial U / \partial L_d = 0$ at $L_d = L_d^*$ in this case. For $\delta > \bar{\delta}$, we have $L_d^{**} > L_d^*$ because $\partial L_d^{**} / \partial \delta > 0$. Taking the log of (20) and then differentiating with respect to L_d^* yields

(A8) $$\frac{\partial \ln \bar{\delta}}{\partial L_d^*} \equiv -\underbrace{\frac{1}{\partial L_x / \partial L_d^*}}_{<0}\underbrace{\left(\frac{\partial^2 L_x}{\partial L_d^{*2}}\right)}_{>0} - \frac{1}{L_x}\underbrace{\left(\frac{\partial L_x}{\partial L_d^*}\right)}_{<0} - \underbrace{\frac{f''}{f'}}_{<0} > 0 .$$

The rest of the proof considers the concavity of U . In (17), g_c / ρ is concave in L_d from the proof of Proposition 1, and $\delta f(L_d)/\rho$ is weakly concave in L_d . Therefore, if $\ln L_x$ is also concave in L_d , then U must be concave in L_d . Using (19), differentiating $\ln L_x$ with respect to L_d twice yields

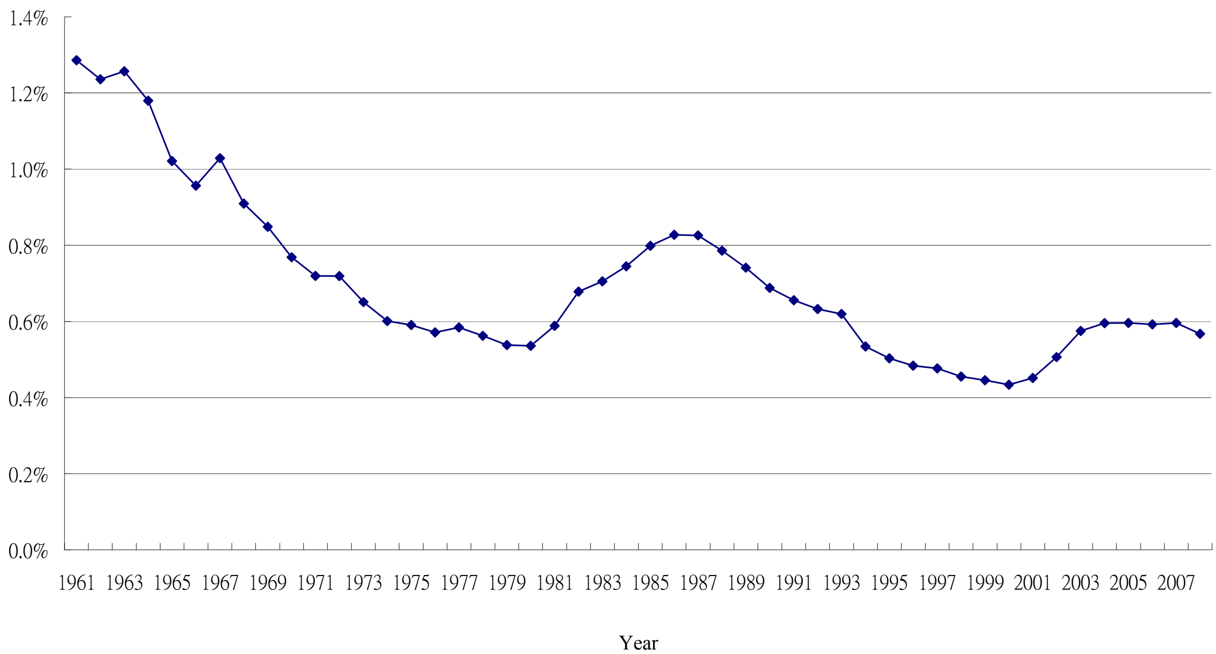
(A9) $$\frac{\partial^2 \ln L_x}{\partial L_d^2} = -\frac{1}{L_x}\left(\frac{1}{L_x}\left(\frac{\partial L_x}{\partial L_d}\right)^2 - \frac{\partial^2 L_x}{\partial L_d^2}\right) = -\frac{1}{L_x}\left(\frac{1}{L_x}\left(\frac{1}{z} + \frac{\phi\rho}{z\varphi L_d^{1+\phi}}\right)^2 - \frac{(1+\phi)\phi\rho}{z\varphi L_d^{2+\phi}}\right),$$

in which $\partial^2 L_x / \partial L_d^2 > 0$ because $\phi > 0$. Manipulating (A9) shows that

(A10) $$\frac{\partial^2 \ln L_x}{\partial L_d^2} < 0 \Leftrightarrow \frac{\varphi}{\rho}\left(\frac{L_d^{2(1+\phi)}}{\phi}\right) + 2L_d^{(1+\phi)} > (1+\phi)L_d^{\phi}(1-L_d) + \frac{\rho}{\varphi} .$$

Therefore, there is a large enough φ/ρ above which (A10) holds because the left-hand side is increasing in φ/ρ while the right-hand side is decreasing in φ/ρ .□

Figure 1: Federal R&D on national defense as a percentage of US GDP



Data sources: (a) National Science Foundation, and (b) Bureau of Economic Analysis.

Figure 2: Growth effects of defense R&D

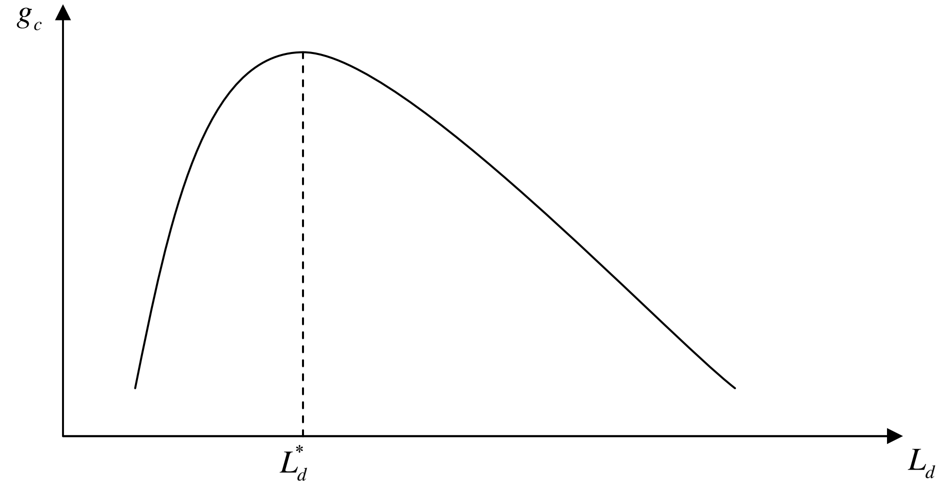


Figure 3: Growth versus welfare maximizing defense R&D

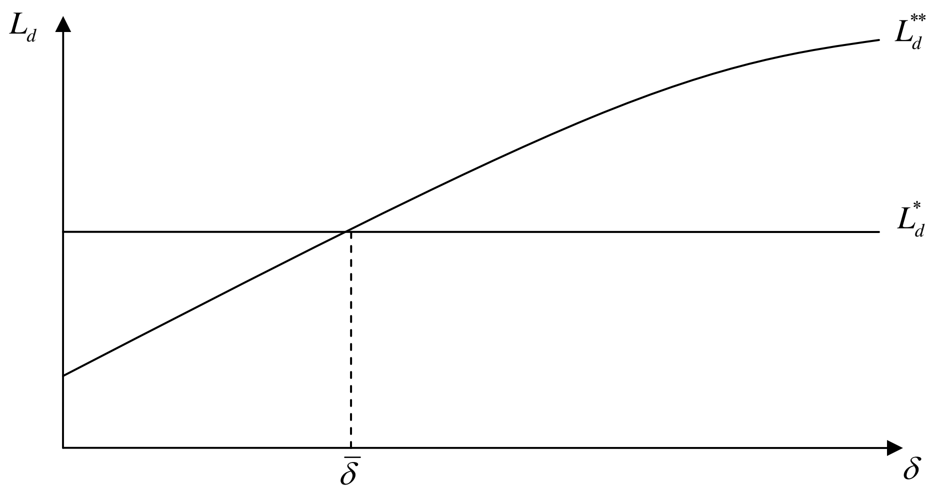


Figure 4: Phase diagram